HANDWRITTEN NOTES

OF

(THEORY OF MACHINE)

BY

ENGGBUZZ.COM

Mechanical Engineering

⇓

Engineering of Mechanics

↓

Study of Motion

Kinematic

Study of Motion without the Consideration of Basic Cause of Motion i.e force

$$\vec{v} = \frac{d\vec{s}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}, \quad \vec{j} = \frac{d\vec{a}}{dt}$$

Dynamics (mass)

Study of Motion with the Consideration of the basic Cause of motion i.e Force.

Simple Mechanisms

Kinematic Link or Element :-

"Every Part of a machine, which is having Some Relative motion with respect to some other part will be Known as, Kinematic Link or element". e.g.:- Man standing on floor

It is necessary for the link, to be the resistant body so that it is capable of transmitting power and motion, from one element to the other element.

Types of Link :

1) Rigid Link :

Deformations are Negligible.

for e.g:

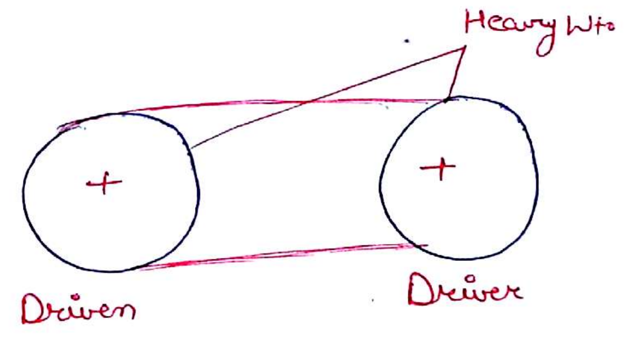
Crank, Connecting Rod
Piston, Cylinder

2) Flexible Link

Deformation's are there, but are in permissible limits.

for e.g:-

Belt, Rope, chain drives

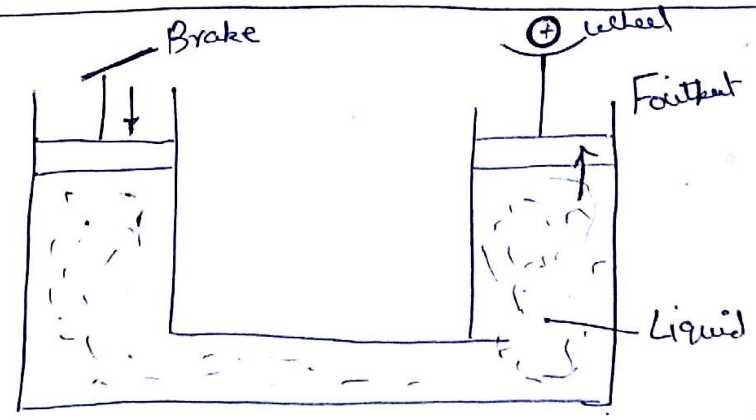


3) Fluid Link:-

When Power is Transmitted bcz of Fluid Pressure.

for e.g:

Hydraulic lift
Hydraulic RAM
Hydraulic Crane etc



Hydraulic Brakes

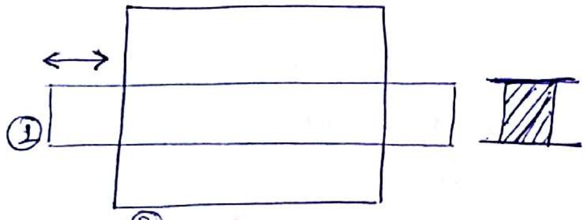
Different Types of Relative Motions :

Relative Motion → For Relative motion system is having Two links.

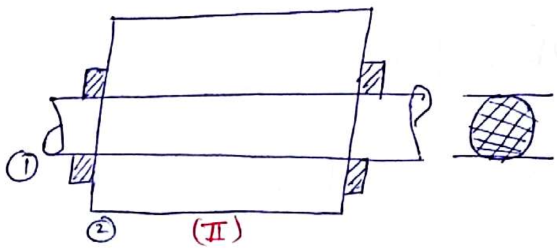
1. Completely Constrained motion
2. Successfully Constrained Motion

] Constrained Motion $x \to y$ (Desired Motion)

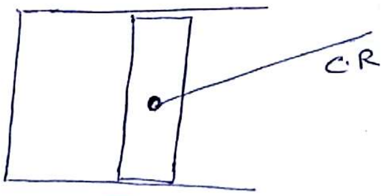
3) Incompletely Constrained Motion → Unconstrained Motion



(I)



(II)



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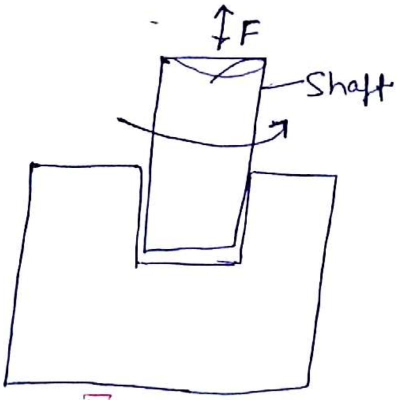

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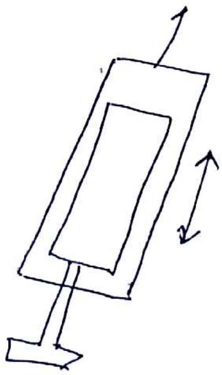
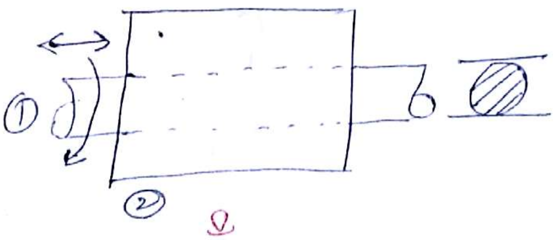




V

i, ii) Completely Constrained Mo^n

iii, IV) Successfully Constrained

(V) Incompletely Contrained

Kinematic Pairs:

" Any Connections b/w two links is always a joint or a pair, but this pair will also be a Kinematic pair, if the Relative motion b/w the links is a Constrained motion.

Classification of Kinematic Pairs:-

A) According to the type of Relative motion:-

Kinematic b/c one independent

i) Turning Pair (Revolute Pair) (Pin Joint):

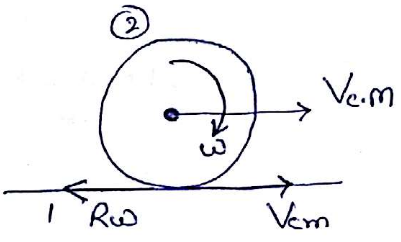
When Relative motion is pure turning.

ii) Sliding Pair (Prismatic Pair):

when Relative motion is pure sliding.

iii) Rolling Pair:

When Relative motion is pure Rolling.
$\Downarrow$
Rolling without slipping.



For Pure rolling
$V_{cm} = R\omega$

iv) Screw Pair:-

When relative motion is over the threads.

for e.g. Nut & Bolt.

v) Spherical Pair (Ball in Socket Joint): 3D Rotation

When Relative motion is (3-D) Rotation. Spherical Motion.

e.g. mirror in vechiles

B) According to Type of Contact :

i) Lower Pair : → Surface Contact

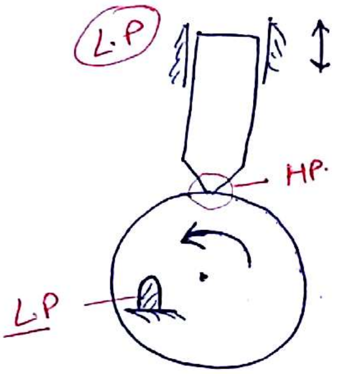
ii) Higher Pair → Point / line Contact

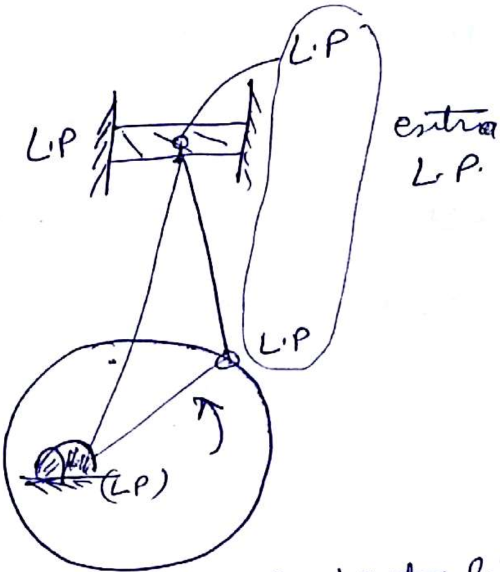
iii) Wrapping Pair → When one link is wrapped over other link

Wrapping Pair is close to Higher Pair .

e.g → Belt Pulley , Shoes Shall , Rope Pulley.

$$\boxed{1\,HP \equiv 2\,LP}$$





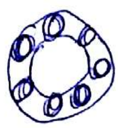
To eleminate 1HP → 1 extra link & 2 L.P

c) According to the type of Closeure :

i) Self closed Pair (closed Pair) :-

Permanent Contact

e.g → Ball is Ball Bearing ,



ii) Forced closed Pair (open Pair) :

forceful Contact.

e.g. HP is Cam & follower

Door closeer

Automatic Clutch Operating System

Centrifugal force

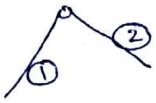
as C.F ↑ clutch operates & gears Changes.

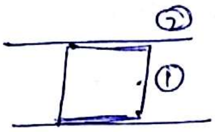
Operation depend on Springs & Gear

Different Types of Joints :

1) Binary Joints :

Where two links are connected.

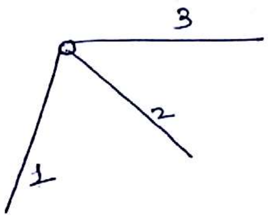




2) Ternary Joints :

When three links are connected.

$\boxed{1T \equiv 2B}$



$(1,2) \rightarrow B$

$(2,3) \rightarrow B$

$\boxed{(1,3) \rightarrow B}$

Dependent.

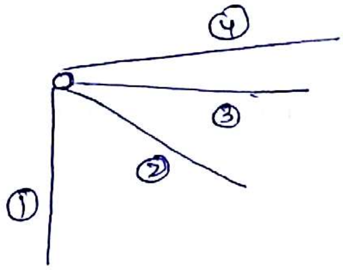
3) Quaternary Joint :

When four links are connected.

(1,2) (2,3) (1,4)

(2,3) (2,4)

(3,4)



$\boxed{1Q \equiv 3B}$

→ Kinematic chain :

" If all the links are Connected in such a way such that first link Connected to last link in order to get the close chain and if all the Relative motion in this close chain are Constrained, then the chain is known as Kinematic chain.

Kinematic Chain
(Constrained chain)

↓ To use this chain one link must be fixed.

Mechanism → Which can give Desired output w.r.to given input.

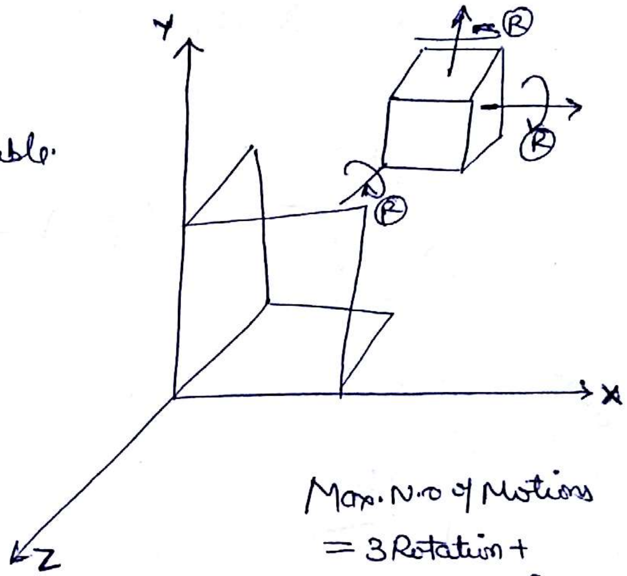
↓ Utilize it

Machine ⟶ Desired output is Obtained

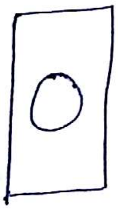
Degrees of Freedom : (Mobility)

"The min. no. of independent Variables requires to defines the position & motion of System is Known as degrees of freedom of the System".

D.O.F = 6 − Restraints
↓
No. of those motion
नहीं हो सकते which are not possible.



Max. N.o of Motions
= 3 Rotation +
3 Translation

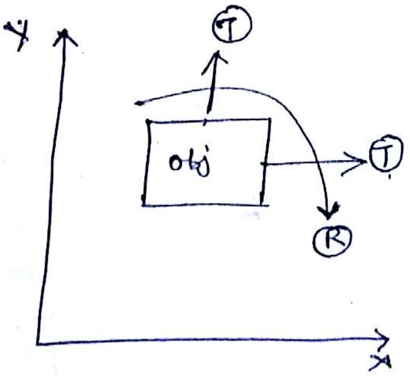
Pair	Restraints	D.OF
	3T+2R = 5	6−5 = 1
	1T = 1	6−1=5

NoTe

1) Lower Pair → 1 D.O.F
2) Higher Pair → 2 D.O.F
3) Sperical Pair → D.O.F = 3
bcz it Can rotate in 3 dir^n.

Aim :

To find out degree of freedom of (2-D) Planer Mechanism.



Motion = 2T + 1R = 3

No, of links = l

No. of Binary joints = j

No. of Higher joints = h

$\boxed{\text{One link fixed}}$

$$\boxed{F = 3(l-1) - 2j - h} \;**$$

[No. of Max. motion in 2-D Planner Mechanism]

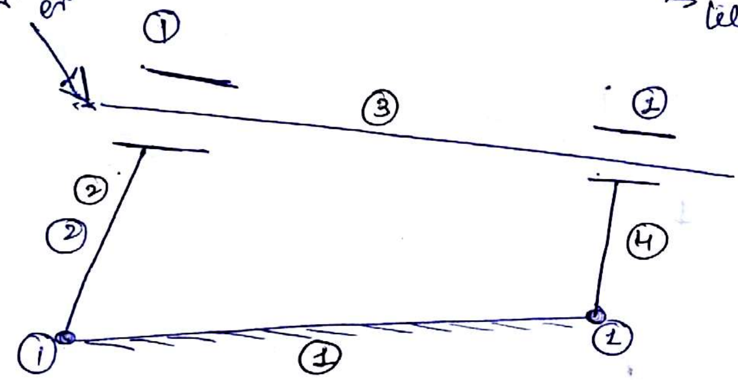
$$\boxed{F = [3(l-1) - 2j - h] - F_r}$$

Where F_r = No. of those machines which are not the Part of Mechanism.

→ Without input - move

Without any input when this link move either external force Applied

1)

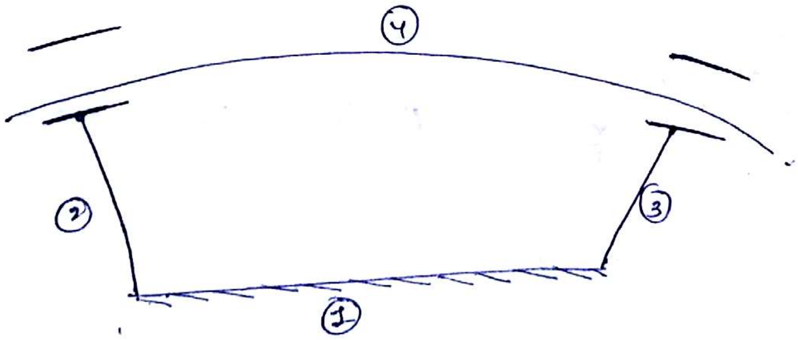


$l = 4$
$j = 4$
$h = 0$
$F_r = 1$

$$\boxed{F = [3\times(4-1) - 2\times4 - 0] - 1}$$

$F = 0$

(2)

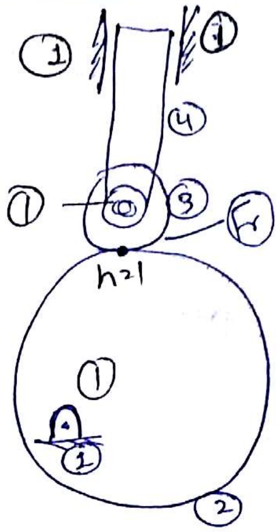


$l = 4$
$j = 4$
$h = 0$
$F_r = 0$

$\boxed{F = 1}$ *

③ $F = [3(4-1) - 2\times3 - 1] - 1$

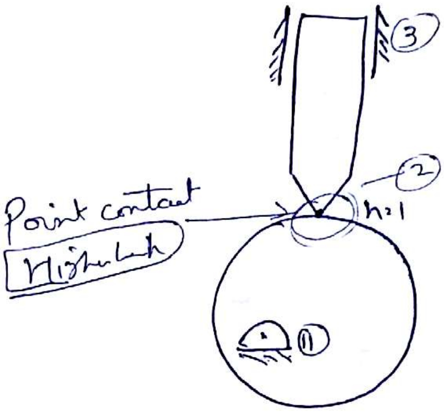
$\boxed{F = 1}$



$l = 4$
$j = 3$
$h = 1$
$f_r = 1$

④ $F = (3(4-1) - 2\times3 - 1)$

$\boxed{F = 1}$

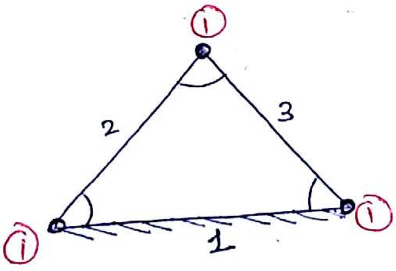


$l = 3$
$j = 2$
$h = 1$
$f_r = 0$

Physical Significance Of Degree of freedom:

1) If $F = 0$ → No Relative Motion (frame / Structure)

e.g.:

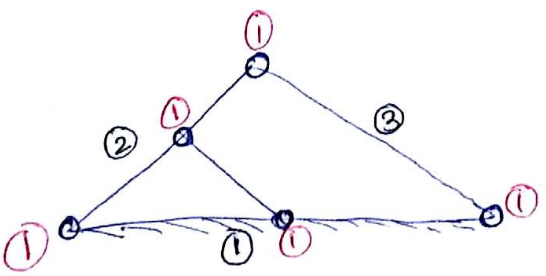


$l = 3$
$j = 3$
$h = 0$

$F = 3(l-1) - 2j - h - F_r$
$= 3(3-1) - 2\times3 - 0 - 0$
$= 6 - 6$
$\boxed{F = 0}$

$\boxed{F = 0}$

2) If $F < 0$; → $-1, -2, -3,$ No Relative motion Super Structure (Intermediate Structure) With great Strength

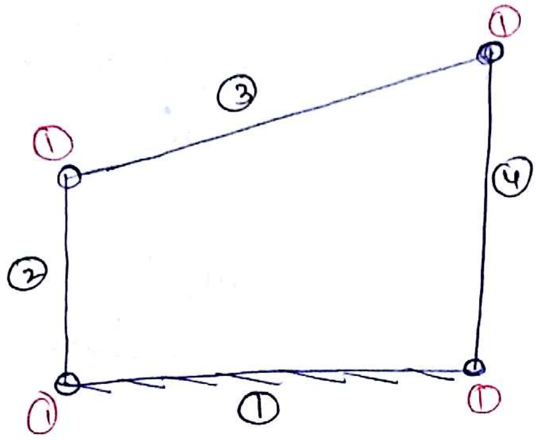


$l = 4$
$j = 5$
$h = 0$

$\boxed{F = -1}$ ✗

3) If F=1 Kinematic chain

e.g



l = 4
j = 4
h = 0

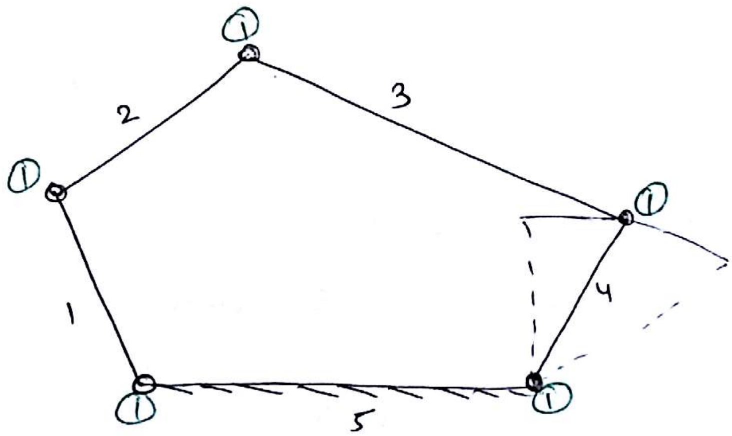
$\boxed{F=1}$ * K.C

Kinematic chain
$\boxed{DOF = 1}$

☆☆☆

4) If F > 1 .2, 3, 4, 5 — Unconstrained Chain

e.g

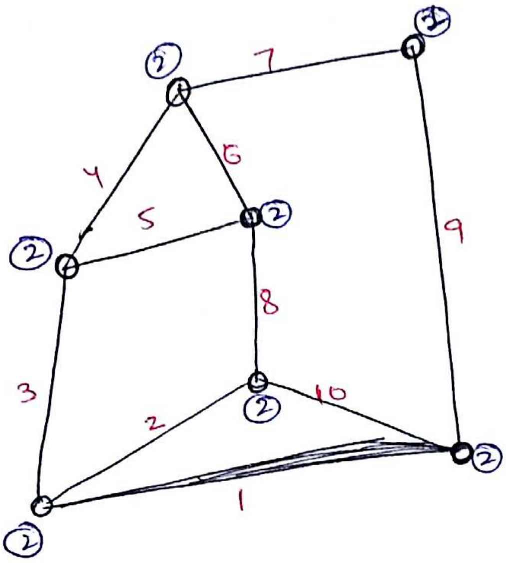


l = 5
j = 5
h = 0

$\boxed{F = 2}$

" DOF is the no. of Input Required to get the unconstrained output in Any chain.

i)



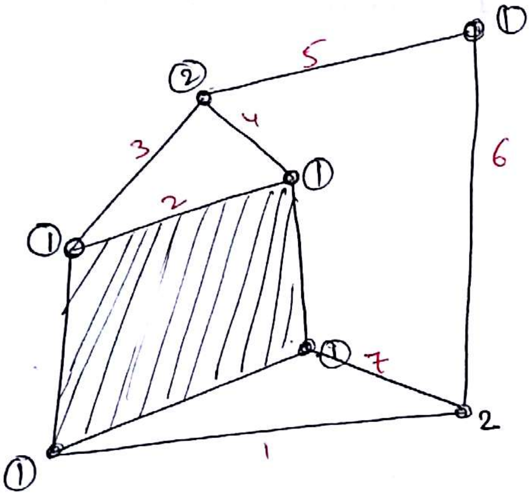
$l = 10$

$j = 13$

$h = 0$

$\boxed{F = 1}$ Kinematic chain

ii)



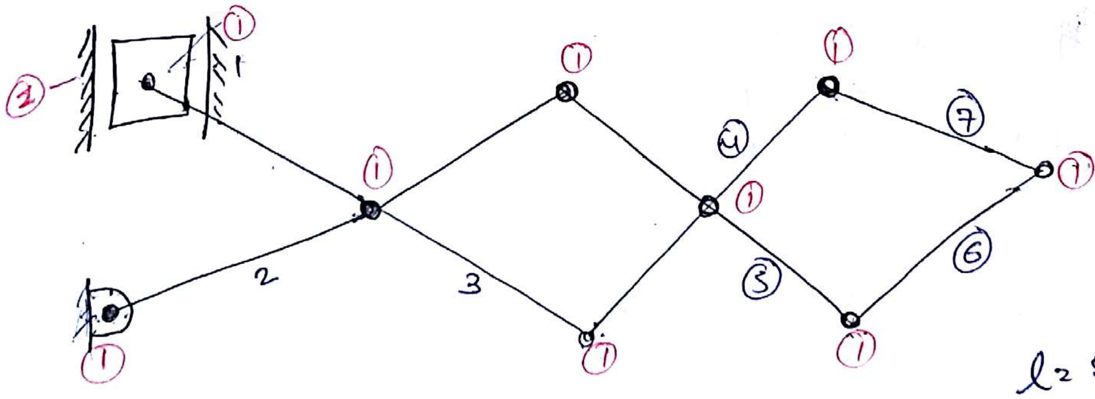
$l = 7$

$j = 9$

$h = 0$

$\boxed{F = 0}$ frame | structure

iii)



$l = 8$

$\rightarrow j = 10$

$h = 0$

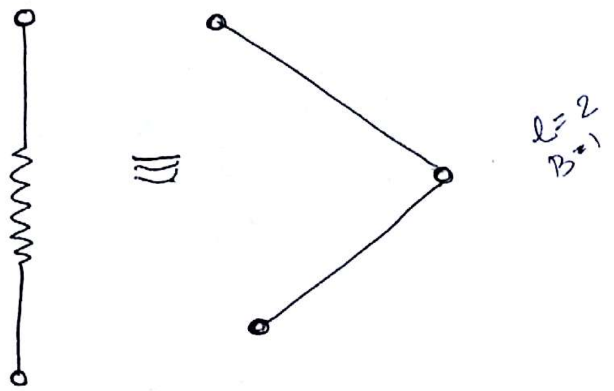
fixed → 1

1 Slider

Slider in cylinder 1 joint

$\boxed{F = 1}$ (K.C)

Spring as a link   Verb



Grubler's Equation :

Grubler's equation is valid only for those mechanism in which

$$\boxed{F=1}$$

$$\boxed{h=0}$$

Applying Kutzback's equ^n :

$$F = 3(l-1) - 2j - h$$

$$1 = 3l - 3 - 2j - 0$$

$$\boxed{3l - 2j - 4 = 0}$$

$(3l)_{always} \rightarrow$ even

$(l)_{always} \rightarrow$ even

$\boxed{(l)_{min} \rightarrow 4}$ first Mechanism in Lower Pair

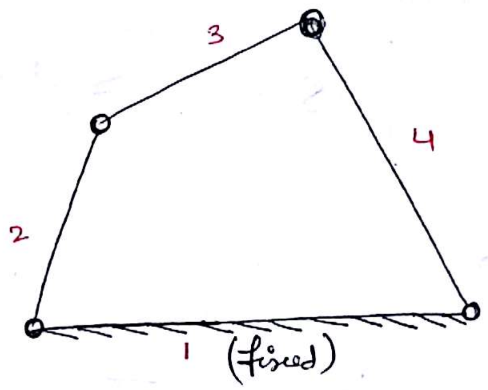
Simple Mechanism :

i) Four bar Mechanism
ii) Single Slider Crank Mechanism
iii) Double Slider Crank Mechanism

] Four link (Lower Pair)

i) Four bar Mechanism :- (4 Links, 4 Turning Pair)

Best Position → fixed because it governs both input and output



input / output
⇓
Complete rotation → Crank (Complete 360°)
Partial Reaction → Rocker / lever (if less than 360°) oscillation

V.imp (NOTe)
If NO. of Links = l
then NO. of inversion $\leq l$

Inversions :-

Mechanism which are obtained by fixing one by one different links

i) Double Crank Mechanism
ii) Crank ↔ Rocker Mechanism
iii) Double Rocker Mechanism

Grashof's Law:

Aim

" For the Continous Relative motion b/w the number of Planes, in a mechanism, the Summission of ~~___~~ length of Shortest & the longestest link Should not be greater than the Summission of the length of other two links."

** For Continous Relative motion

$$\boxed{(S + l) \leq (P+q)}$$

S – Shortested link

l = longest link

Where,

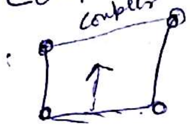
Best Position → fixed < input / output

Best link for Rotation → Sl

Worst Position

↓

Coupler / coupler



Case 1) if $(S+l) < (P+q)$

Law is Satisfied

1) S → fixed < input / output → Double crank.

2) S → Adjacent Link fixed → Crank Rocker

3) S → Coupler → Double Rocker.

Case 2) if $(S+l) = (P+q)$

Law Satisfied

Condition: Not having Pair of equal link

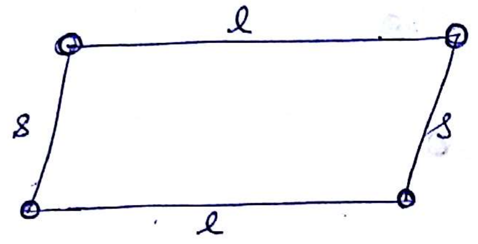
(2, 5, 4, 3)

2 → S, 5 → l.

Case 3 if $(S+l) = P+Q$ but, Having equal links

2, 2, 5 5
S S L L

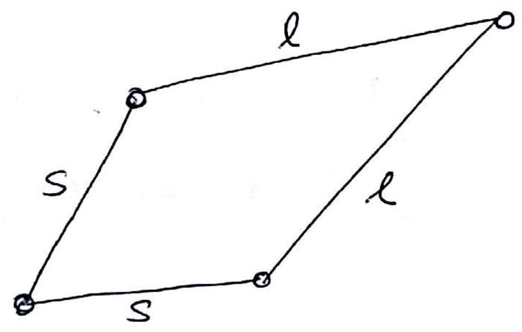
1) Parallelogram linkage:



S – fixed
↓
Double crank

l fixed → Double Crank.
bcz short length can Rotate easily.

2) Deltoid linkage:



Short – fixed
↳ S– fixed
↓
Double Crank

l fixed → Crank–Rocker.

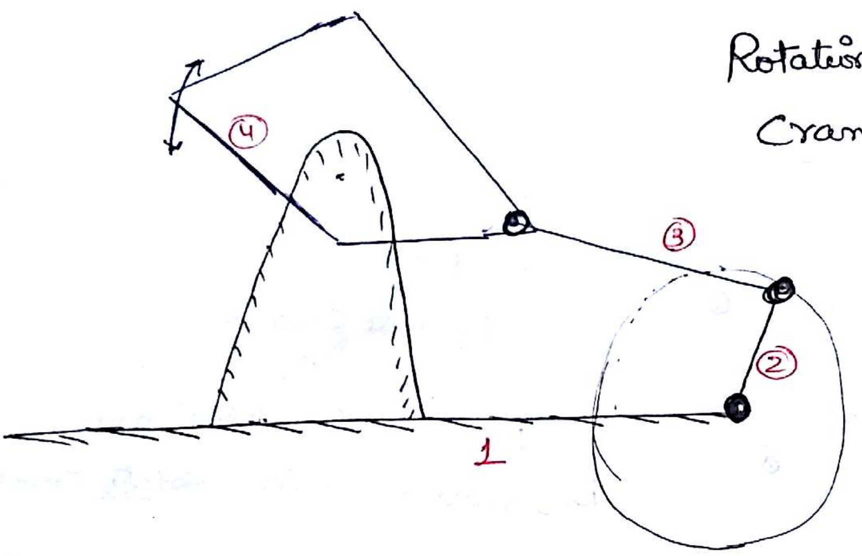
if $(S+L) > (P+Q)$

Law not satisfied

Double – Rocker will Obtained

Some Practical Applications of Four bar Mechanism

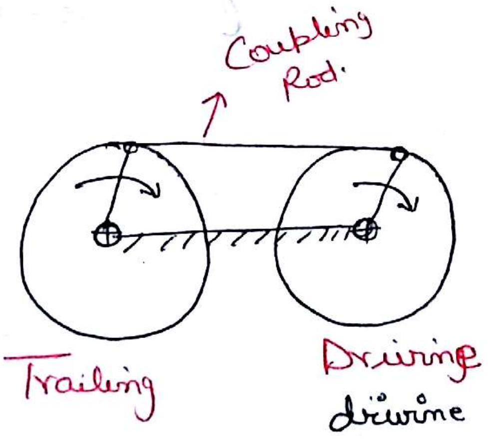
1) Beam Engine Mechanism: By James Watt



Rotation ⟷ oscillation

Crank ⟷ Rocker.

2) Coupling Rod of Locomotive:



[Parallelograme linkage
Double Crank]

Single Slider- Crank Mechanism : 4 link = 3TP + 1SP

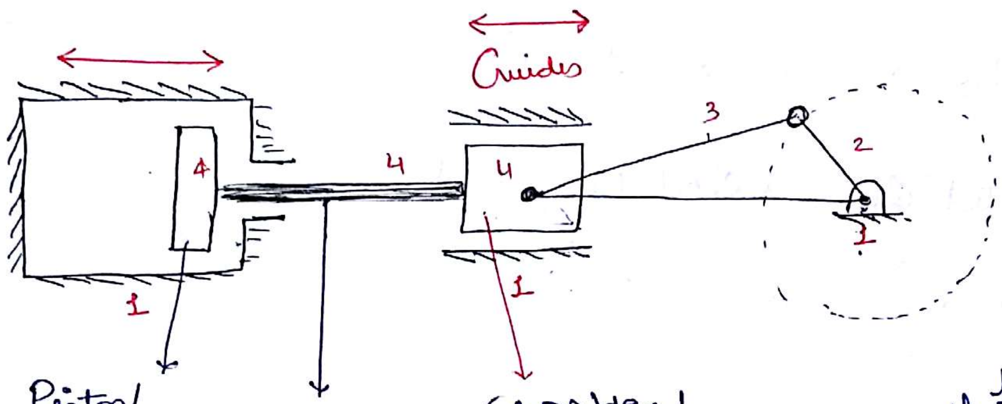
TP → Turning Pair
SP → Sliding Pair

ii) Inversions

(i) I, inversion (Basic)

(Cylinder fixed)

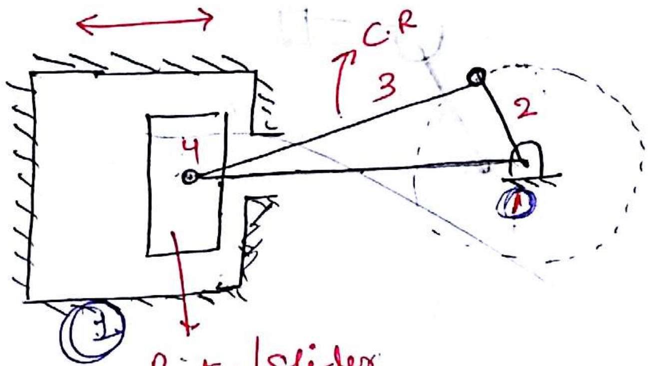
external Combustion engine



all fixed taken as one link
joint may be Same

Same.

Internal Combustion engine



Summary

Rotation ⟷ Reciprocating
(Crank) ⟷ (Piston)

(o) ⟵ i ⟶ Reciprocating engine
(i) ⟶ o ⟶ Reciprocating Compression

ii) II Inversion

(Crank fixed)

i). Whitworth (Quick return Motion Mechanism) QRMM

ii) Rotary I C engine Mechanism (GNOME Engine)

iii) III Inversion

(~~Crank Rod fixed~~) (Connecting Rod fixed)

i) Crank & Slotted lever (QRMM) Best

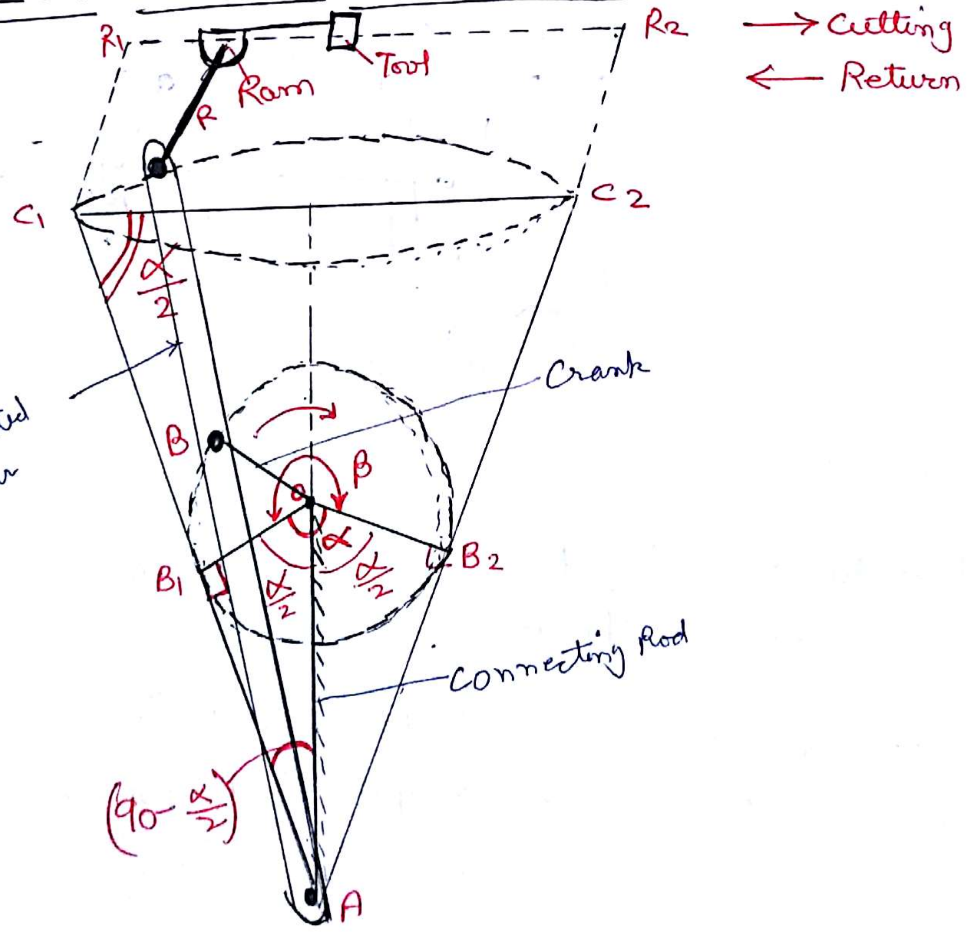
ii) Oscillating cylinder engine Mechanism.

iv) IV Inversion (Slider fixed)

Hand pump (Pendulum Pump) (Bull Engine)

CRANK & SLOTTED Lever (QRMM) :- (C.R fixed)

Connecting Rod fixed



$\beta \rightarrow$ Cutting stroke angle

$\alpha \rightarrow$ Return stroke angle

$$\alpha + \beta = 360^\circ$$

$$\boxed{\alpha < \beta} \quad QRMM$$

$$\frac{(\text{time})\ \text{cutting}}{(\text{time})\ \text{Return}} = \frac{\beta}{\alpha} = \underline{QRMM}$$

अगर $\frac{\beta}{\alpha}$ less than 1 given है तो वो Ratio $\frac{\alpha}{\beta}$ का है।

$$\boxed{\frac{\beta}{\alpha} > 1} \text{ not}$$

Note → if in Question it is given less than one then it is Ratio of $\frac{\alpha}{\beta}$ is given.

Stroke

$$= R_1 R_2$$

$$= C_1 C_2$$

$$= 2(C_1 M)$$

$$= 2(AC_1) \cdot \cos\frac{\alpha}{2}$$

$$= 2(AC_1)\left(\frac{OB_1}{OA}\right)$$

$$= \frac{2(AC)(OB)}{(OA)}$$

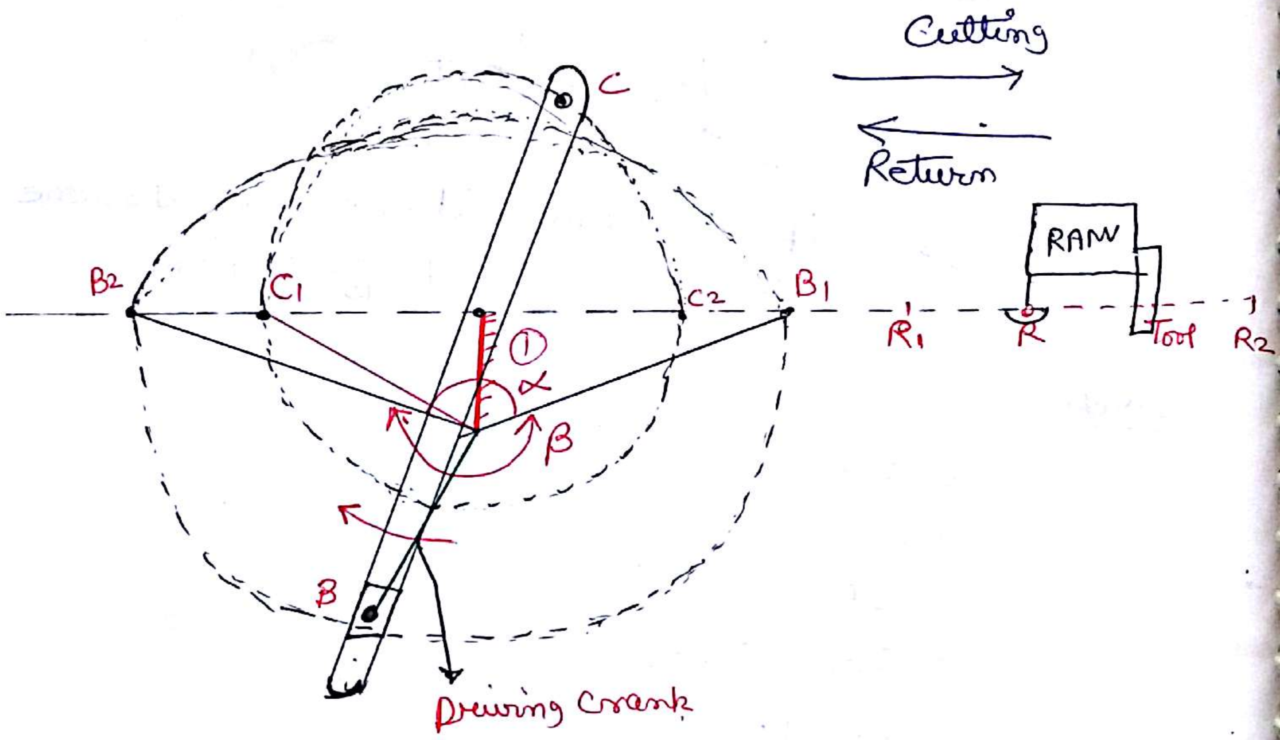
$$= \frac{2\,(\text{Length of Slotted bar}) \times (\text{Length of Cranks})}{\text{length of Connecting Rod}}$$

Withworth Quick Return Motion Mechanism :-

(Crank fixed)

Rotation → Rotation

Double Crank.



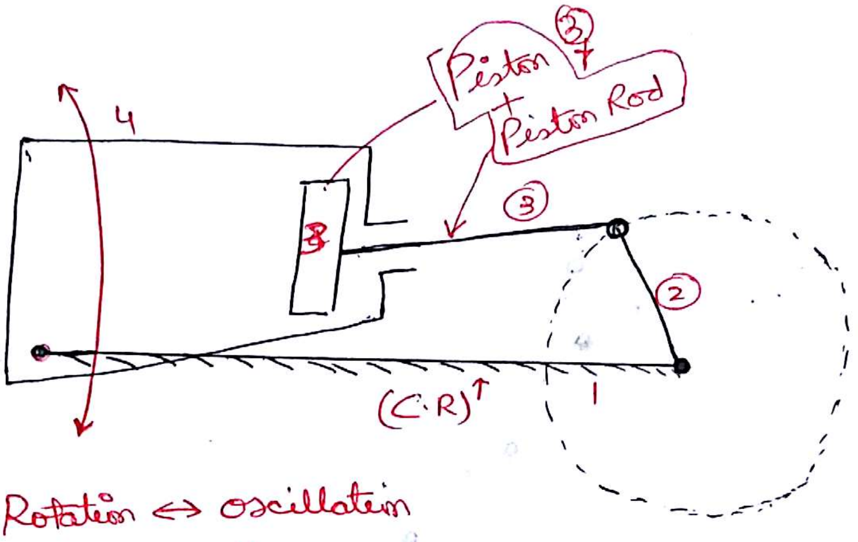
Stroke

= R_1R_2

= C_1C_2

= $2OC$

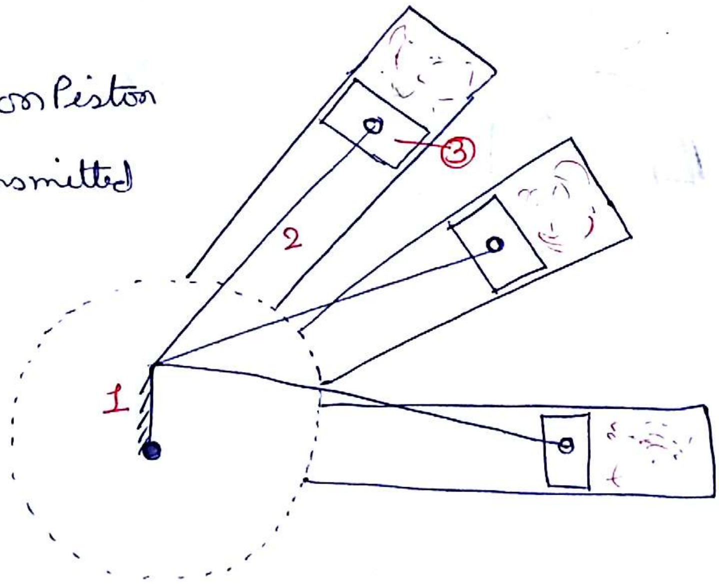
Oscillating Cylinder engine Mechanism:
(Connecting Rod fixed)



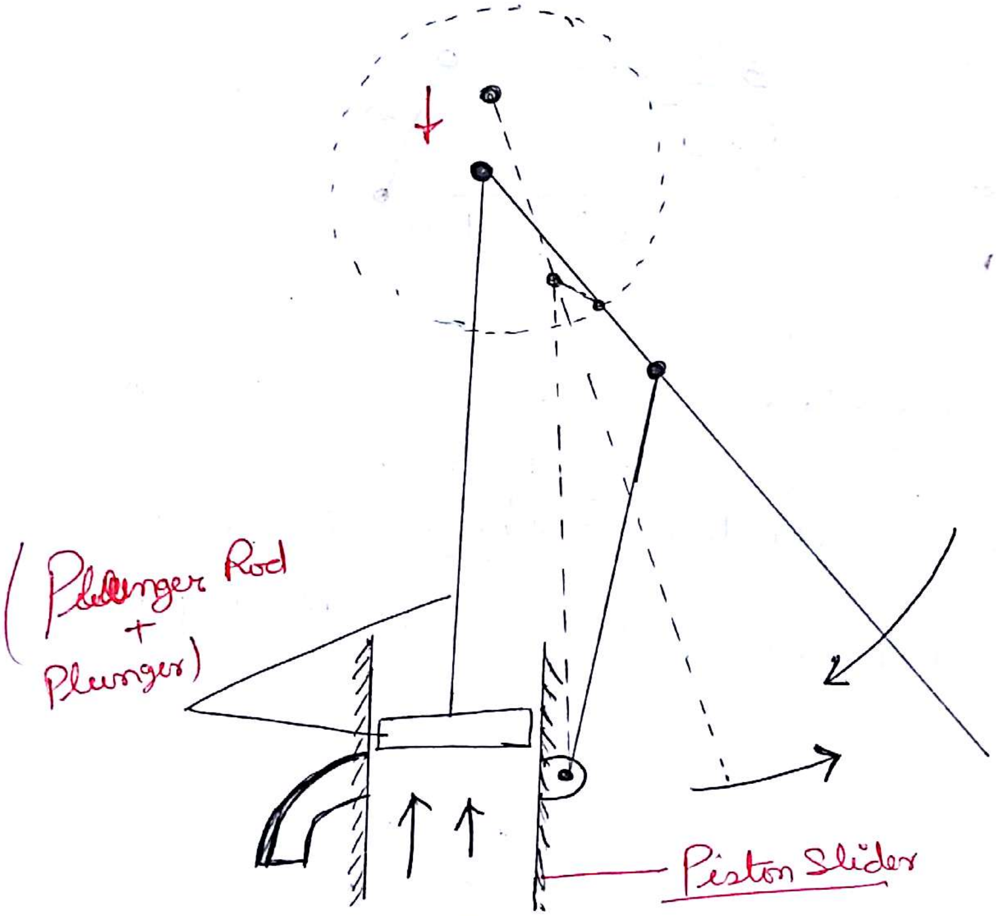
Rotation ↔ oscillation
Crank ↔ Rocker

Rotary Internal Combustion Engine: (GNONE Engine)
(Crank fixed)

When Combustion takes place inside the cylinder,
Input force comes on Piston
then this force transmitted to Connecting Rod.
then, Connecting Rod ~~gain this force~~.
and Piston both Rotate
↓
Cy. Block Rotates (output)

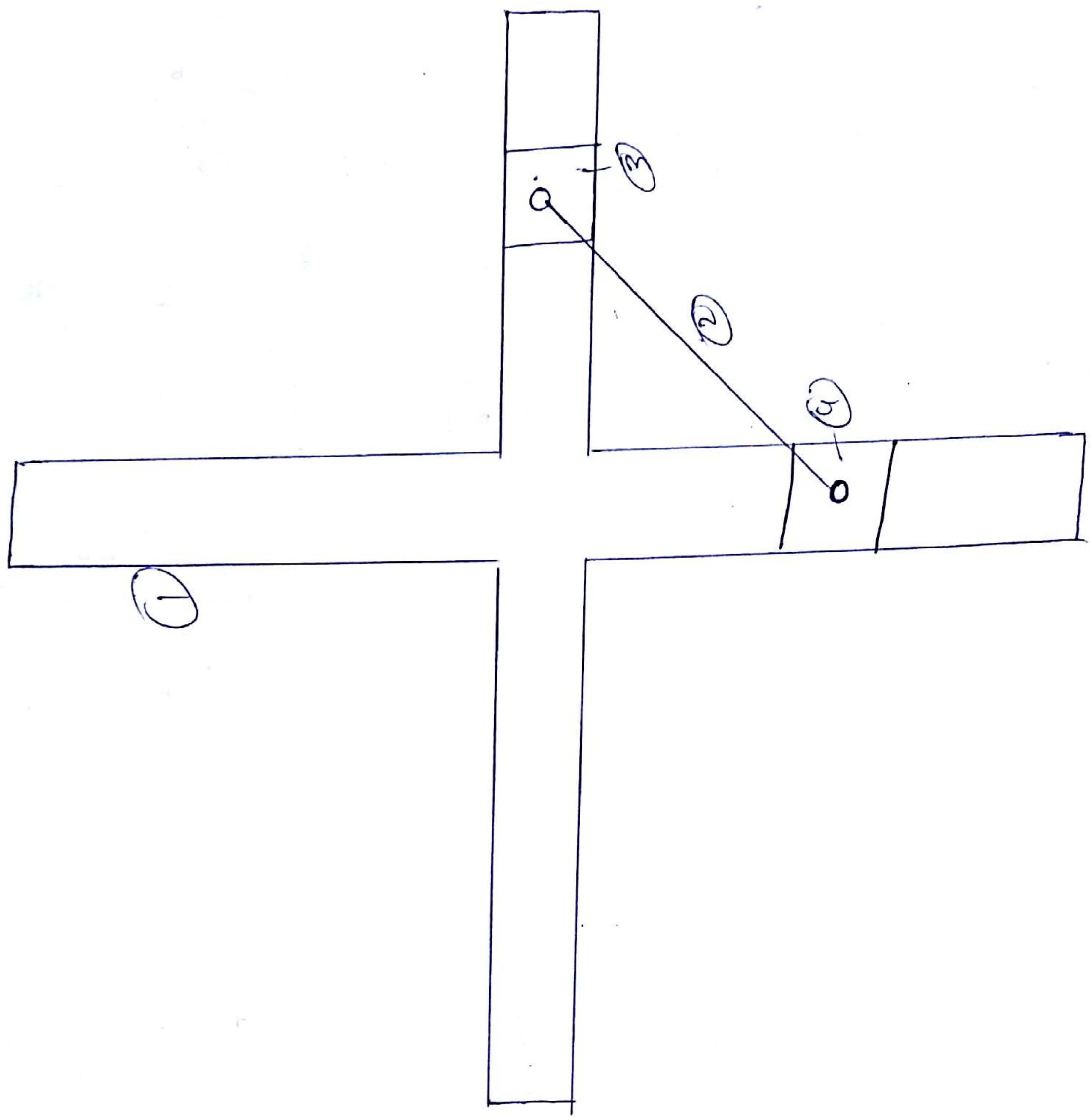


Hand Pump (Slider Pump) :-



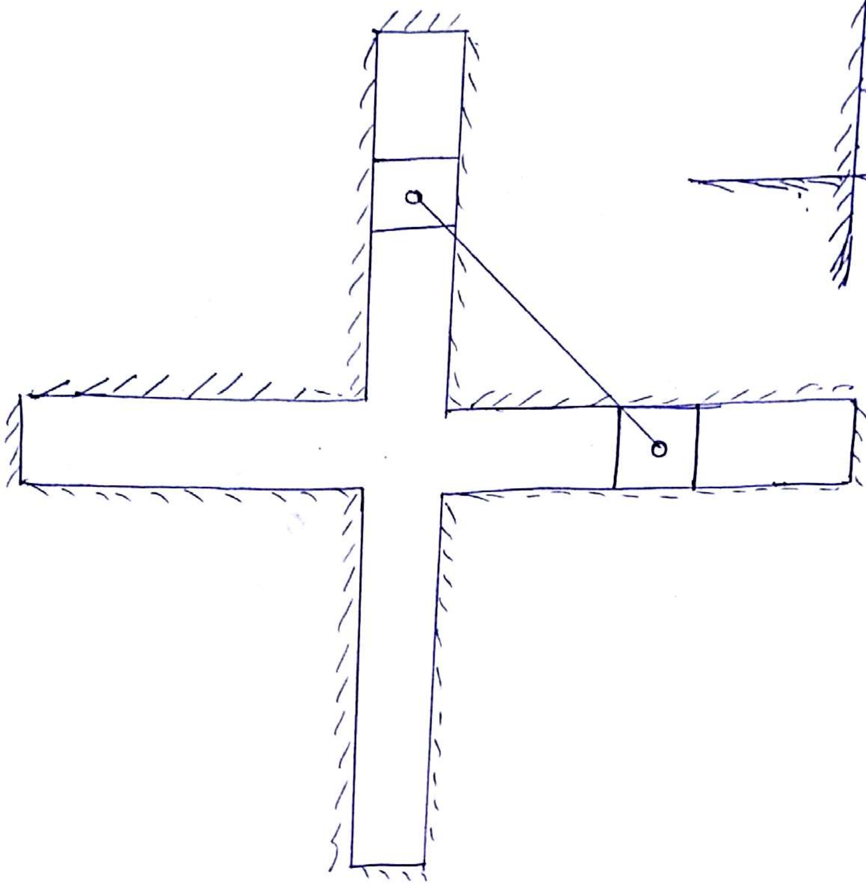
③ Double Slider-Crank Chain:

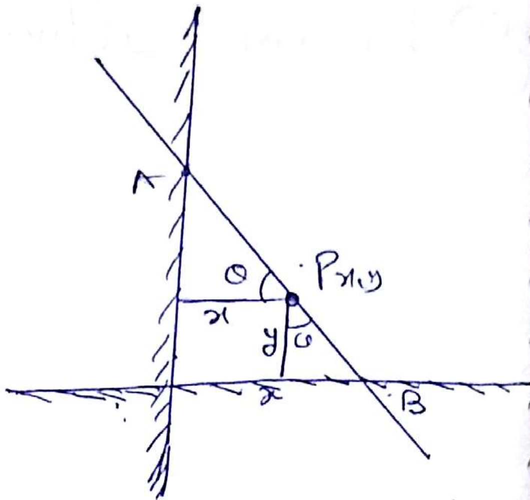
4 link → 2TP + 2SP



1) Slotted Plate fixed:

Elliptical Trammels:





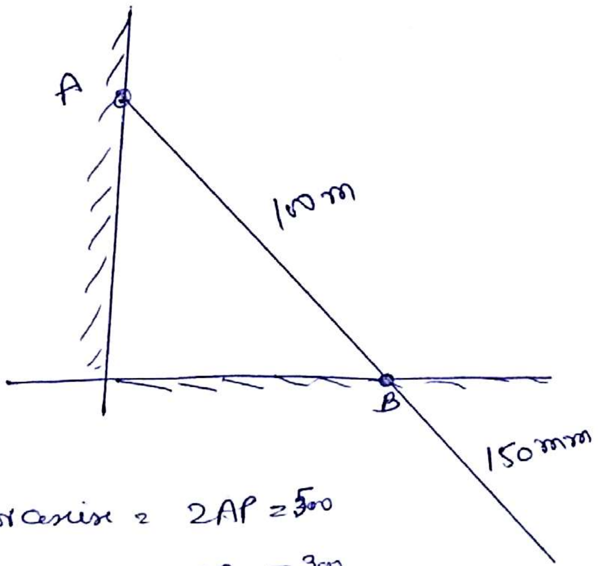
$\cos\theta = \frac{x}{AP}$

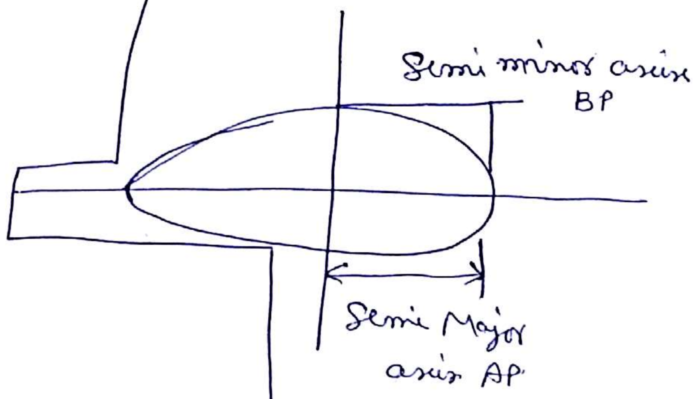
$\sin\theta = \frac{y}{BP}$

$$\boxed{\frac{x^2}{AP^2} + \frac{y^2}{BP^2} = 1}$$

Ellipse

Qn)





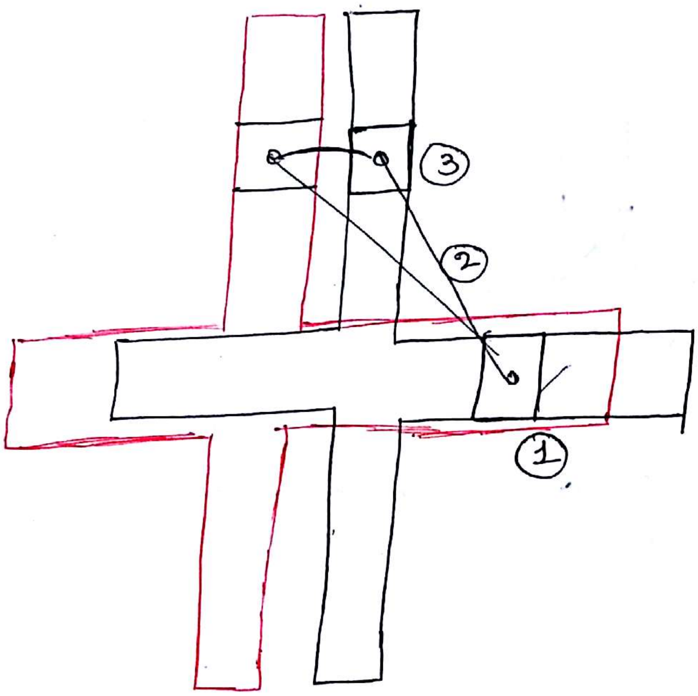
major axis = 2AP = 300

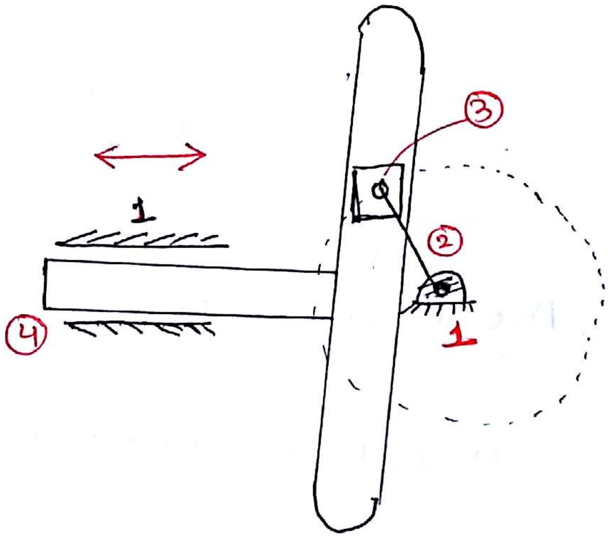
minor axis = 2BB = 300

2) If Any of the Slides is fixed :-

Scotch-Yoke Mechanism

Rotation → Reciprocating



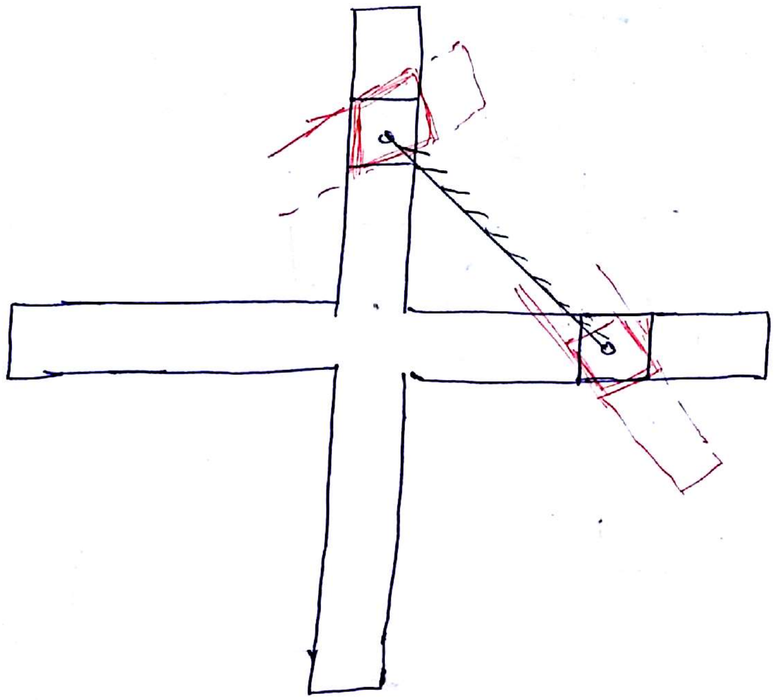


3) If Link Connecting Slides is fixed:

Oldham's Coupling:

⇓

used to Connect the Shafts having lateral misallignment.



