## 2.2 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

**2.** *Derive an expression for the electric potential at a distance r from a point charge q. What is the nature of this potential ?*

**Electric potential due to a point charge.** Consider a positive point charge $q$ placed at the origin $O$. We wish to calculate its electric potential at a point $P$ at distance $r$ from it, as shown in Fig. 2.2. By definition, the electric potential at point $P$ will be equal to the amount of work done in bringing a unit positive charge from infinity to the point $P$.

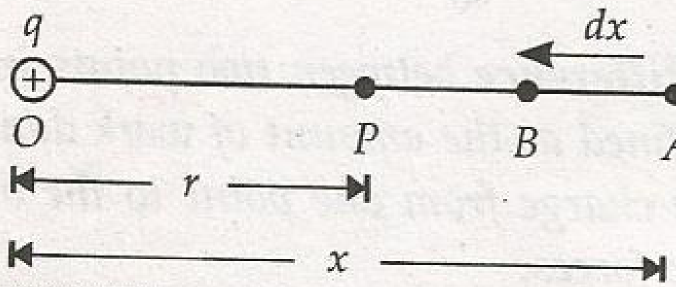


**Fig. 2.2** Electric potential due to a point charge.

Suppose a test charge $q_0$ is placed at point $A$ at distance $x$ from $O$. By Coulomb's law, the electrostatic force acting on charge $q_0$ is

$$F = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q q_0}{x^2}$$

The force $\vec{F}$ acts away from the charge $q$. The small work done in moving the test charge $q_0$ from $A$ to $B$ through small displacement $\vec{dx}$ against the electrostatic force is

$$dW = \vec{F} \cdot \vec{dx} = F dx \cos 180° = - F dx$$

The total work done in moving the charge $q_0$ from infinity to the point $P$ will be

$$W = \int dW = -\int_{\infty}^{r} F dx = -\int_{\infty}^{r} \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{qq_0}{x^2}\, dx$$

$$= -\frac{qq_0}{4\pi\,\varepsilon_0} \int_{\infty}^{r} x^{-2}\, dx = -\frac{qq_0}{4\pi\,\varepsilon_0} \left[ -\frac{1}{x} \right]_{\infty}^{r}$$

$$= \frac{qq_0}{4\pi\,\varepsilon_0} \left[ \frac{1}{r} - \frac{1}{\infty} \right] = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{qq_0}{r}.$$

Hence the work done in moving a unit test charge from infinity to the point $P$, or the electric potential at point $P$ is

$$V = \frac{W}{q_0} \quad \text{or} \quad V = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q}{r}$$

Clearly, $V \propto 1/r$. *Thus the electric potential due to a point charge is spherically symmetric* as it depends only on the distance of the observation point from the charge and not on the direction of that point with respect to the point charge. Moreover, we note that the potential at infinity is zero.

Fig. 2.3 shows the variation of electrostatic potential ($V \propto 1/r$) and the electrostatic field ($E \propto 1/r^2$) with distance $r$ from a charge $q$.

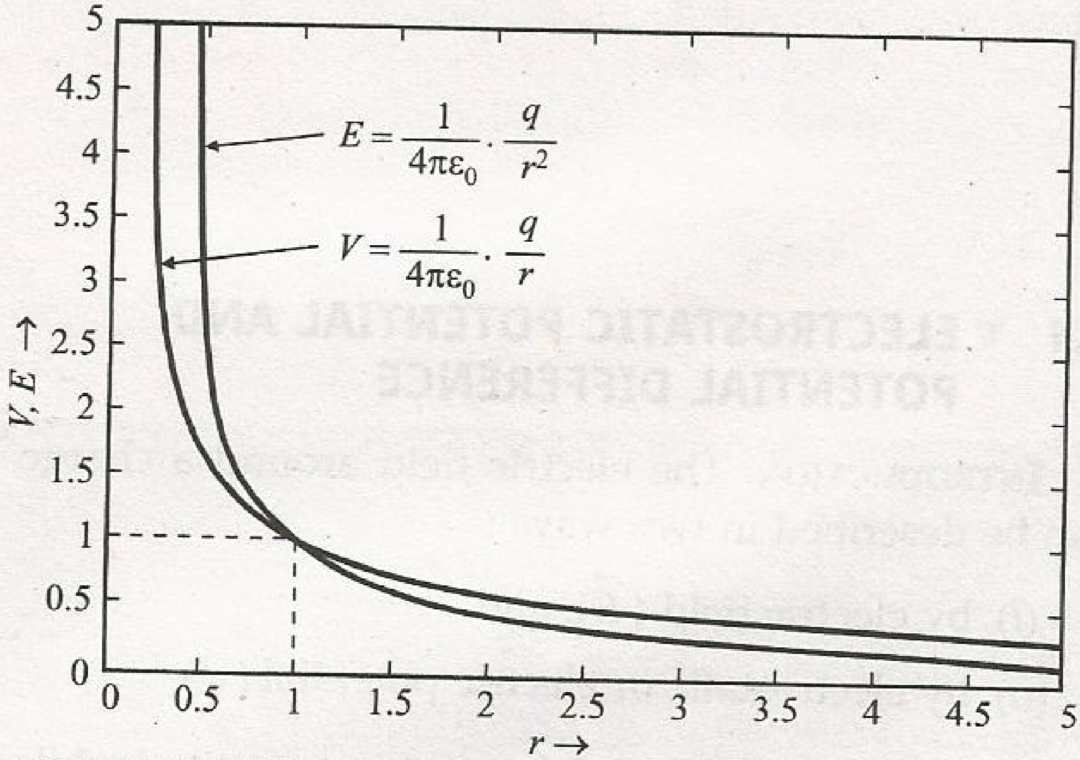


**Fig. 2.3** Variation of potential $V$ and field $E$ with $r$ for a point charge $q$.

## 2.3 ELECTRIC POTENTIAL DUE TO A DIPOLE

**3.** *Derive an expression for the potential at a point along the axial line of a short dipole.*

**Electric potential at an axial point of a dipole.** As shown in Fig. 2.4, consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by distance $2a$. Let $P$ be a point on the axis of the dipole at a distance $r$ from its centre $O$.

