Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions
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Source: *Music Analysis*, Vol. 15, No. 1 (Mar., 1996), pp. 9-40
Published by: Blackwell Publishing
Stable URL: http://www.jstor.org/stable/854168
Accessed: 15/04/2011 20:32

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Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions

> To be adequate, a description of Wagnerian harmony would have to clarify its position floating between a tonality that has been attacked by the weakening of the root progressions but not yet completely destroyed, and an atonality anticipated in the increased independence of semitonal motion but not yet reached; and this [position] would have to be defined as clearly as the relationship between the technical features and their expressive role.
>
> Carl Dahlhaus[1]

I Problematising triads in neo-Riemannian theory

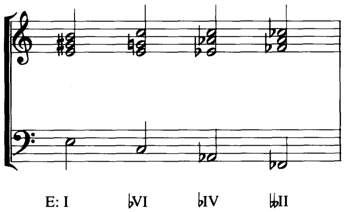
This article proposes a framework for exploring the idiosyncratic behaviour of major and minor triads in the music of Wagner, Franck, Richard Strauss and their contemporaries. The central position of these structures constitutes the strongest bond of continuity that connects eighteenth-century music to the progressive music of the last quarter of the nineteenth century, and the strongest motivation for the prevailing practice of adapting analytical approaches to late Romantic music from models forged for music of the Classical era. But the diatonic indeterminacy of triadic progressions in late Romantic music often limits the efficacy of these diatonically based models.

These limitations leap most rapidly into view in the familiar case of triadic progressions whose roots divide the octave into equal parts. Ex. 1 presents five enharmonically equivalent notations for a progression of major triads whose roots trisect the E octave. The standard term 'equal division of the octave' captures intuitions about two types of equivalence in Ex. 1: between the individual segments of the span, and between its two boundary events. Classical methods of analysis can capture either of these intuitions individually, but not both simultaneously. The fundamental-bass scale-step analyses at (a), (c) and (d) capture the equivalence of the boundary events, but not of the local segments: they assert a mixture of two kinds of root motion, major thirds and diminished fourths. The analyses at (b) and (e) restore the equivalence of the local segments, but undermine the equivalence of the

Ex. 1 Five notations for an equal division

a)

b)



c)

d)

e)

boundary events, asserting a scale-degree progression rather than a prolongation. Either the divisions are unequal, or they are not divisions of an octave.

The problem cannot be laid solely at the door of the fundamental bass tradition. Equal divisions are equally paradoxical from a Schenkerian/linear perspective, in part because they erode the fundamental distinction between consonance and dissonance. If we intuit the surface harmonic motions as equivalent, then we intuit them as equivalently consonant. Yet this latter intuition is violated at (a), (c) and (d), which assert a mixture of consonant and dissonant intervals in the bass. The intuition that the entire passage composes out a consonance is violated at (b) and (e), which assert that what is heard in the bass as a consonant octave is conceived as a dissonant augmented seventh.

It is problems of this type, undoubtedly, that led Felix Salzer and Carl Schachter to state that 'we register the equal intervallic progressions without referring them to a supposed diatonic original. This temporary lack of a diatonic frame of reference creates, as it were, a suspension of tonal gravity.'[2] The conceptual problem springs from the same source as the notational problem: the culprit, in both cases, is the diatonic scale. Classical analytical methods, whether oriented towards *basse fondamentale* or towards *Auskomponierung*, follow the notational system in assuming the diatonic determinacy of events. Yet the relationship between the constituents of a symmetrical division and the diatonic *Stufen* is fundamentally indeterminate. A symmetrical division of the chromatic twelve cannot also be a symmetrical division of the diatonic seven without engaging in some enharmonic sleight-of-hand. In his 1984 article 'Amfortas's Prayer to Titurel and the Role of D in *Parsifal*', David Lewin emphasised the paradoxical and illusory aspects of such motions, which at once divide their space equally and unequally, inducing a mild type of vertigo.[3] The enharmonic shift can't be located: it occurs everywhere, and it occurs nowhere.[4]

In 'Amfortas's Prayer', the diatonic indeterminacy of Wagnerian harmony led Lewin to resurrect Riemann's theory of harmonic functions, whose 'great virtue and power ... is precisely its ability to avoid assigning letter names (i.e. implicit scale-degree functions) to its objects'.[5] Although Riemannian functions make no appeal to a governing diatonic collection, they nonetheless classify triads in relation to governing tonic triads, and thus have limited application to triadic music that 'suspends tonal gravity'. Furthermore, Riemann's commitment to just intonation, although not as tenacious as that of some of his contemporaries, was sufficient to prevent him from pursuing the implications of enharmonic equivalence,[6] further limiting the application of his function theory to chromatic music.

In publications subsequent to 'Amfortas's Prayer', Lewin has responded to these limitations by detaching Riemannian triadic theory from its tonally centred origins, and explicitly adapting Riemannian functions to the group-theoretic framework already implicit in Riemann's well-known table of tonal relations.[7] This neo-Riemannian adaptation has three significant components. The first is the removal of the habitually problematic dualist foundation, which Lewin tacitly eliminates by treating consonant triads as given *a priori* rather than acoustically generated, hence dispensing with the overtone series and its apocryphal dual in a single stroke.[8] A second component is the assumption of enharmonic equivalence and equal temperament, which allow group closure. The third component is the reconception of Riemann's functions as dynamic transformations that relate triads directly to each other, along the lines of algebraic functions whose arguments and values are triads.[9] This reconception eliminates the need to confer tonic status on one of the so-related triads, or to impose some third entity, a putative tonic which may not be perceived as abstractly present in the passage at hand.

The most detailed investigation of the group structure of neo-Riemannian transformations is presented in Brian Hyer's dissertation.[10] Much of Hyer's work focuses on the group structure of three neo-Riemannian transformations: P, which maps a triad to its parallel major or minor; R, which maps a triad to its relative major or minor; and L, which maps a triad to its *Leittonwechsel* ('leading-note exchange').[11] The three transformations share several properties: each is an involution; each is mode-altering (it maps a triad to its pitch-class inversion); and each transformation preserves two common tones, replacing the third with a pc a semitone away (in the case of P and L) or a whole tone away (in the case of R). This last property is particularly significant for the analysis of late Romantic music, where common-tone preservation and smooth voice-leading are characteristic stylistic features. These concerns will emerge as central to the approach developed in Parts II and III of this article, which focus on neo-Riemannian L and P relations from a slightly different perspective.

In assuming the *a priori* status of consonant triads, neo-Riemannian theory leaves unaddressed the reasons that late nineteenth-century composers continued to favour triads as harmonic objects. Indeed, the adoption of a group-theoretic approach to relations between triads suggests that the internal structure of the individual triads might also be viewed group-theoretically, as a complex of equally weighted pitch-classes and intervals.[12] This then raises the question of why late-Romantic composers would privilege triads as group-theoretic structures, since, by default, one is inclined to assume that triads are privileged in Romantic music, as in Classical music, solely for their acoustic properties. Why should triads have any status at all, except as optimal acoustic objects?

This article proposes a solution to this problem, by demonstrating that consonant triads have two sets of unique properties that are apparently independent of each other. One set may be characterised in terms of acoustics, and is the primary basis of the syntactic routines of diatonic tonality. These properties are the basis of prevailing analytical theories of eighteenth- and nineteenth-century music, and are sufficiently familiar and understood as to need no further attention here. The other set concerns the voice-leading potential of motion between triads, may be characterised in group-theoretic terms without any appeal to tonal centres, diatonic collections, harmonic roots and the like, and is the basis of many of the syntactic routines of chromatic music. Part II of this article characterises these group-theoretic properties, showing how consonant triads (together with their nine-note complements) uniquely fulfil them. The properties discussed in Part II suggest a formal layout of triadic relations, which is introduced in Parts III and IV. Paths through the formal layout are illustrated with some analyses of passages from music composed by Wagner, Mahler, Liszt and Franck between 1875 and 1895. Part V speculates on the scope and limitations of the analytical method.

