Dots and Boxes 5x5 guide

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***Abstract*— A hopefully easy to understand guide on Dots and Boxes strategy covering basics as well as more advanced knowledge.**

CONTENTS

I. INTRODUCTION

Dots and Boxes is a game that many people probably remember having played in school, yet very few know that it involves any strategy at all. While there's lots of math to Dots and Boxes this guide shall focus merely on aspects that are relevant for playing the game and deliver them short and simple.

A. Rules

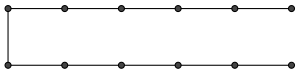
On a grid of dots, players take turns drawing lines from one dot to another horizontally or vertically adjacent dot. In the process of doing so they will draw the walls of boxes. Drawing the fourth wall of a box will earn the player a point and forces him to draw another line. The player with the most points wins.

B. The third wall

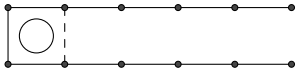
An obvious conclusion derivated directly from the rules is that you don't want to draw the third wall of a box as your opponent can thereafter draw the fourth wall earning him a point. As it is often the case with interesting games, we will see exceptions to this simple statement.

C. Multi-captures

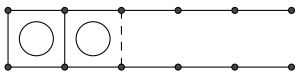
It is oftenly possible to capture more than just one box per turn.



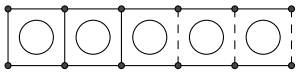
The leftmost box of aboves position has three walls, hence the box can be completed by drawing the fourth wall as seen below.



The rules state that upon completing a box you're forced to draw another line, hence the second box from the left, which now has three walls, can be completed in the same turn.



As a box has been completed once again the turn is still ongoing. Analougously the rest of the boxes can be taken.

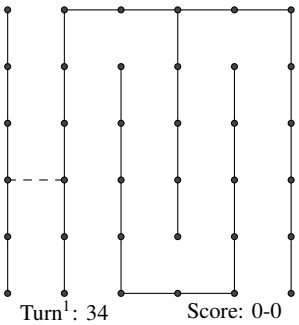


II. STRATEGY BASICS

If you want to beat your clueless buddies understanding the following two subsections should suffice in order to beat them almost all the time.

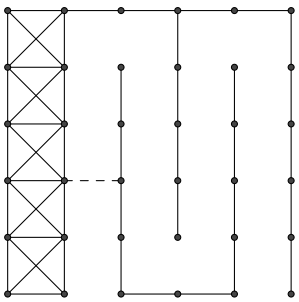
A. Doublecross-strategy

Key to playing Dots and Boxes strategically is an “all but 2”-trick used during the endgame.



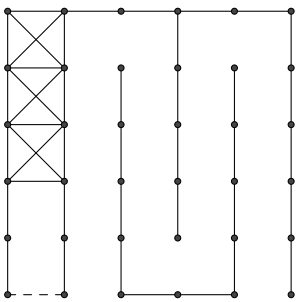
Turn[1]: 34 Score: 0-0

Given endgame position consists of two chains one of length 5 the other of length 20, there's no free move that doesn't give away a box. As you want to offer as few boxes as possible O has just played the dashed line into the shorter 5-chain. The clueless reaction by X on the dashed move would be following:



Turn: 35 Score: 5-0

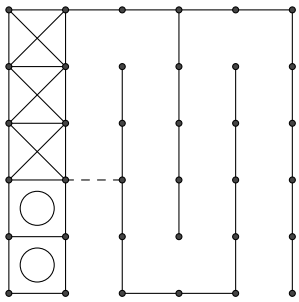
He just took the five boxes and played into the 20-chain. O will take those 20 boxes and win the game. However, there is a better move!



Turn: 35 Score: 3-0

Instead of taking all five boxes he uses the "all but 2"-trick finishing his move early as the dashed line does not complete a box. X loses two boxes by letting O doublecross two boxes of the 5-chain but gets the entire 20-chain in return.

[1]Turn includes the last played, dashed move

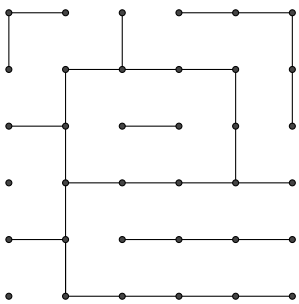


Turn: 36 Score: 3-2

B. The long-chain-rule

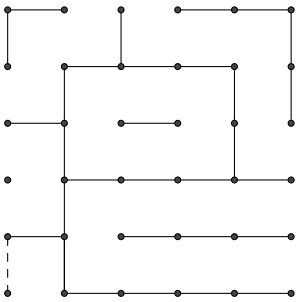
X won previous endgame by using doublecross-strategy, O lost as he had to play into either 5 or 20-chain not having any chance to defend himself. We conclude that we don't want to be the player who has to play into a chain first, because if we do then doublecross-strategy will be used against us. This leads us to the question answered by the long-chain-rule: How can we avoid having to play into a long-chain first?

The long-chain-rule for the 5x5 game of Dots and Boxes states that if the number of long-chains is even player 2 will have to play the first move into a long chain and that if the number of long-chains is odd it will be player 1. In other words: **Player 1 should aim to build an even number of long-chains, while player 2 should aim to build an odd number of long-chains**. Our previous endgame had two long-chains, an even number, hence it was O, player 2, who had to play into one of the long-chains.
Not all regions count as long-chains, a chain is long if it's length is ≥ 3 otherwise it's a short-chain and then there's loops. A more realistic endgame position could look like this:

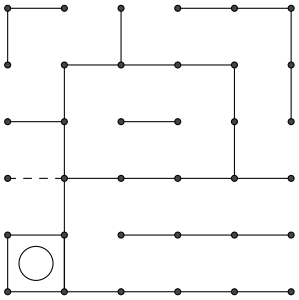


Turn: 30 Score: 0-0

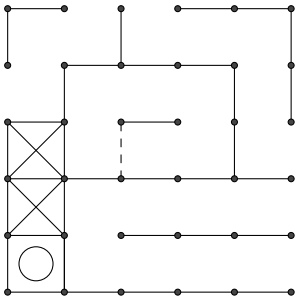
The number of long chains is 3, there's a 3-chain in the top-left, a 5-chain in the top-right and an 8-chain in the bottom right. And then we have two short-chains, a 1-chain in the bottom-left corner and a 2-chain above the 1-chain, as well as a 6-loop. As the number of long-chains is odd it should be player 1 who has to play into a long-chain first. Let's play some moves and see what happens.