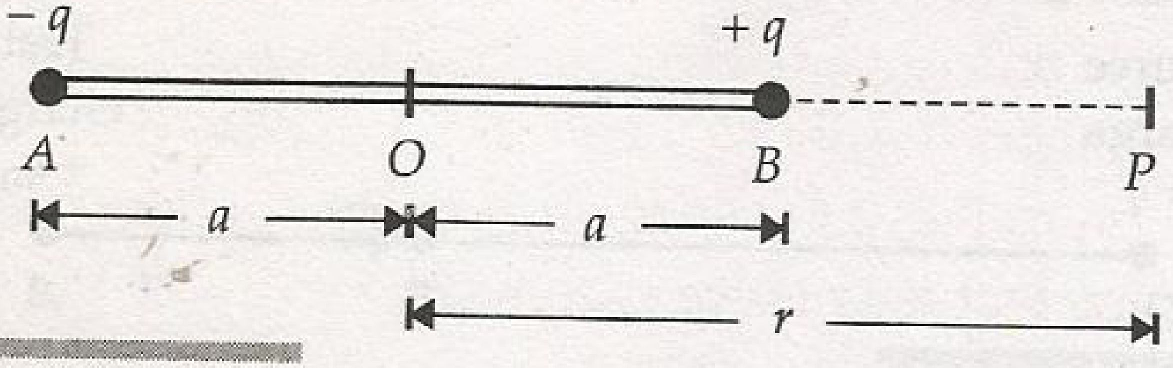


**Fig. 2.4** Potential at an axial point of a dipole.

Electric potential at point $P$ due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r+a} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)-(r-a)}{r^2-a^2}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times 2a}{r^2-a^2}$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2-a^2} \qquad [\because p = q \times 2a]$$

For a short dipole, $a^2 << r^2$, so $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$.

**4.** *Show mathematically that the potential at a point on the equatorial line of an electric dipole is zero.*

**Electric potential at an equatorial point of a dipole.** As shown in Fig. 2.5, consider an electric dipole consisting of charges $-q$ and $+q$ and separated by distance $2a$. Let $P$ be a point on the perpendicular bisector of the dipole at distance $r$ from its centre $O$.

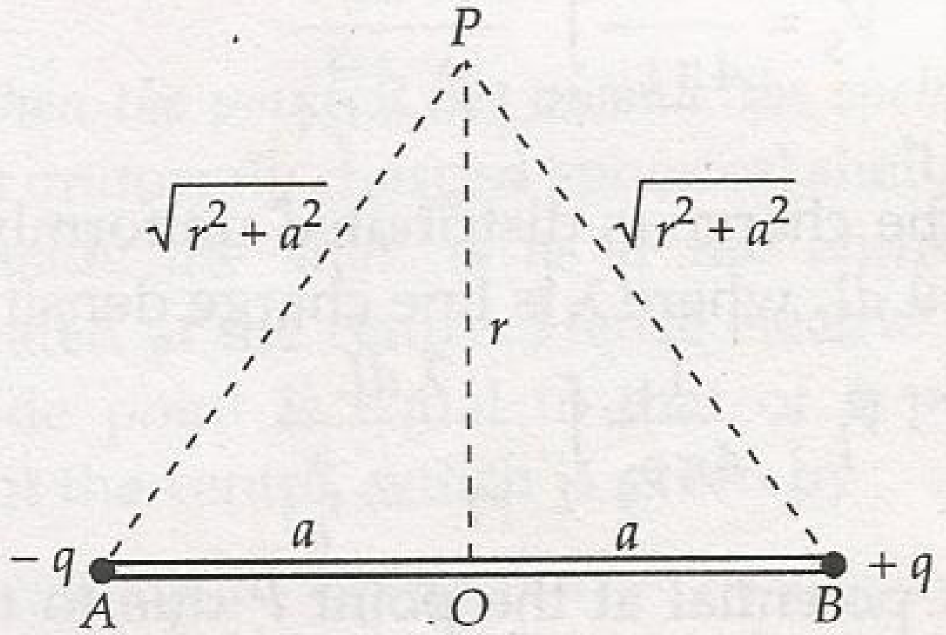


**Fig. 2.5** Potential at an equatorial point of a dipole.

Electric potential at point $P$ due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{r^2+a^2}} = \mathbf{0}.$$

**5.** *Derive an expression for the electric potential at any general point at distance r from the centre of a dipole.*

**Electric potential at any general point due to a dipole.** Consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by distance $2a$, as shown in Fig. 2.6. We wish to determine the potential at a point $P$ at a distance $r$ from the centre $O$, the direction $OP$ making an angle $\theta$ with dipole moment $\vec{p}$.

Let $AP = r_1$ and $BP = r_2$.

Net potential at point $P$ due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

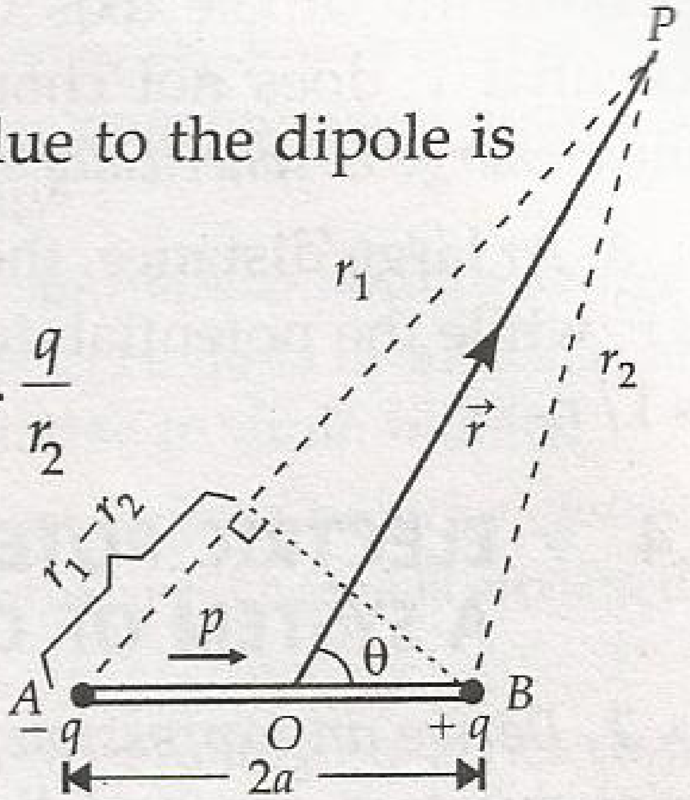


**Fig. 2.6**

If the point $P$ lies far away from the dipole, then

$$r_1 - r_2 \simeq AB\cos\theta = 2a\cos\theta \quad \text{and} \quad r_1 r_2 \simeq r^2$$

$\therefore$
$$V = \frac{q}{4\pi\varepsilon_0} \cdot \frac{2a\cos\theta}{r^2}$$

or
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$$

or
$$V = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

Here $p = q \times 2a$, is the dipole moment and $\hat{r} = \vec{r}/r$, is a unit vector along the position vector $\vec{OP} = \vec{r}$.

## 2.4 ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES

**7.** *Derive an expression for the electric potential at a point due to a group of N point charges.*

**Electric potential due to a group of point charges.** As shown in Fig. 2.7, suppose $N$ point charges $q_1, q_2, q_3, \ldots, q_N$ lie at distances $r_1, r_2, r_3, \ldots, r_N$ from a point $P$.