Throughout the article, 'triad' refers restrictively to major and minor triads, not to augmented and diminished. Although the approach adopted here deliberately avoids appeals to acoustic phenomena, for the comfort of reader and writer alike this article adopts the convention of invoking acoustic roots for their mnemonic value. Accordingly, triads will named by their traditional (not Riemannian!) roots, with the suffixed symbols + and – designating major and minor triads respectively. Use of these labels should not be interpreted as implying generative status on the part of the named pitch-class. On the contrary, the component pcs should in all cases be considered as equally weighted.[13]

II Maximally smooth cycles

Ex. 2a, which includes an archetypal specimen of an equal division of the octave by major thirds, occurs just before the recapitulation of the first movement of Brahms's Concerto for Violin and 'Cello in A minor, Op. 102. The passage begins with a perfect cadence at bar 270 tonicising A♭ major (enharmonically, the leading note of A minor), and leads to the retransitional dominant at bar 278. Although bars 270–77 thus have a role to play in the larger A minor context of the movement as a whole, the internal structure of the passage does not follow the logic of Classical harmonic progression, for all of the reasons outlined in the discussion of Ex. 1. Nonetheless, the progression does have an exquisitely patterned logic, which may be demonstrated on group-theoretic grounds alone. Ex. 2b provides a reduction of bars 270–78, with voice-leading normalised. Filled note-heads indicate the appearance of a pc not present in the previous harmony. Three properties of this passage are salient to the present discussion:

Ex. 2 (a) Brahms, Concerto for Violin and 'Cello, first movement, bars 268–79

Ex. 2 (b) reduction of (a)

- The harmonies of the passage form a *cycle*, defined for present purposes as an ordered set of at least four elements whose initial and terminal elements are identical and whose other elements are distinct.[14] More precisely, bars 270–76 constitute a closed cycle, and bar 277 begins a second traversal which is aborted in the following bar.

- The passage exhibits *set-class consistency*: each chord is a consonant triad (i.e. a member of set-class 3–11).

- All transitions between adjacent chords are *maximally smooth*: only one voice moves, and that motion is by semitone. Stated differently: the symmetrical difference between the adjacent chords is always a member of dyad class [0, 1].

In isolation, these properties are not particularly remarkable. Indeed, journeys that return to their origins, circumscription of a harmonic lexicon, maximisation of common tones and incremental voice-leading are among the most characteristic features of our musical tradition. What is exceedingly rare is their combination into a single harmonic routine.

In order to show this formally, the above properties are consolidated into the formal construct of the *maximally smooth (MS) cycle*. Using the preliminary definitions

(a) SC(A) is the T_n/T_nI set-class of which pc set A is a member

(b) #A is the cardinality of set A

(c) $A \Delta B = (A \cup B) - (A \cap B)$, the symmetric difference of sets A and B

an *MS-cycle* is defined as an ordered set X of pc sets $<X_1, X_2, X_3 \ldots X_n>$ such that

(d) $\#X > 3$	min. cycle-length constraint
(e) $X_1 = X_n \neq X_2 \neq \ldots \neq X_{n-1}$	definition of a cycle
(f) $SC(X_j) = SC(X_k)$, for all $j, k \leq n$	set-class consistency

(g) $SC(X_j \Delta X_{j+1}) = SC([0, 1])$, for all $j < n$ maximal smoothness

The sets capable of participating in an MS-cycle are limited to the members of six set-classes, which fall into three pairs of complements, as shown in Table 1. One pair is trivial, both musically and mathematically: the single set-class of cardinality one and its complement of cardinality eleven. The remaining four set-classes constitute a society of exceptional pedigree. The consonant triad is joined by the pentatonic collection and the diatonic collection, the most prominent scalar types in our musical tradition (and in others as well). Only the set-class of cardinality nine lacks high breeding, but it is easily provable that the complement of a cycle-inducing set-class is itself cycle-inducing. If one wants to invite the consonant triad, one is obliged to invite her mate, whether or not he is sufficiently lean and nimble to be of any value.[15]

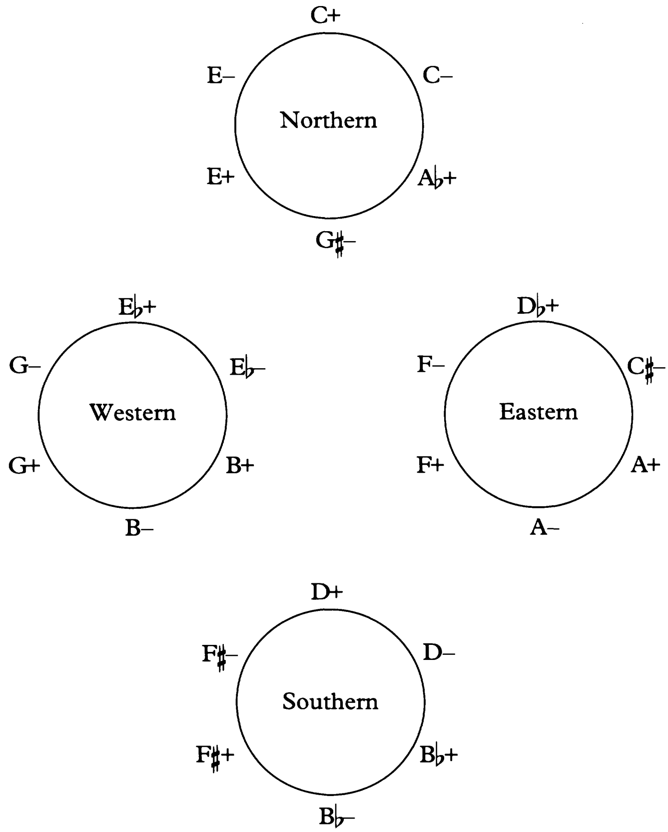
Table 1 Set-classes that can participate in a maximally smooth cycle

Forte name	prime form
1–1	[0
3–11 (the consonant triad)	[0, 3, 7]
5–35 (the usual pentatonic)	[0, 2, 4, 7, 9]
7–35 (the usual diatonic)	[0, 1, 3, 5, 6, 8, 10]
9–11	[0, 1, 2, 3, 5, 6, 7, 9, 10]
11–1	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

That the MS-cycle induced by the diatonic collection is none other than the chromatic 'circle of fifths' suggests not only how familiar MS-cycles are, but also how fundamental to our musical tradition: the maximal smoothness of a pair of T_7-related diatonic collections underlies the system of key signatures![16]

Although Table 1 shows that the triad keeps exclusive company, it does not yet fulfil the claim of uniqueness. This stronger claim is made good by exploring the periodicity of the MS-cycles induced by the set-classes in Table 1. The sets of cardinalities 1, 5, 7 and 11 each form a single MS-cycle, which includes all twelve members of the set-class. (In each case there are only twelve members instead of the usual 24, because the member sets map into themselves under inversion.) By contrast, an MS-cycle formed by consonant triads (e.g. Ex. 2b) traverses only six of the 24 members of the set-class. This cycle is one of four co-cycles which partition the 24 triads.[17] The ensemble of co-cycles is shown in Fig. 1, where they are furnished with labels matching their geographic placement on the page.

Fig. 1 The four hexatonic systems



To summarise, what is unique about set-class 3–11, together with its nine-note complement, is the capacity of its member sets to form an ordered set of maximally smooth successions that is long enough to be perceived as a cycle (i.e., longer than two distinct members, so that one can enter and depart through different portals), yet short enough that it does not exhaust all the members of its set-class. The following sections of this article demonstrate why this last property is of compositional and analytical significance: it ensures that, in the universe of triadic relations, the forces of unity (six triads) and diversity (four cycles) are appropriately balanced. Part III takes unity as its topic, by charting motions *within* the cycles. Part IV explores diversity, by investigating the treatment of cycles as harmonic regions, and the capacity of motions *between* the cycles to form coherent modulatory patterns.

III Hexatonic systems

Each of the four co-cycles of Fig. 1 constitutes a *hexatonic system*. The two halves of this term require separate explanation.

'Hexatonic' is a nickname for set-class 6–20, whose prime form is [0, 1, 4, 5, 8, 9].[18] One of the all-combinatorial hexachords, 6–20 maps into itself under three distinct transpositions and three distinct inversions. Consequently, there are only four distinct hexatonic pc sets, each of which supplies the fund of pcs from which the triads in a single hexatonic system are drawn. Table 2 lists the four hexatonic pc sets in both integer and pc notation. The final column indicates the set's affiliation (by pc inclusion) with a hexatonic system of triads. For example, the six triads in the Northern system draw only on [D♯, E, G, G♯, B, C] and their enharmonic equivalents.