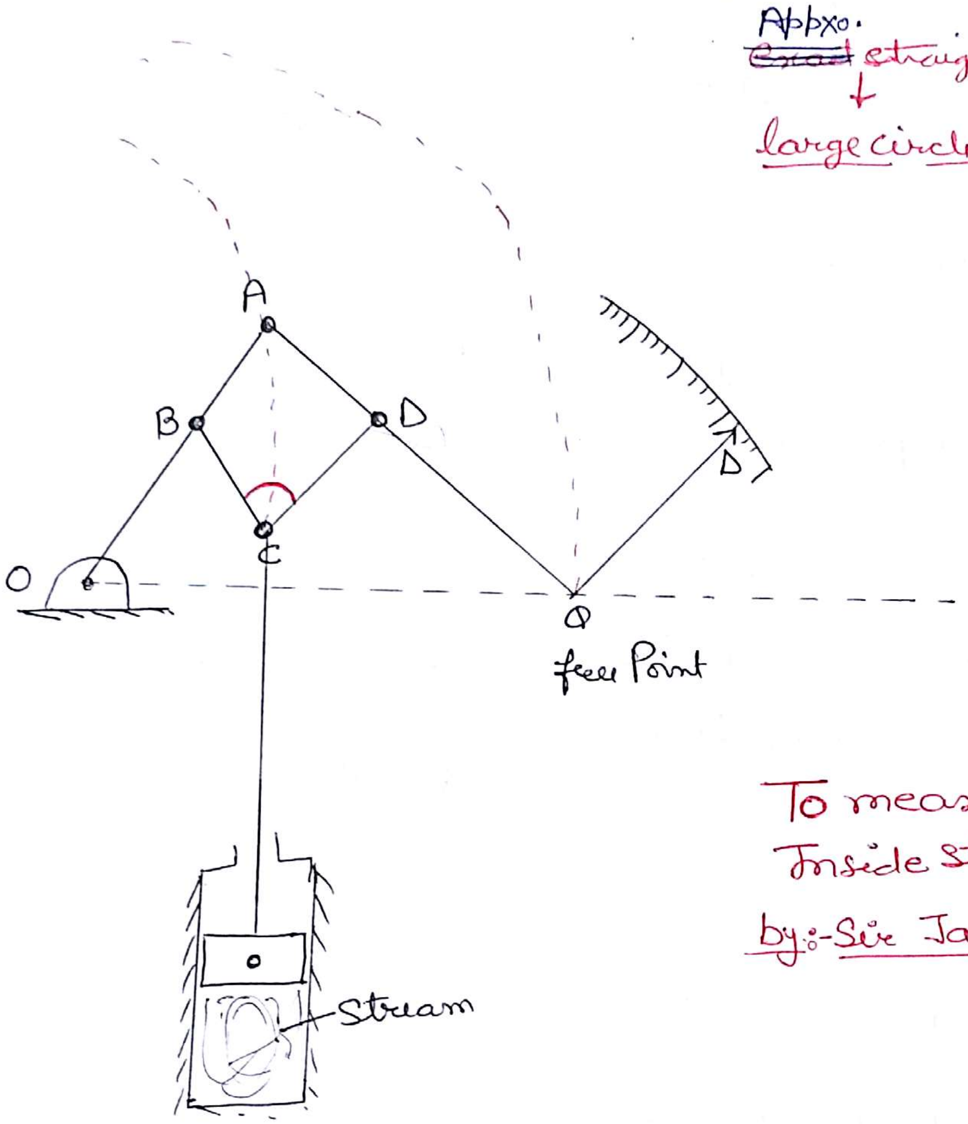
Complex Mechanism With Lower Pair:

→ Exact Straight Line Motion Mechanism.

→ Approximate Straight Line Motion Mechanism.

Watt's Indicator Mechanism:-

Appxo. ~~Exact~~ straight line
↓
large circle with ↑ radius



To measure Pressure
Inside steam Chamber
by:- Sir James Watt

Observation:

→ Point C and Point Q, both moves in Approximate Straight line motion.
[Approximate st. line motion Mechanism]

→ There is no relative motion b/w link BC & CD hence BCD → one link.

→ Link BCD → levers → Double crank Mechanism
Link AQ →

Steering Gear Mechanism:

⇓

Changing direction of motion.

i) Davis Steering gear Mechanism:

→ having 2 Turning pair + 2 Sliding pair (life is less)

→ Exact at all positions.

ii) Acerman steering gear Mechanism:

→ having only turning pairs (life is very high)

→ Exact at all three position.
(mid, extreme left, extreme right)

iii) Rapson's Slide:

→ uses in Ships

Intermittent Motion Mechanism:

⇓

Provide the periodic motions with breaks at output w.r.to given continous input.

i) Geneva Mechanism:

→ used in Indexing in milling etc

ii) Ratchet Mechanism:

→ used in clocks

Mechanical Advantage of A Mechanism:

$$M.A = \frac{F_{output}}{F_{input}} \longrightarrow \boxed{M.A = \frac{V_{input}}{V_{output}} \times \eta_{mechanism}}$$

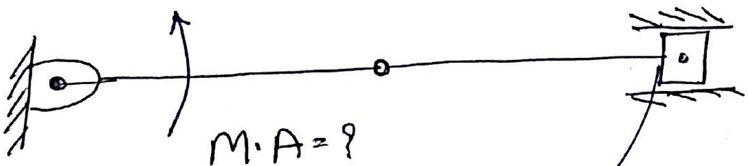
or

$$M.A = \frac{T_{output}}{T_{input}} \longrightarrow \boxed{M.A = \frac{\omega_{input}}{\omega_{output}} \times \eta_{mechanism}}$$

Efficiency

$$\eta_{mechanism} = \frac{P_{output}}{P_{input}} = \frac{F_{output} \times V_{output}}{F_{input} \times V_{input}} = \frac{T_{output} \times \omega_{output}}{T_{input} \times \omega_{input}}$$

Qn) find Mechanical Advantage of fig.?



$M.A = ?$

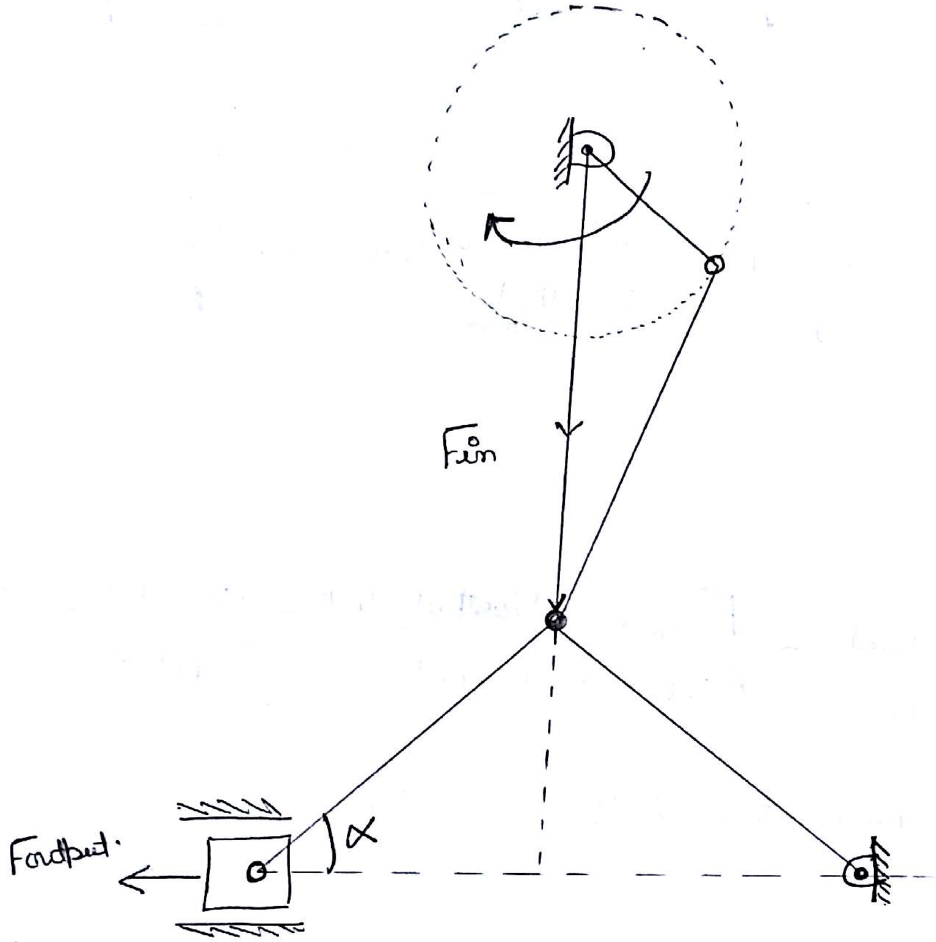
Given that, $\omega_{input} = \omega$

$\omega_{output} = 0$ ← Pure sliding

$$\boxed{M.A = \frac{\omega}{0} = \infty}$$

BCZ, Rotation ⟶ Resiprocating
↓
Rotation of infinite Radius circle
Means Resiprocation.

Toggle Mechanism :-



$$\tan\alpha = \frac{F_{input}}{F_{output}}$$

$$F_{output} = \frac{F_{input}}{\tan\alpha}$$

As $\alpha \longrightarrow 0$

$\tan\alpha = 0$

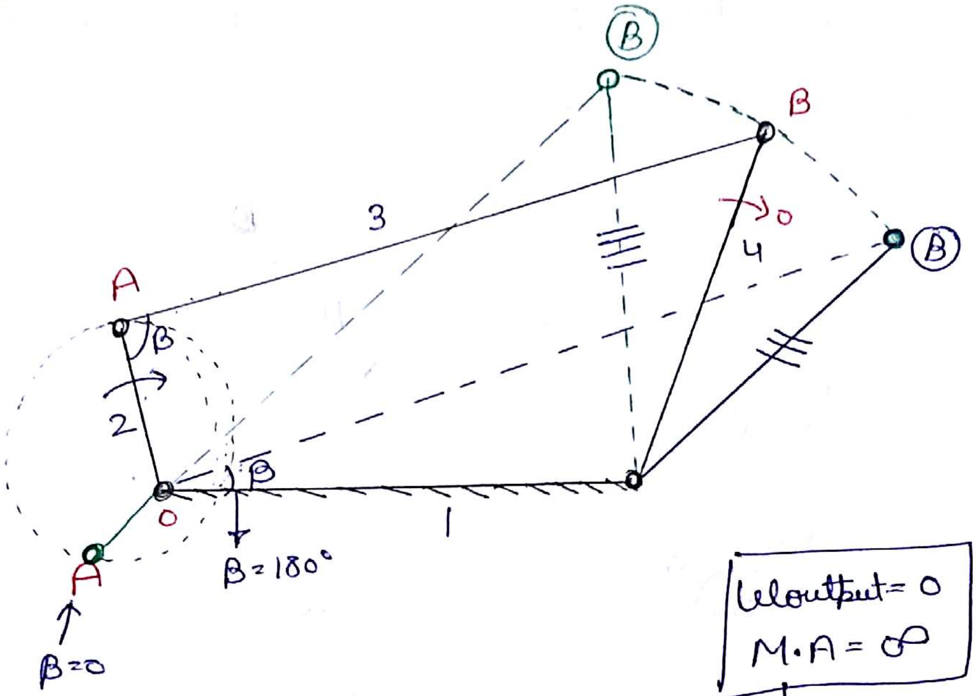
$F_{output} \to \infty$

$M.A = \alpha$

$\boxed{F_{output} >>>>>>> F_{input}}$ *

Toggle's Position :

It is the extreme positions of output link, as Rocker in four Bar Mechanism.



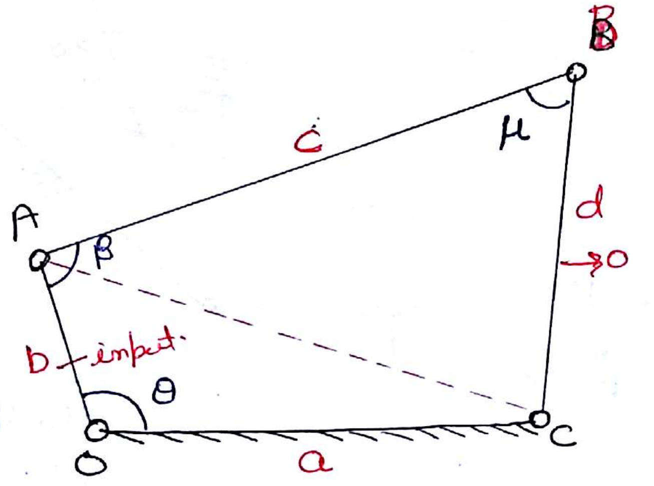
$\omega_{output} = 0$
$M.A = \infty$

Toggle Position

Transmission Angle in Four bar Mechanism :(μ)

Representation of Transmission angle → μ

→ The Angle b/w the output link & Coupler link in Four bar Mechanism is Known as Transmission Angle.



$$AC^2 = b^2 + a^2 - 2\,ba\cos\theta = c^2 + d^2 - 2cd\cos\mu$$

Diffrentiating both side

$$(-\cancel{2}\,ba)(-\sin\theta\cdot d\theta) = (-\cancel{2}\,cd)(-(\sin\mu\cdot d\mu)$$

$$\frac{d\mu}{d\theta} = \frac{ba}{cd} = \frac{\sin\theta}{\sin\mu}$$

for μ, to be max. or min.

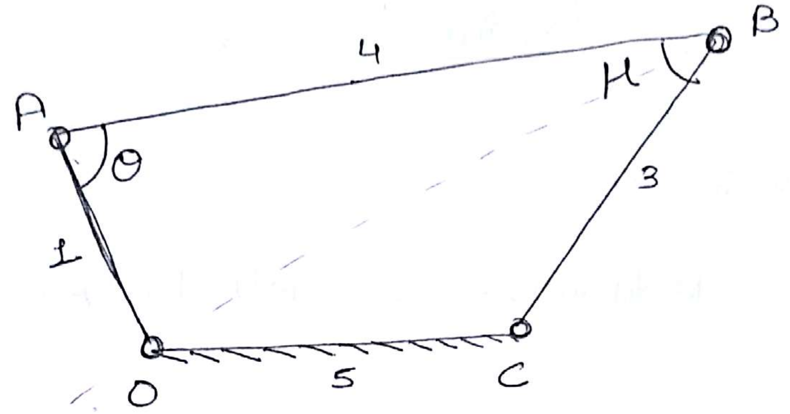
$$\frac{d\mu}{d\theta} = 0$$

$$\Rightarrow \sin\theta = 0$$

$$\theta = 0^\circ, 180^\circ$$

~~Max~~ μ_{min} μ_{max}.

Problem:



find, $H_{min} = ?$

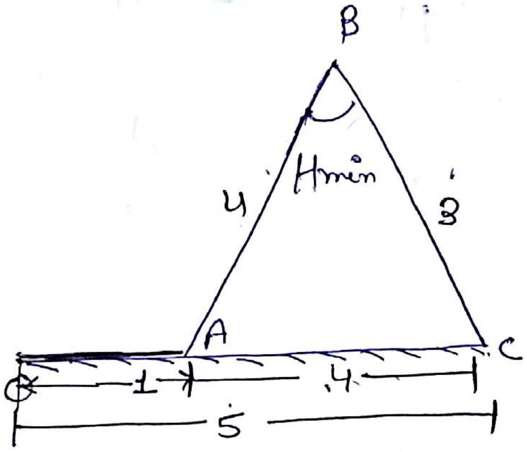
$H_{max} = ?$

first check whether triangle law is valid or not i.e. sum of lowest & largest must be equal or less

for Hmin $(\theta = 0^\circ)$

$(4)^2 = (4)^2 + (3)^2 - 2(4)(3)\cos H_{min}$

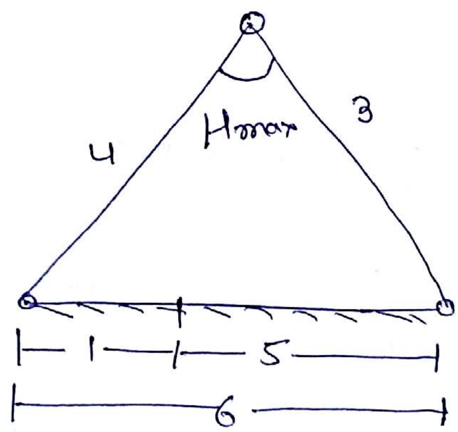
$\boxed{H_{min} =}$



for Hmax = $\theta = 180^\circ$

$(6)^2 = (4)^2 + (3)^2 - 2(4)(3)\cdot\cos H_{max}$

$\boxed{H_{max} =}$



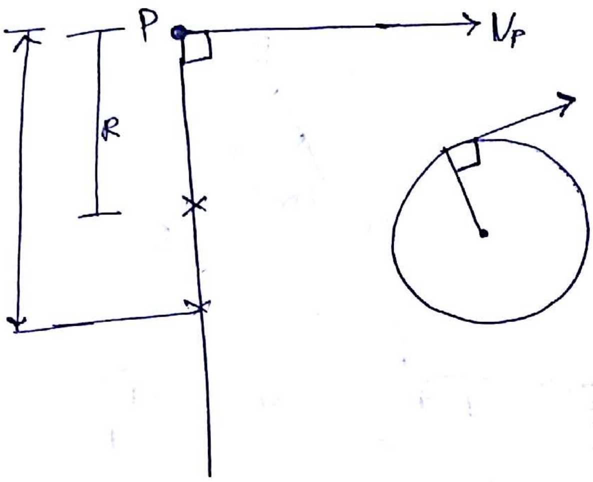
Motion Analysis

Velocity Analysis :-

Instantaneous Centre Method Application

(by: Sir, ARNHOLD)

Instantaneous Centre of Rotation :-



$$\frac{V}{R} = \omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \frac{V_D}{DI} = \frac{V_E}{EI} = \text{.........}$$

for e.g.

$I_{23} \rightarrow$ Instantaneous centre for the Relative motion b/w two link i.e b/w link 2 & 3

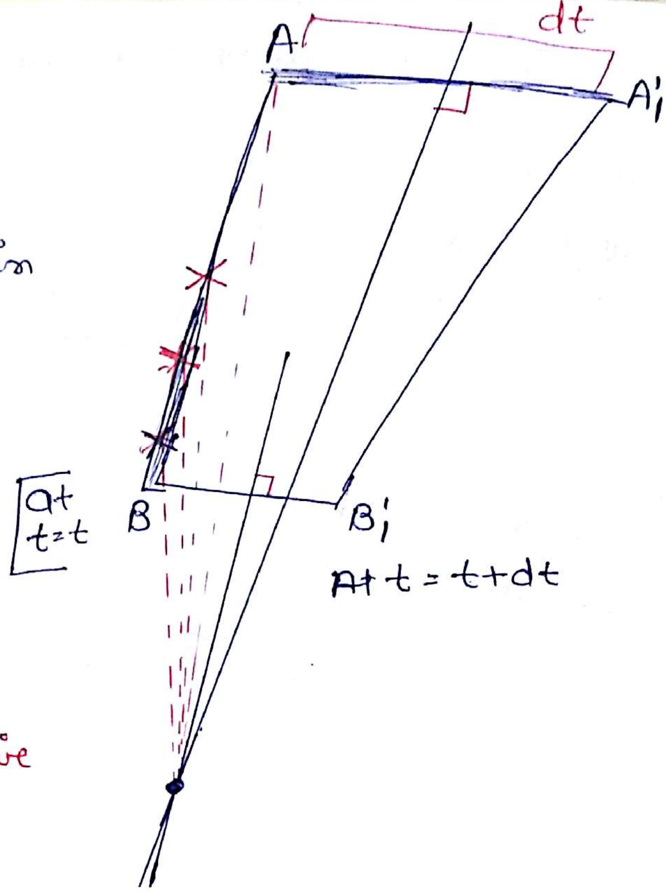
$\boxed{I_{23} \text{ or } I_{32} \text{ both are Same. 2,3 represent link only}}$

Motion	Centrode	Axode
General Motion	Curve	Curved Surface
Pure Translation	Straight line	Plane Surface
Pure Rotation	Point	Straight line

In Reality

$$\left.\begin{aligned} AA_1 &\to 0 \\ BB_1' &\to 0 \end{aligned}\right] \text{Diffrential}$$

This link AB at this instant is in general motion



In general, when the link moves then its instantaneous centre of Relative motion changes its position.

Locus of I-Centre for the Relative motion b/w the Centrode link.

Locus of I-axis of Relation for the Relative motion ⇒ Axode b/w the links

~~Motion~~

" In General, the motion of link in mechanism is neither Pure translation nor pure rotation. It is a Combination of Translation and rotation, which we normally say, that the link is in general motion but, any link at any instant, Can be assume to be in pure rotation with respect to point in the Space Known as Instantaneous Centre of Rotation. "
This Centre is also Known as the Virtual Centre.

No. of Instantaneous Centre in Mechanism :

If No. of link = l

No. of Instantaneous Centre = ${}^{l}C_2$

$$= \frac{l(l-1)}{2}$$

for e.g

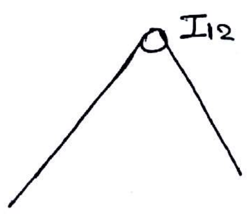
$l = 5$

I.C = 10

$$\left.\begin{array}{cccc} I_{12} & I_{13} & I_{14} & I_{15} \\ & I_{23} & I_{24} & I_{25} \\ & & I_{34} & I_{35} \\ & & & I_{45} \end{array}\right]$$

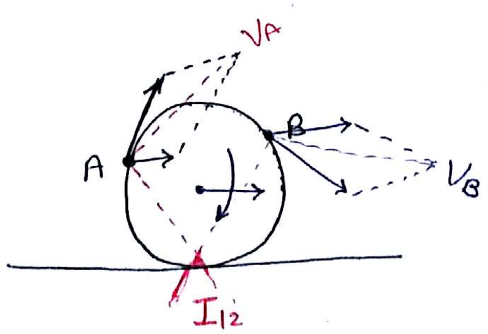
Basic Instantaneous Centre in a Mechanism :

1) Turning Pair :



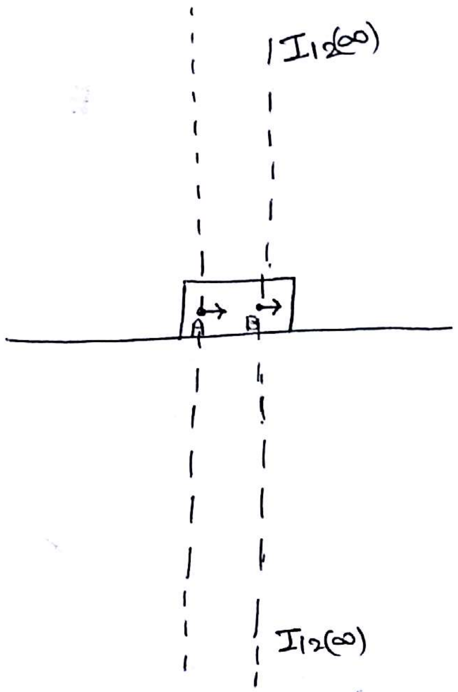
I.C at Centre / Point of Pin .

② Rolling Pair :



③ Sliding Pair

i) on Plane Surface:

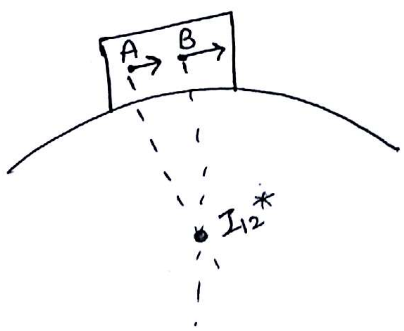


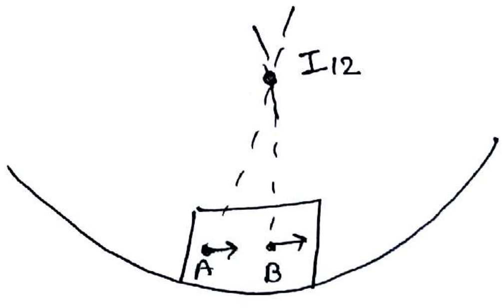
Ic will go at ∞, but in the opposite dirⁿ, ⊥ to the Sliding Surface.

ii) Curved Surface:

Convex

Concave





l = 6

Ic = 15

I12 I13 I14 I15 I16

I23 I24 I25 I26

I34 I35 I36

I45 I46

I56

13 < 12,23 ; 14,43

15 < 14,45 ; 16,65

24 < 23,34 ; 21,14

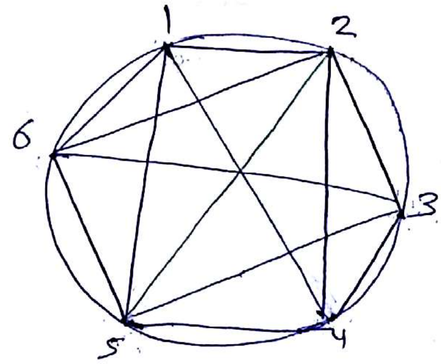
25 < 24,45 ; 21,15

26 < 25,56 ; 21,16

35 < 34,45 ; 32,25 ; 31,15 same

36 < 35,56 ; 32,26 ; 31,16

46 < 41,16 ; 42,26 ; 43,36 ; 45,56



Given $N_{OA} = 120$ rpm Clockwise

$$\omega_{OA} = \frac{2\pi \times 120}{60} = 4\pi \ \frac{Rad}{Sec}$$

$$V_A = 0.2 \times 4\pi = 2.5132 \ m/s$$

link - 3, (A, B) (I_{13}) bcz ground (Absolute Vel. obtain w.r.t. to ground)

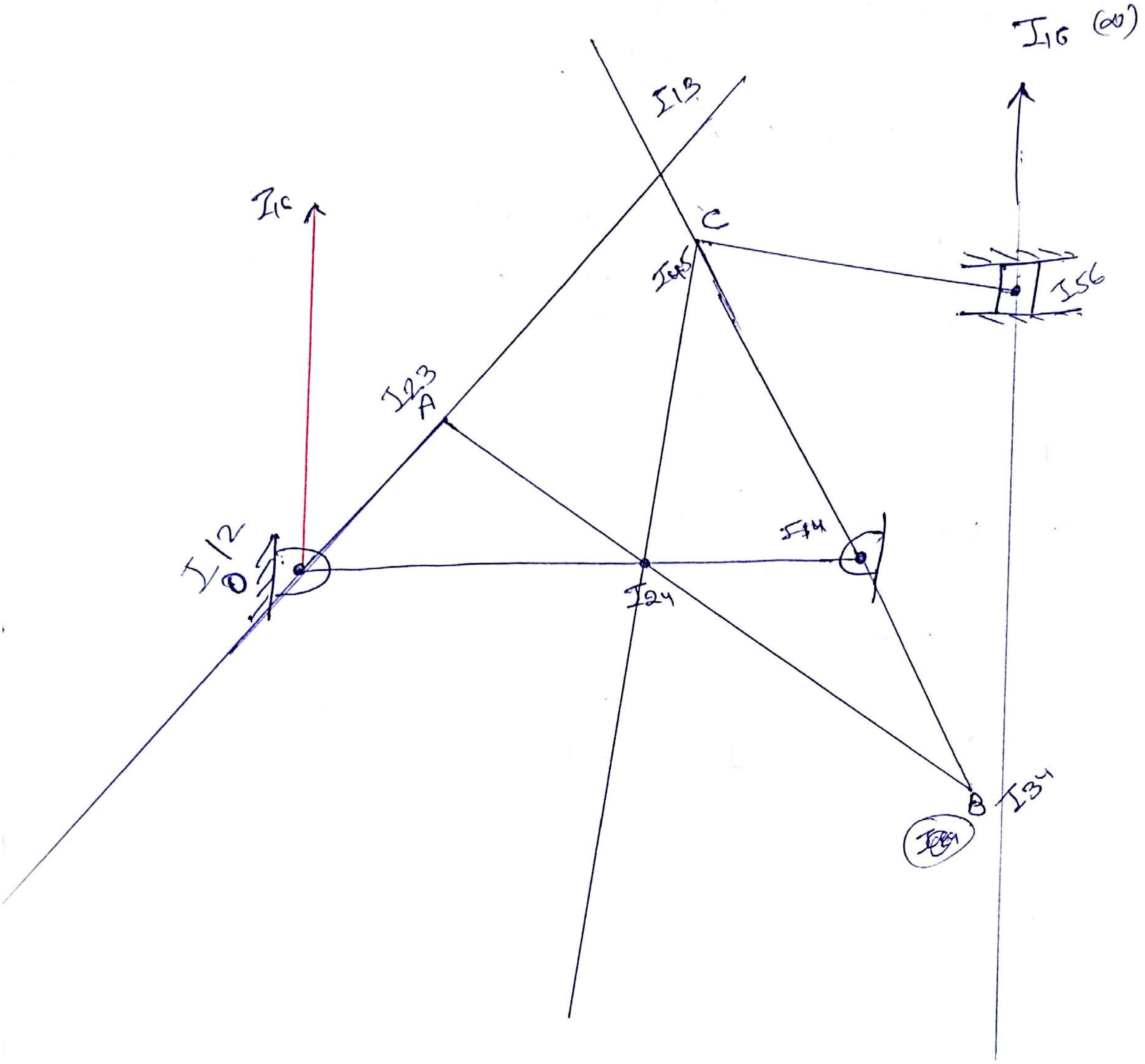
$$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$$

link 4 (B, C) (I_{14})

$$\omega_4 = \omega_{BC} = \frac{V_B}{I_{14}B} = \frac{V_C}{I_{14}C}$$

link 5 (C, D) (I_{15}).

$$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15}C} = \frac{V_D}{I_{15}D}.$$


I16 (∞)
I13
I16
C
I45
I56
I23
A
I34
I12
O
I24
B
I34

Kennedy's Theorem :

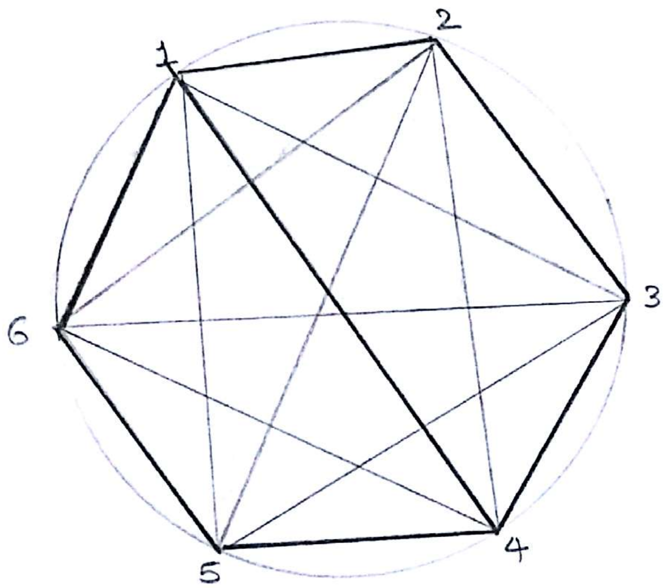
→ For the Relative motion b/w the No. of links in a Mechanism any three links, their three Instantanous centre must lie in straight line.

Qn) 28)

Given link = 6

$$IC = \frac{6(6-1)}{2} = \frac{6\times5}{2} = \frac{30}{2} = 15$$

$\boxed{IC = 15}$



I_{12} I_{13} I_{14} I_{15} I_{16}
I_{23} I_{24} I_{25} I_{26}
I_{34} I_{35} I_{36}
I_{45} I_{46}
I_{56}

① $\omega_3 = \omega_{AB} = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B} \Rightarrow V_B = \frac{2.5132\times1.1}{.8}$
$I_{13}A \to 0.8m$, $I_{13}B \to 1.1m$
$= 3.455 \, m/s$

$\omega_{AB} = \frac{2.5132}{.8} = 3.14 \, rad/s$

② $\omega_4 - \omega_{BC} = \frac{V_B}{I_{14}B} = \frac{V_C}{I_{14}C} = V_C = \frac{3.455\times.2}{.4} = 1.727 \, m/s$
$I_{14}B \to 0.4m$, $I_{14}C \to 0.2m$

$= \omega_{BC} = \frac{3.455}{.4} = 8.637 \, rad/s.$

$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15}C} = \frac{V_D}{I_{15}D} = V_D = \frac{1.725\times.45}{.7} = 1.11 \, m/s$
$I_{15}C \to .7m$, $I_{15}D \to .45m$

$\omega_D = \frac{1.727}{.7} = 2.467 \, rad/s.$

$V_B = 3.2 \, m/s$
$V_C = 1.6 \, m/s$
$V_D = 1.08 \, m/s$
$\omega_{AB} = 2.99$
$\omega_{BC} = 8$
$\omega_{CD} = 2.16$

GRAPHICAL Representation

Scale 1 cm = 100 mm

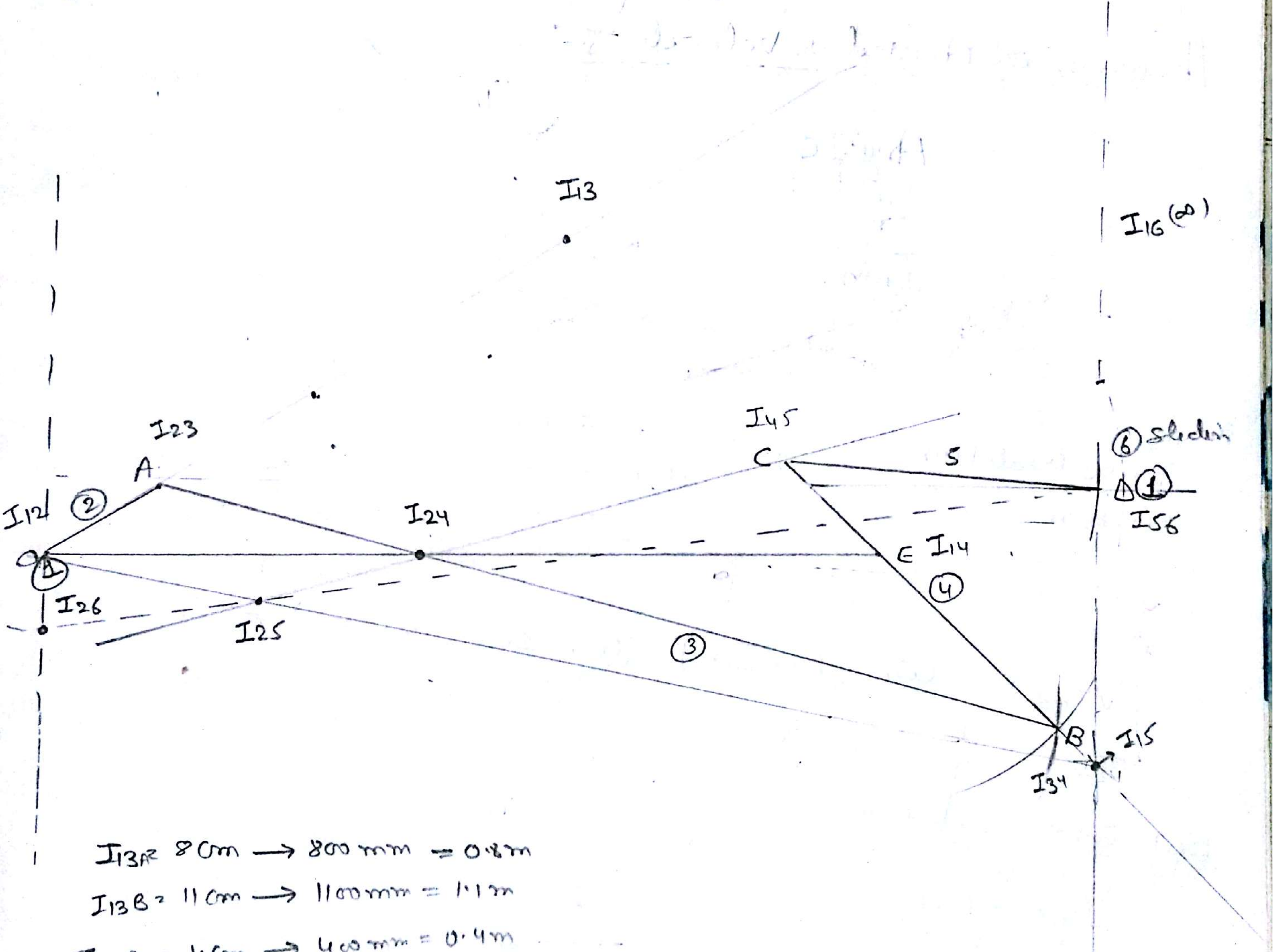
AB = 1.5 m = 1500 mm → 15 cm

BC = 600 mm → 6 cm

CD = 500 mm → 5 cm

BE = 400 mm → 4 cm

OE = 1.35 m → 1350 mm → 13.5 cm.



$I_{13}A$ = 8 cm → 800 mm = 0.8 m

$I_{13}B$ = 11 cm → 1100 mm = 1.1 m

$I_{14}B$ = 4 cm → 400 mm = 0.4 m

$I_{14}C$ = 2 cm → 200 mm = 0.2 m

$I_{15}C$ = 7 cm → 700 mm = 0.7 m

$I_{15}D$ = 4.5 cm → 450 mm = 0.45 m

Kennedy's Theorem :

only for Absolute Results

For the Relative motion b/w the no. of links in a Mechanism, Any three links, their three instantanious Centre must lie in a Straight Line.

Theorem of Angular velocities :-

This Theorem is applied at

$\Downarrow$

$\boxed{I_{mn}}$

Any IC

$\downarrow$

I_{mn}

Can be treated on link m

Can be treated on link n

***

$$\boxed{V_{I_{mn}} = \omega_m (I_{mn} I_{1m}) = (\omega_n I_{mn} I_{1n}) = V_{I_{mn}}}$$

But total IC in use :-

I_{mn} — link 1

I_{1m} — link 2

I_{1n} — link 3

If I_{1m}, I_{1n} lie at the Same side of I_{mn} $\rightarrow$ Dir^n will be Same.

If I_{1m}, I_{1n} dont lie at same side of I_{mn} $\rightarrow$ Dir^n will be different.

ω_2 = Given in previous Question (Clockwise dirⁿ) of ② link

I_{25} :

$$\omega_2 \underbrace{(I_{25} I_{12})}_{\text{Distance}} = \omega_5 (I_{25} I_{15}) \quad \text{(A·C)}$$

I_{24} :-

$$\omega_2 (I_{24} I_{12}) = \omega_4 (I_{24} I_{14}) \quad \text{AC}$$

I_{45} :-

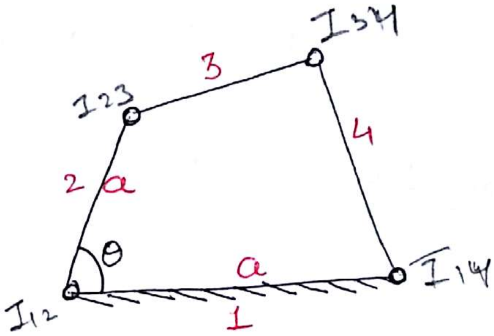
$$\omega_4 (I_{45} I_{14}) = \omega_5 (I_{45} I_{15}) \quad \text{AC}$$

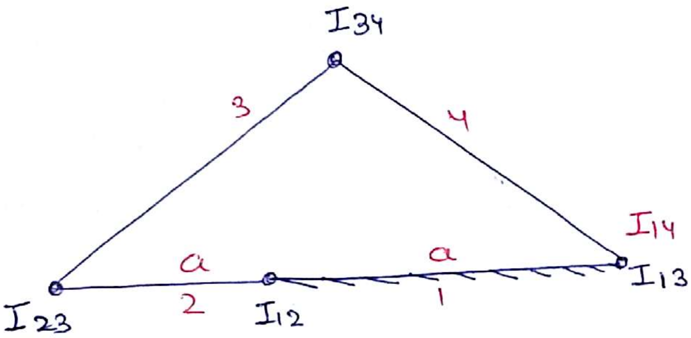
Problem 1

When $\theta = 180°$

$\omega_2 = 5 \text{ rad/s}$

$\omega_3 = ?$

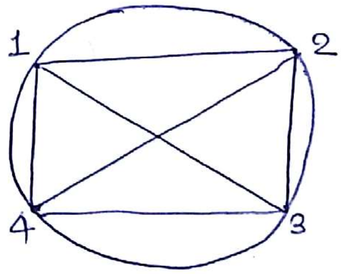




$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$5(\cancel{a}) = \omega_3 \, 2(\cancel{a})$$

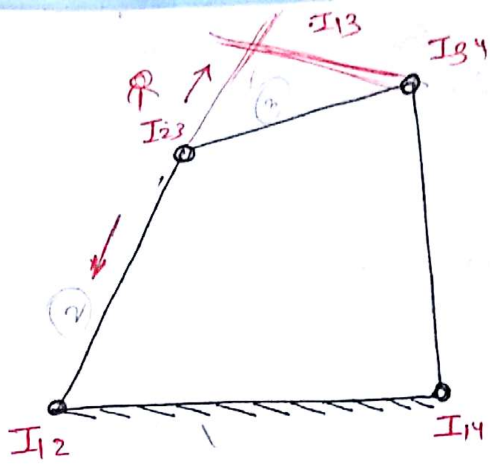
$$\boxed{\omega_3 = 5 \text{ rad/sec}} \quad \text{Ans}$$



13 < 12, 23 ; 14, 43

24 < 23, 34 ; 21, 14

Qn) -



$\omega_2 = 5$ rad/s (clock)

$\omega_3 = 14$ rad/s

find the Angular velocity of link 2 w.r.to link 3   clockwise (+)

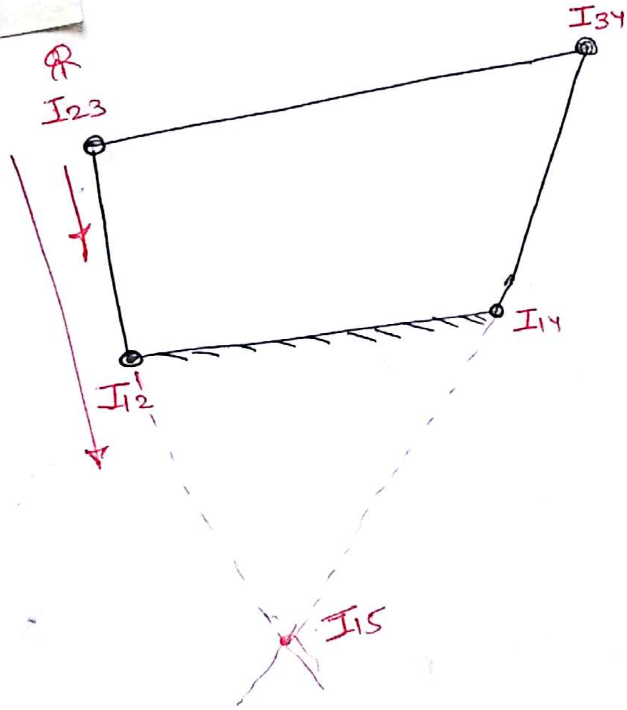
$\vec{\omega}_{23} = \vec{\omega}_2 - \vec{\omega}_3$

$= (+5) - (-14)$

$= 5 + 14$

$= 19$ rad/sec. clock.

Qn)



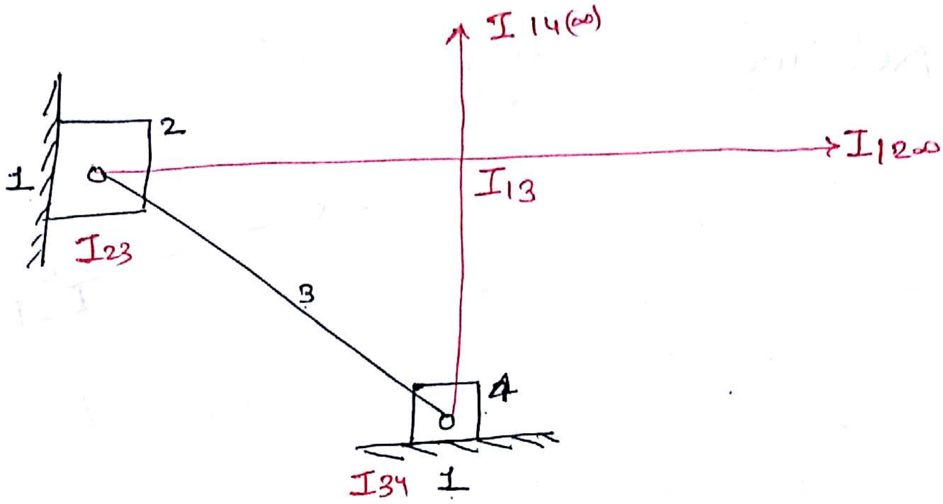
$\vec{\omega}_{23} = +5 - (+14)$

$= -9$

$\vec{\omega}_{23} = -9$ Rad/sec

$\vec{\omega}_{23} = 9$ rad/sec (AC)

Pb ... rk



Where is I 24?

23, 24

24

21, 24
(∞)(∞)

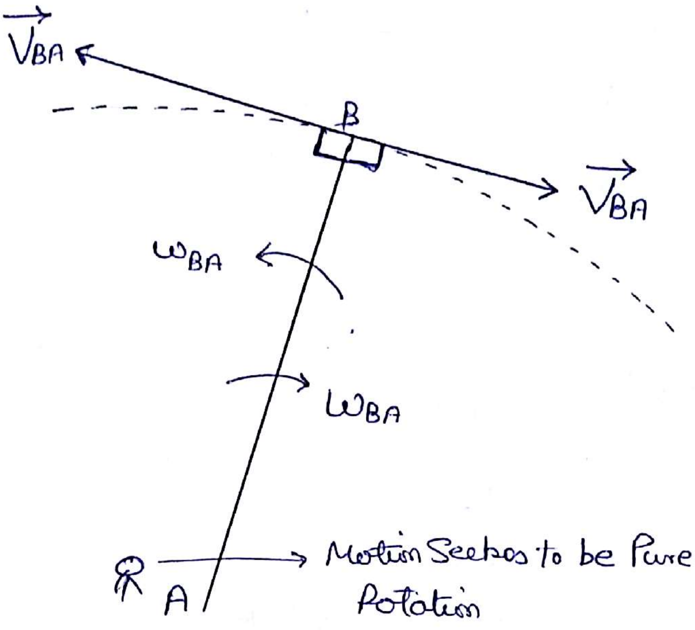
will lie at (∞)

infinitely only displaced line

Ans

Relative Velocity Method :-

The Velocity of Point B, w.r.to. point A will be in the dirⁿ ⊥ to the link AB.



Point	W.r.to	Procedure
A	O	Plot link ⊥ to link OA
B	A	Plot link ⊥ to link AB
B	E	Plot link ⊥ to link EB
C	$BC/BE = bc/be$	bc/be
D	C	line ⊥ to link CD
D	fixed	line ∥el to the motion of Slider

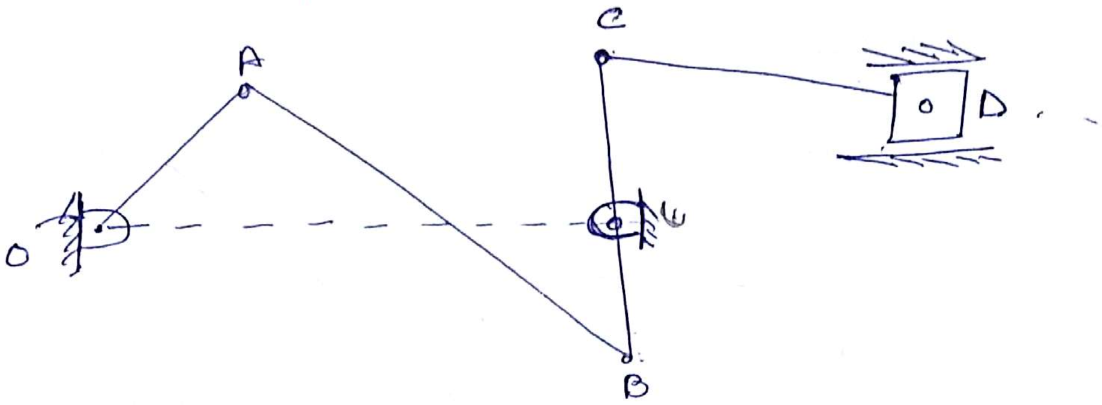
$$\omega_{AB} = \frac{V_{AB}}{AB} \text{ Rad/s (AC)}$$

$$\omega_{BC} = \frac{V_{BC}}{BC} \text{ Rad/s (AC)}$$

$$\omega_{CD} = \frac{V_{CD}}{CD} \text{ Rad/s (AC)}$$

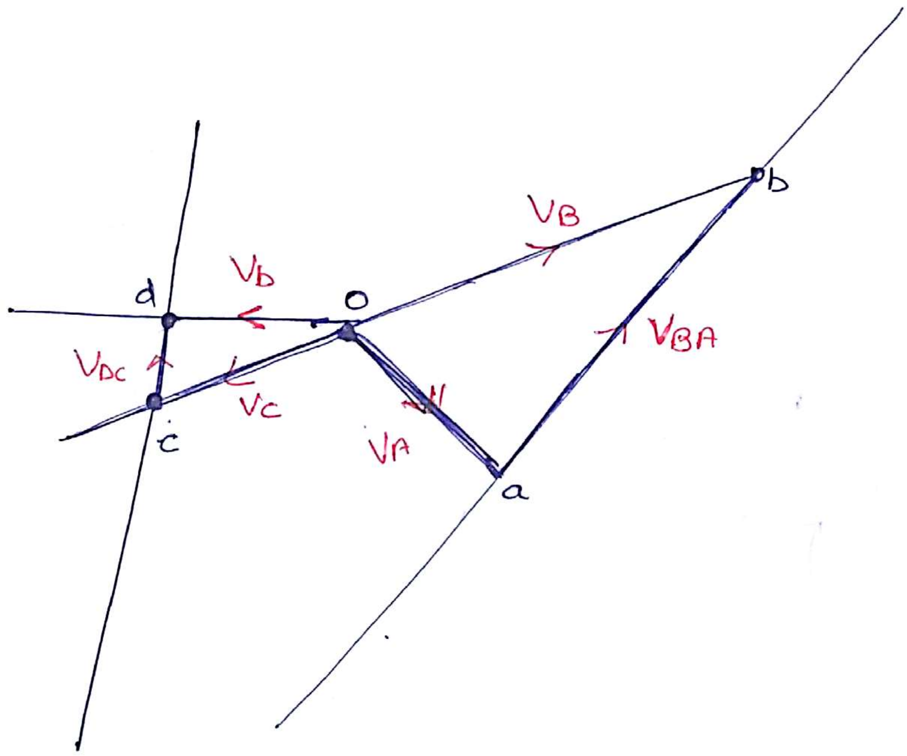
Graphical
Scale.

All letters in Configuration dia. will be capital



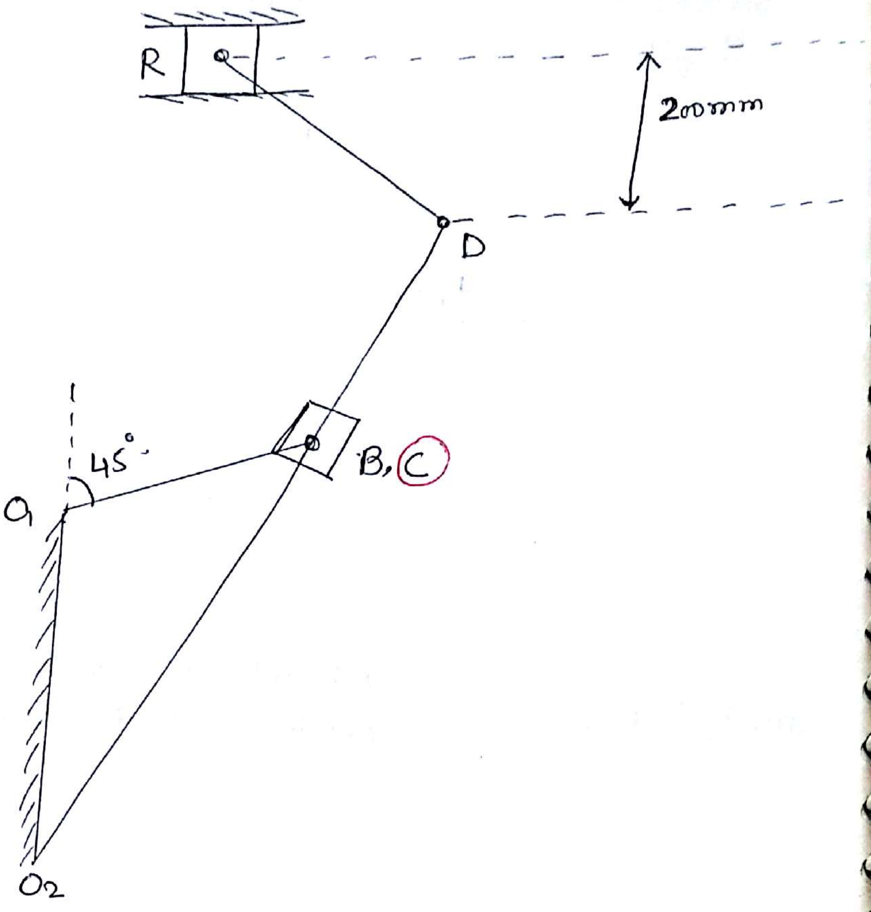
Velocity Profile

Absolute → motion with Respect to fixed
Relative → Motion w.r.to to another



Pb.

If Crank O_1B
↳ 40 rpm
(AC)



Given data

$V_B = (O_1B)\,\omega_{O_1B}$

$V_B = 0.25 \times \frac{2\pi \times 40}{60}$ m/s

↓

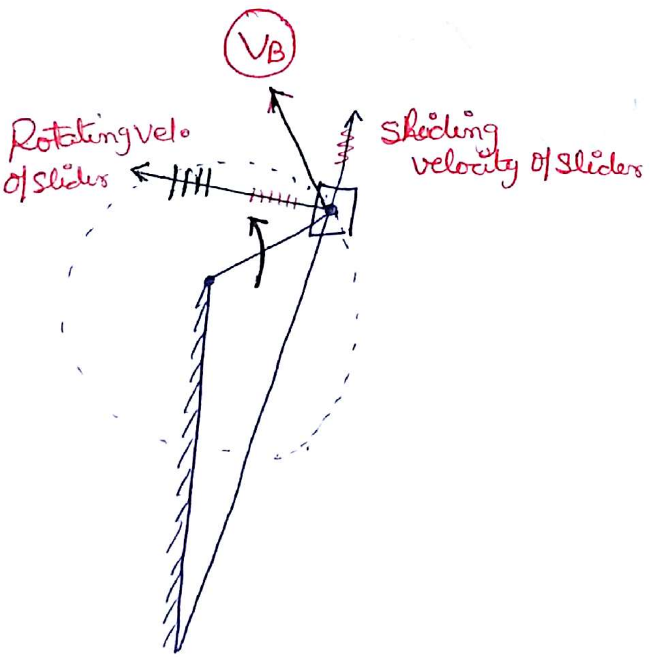
Scale of Velocity

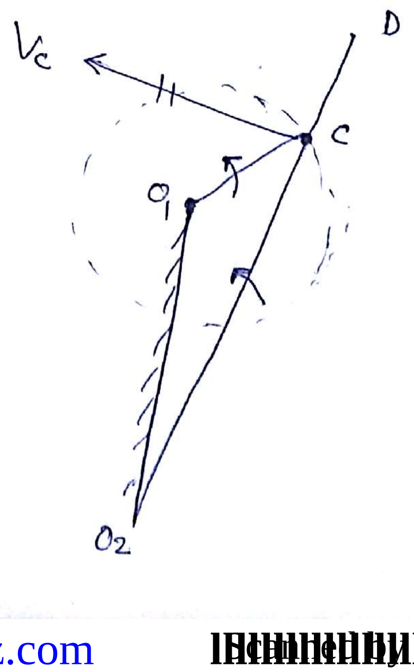
Point C

↓

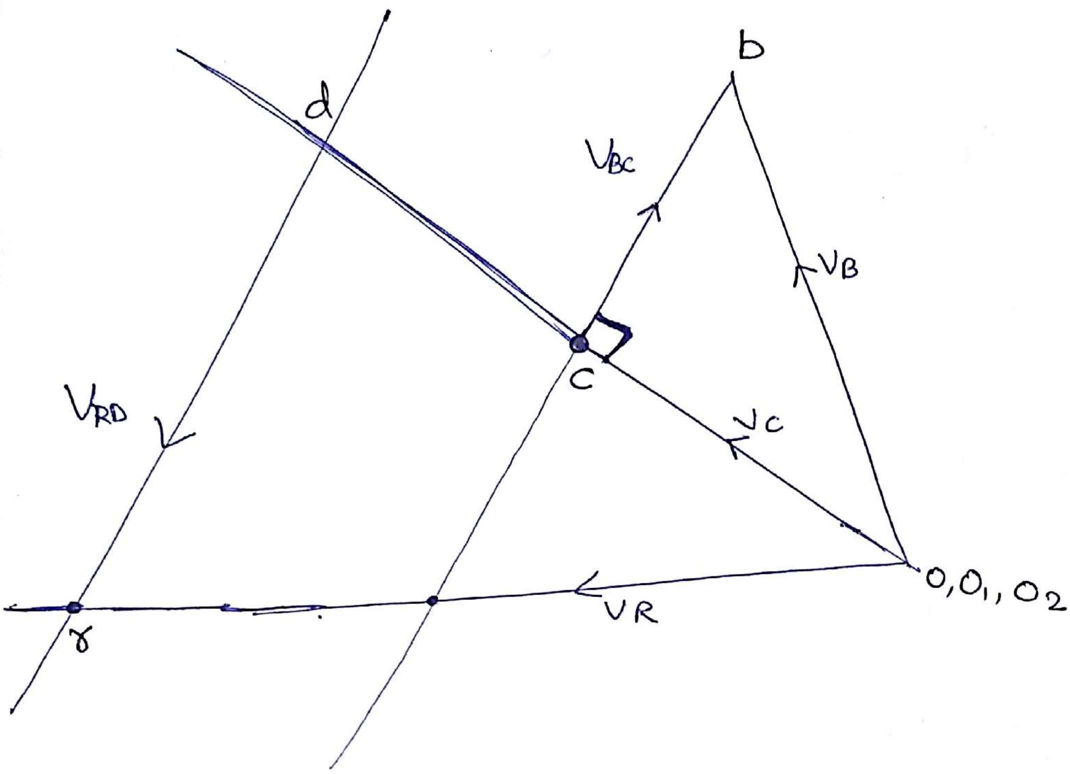
Coincident Point of Slider B, But on Slotted bar.

