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Some New Results Of The integrals in the groups Mod-n

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Abstract.
Let G = Zn be a belian group Mod n. we shall define a new integral of the all elements of the G = Zn (n is a finite number , n∈ N , n ≥2).

We shall study a new – results of the integrals properties in the group Z_n. We gave the definitions of integrals in the Zn, and we shall gave the anew- definitions of the neat (semi) integrals, and the some a new results of this integrals.

Introduction

Let G = Z_n be abelian group Mod n , n ≥ 2. We shall starts with the a new – definition of the integrals, by the following :

Definition A : The integrals in the group Zn define b

$$\forall n \geq 2\,,\ \int_0^k \bar{x}\, dZn = \bar{x}\, Zn \int_0^{kr} = k\bar{x}$$

$$= \{0\bar{x}, 1\bar{x}, 2\bar{x}, 3\bar{x}....., (n-1)\bar{x}\}$$

With K = { 0 , 1, 2 , …… , n - 1 }

Example 1 :Take G = Z_3 , So

$$< \bar{0} > \int_0^k \bar{0}\, dz_3 = \int_0^2 \bar{0}\, dz_3 = \{0\bar{0}\ , 1\bar{0}\ , 2\bar{0}\} = \{\bar{0}\} =$$

$$\int_0^2 \bar{1}\, dz_3 = \{0\bar{1}\ , 1\bar{1}\ , 2\bar{1}\} = Z_3$$

$$\int_0^2 \bar{2}\, dz_3 = \{0\bar{2}\ , 1\bar{2}, 2\bar{2}\} = \{0, \bar{2}, \bar{1}\} = Z_3$$

So $\int_0^2 \bar{1}\, dz_3 = Z_3 = \int_0^2 \bar{2}\, dz_3$

Clearly , $\int_0^k dz\mathrm{n} = Zn \qquad \forall n \geq 2$

Example 2 .Take G = Z_6 It's easily to show that

$$\int_0^k \bar{0}\, dz_6 = \int_0^5 dz_6 = <\bar{0}>$$

$$\int_0^5 \bar{1} dz_6 = Z_6$$

$$\int_0^5 \bar{2}\, dz_6 = \{0\bar{2}, 1\bar{2}, 2\bar{2}, 3\bar{2}, 4\bar{2}, 5\bar{2}\}$$

$$= \{\bar{0}, \bar{2}, \bar{4}, \bar{0}, \bar{2}, \bar{4}\} = <\bar{2}>$$

$$= \{\bar{3}\},$$

So

$$\int_0^5 \bar{3}\, dz_6 = \{0\bar{3}, 1\bar{3}, 2\bar{3}, 3\bar{3}, 4\bar{3}, 5\bar{3}\}$$

$$\int_0^5 \bar{5} \mathrm{dz}_6 = Z_6$$

Here,

1 - $\int_0^5 \bar{1} \mathrm{dz}_6 = \int \bar{5}\, dz_6 = Z_6$

2 - $\int_0^5 (\bar{1} +_6 \bar{2})\, dz_6 = \int_0^5 \bar{3}\, dz_6 = <\bar{3}>$

$$\int_0^5 (\bar{1} +_6 \bar{2})\, dz_6$$

Definition B :

The odd integrals, between two different Mod – group , define by

$$\int_{0}^{k1+k2} (Zn + Zm)\, dz_{n+m} = \int_{0}^{k1} Zn\, dz_{n+m} + \int_{0}^{k2} Zm\, dz_{n+m}$$

$$= Zn + Zm$$

Example 3 :

$$\int_{0}^{k1+k2} (Z_2 + Z_3)\, dz_{2+3}$$

$$\int_{0}^{k1} Z_2 + dz_2 + \int_{0}^{k2} Z_3\, dz_3 =$$

$$= \int_{0}^{1} Z_2\, dz_2 + \int_{0}^{2} Z_3\, dz_3$$

$$= Z_2 + Z_3 = \{(\bar{0},\bar{0}),(\bar{0},\bar{1}),(\bar{0},\bar{2}),(\bar{1},\bar{0}),(\bar{1},\bar{1}),(\bar{1},\bar{2})$$

We are ready to gave the new-definition to the neat – integrals subgroups –

Definition C : [2]

Let G = Z_n be a group Mod – n and H be a subgroup to G.

Then the integrals of the elements of the H. Is said to be neat – integrals element, for all prime number P , for all $h \neq 0 \in H$

If $\int_{0}^{p} x\, d\, Z_n = Px = h$ for some Z_n then $\int_{0}^{p} ho\, dH = Pho = h$ for some $ho \in H$.