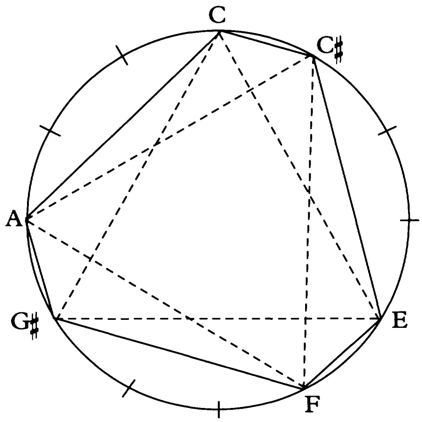
Table 2 The four hexatonic pitch-class sets

integers	pitch-classes	location in Fig. 1
[0, 1, 4, 5, 8, 9]	[C, C♯, E, F, G♯, A]	Eastern system
[1, 2, 5, 6, 9, 10]	[C♯, D, F, F♯, A, A♯]	Southern system
[2 ,3, 6, 7, 10, 11]	[D, D♯, F♯, G, A♯, B]	Western system
[3, 4, 7, 8, 11, 0]	[D♯, E, G, G♯, B, C]	Northern system

Fig. 2 represents the symmetry of 6–20.[19] One symptom of this symmetry is the strict alternation of semitones and minor thirds (represented by the continuous line around the periphery), which has led Ernö Lendvai to refer to 6–20 as the 1:3 collection, analogous with the 1:2 alternation of the octatonic collection.[20] Another symptom is the ability of the collection to be partitioned into two adjacent T4-cycles (represented by the pair of broken triangles), analogous with the partitioning of the octatonic into two adjacent T3-cycles. It is these analogies with the octatonic that makes the 'hexatonic' label appropriate.

'System' is used because each of the four co-cycles is a Generalized Interval System (GIS), as defined by David Lewin.[21] Although this need not be formally explored here, it does require some explanation, since what is meant by 'interval' may initially seem counter-intuitive to those unfamiliar with Lewin's work. When applied to harmonies, intervals conventionally characterise the relation between pitch-class roots (e.g. 'third-related harmonies'). As used in the present context, 'interval' refers to the distance between two triads within a hexatonic system.[22] In the Northern system of Fig. 1, the directed (clockwise) interval from C major to C minor is one, reflecting the adjacency of these triads on the circle; from C minor to E major

Fig. 2 Symmetry of 6–20



is three, reflecting their diametric relation; and so forth. Undirected hexatonic intervals are measured by *interval-classes* whose values reflect not only the proximity of the triads around the circle, but, more meaningfully, the number of pitch-classes displaced (by semitones) when a move between the two triads is executed.

- IC1-related triads are represented as adjacencies in the cycle, have a single pc displacement (hence two common tones), and are modally mismatched (i.e., one is major, the other minor).
- Triads related by IC2 are represented as next-adjacencies, have two pc displacements (hence one common tone), and are modally matched.
- IC3-related triads are represented diametrically, have three pc displacements (hence no common tones), and are modally mismatched.

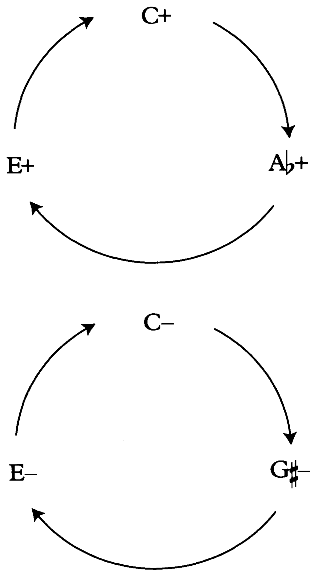
The relation between IC3-related triads in a hexatonic system is distinctive enough to warrant a special label: any two so-related triads serve as each other's *hexatonic poles*. Two triads in a hexatonic polar relation are pc-complementary with respect to their source hexatonic collection: they are maximally disjunct, but together they efficiently define the entire collection of six pcs from which they are drawn.[23]

Lewin has shown that any GIS converts into a transformational apparatus, with the value of a directed interval translating directly into the value of a transposition operation.[24] 'Transposition' here refers not to the standard mapping of pcs in chromatic space, but rather to the mapping of triads through a hexatonic system. Thus, C major won't map to C minor under transposition in pc space, but will do so *via* T_1 in hexatonic triadic space. Hexatonic transposition may be conceived as a set of clockwise clicks by a pointer at the centre of one of the cyclic representations in Fig. 1.

Transpositional operations act as generators in cyclic systems.[25] Ex. 2b provides a textbook example of T1-generation, with its incremental motion through the entire system of six triads. Leaving the initial A♭ major, the notes of the initial triad drop away one by one until the pole of E minor is reached, at which point they begin to return one by one in the order that they disappeared. T5 generates the same progression in retrograde.

T2, T3 and T4 partition hexatonic systems into co-cycles. The pair of T2-generated co-cycles, illustrated for the Northern system in Fig. 3, generate progressions of exclusively major and exclusively minor triads respectively. T4 generates retrograde versions of the same co-cycles. Such co-cycles are particularly abundant throughout the nineteenth century, probably because their modal consistency permits verbatim transposition of material from one station to the next without the adjustments often necessitated by the modal conversion of thematic and harmonic material.[26]

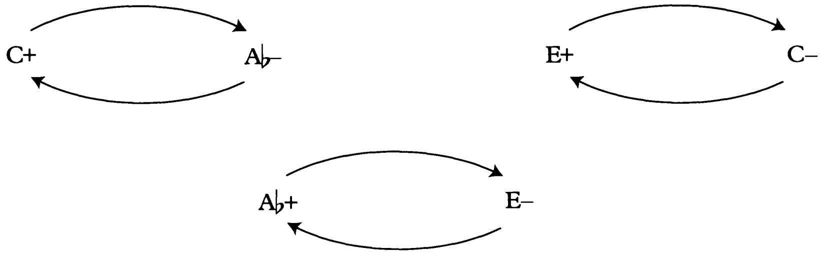
Fig. 3 T2-generated co-cycles



The trio of T3-generated co-cycles is illustrated in Fig. 4. Each such cycle consists of a direct motion from a triad to its hexatonic pole and back.

More than any other triadic pairing, the hexatonic polar relation resists interpretation in terms of diatonic tonality. Ernö Lendvai's *The Workshop of Bartók and Kodály* has drawn attention to the special power of the hexatonic polar relation to 'annihilate tonality'.[27] Lendvai stresses that this capacity

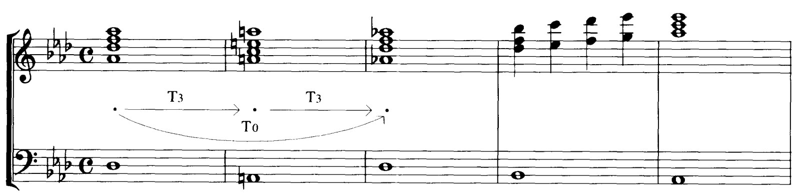
Fig. 4 T3-generated co-cycles



results from the high degree of symmetry in the set formed by the union of the two triads. Indeed, no other union of triadic pairs has a degree of symmetry higher than two. Recognition of two additional (but related) special features of the hexatonic polar relation will enhance our understanding of its paradoxical quality from the viewpoint of diatonic tonality. First, contrary motion is involved in the voice-leading between juxtaposed polar triads in closed position. (This is true of no other types of closed-position triadic juxtapositions where common tones are absent.) The contrary motion allows the second special feature of the polar relation to emerge: each triad contains the other's two most piquant tendency tones, the raised seventh and the flattened sixth degree (or some enharmonic version(s) thereof). Polar triads are a musical analogue of a Lewinian trope, the recursive gaze of Sieglinde and Siegmund.[28]

As a consequence of their diatonically paradoxical aspects, hexatonic polar progressions are frequently affiliated, by both composers and listeners, with an ethos of uncanniness. Ex. 3, the final chromatic event in *Parsifal* (1883; Schirmer edn, p. 276), is a poignant example: Wagner introduces the progression into an otherwise diatonic environment to depict Kundry's *Entseelung*, the removal of her soul from her body.

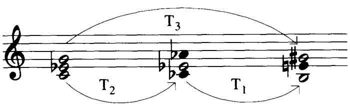
Ex. 3 Wagner, *Parsifal*, Act III, bars 1122–6 (Kundry's *Entseelung*)



Ex. 4a provides an example, from the first movement of Mahler's Symphony No. 2 (1894), of a mixed (i.e. non-cyclic) motion through a hexatonic system.[29] The passage connects the C minor tonic with its hexatonic pole, E major, the site of the *Gesang* theme. This T3 motion is composed of two smaller motions: T2 takes C minor to A♭ minor, which is taken to E major *via* T1. Ex. 4b sketches this motion as a network of transpositions. Since the GIS is commutative, the arithmetic works out in the ordinary way: 2 + 1 = 3 (mod 6). It is unclear whether Mahler, whose symphony takes Resurrection as its topic, had Kundry's *Entseelung* in his ear when he composed this hexatonically polar relation, but a recent critic, Carolyn Abbate, has written that the *Gesang* is 'marked by musical *blankness*, by a sense of substance leached away',[30] a characterisation that is hauntingly reminiscent of the events depicted by Wagner in Ex. 3.