Slotted bar → O_2CD



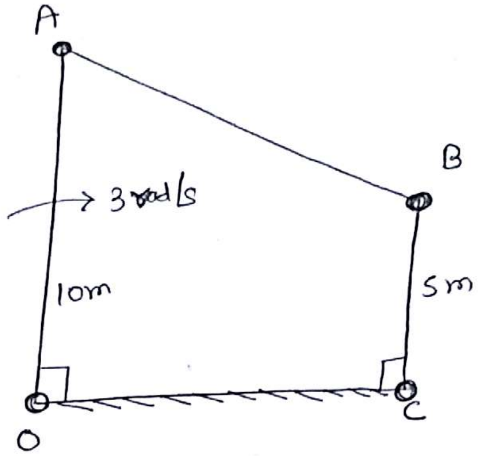


Point	w.r.t	Procedure
C	O_2	line ⊥ to link O_2C
C	B	line ∥el to link O_2C

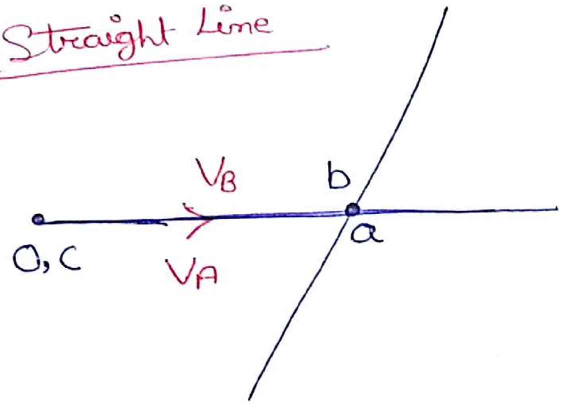


Pb. 1) (2M)

The velocity diagram for the given configuration at this instant will be :-



Straight Line



Pb2) $V_B = ?$

(2M)

$V_B = V_A = 10 \times 3 = 30 m/s$

Pb. 3) $\vec{V}_{BA} = ?$ $\boxed{V_B = V_A}$

Zero.

Pb 4) The motion of link AB at this Instant will be,

$$\omega_{AB} = \frac{\cancel{V_{AB}}^{0}}{AB} = 0$$

Pure Translation

Pb. 5

2M

$\omega_{BC} = ?$ *

$V_B = V_A$

$5 \times \omega_{BC} = 10 \times 3 \Rightarrow \omega_{BC} = \frac{30}{5} = 6$ rad/sec

Q.6 2M

1 ——
2 ——
3 ——
4 ——
5 ——

a) 1, 2, 3

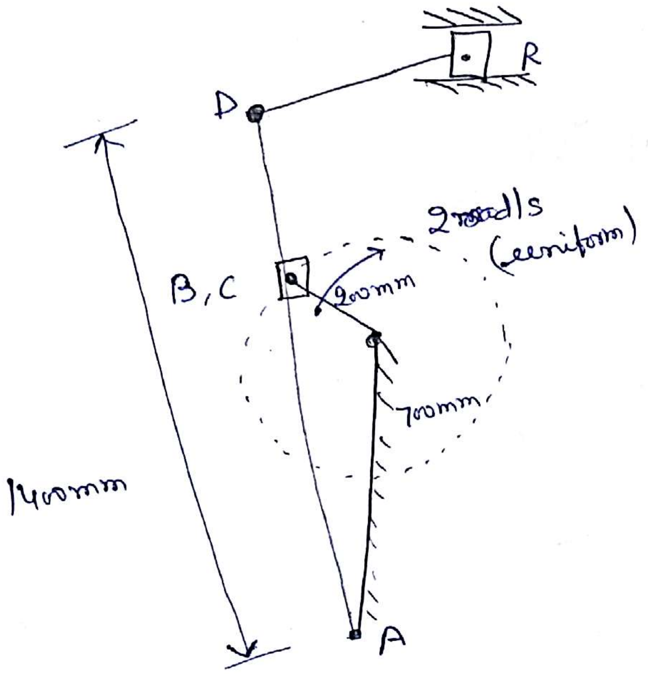
b) 2, 4 - - -

c) 1, 3 —

D) - - NOT.

Pb

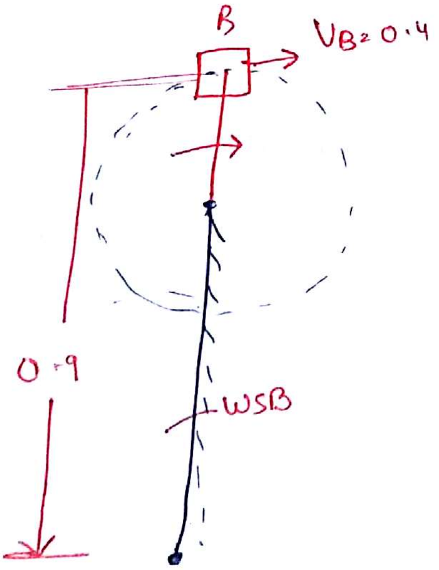
$V_B = 0.2 \times 2 = 0.4$ m/s.
uniform)

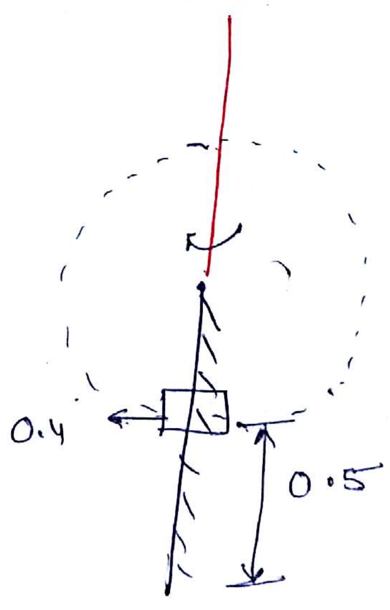


find the Avg. Velocity of Slotted Bar, when Vel. of RAM is Max.

Return Stroke

Cutting Stroke





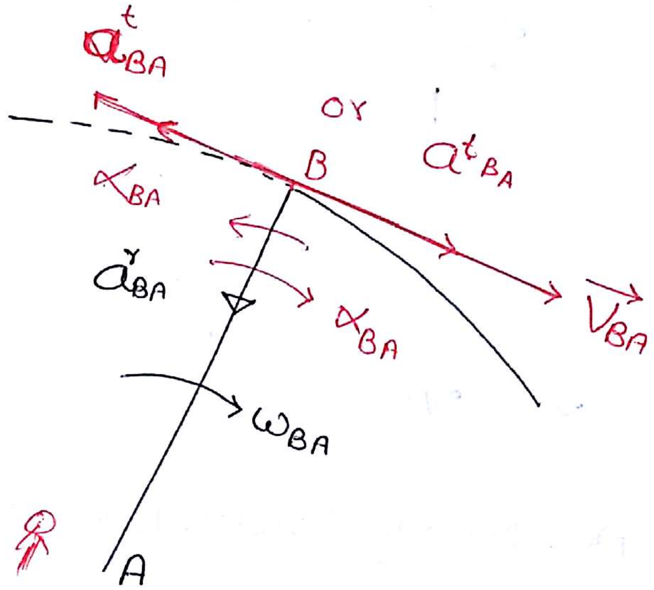
$\omega_{SB} = \frac{0.4}{0.9} = 4/9$ rad/sec

$\omega_B = \frac{0.4}{0.5}$ 4/5 rad/sec.

Accelaration Diagram:

(only objective)

Accelaration Analysis:-



Because of the change in dir^n of velocity

$$\vec{a}_{BA} = \frac{d}{dt}\vec{V}_{BA}$$

$a^r_{BA} = \frac{V_{BA}^2}{BA}$ (B→A)

Known for all link (find via vel. dig)

$a^t_{BA} = (BA)\alpha_{BA}$

Bcz of change in magnitude of velocity.

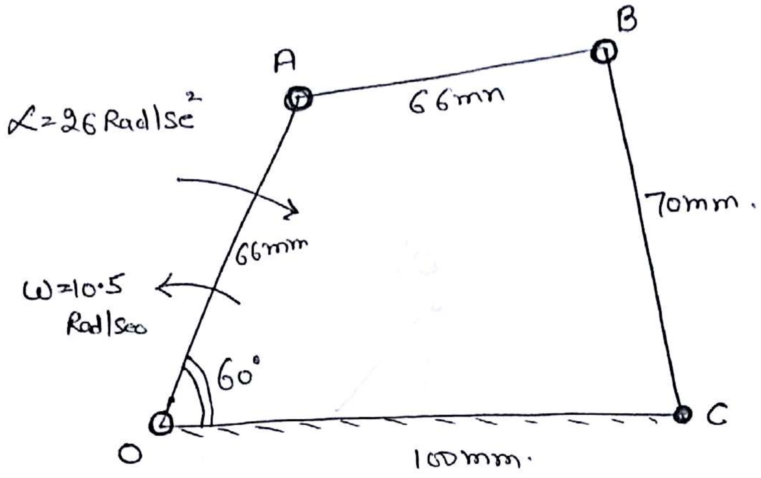
May or May not be zero

→ only for Known input link.

→ $\perp$ to the Radial dir^n

→ not mentioned in Ques assumed to be 0.

Qn)



find → $a_B = ?$ $\quad \alpha_{BA} = ?$

$a_{BA} = ?$ $\quad \alpha_{BC} = ?$

ans)

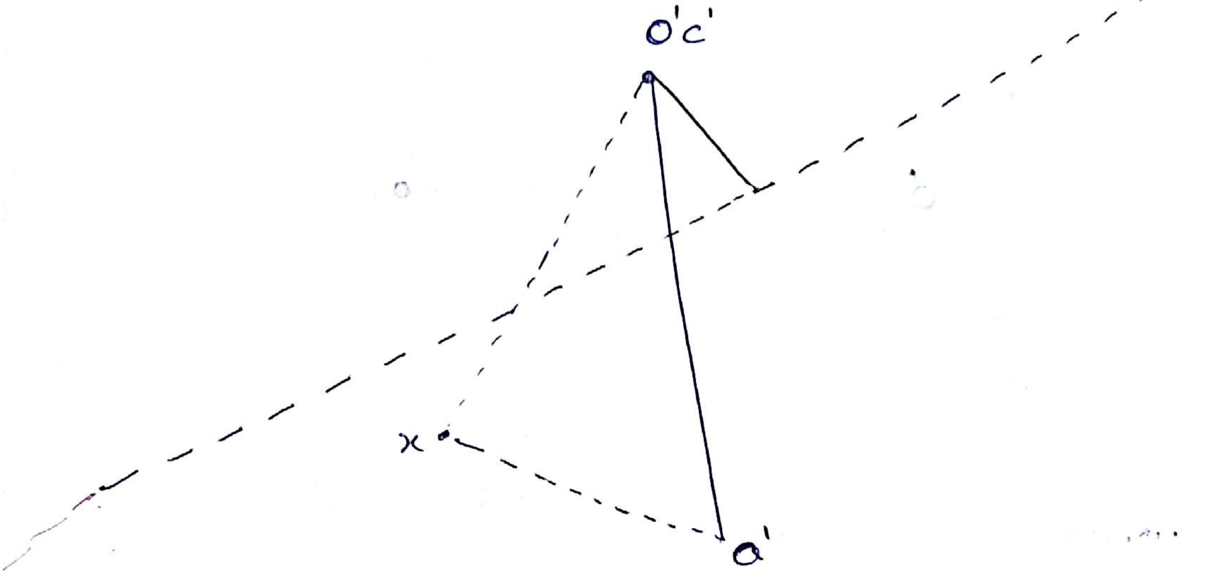
Point	w.r.to	Procedure
A	O	$a^r_{AO} = \frac{V^2_{AO}}{AO}$ (A → O) (known) $a^t_{AO} = (AO)\,\alpha_{AO}$ (Known) (⊥ to dirⁿ of Radial)
B	A	$a^r_{BA} = \frac{V^2_{BA}}{BA}$ (B → A) $a^t_{BA} = (BA)\,\alpha_{BA}$ (unknown)
B	C	$a^r_{BC} = \frac{V^2_{BC}}{BC}$ (B → C) (known) $a^t_{BC} = (BC)\,\alpha_{BC}$ (unknown) ⊥ to dirⁿ of Radial

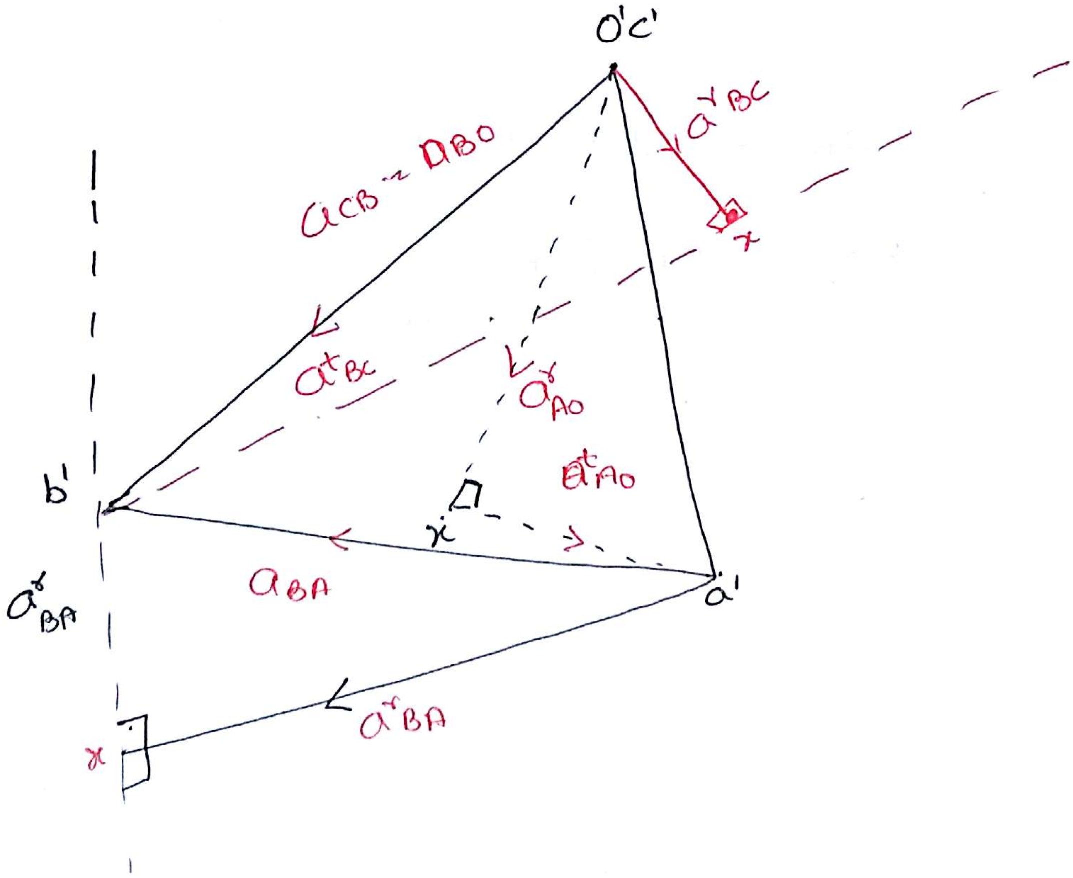
$$a^r_{AO} = \frac{V^2_{AO}}{AO}$$

$$a^r_{BA} = \frac{V^2_{BA}}{BA}$$

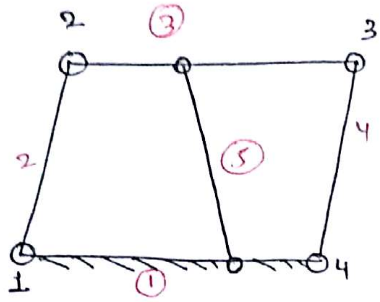
$$a^r_{BC} = \frac{V^2_{BC}}{BC}$$

$$a^t_{(AO)} = (AO)\,\alpha_{AO}$$


o'c'
x
a'

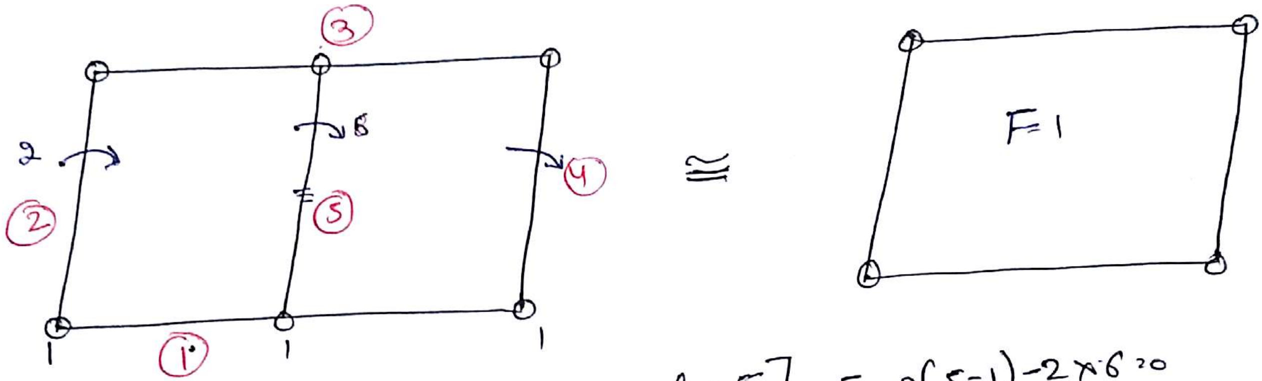

o'c'
a^r_{BC}
x
$a_{CB} = a_{BO}$
a^t_{BC}
a^r_{AO}
a^t_{AO}
b'
x
a_{BA}
a'
a^r_{BA}
a^r_{BA}
x

Note:



$l = 5$
$j = 6$
$h = 0$

$F = 3(5-1) - 2\times 6$
$F = 0$ frame structure



$l = 5$
$j = 6$
$h = 0$

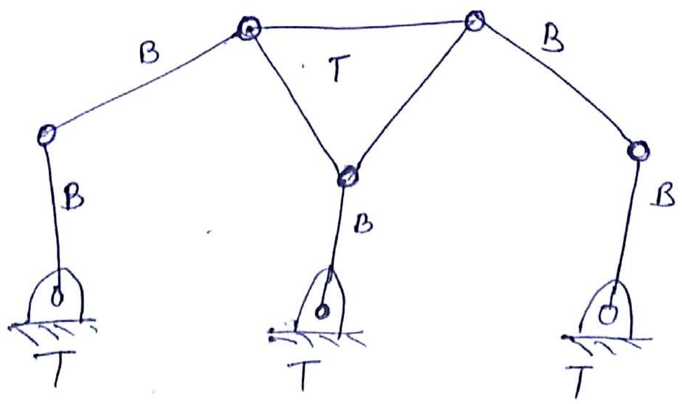
$F = 3(5-1) - 2\times 6 = 0$

$\boxed{F=0}$ $\boxed{F=1}$ **

Binary link → Connected at two place

Ternary link → Connected at three place

Quaternary link → Connected at four place.



KLEIN'S CONSTRUCTION

"It is only applied in single slider crank ~~Mech~~ Mechanism (BASIC)"

When

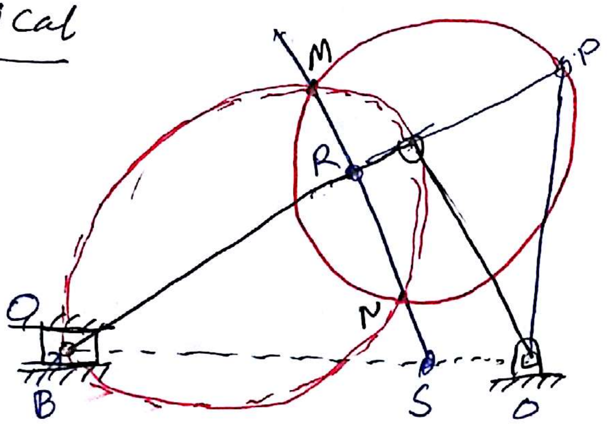
$\alpha_{crank} = 0$

Given

→ ω_{crank}

GRAPHICAL

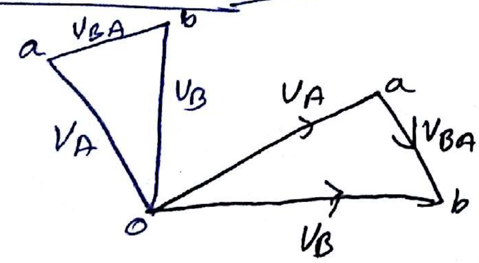
Scale



Δ OAP

→ velocity Δ

Velocity Dig (Rough)



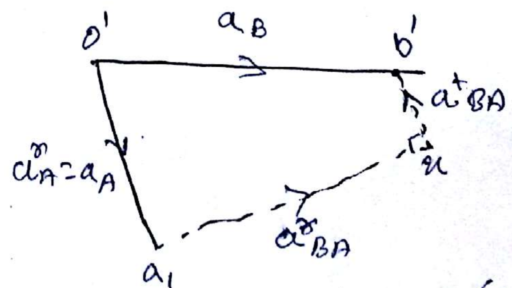
Given

$$\frac{\overset{?}{V_A}}{OA} = \frac{\overset{?}{V_B}}{OP} = \frac{\overset{?}{V_{BA}}}{AP} = \omega_{crank}$$

Δ OARS

→ Acce^n Δ

Acce^n Dia (Rough)



Given

$$\frac{\overset{?}{a_A}}{OA} = \frac{\overset{?}{a^t_{BA}}}{RS} = \frac{\overset{?}{a_B}}{OS} = \frac{\overset{?}{a^r_{BA}}}{AR} = \omega_{crank}$$

Coriolis Accⁿ (a^c):-

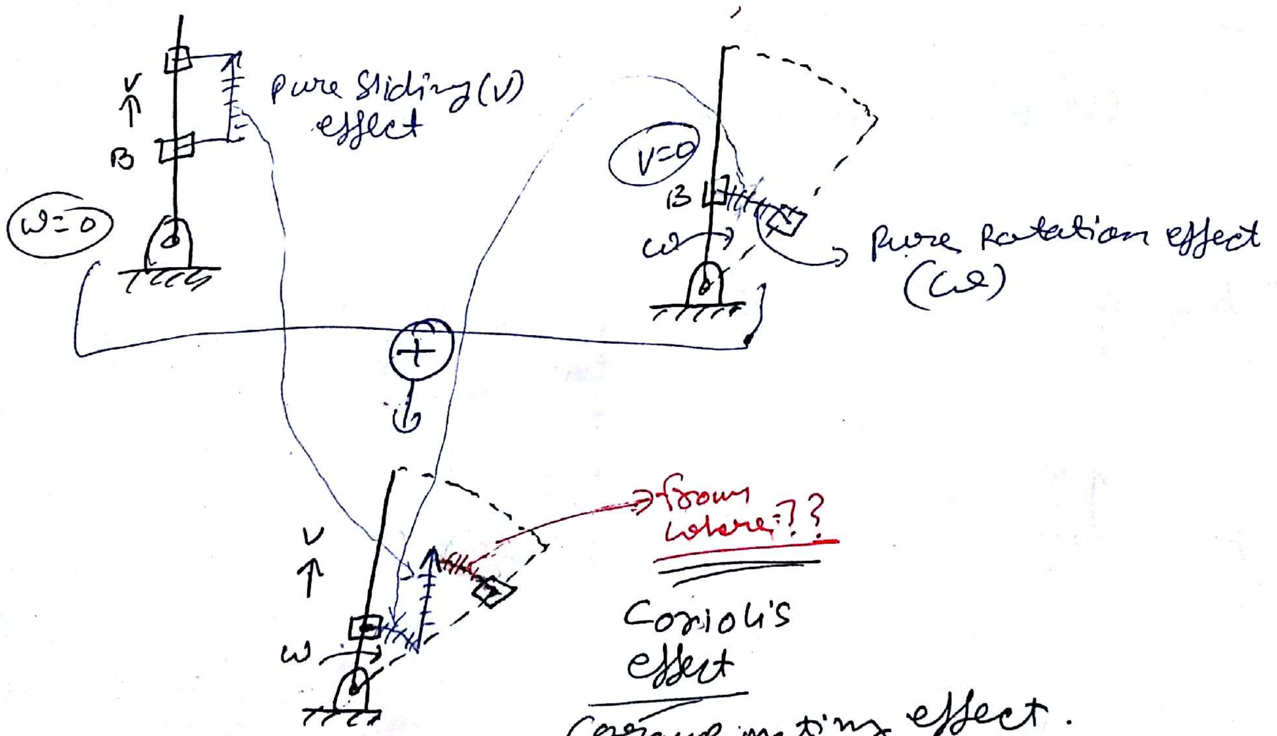
"This accⁿ will always be associated with the slider when the slider is sliding on the rotating body"

The magnitude is (a^c)

$$\boxed{a^c = 2v\omega}$$

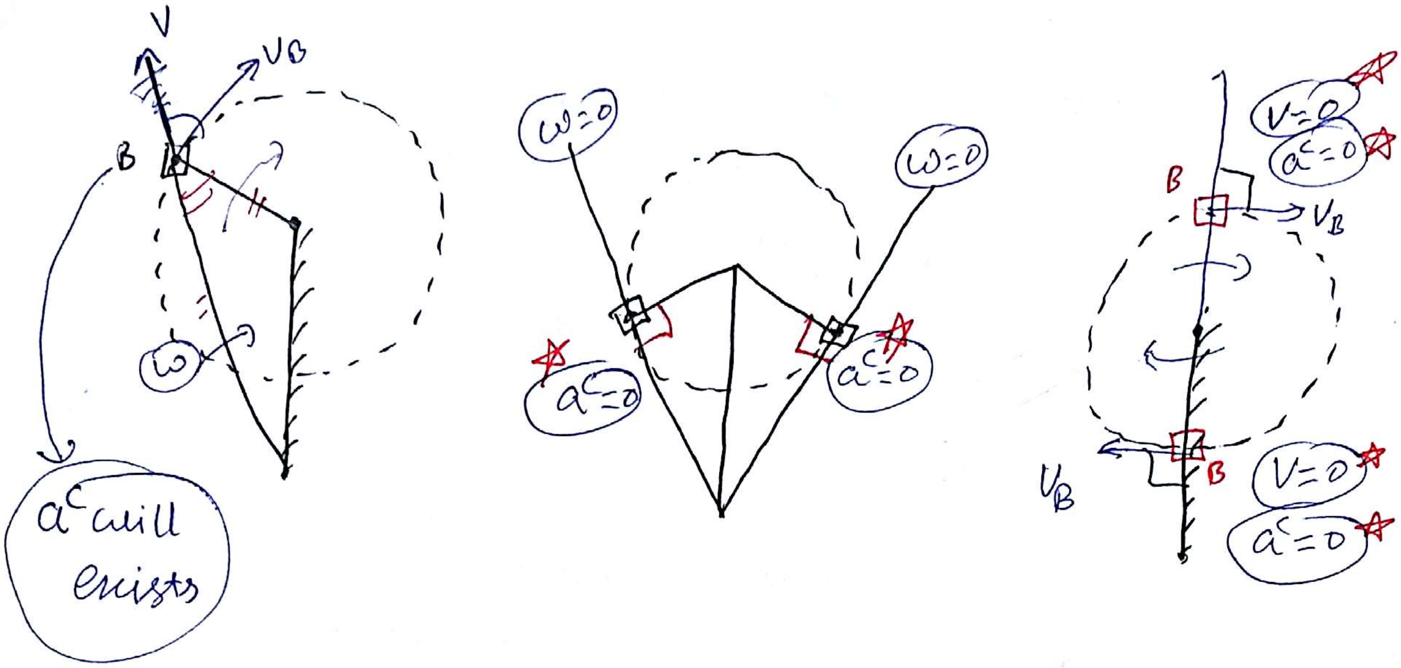
$V \Rightarrow$ Sliding vel. of slider

$\omega \to$ Ang. vele of body on which slider is sliding.



Coriolis effect

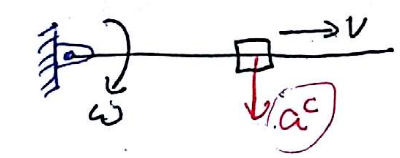
(Group mating effect.



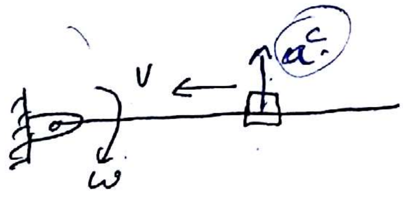
Direction of (a^c):

(i) Take the sense of ω.
(ii) Rotate the $\vec{V}$ in that sense by 90°.

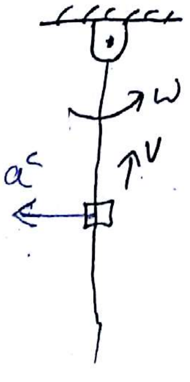
1)



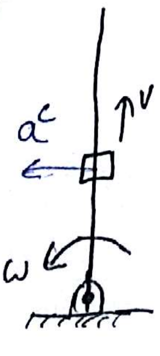
2)



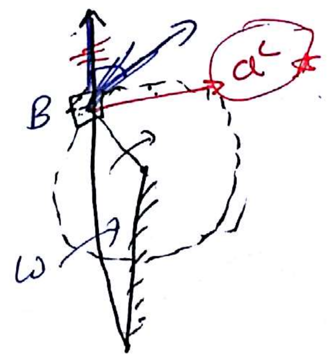
3)



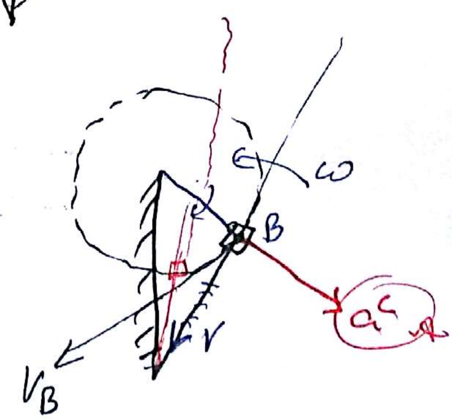
4)



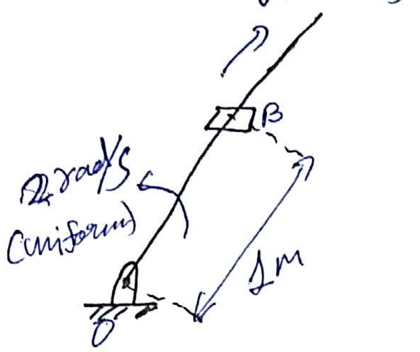
5)



6)



Prob



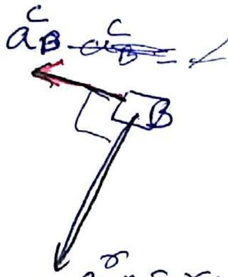
Find the Acce^n of slider $a_B = ?$

$\omega = 2$ rad/s $\quad$ V/2

$V = 0.75$ m/s $\quad$ $2V\omega$

$a_B^c = 2V\omega$

$= 3\ m/s^2$



$a_B^r = r\omega^2$

$=(1)(2)^2$

$= 4\ m/s^2$

$a_B = \sqrt{(3)^2 + (4)^2}$

$= 5\ m/s^2$ Ans

INSTANTANEOUS FLUCTUATION OF SPEED CONTROL DEVICES :-

- Intra-cycle fluctuation → Flywheel
- Inter-cycle fluctuation → governors

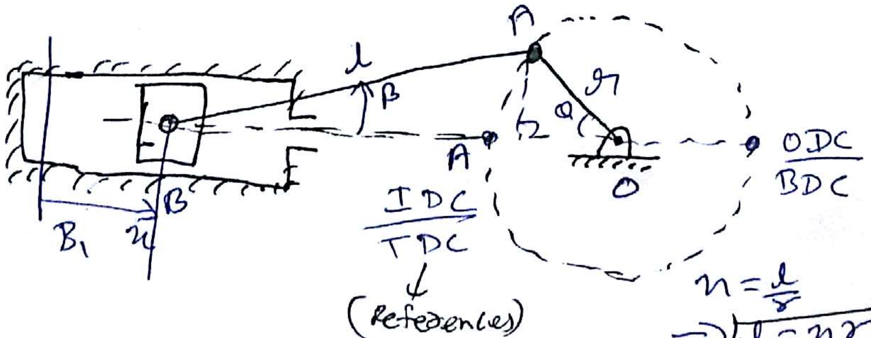
Engine

- S → 8000 rpm
- C → 300 rpm ← Suddenly change
- P → 200 J → Crank Shaft → 10000 rpm
- E → 900 rpm

Kinematic Analysis of Single Slider Crank Mechanism :-

(Inertia of connecting rod is not considered in this analysis for being the lightest body)

m



$n = \frac{l}{r}$

$\Rightarrow \boxed{l = nr}$

$\text{Am} \left\{ l \sin\beta = r\sin\theta \right.$

$\sin\beta = \frac{\sin\theta}{n}$

$$\cos\beta = \sqrt{1-\frac{\sin^2\theta}{n^2}} = \frac{\sqrt{n^2-\sin^2\theta}}{n}$$

$m \rightarrow$ Mass of Reciprocating Parts.

$l \rightarrow$ length of Conecting rod (C.R)

r = radius of crank.

$n = \frac{l}{r} \Rightarrow$ obliquity Ratio (large)

$\omega \Rightarrow$ Ang. vel. of Crank

$\theta \rightarrow$ Angle turned by crank from IDC/TDC

$$\boxed{\omega = \frac{d\theta}{dt}}$$

PISTON MOTION

Displacement

$$x = B_1B$$

$$= B_1O - BO$$

$$= (r+l) - (BM+MO)$$

$$= (r+l) - (l\cos\beta + r\cos\theta)$$

$$= r + nr - \cancel{n}r \cdot \frac{\sqrt{n^2-\sin^2\theta}}{\cancel{n}} - r\cos\theta$$

$$= r(1-\cos\theta) + r(n - \sqrt{n^2-\sin^2\theta})$$

$$\boxed{x = r\left[(1-\cos\theta) + (n - \sqrt{n^2-\sin^2\theta})\right]}$$

VELOCITY (v)

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} \rightarrow \omega$$

$$v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

$$\boxed{v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]}$$ (n large)

Acce^n (a) :-

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} \rightarrow \omega$$

$$\boxed{a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$ (n large)

Connecting Rod Motion :-

Ang. Velocity (ω_{CR})

$$\omega_{CR} = \frac{d\beta}{dt}$$

$$\sin\beta = \frac{\sin\theta}{n}$$

Diff. Both side

$$\cos\beta \cdot \frac{d\beta}{dt} \rightarrow \omega_{CR} = \frac{\cos\theta}{n}\cdot\frac{d\theta}{dt} \rightarrow \omega$$

$$\omega_{CR} = \frac{\omega\cos\theta}{\cancel{n}\cdot\frac{\sqrt{n^2 - \sin^2\theta}}{\cancel{n}}} \rightarrow 0$$

$$\boxed{\omega_{CR} = \frac{\omega\cos\theta}{n}}$$

Ang. Acce^n (α_{CR})

$$\alpha_{CR} = \frac{d\omega_{CR}}{dt} = \frac{d\omega_{CR}}{d\theta} \times \frac{d\theta}{dt} \rightarrow \omega$$

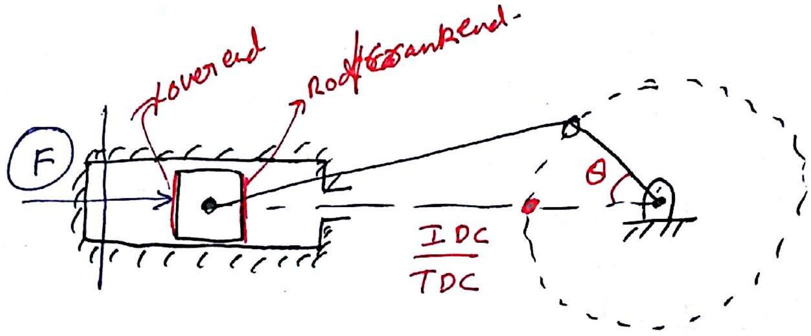
$$\boxed{\alpha_{CR} = -\frac{\omega^2}{n}\sin\theta}$$

Dynamic analysis of Single Slider Crank Mechanism :-

1) Effetive Driving Force to Drive the Piston :-

(Piston Effort) (F)

(Calculated from cover end to crank end)



Support

Gas pressure Force

$P_1, P_2 \rightarrow$ Pr. of gas at cover end & Crank end side of piston

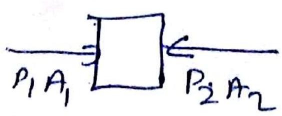
$A_1, A_2 \rightarrow$ Cross-sec. areas of piston at Cover end & crank end side.

$D \rightarrow$ Piston Dia.,

$d \rightarrow$ Piston Rod dia.

$$A_1 = \frac{\pi}{4} D^2$$

$$A_2 = \frac{\pi}{4} (D^2 - d^2)$$



$$F_{gas} = (P_1 A_1 - P_2 A_2)$$

Oppose :-

1) Inertia Force

$$F_I = m.a = m.r\omega^2\left\{\cos\theta + \frac{\cos 2\theta}{n}\right\}$$

(ii) Kinetic Force (friction (f))

If $\theta \in [0, 180°] \rightarrow -f]$

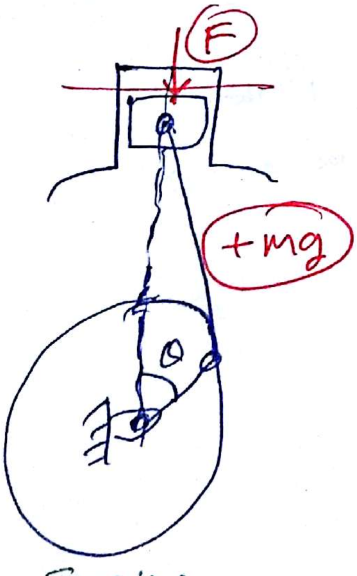
If $\theta \in [180°, 360°] \rightarrow +f]$

PISTON EFFORT

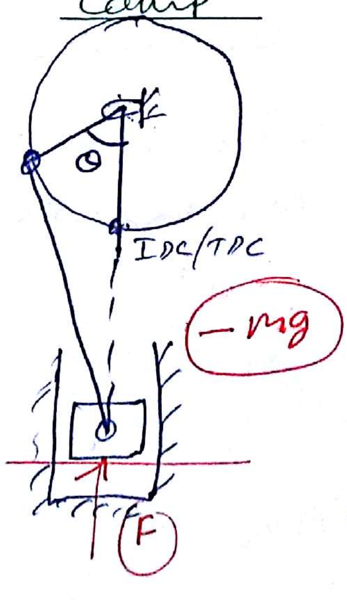
(Cover end to Crank end)

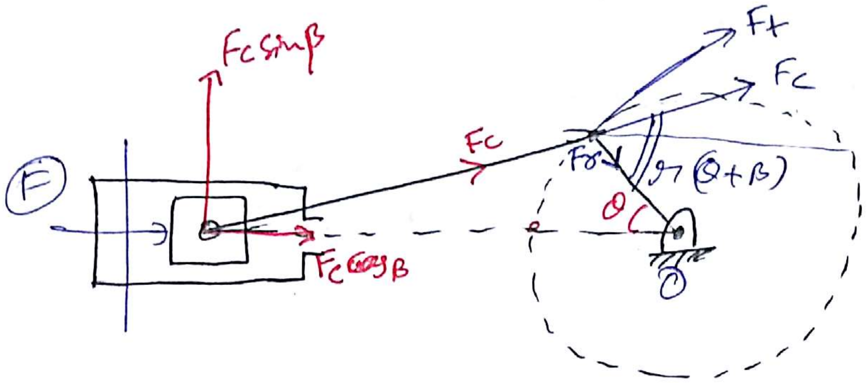
$$\boxed{F = (F_{gas} - F_I \pm f) \pm Mg}$$

use incase of only vertical engine



Engine





② Force Along C.R (F_c)

$$F_c \cos\beta = F$$

$$\boxed{F_c = \frac{F}{\cos\beta}}$$

③ Normal thrust to Cy. Walls (F_n):-

$$F_n = F_c \sin\beta$$

$$\boxed{F_n = F \cdot \tan\beta}$$

④ Crank effect (F_t)

$$F_t = F_c \sin(\theta+\beta)$$

$$\boxed{F_t = \frac{F}{\cos\beta} \sin(\theta+\beta)}$$

⑤ Radial Thrust to Crank Shaft Bearings (F_r):-

$$F_r = F_c \cos(\theta+\beta)$$

$$\boxed{F_r = \frac{F}{\cos\beta} \cdot \cos(\theta+\beta)}$$

⑥ Turning Moment on Crank Shaft (T):-

$$T = F_t \cdot r$$

Output of engine

$$\boxed{T = \frac{F}{\cos\beta} \cdot \sin(\theta+\beta) \cdot r}$$

$T = f(\theta)$
& $\theta = f(\text{time})$

$T = f_n(\text{Time})$

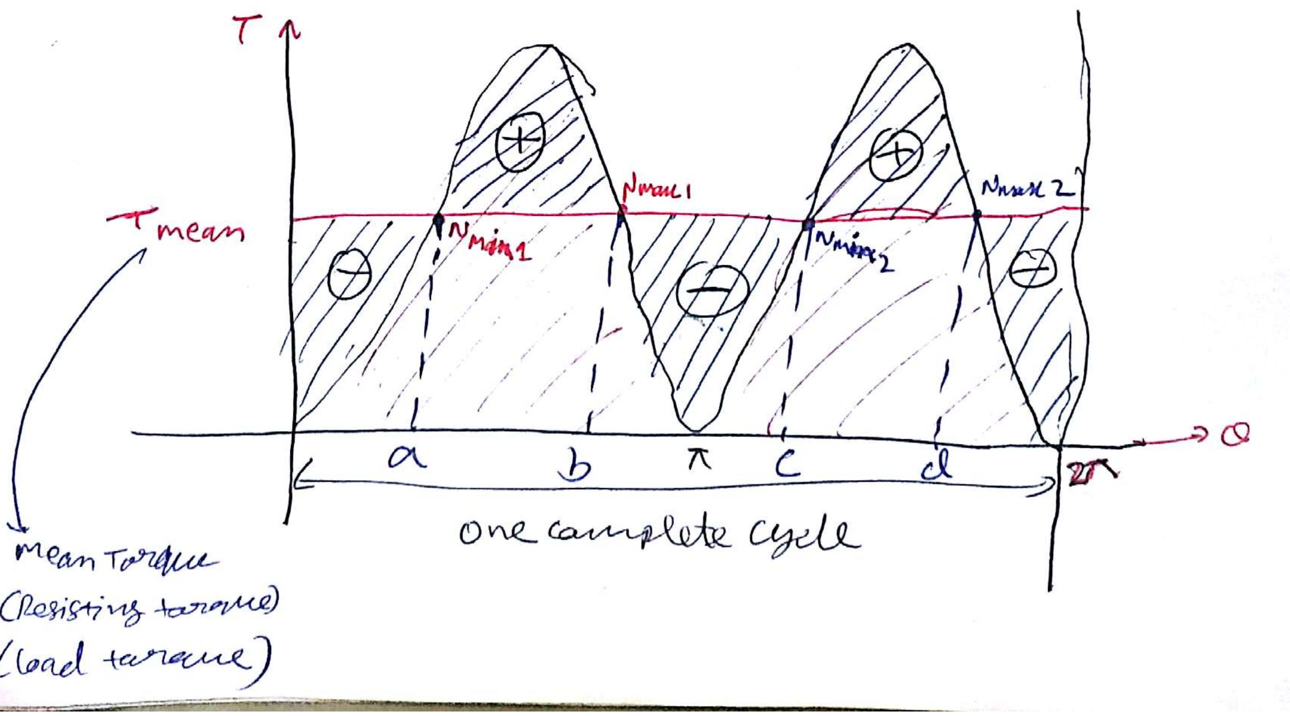
Inertia

$I\alpha = F_n(\text{Time})$

$\Rightarrow \alpha = f_n(\text{time})$

Fluctuations with Jerks

TURNING MOMENT DIG. OF SINGLE ~~CY.~~ DOUBLE ACTING STEAM ENGINE:-



Area under (T-θ) Dig. in a cycle

⇓

$$\boxed{W_{cycle} = T_{mean} \times 2\pi}$$

$$\boxed{T_{mean} = \frac{W_{cycle}}{2\pi}}$$

Even after Installing the flywheel:

min (N_{min_1}, N_{min_2}, - - - -)

→ N_{min}.

Max (N_{max_1}, N_{max_2}, - - - -)

→ N_{max}

$$\frac{(N_{max} - N_{min})}{} \rightarrow \text{Fluctions still left}$$

$$\frac{(N_{max} - N_{min})}{N} = C_s$$

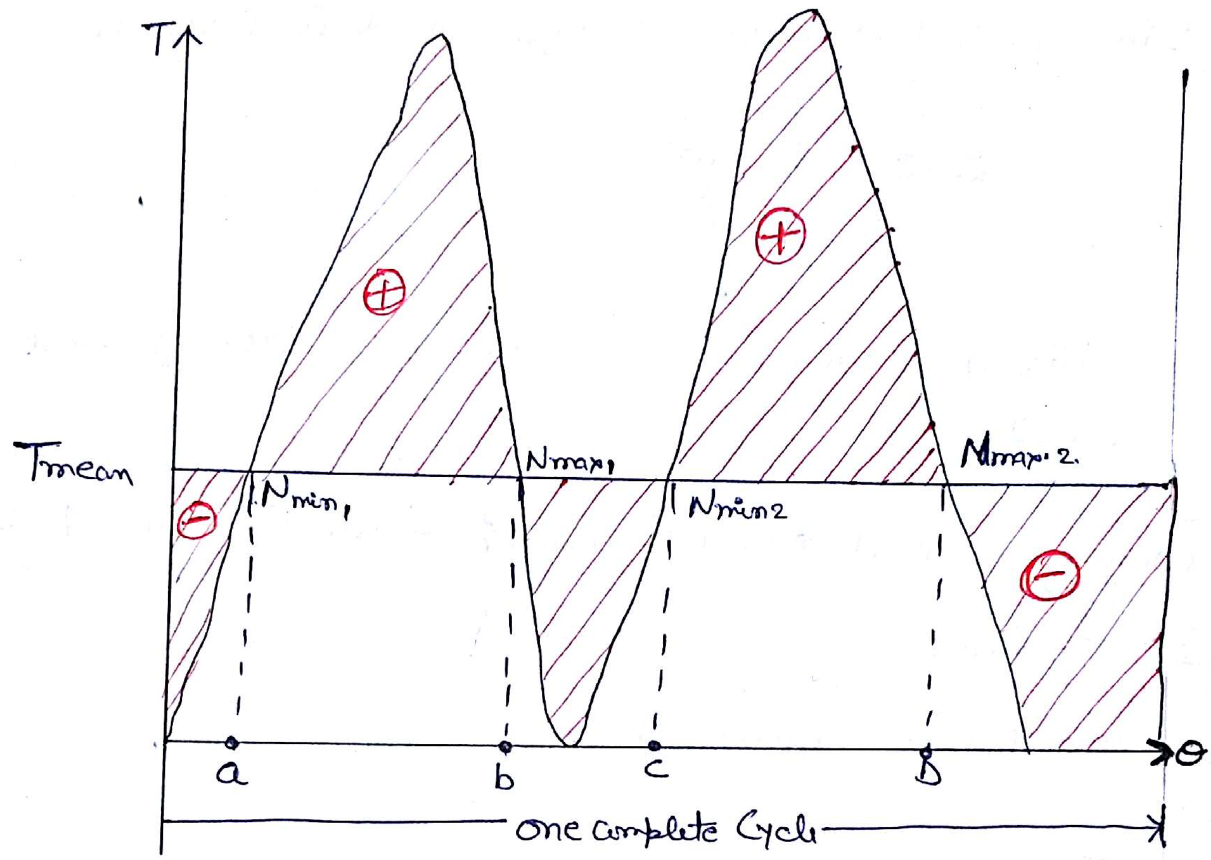
N → (mean speed)

C_s → Coefficient of fluctuations of speed for the flywheel.

For exam

C_s → 2% = 0.02

→ ±2% = 4% ⇒ 0.04



$$C_E = \frac{(E_{max} - E_{min})}{W_{cycle}}$$

$C_E \rightarrow$ Coefficient of fluctuation of energy for the flywheel.

Any (+ve) or (-ve) area above or below <u>Tmean line</u> is Fluctuation of Energy. ΔE.

Sum of all the (+ve) area above Tmean line = Sum of all the (-ve) area below Tmean line

Main function of flywheel :

→ Storing and delivering energies at desired speed.

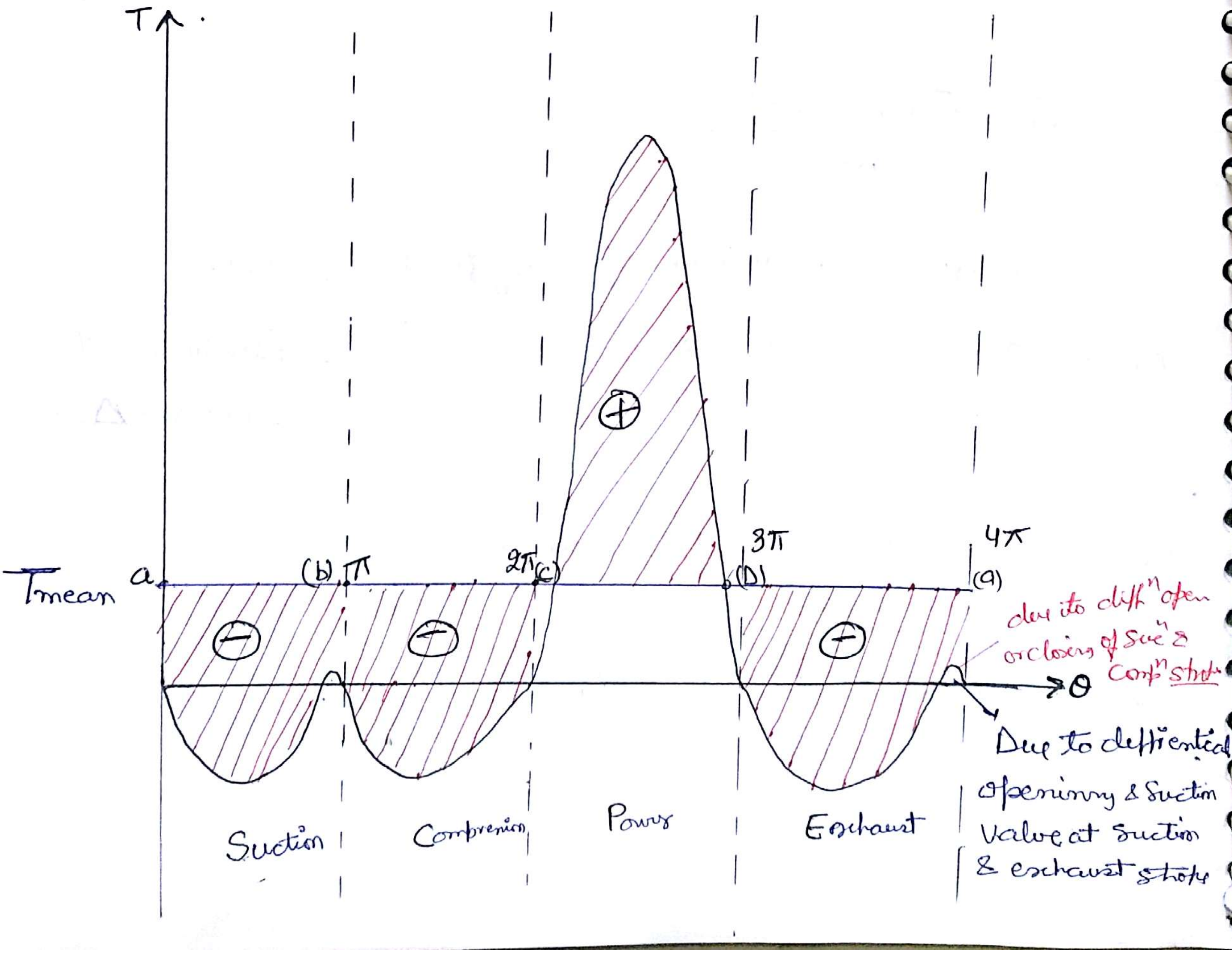
e.g. $200\,J = \frac{1}{2} I \omega^2$

High Speed Running engines are having their flywheel with less I.

Turning Moment form (diagram) of Single Cylinder 4-Stroke IC engine :

$$\boxed{T_{mean} = \frac{W_{cycle}}{4\pi}}$$

$\Delta E \rightarrow$ fluctuation (above & below T_{mean})



Fundamental Equation Of Flywheel :

$m \rightarrow$ mass of Flywheel
$k \rightarrow$ Radius of gyration

Moment of inertia = $\boxed{I = mk^2}$

Ring $\rightarrow k = R \Rightarrow I = mR^2$

Disc $\rightarrow k = \frac{R}{\sqrt{2}} \Rightarrow I = \frac{mR^2}{2}$

Maximum Fluctuation of energy :-

Variation

$$(\Delta E)_{max} = E_{max} - E_{min}$$

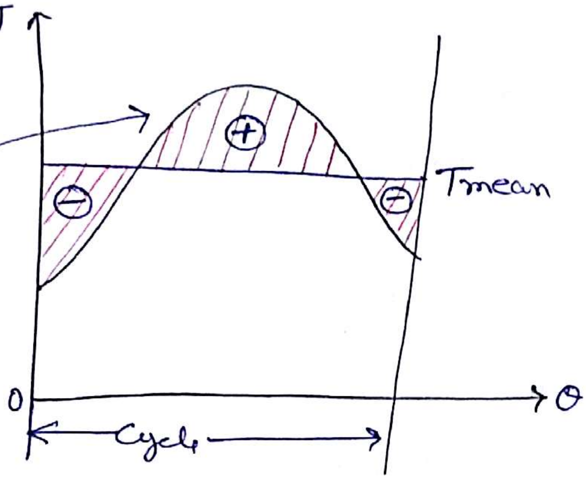
$$= \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$= \frac{1}{\cancel{2}} I \underbrace{\left(\frac{\omega_{max} + \omega_{min}}{2}\right)}_{\omega} \underbrace{\left(\frac{\omega_{max} - \omega_{min}}{\omega}\right)}_{C_s} \cancel{2} \times \omega$$

$$\boxed{(\Delta E)_{max.} = I\omega^2 C_s}$$ ★★

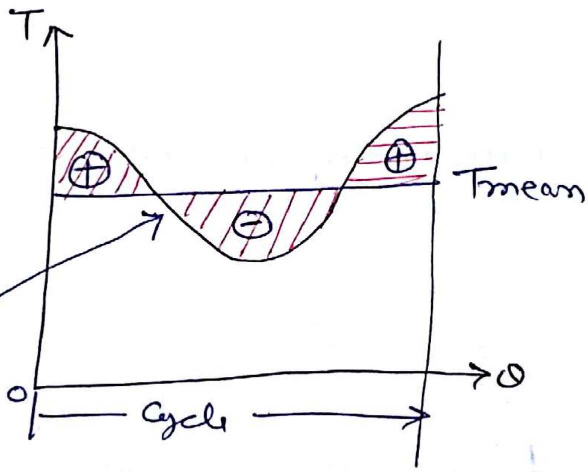
for. e.g

Max. fluctuation at (+ve)



(2)

Max. fluctuation at (−ve)



Period :

$\sin\theta \rightarrow 2\pi$
$\cos\theta \rightarrow 2\pi$
$\tan\theta \rightarrow \pi$

$\sin 3\theta = \frac{2\pi}{3}$

$\cos 3\theta = \frac{2\pi}{3}$

$\tan 7\theta = \frac{\pi}{7}$

$$f(\theta) = \underset{\text{Constant}}{1000} + \underset{\frac{2\pi}{3}}{200\sin 3\theta} + \underset{\frac{2\pi}{5}}{300\cos 5\theta} + \underset{\frac{\pi}{7}}{700\tan 7\theta}$$

$$\boxed{\text{Period} = \frac{\text{L.C.M of Numerator}}{\text{HCF of Denominator}}} = \frac{2\pi}{1} = 2\pi$$

Pb 27 (Pg 29) Flywheel Conventional

Soln) $T = [1000 + 2000 \sin 2\theta - 1800 \cos 2\theta]$ cycle $\rightarrow (0, \pi)$

$C_s = \pm 0.25\% = 0.5\%$
$= 0.05$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60}$ rad/s

$T_{mean} = $ Const.

$W_{cycle} = \int_0^{\pi} T . d\theta$

$= \int_0^{\pi} (10000 + 2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$

$W_{cycle} = 10000\pi$ Joules

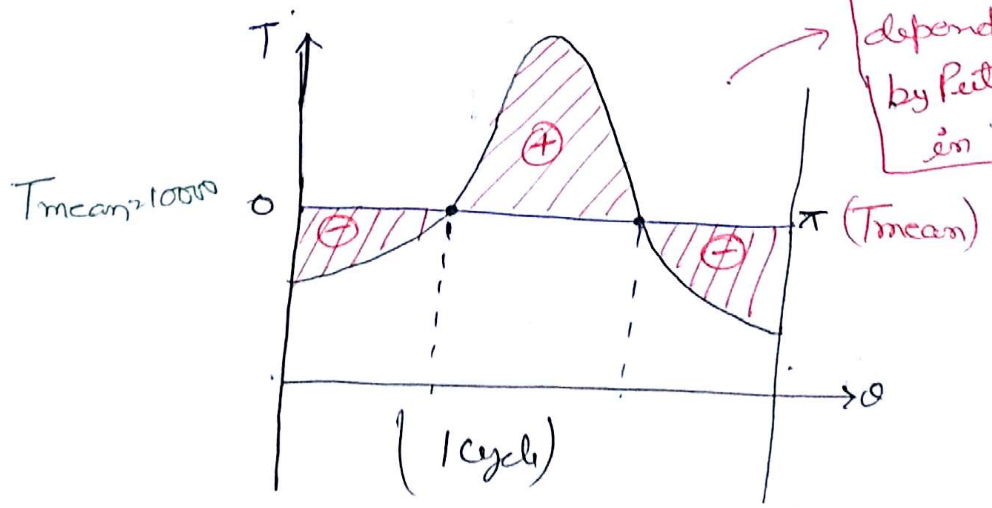
$T_{mean} = \frac{10000.\pi}{\pi} = 10000$ N-m $T_{mean} = \frac{W_{cycle}}{\pi}$

$\boxed{T_{mean} = 10000 \text{ N-m}}$ **

i) Power $\rightarrow$ $P = T_{mean} \times \omega$

$\boxed{P = 10000 \times \frac{2\pi \times 250}{60} \text{ Watt}}$

ii)



dirn of Curve will depend on by Putting Value of θ in T

Those Point, where T curve cut Tmean line,

At Those points:

$$T = T_{mean}$$

$$\cancel{10000} + 2000 \sin 2\theta - 1800 \cos 2\theta = \cancel{10000}$$

as $\sin\theta = 2\pi$, $\cos\theta = 2\pi$, $\tan\theta = \pi$

$$\tan 2\theta = 0.9$$

$$2\theta = 41.8872^\circ,\ \overset{180+41.8872}{221.9872^\circ},\ \overset{221.9872+180^\circ}{401.9872^\circ} - - -$$

$$\theta = 20.9936^\circ,\ 110.9936^\circ,\ 200.9936^\circ$$

$$\Delta E_{max} = \int (T_{mean})\, d\theta \quad (T - T_{mean})$$

$$= \int_{20.9936}^{110.9936^\circ} (2000 \sin 2\theta - 1800 \cos 2\theta)\, d\theta = I\omega^2 C_s$$

this I will put in

iii) $(T_{mean})_{\theta = 45^\circ} = I\alpha$

$$2000 = I\alpha$$

$$\alpha = \frac{2000}{I}$$

$$\boxed{\alpha = \frac{2000}{I}\ \text{rad}/s^2}$$

Q.28)

$T = 5000 + 1500 \sin 3\theta \Big]_{\frac{2\pi}{3}}$

$I = 100\ \text{Kg-m}^2$

$N = 300\ \text{rpm}$

$\omega = \frac{2\pi \times 300}{60}$ rad/s.