We shall said a subgroup H of Zn isa neat – integrals in Zn if for all $h \in H$, h is neat – integral element , that mean H is neat – integrals in Z_n , For all h ∈ H , for all prime number P , $x \in Zn$

If $\int_{0}^{p} x\, dZn = h = \int_{0}^{p} ho\, dh$ for some $ho \in H$.

Example 4 : Take G = Z_6 and $H = <\bar{3}> = \{\bar{0},\bar{3}\}$ then we can show that the all element of H (We have only $\bar{3}$) is a neat – integrals

Take p = 2

Clearly $\int_{0}^{2} \bar{x}\, dz_6 = 2\bar{x} = \bar{3}$

Clearly that, then is no solution in Z_6 , ($\bar{x} \notin z_6$)

Take p = 3

So $\int_0^3 \bar{x}\, dz_6 = 3\bar{x} = \bar{3} \Rightarrow 3\,(\bar{1}) = \bar{3}\ in\ Z_6$

To show $\int_0^3 \bar{x}\, dz_6 = \bar{3}$ it has a solution in H.

$$\int_0^3 \bar{x}\, dz_6 = \bar{3} = \int_0^3 \bar{3}\, dh = 3\bar{3} = \bar{3} = \bar{3} \in H$$

Take p = 5

Clearly $$\int_0^5 \bar{x}\, dz_6 = 5\bar{x} = \bar{3} \in G$$

$$5(\bar{3}) = \bar{3} \in H.$$

So $\int_0^5 \bar{x}\, dz_6 = \int_0^5 \bar{x}\, dh$

Thus , $\forall p,\ p \geq 2 \quad , \quad \forall h \in H$

If $\int_0^p \bar{x}\, dZ_n = h \quad in\ G$

Then $\int_0^p \bar{x}\, dH = Pho = h \quad in\ H$

Therefore , H is integral – neat . We shall denoted H by P- integral neat subgroup of G

Example 5: Take G = Z_{12} and $H = <\bar{2}>$.
and take P = 2

$$\int_0^2 \bar{x}\, dZ_{12} = 2\bar{x} = \bar{2} \quad in\ G$$

We have $2\,(\bar{1}) = \bar{2} \quad in G$

But $\int_0^2 \bar{x}\, dz_{12} \neq \int_0^2 \bar{x}\, dH = \bar{2}$

Which means , That $\int_0^2 \bar{x}\, dz_{12} = \bar{2}$

It has no solution in H

So H is not P- integral neat in G .

Definition D :

A subgroup H of the G is said to be Po – integral neat in G if , $\forall h \in H \int_0^{po} \bar{x}\, dZn = po\, \bar{x} = h$

Then $\int_0^{po} \bar{x}\, dH = po\, \bar{x} = h = po\, ho \qquad for\ some\ ho \in H .$

Example 6 :Take G = Z_8 and $H = <\bar{2}>$

If P = 2 , $h = \bar{2} \in H$

It has solution in Z_8 $\int_0^{2} \bar{x}\, dZ_n = 2\bar{x} = 2$

But $\int_0^{2} \bar{x}\, dZn = \bar{2}$ it has no solution in H .

So H is not P- integral neat in G .

Now , take P = 3 , $h = \bar{2} \in H$

its has solution in Z_8 $(3\bar{6} = \bar{2})$ $\int_0^{3} \bar{x}\, dZ_8 = 3\bar{x} = \bar{2}$

Clearly $\int_0^{3} \bar{x}\, dZ_8 = \bar{2}$ it has solution in H .

$$\int_0^{3} \bar{x}\, dZ_8 = \bar{2} = \int_0^{3} \bar{x}\, dH = 3\bar{6} = \bar{2} \text{ and } ho = \bar{6} \in H$$

Now , test $\bar{4} \in H$

$$\int_0^{3} \bar{x}\, dZ_8 = 3\bar{x} = \bar{4} \quad in \quad G$$

$$3\bar{x} = 3(\bar{4}) = \bar{4} \ \ and \ \ \bar{4} \in H$$

$$\text{So} \int_0^{3} \bar{x}\, dZ_8 = \int_0^{3} \bar{x}\, dH \qquad in \quad H$$

Test $\bar{6} \in H$

$$\int_0^{3} \bar{x}\, dZ_8 = 3\bar{x} = \bar{6} \qquad in \ \ G$$

$$= 3\,(\bar{2}) = \bar{6} \qquad \bar{2} \in H$$

Clearly $\int_0^{3} \bar{x} dz_8 = \bar{6} = \int_0^{3} \bar{x}\, dH \quad in \quad H$ and His 3- neat integrals in G .We are ready to show some results of P- neat integrals .