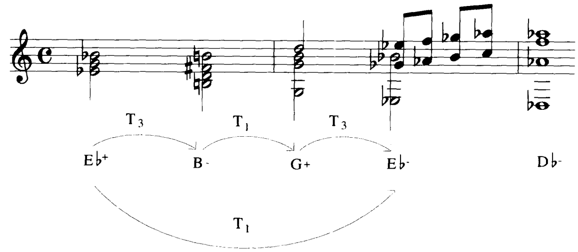
Ex. 4 (a) Mahler, Symphony No. 2 ('Resurrection'), first movement, bars 43–9

(b) transformational network of (a)



Ex. 5, from *Parsifal* (Schirmer edn, p. 271) provides a somewhat more elaborate example of mixed motion through a single hexatonic system. This passage, one of the enharmonic versions of the Grail motive discussed by Lewin in 'Amfortas's Prayer',[31] begins in E♭ major, moves to E♭ minor *via* three moves within the 'Western' hexatonic system, then exits that system at the arrival of D♭ major. The overall T_1 motion from E♭ major to E♭ minor is composed of three separate moves: (i) E♭ major moves to its hexatonic pole of B minor *via* T_3; (ii) B minor moves by T_1 to G major; (iii) G major moves to its hexatonic pole, E♭ minor, again *via* T_3. The composition of the transposition operators reflects addition modulo 6: i.e., 3 + 1 + 3 = 1. Although I offer this passage chiefly in service of the technical exposition, Ernst Kurth's reference to its 'übersinnliche Fremdartigkeit' somewhat reinforces the interpretative point as well.[32]

Ex. 5 Wagner, *Parsifal*, Act III, bars 1098–1102 ('sanfte Erleuchtung des Grales')

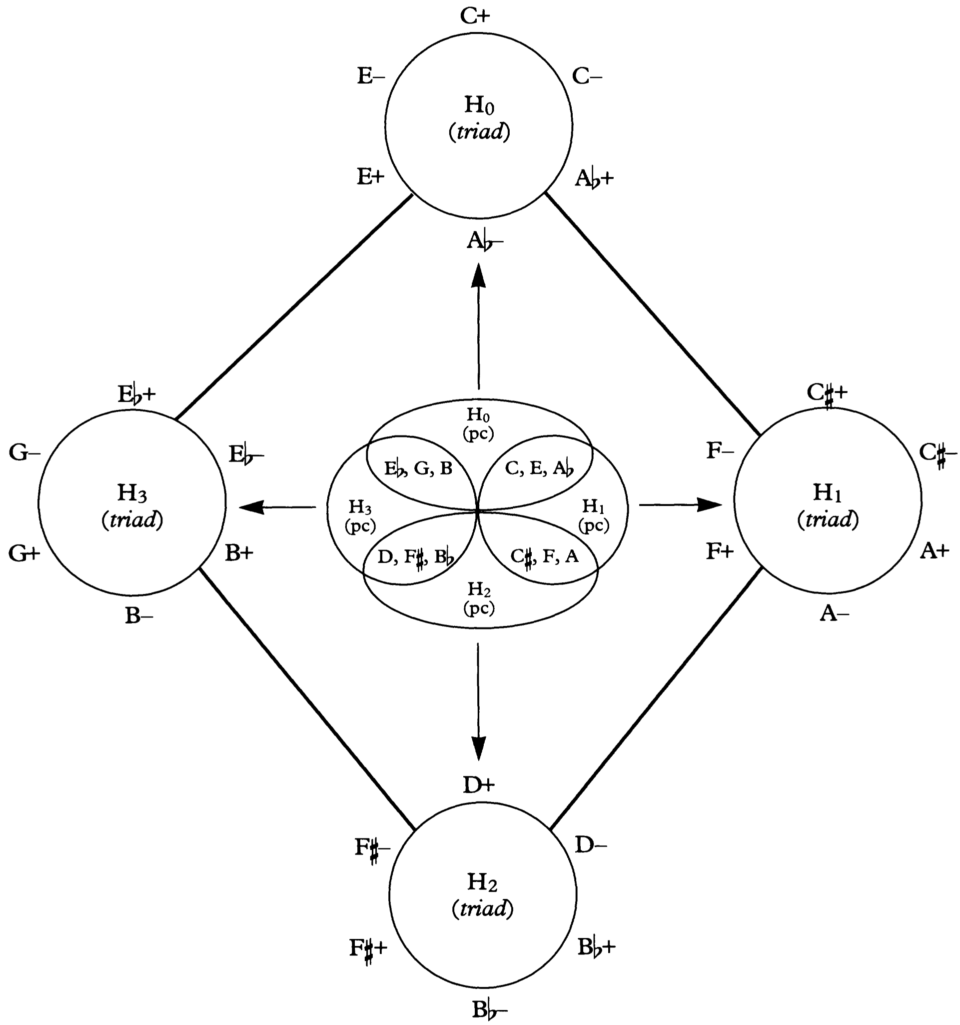


Exs. 3, 4 and 5 have provided a brief inventory of generated and mixed triadic motions within a single hexatonic system. Clearly, such systems are limited in their ability to map musical surfaces (although they often fare better with middlegrounds), since, as the motion to D♭ at the end of Ex. 5 suggests, surfaces rarely maintain allegiance to a single system for very long. Part IV broadens the domain of application by exploring the relationship between the four hexatonic systems.

IV The hyper-hexatonic system

The perimeter of Fig. 5 unifies the four hexatonic systems of Fig. 1 into a single hyper-hexatonic system.[33] The geographical labels are replaced with labels of the form H_n(triad).[34] As the graphical connections between the cycles suggest, each hexatonic system is directly connected to the two neighbouring systems, but is not directly connected to the system opposite. The basis for this cyclic arrangement is discovered at the centre of Fig. 5, where the twelve pitch-classes are partitioned into the four T_4-cycles (augmented triads). The intersecting ovals in which they are enclosed portray the four hexatonic collections of pitch-classes, labelled H0(pc) to H3(pc), each of which includes two T_4-cycles. The arrows from centre to periphery show the affiliations between hexatonic collections and hexatonic systems (cf. Table 2). Neighbouring hexatonic systems (those connected directly) share three pcs, while the pc content of opposite systems is complementary with respect to the twelve-pc aggregate.[35]

Fig. 5 The hyper-hexatonic system



The hyper-hexatonic system is also a rudimentary type of GIS, and can be mapped transformationally. T1 and T3 map a hexatonic system onto its clockwise and anti-clockwise neighbours respectively, and generate the entire hyper-system. T2 maps a system to its opposite (complementary) system, and thus, as a generator, partitions the systems into two co-cycles, pairing H0 with H2, and H1 with H3, in the manner of bridge partners.

Fig. 5 imposes a particular distance metric on the universe of triadic pairs. Although this metric is based somewhat abstractly in the number of common tones shared by the source hexatonic sets from which the paired triads are drawn, it is reinforced by the more intuitive metric of *total voice-leading distance*. Let us imagine two triads whose pcs are paired one-to-one, such that the voice-leading that connects them follows the 'law of the shortest way'[36] – which is to say that there is no other one-to-one pairing of their pcs such that the sum of the (undirected) intervals traversed by the three melodic voices is smaller. This sum, then, constitutes the total voice-leading distance of the triadic pair. Total voice-leading distance is inversely related to *efficiency*, a term which provides an equally intuitive but less cumbersome way of discussing these relations. For a pair of triads that share a hexatonic system, the total voice-leading distance ranges from 0 (for identical triads) to 3 (for polar relations). For a pair of triads in neighbouring systems, the total voice-leading distance ranges from 2 to 4. For a pair of triads in complementary systems, the total voice-leading distance ranges from 5 to 6. Thus, without exception, voice-leading between triads in neighbouring systems is more efficient than between triads in complementary systems. Furthermore, with only a small number of exceptions, voice-leading within a single system is at least as efficient as voice-leading between triads in neighbouring systems. The exceptions involve the hexatonic polar relation, whose efficiency of 3 exceeds that of several relations of efficiency 2 involving neighbouring hexatonic systems.[37]

The distinction between neighbouring and complementary systems is brought out by a comparison of Exs. 6 and 7, which illustrate two contrasting generative paths through the hyper-hexatonic system. Ex. 6, from the first movement of César Franck's Quintet for Piano and Strings in F minor (1879), takes a T3 route, moving anti-clockwise through a series of neighbouring systems. Ex. 7, from Liszt's piano transcription of Polonaise I from his unfinished oratorio *Die Legende der Heiligen Stanislaus* (1875), takes a T2 route, alternating between two complementary systems. Both passages feature a strict pattern of pitch-class transposition through chromatic space.[38] Furthermore, in both passages the transposed material consists of a major triad alternating with its (minor) hexatonic pole. Each alternation engages all six pcs of the hexatonic system, so that the systems engaged serially by each passage are defined with maximum efficiency.