$C_s = ?$

$T_{mean} = 5000 + 600 \sin\theta \Big]_{\frac{2\pi}{1}}$

2π

(ii) (b)

Those point where T curve cuts Tmean curve;

At those point

$T = T_{mean}$

$\cancel{5000} + \cancel{1500}^{5} \sin 3\theta = \cancel{5000} + \cancel{600}_{2} \sin\theta$

$5(3\sin\theta - 4\sin^3\theta) = 2\sin\theta$

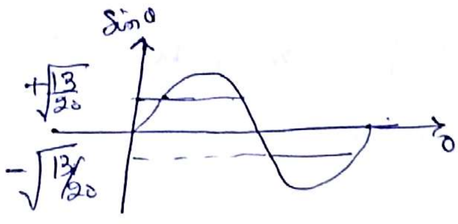
$13\sin\theta - 20\sin^3\theta = 0$

~~Sinθ(13S~~ $\sin\theta(13 - 20\sin^2\theta) = 0$

$\sin\theta = 0 \Rightarrow \cancel{0, \pi, 2\pi}\ 0, \pi, 2\pi$

$\sin^2\theta = \frac{13}{20}$

$\sin\theta = \begin{cases} +\sqrt{\frac{13}{20}} \\ -\sqrt{\frac{13}{20}} \end{cases}$



$\rightarrow 53.7288^\circ$

$\rightarrow 126.2711^\circ$

$\rightarrow 233.7288^\circ$

$\rightarrow 306.2711^\circ$

$0°$ ①

$53.7288°$ ②

126.2711 ③

$180°$ 4

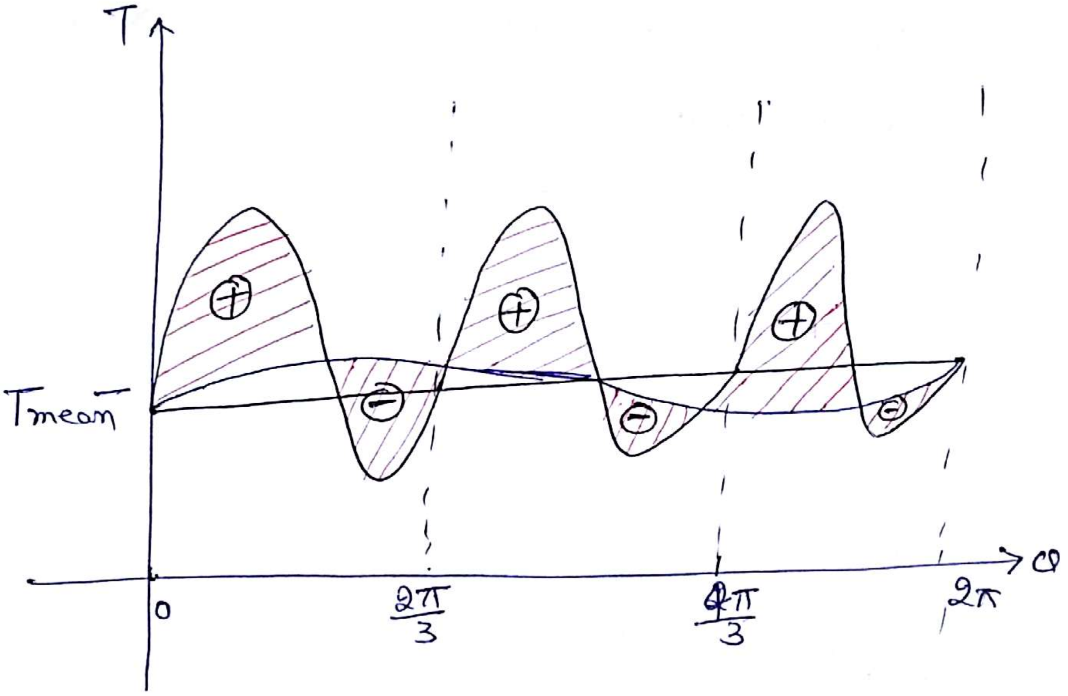
$233.7288°$ 5

$306.2711°$ ⑥

$360°$

$$\int T - T_{mean}$$

$$= \int 1500 \sin 3\theta - 600 \sin\theta$$



$$(\Delta E)_{max} = \int_{233.72880}^{306.2711} (T - T_{mean})\, d\theta = ? = I\omega^2 C_s$$

$C_s = ?$

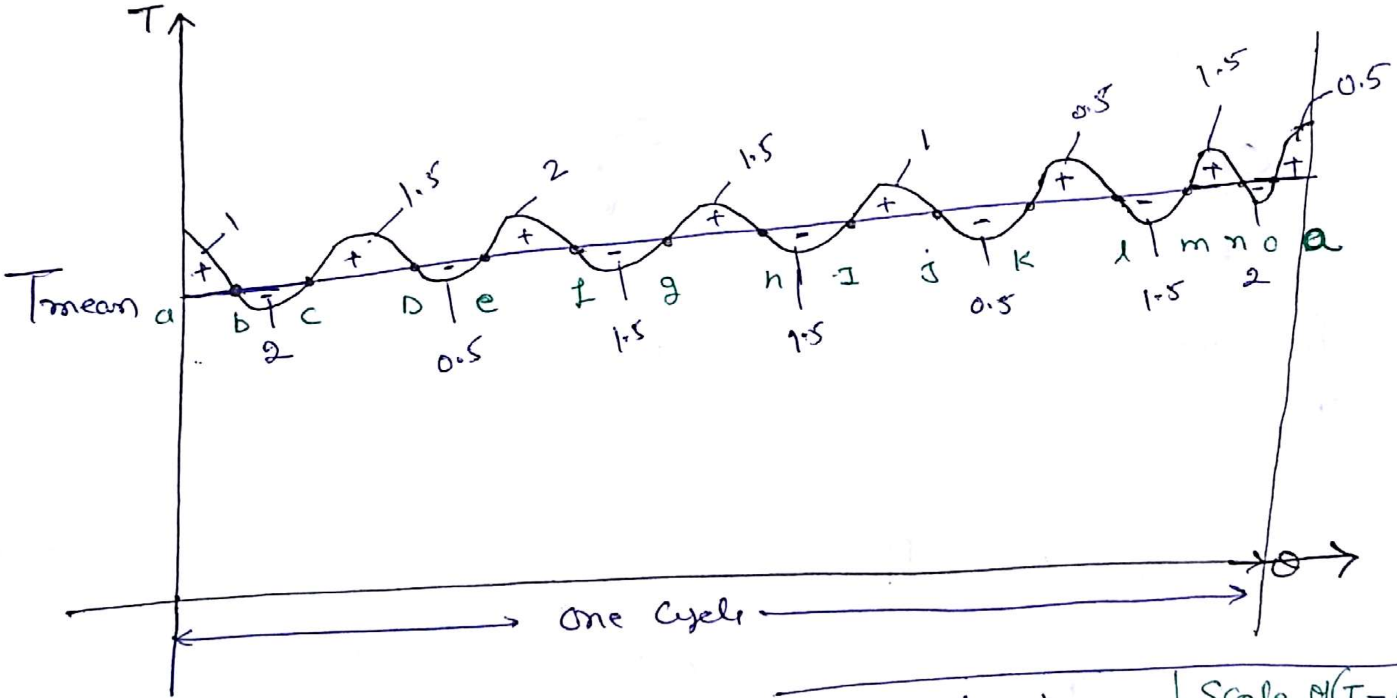
Requirement of Flywheel in multi cylinder Engines :-

→ Multicylinder engine Concept, came into picture, to ↑ the Power as well as to increase the uniformity of the Power. So, that, the fluctuation in the engine performance can be Decreased & the Requirement of flywheel, can be suppressed up to certain limits.

→ This is achieve by imposing a very nice Concept known as firing Order.

→ This Concept is also playing very, very important role in the area of Balancing.

When no. of Cylinder are more than 7 – 8



let us assume

$E_a = E_1$

$E_b = E+1$

$E_c = E-1$

$E_D = E+0.5$

$E_e = E$

$E_f = E+2$

$E_g = E+0.5$

$E_h = E+2$

$E_i = E+0.5$

$E_j = E+1.5$

$E_k = E+1$

$E_l = E+1.5$

$E_m = E$

$E_n = E+1.5$

$E_o = E-0.5$

$E_a = E$

Scale of (T – θ)

$1\,mm^2 = 0.2$ Joules

$E_{max} = (E+2)$

(at f & h)

$E_{min} = (E-1)$ At C

$(\Delta E)_{max} = (E+2) - (E-1)$

$= 3\ mm^2$

$= 3 \times 0.02$ Joul.

then equate to $I\omega^2 C_s$

Requirement Of Flywheel in Power Presses:

Power Press:

⇓

Used in Metal Forming Process.

⇓

Punching
Blanking
Shearing

Power Press Runs by motor

flywheel at CAM shaft — CAM

↓

Follower
(Punch)

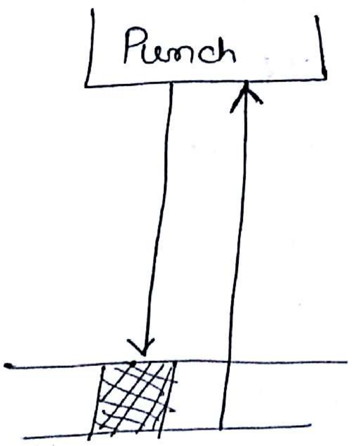
flywheel at CRank shaft. — CRANCK

↓

Piston
(Punch)

One Cycle of Power Press:

↓



→ Two Stroke are Covered

→ Time taken to Complete one cycle.

Cycle time

⇓

Exact operation time << << Cycle Time

↓

very less fraction

In One Cycle

→ CRANK — 2π

→ CAM | Rotation — $\leq 2\pi$ (due to Dwell Stroke)

Due to Dwell Stroke, in which Cam Rotates but follower not.

Q.20) 720 holes/hr

⇒ 720 holes/3600 sec

⇒ 1 hole/5 sec

$\boxed{\text{Cycle time} = 5\ \text{Sec}}$ *

$K = 0.3m$

1 hole / ¼ sec

exact punching time $= \frac{1}{4}$ sec.

During Punching

100 rpm (N_{max})

↓

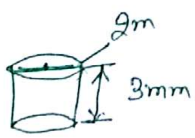
80 rpm (N_{min})

$N = \frac{100+80}{2} = 90\ rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\pi 90}{60}$ rad/sec

$\boxed{C_s = \frac{100-80}{90}}$ *

1 hole



$A_{Sheared} = \pi d T$

$= \pi \times 2 \times 3 = 6\pi\ mm^2$

$E_{hole} = 6\pi \times 20 = 120\pi$ Joules.

Moter

No Punch time used

P_{moter} = energy required /sec

= E_{hole} × No. of holes/sec → (cycle time) *** v. imp

$= 120\pi \times \frac{\cancel{720}}{3600}$

$= 24\pi$ Watt (J/s)

Motor Installed :

Punching = $\left(\frac{1}{4}\text{ sec}\right)$

$E_{available} = 24\pi \times \frac{1}{4}$

$= 6\pi$ Joules

$E_{hole} = 120\pi$ Joules

$120\pi - 6\pi = 114\pi = I\omega^2 C_s$

$I = mk^2$

Variation in above Problem :

(1) Cycle time : 5 Sec

Exact Punching time = ?

Stroke length = 100 mm

---

200 mm — ~~Stro~~ Cycle length

200 mm —— 5 Sec

3 mm —— $\frac{5}{200} \times 3$ sec

↓

thickness of sheet

---

(2) Cycle time = 5 Sec

Exact punching time = ?

ω = ??

Exact punching is done in 20° of crank Rotation.

In one cycle ⟶ Crank Rotation = 2π in 5 Sec

$$\boxed{\omega = \frac{2\pi}{5} \text{ rad/sec}}$$

360° —— 5 Sec

20 —— $\frac{5}{360} \times 20°$

↓

exact Punching time

Q. 29

$$\text{Cycle time} = 2\,\text{sec}$$

$$P_{motor} = 1500\,\text{Watt} = E_{hole} \times \frac{1}{2}$$

$$E_{hole} = 3000\,\text{Joules}$$

$$\omega = \frac{2\pi}{2} = \pi\ \text{rad/s}$$

exact Punching time $= \frac{1}{6}$ sec

$$C_s = 20\% = 0.2$$

Punching ($\frac{1}{6}$ sec)

$$3000 - \left(1500 \times \frac{1}{6}\right) = I\omega^2 C_s$$

$$I = 29$$

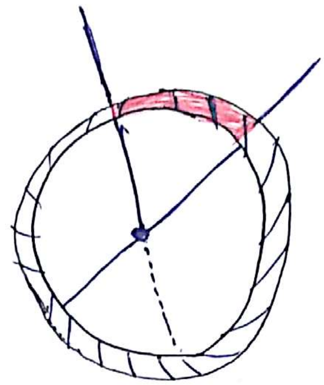
→ Designing of Flywheel Rim :-

$A \rightarrow$ Rim cross section area

$$\boxed{\text{Ring} \rightarrow I = mR^2}$$

↓

Max.



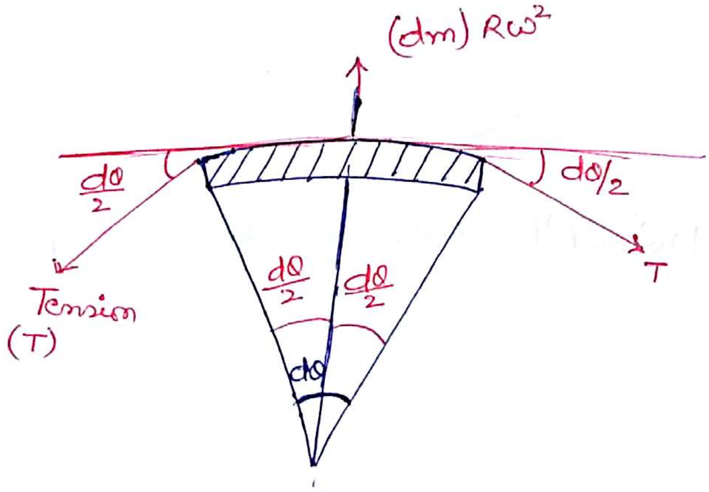
$$2T \sin\frac{d\theta}{2} = (dm)\, R\omega^2$$

$$\cancel{2} T \sin\frac{d\theta}{\cancel{2}} = (R\,d\theta\, A \cdot \rho)\, R\omega^2$$

$$\frac{T}{A} = \rho (R\omega)^2 = \rho V^2$$

⇓

(Hoops stress)



$$\boxed{V = (R\omega) = \sqrt{\frac{\sigma}{\rho}}}$$ **

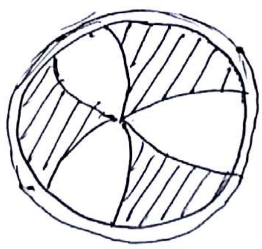
σ → Hoop's stress

$$\boxed{\sigma \le \sigma_b}$$ where σ_b is bearing limit for Hoop's stress

$$\boxed{V_{max} = (R\omega)_{max} = \sqrt{\frac{\sigma_b}{\rho}}}$$ **.

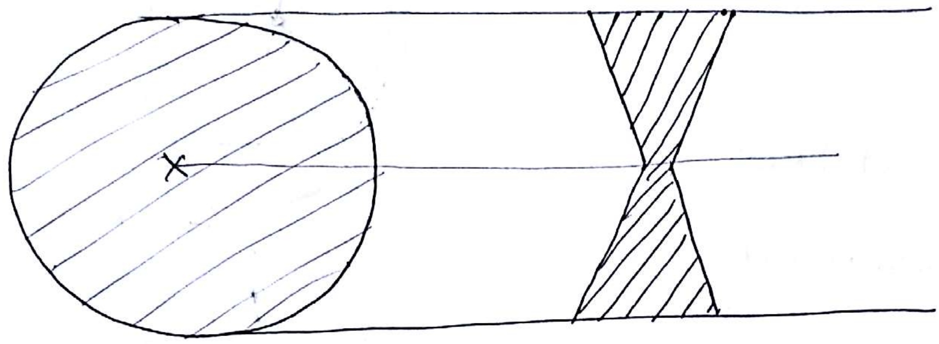
At Medium speed :

Flywheel with spokes.



At high speed :- Disc Shaped flywheel

Best flywheel due to $\boxed{\frac{1}{2} \cdot I \omega^2}$



Having highest energy storing capacity $\rightarrow \frac{1}{2} I\omega^2$

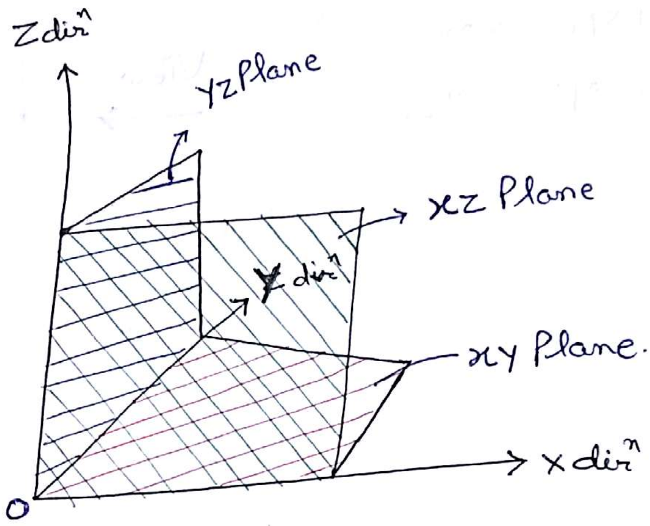
Ring

$I < mR^2$

$> \frac{mR^2}{2}$

Gyroscope

3-D Space Co-ordinate System :-



Concept of Angular velocity ($\vec{\omega}$) :-

⇓

Rate of Change of Angular displacement.

$\vec{\omega}$ → Vector Quantity

Magnitude
(Rate of change of Ang. Displacement)

Direction → (Dir^n of $\vec{\omega}$ is Also Known as Axis of Spin)

⇓

By Right hand thumb Rule.

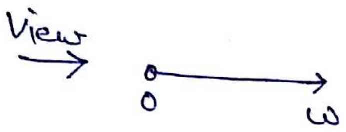
Right hand thumb rule :-

i) Place the Right hand in Such a way, Such that thumb is $\perp^r$ to the fingers

ii) Relate the fingers in the Sense of Rotation, then the direction of thumb will be the dir^n of Angular velocity.

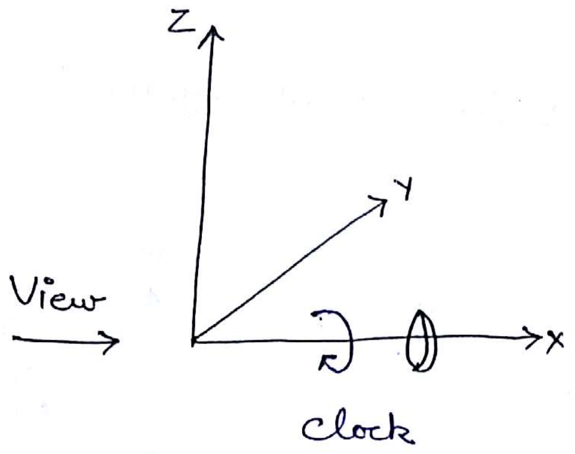
Examples:

1)



Axis of Spin → Ox

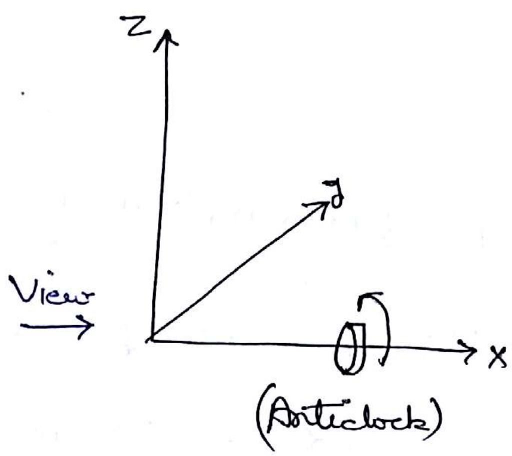
Plane of Spin → YZ



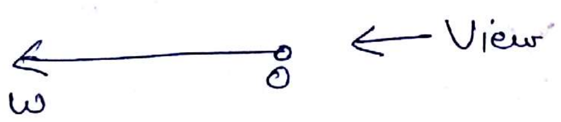
2)

Axis of Spin — Ox

Plane of Spin YZ

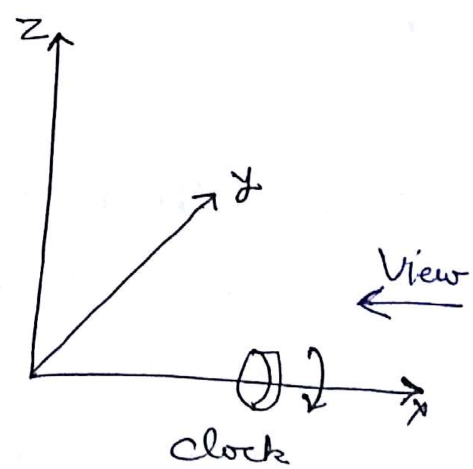


3)



Axis of spin — Ox

Plan of spin — YZ



Angular Accelaration :- $(\vec{\alpha})$

⇓

Rate of change of Angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

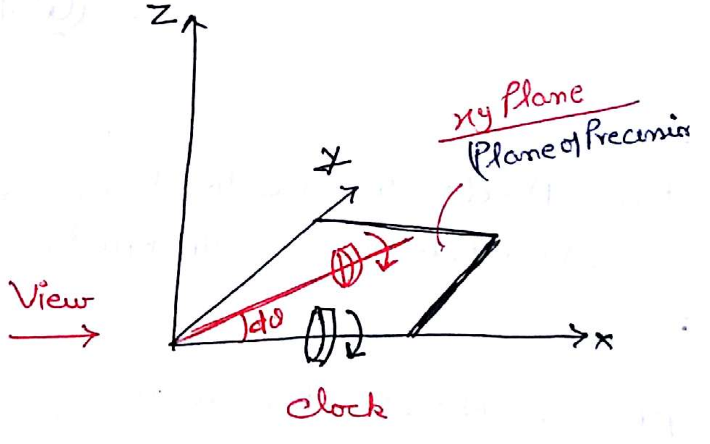
$\vec{\omega} \rightarrow$ Vector Quantity → Mag, → dir^n

α may exist :- bcz of

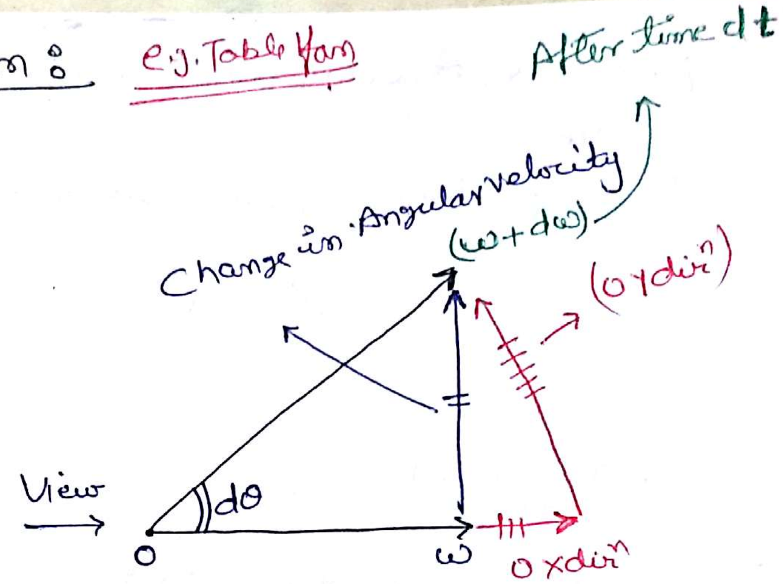
→ Bcz of the change in Magnitude of Angular velocity.

→ Bcz of the change in direction of Angular velocity. → (Precession)

→ Let us consider a case, in which Magnitude of $\vec{\omega}$ & the dir^n of $\vec{\omega}$ both are changing.



Angular Velocity Diagram : e.g. Table Fan



Angular accelaration due to change in Magnitude of Angular velocity :-

$$= \frac{(\omega + d\omega)\cancel{\cos d\theta}^{\,1} - \omega}{dt}$$

$$= \frac{(\cancel{\omega} + d\omega) - \cancel{\omega}}{dt}$$

$$\boxed{\text{Ang. Accelaration due to change in Magnitude of Ang. velocity } (\vec{\omega}) = \frac{d\omega}{dt} \quad \text{OX direction}}$$

Angular Accelaration due to change in dirⁿ of Angular Velocity

$$= \frac{(\omega + d\omega)\sin d\theta - 0}{dt}$$

$$= \frac{(\omega + d\omega)\cdot d\theta}{dt}$$

$$= \frac{\omega \cdot d\theta + \cancel{d\omega \cdot d\theta}^{\,0}}{dt}$$

$$= \omega \frac{d\theta}{dt} \cdot \quad (\text{OY}) \text{ dir}^n$$

Angular Velocity or Precession (ω_p)

$$\boxed{= \omega \cdot \omega_p}$$

OY dirⁿ

Total Average Accelaration :

$$\boxed{\vec{\alpha} = \left(\frac{d\omega}{dt}\right)\hat{i} + \omega\left(\frac{d\theta}{dt}\right)\hat{j}}$$

$$\Rightarrow \quad \vec{\alpha} = \left(\frac{d\omega}{dt}\right)\hat{i} + \underbrace{(\omega \cdot \omega_P)\hat{j}}_{\text{(Gyroscopic accelaration)}}$$

If the magnitude of Angular velocity is not changing, only the dir^n is changing, then;

$$\frac{d\omega}{dt} = 0$$

$$\alpha = \omega \cdot \frac{d\theta}{dt}$$

$$\boxed{\alpha = \omega \cdot \omega_P}$$ * in OY direction

To have the precision, i.e to change the dir^n of $\vec{\omega}$ in order to provide Angular accelaration (α) = $\omega \cdot \omega_P$ in OY dir^n

→ A torque is required; and that requirement of the torque is known as Active Gyroscopic Couple (C)

Active gyroscopic Couple (C) = $I\alpha$

$$\boxed{C = I\,\omega \cdot \omega_P}$$ ** OY direction

Torque required to will be given to the System by external Agency.

e.g to turn Bike
↓
We made to turn Handle

Therefore, the Similar Couple Will be experienced by the external agency in Reverse direction, i.e Known as Reactive Gyroscopic Couple.

$$\boxed{C = I\omega\,\omega_p}$$

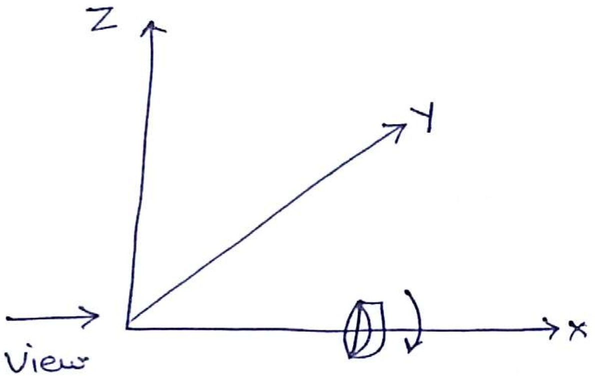
Thus, this Reactive gyroscopic Couple is experienced by the System because that external agency is the part of the System.

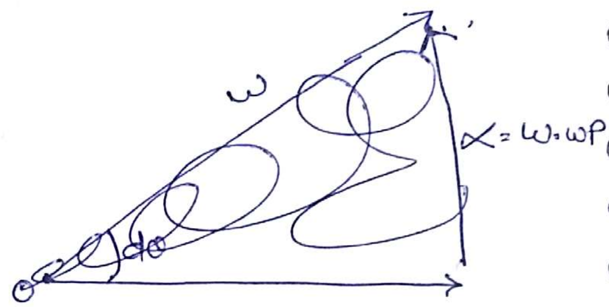
motor-table fan

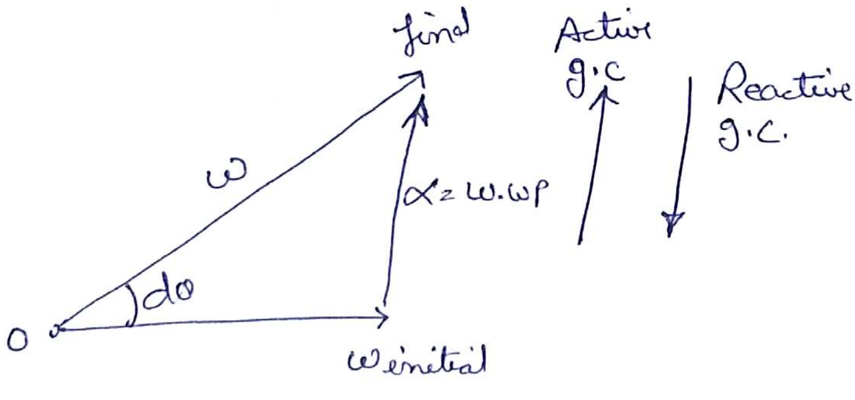
After Effect Of Gyroscopic Actions on the System :-

⇓

Reactive gyroscopic Couple.





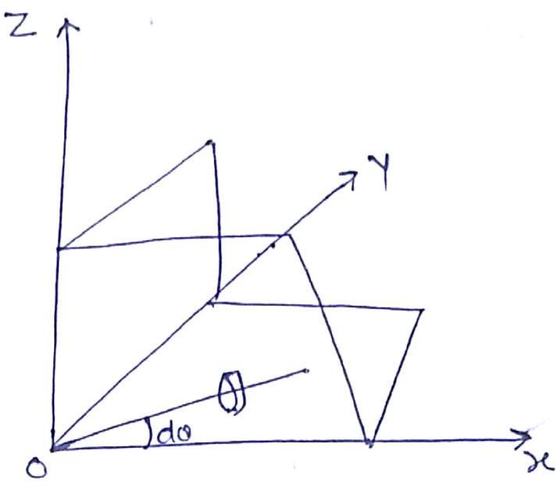


OX — Axis of Spin
yz — Plane of Spin

OZ - Axis of Precession
xy - Plane of Precession

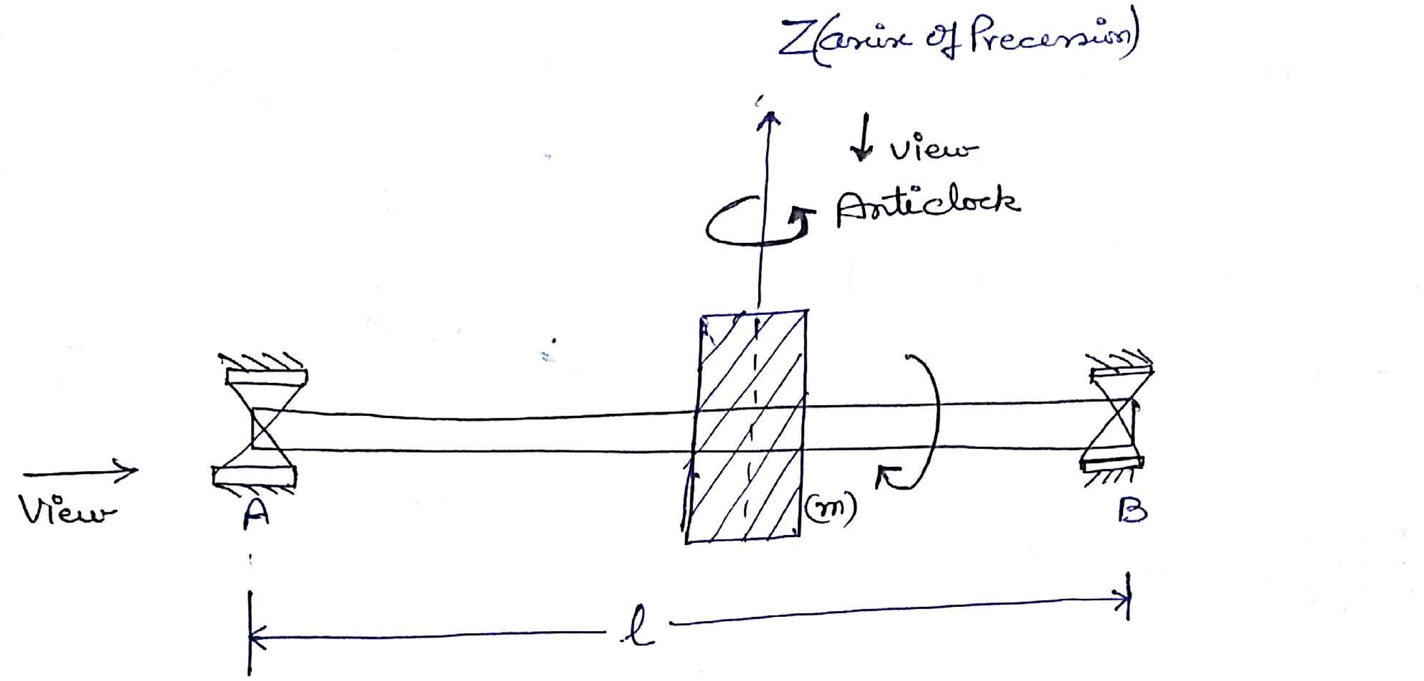
Oy - Axis of Gyroscopic Couple

xz - Plane of Gyroscopic Coupl.

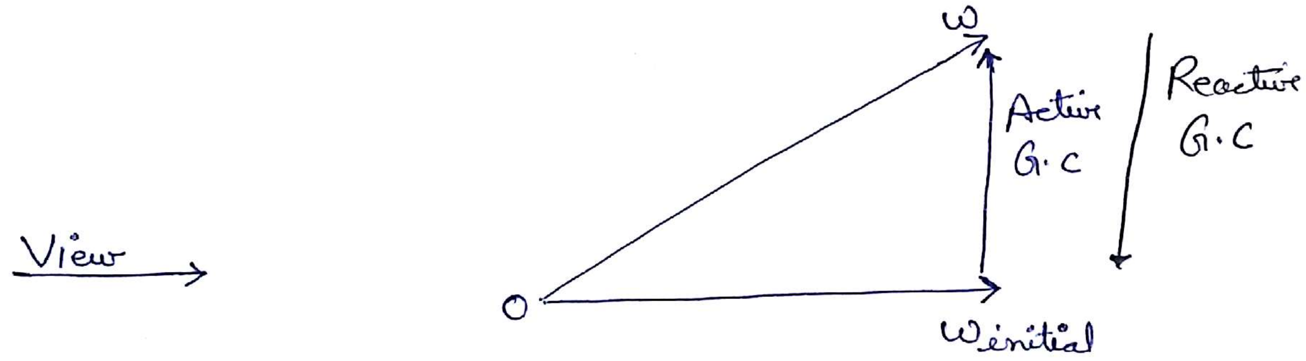


Application :-

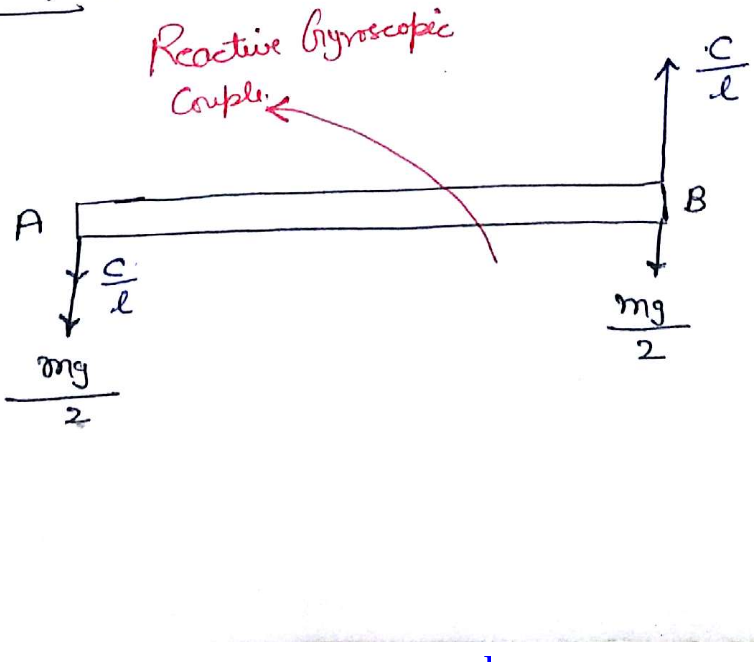
→ Reaction On bearings Under Gyroscopic Effect :-



$$\boxed{C = I\omega \cdot \omega_p}$$



Forces Of Shaft :



$$F_A = \left(\frac{C}{l} + \frac{mg}{2}\right) \text{ (downward)}$$

$$F_B = \left(\frac{C}{l} - \frac{mg}{2}\right) \text{ (upward)}$$

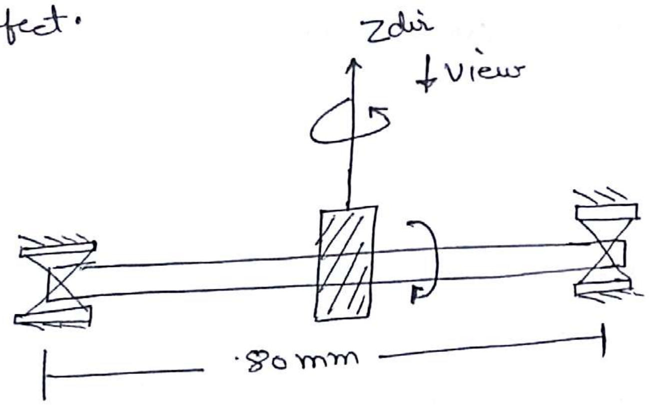
Reaction on bearing

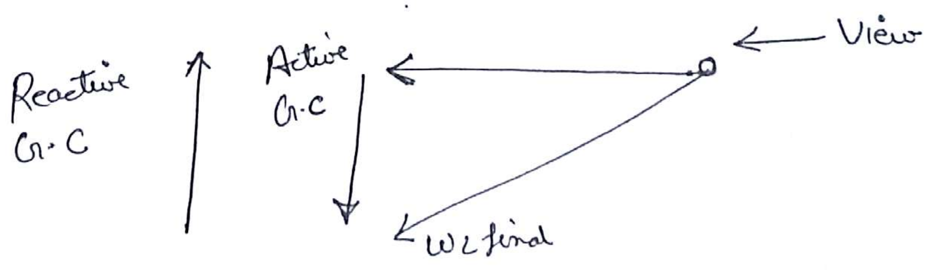
$$R_A = \left(\frac{C}{l} + \frac{mg}{2}\right) \text{ (upward)}$$

$$R_B = \left(\frac{C}{l} - \frac{mg}{2}\right) \text{ (downward)}$$

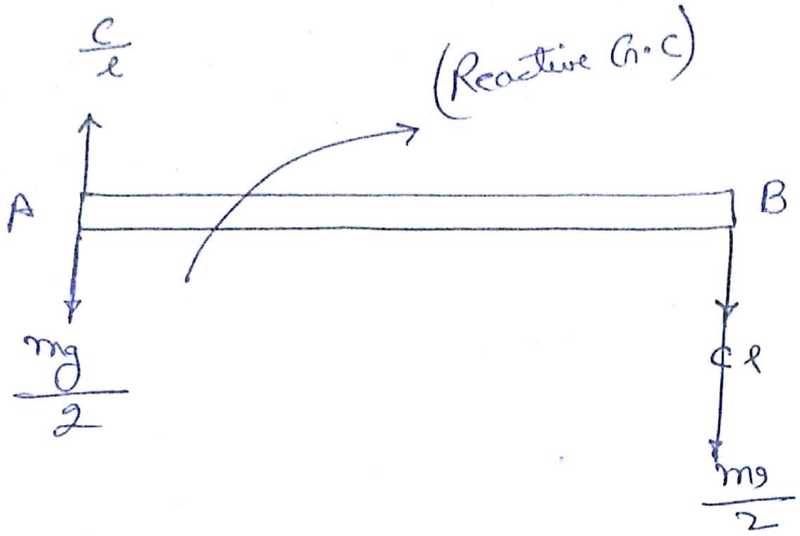
Qn) A disc with Radius of gyration 60 mm & mass 4 kg is mounted Centrally on Horizontal axel of 80 mm length b/w the bearings. It Spins about the axle at 800 rpm (clockwise) when viewed from the Right hand side bearing. The axle precessises about a vertical axis at 50 rpm in anticlockwise direction when viewed from above. Determine the Resultant Rxn. in each bearing due to the mass & gyroscopic effect.

Soln)





Forces on Shaft



$$F_B = \left(\frac{C}{l} + \frac{mg}{2}\right) \quad (\text{downward})$$

$$F_A = \left(\frac{C}{l} - \frac{mg}{2}\right) \quad \text{upward}$$

Reactions on Bearings :

$$R_B = \left(\frac{C}{l} + \frac{mg}{2}\right) \quad \text{upward}$$

$$R_A = \left(\frac{C}{l} - \frac{mg}{2}\right) \quad \text{downward.}$$

Given that;

$m = 4\,kg$

$K = 0.060\,m$

$I = mK^2 = 4 \times (0.060)^2 = 0.0144$

$N = 800\,rpm$

$\omega = \frac{2\pi\, 800}{60} = \quad rad/s$

$N_p = 50\,rpm$

$\omega_p = \frac{2\pi\, 50}{60}$

$C = I\omega \cdot \omega_p = ?$

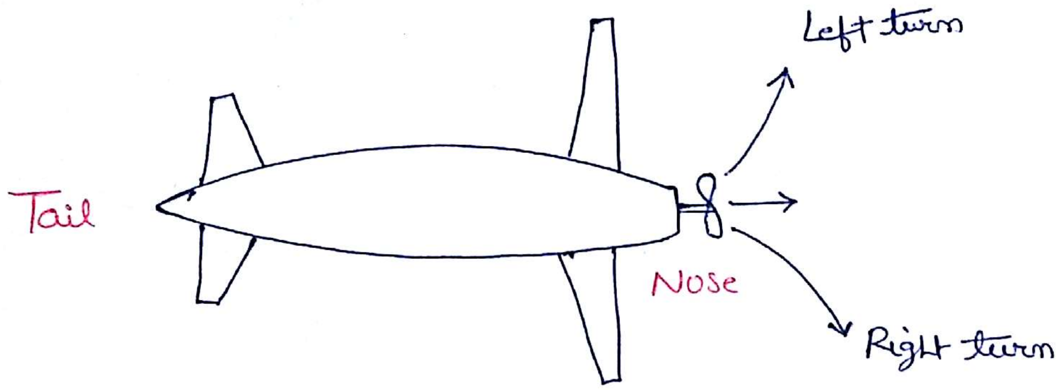
$l = 0.080\ m$

$\frac{C}{l} = ??$

Gyroscopic Effect on Aircraft turning :- (left or right)

→ Front end of Aircraft is known as Nose.

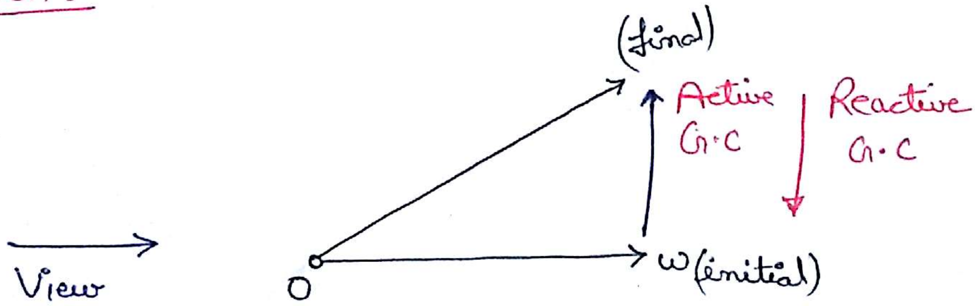
→ Rear end of Aircraft is known as Tail.



→ View from Tail (Rear end) :

Engine Rotate Clockwise

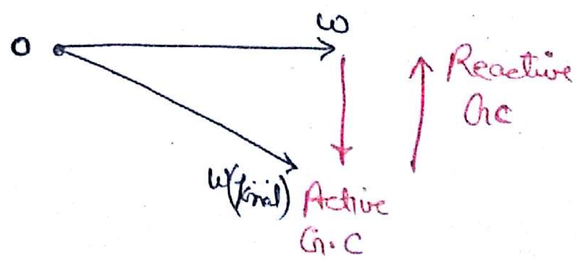
1) Left turn :



Effect :- Nose will go up & tail will go down.

2) Right turn :

Effect :- Nose will go down & Tail will go up.

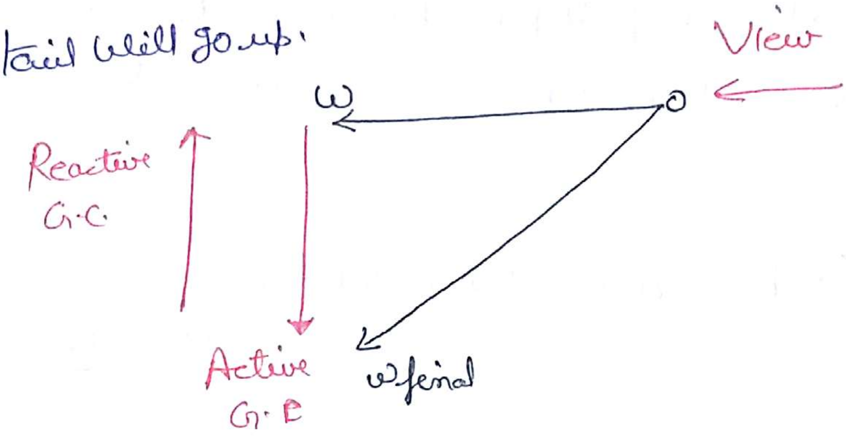


View from Nose (Front end):

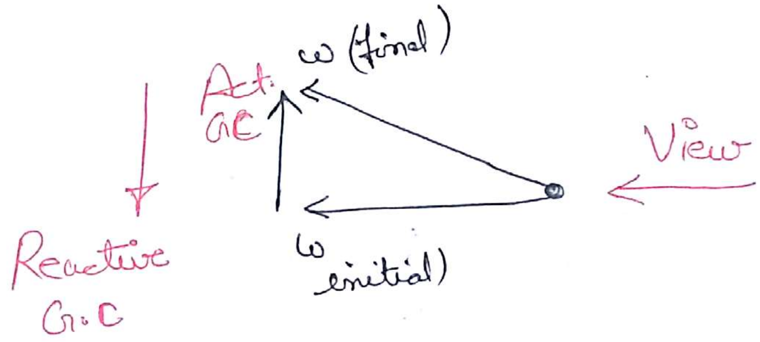
Engine Rotates clockwise

1) Left turn :

effect = Nose will go down & tail will go up.



2) Right turn :



Effect :- Nose will go up & Tail will go down.

Gyroscopic effect on turning of Ship:

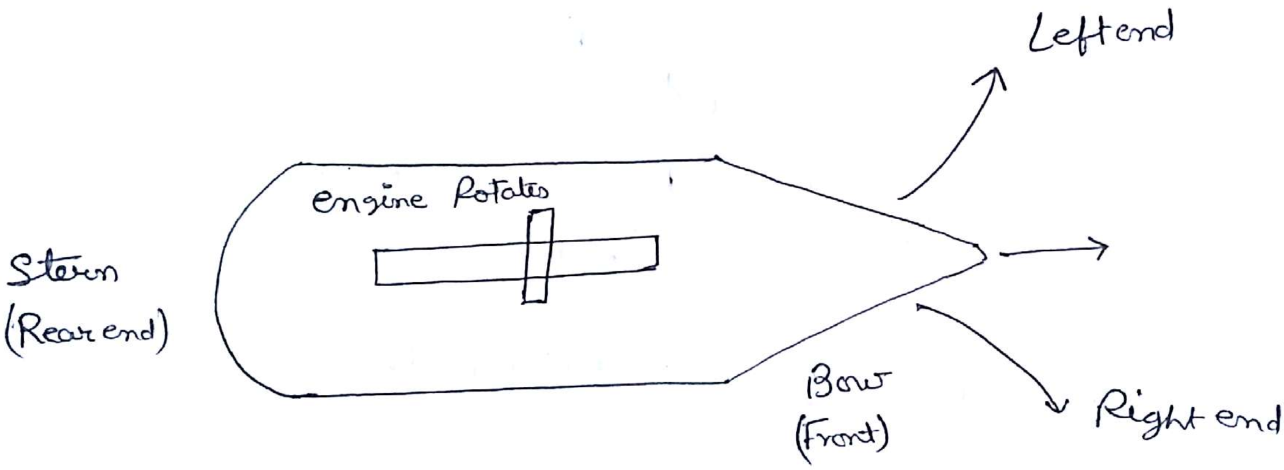
Nomenclature:-

Front end of Ship is known as bow.

Rear end of Ship is known as Stern.

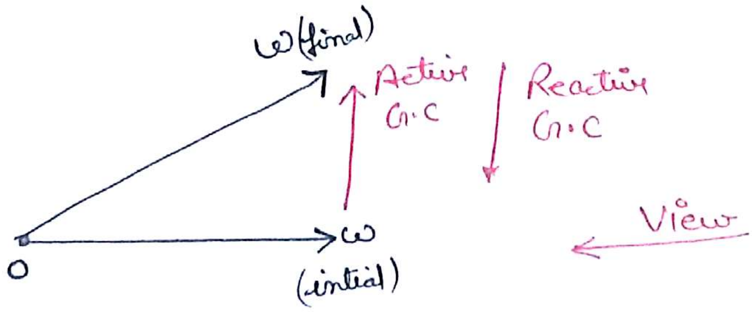
Looking from Stern Side :- The Right hand Side of Ship Portion is ~~Ship~~ Starboared.

Left hand Side of Ship Portion is Port.



1) View from Bow (Front): engine Rotates Anti clockwise

1) Left turn:

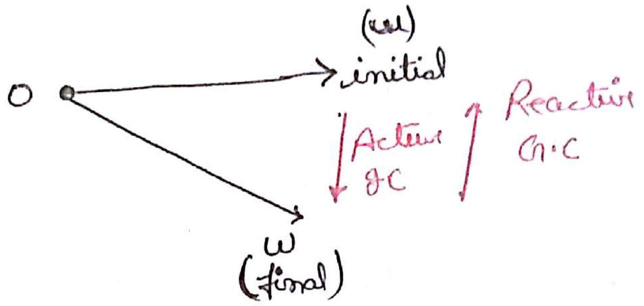


effect → Bow will move up & Stern will go/move down.

2) Right turn:

View ←

Effect:- Bows will move down & Stern will moves up.

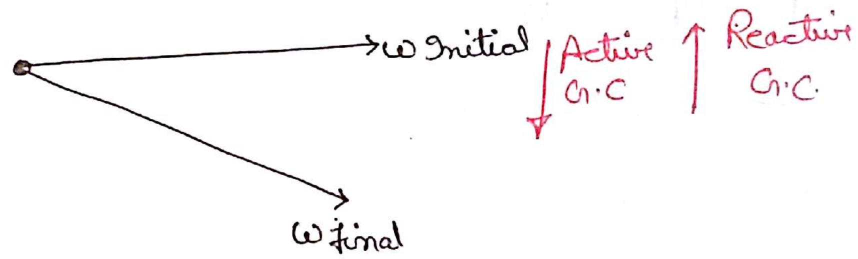


Gyroscopic Effect on Ship During Pitching: engine Rotates Clockwise.

(View from Stern)

1) Bow moves down (stern moves up) during Pitching:

View →



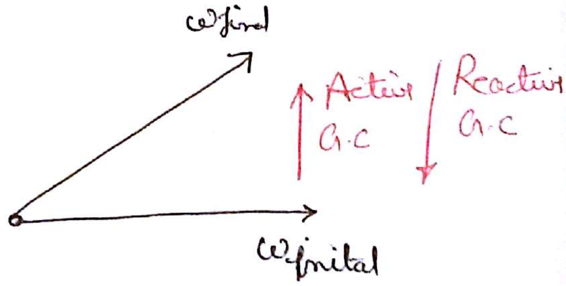
effect:- Ship will try to turn towards Port Side

2) Bow moves up (stern moves down) during Pitching:-

effect:

Ship will try to turn towards Star board Side.

View →



Gyroscopic effect on Rolling of the Ship :-

During Rolling, of the ship, the direction of angular velocity (ω) of rotor of ship is not changing.

There is no Precessional at all.

Hence gyroscopic Couple ($I\omega \cdot \omega_P$) is zero, bcz ω_P is zero.

$\therefore$ Ship will not experience any gyroscopic effect during Rolling

→ Gyroscopic effect on 4 Wheeler during turning :-

$I_E \rightarrow$ M.I of Engine Part (Rotating)

$\omega_E \rightarrow$ Speed of Engine

$I_\omega \rightarrow$ M.I of wheel

$\omega_\omega \rightarrow$ Speed of Wheel

$\omega_P \rightarrow$ Speed of Precession

Gyroscopic effect on engine:

$$\boxed{C_E = I_E \omega_E \omega_P}$$

Gyroscopic effect on Wheel:

$$\boxed{C_\omega = 4 I_\omega \omega_\omega \omega_P}$$

Total Gyroscopic effect on 4-wheeler :-

$$C = C_E + C_W$$

$$C = I_E \omega_E \omega_P + (I_W \omega_W \omega_P)4$$

$$\boxed{C = [I_E + \omega_E + 4 I_W \omega_W]\, \omega_P} \ *$$

During Turning : Gyroscopic effect

Force on outer wheel (each) = $\frac{C}{2l}$ downward $l \to$ axle length b/w inner & outer wheel

Force on inner wheel (each) = $\frac{C}{2l}$ upward.

Problem

Qn) A High Speed Ship is driven by turbine motor having m.o.I, $20\,Kgm^2$ & is running at 3000 rpm in clockwise dirn; when viewed from the Bow. the ship is speeding at 72 Km/hr & taking a right turn round a curve of 600 mt. Radius.
Determine the G.C applied to ship & its effects.

$I = 20\ Kg\text{-}m^2$

$N = 3000$ $\omega = \frac{2\times\pi\times 3000}{60}$

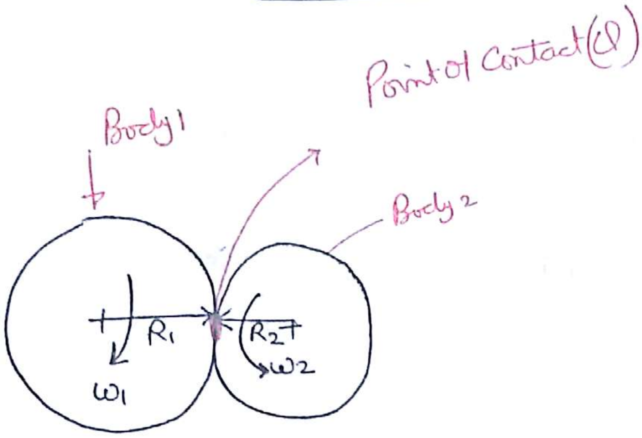
$\omega = 314.16\ rad/sec$

$\omega_P = \frac{72 \times \frac{5}{18}}{600} = 0.0333\ rad/sec$

$C\ \ \cancel{M}I = I.\omega.\omega_P$

$C = 209.23$

Gears



Point of Contact

P

By Body ①

⇓

$R_1\omega_1$

By Body 2

⇓

$R_2\omega_2$

$$\boxed{R_1\omega_1 = R_2\omega_2}$$

Pure Rolling (No Slip)

Pure rolling :

→ $\frac{\omega_1}{\omega_2} = \frac{R_1}{R_2}$

→ Static friction is there.

$$\boxed{0 \leq f_s \leq \mu N}$$

All those Drives in which slips is possible
$\Downarrow$
negative drives

<u>for example:</u>

→ Belt Drives
→ Chain Drives
→ Rope Drive

<u>In Case of Slip</u>

→ $\frac{\omega_1}{\omega_2} \neq$ Constant

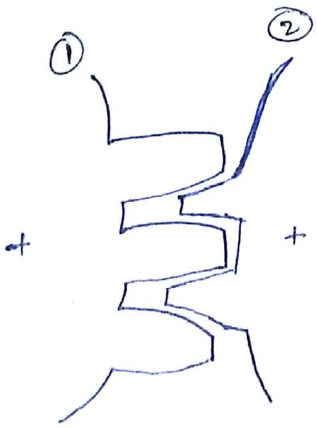
→ Kinetic friction.
(Some power loss)

Requires accuracy
[20–30%. of Power Transmission]

<u>In Some Cases;</u> Positive drives → Gear Drive

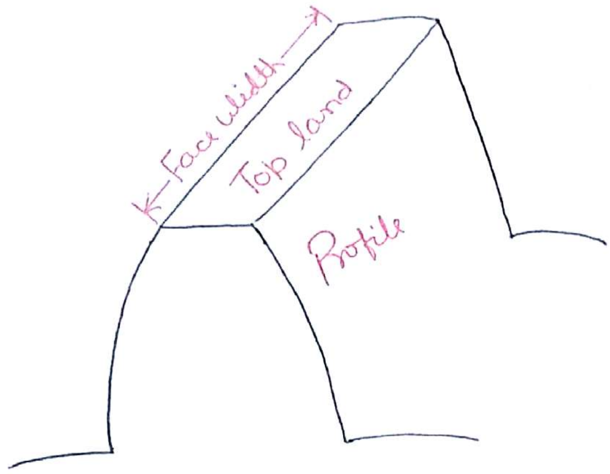
In Power Transmission, a very high level of Accuracy is demanded for the Velocity Ratio to be Constant.

$$\boxed{\frac{\omega_1}{\omega_2} = \text{Constant}}$$



→ Slip is impossible
$\Downarrow$
Positive drives
$\Downarrow$
Gear Drive

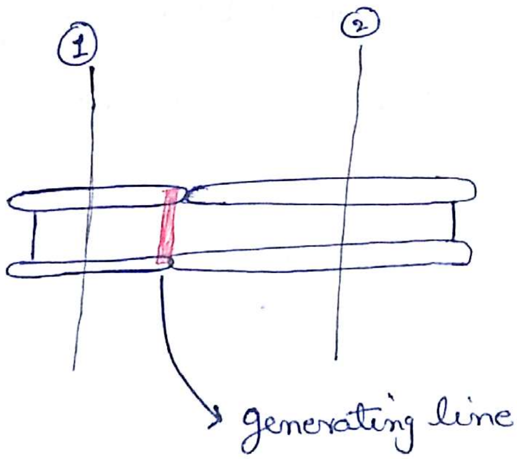
Gear in 3D



Classification of Gears :-

A) According to the Axes of Shaft Connected :-

i) Both Axes are Parallel :



Pure Rolling motion can be Transmitted b/w two Cylindrical Surface in Contact.

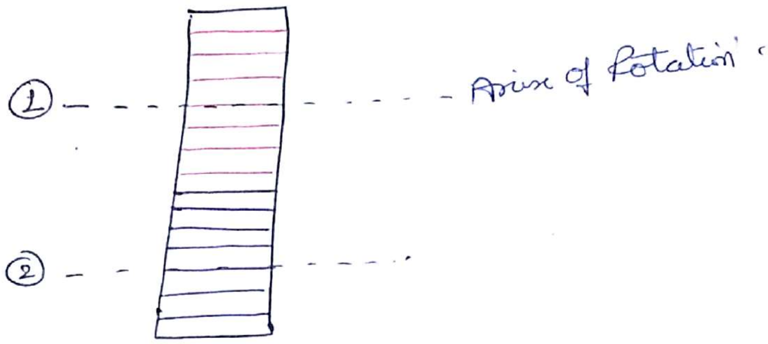
Spur Gear:

Teeth are Straight and ||el to axis of Rotation.

Use:— 99% failed & Only 1% is used. In very low Power Transmission at very Low Speed

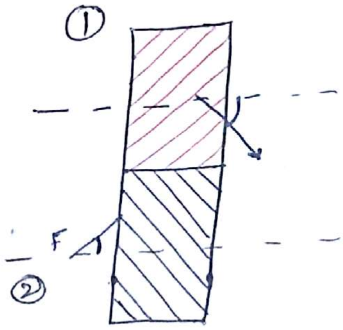
Reason— Instantaneous engagement & Disengagement due to which Impact stresses on the profile.

But having No Axial thrust



Helical Gear: 98% used "Axial thrust is there"

Modification of Spur Gear. "Teeth are straight but inclined to axis of Rotation.



There are two Type of Helical Gear:-

→ Always Opposite hand helical gear must Come in Contact.