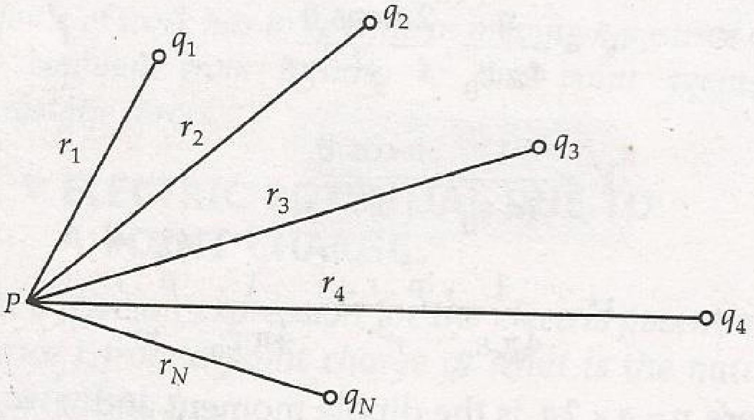


**Fig. 2.7** Potential at a point due to a system of $N$ point charges.

Electric potential at point $P$ due to charge $q_1$ is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1}$$

Similarly, electric potentials at point $P$ due to other charges will be

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2},\ V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_3},\ \ldots,\ V_N = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_N}{r_N}$$

As electric potential is a scalar quantity, so the total potential at point $P$ will be equal to the algebraic sum of all the individual potentials, *i.e.*,

$$V = V_1 + V_2 + V_3 + \ldots + V_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_N}{r_N}\right]$$

or

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ are the position vectors of the $N$ point charges, the electric potential at a point whose position vector is $\vec{r}$, would be

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|}.$$

## 2.6 ▼ ELECTRIC POTENTIAL DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL

**9.** *Write expression for the electric potential due to a uniformly charged spherical shell at a point (i) outside the shell, (ii) on the shell and (iii) inside the shell.*

**Electric potential due to uniformly charged thin spherical shell.** Consider a uniformly charged spherical shell of radius $R$ and carrying charge $q$. We wish to calculate its potential at point $P$ at distance $r$ from its centre $O$, as shown in Fig. 2.8.

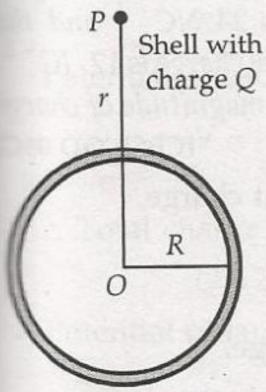


**Fig. 2.8** Potential due to a spherical shell.

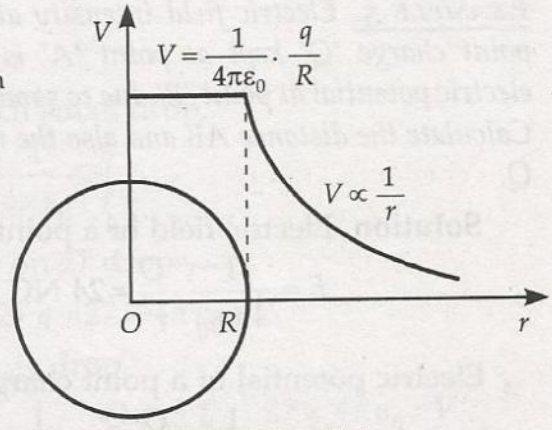


**Fig. 2.9** Variation of potential due to charged shell with distance $r$ from its centre.

(i) *When the point P lies outside the shell.* We know that for a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Hence electric potential at an outside point is equal to that of a point charge located at the centre, which is given by

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \qquad [\text{For } r > R]$$

(ii) *When point P lies on the surface of the shell.* Here $r = R$. Hence the potential on the surface of the shell is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \qquad [\text{For } r = R]$$

(iii) *When point P lies inside the shell.* The electric field at any point inside the shell is zero. Hence electric potential due to a uniformly charged spherical shell is constant everywhere inside the shell and its value is equal to that on the surface. Thus,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \qquad [\text{For } r < R]$$

Figure 2.9 shows the variation of the potential $V$ due to a uniformly charged spherical shell with distance $r$ measured from the centre of the shell. Note that $V$ is constant $(= q/4\pi\varepsilon_0 R)$ from $r = 0$ to $r = R$ along a hori- zontal line and thereafter $V \propto 1/r$ for points outside the shell.

## 2.7 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

**10.** *Show that the electric field at any point is equal to the negative of the potential gradient at that point.*

**Computing electric field from electric potential.** As shown in Fig. 2.20, consider the electric field due to charge $+q$ located at the origin $O$. Let $A$ and $B$ be two adjacent points separated by distance $dr$. The two

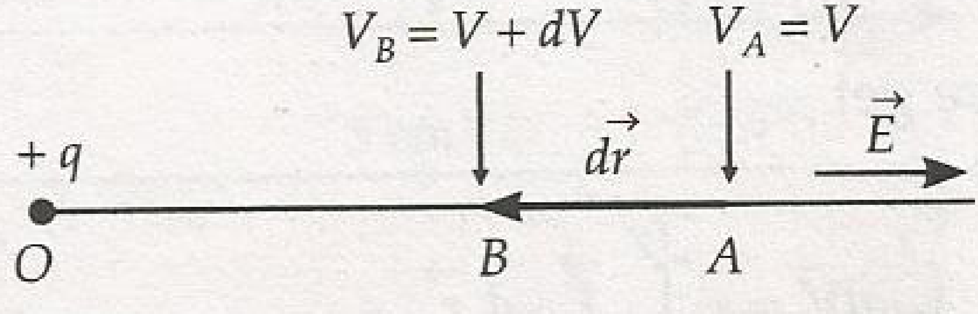


**Fig. 2.20** Relation between potential and field.

points are so close that electric field $\vec{E}$ between them remains almost constant. Let $V$ and $V + dV$ be the potentials at the two points.

The external force required to move the test charge $q_0$ (without acceleration) against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0 \vec{E}$$

The work done to move the test charge from $A$ to $B$ is

$$W = F \cdot dr = -q_0 \, E \cdot dr$$

Also, the work in moving the test charge from $A$ to $B$ is

$$W = \text{Charge} \times \text{potential difference}$$
$$= q_0 (V_B - V_A) = q_0 \, dV$$

Equating the two works done, we get

$$-q_0 E \cdot dr = q_0 \cdot dV$$

or

$$E = -\frac{dV}{dr}$$

The quantity $\frac{dV}{dr}$ is the rate of change of potential with distance and is called *potential gradient*. Thus *the electric field at any point is equal to the negative of the potential gradient at that point*. The negative sign shows that the direction of the electric field is in the direction of decreasing potential. Moreover, the field is in the direction where this decrease is steepest.

From the above relation between electric field and potential, we can draw the following important conclusions :

(*i*) Electric field is in that direction in which the potential decrease is steepest.

(*ii*) The magnitude of electric field is equal to the change in the magnitude of potential per unit displacement (called potential gradient) normal to the equipotential surface at the given point.