In Ex. 6, the first two transpositions are centred around C♯ major and E major respectively. The final two transpositions, around G major and B♭ major, are durationally compressed. The T3 transpositions tour the four hexatonic systems. Each transposition shares half of its pcs with the previous iteration, and half with the subsequent iteration. The sequence concludes with the reappearance of the initial system, represented by an A major triad rather than the C♯ major triad that our tonal ears demand.

Ex. 6 Franck, Quintet for Piano and Strings, first movement, bars 90–106

Although the passage is dominated by the piano, the strings are twice summoned to mark a transition between hexatonic systems. The first transition connects C♯ major with E major through two tetrachords, the first of which is interpretable as ii$^{\varnothing 7}$ of C♯ minor, the second of which acts as a German sixth of E major, resolving directly to the new tonic in root position. Although these references to Classical syntax are surely a component of the transition's effectiveness, there are important group-theoretic connections as

Ex. 6 *(cont.)*

well. The tetrachordal pairing shares a remarkable number of features with the trichordal pairs that it connects:

- The paired sets in each instance are inversionally related: the major/minor triads pair inversionally related members of 3–11, while the half-diminished/dominant sevenths pair inversionally related members of 4–27.
- The paired tetrachords, like the paired triads, share no common tones.
- In both progressions the voices that connect the paired harmonies all move semitonally.
- Each pairing forms a symmetrical mode: the two triads form a complete hexatonic collection, abstractly combining semitonally adjacent T4-cycles, while the two tetrachords form a complete octatonic collection, abstractly combining semitonally adjacent T3-cycles.[39]

The strength of the analogy between the two progressions suggests that, even though tetrachord-classes are incapable of forming MS-cycles, they may nonetheless participate in progressions that are in some significant sense analogous to those triadic progressions that arise in hexatonic contexts.[40]

The second transition, from E major to G major, transposes the first transition at T3. Because of the T3-invariance of octatonic collections, the pc content of the two transitions is identical. The general acceleration of events at the end of the sequence confines each of the final two transitions to a single beat at the end of bars 99 and 101. These transitions are simplified accordingly to include only five pcs, which in both cases are subsets of the same octatonic collection heard in the transitions at bars 93 and 97. The significant presence of this octatonic collection is confirmed by its prolongation *via* dominant seventh chords built on B♭♭ and E♭ at bars 104–6, and suggests why the hyper-hexatonic cycle ends on an A major triad rather than achieving a more standard tonal closure at C♯: A major is the only major triad to stand at the intersection of the orienting hexatonic collection with the invariant octatonic collection.[41]

The passage from the Liszt Polonaise given as Ex. 7 shares with Ex. 6 not only the local pairing of hexatonic poles but also a similar middleground profile: a series of rising transpositions which, after two complete iterations, are intensified through durational compression. Whereas Franck is at pains to smooth the transitions between successive iterations of the hexatonic systems, Liszt's transitions are unmediated. The sense of disjunction is exacerbated by the pitch-class complementarity of the alternated hexatonic systems, H0 and H2.

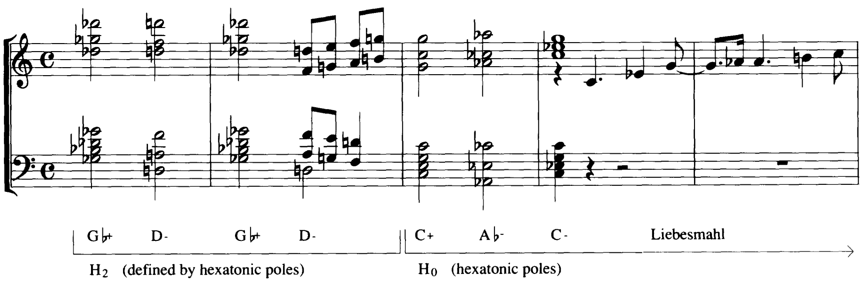
Complementation is an important thematic aspect of Ex. 7, occurring in three different ways. The complementary relationship between the two hexachords, with respect to the pitch-class aggregate, nests a complementary

Ex. 7 Liszt, Polonaise I from *Die Legende der heiligen Stanislaus*, bars 98–110

relationship between the pairs of hexatonic-polar triads with respect to each articulated hexatonic system.[42] Finally, there is a complementary relationship between the six triads in the 'home' (H_0) system: each is sounded exactly once, and the sounding of the last triad, G♯ minor, terminates the sequence. Liszt now exploits the membership of G♯ minor in an E major diatonic system by moving to a cadence in the time-honoured manner. The effectiveness of the passage is largely attributable to the way that Liszt 'modulates' from H_0 hexatonic to E-major diatonic space, bookending the sequence with the only two triads that participate in both spaces.[43]

Ex. 7 realises the T_2 hexatonic relations as T_2 relations in pc space. In contrast, Ex. 8, from Act II of *Parsifal* (Schirmer edn, p. 187), illustrates the other combinatorial possibility that 6–20 makes available: realisation of a T_2 hexatonic relation through a T_6 pitch-class relation. At the beginning of Ex. 8, Parsifal begins to sing 'Es starrt der Blick dumpf auf das Heilgefäss: Das heil'ge Blut erglüht: Erlösung'swonne, göttlich mild, durchzittert weithin alle Seelen' ('The gaze is fixated on the cup of healing; the holy blood glows; the rapture of redemption, holy and mild, reverberates through all souls'). The passage is dramatically related to Ex. 5: this is the moment in the opera when Parsifal first envisages the act of redemption that he will perform at the end of Act III. Wagner binds this dramatic correspondence by means of the musical image of the polar relation between G♭ major and D minor triads. D minor, acting as a supertonic, moves diatonically to C major, and in so doing shifts into the complementary hexatonic system H_0. Wagner now repeats the progression at T_6, juxtaposing C major and A♭ minor to complete the pitch-class aggregate. Rather than continuing the transposition, thereby returning simultaneously to G♭ major and to H_2, Wagner sounds the hexatonic 'Liebesmahl' theme, thereby confirming the H_0 hexatonic region as a significant presence.[44]

Ex. 8 Wagner, *Parsifal*, Act II, bars 1049–54 ('Es starrt der Blick dumpf auf das Heilgefäss: Das Heil'ge Blut erglüht: ...')



V The scope and limitations of hexatonic systems

The passages discussed in this article are particularly susceptible to a hexatonic approach, for both positive and negative reasons. The hexatonic poles featured locally in most of the examples stubbornly resist interpretation in terms of standard diatonically based models. Perhaps more than any other type of triadic pairing, the juxtaposition of hexatonic poles liberates listeners from the cognitive noise of their over-learned habits. But the model of triadic relations given in Fig. 5 is effective for these passages not by its mere opportunistic presence. In Exs. 6, 7 and 8, the surface articulation of hexatonic poles allows the hexatonic systems to be defined with unusual clarity and efficiency. The hexatonic pc sets achieve identities, so that their returns are recognised even when represented by different triads. The coherent identities of the sets in turn encourage a focus on the coherence of the motions through the hexatonic systems.