→ Gradual engagement

→ No impact stresses

→ Axial thrust at very high speed then it failed

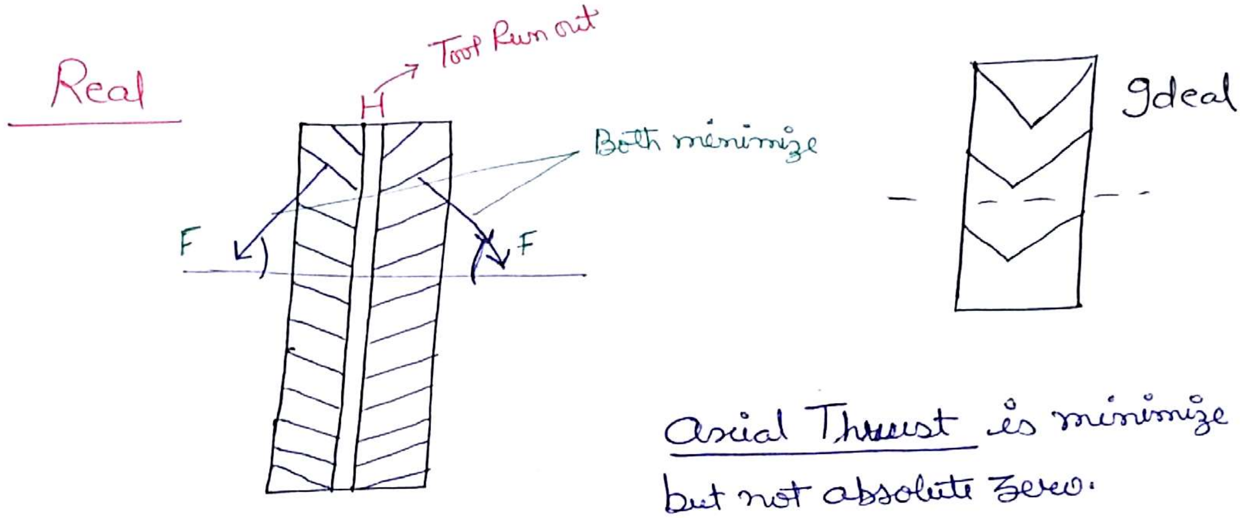
NOTE:-

due to high torque Axial thrust is there as axial thrust is there, overcome by bearing but at high Torque axial thrust is high and bearing were failed

Double Helical Gear: For Very high Torque

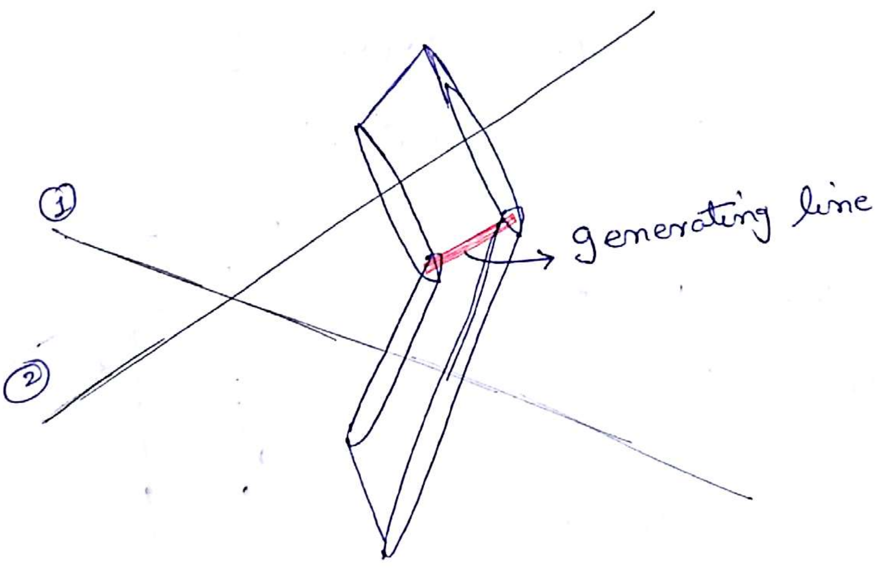
Used to minimize the Axial thrust.

Given by Herringbone & Named Herringbone Gear.



Axial Thrust is minimize but not absolute Zero.

ii) Axes are Non Parallel but Intersecting :-



→ Pure Rolling motion, can be transmitted b/w two Conical Surfaces in Contact.

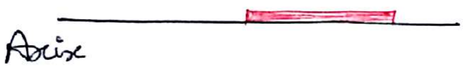
Bevel Gear :-

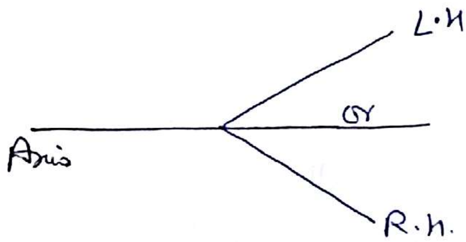
Bevel Gear
- Straight Bevel gear
- Zero Bevel Gear
- Helical Bevel Gear
  - Left Handed
  - Right Handed

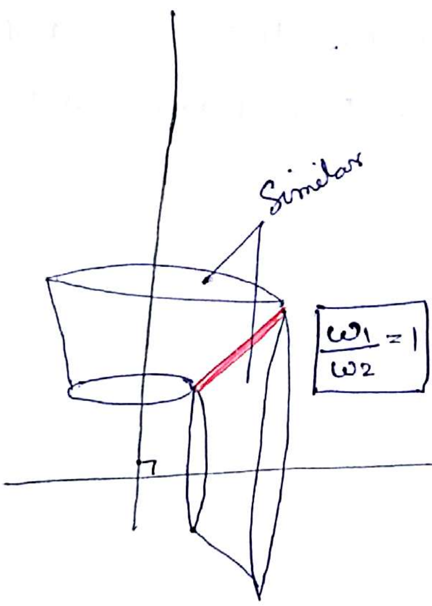
Straight Bevel gear and Zero Bevel Gear: Almost Same

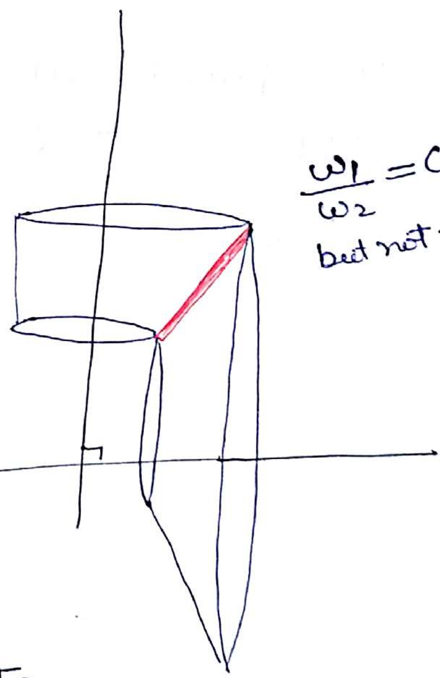
→ Impact stress are there
1% used.

PLAN (TOP View)









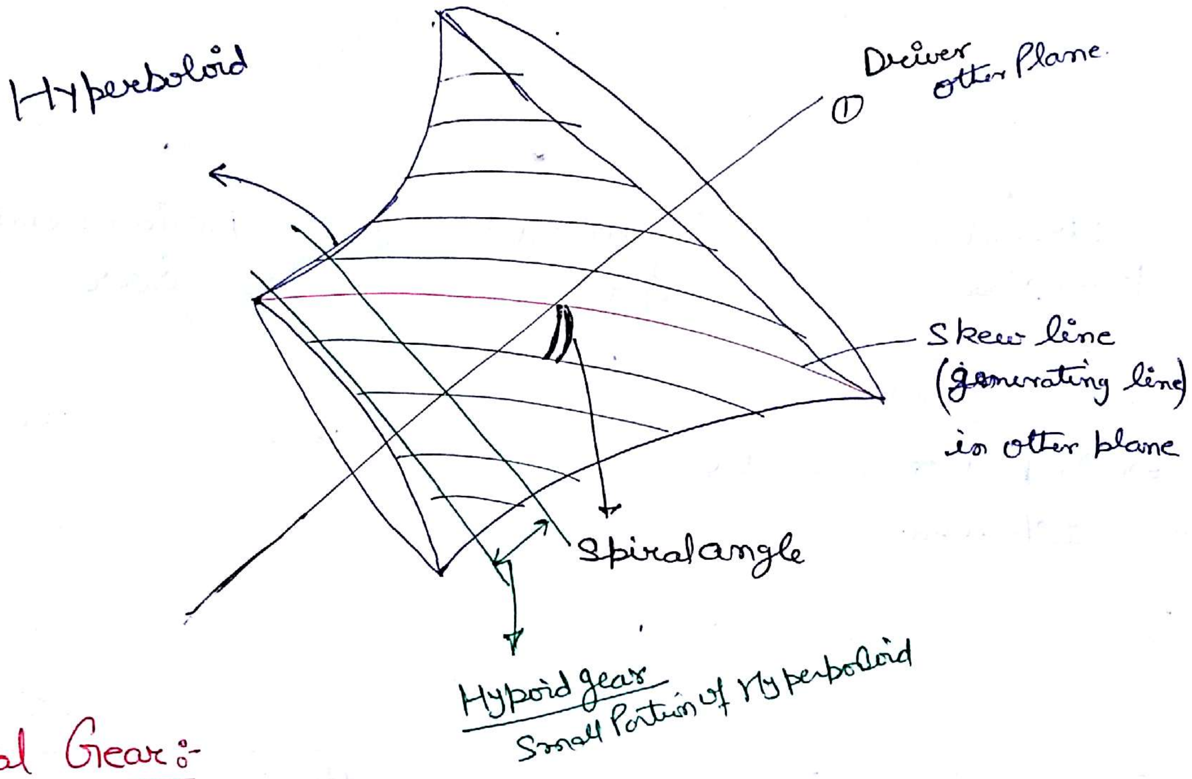
$\frac{\omega_1}{\omega_2}$ = Constant
but not $\neq 1$

$\boxed{\text{Mitre Gear}}$ → Use to Couple the governor with the engine

iii) Axes are Neither Parallel nor Intersecting :

→ Pure Rolling is not Possible.

→ Rolling is possible.
  └→ (Rotation + Partial Sliding)



Spiral Gear :-
⇓
Skew Bevel Gear

When the space b/w the Shafts is very less then Some Portion of Hyperboloid is used to form Spiral gear. Which are Called HyPoid Gear.

Worm & Worm Wheels :

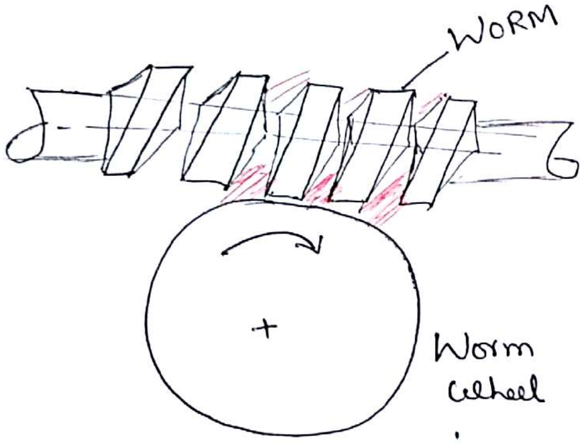
→ Very less diameter
→ Very high Spiral Angle.

→ Very high diameter
→ Very less spiral angle.

Worm is driver

Used in high Speed Reduction Ratio's

W	WW
10	1
30	1
300	1
1000	1
1250	1



Note

① → WORM — Rotates
WORM wheel bcz
Worm overcom
Starting torque of Worm wheel so that it Rotates & Slides

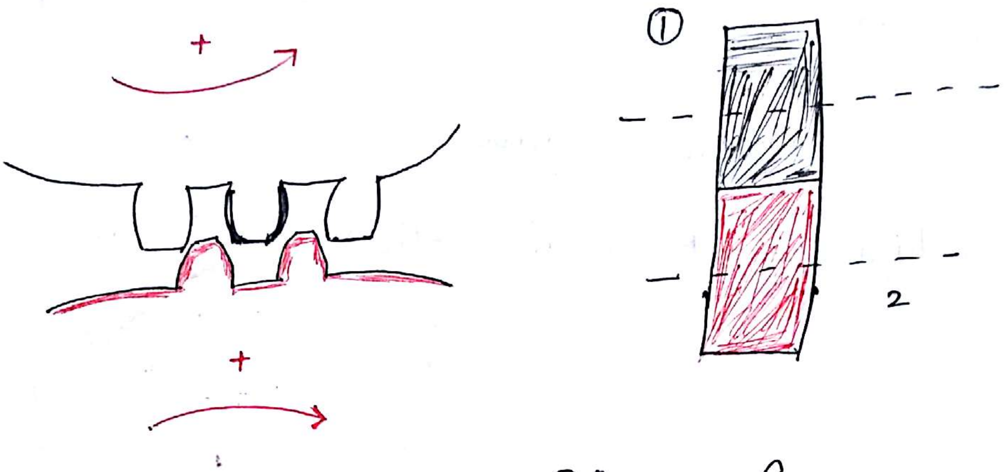
② but in Worm wheel
Can't rotate WORM
So it happen called
Self locking

used of self locking

③ → Screw Jack

B) According to the type of Gearing :-

External Gearing :

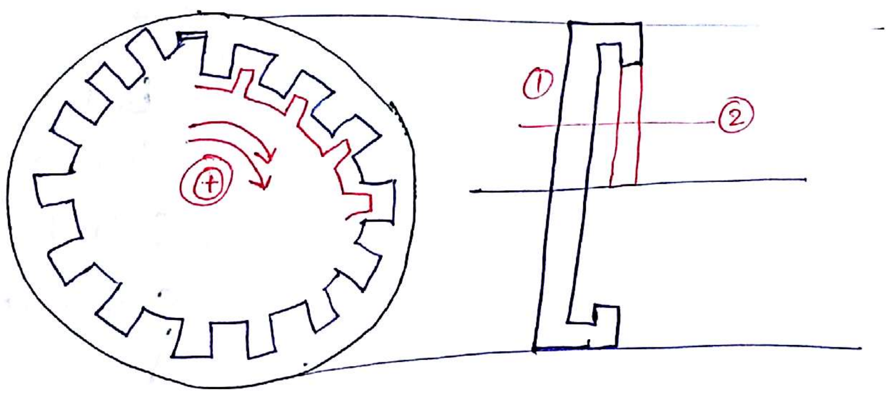


Driver → Smaller one (Pinion)

Bigger — Gear

Smaller — Pinion

Internal Gearing



Bigger → Annular (Ring)

Smaller → Pinion

Note:

→ If More than one gears are mounted on Same Shaft.

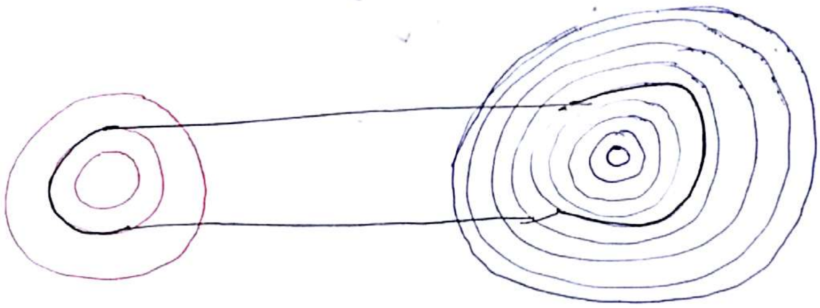
- Compound Gears
- Speed Same

→ Generally in power Transmissions Smaller bodies are made as Drivers.

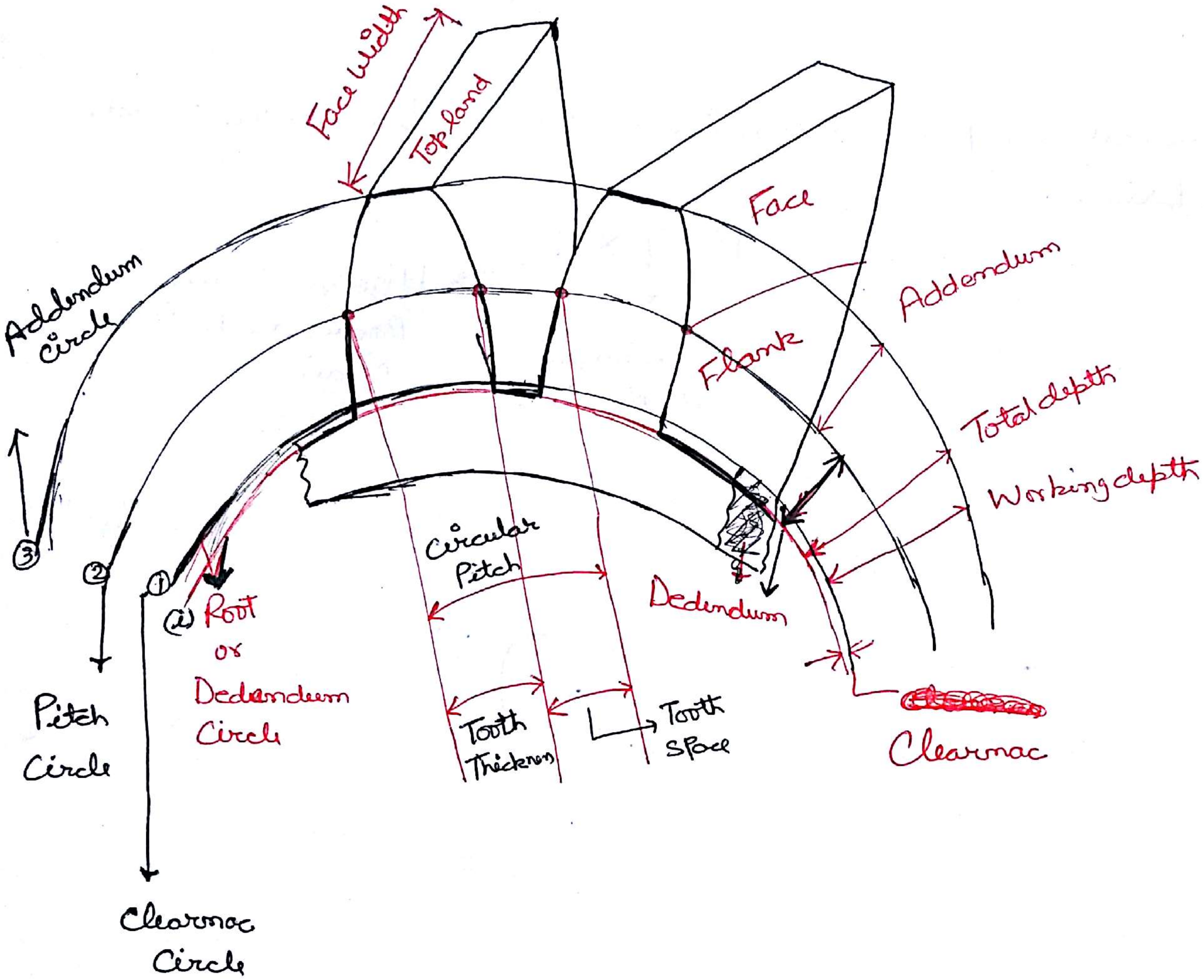
$P = T \times \omega$

T ↓ less Torque is Required.

ω → High for Smaller Bodies due to less Radius.

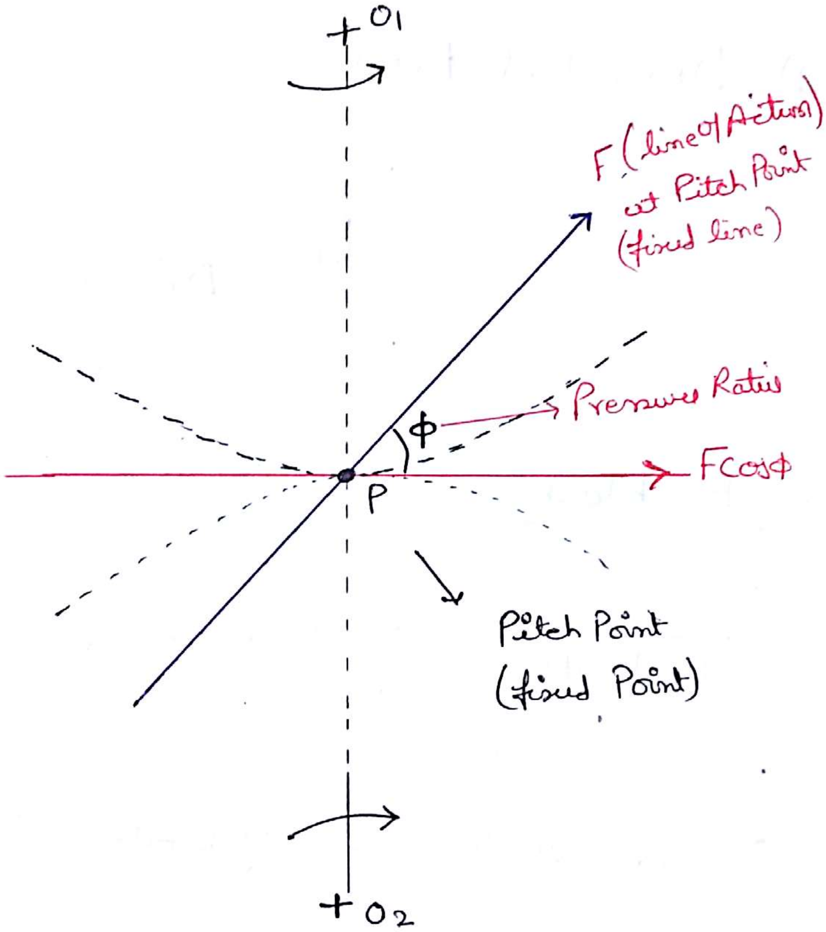


General Gear Terminology :-



Pitch Circle :- Most imp. circle → where Pure Rolling seen.

"It is the imaginary circle in the gears, where the pure rolling motion is observed, when the mating gears are transmitting Power. Being an imaginary circle, it can't be the physical characteristic of gear, but being the most important circle, it is one of the biggest Specification of the Gears. The size of any Gear, is Specified by the diameter of pitch circle."



2) Circular Pitch (Pc) :

Pitch circle diameter = D

No of teeth = T

$$\boxed{P_c = \frac{\pi D}{T}}$$

For two Mating Bodies :

$$P_{c1} = P_{c2}$$

$$\frac{\cancel{\pi} D_1}{T_1} = \frac{\cancel{\pi} D_2}{T_2}$$

$$\boxed{\frac{D_1}{T_1} = \frac{D_2}{T_2}}$$

3) Module (m)

fixed Parameter in Design

$$\boxed{m = \frac{D(mm)}{T}}$$

For two Mating Bodies

$$\boxed{m_1 = m_2}$$

Imp.

Qn) $P_c \cdot P_d = \frac{\pi \cancel{D}}{\cancel{T}} \times \frac{\cancel{T}}{\cancel{D}}$

$P_c \cdot P_d = \pi$

4) Diametral Pitch :

$$\boxed{P_d = \frac{T}{D(\text{inches})}}$$

5) Backlash :

Tooth Space − Tooth thickness of mating Gear = Backlash

To Prevent Jamming of ~~tooth~~ teeth ~~diameter~~ due to thermal expansion.

Both gear & Pinion are have to provide Backlash

6) Pressure Angle (Φ)

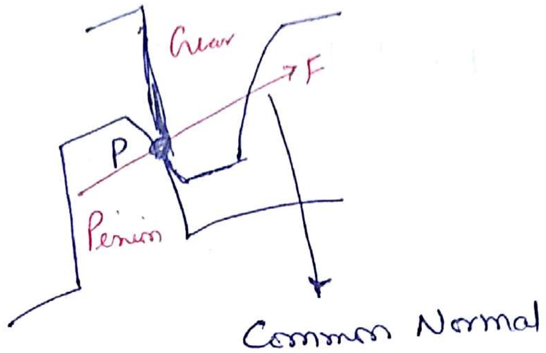
Φ → Pressure angle

⇓

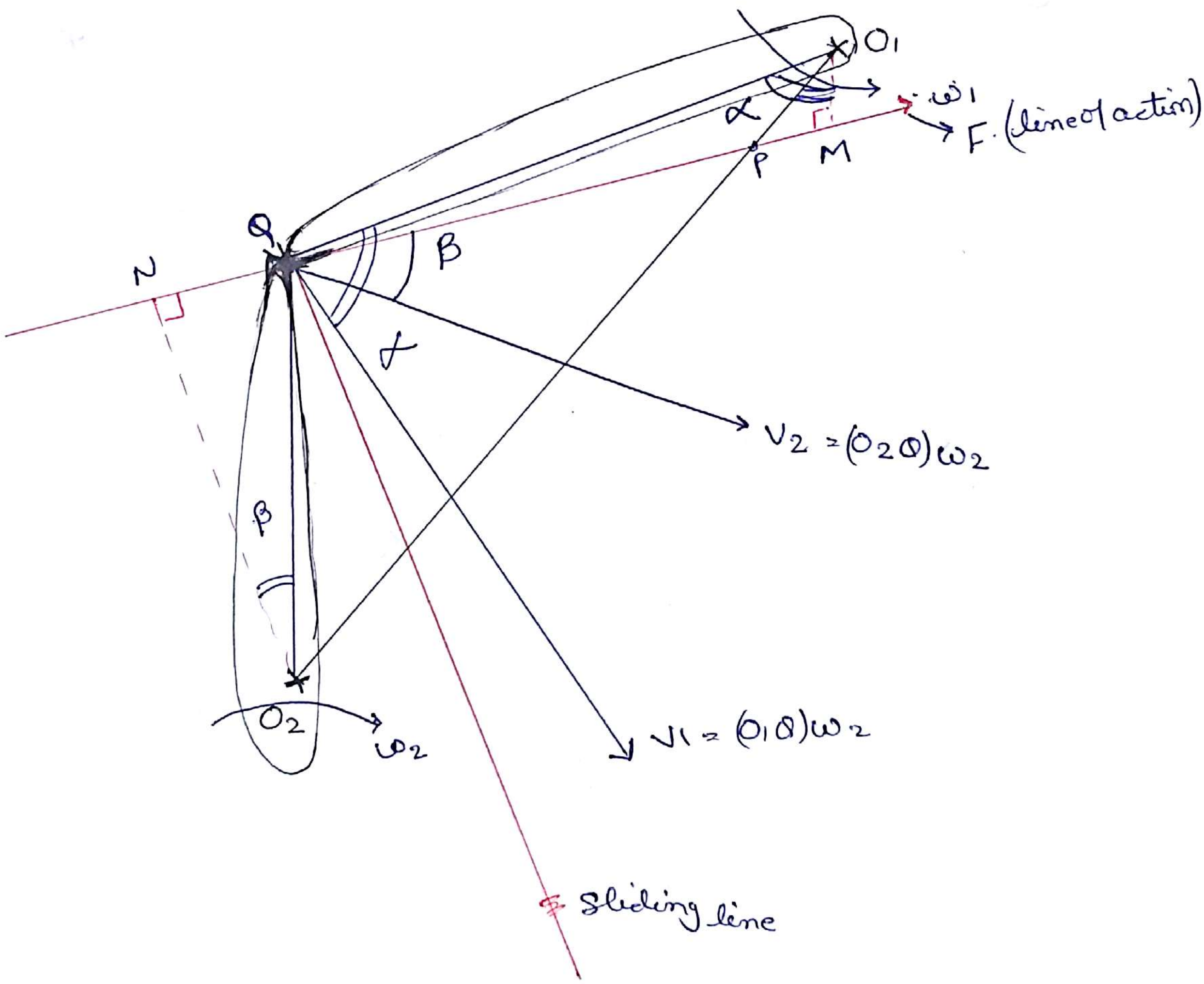
* Angle b/w line of action & Common tangent at P

$\boxed{\phi \in 20^\circ - 25^\circ}$

Most imp. Parameter



Law Of Gearing :-



For Proper Contact

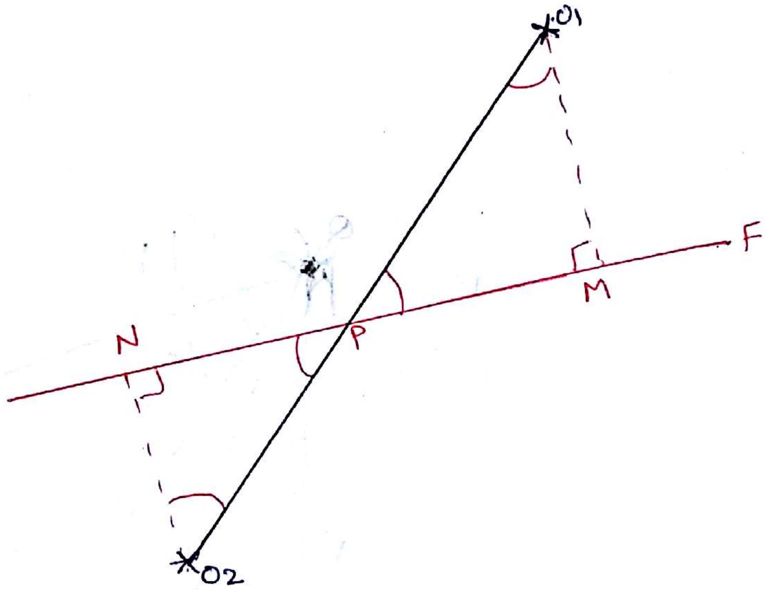
$$V_1 \cos\alpha = V_2 \cos\beta$$

$$(\cancel{O_1 Q})\,\omega_1 \frac{O_1M}{\cancel{O_1Q}} = (\cancel{O_2 Q})\,\omega_2 \frac{O_2N}{\cancel{O_2Q}}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \quad **$$

In $\triangle O_1PM \sim \triangle O_2PN$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}$$



If these two Body are Gear :

$$\boxed{\frac{\omega_1}{\omega_2} = \text{Constant}}$$

$$\frac{O_2P}{O_1P} = \text{Constant}$$

$\left\{ \begin{array}{l} O_1 O_2 \text{ Already fixed} \\ P - \text{ Fixed Point} \end{array} \right.$

$P \rightarrow$ fixed Point

" Line of Action must Always Pass through the fixed Point (Pitch Point) on the line joining the centre of rotation of gears.

For a Body to be a Gear :

$\Downarrow$

Line of Action must always pass through P

$\Downarrow$

Line of Action Common normal at Q.

$\Downarrow$

"Mating Profile must be designed in such a way, such that always Law of gearing is satisfied."

$\Downarrow$

→ Conjugate Profile :

- Involute
- Cycloidal

Velocity of Sliding :

$$V_{sliding} = \left| V_1 \sin\alpha - V_2 \sin\beta \right|$$

$$= \left| (O_1Q)\omega_1 \cdot \frac{QM}{O_1Q} - (O_2Q)\omega_2 \frac{QN}{O_2Q} \right|$$

$$= \left| \omega_1 (QP + PM) - \omega_2 (PN - QP) \right|$$

$$= \left| \omega_1 QP + \omega_1 PM - \omega_2 PN + \omega_2 QP \right|$$

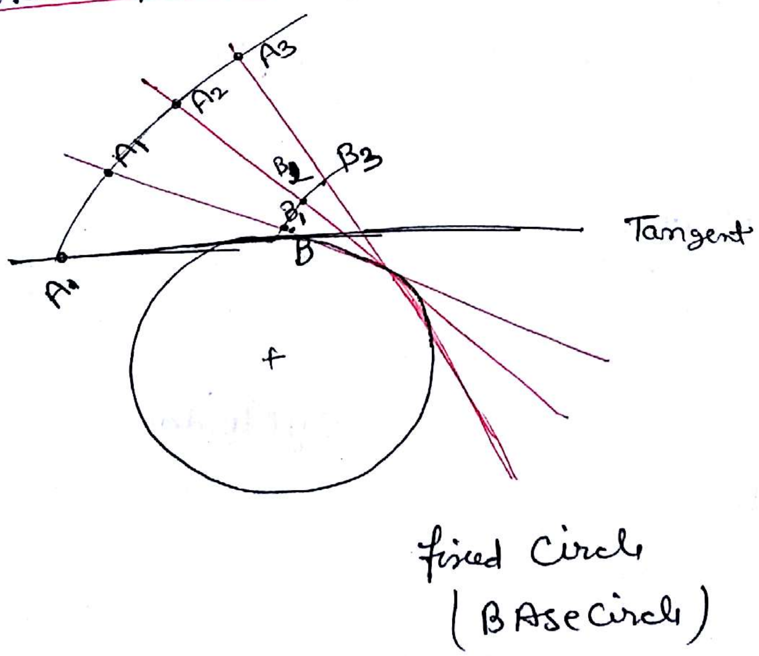
$$= (\omega_1 QP + \omega_2 QP)$$

$$\boxed{V_{sliding} = QP(\omega_1 + \omega_2)} \; **$$

Involute Profile (By nature Conjugate) :-

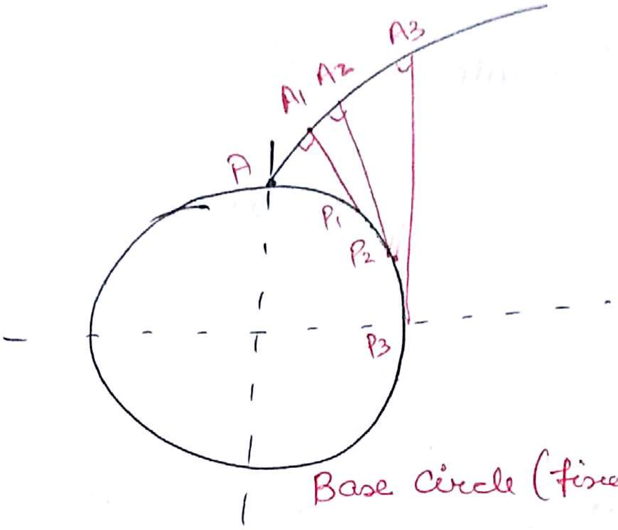
" It is defined as the locus of a point, on the line which rolls without sleeping on the fixed circle". And this fixed circle is known as Base circle, which is basically the generator of involute profile.

Involute Profile →



In Reality :

$$\left.\begin{matrix} AP_1 \\ P_1P_2 \\ P_2P_3 \\ \vdots \end{matrix}\right] \longrightarrow 0 \quad \text{Diffrential}$$



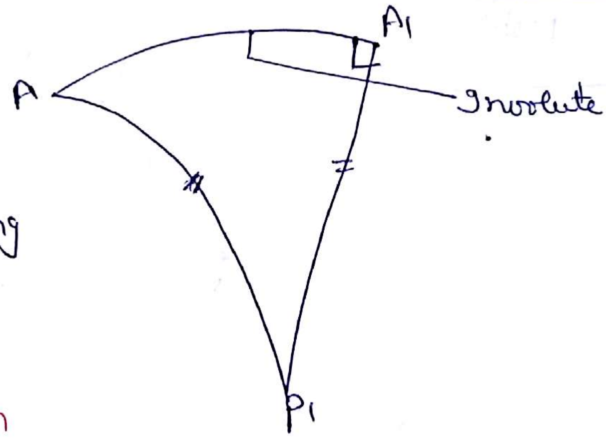
$Arc(AP_1) = P_1A_1$

$Arc(AP_2) = P_2A_2$

$Arc(AP_3) = P_3A_3$

$\vdots$

Now Profile shown is get into straight



Diffrential Portion of circle having Centre P_1 & Radius P_1A_1

"Normal drawn at any Point on Involute Curve Will become tangent to its base circle Automatically."

Actual Position of Base circle in an Involute Gear :-

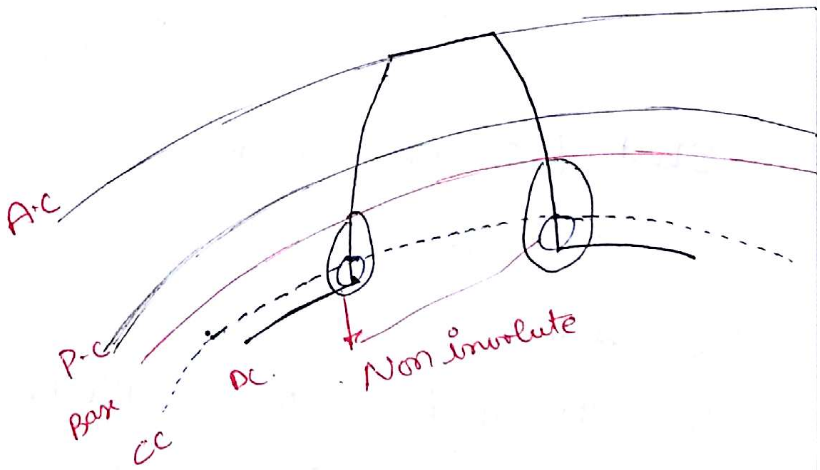
In. external gear

if $R_{Base} \downarrow$

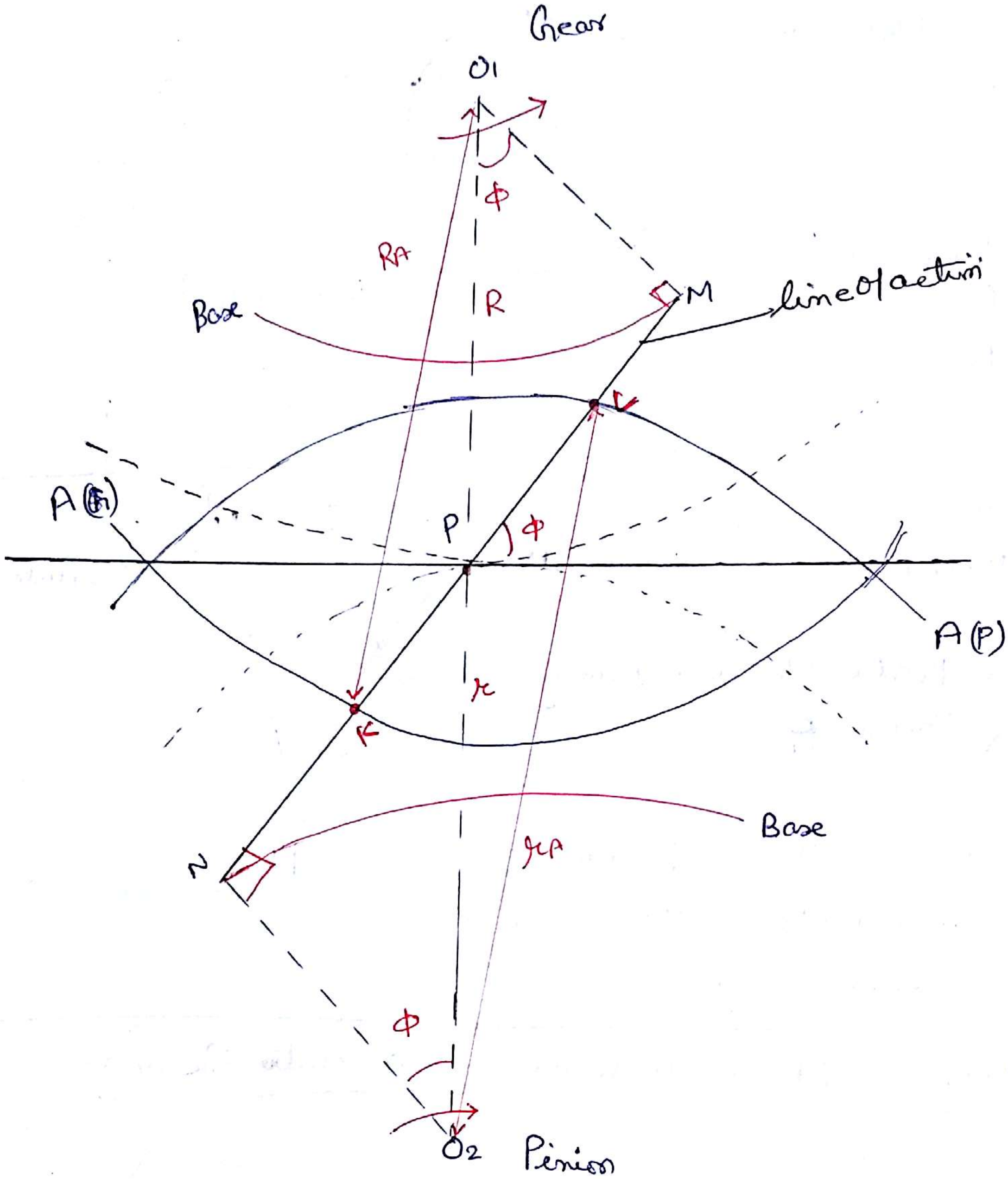
and Pressure angle $\uparrow$

$(20-25^{\circ})$

→ Non-Involute Portion in an external gear Can never be eleminated —— (Reality)



Analysis Of Involute Gears :-



Start of engagement : K

end of engagement : L

Line of Action

1) Pass through Line of Action or (Pitch point)

2) Tangent to both of Base Circle

→ Point of Contact is changing but line of action is not changing.

Hence, $\phi = \text{Constant}$

→ Point of Contact is travelling along the line of action.

Locus of Q (Point of Contact) → Straight Lines

Time interval in which Q is travelling from start to end of engagement.

⇓

One engagement Period

and the distance travelled by Q in this period.

⇓

Path of Contact

KP — Path of Approach

KL — Path of Recess.

$O_1M = R\cos\phi$

$PM = R\sin\phi$

in $\triangle O_1KM$

$$R_A^2 = R^2\cos^2\phi + (KP + R\sin\phi)^2$$

$$\boxed{KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi} \quad \text{—(1)}$$

KP → Path of Approach

Similarly

$$\boxed{P_L = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi} \quad \text{— (2)}$$

Path of Recess

Sum of eqn (1) & (2) is Path of Contact.

Arc of Contact :

When the point of Contact is travelling from the start of engagement & end of engagement, the distance ~~travel~~ travelled by Pinion and gear along their pitch circles, in this period (one engagement period) is known as Arc of Contact.

$$\text{Arc of Approach} = \frac{\text{Path of Approach}}{\cos\phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of Recess}}{\cos\phi}$$

+

$$\boxed{\text{Arc of Contact} = \frac{\text{Path of Contact}}{\cos\phi}}$$ **

Arc of Contact → Travel of Pinion / Gear ~~along~~ their ~~Pitches~~ Pitch circle in one engagement Period.

$$\boxed{\text{Contact Ratio} = \frac{\text{Arc of Contact}}{P_c}}$$

No. of Pair engaged in one engagement Period.

min >1

Contact Ratio is always >1

Contact Ratio lie b/w (1.2 – 1.8)

Imp

Representation

for e.g

Contact Ratio = 1.32

It means, one pair is engaged in full engagement Period, But in 32% time of engagement period. along with this Pair one more Pair is engaged in one engagement Period.

its average Value Comes out to be 1.32.

Pb 47)

$t = 24$

$T = 36$

$m = 8\,mm$

$\phi = 20°$

Addendum (each) = 7.5 mm

$N_P = 450\ rpm$

$$\frac{N_G}{N_P} = \frac{t}{T} = \frac{24}{36}$$

$$\boxed{N_G = 300\ r.p.m}$$

Gear:

$$R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144\,mm$$

$$R_A = 144 + 7.5 = 151.5\,mm$$

Pinion = $r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96\,mm$

$$r_A = 96 + 7.5 = 103.5\,mm.$$

i) Path of Contact:

$$KL = KP + PL$$

$$= \left\{\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi\right\} + \left\{\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi\right\}$$

$$KL = (\quad)\,mm + (\quad)\,mm$$

$$\text{Arc of Contact} = \frac{KL}{\cos 20°} = (\ )\,mm$$

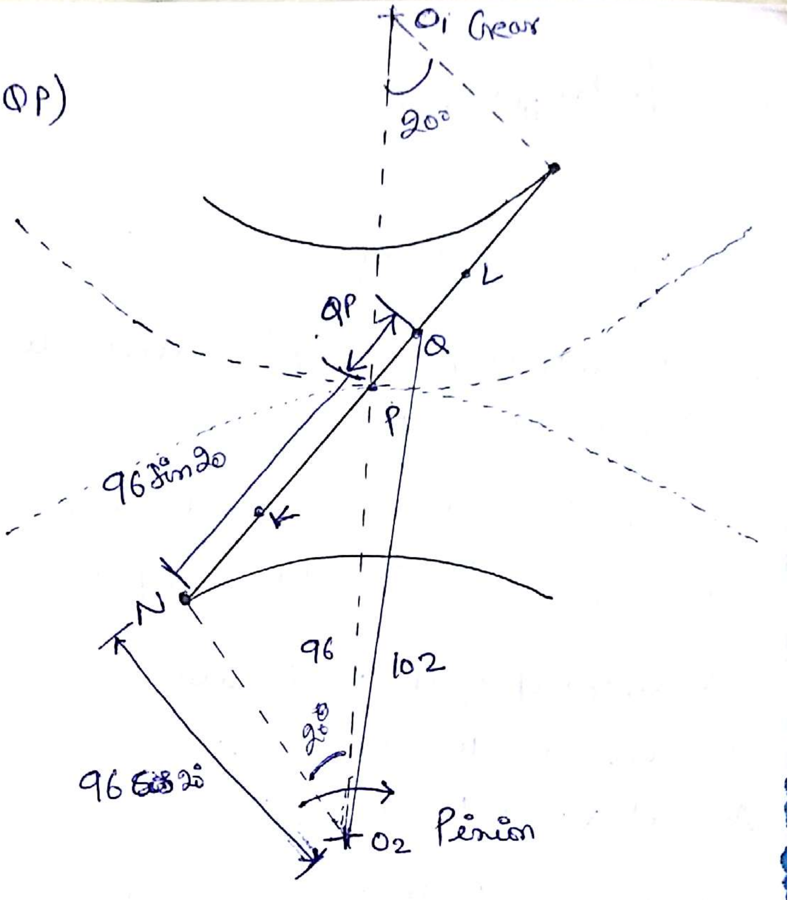
i) $(\text{Angle})_{\text{pinion}} = \frac{\text{Arc of Contact}}{96} \times \frac{180}{\pi} = (\ )°$

ii) $V_{(\text{sliding})} = (\omega_1 + \omega_2)\,QP$

$$= \frac{2\pi}{60}(300 + 450)\,QP. \quad -(1)$$

$$(102)^2 = (96 \cos 20)^2 + (96 \sin 20 + QP)$$

$QP = ?$



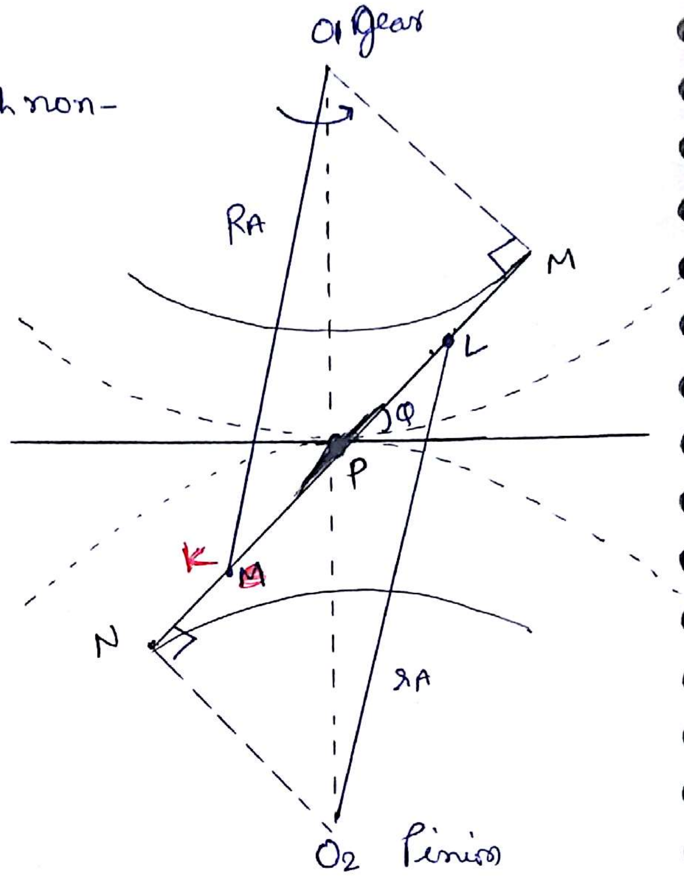
Interference :- due to R_A & r_A ↑

If $r_A > O_2M$

⇒ Involute Tip of the pinion will touch non-Involute flank Position of gear.

⇒ Involute → to non involute Connection

⇒ Law of gearing is not be Satisfied.



Involute tip of the pinion will remove some material from non-Involute flank Position of Gear.

(This Removal of the material is a Process called undercutting.)

Similarly

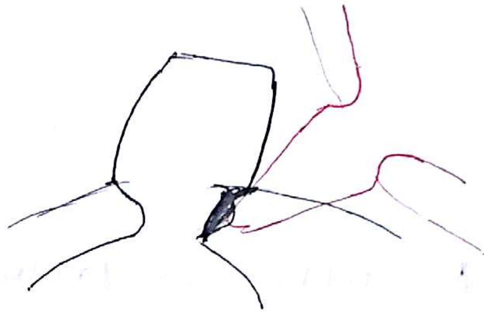
$$\boxed{R_A > O_1N}$$

Last Safety Points of K & L are M and N

Which we can say Critical Point/Interference Point

Methods to Prevent interference :

i) Under Cut Gears :



Under cutting is Provided by the Cutting tool at the time of manufacturing

Strength of tooth is less at root.

Application ~~Limitation~~ → used in low Power Transmission

ii) if Pressure Ratio (ϕ) is increased.

[Used for Medium Power Transmission]

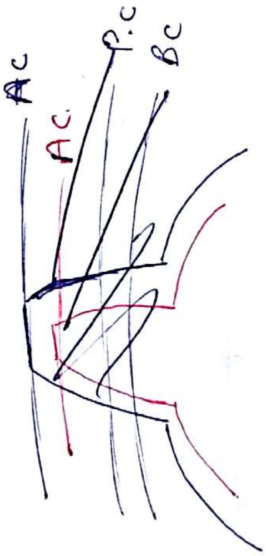
If ϕ is increased by 'decreasing' r_{base}

→ non involute Portion ↓

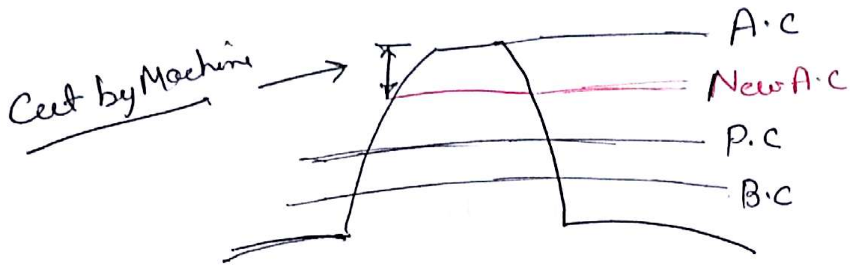
→ Interference ↓↓ (Main Aim)

Limitation of ϕ = $(20° - 25°)$

~~iii) Edge Stubbing the Teeth :-~~



iii) By Stubbing the teeth



By Stubbing ⇒ Addendum ↓
Adendum Circle Radius ↓
Interference ↓↓

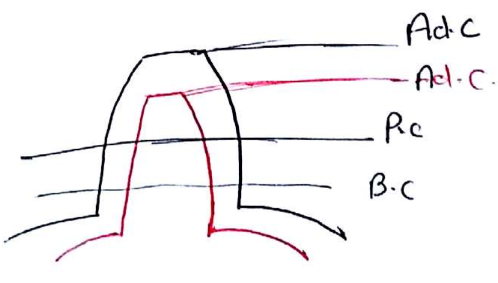
By stubbing → ϕ → Not Change

By Stubbing → Path of Contact of ↓
Arc of Contact ↓
Contact Ratio ↓
min > 1 Limitation

$$\boxed{P_c = \frac{\pi D}{T}}$$
→ No Change.

iv) Increasing number of teeth:

If T ↑ ⇒ ϕ → No Change
⇒ Addendum ↓
⇒ Adendum Circle Radius ↓
⇒ Interference ↓↓



If T ↑ ⇒ Arc of Contact ↓
⇒ $P_c = \frac{\pi D}{T}$ ↓
⇒ Contact Ratio ↑

Velocity Ratio:

$$\frac{\omega_P}{\omega_S} = \frac{T}{t} > 1$$

$$\frac{\omega_S}{\omega_P} = \frac{t}{T} < 1$$

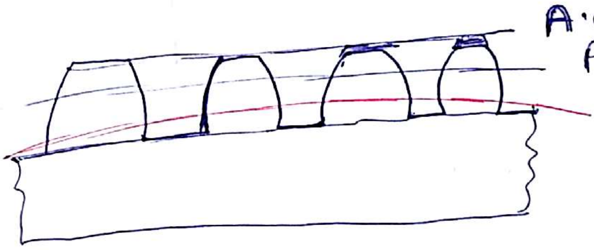
Gear Ratio (s):-

$$\boxed{G = \frac{T}{t}}$$ ** Always ≥ 1

Rack:

Gear of infinite Pitch circle Diameter (P_c)

→ Biggest Gear



Base circle is seems to be straight but it is not straight. it is ~~spherical~~ circle Radius of infinite Dia

NoTe

A → Fractional Addendum for 1 mm module in order to avoid interference.

A_P
A_G
A_R

Therefore, Addendum Required in order to avoid interference.

$= mA$ where (m is module)

$= mA_P$

$= mA_G$

$= mA_R$

for examples:

Addendum = 7.5 mm
module (m) = 8 mm

$mA = 7.5$

$A = \frac{7.5}{8} < 1$ So it is Stub

Involute Gear System:

Full depth Involute: ($14\frac{1}{2}$, 20°)

Addendum = Standard Addendum
= one module vale

$\cancel{m}A = 1\cancel{m}$

$\boxed{A = 1}$

$A_P = 1$
$A_G = 1$
$A_R = 1$

Stub Involute (20°, 25°)

Addendum : < Standard Add.

$\cancel{m}A < 1\cancel{m}$

$\boxed{A < 1}$

$\boxed{\left.\begin{matrix} A_P \\ A_R \\ A_G \end{matrix}\right] < 1}$

→ 20° Involute Stub

Best Gears

1) lesser Interference

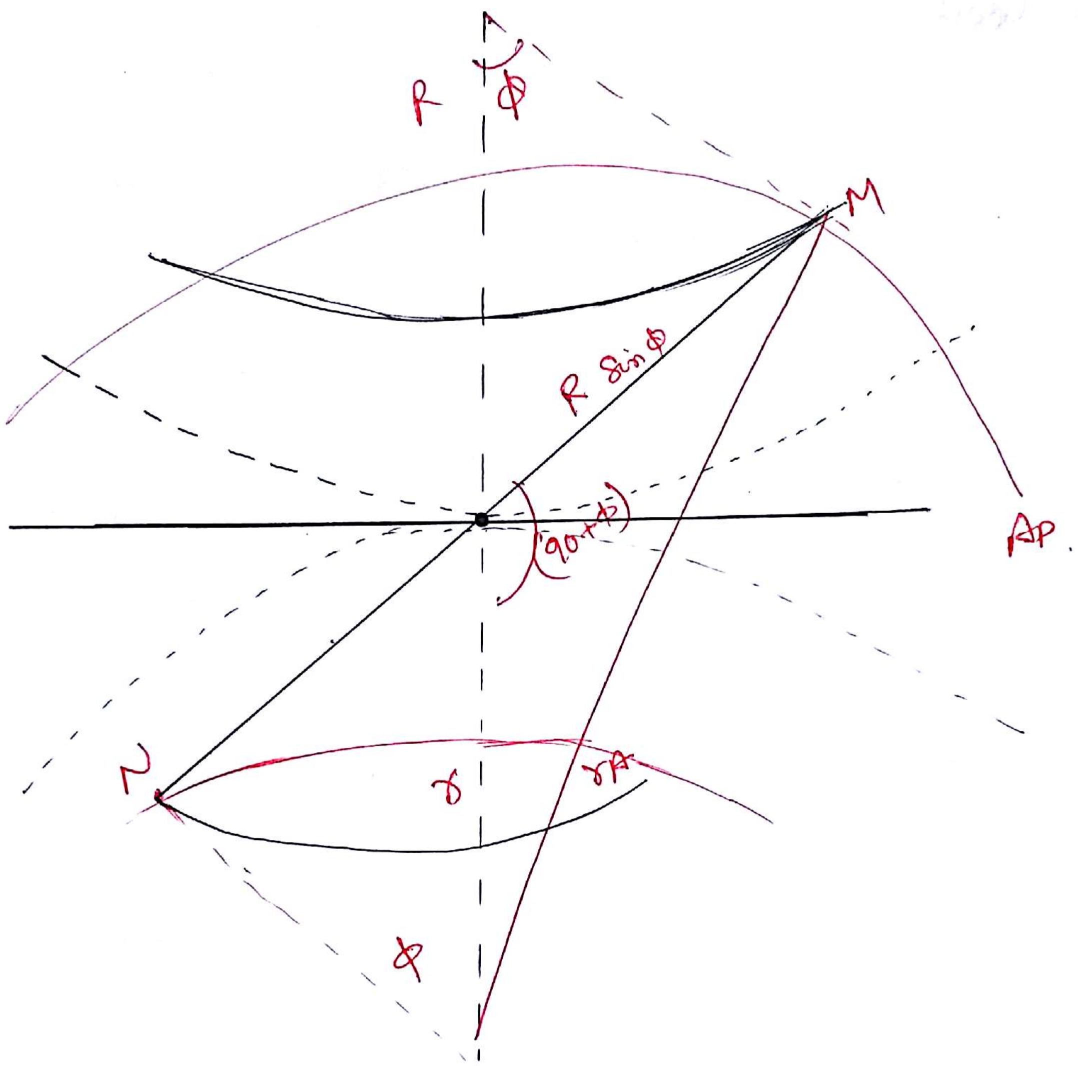
2) Min no. of teeth, requirement is less.