TheoremA : For any P- neat integrals in abelian group G is a Po –neat in G

Theorem B : [1]
Let A and B be two p- neat integrals in G then
i) $A \cap B$ is a P- neat integrals in
ii) A + B is a P – neat integrals in G
Proof :
Let **h** be any element in **A** $\cap$ **B** and for all prime number P

Suppose $\int_0^p \bar{x}\, dG = P\bar{x} = h \quad in \quad G$

So there exist an element $g \in G$ such that

$$\int_0^p \bar{x}\, dG = P\bar{x} = pg = h \in A \cap B$$

Since $h \in A \cap B \quad h \in A \quad and \quad h \in B$

Thus , $\int_0^p \bar{x}\, dG = P\bar{x} = pg = h \in A \quad in \quad G$

But A is p- neat integral in G

$$\text{So} \int_0^p \bar{x}\, dG = \int_0^p \bar{x}\, dA = P\bar{x} = pa = h \quad for\ some\ a \in A$$

and B is p- neat integral in G

We have

$$\int_0^p \bar{x}\, dG = \int_0^p \bar{x}\, dB = P\bar{x} = pb = h \quad in \ B$$

Hence, Pb = h = Pa

So P (a – b) = 0 and thus $a = b \in A \cap B$

Therefore $\int_0^p \bar{x}\, dG = \int_0^p \bar{x}\, d\ A \cap B = h \in A \cap B$

We get $A \cap B$ is p- neat integral in G
ii) To prove , A + B be a p- neat integral in G

Let Z be any element in A + B and suppose that $\int_0^p \bar{x}\, dG = z \ in \ G$

So $P\bar{x} = z$

Since $z \in A + B$, Z = a + b for some $a \in A \ b \in B$

We have $P\bar{x} = a + b \quad \in A + B$

and $\int_0^p \overline{x_1}\, dGn = a$ and $\int_0^p \overline{x_2}\, dgn = b$

But we have A and B are P-neat integrals in G

$$\text{So } \int_0^p \overline{x_1}\, dG = \int_0^p d_o\, dA \qquad for\ some\ d_o \in A$$

$$\int_0^p \overline{x_2}\, dG = \int_0^p d_o\, dB \qquad for\ some\ b_o \in b$$

Thus, $Pd_o = a$

$Pd_o = b$ and we get $P\ (d_o + b_o) = a + b$

So , $P\bar{x} = a + b = pa0 + pb_o = p(d_o + b_o)$

$$\int_0^p \bar{x}\, dG = \int_0^p (d_o + d_o) = a + b$$

Which mean that

It $\int_0^p \bar{x}\, dG = z \in A + B \quad in\ G$

Then $\int_0^p \bar{x}\, d(A + B) = z \in A + B \quad in\ G$

Theorem 3: If A is only neat – integral subgroup of **A** subgroup **B** of **G** then

i) A is a neat- integral of G

ii) B is p-neat integral in G then B/A is a neat – integral of G/A

Proof :/ and for all P it $\int_0^p \bar{x}\, dG = a \quad in\ G$ and for all P it $\int_0^p \bar{x}\, dG = a \quad in\ G$

So $P\bar{x} = a \in A$ .But A is a neat- integral of B so

$$\int_0^p \overline{x_1}\, dB = a \quad \in A \subseteq B$$

So $\int_0^p \overline{x_1}\, dA = a = pd_o = pd_o = for\ some\ d_o \in A.$

Hence , $\int_0^p \bar{x}\, dG = a = \int_0^p d_o\, dA$

Therefore A is p-neat in G

ii) Let $b + A \in B/A\ and\ \forall p(b \in B)$

$$\int_0^p \bar{x}\, d\, G/A = b + A \qquad , \quad \bar{x} \in G/A$$

So $\int_0^p g \, d \, {}^{G}\!/\!_{A} = b + A \quad in \quad {}^{G}\!/\!_{A}$

$$P(g + A) = b + A \in {}^{G}\!/\!_{A}$$

So $Pg + pA = b + A \quad \Rightarrow \quad Pg + A = b + A$

So $Pg = b \Rightarrow \int_0^p g \, dG = b \quad in \ G$

But B is P- neat integral in G

Thus , $\int_0^p g \, dG = b = \int_0^p b_o \quad dB$

P b0 = b some b0 $\in B$

Since we have

$$\int_0^p g + A \quad d \ {}^{G}\!/\!_{A} = \quad P(g + A) = b + A$$

$$P(g + A) = pbo + A = p(bo + A) = b + A$$

$$\text{Thus , } \int_0^p g + A \quad d \ {}^{G}\!/\!_{A} = \int_0^p bo + A \ d \ {}^{B}\!/\!_{A}$$

We get ${}^{B}\!/\!_{A}$ is p- neat integral

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