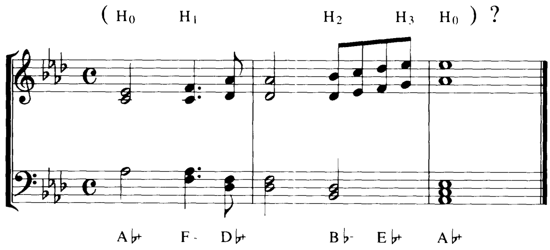
Assuming that the hyper-hexatonic model is granted a position at the music-analytic table, by virtue of the insights it furnishes for Exs. 6–8, what is its domain of application? Potentially, it is quite vast: the hyper-hexatonic model is descriptively adequate for any triadic progression (including the purely diatonic) at any level of structure (e.g. middleground tonicisations). Under what circumstances will we wish to gaze at a composition through a hexatonic lens?[45]

One way to constrain the domain of hexatonic analysis is chronologically. The title of this article, the inflection of its presentation, and the chronological distribution of its examples, all suggest that composers began to 'compose hexatonically' around 1875. But blunt chronological barriers have not served musical understanding well in the past. A more cautious and nuanced formulation emerges if we remember that hexatonic systems arise from an aggregation of component principles – consistent use of consonant triads, incremental voice-leading, common-tone preservation – that are, individually, pan-stylistically fundamental to European music through much of its history. In Part II, I demonstrated that these components can only be combined in one way: as a maximally smooth cycle that traverses a hexatonic system. The dominant position of the diatonic collection prevents these components from mingling in most musical sub-traditions. But the absolute integrity of diatonic collections has been under constant pressure from a variety of forces throughout musical history, although the force of that pressure has fluctuated. When force is sufficient to cause the diatonic barriers that segregate these components to overflow, hexatonic triadic progressions begin to emerge. Juxtapositions of hexatonic poles occur in Solage's late fourteenth-century chanson 'Fumeux fume', and in a striking manner at the opening of Gesualdo's 'Moro lasso', a madrigal whose *topos* is not unrelated to those discussed in Exs. 3 and 4.[46] Bach's A minor Prelude from Book Two of the *Well-Tempered Clavier* juxtaposes complementary hexatonic pole progressions so as to exhaust the aggregate, in the manner of Ex. 7.[47] Stepwise motion through a single hexatonic system occurs in the finales of Mozart's Symphony K. 543 (1788) and Haydn's Symphony No. 98 (1792),[48] in the second movement of Beethoven's Sonata for Violin and Piano Op. 24 (1801) and in the coda of the first movement of Schubert's Piano Trio in E♭ major, Op. 100 (1828).[49] In these latter passages, the triads in the hexatonic system are lightly tonicised using the standard resources of diatonic tonality, so that the systems are present not 'on the surface', but at a thinly veiled level of the middleground. Together, all of these passages suggest that, whereas a greater focus on surface hexatonic relations may have emerged in the last quarter of the nineteenth century, the combination of forces and tendencies that gave rise to that focus also had an impact on earlier repertoires.

A related way to constrain the domain of hexatonic analysis would be to acknowledge that hexatonic elements might infiltrate compositions that otherwise operate according to the principles of diatonic tonality, but to limit the application to elements of those compositions that fail the standard test of diatonic coherence. Although such a limitation is adopted in the present article as an expository tactic, I find it unnecessarily restrictive as an ultimate research strategy, since it seems based on the assumption that coherent music is coherent in exactly one way. This assumption leads to a view of music-theoretic interpretation as a zero-sum game, a view that has been vigorously challenged in Lewin's methodological writings.[50] Such a view seems particularly inappropriate in the case before us, since the principle of maximal smoothness that gives rise to the hexatonic formulation of chromatic space is also fundamental to diatonic voice-leading. Because full development of this topic will take us too far afield, I note only that both generic triads and generic seventh-chords have the capacity to enter into maximally smooth cycles in mod-7 diatonic space, and that the MS-cycle induced by the generic triad, which usually takes the form of a root progression by descending diatonic thirds, is particularly common in Classical music.[51]

More concretely, there may be situations where it is fruitful to contemplate even purely diatonic progressions from a hexatonic perspective. As an illustration, consider Ex. 9, the diatonic version of the Grail theme from *Parsifal*. Although the passage is completely triadic, and thus falls within the descriptive reach of the hyper-hexatonic model, its limitation to a single diatonic collection would seem to place it well outside the model's explanatory domain. Running Ex. 9 through the hyper-hexatonic mill returns the information that the passage prolongs H_0 by means of a T_1-generated cycle through the four hyper-hexatonic systems, with H_1 articulated by the internal hexatonic T_1 move <F-, D♭+>, and all other systems articulated by a single triad.[52] At first glance, such information has no more promise than, say, the information provided by a pc-set inventory of a Vivaldi concerto. And yet, something of value may be saved for it if we consider that the last half-bar of this music is the agent of the T_2 hyper-hexatonic motion of Ex. 8. The diatonic Grail is not hexatonic music, but (to borrow again from Lewin's methodological writings) Ex. 8 and related passages furnish a 'sufficiently developed and extended perceptual context' that might fulfil the potential of a hexatonic interpretation to be meaningful.[53] At the same time, whether spectacularly fruitful or worthless, the value of a hyper-hexatonic reading of the diatonic Grail has no impact one way or the other on the value of the claim that the same passage is in A♭ major, has a root progression of descending thirds, etc. – claims which few theorists would be willing to surrender in any case.

Ex. 9 Wagner, *Parsifal*, Act I, bars 39–41



If Ex. 9 suggests that a hexatonic model has potential for even the most diatonic music, Exs. 6, 7 and 8 suggest the converse. Although these are among the most concentratedly hexatonic progressions of triads that I have located in the literature, tonal/diatonic vestiges are still present in them. We have just acknowledged how Wagner uses a traditional cadential figure to modulate between complementary hexatonic systems in Ex. 8. In Ex. 6, Franck connects neighbouring hexatonic systems to a mediating octatonic collection by means of simple mixture, a basic operation of diatonically contextualised chromaticism. In Ex. 7, Liszt embeds a prolonged hexatonic system into an E major diatonic framework, pivoting into and out of the diatonic system through the pair of triads that it shares with the hexatonic system.

These interpenetrations of diatonic and hexatonic principles suggest that the hexatonic model is likely to achieve the broadest scope and deepest insight into nineteenth-century music if used not in isolation from standard diatonic models, but rather in conjunction with them. Relations well modelled by acoustic theory – tonal centricity, diatonic determinacy, *Auskomponierung*, and all that they entail – latently coexist with relations well modelled by group-theoretic models of chromatic space, such as the transposition operation, smooth voice-leading, and the hexatonic constituency of triads. In much of the most interesting music of the nineteenth century, these two sets of relations, when not in actual equilibrium on the musical surface, are in an equilibrium of potential. Ernst Kurth wrote that 'Everywhere, Romanticism exploits the ability to hear one and the same phenomenon in two or more ways; it is fond of this coexistence and its indefiniteness.'[54] I am most comfortable with this formulation when I understand Kurth to mean not that all phenomena are essentially *mehrdeutig*, but rather that their potential to be so is there for the exploiting. Although extended triadic progressions may simultaneously be interpretable in diatonic and hexatonic (or more generally, chromatic) spaces, more frequently one or the other of these spaces will 'control' or even monopolise a given span or level of a composition.[55] In such a context, 'phenomena hearable in two or more ways' function as pivots through which the music modulates between its conceptual spaces.

The topic of pivots between diatonic and chromatic space returns us to the conceptual framework advanced in 'Amfortas's Prayer to Titurel'. For Lewin, the role of these pivots throughout *Parsifal* ('flaws, splices, and hidden seams', p. 347) is assumed by enharmonically indeterminate pitch-classes. Yet Lewin also calls attention to the over-determined potential of triads to exist coherently in multiple contexts:

> The musical objects and relations that Riemann isolates and discusses are *not* simply the old objects and relations dressed up in new packages with new labels; they are essentially different objects and relations, embedded in an essentially different geometry. This is so even if in some contexts the two spaces may coexist locally without apparent conflict. (p. 345)

The work presented in this article emphasises the role of the triad as the seam or pivot that mediates hexatonic and diatonic space. What allows the triad to fulfil this role is its dual capacity to serve simultaneously as (i) an acoustically optimal structure in the ways that the music-theoretic tradition has led us to conceive it, and (ii) a group-theoretically optimal structure from the point of view of its unique capacity to participate in maximally smooth cycles that, because they do not exhaust the set-class, allow a balance to be struck between the intra-hexatonic unity and inter-hexatonic variety that are mapped by the hyper-hexatonic system. By keeping in mind the distinction between its two natures, at the same time as we acknowledge the capacity for interaction between them, our recognition of the triad's duality will help us approach the clear definition that Carl Dahlhaus, in the passage given at the head of this article, suggested as a standard for descriptions of Romantic harmony.

NOTES

1. Stanley Sadie (ed.), *The New Grove Dictionary of Music and Musicians* (London: Macmillan, 1980), s.v. 'Wagner, Richard' (Vol. 20, p. 123).

2. Felix Salzer and Carl Schachter, *Counterpoint in Composition* (New York: McGraw-Hill, 1969), p. 215. For expressions of similar judgement by earlier theorists, see Rudolf Louis & Ludwig Thuille, *Harmonielehre*, 7th edn (Stuttgart: Carl Grüninger Verlag, 1920), p. 345; Ernst Kurth, *Romantische Harmonik und*

ihre Krise in Wagners 'Tristan', (Bern: Paul Haupt, 1920), p. 248; Donald Francis Tovey, 'Tonality in Schubert', in Tovey, *The Mainstream of Music and Other Essays* (London: Oxford University Press, 1949), pp. 154–5 (originally published as 'Tonality', *Music & Letters*, 9 (1928), pp. 341–63); Adele Katz, *Challenge to Musical Tradition* (New York: Alfred A. Knopf, 1945), p. 213.