Turn: 31 Score: 0-0

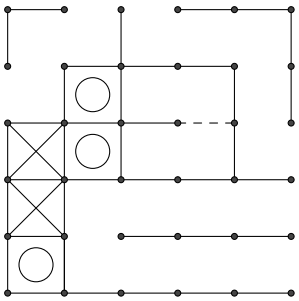


Turn: 32 Score: 0-1



Turn: 33 Score: 2-1

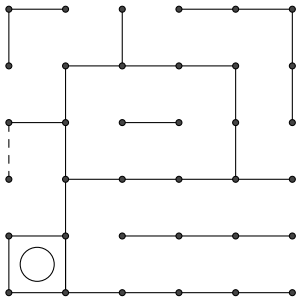
Player 1 has now played into the loop. Similarly to the ”all but 2”-trick for chains there’s an ”all but 4”-trick for loops:



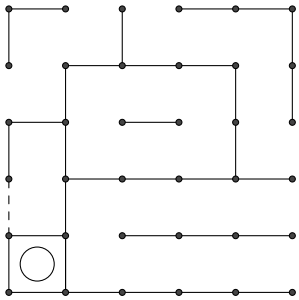
Turn: 34 Score: 2-3

Player 1 will now take his 4 boxes, but is the one who has to play into a chain first which happens as according to the long-chain-rule. A curiosity however is that if the loop would’ve been taken completely, without applying the ”all but 4”-trick to it, it would have been player 2 playing into the first long-chain. Loops do not count as chains, but **every loop that gets taken completely without having the ”all but 4”-trick applied to it inverts the long-chain-rule**!

Another curiosity could occur in turns 32, 33:



Turn: 32 Score: 0-1



Turn: 33 Score: 0-1

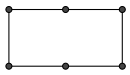
Player 2 made a mistake in turn 32 very common among beginners. Always play into the center of a 2-chain! The reason that a 3-chain is long and a 2-chain short, is that you can always apply the ”all but 2”-trick after a move into a 3-chain is played while the center move into a 2-chain prevents it. Decisive is in fact not how many long-chains exist but rather the number of times the ”all but 2”-trick will be applied which is why a long-chain that gets taken completely doesn’t add to the long-chain count.

III. Formalities

Quick linguistic input to make further writing a bit easier.

A. Technical terms

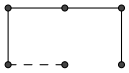
Using the ”all but 2” or ”all but 4”-trick is not mandatory, it is oftenly better not to use it, we call the decision of whether to use it or not a ”domino decision”. A domino is the shape of 2 boxes created after the ”all but 2”-trick is used:



Domino

Any move that gives away one or more boxes is called a ”sacrifice”, while a move that also creates a domino decision is called a ”loony move”.

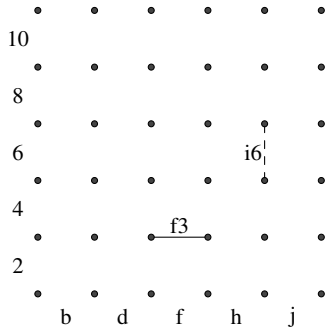
Sacrifice



Loony move

B. Notation

Showing a position as a picture takes a lot of space, further on a notation will be used to tell which moves are played in order to skip showing every trivial turn. The notation that has become somewhat established assigns each line, box and dot a name from a1 to k11.



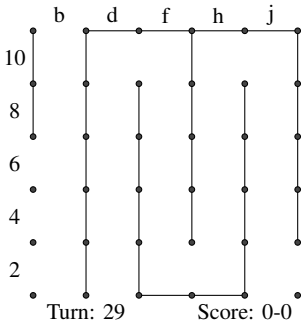
The notation for these two moves would be 1.f3 2.i6.

IV. Advanced strategy

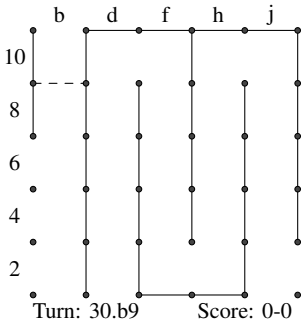
Knowing the long-chain-rule the next step to learning the game is figuring out how to aim for the right number of chains.

A. Preventing chains

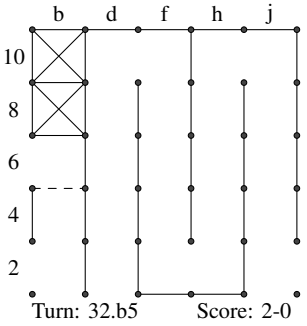
We are taking a look at following position:



There is one very long 19-chain and whoever gets it will be the winner of the game. 29 moves have been played, it is the turn of player 2 and according to the long-chain-rule player 2 should aim for an odd number of chains. There is a 2-chain at the top left which could extend into a 3-chain with a move like a6. If that 2-chain becomes a 3-chain it will be two long-chains in total, an even number. Not what player 2 wants! The coming turn will prevent the short 2-chain turning into a long 3-chain:

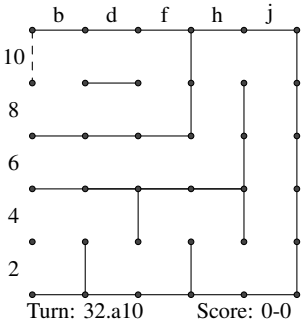


Player 2 played a sacrifice preventing the 2-chain from growing into a 3-chain. Player 1 will now keep trying to build another chain as he needs another one and plays 31.a4. Again player 2 will play a sacrifice to prevent the second chain:



No second long-chain was created, there was two 2-chains but those don't lead to the creation of a domino thus don't affect the long-chain-rule. The only long-chain is the long 19-chain, the only one, odd. Player 2 wins the game by sacrificing four boxes.

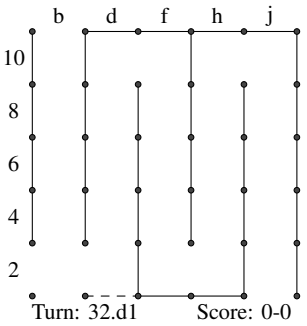
Sacrificing boxes is not the only way to prevent chains.



Player 2 plays 32.a10 creating a 6-loop at the top left preventing the 6-chain that could otherwise be created by b9.

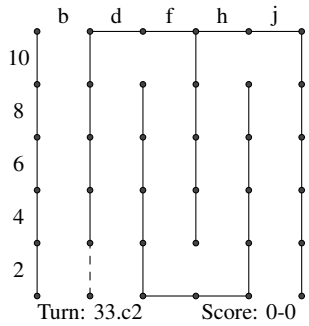
Note that sacrificing and building loops are both expensive ways to prevent chains. Sacrificing gives boxes away while loops will require the winner of the chain-battle to give away two dominos which is four lost boxes.