3) Cost is less

4) Stronger teeth

Minimum Number of teeth requirement on Pinion or Gear, in order to avoid Interference:-

(K-L)
↓
Safety limit



$\triangle O_2PL$

Cos Rule $\rightarrow$

$$r_A^2 = r^2 + R^2 \sin^2\phi - 2(r)(R\sin\phi)\cdot\cos(90+\phi)$$

$$= r^2\left(1 + \left(\frac{R}{r}\right)\cdot\left(\frac{R}{r}\right) + 2\right)\sin^2\phi\Big)$$

$$r_A = r\sqrt{1 + G(G+2)\sin^2\phi}$$

$$\rightarrow r_A - r = \text{Add. Pinion}$$

$$= r\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$

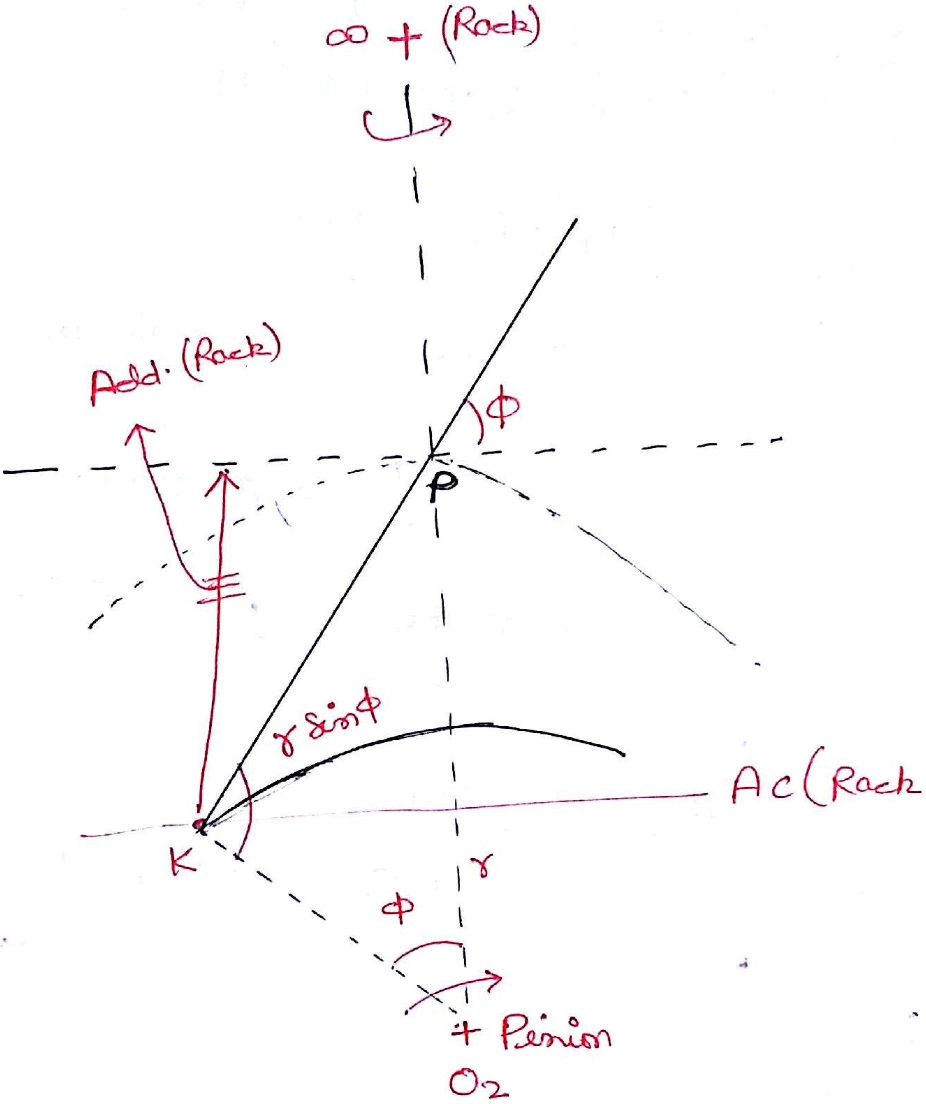
$$= \frac{\cancel{m}\, t_{min}}{2}\left(\sqrt{1 + G(G+2)\sin^2\phi} - 1\right) = \cancel{m}\, A_P$$

$$\boxed{t_{min} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$$

Similarly

$$\boxed{T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}} \ast$$

Minimum number of teeth requirement on the Pinion in Order to avoid Interference in involute Rack & Pinion arrangement :



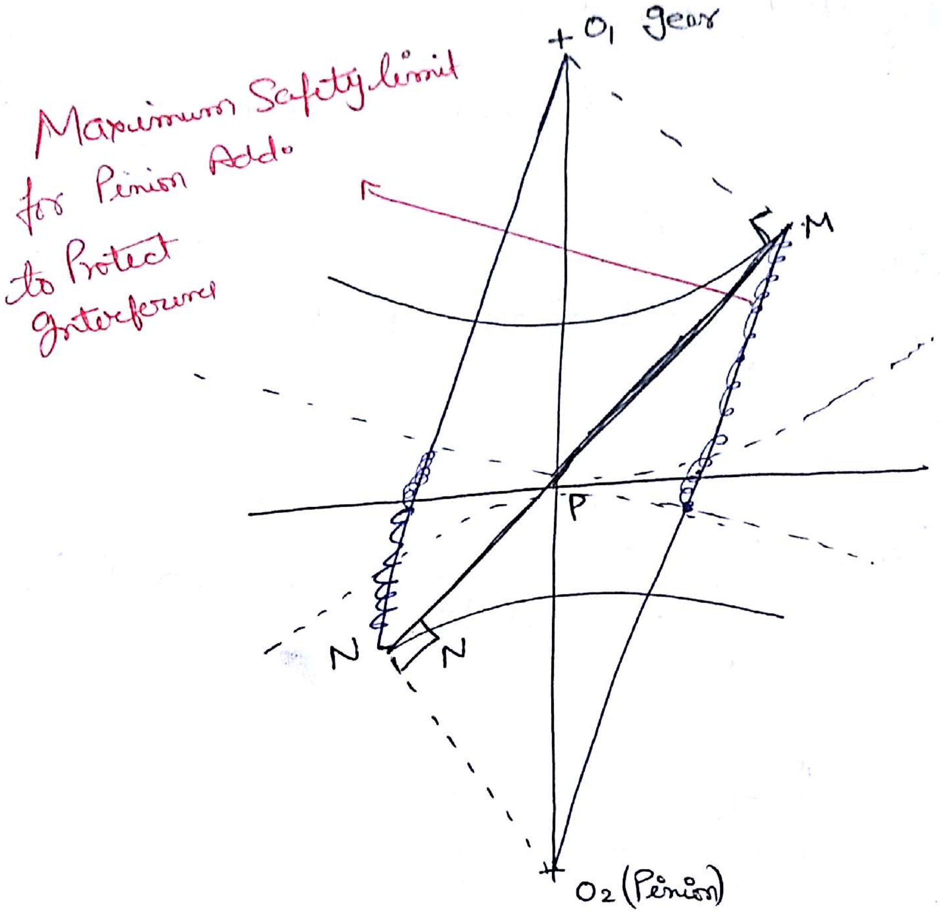
$$\sin\phi = \frac{\text{Add. (Rack)}}{r\sin\phi}$$

$$\text{Add Rack} = R\sin^2\phi$$

$$\cancel{\text{xmt sin}} = \frac{\cancel{m}\, t_{min}}{2}\sin^2\phi = \cancel{m} A_R$$

$$\boxed{t_{min} = \frac{2A_R}{\sin^2\phi}} ** $$

NOTE ***



If Gear | Pinion
→ Addendum Same.

1) first Gear must be Safe

$$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

2) Pinion Will automatically Safe.

$$\boxed{t_{min} = \frac{T_{min}}{G}}$$

If Examiner ask to Work out: $G = 3$

28 → 30 to get

3

$\frac{28}{3} = 9.9.$

10

ii) If Pinion / Gear

→ ~~Gear~~ Add s - Different

$\boxed{G = 3}$

$$T_{min} = \frac{2AG}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} \rightarrow 40 - 42$$

$$t_{min} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} \rightarrow 13 - 14$$

$$\boxed{\frac{T_{min}}{t_{min}} = G}$$

Q) 48 $G = 3$ $A_p = A_G = 1$ $\phi = 20°$ $t_{min} = ?$

$$T_{min} = \frac{2A_G}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1} = 45^*$$

$$\boxed{t_{min} = \frac{45}{3} = 15^{**}}$$

$G = 3 \rightarrow T_{emin} = 12 \times 36 = 36^*$

→ Yes interference occurs

$$T_{min} = 36 = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1} =$$

$A_g = 0.8$

20% Stubbing Ag

Qn -

$$T_{emin} = 36 = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

3

$\phi = ?$

Pb Q6

$\phi = 20^\circ$

$m = 10\,mm$

$A_p = A_G = 1$

$\left.\begin{array}{l} T = 50 \\ t = 13 \end{array}\right] \quad G = \frac{50}{13}$

i) $T_{min} = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1} = 60$ Yes

iv) $T_{min} = 50 = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$

$\phi = ?$

Qn

$G = 4$ find $T_{min} = ?$

$A_p = A_G = 1$

$\phi = 20^\circ$

$T_{min} = \dfrac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1} = \cancel{62}$ 64

$t_{min} = \frac{62}{4} = 15.5$ 16

$\boxed{G = \frac{64}{16} \; 4}$

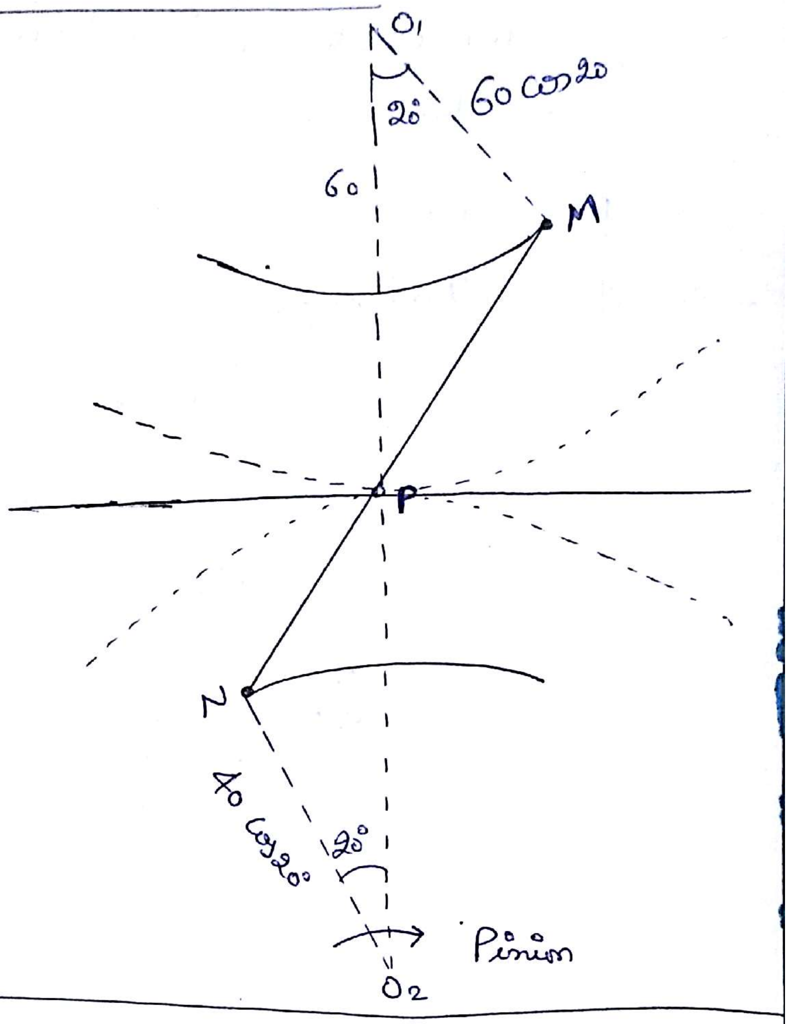
T_{min} for Pinion use करना जरूरी है।

G = Gear Ratio के लिये।

as Gear safe Pinion Safe

Check Pinion also

Effect of the Centre Distance Variation Due to Vibrations On the performance of involute gears :-



Centre distance = 100mm

$R = 60\,mm$

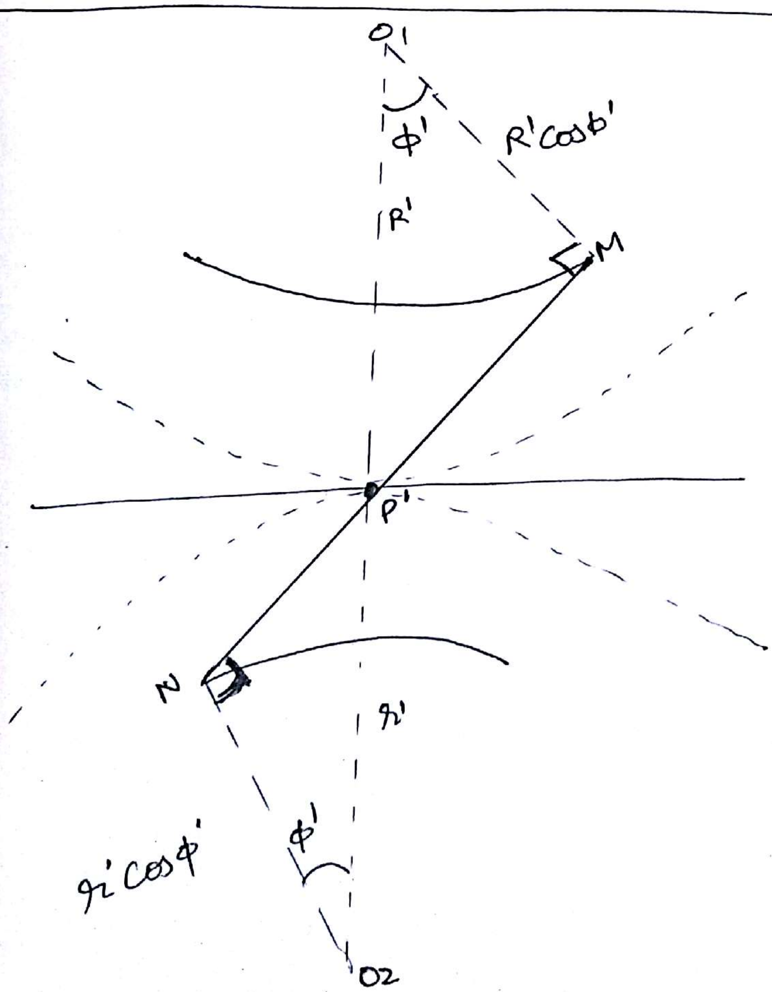
$r = 40\,mm$

$\phi = 20°$

At a moment

Centre distance is ↑ by 2%

100mm
↓
102mm



$R' \cos\phi' = 60 \cos 20$

$r' \cos\phi' = 40 \cos 20$

$\underbrace{(R'+r')}_{102} \cos\phi' = 100 \cos 20°$

$\cos\phi' = \frac{100 \cos 20°}{102}$

$\phi - ?$

Bcz of Vibration :

→ Centre distance is Changing

→ $\underline{Pc}$ Changing

→ P Changing

→ ϕ Changing

r_{Base} → No Change

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}}$$

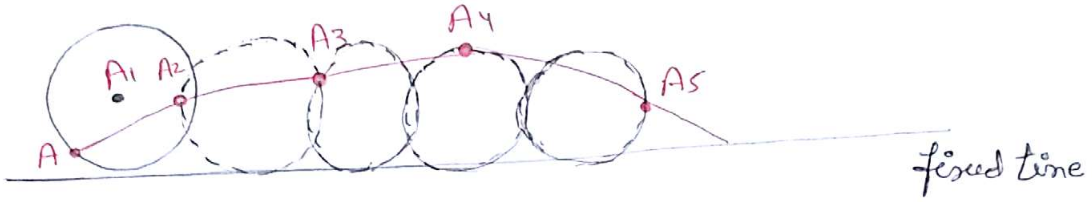
⇓

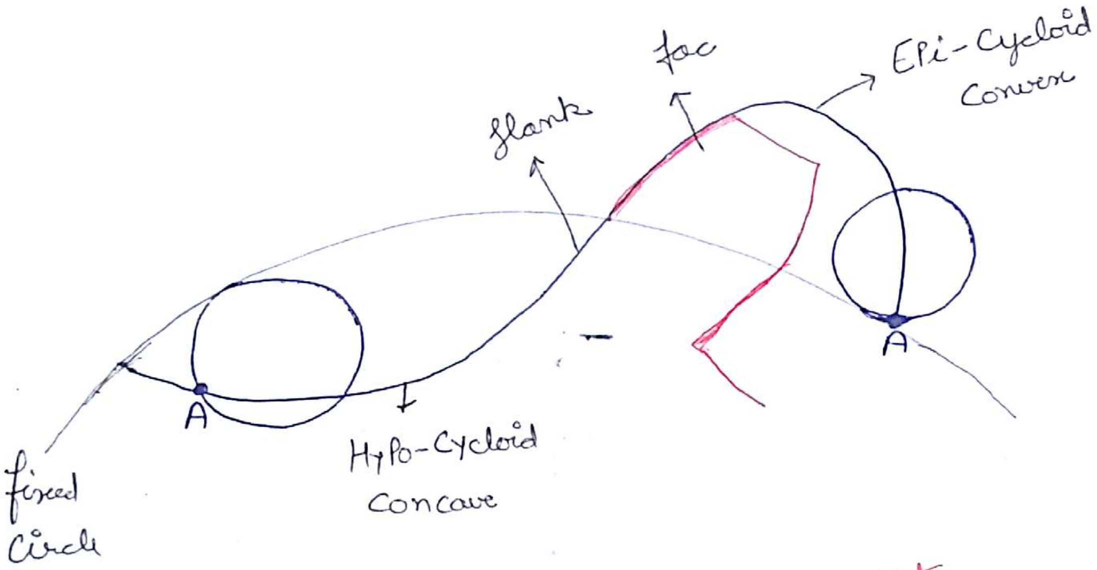
Constant

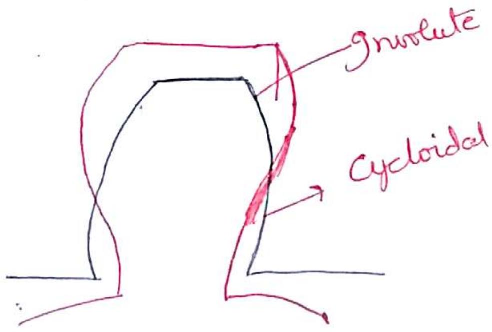
After Vibration Constant Velocity Ratio obtained

Cycloidal Profile (By Natieri Conjugate) :

" It is defined as locus of the point on the circumference of the Circle, which rolls without Sleeping, on the fixed Straight line.







1) Per tooth Cost is more.

Overall Cost of Gear - less (Almost equal) (No machining or extra removal of tooth bottom surface)

Interference is absent

2) Flanks wide → Stronger teeth

3) Convex - Concave Connection

↳ Less Wear & tear

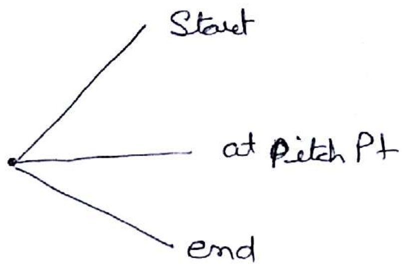
Life is 5 times more than involute (1000 years)

ϕ changing

ϕ_{max} → At Start of engagement

0 → At Pitch Point

ϕ_{max} → at end of engagement (in Reverse dirⁿ)



NOTE:

Power Component of force :- $F\cos\phi$

for. example

The Power Component of force is 0.94 times the Normal Thrust Force.

It means

$$\cancel{F}\cos\phi = 0.94\cancel{F}$$

$$\cos\phi = 0.94$$

So we Can Calculate Value of ϕ i.e Pressure angle

Torque

$$T_{Pinion} = F\cos\phi \times r$$

$$T_{Gear} = F\cos\phi \times R$$

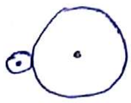
$$\boxed{Power = T \times \omega}$$

Gear Trains

Gear Train is Combination of Gears.

→ Why it is Required?

⇓



teeth will gone.

i) Large Centre distance

ii) Very high / very Low Requirement of velocity Ratio → $\frac{\omega_1}{\omega_2} = \frac{10}{1} = \frac{R_2}{R_1}$

iii) Multiple velocity Ratios are required

Any gear train is Combination of following

⇓

Max.

i) Main Driver

ii) Main Driven

iii) Intermediate Gear

min

iv) Arm. (For epi-cyclic Gear Train) used in Diffrential Box.

**

$$\boxed{\frac{\omega_{\text{main Driver}}}{\omega_{\text{main Driven}}} = \text{Speed Ratio (SR) of Gear Train}}$$

**

$$\boxed{\frac{\omega_{\text{main DVN}}}{\omega_{\text{main DVR}}} = \frac{1}{S.R} = \text{Train Value}}$$

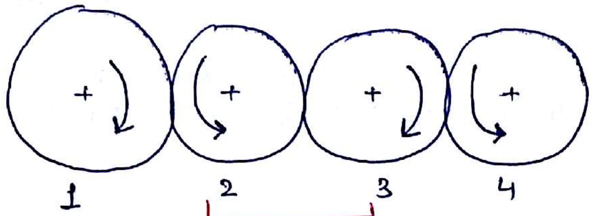
Simple Gear Train :

⇓

Every Shaft is having only one Gear in use.

$m_1 = m_2 = m_3 = m_4$

Mall → Same



Main DVR

Main DVN

Intermediate Gear or Idlers → No Contribution in Speed Ratio

(1,2) $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$ — ①

(2,3) $\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$ — ②

(3,4) $\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$ — ③

If No. of Idlers
- even → Dir^n opposite
- odd → dir^n Same

Product of ① ② ③

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{T_4}{T_1}}$$ ** — Speed Ratio

Gear which mating having Same module always

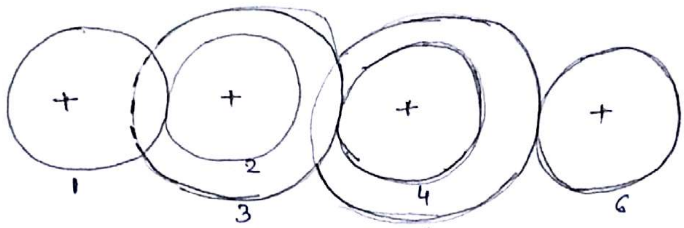
Pinion (Small one) is taken as Driver.

Compound Gear Train :

At least One of the intermediate Shaft must have more than one gear in use.

$m_1 = m_2$
$m_3 = m_4$
$m_5 = m_6$

DVR = (1, 3, 5)
DVN = (2, 4, 6)



Main DVR

2-3 Comp. $(\omega_2 = \omega_3)$

4-5 Comp. $(\omega_4 = \omega_5)$

Main DVN

(1-2) $\dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$ — ①

(3,4) $\dfrac{\omega_3}{\omega_4} = \dfrac{T_4}{T_3}$ — ②

(5,6) $\dfrac{\omega_5}{\omega_6} = \dfrac{T_6}{T_5}$ — ③

Product of ① ② ③

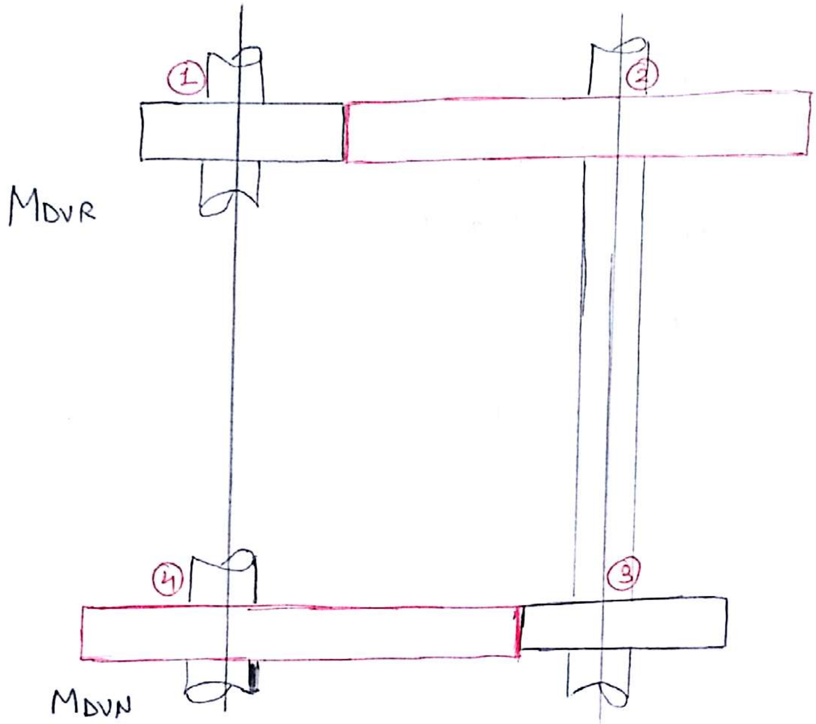
$$\boxed{\dfrac{\omega_1}{\omega_6} = S.R = \dfrac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$ ***

$$\boxed{\text{Speed Ratio} = \dfrac{\text{Product of the no. of Teeth on DVN}}{\text{Product of the no. of teeth on DVR}}}$$

Reverted Gear Trains :-

⇓

→ That Compound Gear Train which is used to Connect Co-axial Shaft.



$m_1 = m_2 = m$

$m_3 = m_4 = m'$

DVR = (1, 3)

DVN = (2, 4)

$$\boxed{S.R = \frac{\omega_1}{\omega_4} = \frac{T_2\,T_4}{T_1\,T_3}}$$ **

NOTE :

If in the Problem of Reverted gear train, Speed reduction is given same

Then Take

$$\boxed{\frac{T_2}{T_1} = \frac{T_4}{T_3}}$$

A General Concept :-

$$(r_1 + r_2) = (r_3 + r_4)$$

$$\boxed{r = \frac{mT}{2}}$$

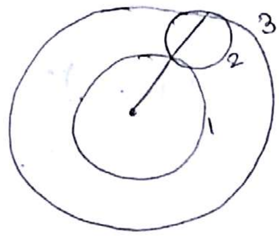
$$\frac{mT_1}{\cancel{2}} + \frac{mT_2}{\cancel{2}} = \frac{m'T_3}{\cancel{2}} + \frac{m'T_4}{\cancel{2}}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

if all gears have same Modules; (All m same)

$$\boxed{T_1 + T_2 = T_3 + T_4}$$

For. example



$$r_1 + 2r_2 = r_3$$

m all same

$$T_1 + 2T_2 = T_3$$

Q.44)

Given : $m = 2mm$

$m' = 3mm$

$\omega_4 < \frac{\omega_1}{12}$

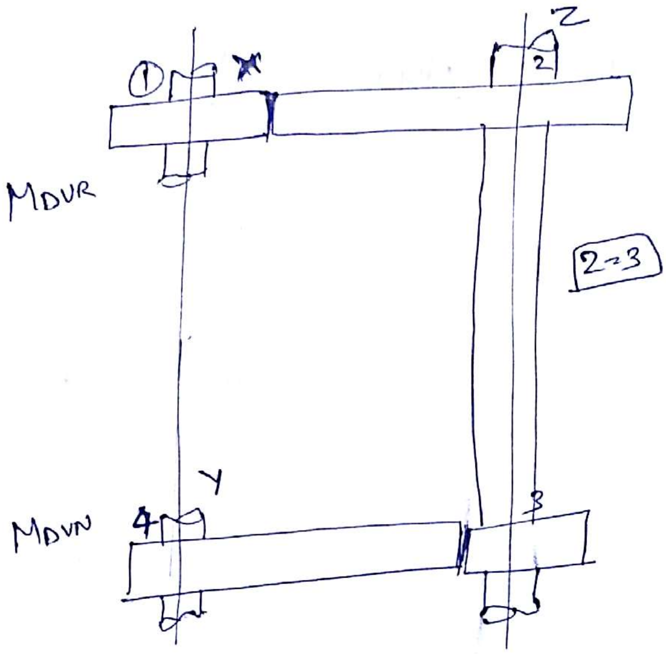
$\boxed{T_1 = T_3 = 24 \text{ given}}$

$T_2 = ?$

$T_4 = ?$

$T_1 = ?$

$T_3 = ?$



$\frac{\omega_1}{\omega_2} > 12$

$\frac{T_2 \ T_4}{24 \times 24} > 12$

$T_2 T_4 > 6912$ — 1

$m(T_1 + T_2) = m'(T_3 + T_4)$

$2(\underset{T_1}{24} + T_2) = 3(\underset{T_3}{24} + T_4)$

$48 + 2T_2 = 72 + 3T_4$

$3T_4 = 2T_2 - 24$

$T_4 = \frac{2}{3}(T_2 - 12)$ — 2

Putting 2 in 1

$$\frac{2}{3}(T_2-12)T_2 > 6712$$

$$T_2^2 - 12T_2 - 10638 > 0$$

$$T_2 > 108$$

Assume

$T_2 = 109 \Rightarrow T_4 = \frac{2}{3}(109-12)$

$$\boxed{T_4 = 64.66}$$

Hit and trial Method;

Assume $T_4 = 65$

$T_3 = 24 - 1 = 23$

$m(T_1+T_2) = m'(T_3+T_4)$

$2(24+T_2) = 3(23+65)$

$T_2 = 108$

$T_1 = 24$
$T_2 = 108$
$T_3 = 23$
$T_4 = 65$

$\frac{\omega_1}{\omega_4} \cdot \frac{T_2 T_4}{T_1 \cdot T_3} = \frac{108 \times 65}{24 \times 23} = 12.718$

Central distance

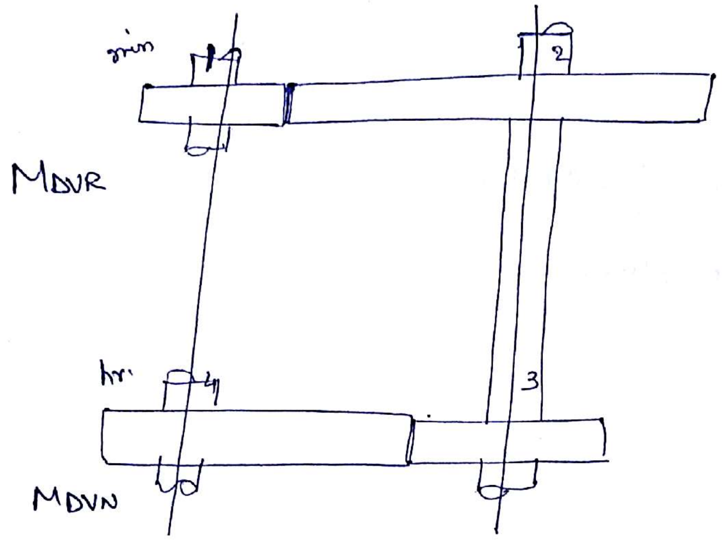
$= (r_1 + r_2)$

$= \frac{mT_1}{2} + \frac{mT_2}{2}$

$= \frac{\cancel{2}}{\cancel{2}}(24+108)$

Centre Distance $= 132$ mm

Qn) A reverted gear train is used in a clock to drive the hour hand with help of minute hand. Find the suitable NO of the teeth for the gear. any gear should not have less than 11 teeth.



$T_1, T_2, T_3, T_4 < 11$

$m_1 = m_2 = m_3 = m_4$

Assume all module same

$$\cancel{m}(T_1 + T_2) = \cancel{m}(T_3 + T_4)$$

New Trics

$$T_1 + T_2 = T_3 + T_4$$

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{T_2 \times T_4}{T_1 \times T_3}}$$

$$\omega_1 = \frac{2\pi}{60 \times 60} = \text{Rad/sec}$$

$$\omega_4 = \frac{2\pi}{12 \times 60 \times 60} = \text{rad/sec}$$

$$\frac{\omega_1}{\omega_4} = 12 = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$\boxed{\frac{\omega_1}{\omega_4} = 12}$$

all m are same

$$T_1 + T_2 = T_3 + T_4$$

$\boxed{\text{Assume } T_1 = 12}$

Assume, $\frac{T_2}{T_1} = 4$ $\qquad \frac{T_4}{T_3} = 3$

$T_2 = 48$

$$(12 + 48) = T_3 + T_4 = 3T_3 + T_3 = 4T_3$$

$$T_3 = \frac{60}{4} = 15, \quad T_4 = 3T_3 = 45$$

$T_1 = 12$
$T_2 = 48$
$T_3 = 15$
$T_4 = 45$ Ans

51

71.

Assume

$\frac{T_2}{T_1} = 6$ $T_2 = 6T_1 = 72$

$12 + 72 = T_3 + T_4$

$84 = 3T_3$

$T_3 = 28$

$T_4 = 56$

$T_1 = 12$
$T_2 = 72$
$T_3 = 28$
$T_4 = 56$

$\frac{T_2}{T_1} = 2$ $\frac{T_4}{T_3}\ 6$

$T_2 = 2T_1$ $T_4 = 3T_3$

$\cancel{12 + 2T_1}$.

Epi-Cyclic Gear Cycle :-

$\boxed{DOF = 2}$

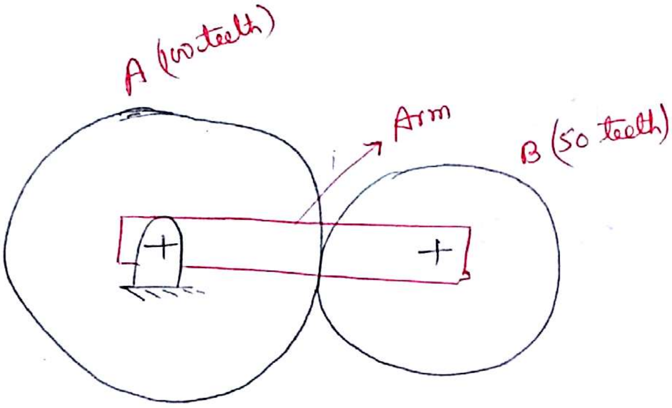
Arm speed is same for all gears

Epi → Axis, Cyclic → Rotation

" A Part from the rotation of the gears, if any gear axis is also rotating w.r.to some other axis, the train is known as Epi-Cyclic gear Train."

It may be simple epicyclic, Compound epicyclic, Reverted epicyclic, Bevel epicyclic & so on.

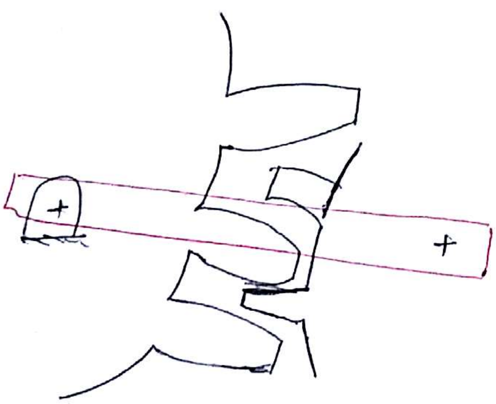
To rotate the Axis of the gear, a link is used which is known as Arm or Carrier. Arm is not a gear. it is simply a link & most imp Part of epi-cyclic gear Train.

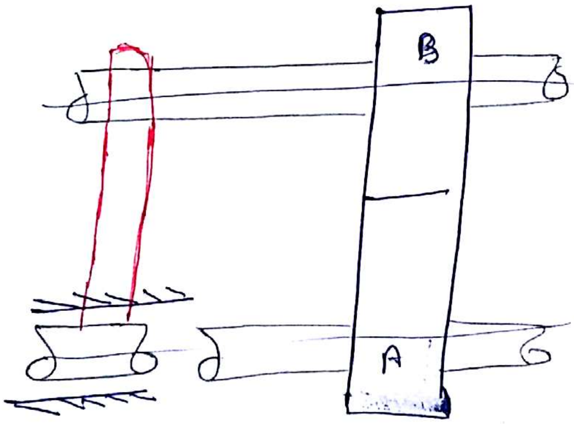


Law of gearing

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}$$

$$\omega_2 = \frac{T_1 \omega_1}{T_2}$$





Let us take Gear A → 100 RPM (AC) → 300 rpm (clock)

Gear B
- By Gear A – 200 r.p.m (clock) → 300 rpm (clock)
- Arm – 100 RPM clock → Zero
- Arm – 200 RPM (AC) → Zero
- Arm – 300 RPM AC → 100 rpm (AC)

Pb?

$T_A = 20$

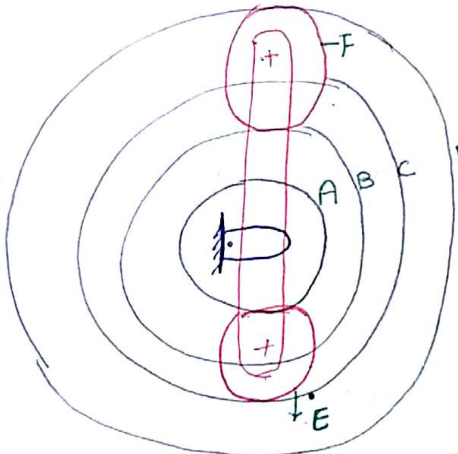
$T_B = 30$

$T_E = T_F = 10$

And All gear have same Module.

$T_A + 2T_E = T_C$ → 40

$T_B + 2T_F = T_D =$ → 50



Motions	Arm	A/B (20/30)	E_{10}	C_{40}	F_{10}	D_{50}
1) Arm fixed Let Gear A rotates by $+x$ rpm (Clock)	0	$+x$	$-x\frac{20}{10}$	$-x\cdot\frac{20\cdot10}{10\cdot40}$	$-x\frac{30}{10}$	$-x\frac{30}{10}\frac{10}{50}$
2 Arm free	y	$(y+x)$	$y-2x$	$y-\frac{x}{2}$	$y-3x$	$y-\frac{3x}{5}$

Soln ②

$y - \frac{3x}{5} = 0$ — ①

$y = -100$ — ②

$x = ?$

$y = ?$

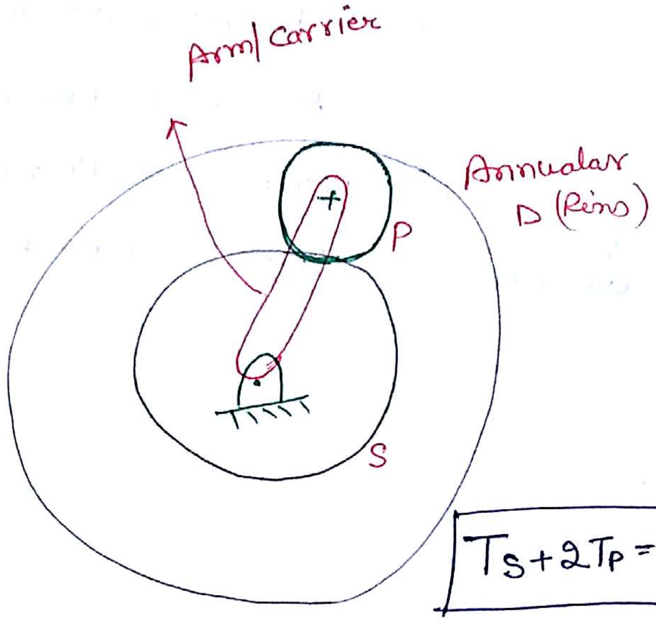
Planetary Gear Trains :

I Input

Sun	Ring
fixed	Input
Input	fixed

II Input

ARM



$$\boxed{T_S + 2T_P = T_D}$$

* Generally in Planetary gear train, no. of Planet gear are more than one

⇓

why

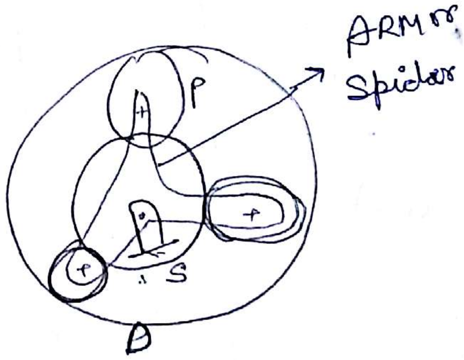
⇓

i) For the Balancing of System

ii) For the load distribution among the Planets in high Power Transmission

Qn) 45 Pg 24

Arm (A)	S (T_S)	P (T_P)	D (72)
0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_P}\cdot\frac{T_P}{72}$
y	$(y+x)$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{72}$



$\frac{252}{3.5} = T_D$, $T_D = 72$, $T_S + 2T_P = 72$ $\quad N_D = 0 \quad N_S = 5N_A$

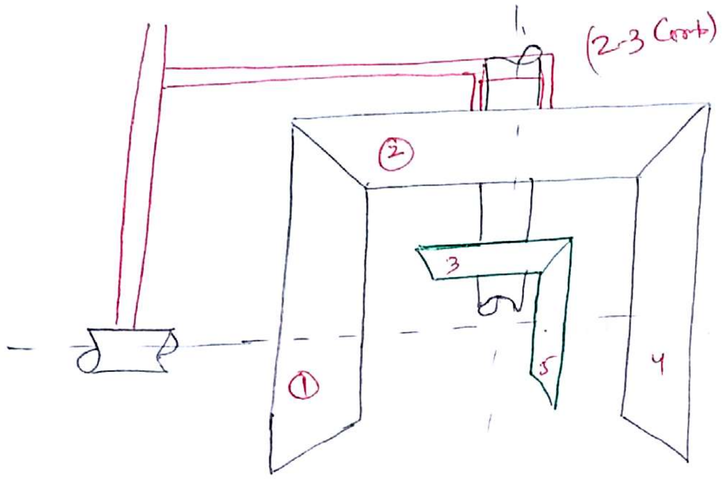
$$y + x = 5y$$

$$x = 4y$$

$$y - x\frac{T_S}{72} = 0 \qquad y\left(1 - \frac{T_S}{18}\right) = 0$$

$$T_S = 18$$

Direction Consideration in Bevel Epi-cyclic Gear Trains :-



Arm	1	2/3	4	5
0	$+x$	$\pm x\frac{T_1}{T_2}$	$\ominus x \frac{T_1}{\cancel{T_2}} \cdot \frac{\cancel{T_2}}{T_4}$	$\ominus x \frac{T_1}{T_2} \cdot \frac{T_3}{T_5}$
y	$y+x$	$\left(y \pm x\frac{T_1}{T_2}\right)$	$\left(y - x\frac{T_1}{T_4}\right)$	$\left(y - x\frac{T_1}{T_2}\frac{T_3}{T_5}\right)$

Fixing/Holding Torque in an epi-cyclic Gear Train :-

Total Torque in an epicyclic gear Train ;

$$\Sigma T = T_{input} + T_{output} + T_{fixing} = 0 \quad -(1)$$

Power Conservation ;

$$T_{input} \cdot \omega_{input} + T_{output} \cdot \omega_{output} = 0 \quad -(2)$$

(1), (2) → T_{fixing}

$$\boxed{\eta_{gear\ Train} = 1}$$

On

$N_{input} = +100$

$N_{output} = +250$

$T_{input} = +50$

$T_{fixing} = ?$

$(50)(100) + T_{out}(250) = 0$

$T_{output} = -20$

$(50) + (-20) + T_{fixing} = 0$

$T_{fixing} = -30$

$= 30\ KN-m\ AC$

Balancing

⇓

Vibrations causing F unbalanced

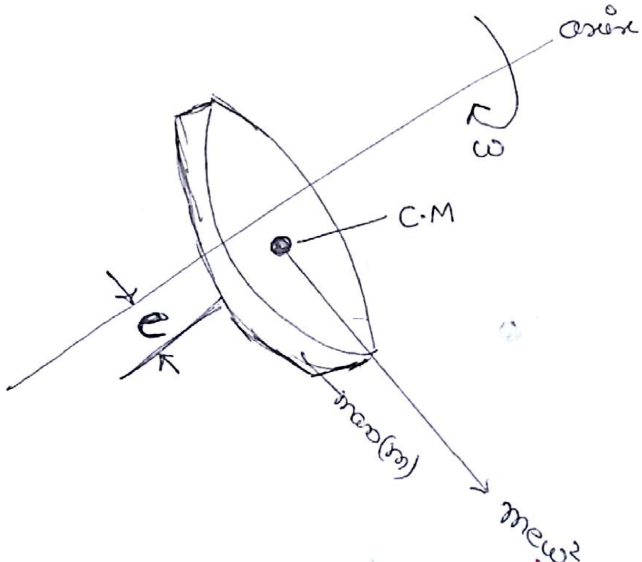
Rotating unbalance
(By rotating mass)

Resiprocating unbalance
(By Resiprocating mass)

Balancing

$$\sum \vec{F} = 0$$

$$\sum \vec{M} = 0$$



Always Constant in Magnitude but changing in dir^n.

Rotating unbalance

Static Balancing

⇓

Here We Balance only forces, Moments are automatically Balanced

⇓

Applied for

⇓

When all the masses are Rotating in Same plane.

Dynamic Balancing

⇓

Here We Balance both forces as Well as moment

⇓

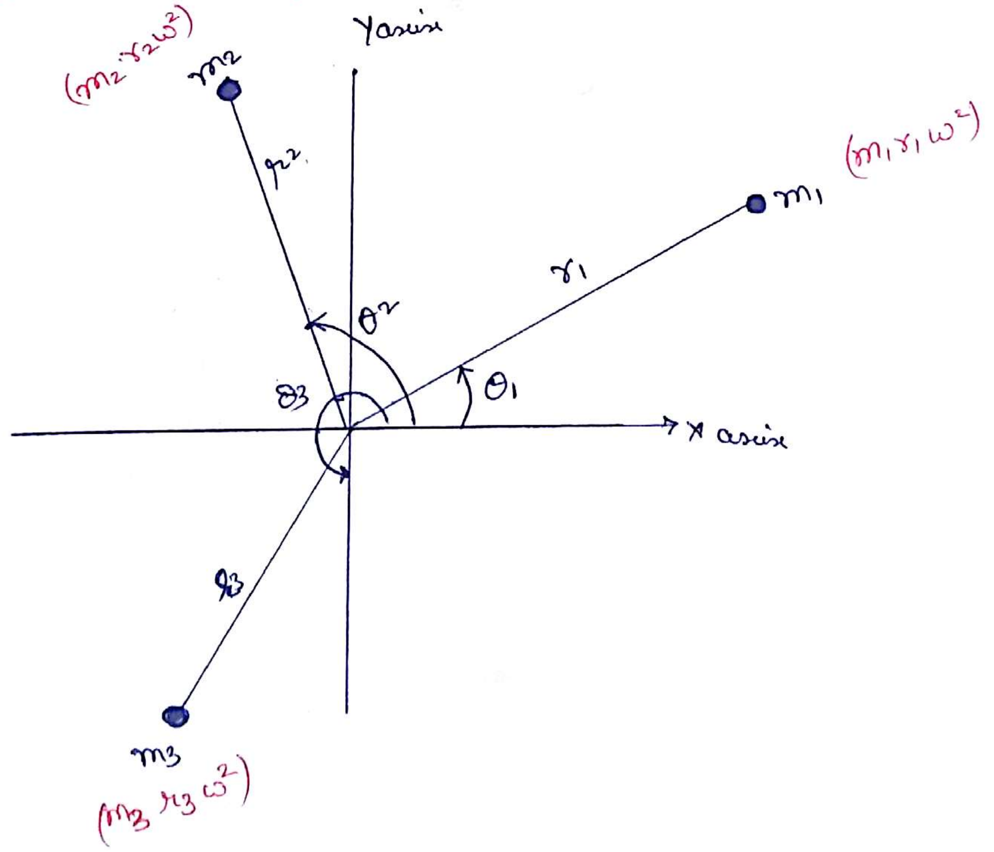
Applied for

⇓

When all the masses are not Rotating in Same Plane

Static Balancing Of Rotating Masses :

All the masses are rotating in a same plane.



After Balancing

$$\sum \vec{F} = 0$$

$$F_x = 0 \quad ; \quad F_y = 0$$

$F_x = 0$

$$(m_1 r_1 \omega^2)\cos\theta_1 + (m_2 r_2 \omega^2)\cos\theta_2 + (m_3 r_3 \omega^2)\cos\theta_3 + (m_b r_b \omega^2)\cos\theta_b = 0$$

as Balancing doesn't depend on speed.

$$(m_B r_b)\cos\theta_B = -\sum m r \cos\theta \quad -(1)$$

$$(m_B r_B)\sin\theta_B = -\sum m r \sin\theta \quad -(2)$$

Squaring (1) and (2) and then adding them, ***

$$\boxed{m_B r_B = \sqrt{\left(-\sum m r \cos\theta\right)^2 + \left(-\sum m r \sin\theta\right)^2}}$$

for angle fo there mass to be attached for Balancing

**

$$\boxed{\tan\theta_B = \frac{-\sum m r \sin\theta}{-\sum m r \cos\theta}}$$

NoTe

The final Sign of Numerator & denominator will decide the Quadrant of Balance mass.

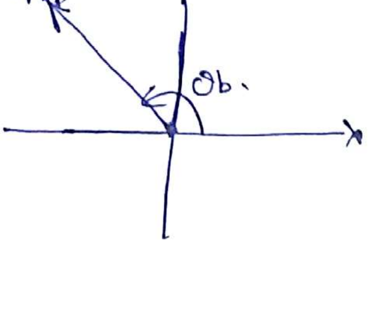
for example

$$\sum m r \cos\theta = +4$$

$$\sum m r \sin\theta = -5$$

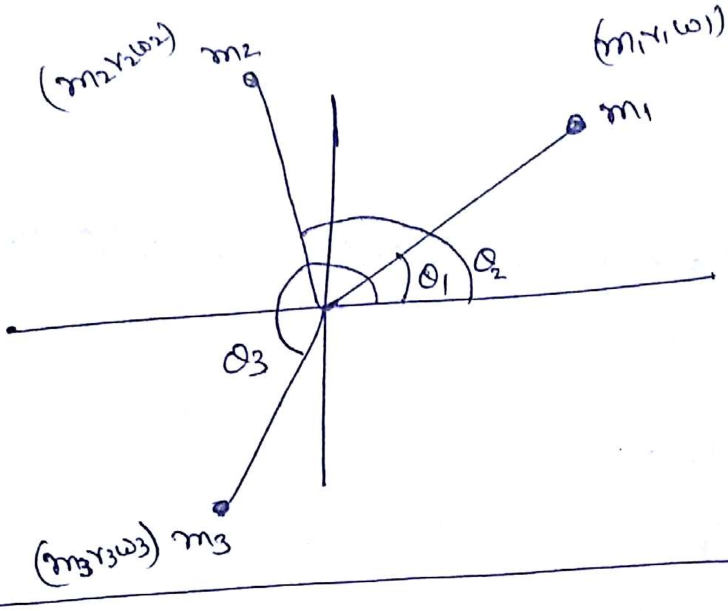
$\cancel{\tan\theta_B = (+5)}$ $= +$

$$\tan\theta_B = \frac{-(-5)}{-(+4)} = \frac{5}{-4} = -\frac{5}{4}$$



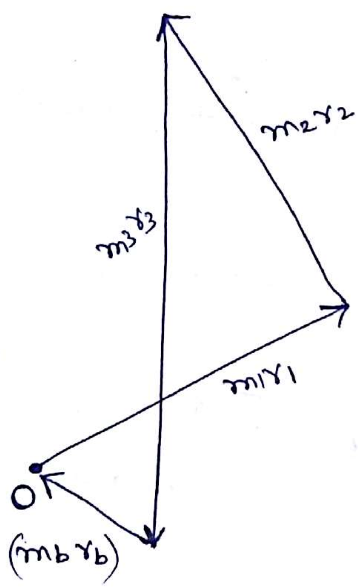
Graphical Method

$\boxed{F/\omega^2}$



$\left.\begin{array}{l} m_1 r_1 \\ m_2 r_2 \\ m_3 r_3 \end{array}\right]$ Known

$m_B r_B \rightarrow$ unknown



Dynamic Balancing :

(All the masses are not rotating in same Plane)

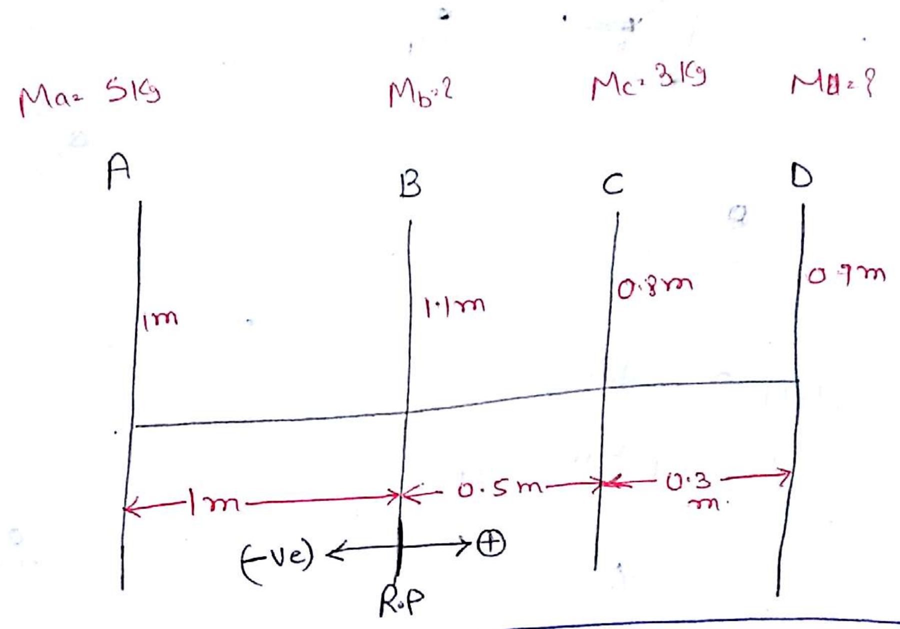
WorkBook Question 30 Pg. 40

$M_B = ?$

$M_D = ?$

$\theta_B = ?$

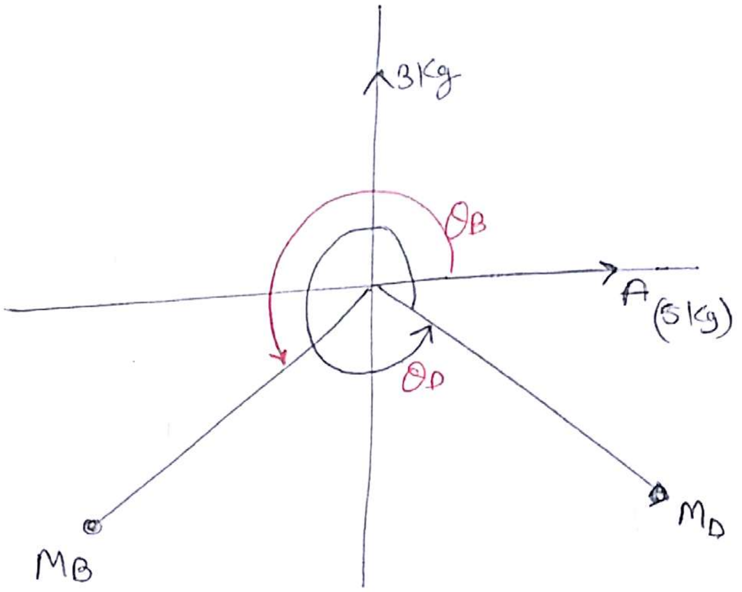
$\theta_D = ?$



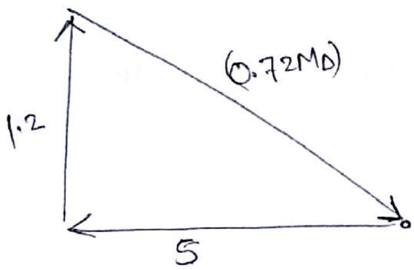
imp* $\boxed{\begin{array}{l} \text{Force} = mr\omega^2 \\ \frac{F}{\omega^2} = mR \end{array}}$

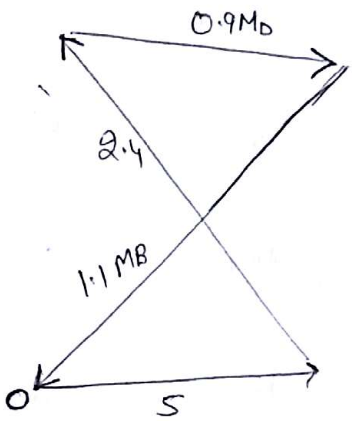
Plane	m	r	m.r (Force/ω^2)	Distance from (R.P.) (l)	Moment m.R l
A	5	1	5	−1	−5
B	M_B	1.1	$1.1 M_b$	0	0
C	3	0.8	2.4	0.5	1.2
D	M_D	0.9	$0.9 M_D$	0.8	$0.72 M_B$

Configuration Dia



Moment Polygon





For Moment Polygon Value :

① Reference Plane Select करना है dirn Shown Hai.
② Main target if one value become zero.

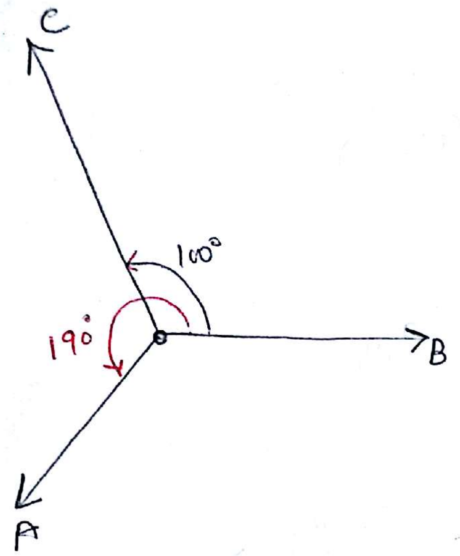
(m. r). l
Distance from Reference Plane.

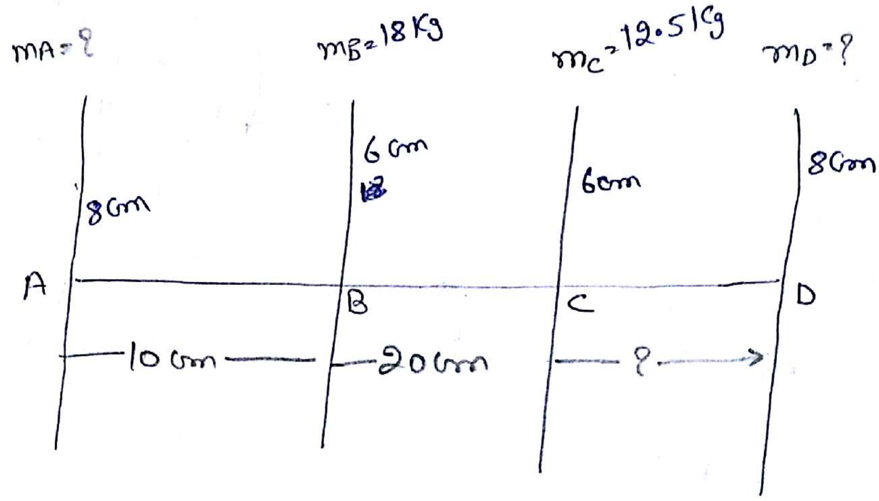
जो Value -ve आएगी उसे out बनाना है like

and +ve as usual

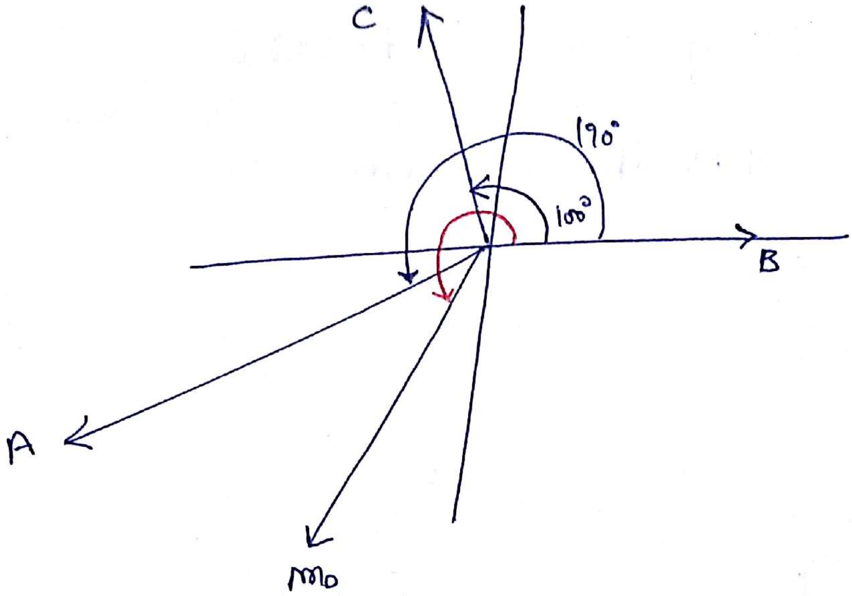
$\boxed{\frac{Force}{\omega}} = m.R$ Always out. never be in.

Qn) T5) [

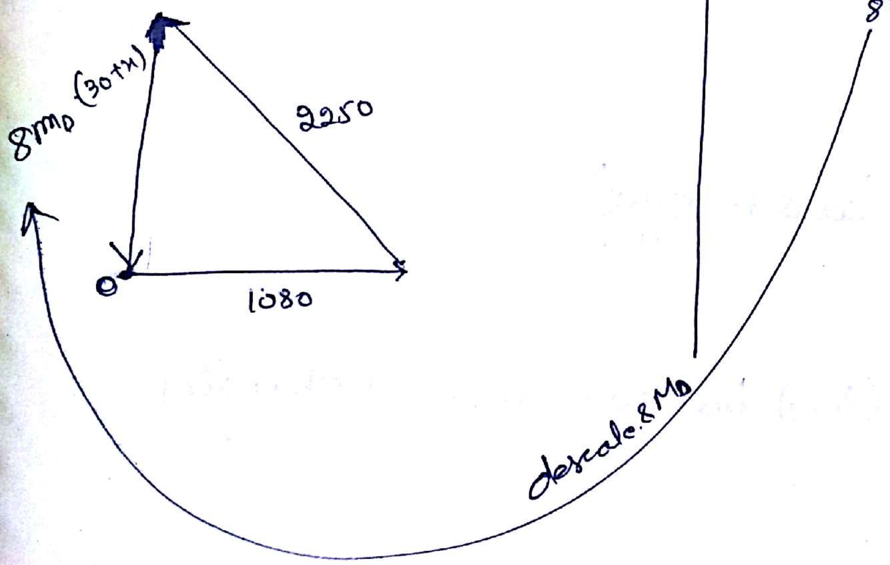




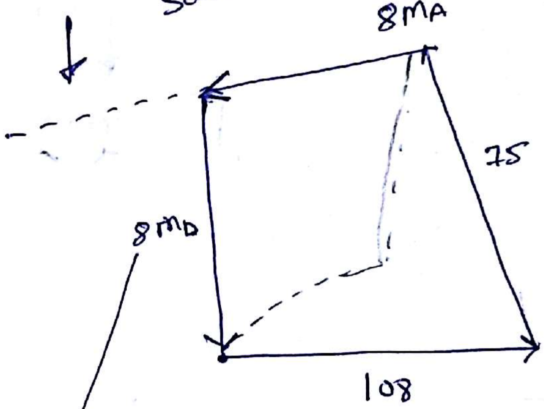
Plane	m	r	m.r	Distance from (R.P) (l)	m.rL
A (R.P)	m_A	8	$8m_A$	0	0
B	18	6	108	10	1080
C	12.5	6	75	30	2250
D	m_D	8	$8m_D$	30+x	$8m_D(30+x)$



Moment Polygon

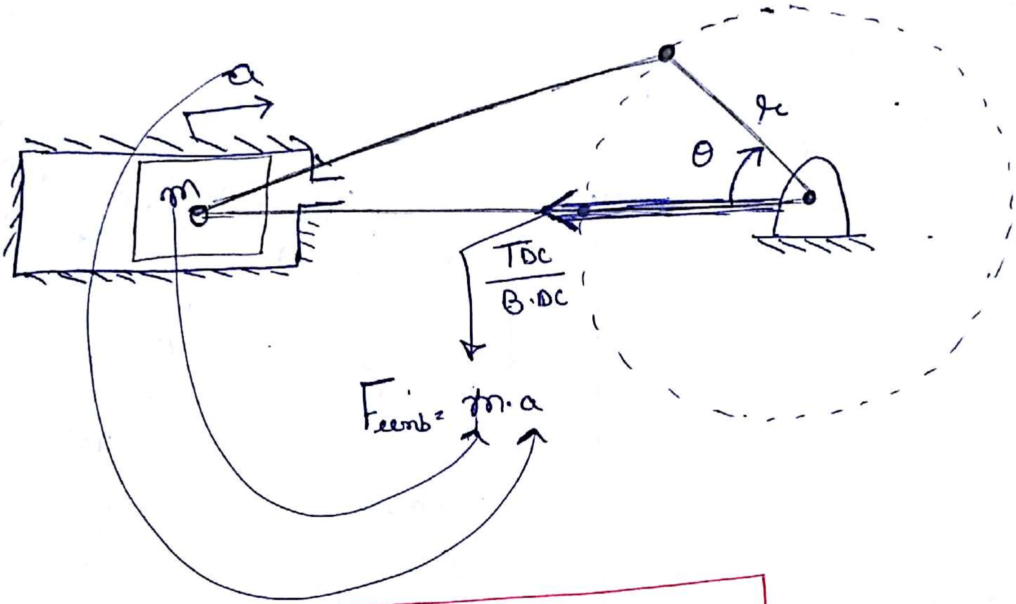


यहाँ A का dirn Pta a length नहीं है
Same as D so जिस line पर काटेगा
वो होगा हमारा Plane.