**11.** *How can we determine electric potential if electric field is known at any point ?*

**Computing electric potential from electric field.** The relation between electric field and potential is

$$\vec{E} = -\frac{dV}{d\vec{r}} \quad \text{or} \quad dV = -\vec{E}\,d\vec{r}$$

Integrating the above equation between points $\vec{r}_1$ and $\vec{r}_2$, we get

$$\int_{V_1}^{V_2} dV = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\,.\,d\vec{r}$$

or $$V_2 - V_1 = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E} \cdot d\vec{r}$$

where $V_1$ and $V_2$ are the potentials at $\vec{r_1}$ and $\vec{r_2}$ respectively. If we take $\vec{r_1}$ at infinity, then $V_1 = 0$ and put $\vec{r_2} = \vec{r}$, we get

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{r}$$

Hence by knowing electric field at any point, we can evaluate the electric potential at that point.

**18.** *Deduce expressions for the potential energy of a system of two point charges and three point charges and hence generalise the result for a system of N point charges.*

**Potential energy of a system of two point charges.** Suppose a point charge $q_1$ is at rest at a point $P_1$ in space, as shown in Fig. 2.29. It takes no work to bring the first charge $q_1$ because there is no field yet to work against.

$$\therefore \quad W_1 = 0$$

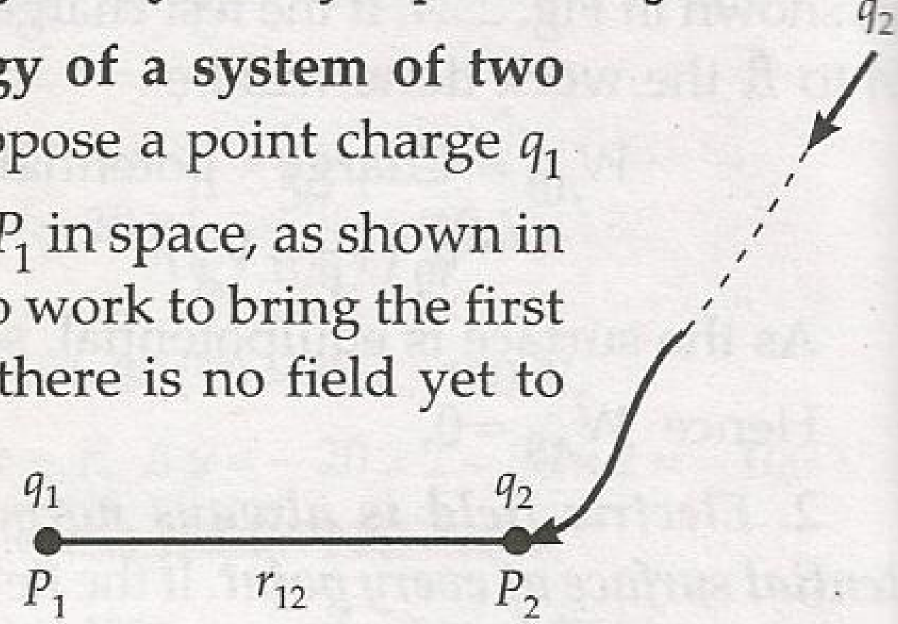


**Fig. 2.29** P.E. of two point charges.

Electric potential due to charge $q_1$ at a point $P_2$ at distance $r_{12}$ from $P_1$ will be

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{12}}$$

If charge $q_2$ is moved in from infinity to point $P_2$, the work required is

$$W_2 = \text{Potential} \times \text{charge}$$

$$= V_1 \times q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

As the work done is stored as the potential energy $U$ of the system $(q_1 + q_2)$, so

$$U = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}.$$

**Potential energy of a system of three point charges.** As shown in Fig. 2.30, now we bring in the charge $q_3$ from infinity to the point $P_3$. Work has to be done against the forces exerted by $q_1$ and $q_2$.

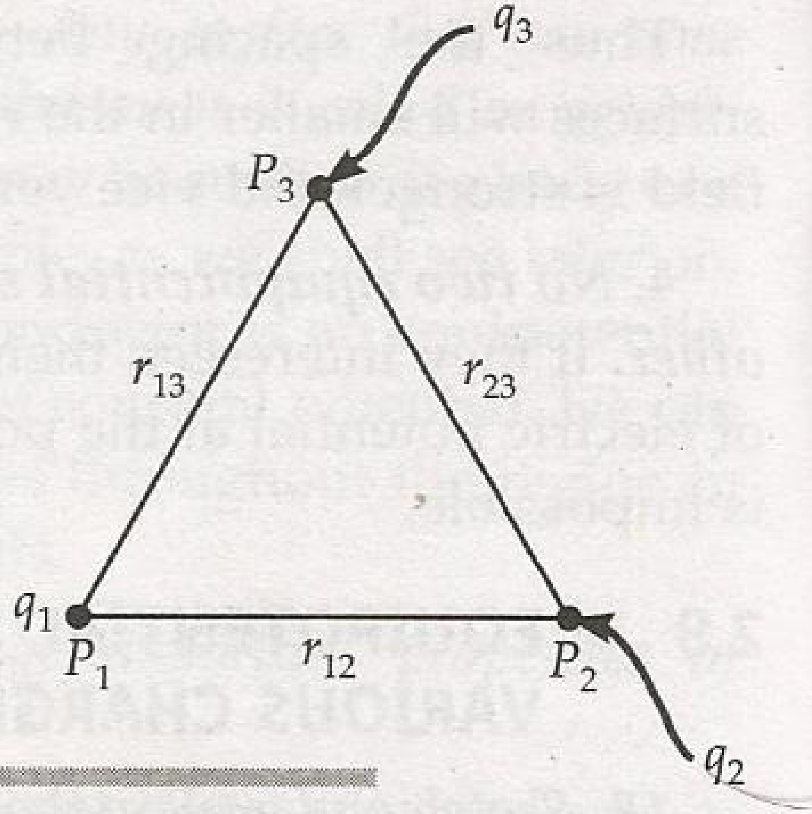


**Fig. 2.30** P.E. of three point charges.

Therefore

$W_3$ = Potential at point $P_3$ due to $q_1$ and $q_2$ × charge $q_3$

or $$W_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right] \times q_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Hence the electrostatic potential energy of the system $q_1 + q_2 + q_3$ is

$U$ = Total work done to assemble the three charges

$= W_1 + W_2 + W_3$

or $$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

**Potential energy of a system of $N$ point charges.** The expression for the potential energy of $N$ point charges can be written as

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all pairs}} \frac{q_i q_j}{r_{ij}} = \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{r_{ij}}$$

As double summation counts every pair twice, to avoid this the factor 1/2 has been introduced.