The least expensive way to prevent an additional chain is to merge the upcoming chain with an already existing one.



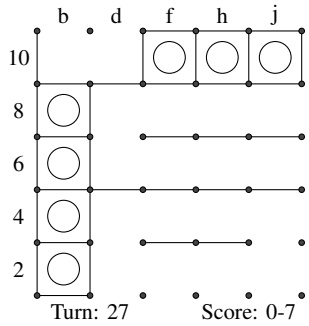
Player 2 has merged the 19 and the 4-chain, creating a long 25-chain. If he hadn't done that but played 32.a2 instead then

player 1 could have played 33.c2 in order to seperate the two chains.

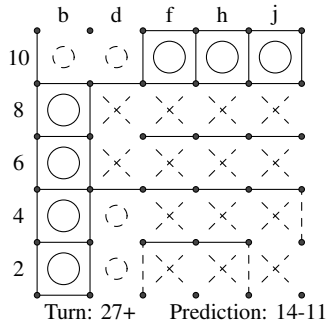


B. Preemptive Sacrifices

Given the position:

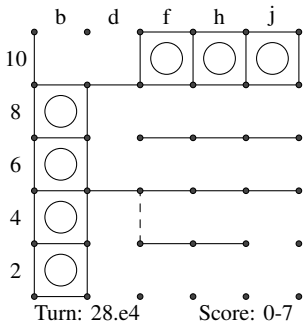


No dominos have been created yet, it is the turn of player 2 who is seven boxes ahead but he has lost the long-chain-battle, the total number of long-chains is two, even. In order to create a third chain player 2 could try to build space for another chain at the bottom right thus playing 28.e2. Player 1 would now merge the bottom chain with that space preventing the additional chain by playing 29.k4 and then player 2 would play 30.i2 to prevent the bottom chain from growing even longer. Let's try to predict the score resulting from 28.e2 29.k4 30.i2.

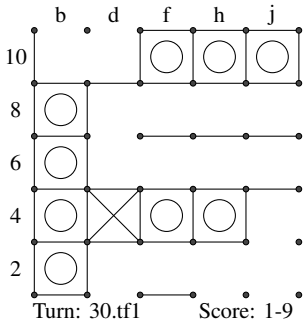


Player 2 gets one of the two 2-chains and a domino from the bottom 6-chain, the rest of the boxes goes to player 1.

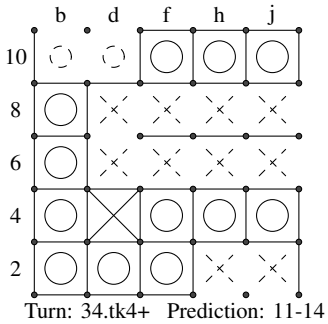
The better move to get additional space for a third chain is the loony move 28.e4.



Player 1 will take the d4 box and is then given a domino decision. As he's already seven boxes behind winning the long-chain battle is his only chance of victory, he plays 29.ti4 (t for the taken d4 box). Player 2 now has lots of space at the bottom to try building another chain with f1.



Player 1 now has to sacrifice three more boxes at the bottom. The next moves will be 31.e2 32.tk2 33.j3 34.tk4. Aside from the three sacrificed boxes player 2 will also get two boxes from one of the 2-chains and wins with a total of 14 boxes.

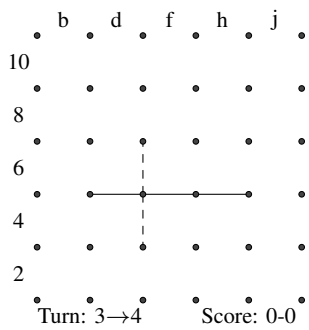


Worth noting: A preemptive sacrifice into a n-chain can only be a winning move if the sacrificing player is ahead by at least $n-1$ boxes (e.g. if you're one box ahead and play into a 3-chain, your opponent will take one box and is then given a domino-decision so he'll definitely win if he makes the right choice, hence an advantage by two boxes is required). Likewise you need to be $m-3$ boxes ahead when playing into a m-loop.

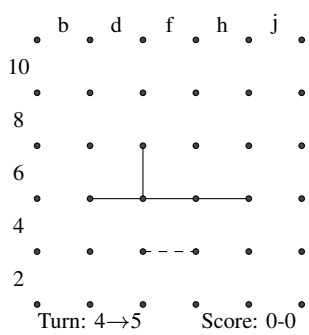
C. Dividing the board

Aiming for the right number of chains starts with aiming for a number of regions that will later on contain chains. **Player 1 should** start off trying to **aim for two chains** (going for four chains doesn't work out for reasons that will

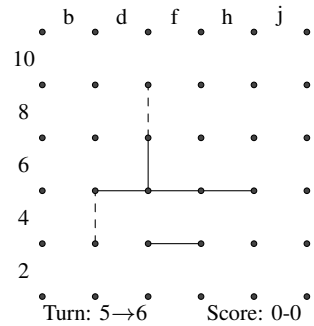
become more clear at subsection IV-E) while player 2 should try to aim for three chains (or one, see subsection V-A). Here's an example on what a thinking pattern could look like.



The board is split into a top and a bottom half. Player 1 is going for two chains, one in the bottom half the other in the top half. It is the turn of player 2 who is aiming for three chains. Aiming for three chains would mean to aim for three regions, because first you need the regions and then you can build chains inside of them. The two dashed moves would be logical choices for move four. 4.e4 would split the bottom half in two going for two chains along the bottom and one at the top. 4.e6 on the other hand would aim to split the top half in two, aiming for two chains at the top and one at the bottom. Those are both logical choices that tend to increase the chain count. The more intuitive one would be 4.e6 though as the top half is bigger. We assume that 4.e6 is the move played, what are viable responses for move 5?

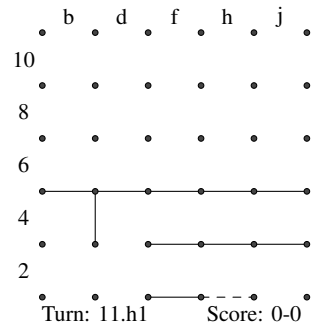


5.f3 would be a logical choice as we are aiming for two chains, it is building a connection between bottom left and bottom right in order to avoid these being split by a move like g4. Again: we really don't want to aim for four chains. In move 6 you'd want to separate the top left further apart. Logical choices would be 6.e8 splitting top left and top right even further or 6.c4 strengthening the separation between top left and bottom.