3. David Lewin, 'Amfortas's Prayer to Titurel and the Role of D in *Parsifal*: The Tonal Spaces of the Drama and the Enharmonic C♭/B', *19th-Century Music*, 7/iii (1984), pp. 345–9 and *passim*. Lewin affiliates these paradoxes with the magical qualities of the opera, and by implication with the entire nineteenth-century tradition of associating equal divisions with magic, a tradition that goes back to Schubert's *Die Zauberharfe* and Weber's *Der Freischütz*, and forward to Stravinsky's *Scherzo Fantastique* and beyond. For more on this tradition, see Richard Taruskin, 'Chernomor to Kashchei: Harmonic Sorcery; or, Stravinsky's "Angle"', *Journal of the American Musicological Society*, 38 (1985), pp. 72–142. For evidence suggesting that the affiliation of enharmonicism with altered states of consciousness may have deeper historical roots, see Edward Lowinsky, 'Adrian Willaert's Chromatic "Duo" Re-examined', *Tijdschrift voor Muziekwetenschap*, 17 (1956), pp. 1–36, reprinted in Lowinsky, *Music in the Culture of the Renaissance and Other Essays*, ed. Bonnie J. Blackburn (Chicago: University of Chicago Press, 1989), pp. 681–98.

4. For a penetrating discussion of the problems encountered in assigning diatonically based interpretations to enharmonically ambiguous events, see Gregory Proctor, 'Technical Bases of Nineteenth-Century Chromatic Tonality' (PhD diss., Princeton University, 1978), esp. pp. 140–43, 177–8.

5. Lewin, 'Amfortas's Prayer', p. 344.

6. Brian Hyer, 'Reimag(in)ing Riemann', *Journal of Music Theory*, 39/i (1995), pp. 101–38, discusses Riemann's views on intonation, and their implications for enharmonic problems. See also Robert W. Wason and Elizabeth West Marvin, 'Riemann's "Ideen zu einer Lehre von den Tonvorstellungen": An Annotated Translation', *Journal of Music Theory*, 36/i (1992), pp. 69–117. For a general discussion of the impact of just intonation on nineteenth-century harmonic theory, see Wason, 'Progressive Harmonic Theory in the Mid-Nineteenth Century', *Journal of Musicological Research*, 8 (1988), pp. 55–90.

7. Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), pp. 175–80; 'Some Notes on Analyzing Wagner', *19th-Century Music*, 16/i (1992), pp. 49–58. Riemann's table took several forms. The one that is reproduced in Wason and Marvin, 'Riemann's "Ideen"', p. 102 was first introduced in Ottokar Hostinsky, *Die Lehre von den musikalischen Klangen: Ein Beitrag zur aesthetischen Begrundung der Harmonielehre* (Prague: H. Dominicus, 1879), p. 67. On the development of the table, see Martin Vogel, *On the Sensations of Tone*, tr. Vincent Jean Kisselbach (Bonn, 1993), pp. 108–10. On its potential for interpretation as a group structure, see Hyer, 'Tonal Intuitions in *Tristan und Isolde*' (PhD diss., Yale University, 1989), pp. 214–15 and *passim*; and 'Reimag(in)ing Riemann'.

8. For an overtly neo-Riemannian consideration of the dispensability of dualism, see Hyer, 'Reimag(in)ing Riemann'. For a recent defence of Riemann's dualism, see Daniel Harrison, *Harmonic Function in Chromatic Music* (Chicago: University of Chicago Press, 1994), Chap. 1.

9. Lewin notes that a transformational perspective is implicit in Riemann's conception of functions (*Generalized Musical Intervals*, p. 177). In a review of *Generalized Musical Intervals* (*Intégral*, 2 (1988), pp. 161–77) Bo Alphonce has suggested that such a perspective is already present in much post-Riemannian work in Northern Europe, and perhaps even explicit (though not fully formalised) in Riemann's own writings. David Kopp has called attention to a passage in Riemann's writings where he explicitly 'argues for an essential identity of chord progression types existing independently of the character which they take on in the context of a key'. See Kopp, 'The Function of Function', *Music Theory Online*, 1/iii (1995), paragraph 9.

10. Hyer, 'Tonal Intuitions', pp. 175–226.

11. These transformations are introduced in Lewin, *Generalized Musical Intervals*, p. 178, using the labels PAR, REL, and LT respectively. The single-letter labels are from Hyer, 'Tonal Intuitions'.

12. This conception of triadic structure resembles the early twentieth-century move towards a decentred view of the diatonic collection, captured under the rubric of 'pandiatonicism'.

13. Some readers may be struck, as I have been, by the kinship this caveat bears with similar formulations by scholars of 'pre-tonal' music. See, for example, Don Randel, 'Emerging Triadic Tonality in the Fifteenth Century', *Musical Quarterly*, 57 (1971), pp. 73–86.

14. The four-member minimum excludes sequences of the form <A, B, A>, which fit the technical requirements for a cycle but violate an intuition associated with circularity: that the path home ought to traverse different territory than the path of departure.

15. There is much more to explore about the structure of this list. For example, why does it contain one representative of each odd cardinality, but no representatives of even cardinality? I have investigated abstract questions of this type with a team of researchers at the State University of New York at Buffalo, including David Clampitt, John Clough, Jack Douthett and David Lewin. The findings of the Buffalo group are as yet unpublished.

16. This re-frames an observation of Gerald Balzano, 'The Group Theoretic Description of 12-Fold and Microtonal Pitch Systems', *Computer Music Journal*, 4/iv (Winter 1980), pp. 66–84.

17. The Buffalo group has found generally that, for any chromatic universe of size **c**, cycles that exhaust the set-class are induced only by set-classes with cardinality co-prime to **c**; co-cycles are induced only by set-classes whose cardinalities share a divisor with **c**.

18. The name was suggested to me by Easley Blackwood. Set-class 6–20 has borne many other rubrics in the music theoretic literature, including 'Wunderreihe' (sometimes translated as 'Miracle hexachord'), 'Ode to Napoleon' (due to its prominence in Schoenberg's piece of that name), 'source-set E', 'augmented scale' and 'Liszt mode'. See Donald J. Martino, 'The Source Set and its Aggregate Formations', *Journal of Music Theory*, 5 (1961), pp. 224–73; Lewin, 'Intervallic Content of a Collection of Notes and its Complement: an Application to Schoenberg's Hexachordal Pieces', *Journal of Music Theory*, 4 (1960), pp. 98–101; Jeff Pressing, 'Pitch Class Set Structures in Contemporary Jazz', *Jazzforschung*, 14 (1982), pp. 133–72; Henri Pousseur, 'Stravinsky by Way of Webern: The Consistency of a Syntax', *Perspectives of New Music*, 10/ii (1972), p. 33.

19. Although such representations are associated with Ernst Krenek's name, they originate in nineteenth-century writings. See Wason, 'Progressive Harmonic Theory'.

20. Ernö Lendvai, *Béla Bartók: An Analysis of his Music* (London: Kahn & Averill, 1971), pp. 51–4, and *The Workshop of Bartók and Kodály* (Budapest: Editio Musica, 1983), pp. 370–81. For further discussion of the properties of 6–20, see Wason, 'Tonality and Atonality in Frederic Rzewski's Variations on "The People United Will Never be Defeated"', *Perspectives of New Music*, 26 (1988), pp. 108–43; John Schuster-Craig, 'An Eighth Mode of Limited Transposition', *The Music Review*, 51 (1990), pp. 296–306; Richard Cohn, 'Bartók's Octatonic Strategies: A Motivic Approach', *Journal of the American Musicological Society*, 44 (1991), pp. 262–300.

21. Lewin, *Generalized Musical Intervals*, p. 26 and *passim*.

22. Lewin discusses this sense of intervals between triads (*ibid.*, pp. 179–80), although his GIS is different from the one proposed here.

23. The tri-partition of third-relations, on the basis of common-tone retention and modal correlation, is suggested in Louis & Thuille, *Harmonielehre*, 7th edn, p. 347. Richard Isidore Schwartz's dissertation, 'An Annotated English Translation of "Harmonielehre" of Rudolf Louis and Ludwig Thuille' (Washington University, 1982), p. 413, indicates that the passage was included in the 4th edn of 1913. He does not indicate whether it was passed on from earlier editions, which are unavailable to me for examination. For a similar taxonomy, see Hans Tischler, 'Chromatic Mediants: A Facet of Musical Romanticism', *Journal of Music Theory*, 2 (1958), p. 95.