## 2.11 POTENTIAL ENERGY IN AN EXTERNAL FIELD

**19.** *Write an expression for the potential energy of a single charge in an external field. Hence define electric potential.*

**Potential energy of a single charge.** We wish to determine the potential energy of a charge $q$ in an external electric field $\vec{E}$ at a point $P$ where the corresponding external potential is $V$. By definition, $V$ at a point $P$ is the amount of work done in bringing a unit positive charge from infinity to the point $P$. Thus, the work done in bringing a charge $q$ from infinity to the point $P$ will be $qV$, *i.e.*, $W = qV$

This work done is stored as the potential energy of the charge $q$. If $\vec{r}$ is the position vector of point $P$ relative to some origin, then

$$U(\vec{r}) = qV(\vec{r})$$

P.E. of a charge in an external field

= Charge × external electric potential

As $$V = \frac{U}{q}$$

So we can define ***electric potential*** *at a given point in an external field as the potential energy of a unit positive charge at that point.*

**20.** *Write an expression for the potential energy of two point charges $q_1$ and $q_2$, separated by distance r in an electric field $\vec{E}$.*

**Potential energy of a system of two point charges in an external field.** Let $V(\vec{r}_1)$ and $V(\vec{r}_2)$ be the electric potentials of the field $\vec{E}$ at the points having position vectors $\vec{r}_1$ and $\vec{r}_2$ as shown in Fig. 2.31.

Work done in bringing $q_1$ from $\infty$ to $\vec{r}_1$ against the external field

$$= q_1 V(\vec{r}_1)$$

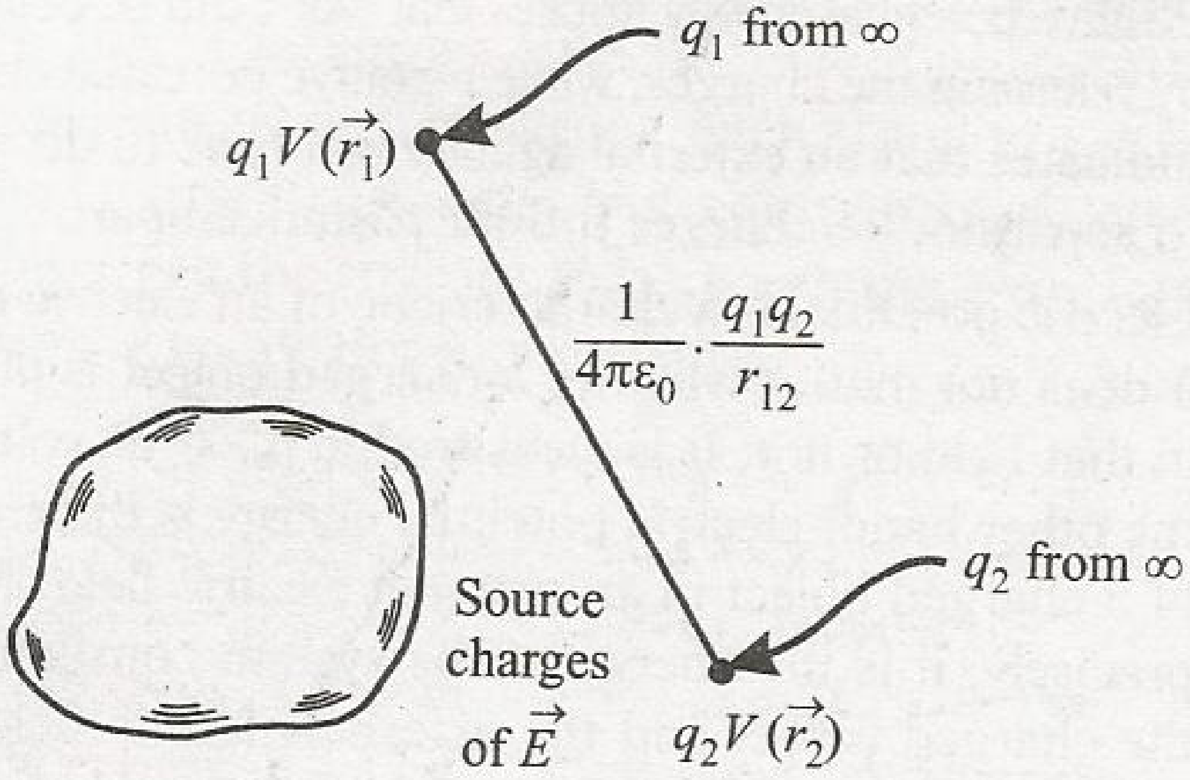


**Fig. 2.31** P.E. of two charges in an external field.

Work done in bringing $q_2$ from ∞ to $\vec{r_2}$ against the external field

$$= q_2 V(\vec{r_2})$$

Work done on $q_2$ against the force exerted by $q_1$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

where $r_{12}$ is the distance between $q_1$ and $q_2$.

Total potential energy of the system = The work done in assembling the two charges

or $$U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

## 2.12 POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM ELECTRIC FIELD

**22.** *Derive an expression for the potential energy of a dipole in a uniform electric field. Discuss the conditions of stable and unstable equilibrium.*

**Potential energy of a dipole placed in a uniform electric field.** As shown in Fig. 2.32, consider an electric dipole placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ making an angle $\theta$ with the field. Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends. The two forces form a couple. The torque exerted by the couple will be

$$\tau = qE \times 2a \sin\theta = pE \sin\theta$$

where $q \times 2a = p$, is the dipole moment.

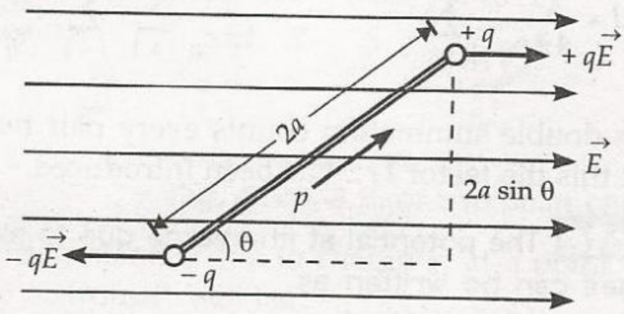


**Fig. 2.32** Torque on a dipole in a uniform electric field.

If the dipole is rotated through a small angle $d\theta$ against the torque acting on it, then the small work done is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

The total work done in rotating the dipole from its orientation making an angle $\theta_1$, with the direction of the field to $\theta_2$ will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$

$$= pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2} = pE(\cos\theta_1 - \cos\theta_2)$$

This work done is stored as the potential energy $U$ of the dipole.

$$\therefore \quad U = pE(\cos\theta_1 - \cos\theta_2)$$

If initially the dipole is oriented perpendicular to the direction of the field ($\theta_1 = 90°$) and then brought to some orientation making an angle $\theta$ with the field ($\theta_2 = \theta$), then potential energy of the dipole will be

$$U = pE(\cos 90° - \cos\theta) = pE(0 - \cos\theta)$$

or $$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

## 2.20 PARALLEL PLATE CAPACITOR

**33.** *What is a parallel plate capacitor ? Drive an expression for its capacitance. On what factors does the capacitance of a parallel plate capacitor depend ?*

**Parallel plate capacitor.** The simplest and the most widely used capacitor is the parallel plate capacitor. *It consists of two large plane parallel conducting plates, separated by a small distance.*

Let $A$ = area of each plate,

$d$ = distance between the two plates

$\pm\sigma$ = uniform surface charge densities on the two plates

$\pm Q = \pm\sigma A$ = total charge on each plate.