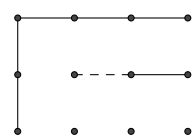
D. Indirect chains, wasting moves

Moves are valuable! You can only play one move per turn, make sure to focus on the right thing. What is wrong with the following move?

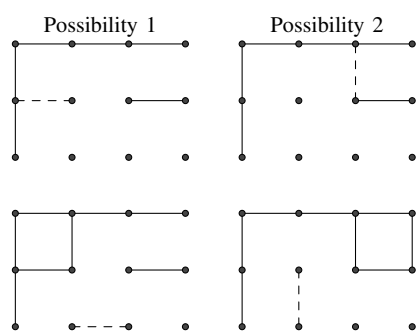


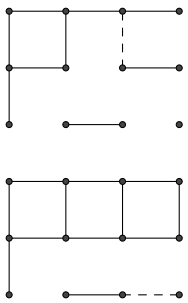
The line itself is not the problem, there is nothing wrong with extending the chain at the bottom. However it is not the right thing to focus on in this situation. The top half is yet completely empty, rather play something that benefits your aim of turning the top half into just one chain instead of increasing the length of that bottom chain which will only turn out helpful in case the top half turns out right which has yet to be confirmed.

How do you prevent following region from turning into a chain?

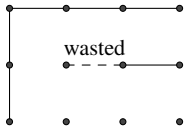


The dashed line will create a 3-chain, you have to sacrifice in order to prevent that:



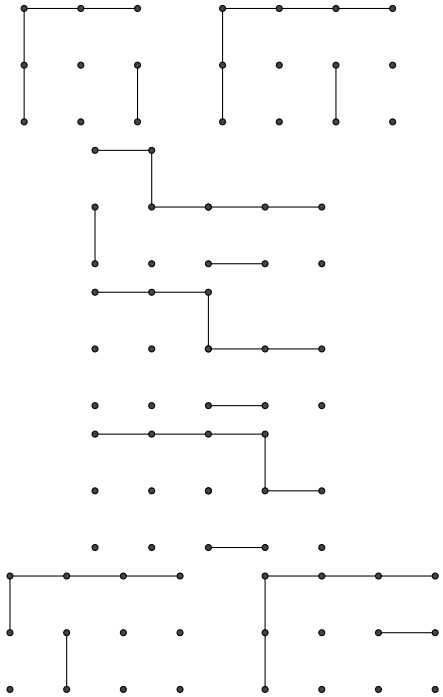


It is not possible to prevent the chain. The chain hasn't been build yet, but indirectly it's already there. Don't waste moves turning an indirect chain into an actual chain if there's something else left to do.



Moreover, indirect chains have the advantage that they can't be sacrificed preemptively.

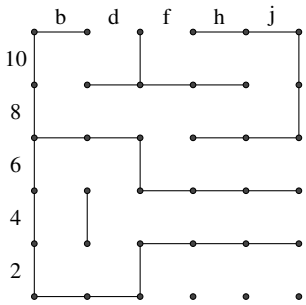
Here are a couple regions that are already chains:



Remembering those will turn out helpful for sure. Also, note that even if a region isn't a chain yet, it might be too expensive to sacrifice and prevent the chain. In that case you can just pretend that a region is a chain.

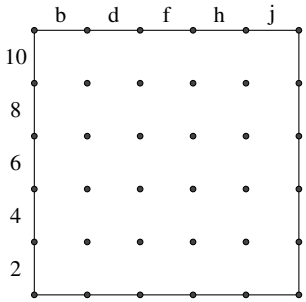
E. Ladders and edge moves – force loops/merge regions

Chains end at the edge of the board or inside another chain or loop.



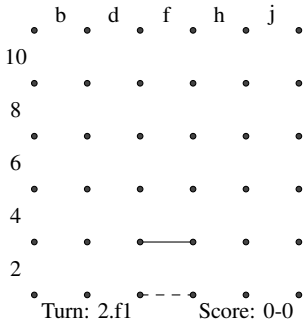
At the bottom is a 3-chain with one end at the edge of the board and the other running into a 6-loop, above is a 10-chain as well as a 3-chain with one end running into the 10-chain (you could also see it as a 4-chain running into a 9-chain or a 5-chain running into a 8-chain, but I prefer to name the smallest branch specially).

The following position is called the swedish game of 5x5.

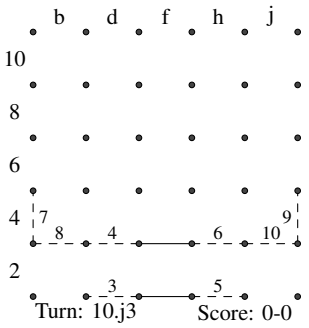


All the edge moves are played here. As a result chains can't end at the edge, only run into loops and other chains. Chains that could otherwise end at the edge will instead continue running into the board and take up more space. As an effect the number of chains will decrease. The number of loops on the other hand will likely increase.

Edge moves in general oftenly have this effect of compressing the board, they are very suited for turning regions into loops and connecting regions.

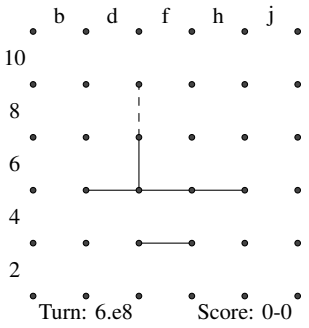


2.f1 is a frequently seen attempt to start building a chain at the bottom, leaving the top 5x4 field for another two chains. The first players strategy of defense against it shouldn't be the sacrifice 3.e2, but a ladder strategy using 3.d1 and further edge moves.

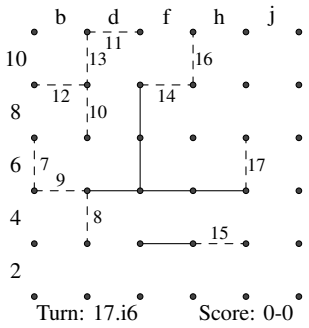


3.d1 is a threat to direct the bottom chain that is supposed to be created by 2.f1 to the north by using c2. 4.d3 is played to prevent c2. The same happens with 5.h1 and 6.h3. 7.a4 also threats to direct the now existing 3-chain to the north by using b5 or c4 which is prevented by 8.b3, same with k4 and h3. These moves leave us with a position in which the second player succeeded in creating his chain along the bottom edge, he will now try to create two chains within the top 5x4 field. Player 1 however has made his preparations to successfully aim for just one chain at the top: the pincer-like shape at the bottom edge of the upper 5x4 field is holding the top together. Player 1 will continue to play edge moves and there will be barely any edge left making it very hard to create chains, since chains need free edges for their ends. In a well played game there will be a loop somewhere and just one chain spreading along the top. Some sacrifices might be necessary to keep the chain count at two in total, yet player 1 has a huge advantage. The position has been computer analysed and is a 13-12 victory for player 1 with perfect play.