Balancing Of Reciprocating Masses :-

$m \rightarrow$ mass of Reciprocating ~~mass~~ Parts



$F_{unb} = m \cdot a$

$$\boxed{F_{unbalanced} = mr\omega^2 \left\{ \cos\theta + \frac{\cos 2\theta}{n} \right\}}$$

$\Downarrow$

Always Constant in dir^n (line) but every moment changing in magnitude.

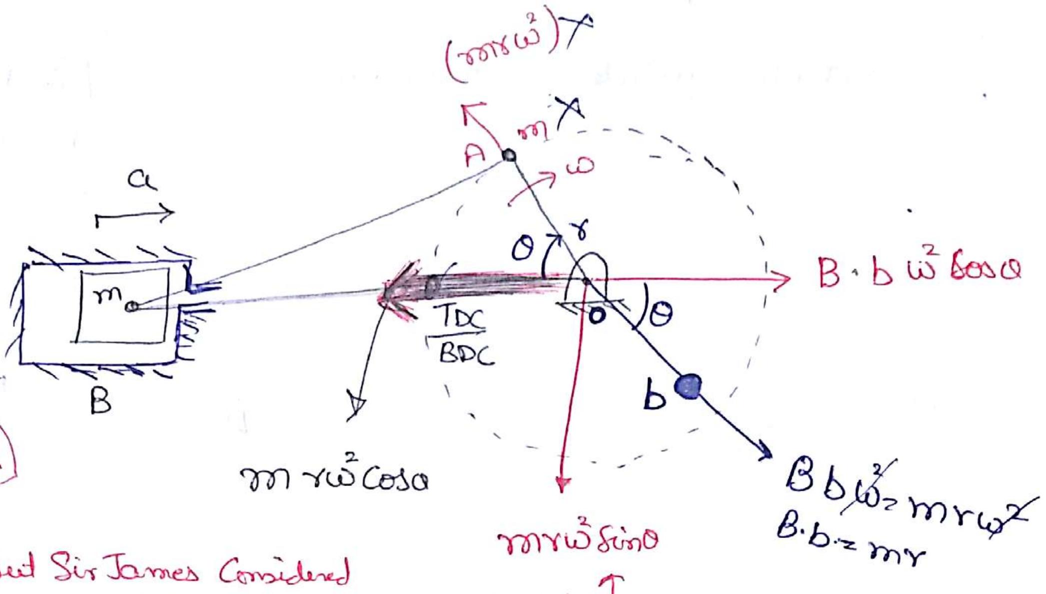
$$F_{unb} = \underbrace{(mr\omega^2\cos\theta)}_{F_{unb}\text{ (Primary)}} + \underbrace{\left(mr\omega^2\frac{\cos 2\theta}{n}\right)}_{F_{unb}\text{ (Secondary)}}$$

Primary Balancing of Reciprocating Masses :

Sir James Watt & Team

Artificial thinking

①



Vibration due to line of Reciprocation

But Sir James Considered it as Rotational, he sucued by New developed

They failed in the Balancing of Reciprocating masses.

We will not Balance the Reciprocating masses Completely, We will balance them Partially

Partial Balancing of Reciprocating masses (Primary) (By Mr. Allen Causley)

Let $C \rightarrow$ fraction of Reciprocating mass to be balanced. $0 < C < 1$

$$\boxed{B \cdot b = C m r} < m \cdot r$$

After Partial Balancing

$$\boxed{F_{unb}\,(\text{Along LOS}) = m r \omega^2 \cos\theta - B \cdot b\,\omega^2 \cos\theta = (1 - C)\, m r \omega^2 \cos\theta}$$

$$\boxed{F_{un}\,(\perp \text{ to LOS}) = B b \omega^2 \sin\theta = C m r \omega^2 \sin\theta}$$

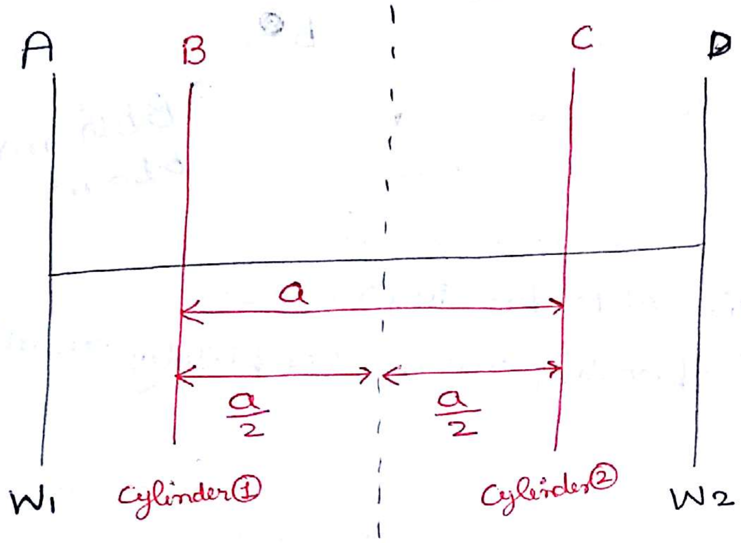
first use in Locomotive

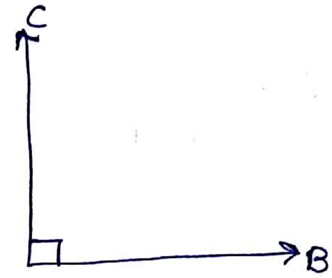
→ Vibration are there but intensity is less

Two Cylinder Locomotive :

- → Inside Cylinder Locomotive ✓
- → Outside Cylinder Locomotive X

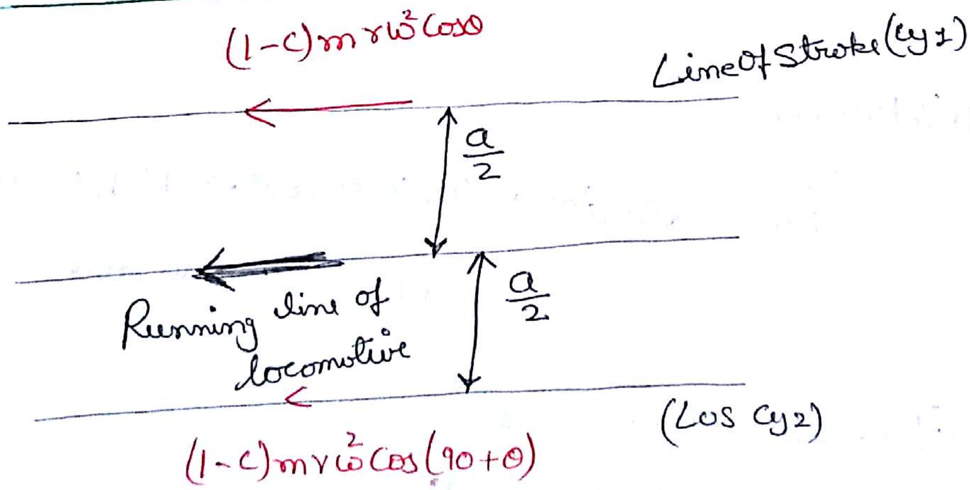
"First time to Start the engine both of the Cranks were Kept 90° to each other".





Effect of Partial Balancing on two cylinder locomotive :-

i) Variation in Tractive forces : → Pulley



Tractive force = $(1-c)\, mr\omega^2 \sin\theta + (1-c)\, mr\omega^2 \cos(90+\theta)$

$$= (1-c)\, mr\omega^2 (\cos\theta - \sin\theta)$$

for Maximum differentiate it,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$

$$\boxed{\tan\theta = -1} \qquad (\text{at } \theta = 135°, 315°)$$

ii) Variation in Swaying Couple : → लहरना

Swaying Couple = $(1-c)\, mr\omega^2 \cos\theta \frac{a}{2} - (1-c)\, mr\omega^2 \cos(90+\theta)\frac{a}{2}$

$$= \frac{a}{2}(1-c)\, mr\omega^2 (\cos\theta + \sin\theta)$$

For Max; differentiate it,

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$$

$$\boxed{\tan\theta = 1} \qquad \theta = 45°, 225°$$

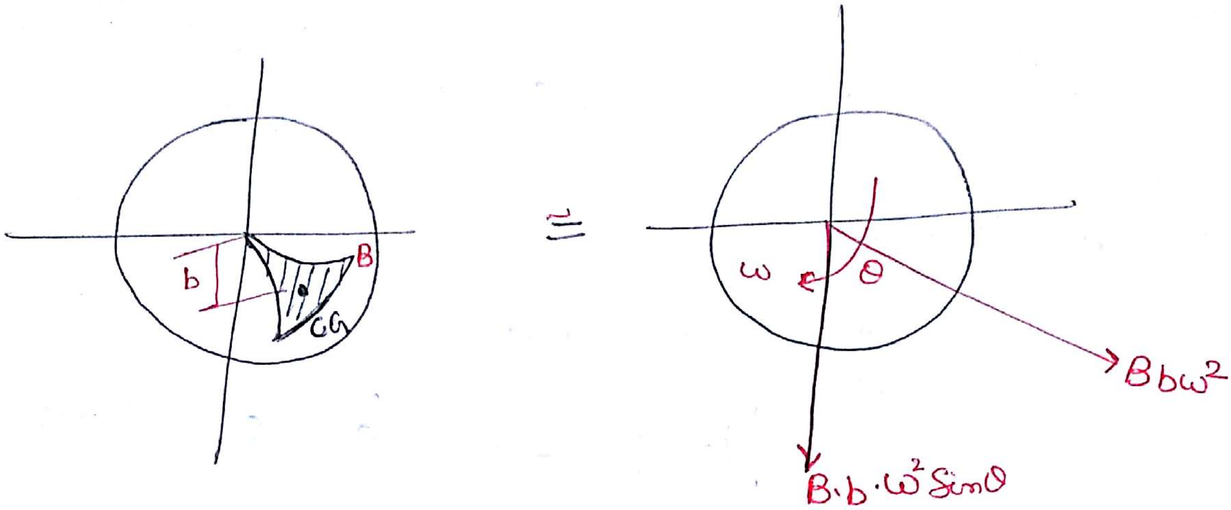
$$\boxed{\text{Variation in Swaying Couple} = \pm \frac{a}{\sqrt{2}}(1-c)\, mr\omega^2}$$

iii) Hammer Blow :

In Reality the balance masses cannot be attached reverse to the crank.

In reality Balance masses are attached on the wheels.

$$\boxed{B \cdot b \neq Cmr}\ *$$



$(Bb\omega^2 \sin\theta)_{max} = Bb\omega^2$

$\Downarrow$

Hammer blow

$$\boxed{(Bb\omega^2) \leq \text{Per Wheel static load}}$$

Speed limited

10% of total Power of engine is used

Hammer Blow = (B). b ω^2. (per wheel)

??

only that Balance Mass will be responsible for Hammer Blow which is required for Reci. balancing.

Prob

31

Cy. Planes gap = 0.7 m

$r = 0.3$ m

Cranks are at 90°

Rot. Mass/cy. = 160 kg

Reci. mass/cy. = 180 kg

Balance → Rot. + $\frac{2}{3}$ Reci.

$b = 0.80$ m

wheels planes gap = 1.5 m.

N = 300 r.p.m.

~~Blance~~ BAlance

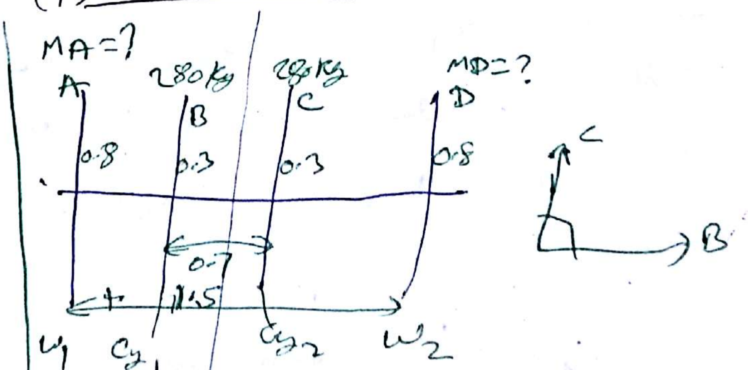
Rot. + $\frac{2}{3}$ Reci.

$= 160\text{ kg} + \frac{2}{3} \times 180\text{ kg}$

$= 160\text{ kg} + 120\text{ kg}$

$= 280\text{ kg/cy.}$

(i) BALANCING



m_A = ? Same

m_D = ?

Q_A = ? Different

Q_D = ?

for exam

$m_A = m_D$

= 80 kg

(II) Hammer Blow :- ?

B $\quad b \cdot \omega^2$

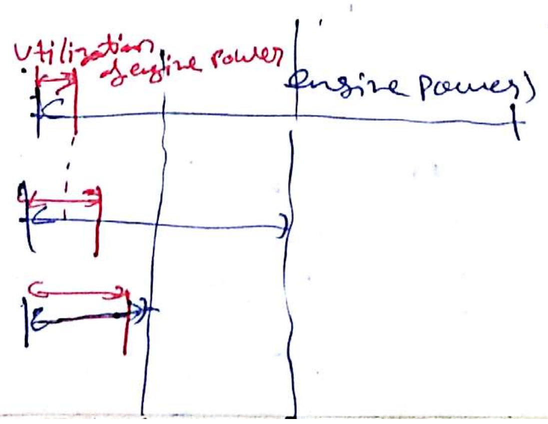
(0.8) $\quad \omega = \frac{2\pi \times 300}{60}$

280 kg ⟶ 80 kg

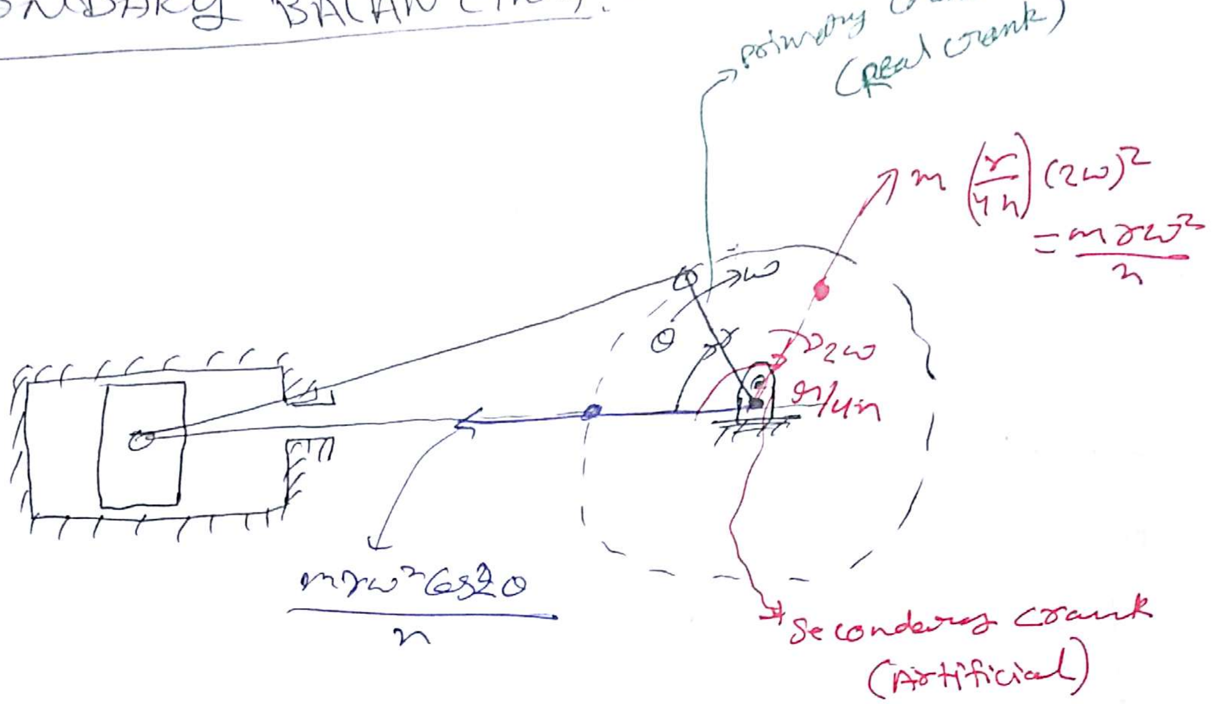
120 kg Reci. ⟶ $\frac{80}{280} \times 120$ kg ⟶ B

INTRODUCTION OF COUPLING ROD IN TWO CY. LOCOMOTIVES

Coupling Rod was Basically Introduced to decrease the amount of Hammer blow by ↓ing the BALANCE mass requirement for the reciprocating. Balancing through spliting the reciprocating Balancing b/w the driving & trailing wheels. this was the development from passenger locomotives to the Express locomotives (In which ⊕ 2-2 wheels were coupled) to the Super fast locomotives (In which ⊕ 3-3 wheels were coupled).

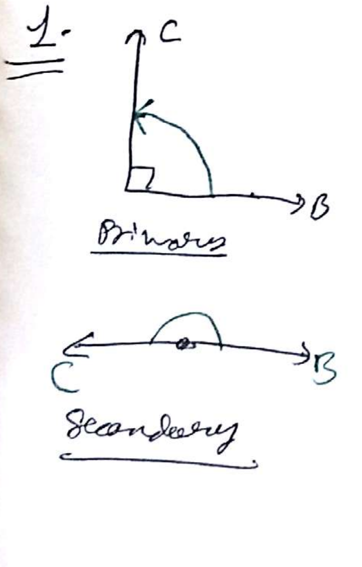


SECONDARY BALANCING :-

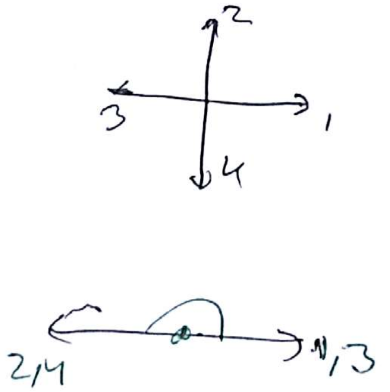


Primary	Secondary
$mr\omega^2 \cos\theta$	$\frac{mr\omega^2 \cos 2\theta}{n}$
	$= m\left(\frac{r}{4n}\right)(2\omega)^2 \cos 2\theta$

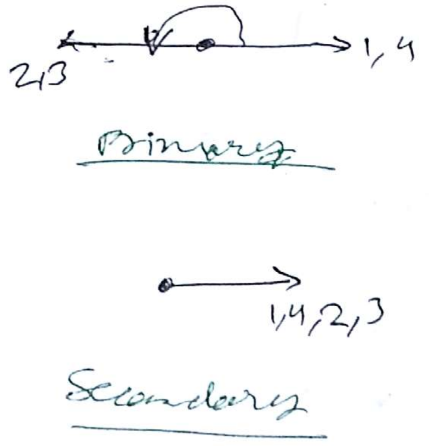
1.



2) 4-cy. Inline engine



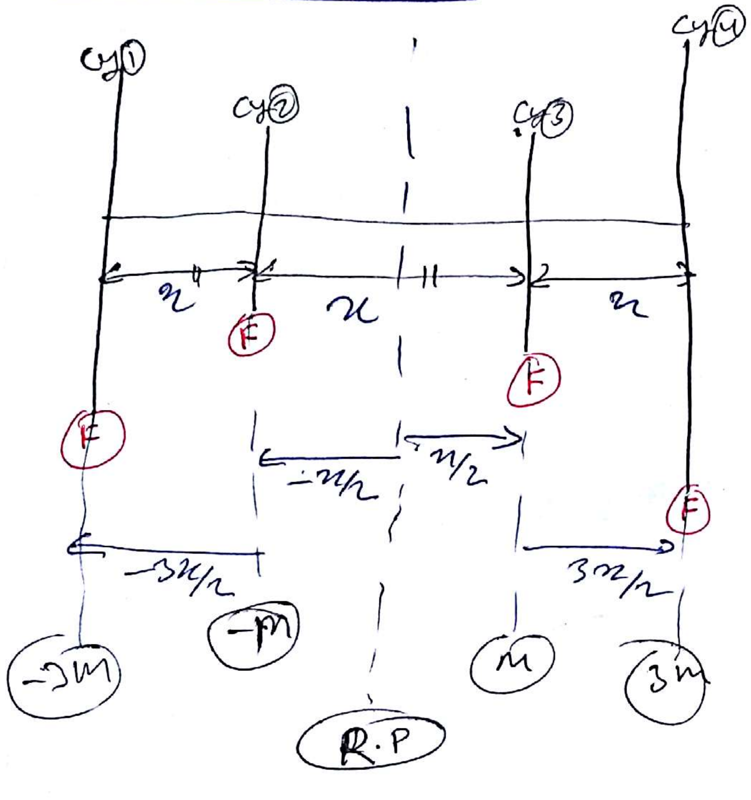
3) 4-cy. Inline engine

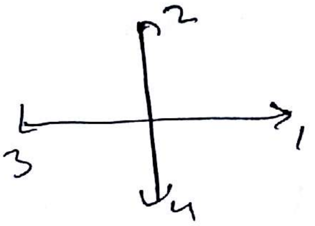


ROLL OF THE FIRING ORDER IN MULTICYLLINDER INLINE ENG. IN BALANCING

PFP → primary force polygon
PMP → " Moment "
SFP → Secondary Force "
SMP → " moment "

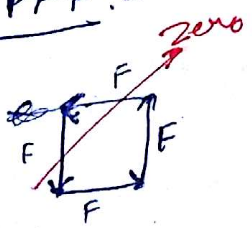
4-cy. Inline Engine :-



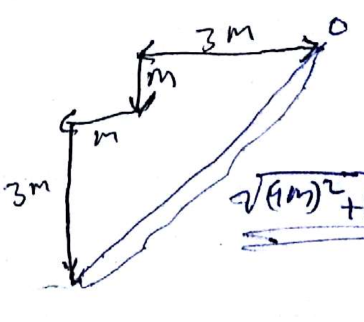


Primary

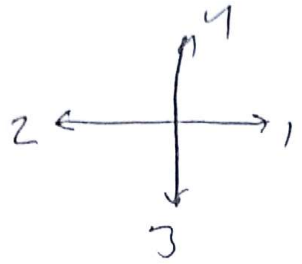
PFP :-



PMP :-

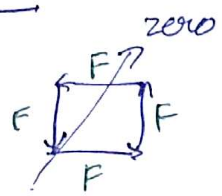


$\sqrt{(4m)^2 + (4m)^2}$

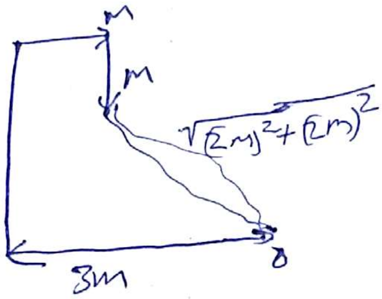


Primary

PFP



PMP :-



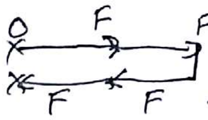
The real Firing order of 4-cy. Inline engine in practical is

Best. in 4-cy.leng.

Primary

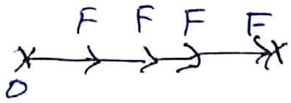
Secondary

PFP



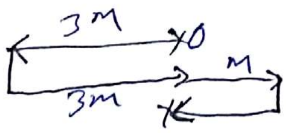
Zero

SFP :-



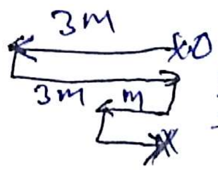
Non-zero

PMP :-



zero

SMP :-



zero

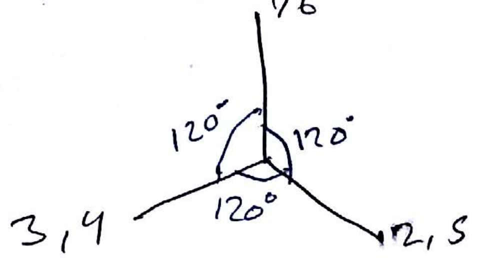
NOTE

4-Cy. Inline engine

↳ Not Completely Balanced.

6-Cy. Inline engine

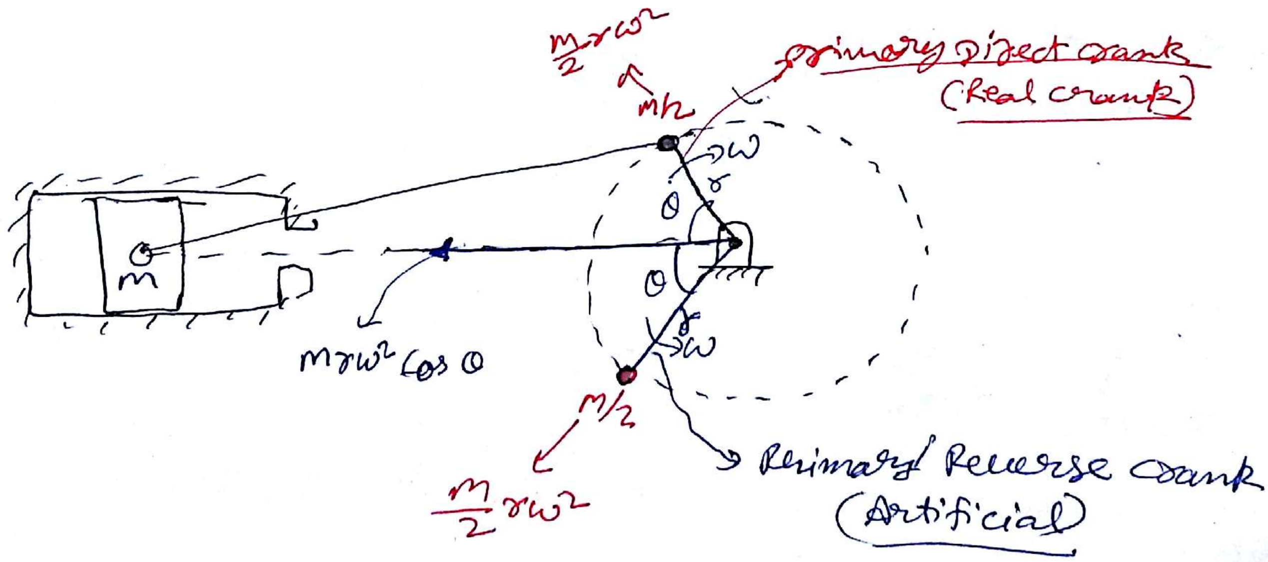
Completely Balanced



← This F. order used in Highly costly cars.
Highly (BMW, Farari)

DIRECT & REVERSE CRANK METHOD:-

"This method is basically applied ~~it~~ in multi cy. radial ~~engine~~ eng. to get the magnitude & dirⁿ of Primary & secondary unbalanced forces"!



(M) Reciprocating mass

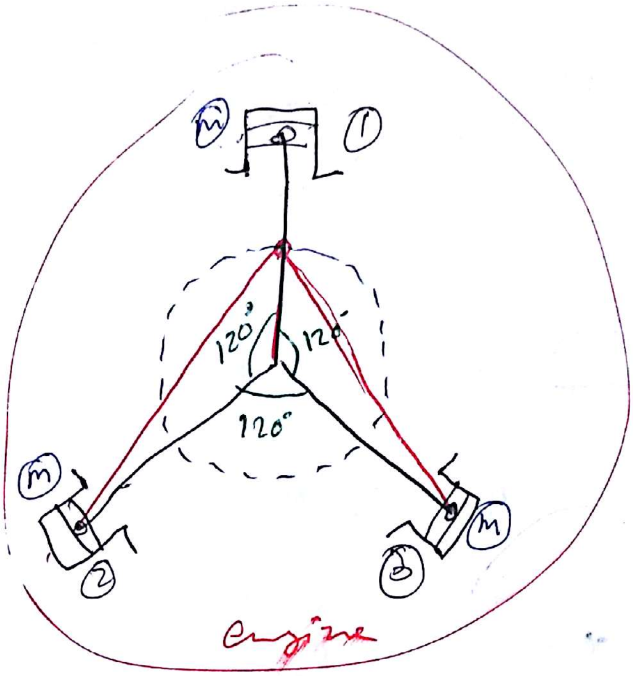
$\frac{m}{2}$ → Direct → Draw Forces → x, y Components

$\frac{m}{2}$ → Reverse → Draw Forces → x, y Components

Add → Total X ?, Total Y ?

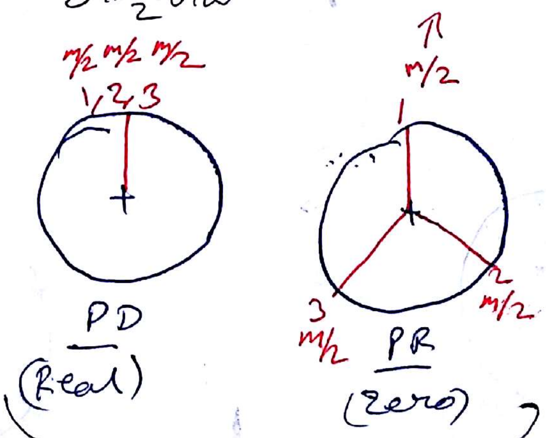
Three cy. Radial engine

in Radial there is only one crank.



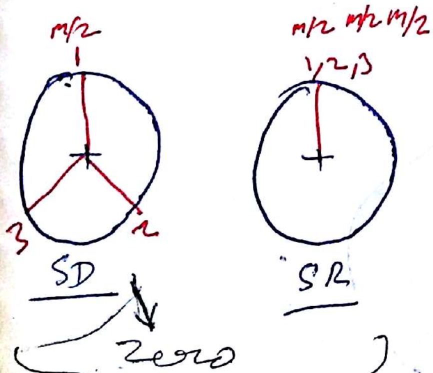
$\vec{F}_{primary} = ?$

$3 \times \frac{m}{2} r \omega^2$



$\uparrow \frac{3}{2} m r \omega^2$

$\vec{F}_{secondary} = ?$



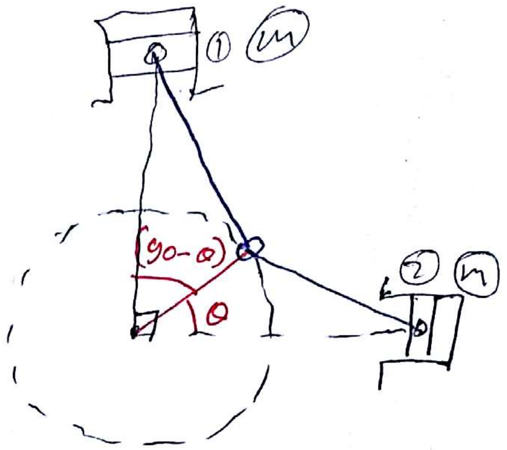
$\uparrow \frac{3}{2} \frac{m r \omega^2}{n}$

Prob

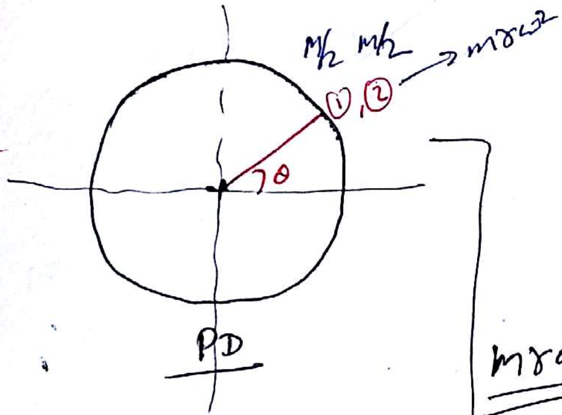
90° – V-engine

$\vec{F}_{primary}$ → max ? → min ?

$\vec{F}_{secondary}$ → max ? → min ?

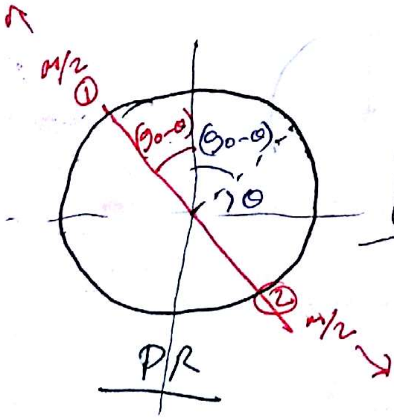


Primary



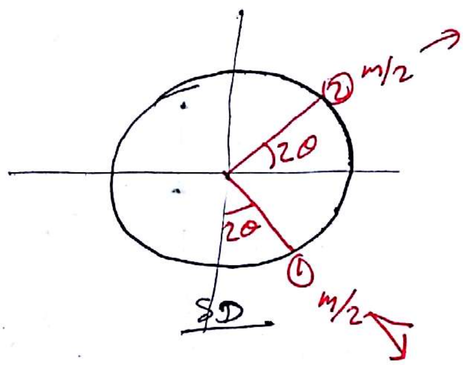
PD

$\underline{\underline{mr\omega^2}}$

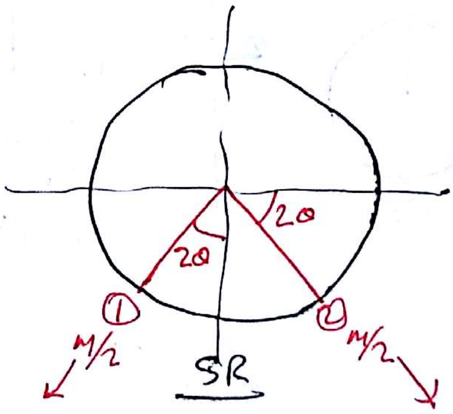


P/R

Secondary



SD



SR

$$F_x = \frac{m}{2}\frac{r\omega^2}{n}\left[\cos^2\theta + \cancel{\sin 2\theta} + \cos 2\theta - \cancel{\sin 2\theta}\right]$$

$$\rightarrow \underline{F_x = \frac{mr\omega^2}{n}\cos 2\theta}$$

$$F_y = \frac{m}{2}\frac{r\omega^2}{n}\left[\sin 2\theta - \cos 2\theta - \sin 2\theta - \cos 2\theta\right]$$

$$\rightarrow \underline{F_y = -\frac{mr\omega^2}{n}\cos 2\theta}$$

$$F_{secondary} = \sqrt{2}\,\frac{mr\omega^2}{n}\cos 2\theta \rightarrow max$$

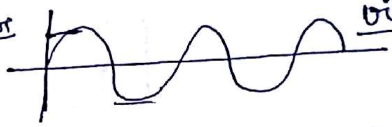
Vibrations

Vibrations
↓
To & Fro Periodic motions about their mean/equilibrium Position
→ Harmonic Motion

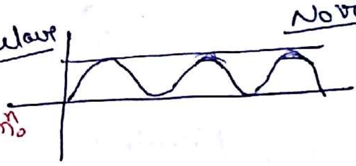
Equilibrium Position
Mean Position
Zero Position
] having Force(Net) & Net Moment Zero.

→ Vibrations are introduced b/c of some initial disturbances.

Sin Wave — vib^n



Sin^2 Wave — No vib^n



Any vibration System

i) K.E storing device (mass)

ii) P.E Storing device (stiffness) → Resistance to deform^n

iii) Energy loss due to Kinetic friction $\neq 0$

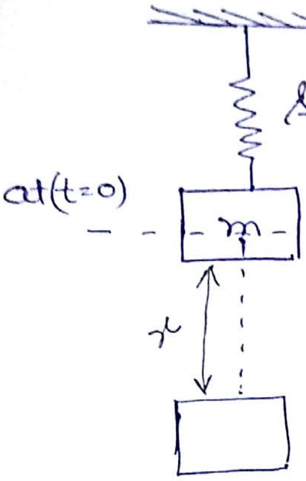
iv) Unbalanced forces $F_{un} \neq 0$

i, ii, iii are Basic cause of Non-Running vibrating System.

i, ii, iii including iv are Basic Cause of Running vibrating System.

Natural Vibrations :- By Sir Galileo →

"The vibrations in which there is no Kinetic friction at all, as well as there is no external force, after the initial Release of the System, are Known as Natural vibrations".

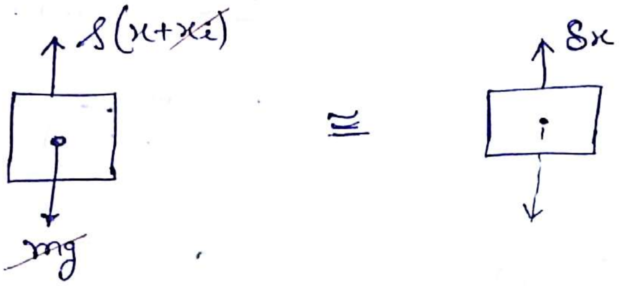


$s \rightarrow$ Stiffness of the System $\boxed{mg = s x_i}$ Equilibrium eqn

Mean eqn Position

$\rightarrow$ At $(t=t)$ $\quad x = f(t) = ?$

At $t = t$ System Free body diagram



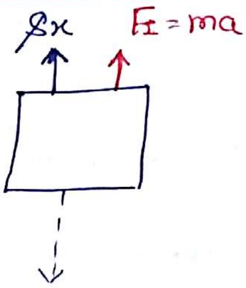
By using Newton's Second Law :

$$0 - sx = ma$$

$$\boxed{ma + sx = 0} \;**$$

D' Alembert Principle : Father of inertia force.

System (F.BD)
At $(t=t)$



$$\boxed{ma + sx = 0} \;**$$

$$a + \left(\frac{s}{m}\right)x = 0$$

$$\boxed{\ddot{x} + \frac{s}{m}x = 0} \;**$$ eqn Of Natural System

$$\ddot{x} + \frac{s}{m} x = 0$$

Solution of above eqⁿ is

$$\boxed{x = R \sin\left(\sqrt{\frac{s}{m}}\, t + \phi\right)}$$

Where R & ϕ are Constant

vibration with frequency

$$\boxed{\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}}$$ Angular frequency

Amplitude

⇓

Constant.

$$\boxed{T_n = \frac{2\pi}{\omega_n} \text{ sec}}$$ time period

$$\boxed{f_n = \frac{\omega_n}{2\pi} \text{ Hz}}$$ linear frequency

→ R, ϕ are found by Initial Condition i.e at t = 0

i) At $t = 0$ $\begin{cases} x = x_0 \\ \dot{x} = 0 \end{cases}$

ii) At $t = 0$ $\begin{cases} x = 0 \\ \dot{x} = V_0 \end{cases}$

iii) At $t = 0$ $\begin{cases} x = x_0 \\ \dot{x} = V_0 \end{cases}$

finally, equation of Natural vibration

$$\boxed{\ddot{x} + \omega_n^2 x = 0}$$

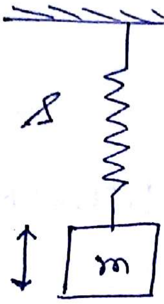
Pb) The Natural vibration System equ" is

Relate with this

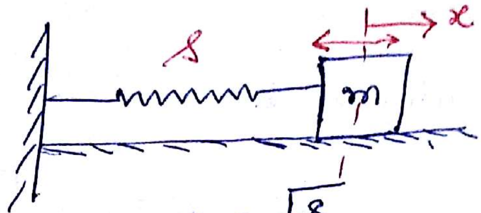
$$5\ddot{x} + 3x = 0$$

$$\ddot{x} + \frac{3}{5} x = 0$$

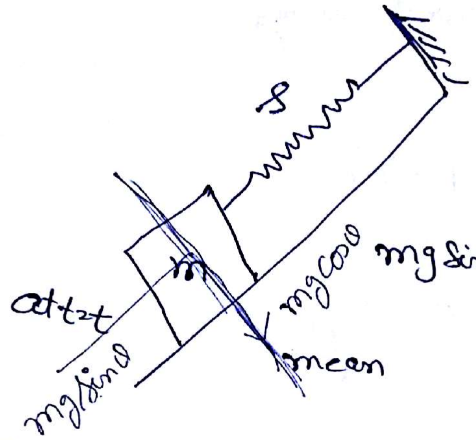
$$\boxed{\omega_n = \sqrt{\frac{3}{5}} \text{ rad/sec}}$$ Ans



$\omega_n = \sqrt{\frac{s}{m}}$



$\omega_n = \sqrt{\frac{s}{m}}$



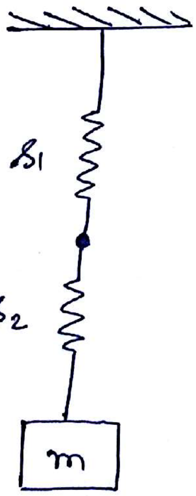
$mg\sin\theta = s x_i$

$s \propto \text{Area}$

$s \propto \frac{1}{\text{Length}}$

Combinations Of Springs

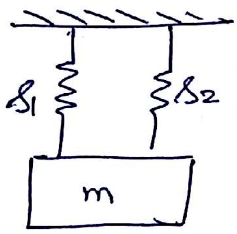
Series

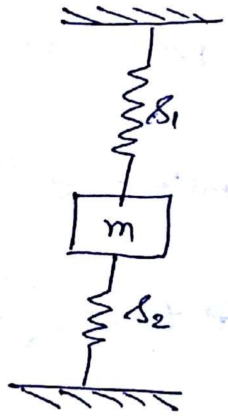


$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2} \Rightarrow s = ?$

$\omega_n = \sqrt{\frac{s}{m}}$

Parallel



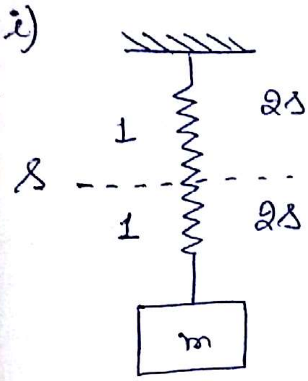


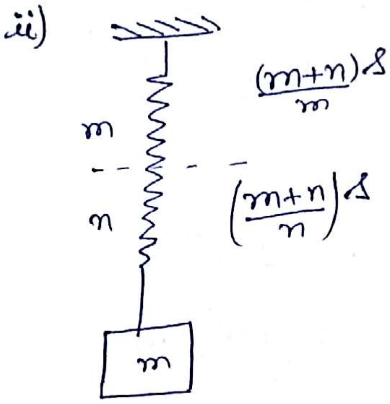
$\boxed{s = s_1 + s_2}$

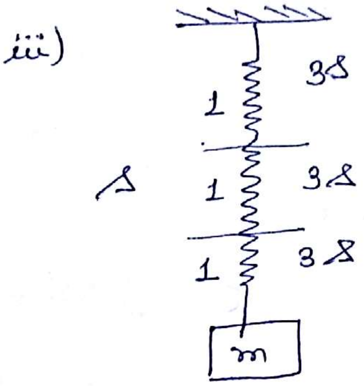
$\omega_n = \sqrt{\frac{s}{m}}$

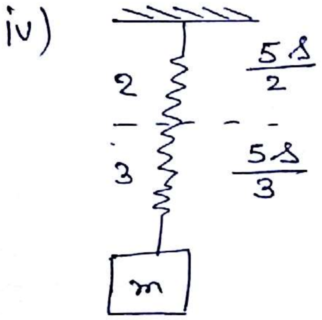
Both same

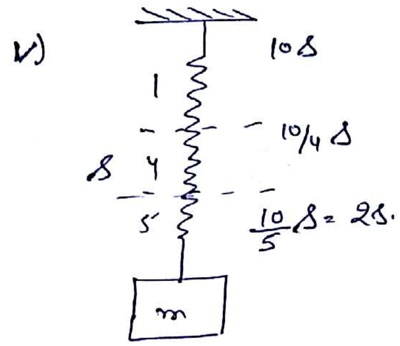
Cutting of Springs :







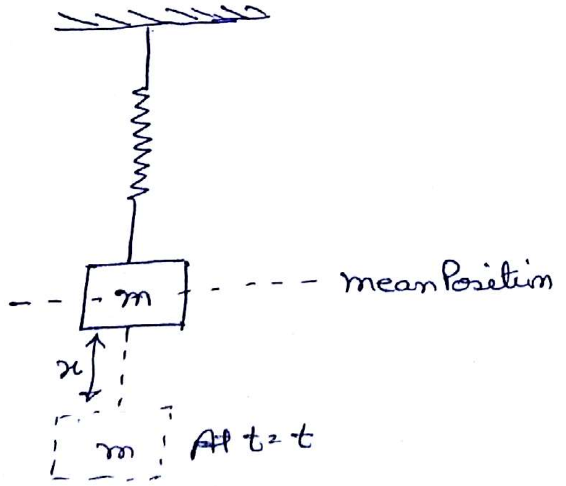




Energy Method :

In Natural vibration, Kinetic friction is 0. but Static friction is there which have no effect.

Total energy (E) = Const. $\boxed{\frac{dE}{dt} = 0}$ ***



$$\frac{dE}{dt} = \frac{1}{\cancel{2}} m \cdot \cancel{2v} \frac{dv}{dt} + \frac{1}{\cancel{2}} s \cancel{2} x \frac{dx}{dt} = 0$$

Acceleration

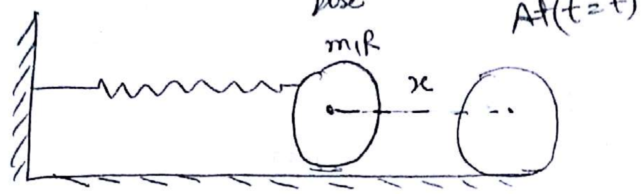
$$m\ddot{x} + sx = 0$$

$$\ddot{x} + \frac{s}{m}x = 0$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m}}}$$ ***

Energy = $\frac{1}{2}mv^2 + \frac{1}{2}sx^2$

Problem 2M



At t = t

$$E = \frac{1}{2} S x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{mR^2}{2}\right) \omega^2$$

$$= \frac{1}{2} (S) x^2 + \frac{1}{2} \left(\frac{3m}{2}\right) v^2$$

$$\omega_n = \sqrt{\frac{S}{\frac{3m}{2}}} \Rightarrow \omega_n = \sqrt{\frac{2S}{3m}}$$

Various Moment of inertia:

Ring / Hollow cylinder $\Rightarrow$ $I = mR^2$

Disc / Solid Cylinder $\Rightarrow$ $I = \frac{mR^2}{2}$

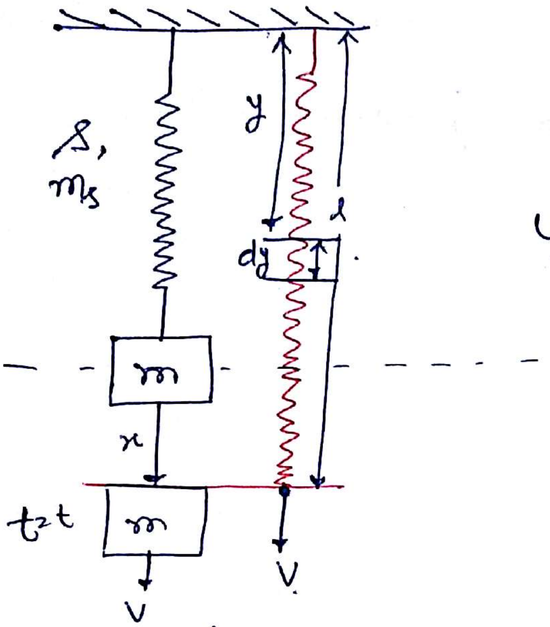
Hollow Sphere $\Rightarrow$ $I = \frac{2}{3} m R^2$

Solid Sphere $\Rightarrow$ $I = \frac{2}{5} mR^2$

Very imp

Spring Mass System:

(Spring is also having mass)



$$u = \frac{V}{l} y.$$

$$K.E = \int_0^l \frac{1}{2} \frac{m_s}{l} dy \cdot \left(\frac{V}{l} y\right)^2$$

$$= \frac{1}{2} \frac{m_s}{l} \cdot \frac{V^2}{\cancel{l^2}} \cdot \frac{\cancel{l^3}}{3}$$

$$\boxed{K.E = \frac{1}{6} m_s V^2}$$

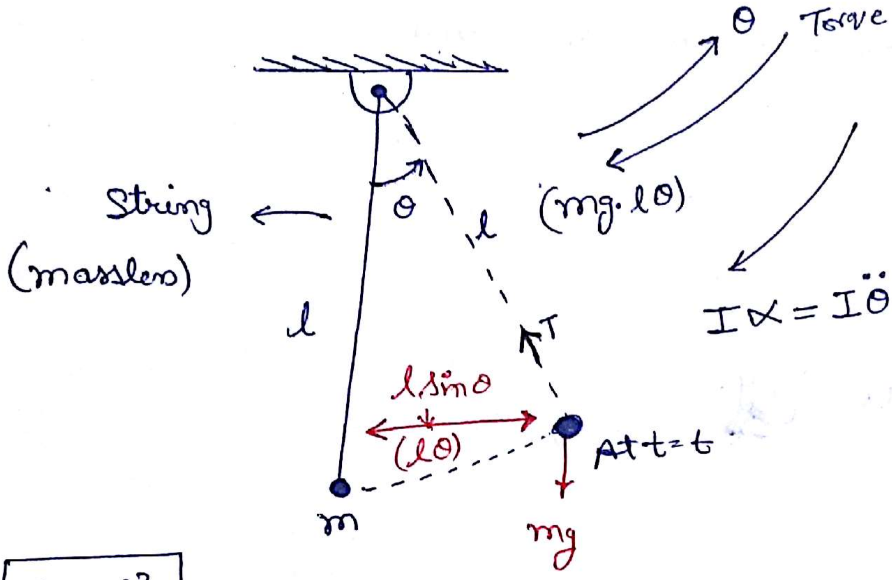
At t=t

$$E = \frac{1}{2} s x^2 + \frac{1}{2} m V^2 + \frac{1}{6} m_s V^2$$

$$= \frac{1}{2} s x^2 + \frac{1}{2}\left(m + \frac{m_s}{3}\right) V^2$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}}$$

Torque Method :- (for Small Oscillations)



$$I\alpha = I\ddot{\theta}$$

$$\boxed{I = ml^2}$$

$$I\ddot{\theta} + (mgl)\theta = 0$$

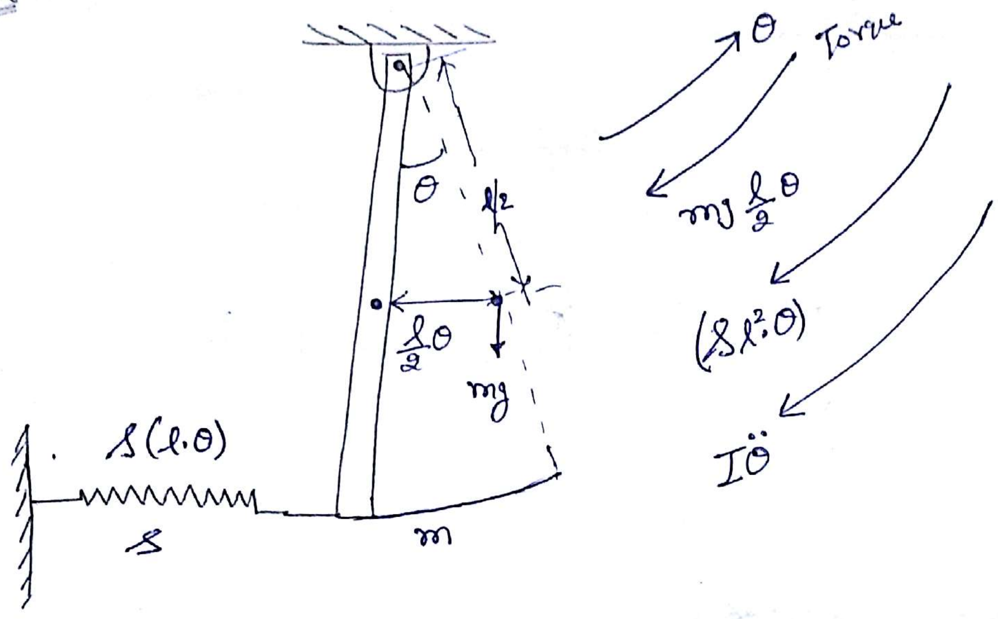
$$\ddot{\theta} + \left(\frac{mgl}{I}\right)\theta = 0$$

$$\ddot{\theta} + \frac{\cancel{m}g\cancel{l}}{\cancel{m}l^{\cancel{2}}}\theta = 0$$

$$\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0}$$

$$\boxed{\omega_n = \sqrt{\frac{g}{l}}} \text{ rad/sec}$$

Ps:



$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

$$= \frac{ml^2}{12} + \frac{ml^2}{4}$$

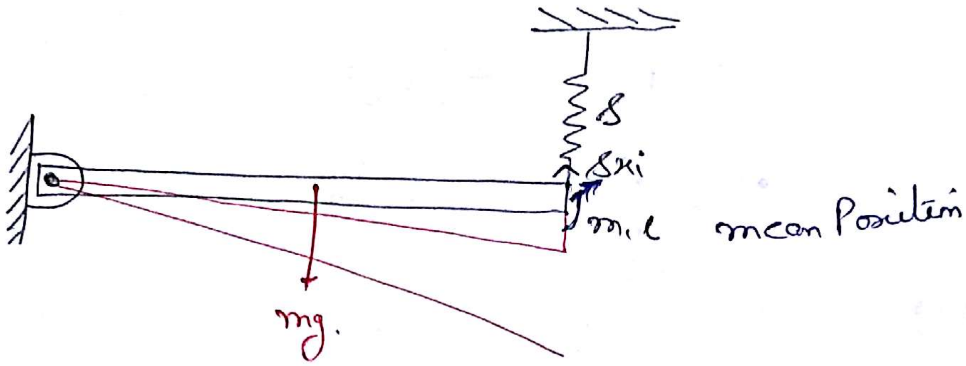
$$\boxed{I = \frac{ml^2}{3}}$$

$$I\ddot{\theta} + \frac{mgl}{2} + Sl^2 = 0$$

$$\ddot{\theta} = \frac{\frac{mgl}{2} + Sl^2}{ml^2/3}\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{\frac{mgl}{2} + Sl^2}{\frac{ml^2}{3}}}}$$ **

NOTE: Horizontal System



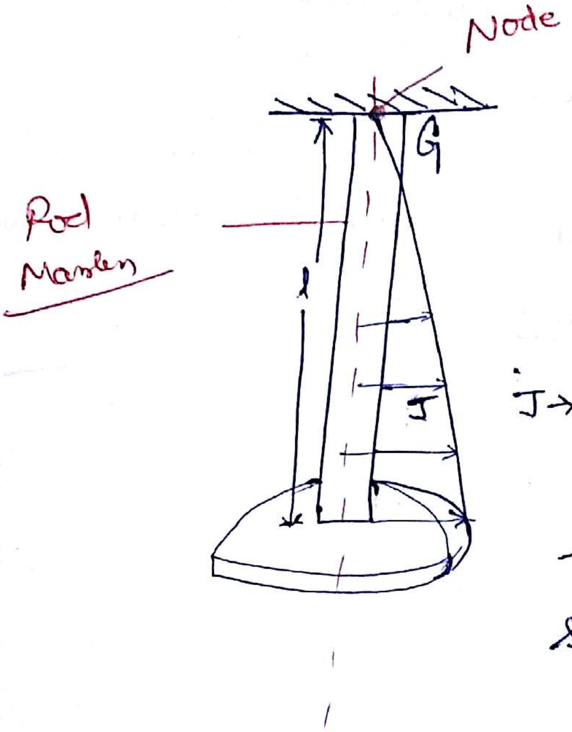
$$mg\frac{l}{2} = Sx_i \times l$$

[mg torque is cancelled by Sx_i torque
mg torque will not be considered

$$\omega_n = \sqrt{\frac{Sl^2}{\frac{ml^2}{3}}}$$

$$\boxed{\omega_n = \sqrt{\frac{3S}{m}}}$$

Torsional vibrations:



Torsional Stiffness

$$\boxed{S_T = \frac{GJ}{l}} \ *$$

J → Polar moment of inertia (By Area)

θ

$S_T \cdot \theta$

$I\ddot{\theta}$

$$I\ddot{\theta} + (S_T)\theta = 0$$

$$\ddot{\theta} + \frac{S_T}{I}\theta = 0$$

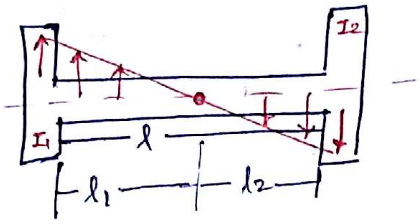
$$\boxed{\omega_n = \sqrt{\frac{S_T}{I}}} \ ***$$

NOTE

If Rod (shaft) is having mass. (I_{shaft})

$$\boxed{\omega_n = \sqrt{\frac{S_T}{I + \frac{I_{shaft}}{3}}}} \ **$$

Two Rotor System :



* If Number of Rotor $= n$
No of Node will always
Always $(n-1)$

Node – where Torsional vibration are zero

$$l_1 + l_2 = l \quad -(1)$$

$$\sqrt{\frac{\delta_{T_1}}{I_1}} = \sqrt{\frac{\delta_{T_2}}{I_2}}$$

$$\frac{\delta_{T_1}}{I_1} = \frac{\delta_{T_2}}{I_2}$$

$$\frac{\cancel{GJ}}{l_1 I_1} = \frac{\cancel{GJ}}{l_2 I_2} \Rightarrow \frac{l_1}{l_2} = \frac{I_2}{I_1} \quad -(2)$$

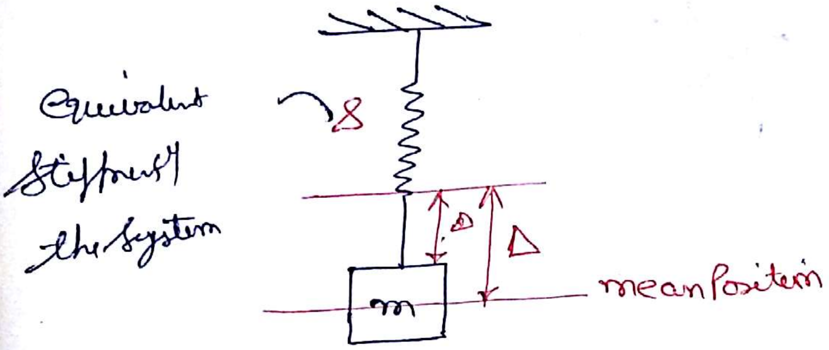
by (1) & (2) we can calculate l_1 & l_2

Rayleigh Method :

Method of Static Deflection of Mass Δ

$$\boxed{\Delta \rightarrow \text{Mass}}$$

Basic Spring-mass System



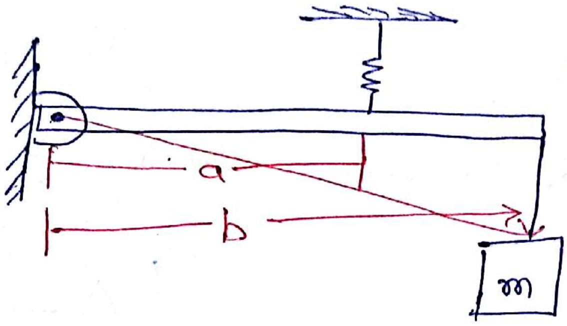
$$\Delta = \frac{mg}{S}$$

$$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{\frac{mg}{S}}} = \sqrt{\frac{S}{m}} = \omega_n$$

**

$$\boxed{\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{S}{m}}}$$

Pb 1)



$F = ?$ $\Delta_1 = \frac{mg(b/a)}{\delta_1}$

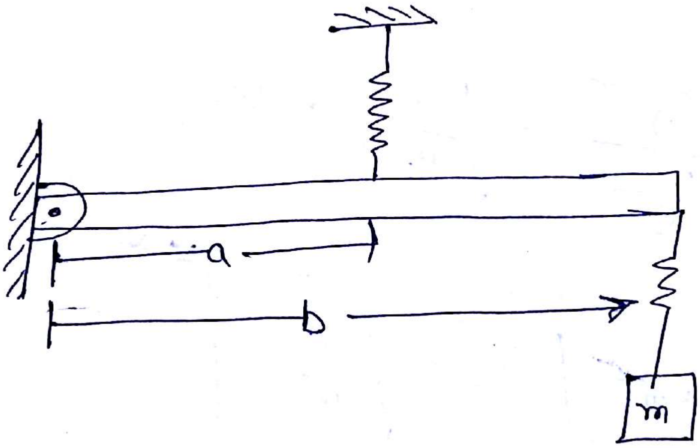
$mg\,b = F \cdot a$

$\boxed{F = \frac{mg \cdot b}{a}}$

$\frac{\Delta}{b} = \frac{\Delta_1}{a} \Rightarrow \Delta = \Delta_1 \frac{b}{a} = \frac{mg}{\delta_1} \cdot \left(\frac{b}{a}\right)^2$

$\boxed{\omega_n = \sqrt{\frac{g}{\Delta}} = ?} = \sqrt{\frac{\delta}{m}} \rightarrow ??$

Pb 2)

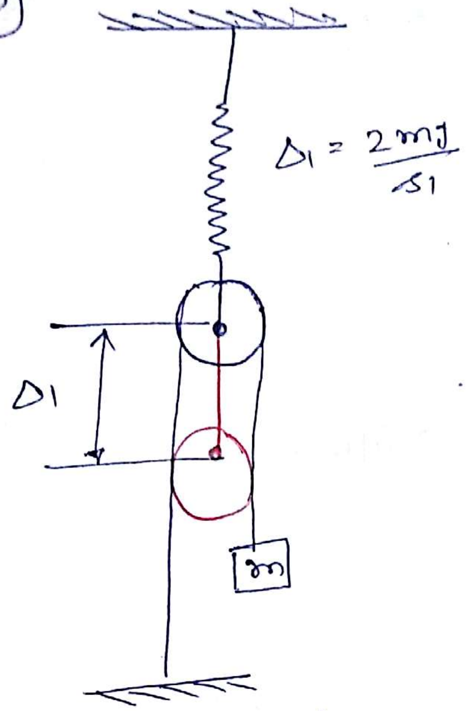


$\Delta_2 = \frac{mg}{\delta_2}$

$\boxed{\Delta = \Delta_1 \frac{b}{a} + \Delta_2}$ xxx.