    In an article that focuses on polar relations between large-scale tonics in sonata form movements, Rey M. Longyear and Kate R. Covington suggest measuring 'degrees of remoteness among third relationships' on the basis of 'the presence or absence of common tones' ('Liszt, Mahler and a Remote Tonal Relationship in Sonata Form', in Anke Bingmann, Klaus Hortschansky and Winfried Kirsch (eds.), *Studien zur Instrumentalmusik: Lothar Hoffman-Erbrecht zum 60. Geburtstag* (Tutzing: Hans Schneider, 1988), pp. 457–69).

24. Lewin, *Generalized Musical Intervals*, pp. 157ff.

25. This is the sense, for example, in which transposition by seven semitones generates the chromatic universe *via* the 'cycle of fifths'. For an introduction to generators in music theory, see Balzano, 'The Group Theoretic Description of 12-Fold and Microtonal Pitch Systems'.

26. Salzer and Schachter observe (*Counterpoint in Composition*, p. 215) that the major cycle is more common than the minor. The explanation for this remains an open question.

27. Lendvai, *Workshop*, pp. 199, 235–8 and 377–81. Lendvai's term for this triadic pairing is 'complementary', i.e. with respect to the '1:3 model'. Lendvai uses illustrations from music by Beethoven, Wagner, Verdi, Bartók and Kodály.

28. See Lewin, 'Music Theory, Phenomenology, and Modes of Perception', *Music Perception*, 3 (1986), pp. 327–92.

29. The passage is discussed in somewhat similar terms in Longyear & Covington, 'Liszt, Mahler', p. 466, and in Leonard Ratner, *Romantic Music: Sound and Syntax* (New York: Schirmer, 1992), p. 289.

30. Carolyn Abbate, *Unsung Voices* (Princeton: Princeton University Press, 1991), p. 152.

31. Lewin, 'Amfortas's Prayer', pp. 345–6.

32. Kurth, *Romantische Harmonie*, p. 246. Translated as 'supernatural strangeness' in Lee Rothfarb (ed.), *Ernst Kurth: Selected Writings* (Cambridge: Cambridge University Press, 1991), p. 124. Further support for the interpretative claim concerning the 'ethical' qualities of hexatonic poles will be provided in 'Uncanny Resemblances', a chapter of a book (as yet untitled) on which I am working.

33. The term 'Hyper-hexatonic' was suggested to me by Robert Cook.

34. The assignment of H_0 to the Northern system is based on the conventional primacy of C. Using a different convention, H_0 could have been assigned to the Eastern system, whose source pc-set is the prime form of the set-class.

35. The relations between members of 6–20 are discussed in detail in Wason, 'Tonality and Atonality', p. 122.

36. The most familiar citation of this 'law' is found in Arnold Schoenberg, *Theory of Harmony*, trans. Roy E. Carter (Berkeley and Los Angeles: University of California Press, 1978), p. 39: 'Each voice will move only when it must; each voice will take the smallest possible step or leap, and then, moreover, just that smallest step which will allow the other voices also to take small steps'.

37. The material in this paragraph was stimulated and clarified by voluminous unpublished work by Jack Douthett, whose formalisation of such matters is far more powerful than I have been able to indicate here.

38. See Proctor, 'Technical Bases of Nineteenth-Century Chromaticism', p. 159.

39. Lendvai (*Workshop*, p. 510) gives other examples of this partitioning of the octatonic collection, from Wagner and Musorgsky. See also Rothfarb (ed.), *Ernst*

Kurth: Selected Writings, p. 113.

40. It is intriguing to speculate that 4–27, the most prominent tetrachordal type in both Classical and Romantic repertoires, may share fundamental properties with 3–11 beyond their obvious inclusion relation, properties that permit 4–27 to partake in smooth (if not maximally smooth) voice-leading. I intend to develop this speculation elsewhere; for now, I merely note that both 4–27 and 3–11 (along with 6–34, the 'Mystic chord') are minimal perturbations of a symmetrical division of the octave.

41. A minor shares the intersection. Generally, the intersection of any octatonic with any hexatonic collection is a member of set-class 4–17 [0, 3, 4, 7]. Hence each such intersection is uniquely represented by a major and minor triad sharing a pitch-class root. This is related to the fact that each pc uniquely intersects a T_3-cycle with a T_4-cycle, a circumstance that was known to nineteenth-century theorists. See Wason, 'Progressive Harmonic Theory', pp. 78–9. Similarly, each triad in a hexatonic system uniquely intersects a T_2- with a T_3-generated co-cycle (see Figs. 3 and 4).

42. This passage lends itself nicely to a proto-serial interpretation: each hexachord both begins an aggregate and ends a previous aggregate. In terms of 'classical' serial procedures, the passage can be characterised as a sequence of hexachordally overlapping twelve-tone rows, each in a T_2 relation with its predecessor. I do not mean by this to suggest that Liszt was thinking about pc relations in the same way as Schoenberg was in the 1940s, but rather that Schoenberg would have had some interest in the way that Liszt was thinking about pc relations in the 1870s. For some music by a contemporary of Schoenberg that follows a plan very similar to Ex. 7, see the refrain of Hugo Distler's 'Fürwahr, er trug unsere Krankheit', his Motet No. 9 for mixed four-voice chorus (1934–6).

43. Liszt's strategies for inter-relating diatonic and chromatic modes of organisation are an important concern of Ramon Satyendra, in 'Chromatic Tonality and Semitonal Relationships in Liszt's Late Style' (PhD diss., University of Chicago, 1992). Satyendra's discussion of the Polonaise (pp. 80–81) serves as a point of departure for my treatment of Ex. 7.

44. The hexatonic character of the C minor version of the 'Liebesmahl' theme has been discussed by Lendvai (*Workshop*, p. 377) and Lewin ('Some Notes', p. 57).

45. The need to constrain the application of super-potent descriptive systems has been a theme of much meta-theoretic writing, including William Benjamin, 'Ideas of Order in Motivic Music', *Music Theory Spectrum*, 1 (1979), pp. 23–4; Richard Taruskin, 'Reply to van den Toorn', *In Theory Only*, 10/iii (1987), pp. 47–57; and Patrick McCreless's review of Warren Darcy, *Wagner's 'Das Rheingold'*, *19th-Century Music*, 18 (1995), pp. 289–90.

46. 'Fumeux fume' is transcribed in Peter M. Lefferts, 'Subtilitas in the Tonal Language of "Fumeux fume"', *Early Music*, 16 (1988), pp. 176–83. See bars 27–9 and 32–3 (other published transcriptions agree with Lefferts for the passages in question). For a discussion of the problems associated with analysis of chromatic third relations in Gesualdo's music, with special attention to hexatonic

poles, see Carl Dahlhaus, 'Zur chromatischen Technik Carlo Gesualdos', *Analecta Musicologica*, 4 (Köln: Bühlau Verlag, 1967), pp. 77–96.

47. Dahlhaus calls attention to this passage in 'Zur chromatischen Technik Carlo Gesualdos', p. 87.

48. These passages are discussed respectively in Mario LeBlanc, 'Franz Schubert: un pas vers l'atonalité', *Canadian University Music Review*, 9/ii (1989), pp. 84–115 and Paula Jean Telesco, 'Enharmonicism in Theory and Practice in the Eighteenth Century' (PhD diss., Ohio State University, 1993), p. 157.

49. I have discussed this passage in 'As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert', unpublished paper presented at a Conference on Schubert's Piano Music, Washington DC, 1995.

50. See especially 'Music Theory, Phenomenology, and Modes of Perception', pp. 357ff.

51. Generic triads and sevenths are discussed in John Clough and Gerald Myerson, 'Variety and Multiplicity in Diatonic Systems', *Journal of Music Theory*, 29 (1985), pp. 249–70. Insight into their maximally smooth potential may be gained from Eytan Agmon, 'Linear Transformations Between Cyclically Generated Chords', *Musikometrica*, 3 (1991), pp. 15–40.

52. Robert Cook discusses the hexatonic potential of the diatonic Grail in 'Alternative Transformational Aspects of the "Grail" in Wagner's *Parsifal*', unpublished paper presented at the annual meeting of Music Theory Midwest, 1994.

53. Lewin, *Generalized Musical Intervals*, p. 87.

54. Rothfarb (ed.), *Ernst Kurth: Selected Writings*, p. 134.

55. This theme is developed in different ways in the dissertations of Hyer ('Tonal Intuitions') and Satyendra ('Chromatic Tonality').