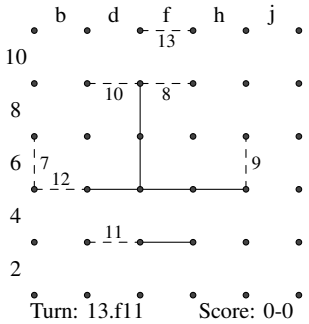
We return to this example from earlier (see IV-C):



6.e8 has been played, trying to split the board into three parts. Now it is the turn of player 1 and he wants not three but two chains and in order to accomplish that he will now attack the top left region trying to force it into either turning into a loop or connecting with the bottom or top right region. He decides to play the edge move 7.a6. The following moves could be 8.c4 9.b5 10.c8 11.d11 12.b9 13.c10 14.f9 15.h3 16.g10 17.i6.



8.c4 tries to prevent a join between bottom and top left, 9.b5 and 11.d11 threat to turn the top left into a loop which is prevented by 10.c8 and 12.b9, but then 13.c10 as well as 15.h3 try to merge the top right with either top left or bottom which is prevented by 14.f9 and 16.g10, however 17.i6 then turns top right into a loop which makes a total of two chains. Player 1 succeeded in turning one of the regions into a loop. A different variation could be 8.f9 9.i6 10.d9 11.d3 12.b5 13.f11



The result is similar, we end up building loops and get two chains: one at the bottom and one at the very top (unless player 2 decides to play the sacrifice e10/g10 which is not recommended). However, this variant is a lot more dangerous, because there is two quads which would require you to give away four boxes as dominos if you want to get the last chain for sure. Forming too many loops is something to be wary of.

V. Expert strategy

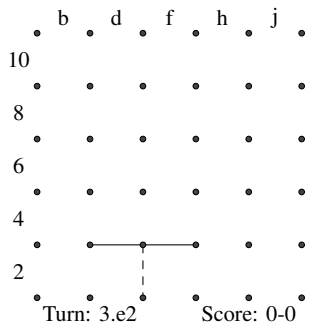
A. *Balance – second player approach*

The guideline so far has been player 1 should aim for two chains and player 2 should aim for three. With edge moves and ladder strategies section IV-E has provided a good technique to keep the chain-count low and if necessary sacrifices can be used as well. The truth is aiming for two chains is a lot easier than aiming for three, at the same time aiming for just one chain from the get-go is seemingly ridiculous. The solution to this mess is a flexible playstyle that allows player 2 to switch between aiming for one and aiming for three chains, which is what this paper refers to as ”balanced” play.

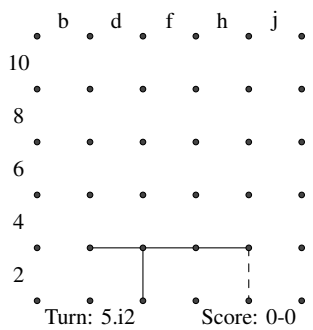
One common approach is to cut off an edge region in which you threat a very tiny chain (expected to be the third chain aside from the other two that will likely be build in

the big region) making player 1 prevent that tiny chain, then extend the cut off region to continue threatening the tiny chain, repeating that cycle while influencing the big area with a compressing wall (see ladder strategy) which makes it possible to turn it into just one chain and then finally balance out the big area and the edge region so that there is either one chain at the edge and two in the big region or no chain at the edge and just one in the big region.

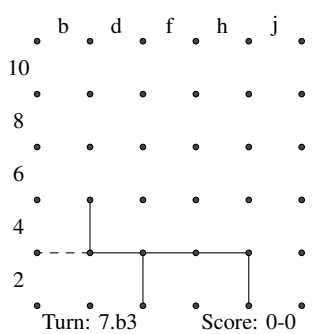
Well, that was an overly complex, terrible sentence so here is a step-by-step example (note that the moves of player 1 are not at all optimal):



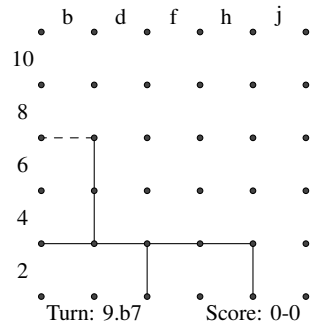
The moves d3 and f3 were played first, as a result there is a tiny region at the bottom where a third chain could potentially exist. To prevent that the move 3.e2 is played.



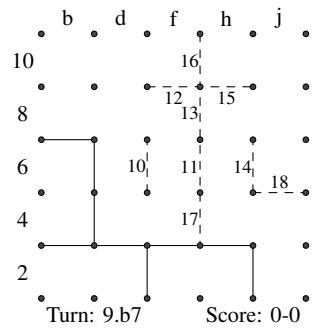
Player 2 has extended the edge region using 4.h3 and in order to once more prevent a tiny chain from coming into being the move 5.i2 is played.



6.c4 has been played creating a corner region which could again lead to a little chain in that bottom left corner and again 7.b3 is played in order to prevent that.



And finally 8.c6 and 9.b7 have been played for known reasons. All those moves played by player 1 were used in order to prevent a third chain, player 2 however is now taking the opposite direction not aiming for three but for one chain and all those moves played make this very easy. Here's an example of how the game could continue:



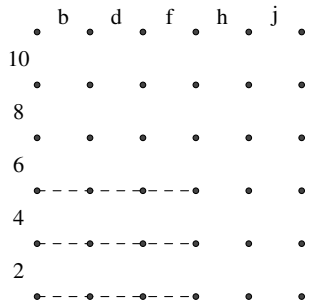
The position is an easy victory for player 2.

This wasn't a well played game but what it shows is the idea behind balanced play. Player 2 tries to abuse the lines of player 1 for his own purpose. A move that makes it hard to aim for three chains makes it more easy to aim for one chain and a move that makes it harder to aim for one makes it easier to aim for three chains. At some point player 1 should have stopped playing those chain preventing moves but if he hadn't played them at all the risk was high that the chain-count could end up being three.

B. Openings

A dots and boxes game can become very complex and probably will if played well from both sides. The first moves played in a dots and boxes game may not change the score of the game that it will result in with perfect play, but you can control the direction that you want it to take and create a setup that will make it easier to play later on.