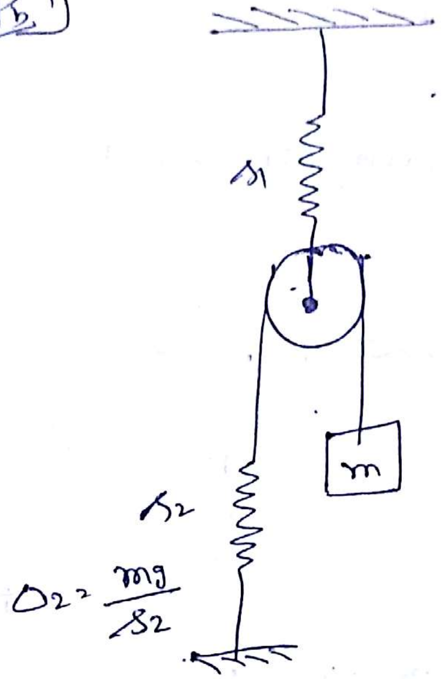
will Remain always same

Pb 3)



$$\boxed{\begin{gathered}\Delta = 2\Delta_1 \\ \Delta = \frac{4mg}{s_1}\end{gathered}}$$

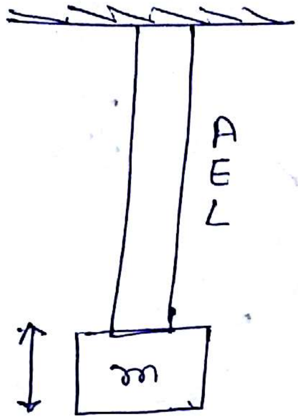
Pb 4)



$$\boxed{\Delta = 2\Delta_1 + \Delta_2}$$

Longitudinal vibrations :

Vibration along the length

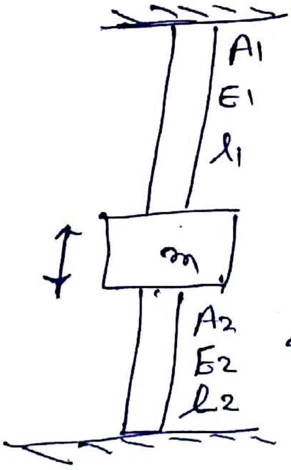


longitudinal stiffness

$$\boxed{s = \frac{AE}{l}}$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m}}}$$ ***

Pb.



$s_1 = \frac{A_1 E_1}{l_1}$

$s_2 = \frac{A_2 E_2}{l_2}$

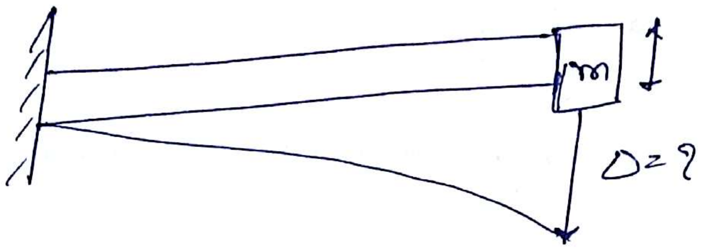
$\therefore s = s_1 + s_2$

$$\boxed{\omega_n = \sqrt{\frac{s}{m}}}$$

Transverse vibration Of Beams

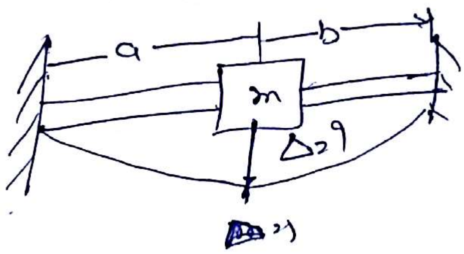
↓

Vibration in the dirⁿ ⊥ to length



$$\omega_n = \sqrt{\frac{g}{\Delta}} = ?? = \sqrt{\frac{S}{m}}$$

stiffness of System for Transverse vib



Damped System (Kinetic friction ≠ 0)

⇓

Technical Name of Kinetic friction in vibration System

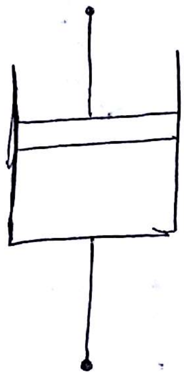
⇓

Damping

⇓

And damping is Represented by the Symbol

⇓



Damping in Any System

friction b/w dry Surfaces (Coulamb Damping) Very high

Viscous Damping (Very low)

Damping force in any system is proportional to $\dot{x}$

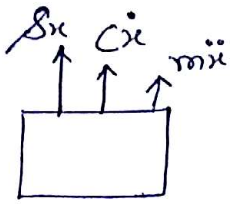
$$(\text{Damping force})_{\text{system}} \propto \dot{x}$$

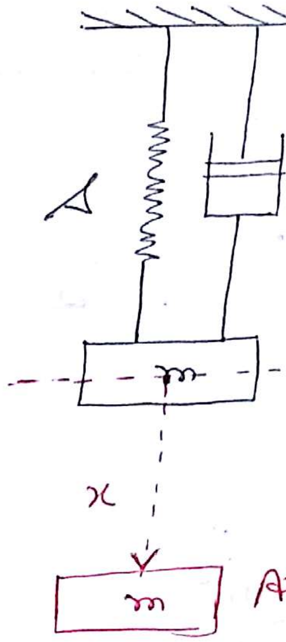
$$= c\dot{x}$$

↳ Coefficient Of Damping

At $t = t$

free Body diagram of System





[mean eq^m Position]

At $t=t$

$$\boxed{m\ddot{x} + c\dot{x} + Sx = 0}$$

$$\boxed{\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \omega_n^2 x = 0} \text{ — (A)}$$

Equation of Damped System

And the Solution of above equ^n is

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \qquad \alpha_1 \neq \alpha_2$$

or

$$x = (A+Bt)e^{\alpha t} \longrightarrow \boxed{\alpha_1 = \alpha_2 = \alpha}$$

$\alpha_{1,2} \rightarrow$ Roots of Auxillary equ^n

$$\Downarrow$$

$$\boxed{\alpha^2 + \frac{c}{m}\alpha + \omega_n^2 = 0}$$

$$\alpha_{1,2} = -\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}$$

$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}$$

$$\frac{\left(\frac{C}{2m}\right)^2}{(\omega_n)^2} = \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{C}{2m}\right)^2}{\omega_n^2}} = \xi \Rightarrow$$ Damping factor or Damping Ratio

$$\xi = \sqrt{\frac{\frac{C^2}{4m^2}}{\frac{s}{m}}} = \frac{C}{2\sqrt{sm}}$$

$$\boxed{2\xi\omega_n} \longrightarrow \cancel{2} \times \frac{C}{\cancel{2}\sqrt{sm}} \times \sqrt{\frac{s}{m}} \Rightarrow \frac{C}{m}$$

$$\boxed{2\xi\omega_n = \frac{C}{m}}$$

finally

equn of Damped System

$$\Rightarrow \boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n^2)x = 0} \ **$$

Sol^n of this equ^n,

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad (\alpha_1 \neq \alpha_2)$$

or

$$x = (A + Bt)e^{\alpha t}$$

where,

$$\alpha_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$$

If $\xi > 1 \Rightarrow$ Over Damped System (over damping Coulamb damping)

If $\xi \doteq 1 \Rightarrow$ Critically Damped system (Critical damping)

If $\xi < 1 \Rightarrow$ Under damped System (under damping / viscous damping)

(i) if $\xi > 1$ Over Damped System

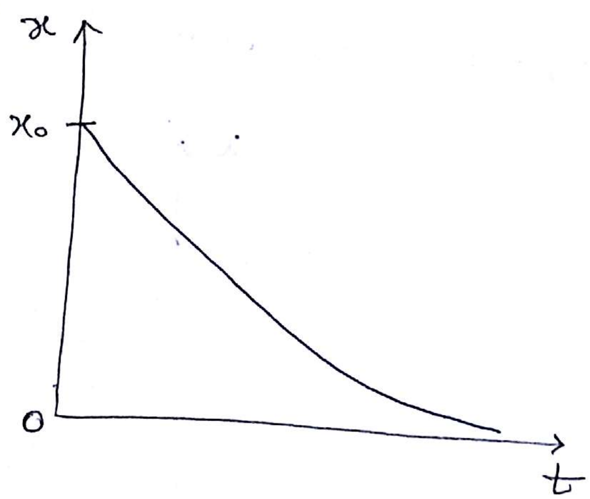
The Solⁿ will be

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$\boxed{x = Ae^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_n t} + Be^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}}$$

$\Downarrow$

No vibration in this System. e.g Door closer



Over Damped System

ii) Critically Damped System ($\xi = 1$)

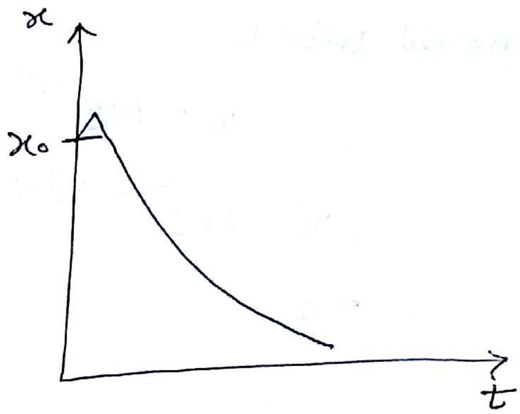
$$\boxed{\alpha_1 = \alpha_2 = \alpha = -\omega_n}$$

The Solⁿ will be;

$$\boxed{x = (A + Bt)e^{-\omega_n t}}$$

No vibⁿ

A, B are Constant

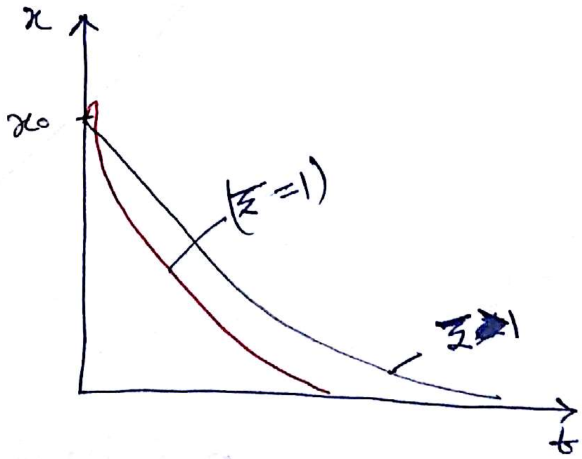


AK-47 example

660/min

NoTe

Critical Damping Response is much faster than over damping System.



iii) Under Damped System ($\zeta < 1$)

$$\alpha_{1,2} = -\zeta\omega_n \pm i\underbrace{\sqrt{1-\zeta^2}\cdot\omega_n}_{\omega_d = \text{Const.}}$$

$$\alpha_{1,2} = (-\zeta\omega_n \pm i\omega_d)$$

This Solⁿ will be ;

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = Ae^{(-\zeta\omega_n + i\omega_d)t} + Be^{(-\zeta\omega_n - i\omega_d)t}$$

$$x = e^{-\zeta\omega_n t}\left[\underbrace{(A+B)}_{X\sin\phi}\cos\omega_d t + \underbrace{i(A-B)}_{X\cos\phi}\sin\omega_d t\right]$$

$$x = e^{-\zeta\omega_n t}\, X\sin(\omega_d t + \phi)$$

$$\boxed{x = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)}$$

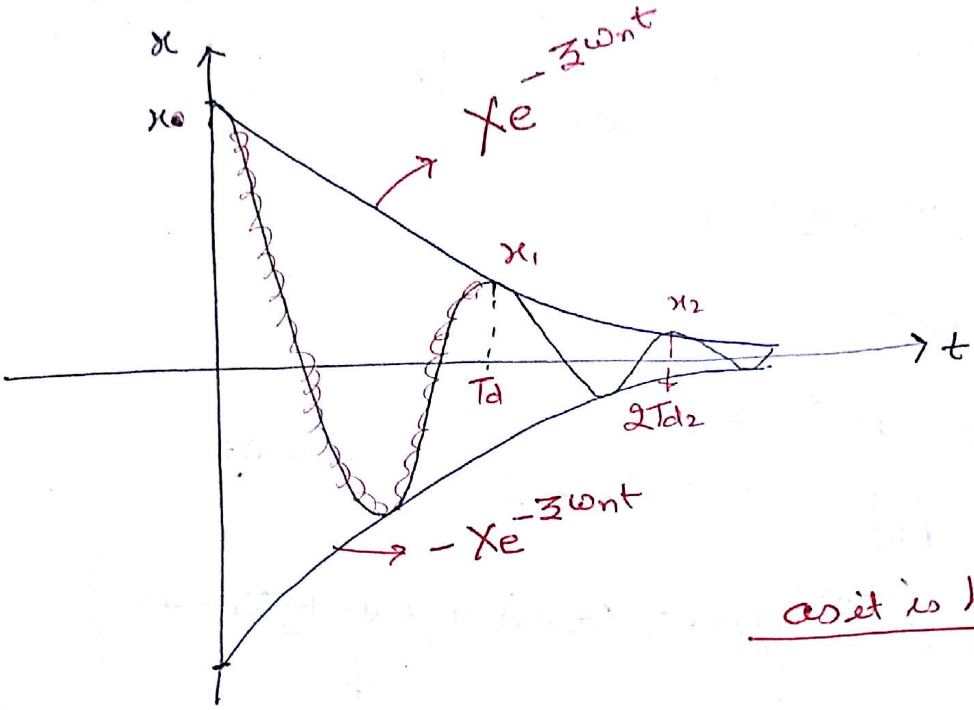
X, ϕ Const

$Xe^{-\zeta\omega_n t}$ → Amplitude → ↓↓ with time

$\sin(\omega_d t+\phi)$ → Vibⁿ with frequency $\boxed{\omega_d = \sqrt{1-\zeta^2}\cdot\omega_n}$ — Const

$$T_d = \frac{2\pi}{\omega_d} \quad \text{Sec} \text{ — Const}$$

$$F_d = \frac{\omega_d}{2\pi} \quad \text{Hz} \text{ — Const}$$



as it is harmonic motion

At t = 0

$$X_0 = X \sin\phi$$

At t = Td

$$X_1 = Xe^{-\xi\omega_n(Td)} \sin\phi$$

At t = 2Td

$$x_2 = Xe^{-\xi\omega_n(2Td)} \sin\phi$$

Decrement Ratio

$$\boxed{\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} - - - - - - - = e^{\xi\omega_n Td} = \text{Const}}$$ **

Logrithmic Decrement (δ) :

$$\delta = \ln e^{\zeta \omega_n T_d}$$

$$= \zeta \omega_n T_d$$

$$= \zeta \cancel{\omega_n} \frac{2\pi}{\sqrt{1-\zeta^2} \cdot \cancel{\omega_n}}$$

$$\boxed{\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}} \;**$$

Critical Damping Coefficient (C_c) :-

$$\frac{\cancel{2} \zeta \cancel{\omega_n} = \frac{C}{\cancel{m}}}{\cancel{2} \times 1 \times \cancel{\omega_n} = \frac{C_c}{\cancel{m}}} \Rightarrow \boxed{\zeta = \frac{C}{C_c}} = \frac{\text{Actual Damping Coeff.}}{\text{Critical Damping Coeff.}}$$

NOTE:

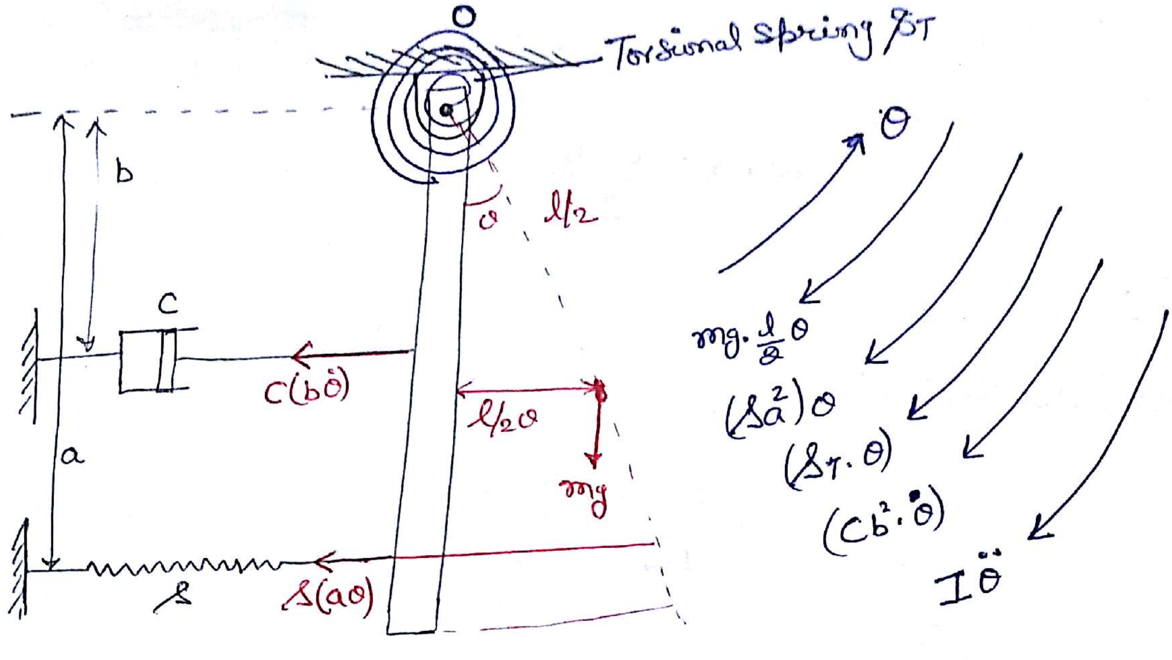
- The Ratio of Displacement of end of 3^{rd} cycle to Start of 8^{th} cycle is 2.5

$$\boxed{\frac{x_3}{x_7} = 2.5}$$

- The Ratio of Displacement of 3^{rd} cycle to 8^{th} cycle is 2.5

$$\boxed{\frac{x_3}{x_8} = 2.5}$$

Pb



$I = \frac{ml^2}{3}$

$$\boxed{I\ddot{\theta} + (Cb^2)\dot{\theta} + \left(\frac{mgl}{2} + Sa^2 + S_T\right)\theta = 0}$$

$$\boxed{m\ddot{x} + c\dot{x} + Kx = 0}$$

Problem

Qn The Damping Coefficient in vib^n equ^n will be $\rightarrow$ Cb^2

Qn) $\omega_n \rightarrow$ $\omega_n = \sqrt{\dfrac{\frac{mgl}{2} + Sa^2 + S_T}{I}}$ $\quad I = \frac{ml^2}{3}$

Qn3) $\xi = ?$ $\rightarrow$ $2\xi\omega_n = \frac{Cb^2}{I}$

Qn4) $\omega_d = ?$

$\omega_d = \sqrt{1-\xi^2}\cdot\omega_n$

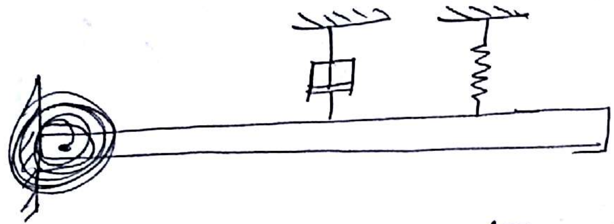
ω_n ?

(5) $C_c = ?$ →

$$2\omega_n = \frac{C_c b}{I}$$

Note :

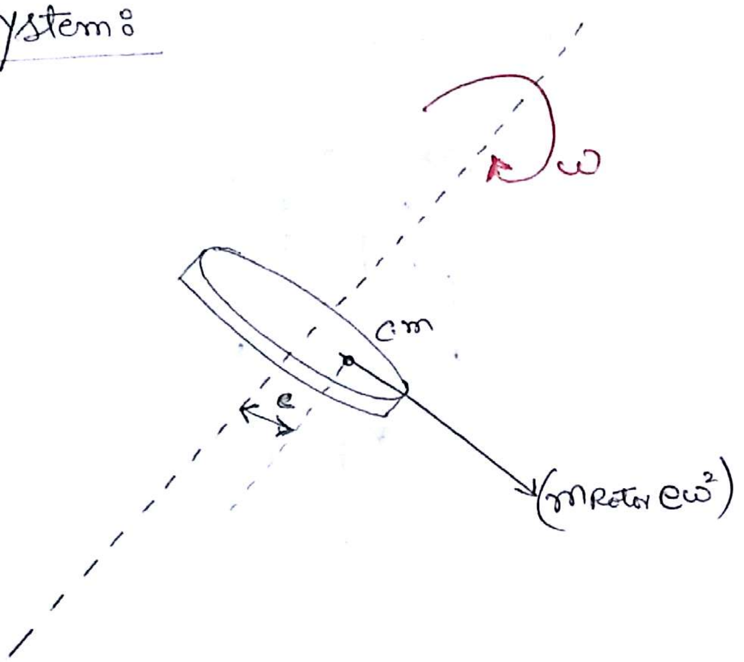
Horizontal System :



mg torque will not be Considered

$$\boxed{I\ddot{\theta} + (Cb^2)\dot{\theta} + (Sa^2 + St)\theta = 0}$$

Vibrations Causing Unbalanced Forces in Running Mechanical System:



At $t = t$

In a Particular dir^n

$F_{un} = (M_{rotor} \cdot e\omega^2) \sin\theta$

$= (M_{rotor} \cdot e\omega^2) \sin\omega t$

$$\boxed{F_{un} = F_0 \sin\omega t}$$

F_0 — Max. Value

$\omega \rightarrow$ Force Frequency.

in Case of Resiprocating unbalance:

At $t = t$

$F_{un} = (M_{resi}\, r\omega^2) \sin\omega t$

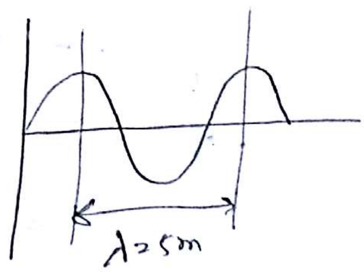
$$\boxed{F_{un} = F_0 \sin\omega t}$$

$F_0 \rightarrow$ max. Value

$\omega \rightarrow$ Force frequency.

Wave form:

$\vec{V} = 20\,m/s$



$V = f\lambda$

$20 = F \times 5 \Rightarrow f = 4\,Hz$

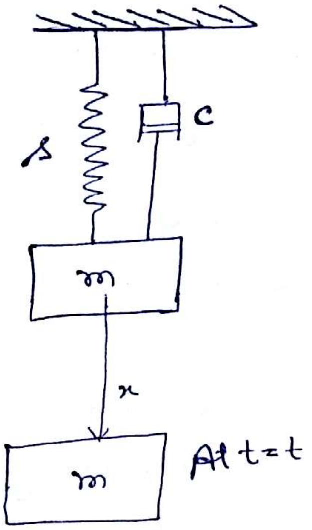
$\omega = 2\pi f$

$$\boxed{\omega = 2\pi \times 4 = 8\pi\ rad/s}$$

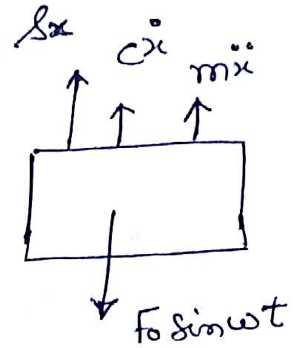
Direct form $\rightarrow$ $F_{un} = 200 \cos 3t$

200 $\rightarrow F_0$, 3 $\rightarrow \omega$

Forced Damped System :-



Free body diagram at t = t



$F_{ext} = F_0 \sin \omega t$

$F_0 \rightarrow$ Max. Value

$\omega \rightarrow$ Force frequency

$m\ddot{x} + c\dot{x} + sx - F_0 \sin \omega t = 0$

$$\boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t}$$

Eqn of damped (Forced) system

Solⁿ of this eqn is ;

$$\boxed{x = CF + PI}$$

$$C.F \rightarrow \begin{cases} \xi > 1 \\ \xi = 1 \end{cases} \text{No vib}^n \qquad \xi < 1 \; \text{– Vib}^n$$

After some time CF → 0

After some time

CF → 0

$$\boxed{x = P.I}$$ ***

P.I

$D \to$ Parameter

$D^2 \to -\omega^2$

$$P.I = \frac{\frac{F_0}{m}\sin\omega t}{D^2 + (2\xi\omega_n)D + \omega_n^2}$$

$$= \frac{\frac{F_0}{m}\sin\omega t}{\{(\omega_n^2-\omega^2)+(2\xi\omega_n)D\}} \cdot \frac{\{(\omega_n^2-\omega^2)-(2\xi\omega_n)D\}}{\{\omega_n^2-\omega^2)-(2\xi\omega_n)D\}}$$

$$= \frac{\frac{F_0}{m}\left[\underbrace{(\omega_n^2-\omega^2)}_{R\cos\phi}\sin\omega t - \underbrace{2\xi\omega\omega_n}_{R\sin\phi})\cos\omega t\right]}{(\omega_n^2-\omega^2)^2 + (2\xi\omega\omega_n)^2}$$

$$= \frac{F_0}{m}\cdot\frac{R\sin(\omega t-\phi)}{\cancel{R^2}}$$

$$P.I = \frac{F_0/m}{(\omega_n^2-\omega^2)^2+(2\xi\omega\omega_n)^2}\sin(\omega t-\phi)$$

~~$P.I = F_0/S$~~

$$P.I = \frac{F_0/S}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}\sin(\omega t-\phi)$$

Amplitude $\Downarrow$ A

Vibration with Frequency ω

Independent of time (more Dangerous)
Never ends

After some time

$CF \to 0$

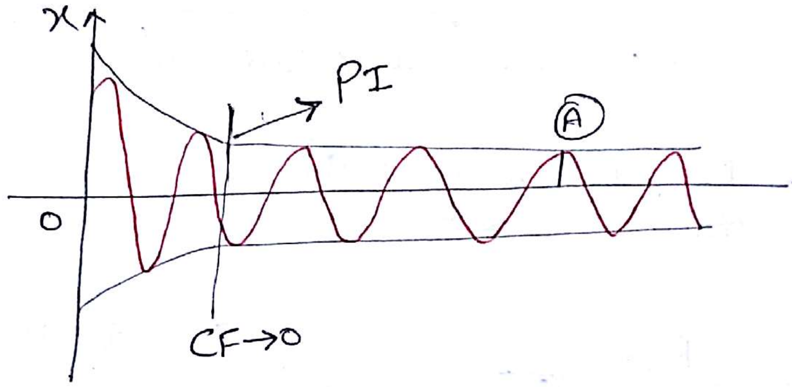
$\boxed{P.I = x}$

$$\boxed{x = A \sin(\omega t - \phi)}$$

Where; $A = \dfrac{F_0/s}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$

Amplitude of Steady ~~State~~ vibration (forced vib^n)

doesn't depend on time



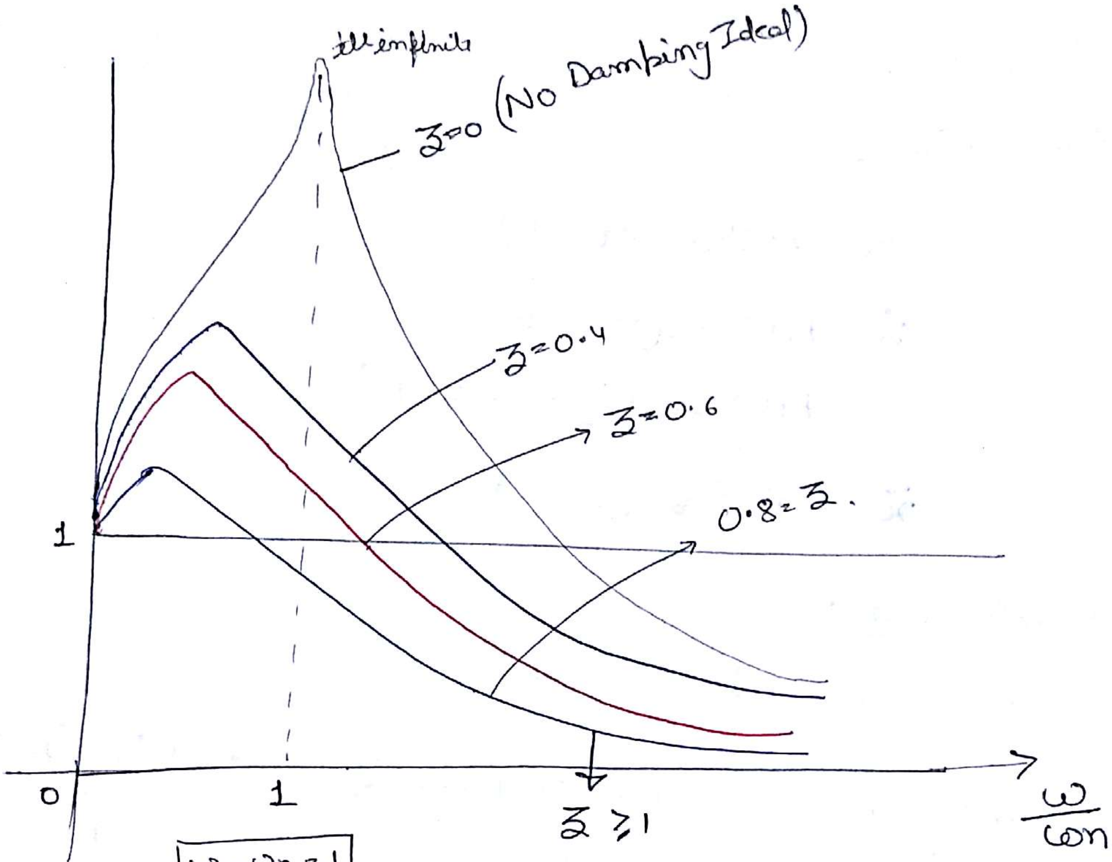
Vibration in Running System Will never Stop.
every mechanical System must have one Running life

$M.F = \dfrac{A}{F_0/s}$

$$\boxed{M.F = \frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

Magnification factor depend on :-

1) $\frac{\omega}{\omega_n}$ 2) ξ



$\boxed{\omega = \omega_n = 1}$

Resonance $\frac{\omega}{\omega_n} = 1$

i) $\uparrow$ underdamping $\Rightarrow \zeta \downarrow$

MF $\uparrow \Rightarrow A \uparrow$ running life $\downarrow\downarrow$

ii) $(A)_{max}$ at (MF_{max} al)

at $= \frac{\omega}{\omega_n} < 1 \rightarrow$ under damping (viscous)

at $= \frac{\omega}{\omega_n} = 1 \rightarrow$ No Damping

at $= \frac{\omega}{\omega_n} = 0 \rightarrow$ over / critical damping (Coulomb).

3) $$\boxed{(A)_{Resonance} = \frac{(F_0/s)}{2\zeta}} \propto \frac{1}{\zeta}.$$

NOTE:

After Some time,

$$x = A \sin(\omega t - \phi)$$

$$\dot{x} = A\omega \cos(\omega t - \phi)$$

$$= A\omega \sin\left\{\frac{\pi}{2} + (\omega t - \phi)\right\}$$

$$\ddot{x} = -A\omega^2 \sin(\omega t - \phi)$$

Basic equation was;

$$F_0 \sin\omega t - m\ddot{x} - c\dot{x} - sx = 0$$

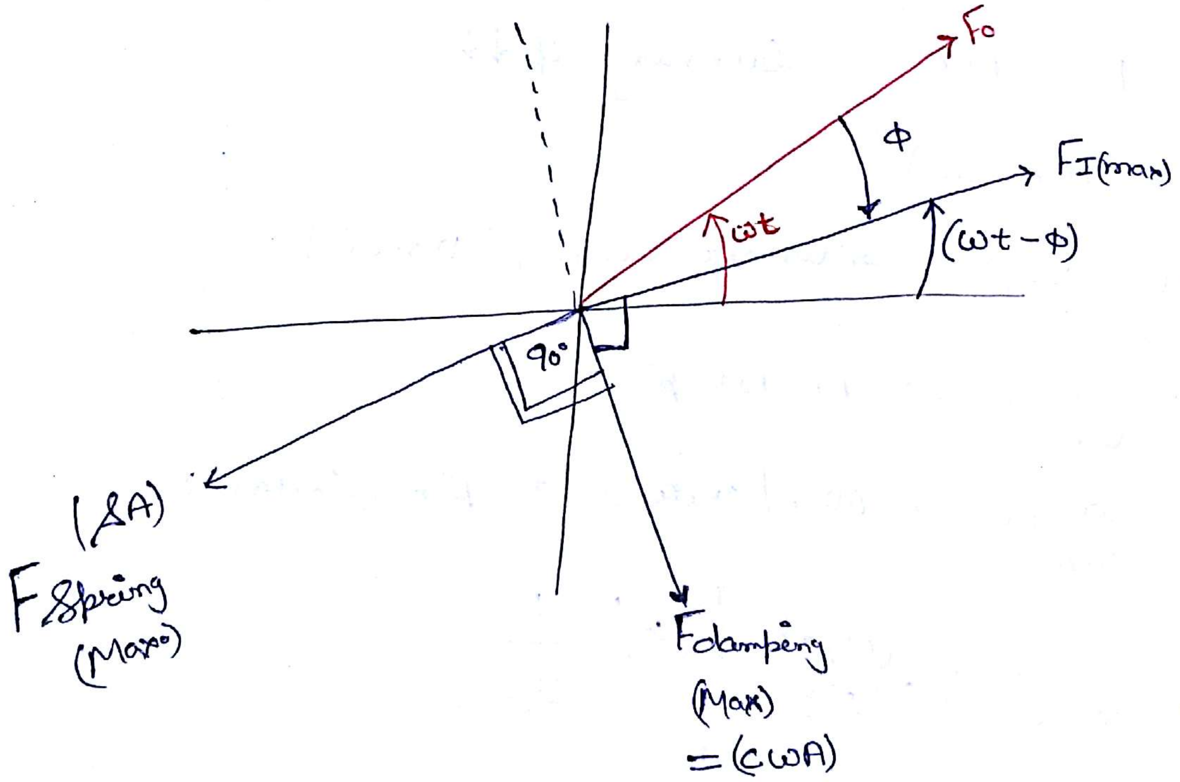
$$F_0 \sin\omega t + m\omega^2 A \sin(\omega t - \phi) - c\omega A \sin\left\{\frac{\pi}{2} + (\omega t - \phi)\right\} - sA \sin(\omega t - \phi) = 0$$

F_0 → F_{un} (Max.)

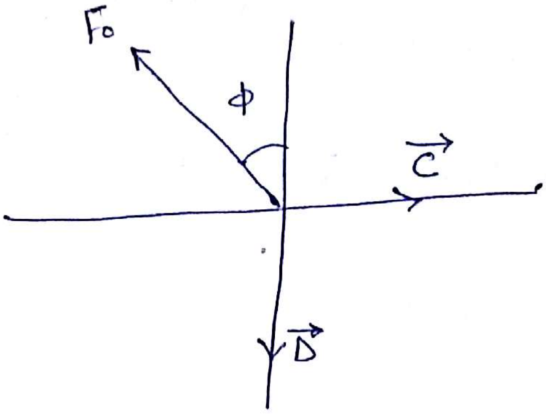
$m\omega^2 A$ → F_I (max.)

$c\omega A$ → $F_{damping}$ (max.)

sA → (F_{spring}/$F_{elastic}$) (max.)



Pb

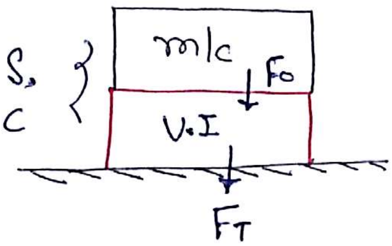


$\vec{C}$ = ? Damping force max.

$\vec{D}$ = Spring force Max.

Vibration Isolation :- (Foundation)

How to Isolate the ground from the vibration of Running Machine.



Where;

$F_T \rightarrow$ Transmitted force to ground. (ϵ)

$$\boxed{F_T <<<<<< F_0}^{*}$$

$$\boxed{\epsilon = \frac{F_T}{F_0}}$$

$$\boxed{0 < \epsilon < 1}$$

$\epsilon \rightarrow 0$ Best

Transmissibility (ϵ) : (Performance of vibration Isolation System)

$$F_T = \sqrt{(SA)^2 + (C\omega A)^2}$$

bcz both are ⊥ to each other.

F_s & $F_c \rightarrow$ Max. Response

$$= SA\sqrt{1 + \left(\frac{\frac{C}{m}\omega A}{\frac{S}{m}A}\right)^2}$$

$$F_T = SA\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}$$

$$F_0 = SA\sqrt{\left(1-\left(\frac{\omega}{\omega n}\right)^2\right)^2+\left(\frac{2\xi\omega}{\omega n}\right)^2}$$

$$\boxed{\epsilon = \frac{F_T}{F_0}}$$

$$\boxed{\epsilon = \frac{\sqrt{1+\left(\frac{2\xi\omega}{\omega n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega n}\right)^2\right)^2+\left(\frac{2\xi\omega}{\omega n}\right)^2}}}$$

$\epsilon \rightarrow$ depends on;

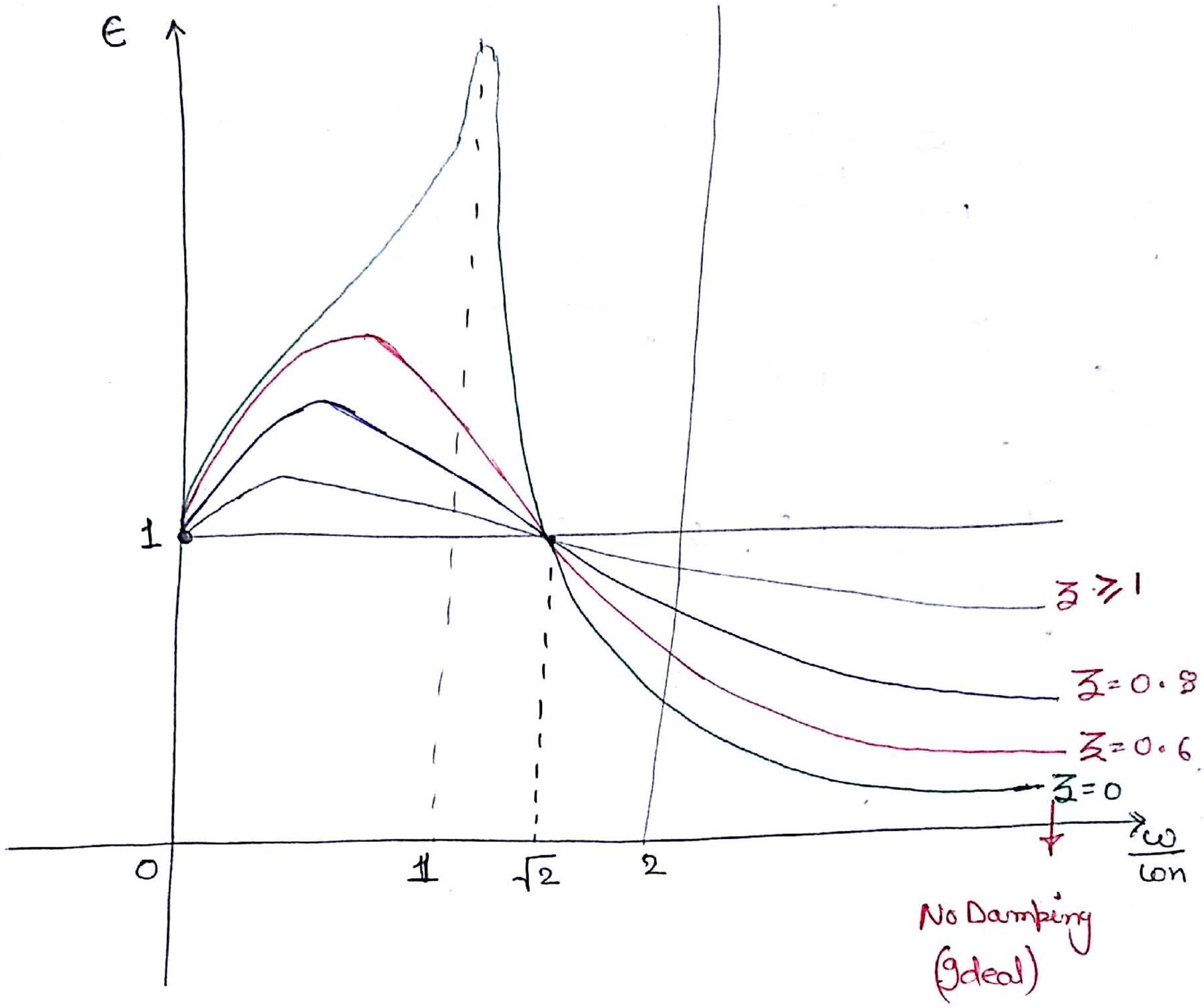
i) $\omega/\omega n$

ii) ξ

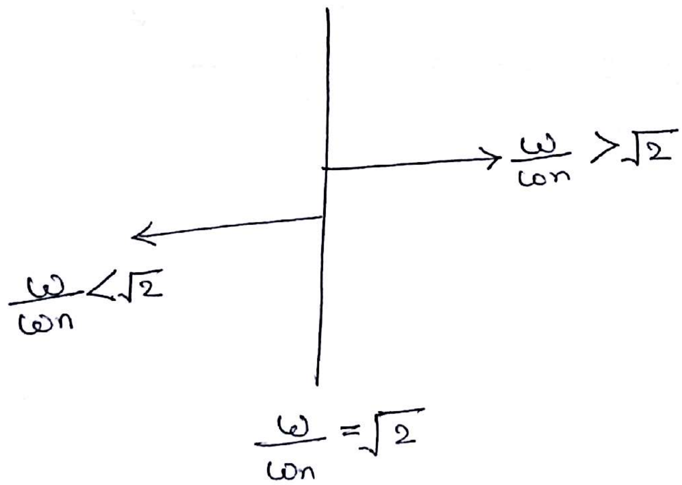
If, $\frac{\omega}{\omega n} = 0$

if $\frac{\omega}{\omega n} = \sqrt{2}$

$e = 1$ for all Values of ξ *



ε
1
0
1
√2
2
ω/ωn
ζ ≥ 1
ζ = 0.8
ζ = 0.6
ζ = 0
No Damping
(Ideal)



ω/ωn > √2
ω/ωn < √2
ω/ωn = √2

i) If underdamping → $\zeta \downarrow$

ϵ will ↑ if $\frac{\omega}{\omega_n} < \sqrt{2}$

ϵ will ↓ if $\frac{\omega}{\omega_n} > \sqrt{2}$

ϵ will Remain Same if $\frac{\omega}{\omega_n} = \sqrt{2}$

ii) Vibration Isolation will be effective,

when $\epsilon < 1$

↓

If $\left(\frac{\omega}{\omega_n} > \sqrt{2}\right)$

iii) If effective V.I Zone

$\frac{\omega}{\omega_n} > \sqrt{2}$ $(\epsilon < 1)$

→ No damping is Best $(\epsilon \to \infty)$

→ Damping becomes determintal (harmful)

Pb) for effective Vibration Isolation the (ω_n)

i) ω ii) 2ω iii) $\omega/4$ iv) $\cdot$ 2ω

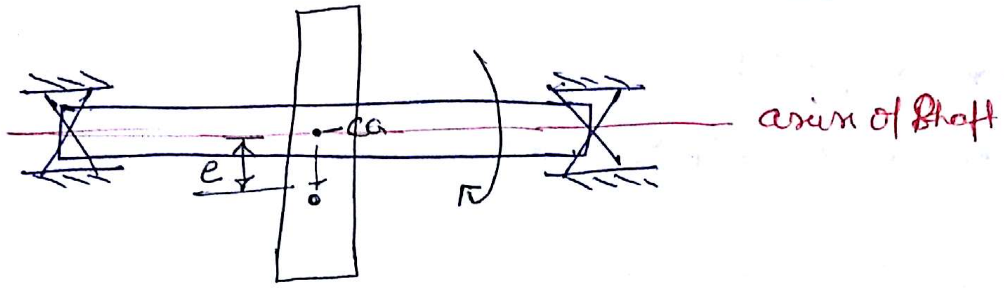
for effective V.I

$\cancel{\frac{\omega}{\omega_n} < \omega}$

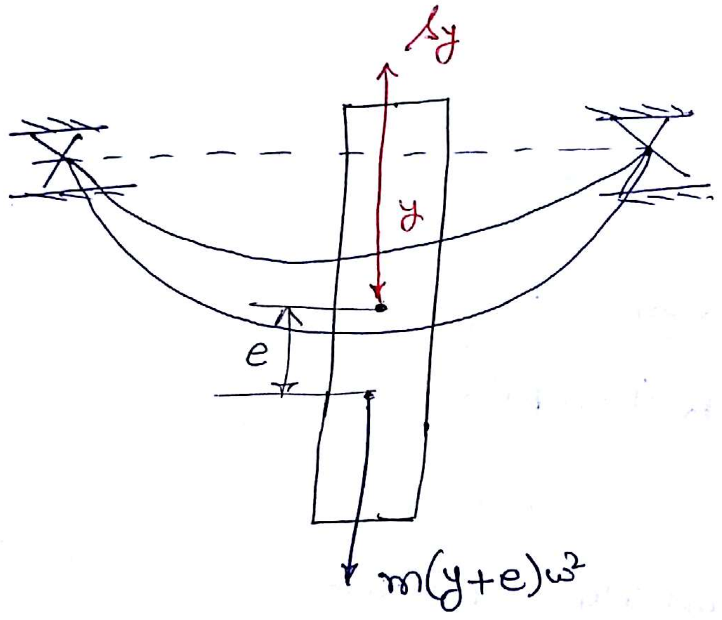
$\frac{\omega}{\omega_n} > \sqrt{2}$

$$\boxed{\omega_n < \frac{\omega}{\sqrt{2}}}$$

Whirling Of Shaft :- (Shaft failure)



After some time



$$m(y+e)\omega^2 = Sy$$

$$my\omega^2 + me\omega^2 = S \cdot y$$

$$Sy - my\omega^2 = me\omega^2$$

$$\cancel{m}y\cancel{\omega^2}\left(\frac{S}{m\omega^2} - 1\right) = \cancel{m}e\cancel{\omega^2}$$

$$\boxed{y = \frac{e}{\left(\frac{\omega}{\omega n}\right)^2 - 1}}$$

deflection in Shaft.

In Some Systems

- Whose Running life is less
- → How to ↑ running life.

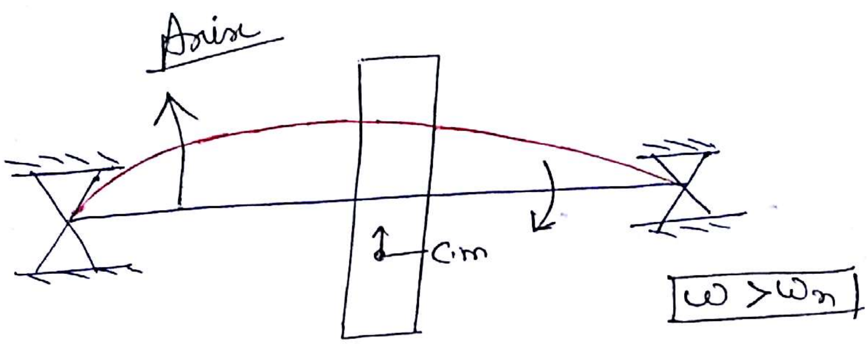
Low life

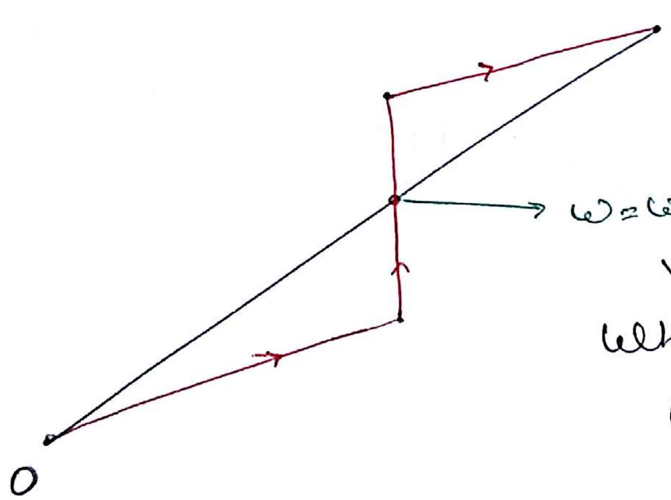
1) Heavy
2) more delicate item

If $\omega > \omega_n$

→ Y will become −ve.

→ Shaft bending will be in opposite dirⁿ.





$\omega = \omega_n$ → (Resonance (Occur is very less time))

⇓

Whirling Speed

(Critical Speed)

⇓

$\sqrt{\frac{S}{m}}$ Rad/sec.

(To stop opposite Torque is used)

Qn)

$m = 17\,Kg$

$S = 1000 \frac{N}{m}$

$\omega_n = \sqrt{\frac{1000}{17}}$ Rad/s

$m_{desi} = 2\,Kg$

$r = \frac{75}{2000}$ ~~meter~~

$N = 500\,rpm$

Imp.

$\omega = \frac{2\pi \times 500}{60}$ Rad/sec

$F_0 = 2 \times \frac{75}{2000} \times \frac{2\pi 500}{60}$ N

$\frac{\omega}{\omega_n} = ?$

$\zeta = 0.20$

1) $A = \dfrac{F_0/S}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$

2) $\epsilon = \dfrac{\sqrt{1+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$, ?

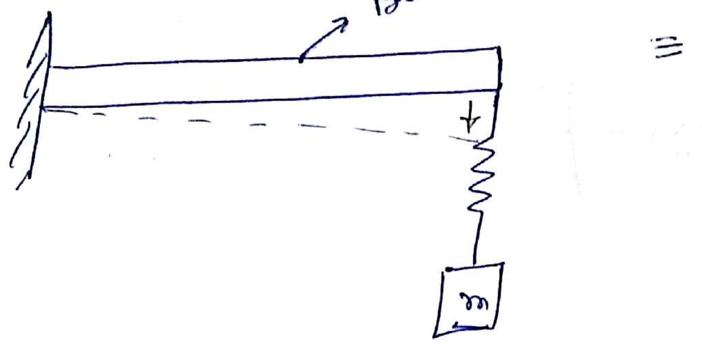
$\epsilon = \frac{F_T}{F_0}$ ∴ F_T ?

Pb

Stiffness is opposite to deformation

Bending stiffness S_1

Q→



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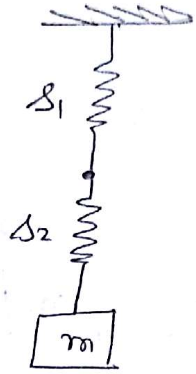
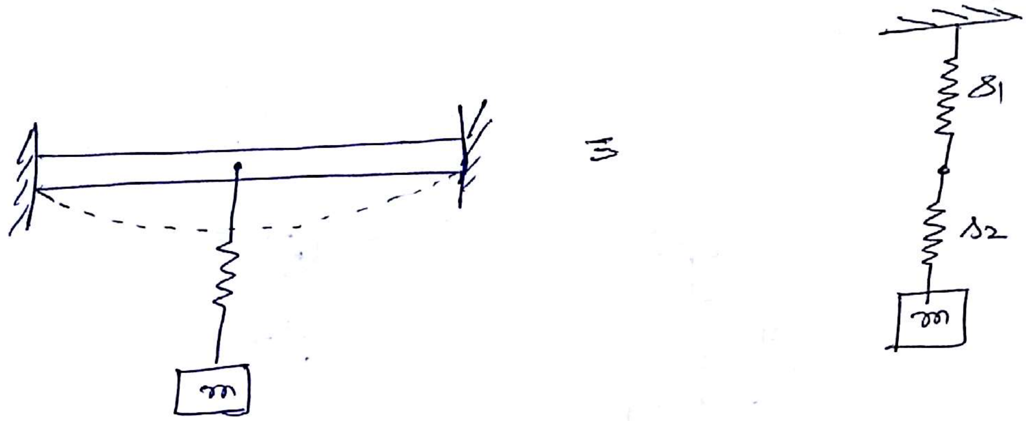
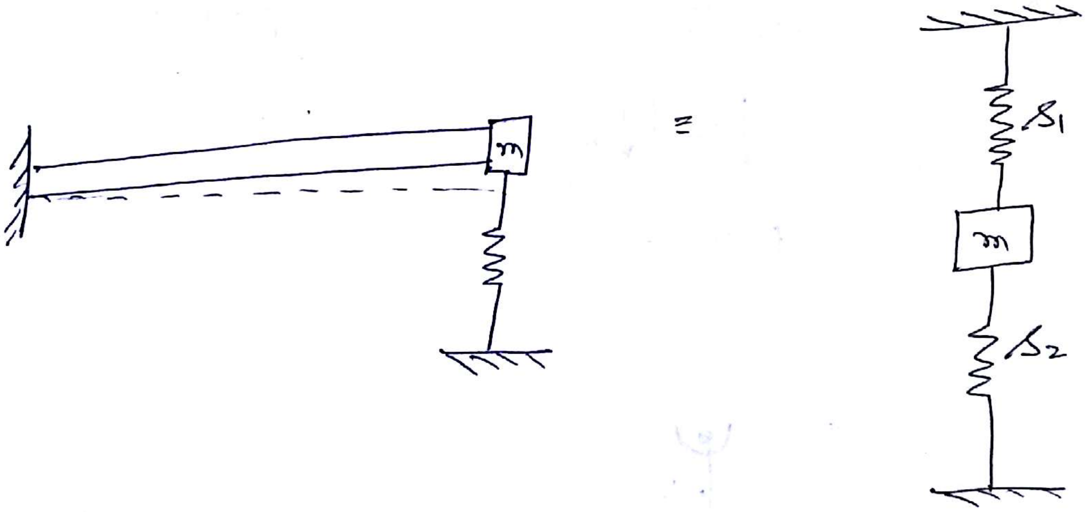


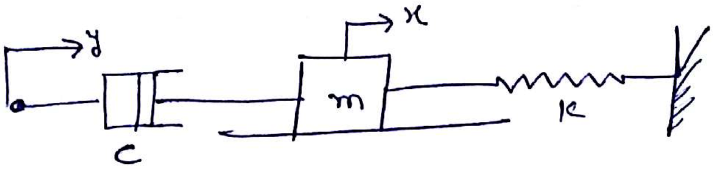
Qn)

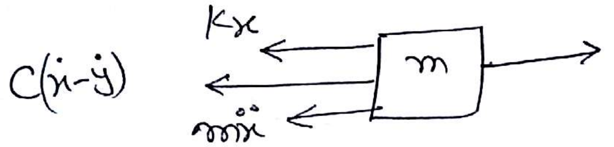


Qn)

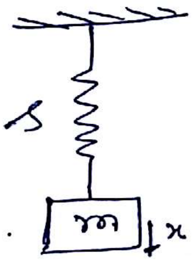


Pb

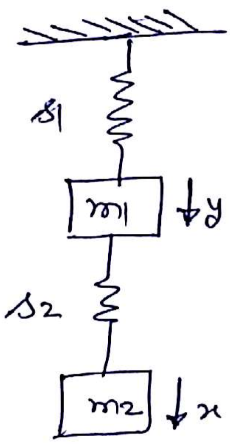




D.O.F of vibrating System

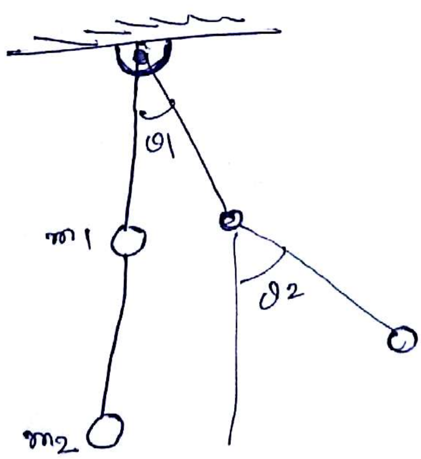


DoF=1

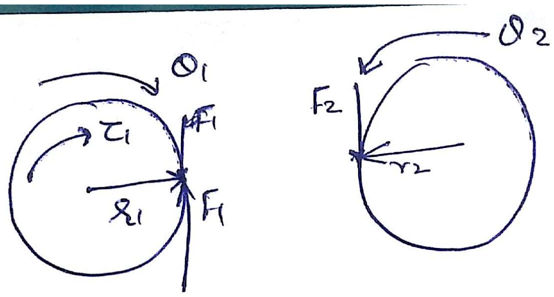


D.OF=2

DOF=1



DoF=2



$$\tau_1 = F_1 r_1 = I_1 \ddot{\theta}_1$$

$$\tau_1 = F_1 r_1 + I_1 \ddot{\theta}_1$$

$$= F_2 r_1 + I_1 \ddot{\theta}_1$$

$$= I_2 \left(\frac{r_1}{r_2}\right)^2 \ddot{\theta}_1 + I_1 \ddot{\theta}_1$$

$$\Rightarrow \tau_1 = \left[I_1 + I_2\left(\frac{r_1}{r_2}\right)^2\right] \ddot{\theta}_1$$

$$F_2 r_2 = I_2 \ddot{\theta}_2$$

$$F_2 = \frac{I_2 \ddot{\theta}_2}{2}$$

$$r_1 \omega_1 = r_2 \omega_2$$

$$r_1 \alpha_1 = r_2 \alpha_2$$

$$r_1 \ddot{\theta}_1 = r_2 \ddot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{r_1}{r_2} \ddot{\theta}_1$$

$$F_2 = \frac{I_2 r_1 \ddot{\theta}_1}{r_2}$$