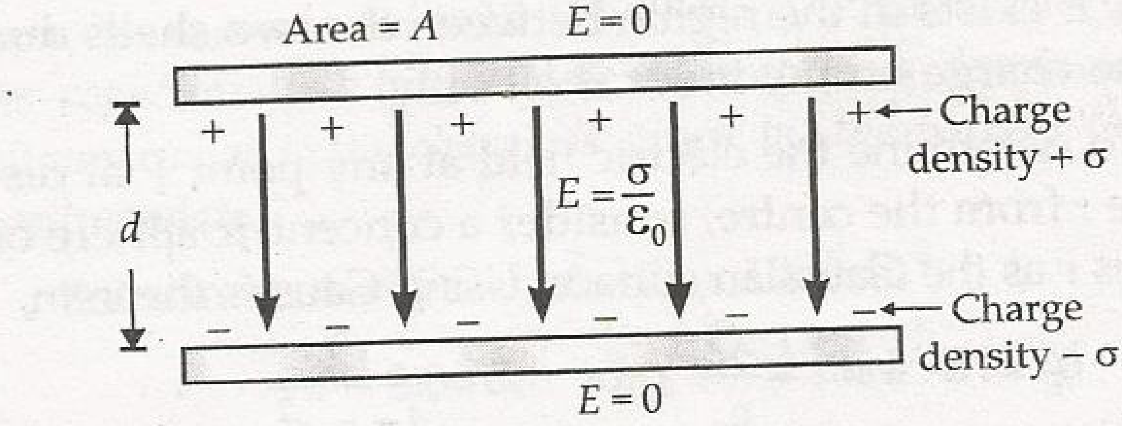


**Fig. 2.47** Parallel plate capacitor.

In the outer regions above the upper plate and below the lower plate, the electric fields due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the two capacitor plates, the electric fields due to the two charged plates add up. The net field is

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The direction of the electric field is from the positive to the negative plate and the field is uniform throughout. For plates with finite area, the field lines bend at the edges. This effect is called ***fringing of the field***. But for large plates separated by small distance $(A >> d^2)$, the field is almost uniform in the regions far from the edges. For a uniform electric field,

P.D. between the plates

= Electric field × distance between the plates

or $$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

Capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d / \varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

## 2.23 COMBINATION OF CAPACITORS IN SERIES AND IN PARALLEL

**36.** *A number of capacitors are connected in series. Derive an expression for the equivalent capacitance of the series combination.*

**Capacitors in series.** *When the negative plate of one capacitor is connected to the positive plate of the second, and the negative of the second to the positive of third and so on, the capacitors are said to be connected in series.*

Fig. 2.51 shows three capacitors of capacitances $C_1$, $C_2$ and $C_3$ connected in series. A potential difference $V$ is applied across the combination. This sets up charges $\pm Q$ on the two plates of each capacitor. What actually happens is, a charge $+Q$ is given to the left plate of capacitor $C_1$ during the charging process. The charge $+Q$ induces a charge $-Q$ on the right plate of $C_1$ and a charge $-Q$ on the left plate of $C_2$, etc.

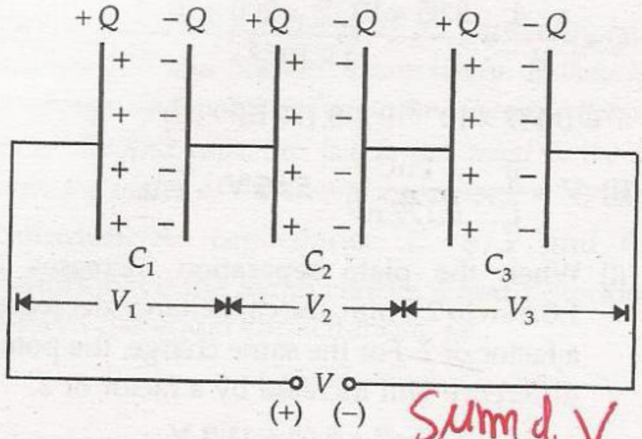


**Fig. 2.51** Capacitors in series.

The potential differences across the various capacitors are

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

For the series circuit, the sum of these potential differences must be equal to the applied potential difference.

$$\therefore\ V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

or $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ...(1)$$

Clearly, the combination can be regarded as an effective capacitor with charge $Q$ and potential difference $V$. If $C_s$ is the equivalent capacitance of the series combination, then

$$C_s = \frac{Q}{V}$$

or $$\frac{1}{C_s} = \frac{V}{Q} \quad ...(2)$$

From equations (1) and (2), we get

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For a series combination of $n$ capacitors, we can write

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + ..... + \frac{1}{C_n}$$

**For series combination of capacitors**

1. The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances.
2. The equivalent capacitance is smaller than the smallest individual capacitance.
3. The charge on each capacitors is same.
4. The potential difference across any capacitor is inversely proportional to its capacitance.

**37.** *A number of capacitors are connected in parallel. Derive an expression for the equivalent capacitance of the parallel combination.*

**Capacitors in parallel.** *When the positive plates of all capacitors are connected to one common point and the negative plates to another common point, the capacitors are said to be connected in parallel.*

Fig. 2.52 shows three capacitors of capacitances $C_1$, $C_2$ and $C_3$ connected in parallel. A potential difference $V$ is applied across the combination. All the capacitors have a common potential difference $V$ but different charges given by

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V$$

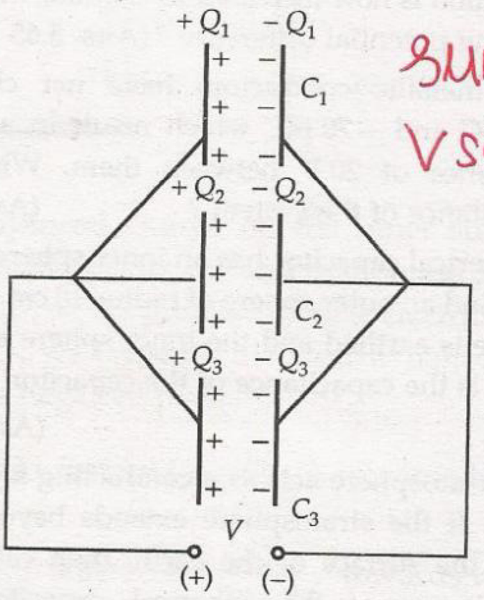


**Fig. 2.52** Capacitors in parallel.

Total charge stored in the combination is

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)\,V \qquad ...(1)$$

If $C_p$ is the equivalent capacitance of the parallel combination, then

$$Q = C_p\,V \qquad ...(2)$$

From equations (1) and (2), we get

$$C_p V = (C_1 + C_2 + C_3)\,V$$

or

$$C_p = C_1 + C_2 + C_3$$

For a parallel combination of $n$ capacitors, we can write

$$C_p = C_1 + C_2 + ..... + C_n$$

**For parallel combination of capacitors**

1. The equivalent capacitance is equal to the sum of the individual capacitances.
2. The equivalent capacitance is larger than the largest individual capacitance.
3. The potential difference across each capacitor is same.
4. The charge on each capacitor is proportional to its capacitance.