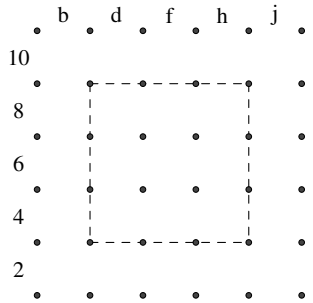
For the very first move there's nine different lines you can choose from if you exclude the identical mirrored and rotated versions.



For player 1, one possible setup includes the edge moves 1.f1 and 1.d1 preparing a ladder along the bottom supporting a decrease in the total number of chains as the bottom turns more likely to contain fewer chains, this will obviously help to defend against the high number of three chains. At the same time however it makes it hard to aim for one chain as a chain part of the bottom-ladder will hardly spread across the entire board up to the very top, it becomes more likely though as a loop gets created which (unfortunately for player 1) is also supported by 1.f1 and 1.d1. In any case, edge moves are good candidates for early moves by player 1.

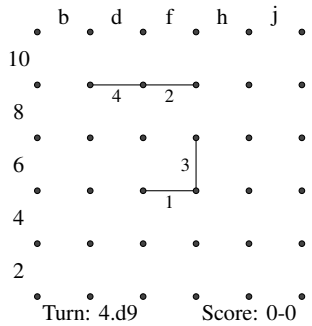
A different approach is to play a line with a small number of viable responses allowing you to memorize the most common opening variations providing you with an environment that is familiar and thus easy to play for you. One such move could be the center line 1.f5.
1.f5 2.e4 3.d3, 1.f5 2.d5 3.f3 and 1.f5 2.i6 3.g6 for example result in very common opening shapes easy for player 1 to play.

The second player is best off trying to apply the balance strategy in any case. The following 12 moves will generally be worth considering most of the time.



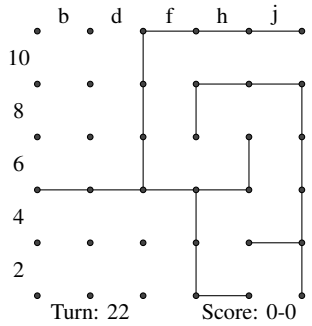
I strongly recommend not to use 1.f3 or 1.d3 as the first line as they are part of the 12.

An example
A well played opening could be 1.f5 2.f9 3.g6 4.d9.

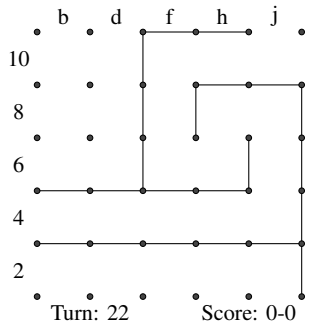


1.f5 divides the board into two halves providing a base structure for an aim of one chain at top and bottom each. 2.f9 is one of the possible 12 balance moves with two effects: 1. It creates space at the top where a chain could be or not and 2. it prepares space between f5 and f9 where a loop (otherwise a chain) could be created or not – a classic balance strategy. 3.g6 is somewhat harder to understand, you can come up with many different strategies at this point but 3.g6 in particular reduces the problematic area between f5 and f9 to the left side of the line saying there will be one chain at the top right for sure. The problem zone can just turn into a loop or connect with the bottom left where the second chain should be. 4.d9 is very similar to the second move: it just increases the danger of having a third chain at the top edge while continuing to build the loop shape.

C. Visual approach to nim

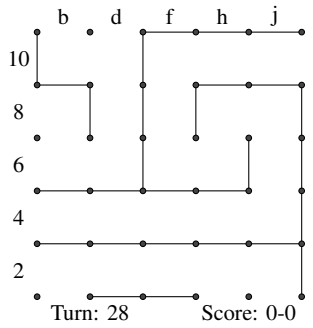


A 13-chain and two undecided regions of the same shape which may or may not turn into a chain. It is the turn of player 1 and player 2 has a very easy strategy to win here: Player 1 is first to play into one of the undecided regions and player 2 is simply going to copy the played moves into the other undecided region, e.g. if player 1 were to play 23.a6 then player 2 would respond with 24.f1. As a result the two undecided regions will end up with the same number of chains n and of course, because they are two regions, together they will contain $2 \cdot n$ chains, even, which along with the 13-chain makes an odd number of chains.

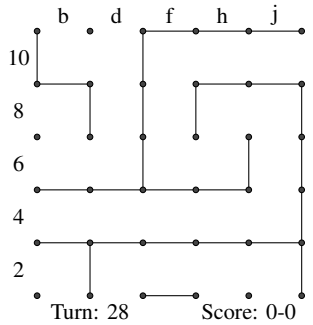


Similar situation to before, except the two undecided regions don't look alike. Then what's the strategy here? Copying the move into the other area doesn't work anymore, or does it? Actually it does. This is important: the two areas may not look the same, but in a more mathematic way they are equivalent, so you can copy moves just fine by playing the move of similar nature to the one played in the first area.

We will take a look at the different natures of moves a little bit further on, first up are two possible follow-up-positions to clarify why the nature matters; can't you just make sure to create/prevent a chain in the second area whenever it is done in the first? No, here is why:



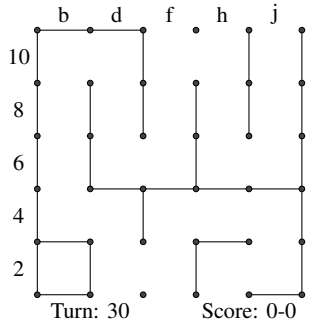
Position 1 – This one is a second player win, because it is the turn of player 1 and any move in one of the undecided regions creates or prevents a chain and then player 2 can play accordingly in the other area. This is the result of copying moves correctly.



Position 2 – This one however is a first player win, because in this case there's a 1-chain in the bottom left corner which can be sacrificed without problems and then player 2 will have to play the critical move into one of the undecided areas. This is what can happen when not paying attention to the nature of undecided regions.

We can see that the lines c2 and d1 are very different from each other even though both do not create a chain at the bottom: they have different "nim-values", although we don't want to enter the value-topic in this guide, but rather use visual keys. The attempt is to differenciate between 2 different types of regions/moves and then figure out how to pair them up correctly.

One more example for when nim matters:

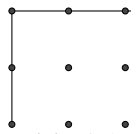


This one is different in the sense that there is no question of whether we want to create or destroy a chain somewhere, this time the question is whether we want to connect or seperate two existing chains. So again we have two critical areas: 1. d1/f1 connect chains at the bottom, while e2 seperates and 2. f11/h11 connect, while g10 seperates. Again it is the one, who doesn't have to play into one of the critical areas first, that wins.

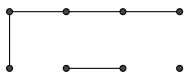
The possibility of connecting or seperating chains will be part of one of the two types of natures we want to define.

Now it is time to define our two types of regions:

Among others, type 1 includes:



minimal: 0



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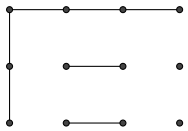

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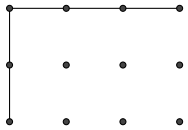

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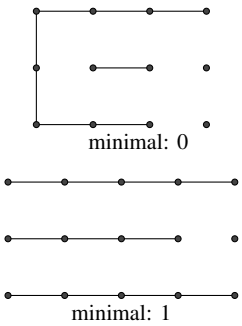
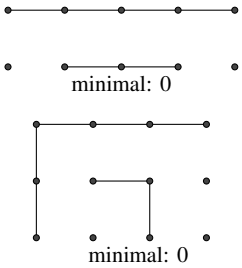


Type 2 includes:



Minimal notes the minimum number of chains that will be created (all existing chains getting connected, all upcoming chains being destroyed). It is zero for the sacrifices since there won't be chains if the upcoming chain in an area gets destroyed and one for the case where it's about connecting two chains since there will be at least one chain.

And now to the interesting part, for positions only with undecided regions of one of the two types, this is how to work out who will win the chain-battle:

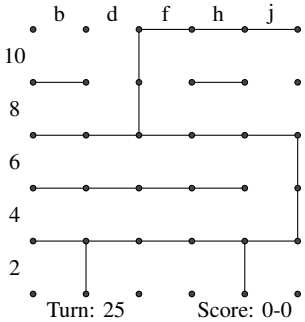
1) Work out the minimum number of chains that will be created on the board.
2) Identify which type each undecided region belongs to
3) Pair up undecided regions of the same type and consider them negating each other

4.a) If there's only one undecided region left, the player whose turn it is can change it the right way

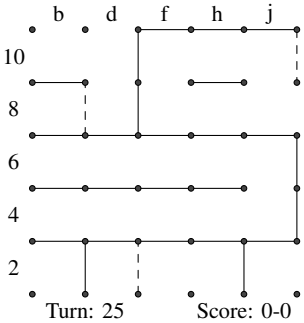
4.b) If there is no undecided region left then the player who would win with the minimal number of chains will win

4.c) If there is two undecided regions left it means one is of type 2 and one of type 1 and in this case the player, who would win with the minimal number of chains, will lose

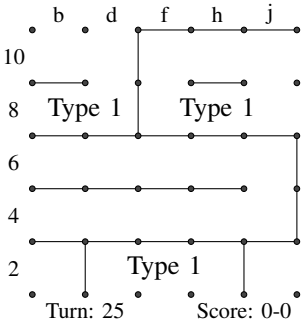
Example 4.a



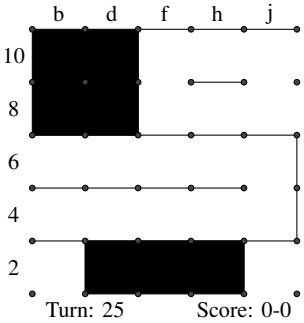
Step 1:
The minimal number of chains is 1.



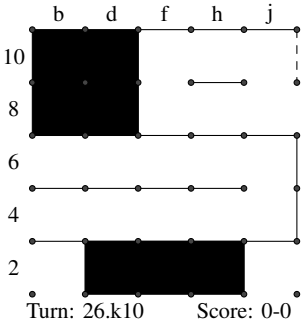
Step 2:



Step 3:

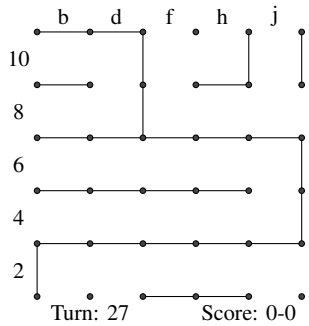


Step 4.a:

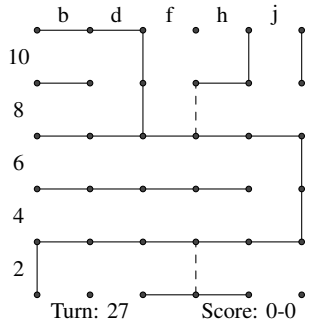


We can see that player 2 is able to win the chain battle by choosing one of the undecided regions and preventing the chain there.

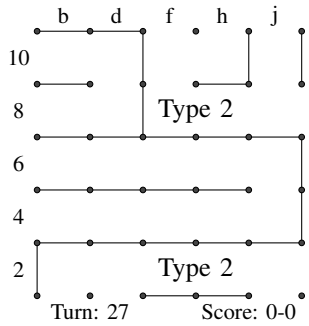
Example 4.b



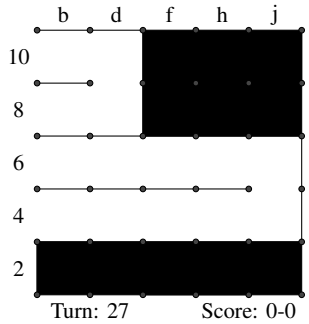
Step 1:
The minimal number of chains is 2



Step 2:

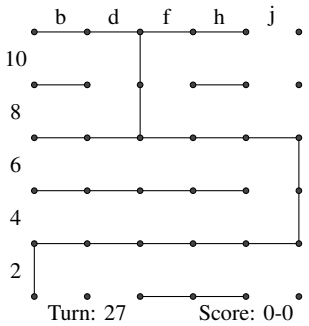


Step 3:

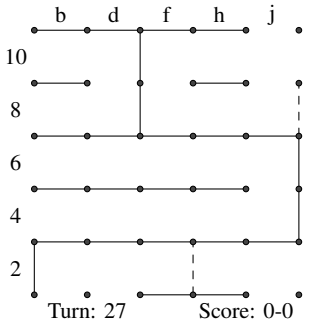


Step 4.b:
Player 1 wins because the minimal number of chains is 2 and after step 3 there is no undecided region left. Taking a look at the position one can see that there is no spot where player 2 wants to play.