## 2.24 ENERGY STORED IN A CAPACITOR

**38.** *How does a capacitor store energy ? Derive an expression for the energy stored in a capacitor.*

**Energy stored in a capacitor.** A capacitor is a device to store energy. The process of charging up a capacitor involves the transferring of electric charges from its one plate to another. The *work done in charging the capacitor is stored as its electrical potential energy.* This energy is supplied by the battery at the expense of its stored chemical energy and can be recovered by allowing the capacitor to discharge.

**Expression for the energy stored in a capacitor.** Consider a capacitor of capacitance C. Initially, its two plates are uncharged. Suppose the positive charge is transferred from plate 2 to plate 1 bit by bit. In this process, external work has to be done because at any stage plate 1 is at higher potential than the plate 2. Suppose at any instant the plates 1 and 2 have charges $Q'$ and $-Q'$ respectively, as shown in Fig. 2.105(*a*). Then the potential difference between the two plates will be

$$V' = \frac{Q'}{C}$$

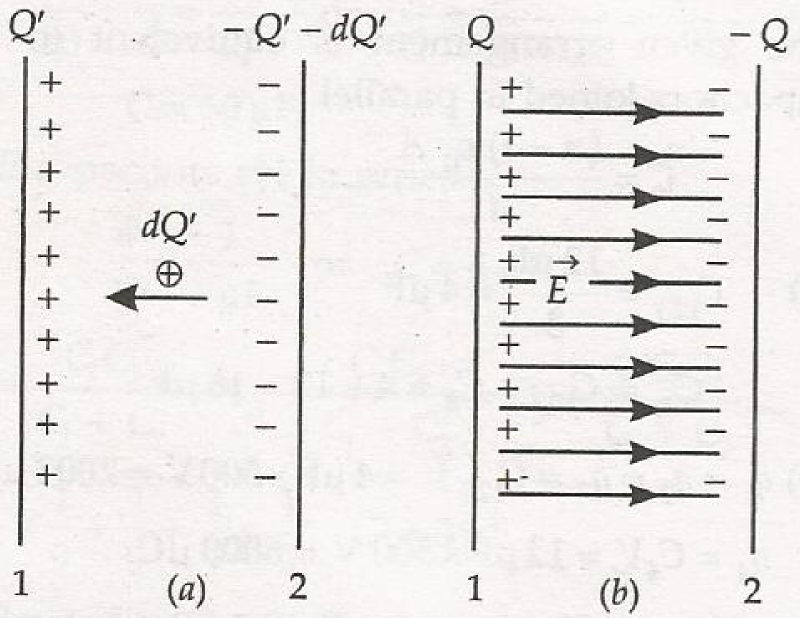


**Fig. 2.105** (*a*) Work done in transferring charge $dQ'$ from plate 2 to plate 1. (*b*) Total work done in charging the capacitor may be considered as the energy stored in the electric field between the plates.

6 Suppose now a small additional charge $dQ'$ be transferred from plate 2 to plate 1. The work done will be

$$dW = V' \, . \, dQ' = \frac{Q'}{C} \, . \, dQ'$$

7 The total work done in transferring a charge $Q$ from plate 2 to plate 1 [Fig. 2.105($b$)] will be

$$W = \int dW = \int_0^Q \frac{Q'}{C} \, . \, dQ' = \left[ \frac{Q'^2}{2C} \right]_0^Q = \frac{1}{2} \, . \, \frac{Q^2}{C}$$

8 This work done is stored as electrical potential energy $U$ of the capacitor.

$$U = \frac{1}{2} \, . \, \frac{Q^2}{C} = \frac{1}{2} \, . \, CV^2 = \frac{1}{2} \, QV \qquad [\because Q = CV]$$

30. If several capacitors are connected in ...

**39.** *If several capacitors are connected in series or parallel, show that the energy stored would be additive in either case.*

**Energy stored in a series combination of capacitors.**

For a series combination, $Q = \text{constant}$

Total energy,

$$U = \frac{Q^2}{2} \cdot \frac{1}{C} = \frac{Q^2}{2} \cdot \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right]$$

$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \ldots$$

or $$U = U_1 + U_2 + U_3 + \ldots$$

**Energy stored in a parallel combination of capacitors.** For a parallel combination, $V = \text{constant}$

Total energy,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} [C_1 + C_2 + C_3 + \ldots] V^2$$

$$= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \frac{1}{2} C_3 V^2 + \ldots$$

or $$U = U_1 + U_2 + U_3 + \ldots$$

Hence *total energy is additive both in series and parallel combinations of capacitors.*

## 2.25 ENERGY DENSITY OF AN ELECTRIC FIELD

**40.** *Where is the energy stored in a capacitor ? Derive an expression for the energy density of an electric field.*

**Energy density of an electric field.** When a capacitor is charged, an electric field is set up in the region between its two plates. We can say that the work done in the charging process has been used in creating the electric field. Thus the presence of an electric field implies stored energy or *the energy is stored in the electric field.*

Consider a parallel plate capacitor, having plate area $A$ and plate separation $d$. Capacitance of the parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

If $\sigma$ is the surface charge density on the capacitor plates, then electric field between the capacitor plates will be

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad \sigma = \varepsilon_0 E$$

Charge on either plate of capacitor is

$$Q = \sigma A = \varepsilon_0 EA$$

$\therefore$ Energy stored in the capacitor is

$$U = \frac{Q^2}{2C} = \frac{(\varepsilon_0 EA)^2}{2 \cdot \frac{\varepsilon_0 A}{d}} = \frac{1}{2} \varepsilon_0 E^2 Ad$$

But $Ad$ = volume of the capacitor between its two plates. Therefore, the *energy stored per unit volume or the energy density* of the electric field is given by

$$u = \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 E^2$$

Although we have derived the above equation for a parallel plate capacitor, it is true for electric field due to any charge configuration. In general, we can say that *an electric field E can be regarded as a seat of energy with energy density equal to* $\frac{1}{2} \varepsilon_0 E^2$. Similarly, energy is also associated with a magnetic field.

*... surfaces with induced surface charge densities* $\pm \sigma_p$.

**Reduced field inside a dielectric and dielectric constant.** In case of a homogeneous and isotropic dielectric, the induced surface charges set up an electric field $\vec{E}_p$ (field due to polarization) inside the dielectric in a direction opposite to that of external field $\vec{E}_0$, thus tending to reduce the original field in the dielectric. *The resultant field* $\vec{E}$ *in the dielectric will be equal to* $\vec{E}_0 - \vec{E}_p$ *and directed in the direction of* $\vec{E}_0$.

The ratio of the original field $\vec{E}_0$ and the reduced field $\vec{E}_0 - \vec{E}_p$ in the dielectric is called *dielectric constant* $(\kappa)$ or *relative permittivity* $(\varepsilon_r)$. Thus

$$\kappa = \frac{\vec{E}_0}{\vec{E}} = \frac{\vec{E}_0}{\vec{E}_0 - \vec{E}_p}$$

**Electric susceptibility.** If the field $\vec{E}$ is not large, then the polarisation $\vec{P}$ is proportional to the resultant field $\vec{E}$ existing in the dielectric, *i.e.*,

$$\vec{P} \propto \vec{E} \quad \text{or} \quad \vec{P} = \varepsilon_0 \chi \vec{E}$$

where $\chi$ (chi) is a proportionality constant called *electric susceptibility*. The multiplicative factor $\varepsilon_0$ is used to keep $\chi$ dimensionless. Clearly,

$$\chi = \frac{\vec{P}}{\varepsilon_0 \vec{E}}$$

*Thus the ratio of the polarisation to $\varepsilon_0$ times the electric field is called the electric susceptibility of the dielectric.* Like $P$, it also describes the electrical behaviour of a dielectric. The dielectrics with constant $\chi$ are called *linear dielectrics*.

**Relation between κ and χ.** The net electric field in a polarised dielectric is

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

But $$\vec{E}_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

∴ $$\vec{E} = \vec{E}_0 - \frac{P}{\varepsilon_0}$$

or $$\vec{E} = \vec{E}_0 - \frac{\varepsilon_0 \chi \vec{E}}{\varepsilon_0} \qquad [\vec{P} = \varepsilon_0 \chi \vec{E}]$$

Dividing both sides by $\vec{E}$, we get

$$1 = \frac{\vec{E}_0}{\vec{E}} - \chi$$

or $$1 = \kappa - \chi \quad \text{or} \quad \kappa = 1 + \chi.$$

## 2.29 CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB

**49.** *Deduce the expression for the capacitance of a parallel plate capacitor when a dielectric slab is inserted between its plates. Assume the slab thickness less than the plate separation.*

**Capacitance of a parallel plate capacitor with a dielectric slab.** The capacitance of a parallel plate capacitor of plate area $A$ and plate separation $d$ with vacuum between its plates is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

When a dielectric slab of thickness $t < d$ is placed between the plates, the field $E_0$ polarises the dielectric. This induces charge $-Q_p$ on the upper surface and

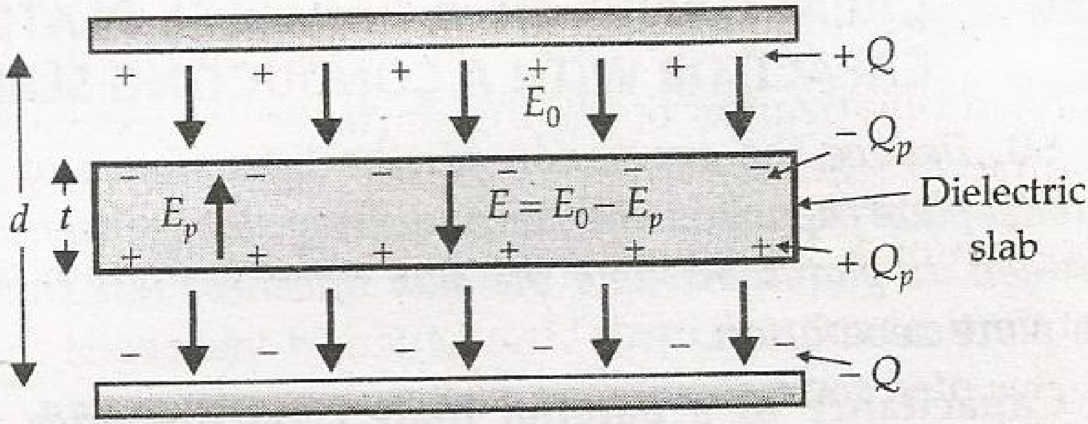


**Fig. 2.118** A dielectric slab placed in a parallel plate capacitor.

$+Q_p$ on the lower surface of the dielectric. These induced charges set up a field $E_p$ inside the dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0} \quad [\sigma_p = \frac{Q}{A} = P, \text{ polarisation density}]$$

The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{\kappa} \qquad \left[\because \frac{E_0}{E_0 - E_p} = \kappa\right]$$

where κ is the dielectric constant of the slab. So between the capacitor plates, the field $E$ exists over a distance $t$ and field $E_0$ exists over the remaining distance $(d-t)$. Hence the potential difference between the capacitor plates is

$$V = E_0(d-t) + Et = E_0(d-t) + \frac{E_0}{\kappa}t \left[\because \frac{E_0}{E} = \kappa\right]$$

$$= E_0\left(d - t + \frac{t}{\kappa}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{\kappa}\right)$$

The capacitance of the capacitor on introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}.$$

## 2.34 COLLECTING ACTION OF A HOLLOW CONDUCTOR

**55.** *A small sphere of radius r and charge q is enclosed by a spherical shell of radius R and charge Q. Show that if q is positive, charge q will necessarity flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge Q on the shell is.*

[NCERT]

**Collecting action of a hollow sphere.** Consider a small sphere of radius $r$ placed inside a large spherical shell of radius $R$. Let the spheres carry charges $q$ and $Q$ respectively.

Total potential on the outer sphere,

$V_R$ = Potential due to its own charge $Q$

+ potential due to the charge $q$ on the inner sphere

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{R}\right]$$

Potential on the inner sphere due to its own charge is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

As the potential at every point inside a charged sphere is the same as that on its surface, so potential on the inner sphere due to charge $Q$ on outer sphere is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$

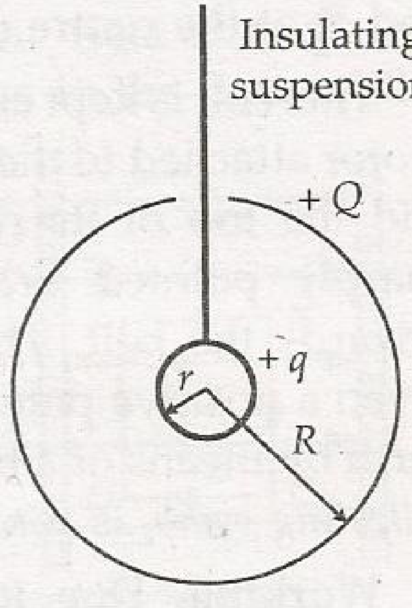


**Fig. 2.127** Small charged sphere suspended inside a charged spherical shell.

$\therefore$ Total potential on inner sphere

$$V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{Q}{R}\right]$$

Hence the potential difference is

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{R}\right]$$

So if $q$ is positive, the potential of the inner sphere will always be higher than that of the outer sphere. Now if the two spheres are connected by a conducting wire, the charge $q$ will flow entirely to the outer sphere, irrespective of the charge $Q$ already present on the outer sphere. In fact this is true for conductors of any shape.