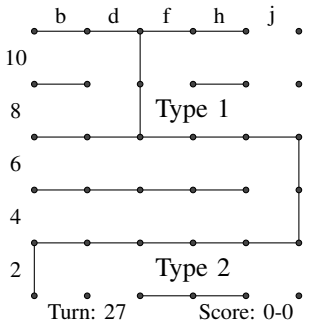
Example 4.c



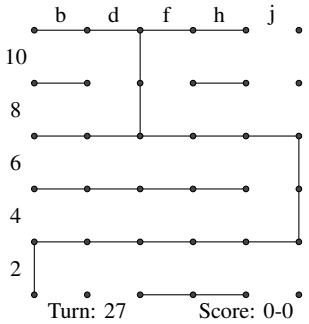
Step 1:
The minimal number of chains is 2



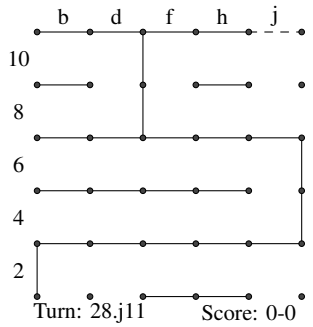
Step 2:



Step 3:

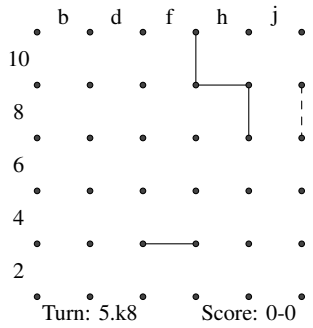


Step 4.c:
Player 2 wins because the minimal number of chains is 2 and after step 3 there is two undecided regions left. Taking a look at the position one can see that player 2 has a move that doesn't cause him any trouble.

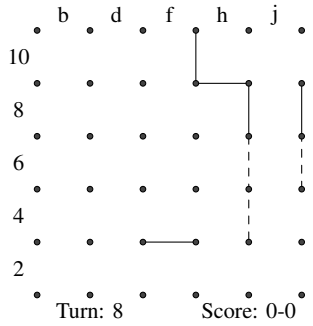


Now it is player 1 who doesn't want to play any of the possible lines.

The next step, after learning about the two types of undecided regions and how they effect each other, is to make use of that knowledge, so here is one example of a very early move that makes use of it.



Nim is actually not too important here, 5.k8 is a good move mostly because it makes for a good ladder approach, but one advantage even if small is also that there is a realistic chance that the top right region will turn into one of the more rare undecided regions of type 2.

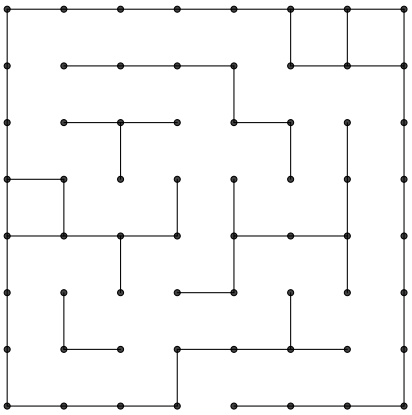


This is good because if the larger region will end up being one chain and another undecided region then it is likely that undecided region will be of type 1 while the minimal number of chains is 1.

VI. Extra

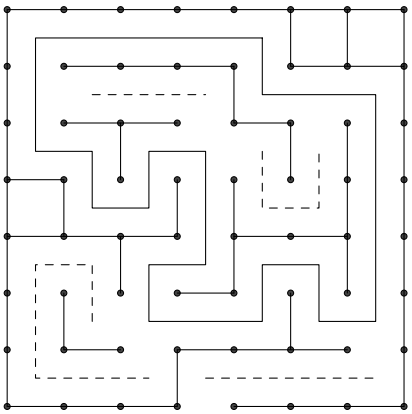
A. Counting chains in a "maze"

Just counting the number of chains in an endgame position may turn out quite confusing and complex at times.

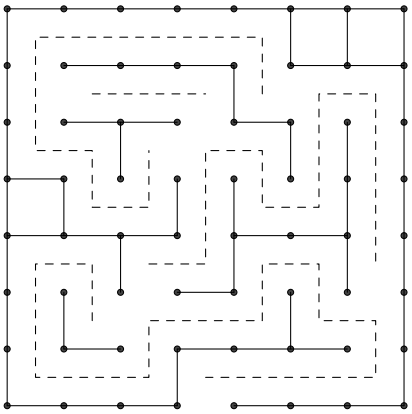


How many chains does aboves position consist of?

Giving an answer upon simply glancing at the board should hardly be doable, yet working out the answer is very easy.



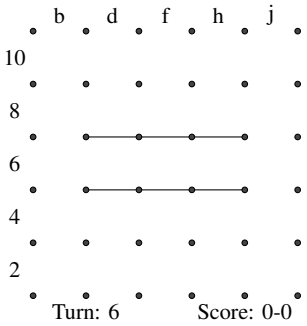
As seen above, all you have to do is draw paths into the board in a way that at most two sides of a box get crossed by a path. Now you just count all the paths that don't make a loop. In aboves example we have 4 chains, visualized as dashed paths, as well as a loop which we don't count.
It doesn't matter how you draw the paths exactly, the result will be the same:

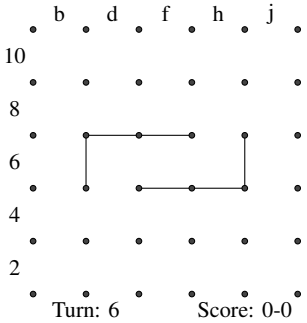


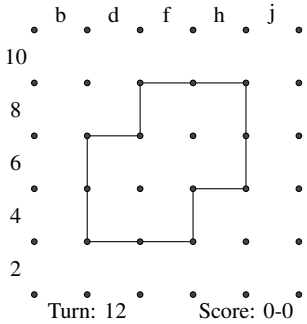
Again we end up with 4 chains as we should.

B. Mirroring

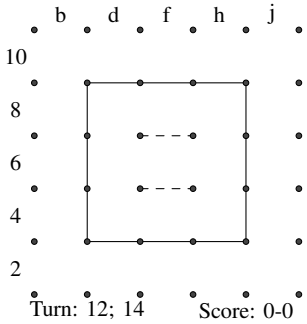
Simply copying your opponents moves right off the bat, applying a mirror strategy, is not something that will always work, but there's some position that you want to avoid getting into. The following three positions are an easy win for player 2, because he can just keep copying player 1's moves. The only way for player 1 to get out would be the sacrifice of the center chain, although as a loony move that obviously doesn't win either.







Certainly, Dots and Boxes wouldn't be a fair game if player 2 could win simply by applying a mirror strategy. The following symmetrical position (with the dashed lines played or unplayed) is a first player win:



Another way to defend against a mirror strategy would be the sacrifice of the center-box: