

12.3 ▼ DISTANCE OF CLOSEST APPROACH : ESTIMATION OF NUCLEAR SIZE

3. *Explain how Rutherford's experiment on scattering of α -particles led to the estimation of the size of the nucleus.*

Distance of closest approach : Estimation of nuclear size. As shown in Fig. 12.5, suppose an α -particle of mass m and initial velocity v moves directly towards

the centre of the nucleus of an atom. As it approaches the positive nucleus, it experiences Coulombic repulsion and its kinetic energy gets progressively converted into electrical energy. At a certain distance r_0 from the nucleus, the α -particle stops for a moment and then begins to retrace its path, i.e., it is scattered through an angle of 180° . The distance r_0 is called the *distance of closest approach*. At this distance r_0 , the entire initial kinetic energy of the α -particle gets converted into electrostatic potential energy.

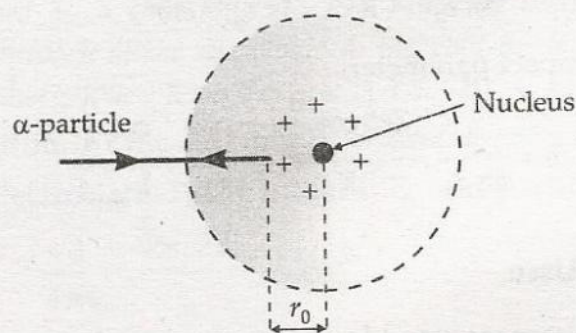


Fig. 12.5 Distance of closest approach.

Now, charge on an α -particle, $q_1 = +2e$

Charge on a scattering nucleus, $q_2 = +Ze$

where Z is the atomic number of foil atoms.

Initial kinetic energy of α -particle, $K_\alpha = \frac{1}{2}mv^2$

Electrostatic P.E. of α -particle and nucleus at distance r_0

$$U = k \cdot \frac{q_1 q_2}{r_0} = k \cdot \frac{2e \cdot Ze}{r_0}$$

By conservation of energy, $K_\alpha = U$

$$\text{or } K_\alpha = \frac{1}{2}mv^2 = k \cdot \frac{2Ze^2}{r_0}$$

$$\therefore r_0 = \frac{2kZe^2}{K_\alpha} = \frac{4kZe^2}{mv^2}$$

$$\text{where } k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}.$$

Clearly, the radius of the nucleus must be smaller than r_0 .

In one of the Rutherford's experiments, α -particles of energy 5.5 MeV were used.

$$\therefore K_\alpha = 5.5 \text{ MeV} = 5.5 \times 1.6 \times 10^{-13} \text{ J}$$

Atomic number of gold, $Z = 79$

$$\therefore r_0 = \frac{2kZe^2}{K_\alpha}$$

$$= \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.5 \times 1.6 \times 10^{-13}}$$

$$= 4.13 \times 10^{-14} \text{ m} = 41.3 \text{ fm}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the α -particle. The radius of a nucleus is of the order of a fermi, where 1 fermi (fm) = 10^{-15} m.

12.6 ▼ BOHR'S QUANTISATION CONDITION

6. Obtain Bohr's quantisation condition on the basis of the wave picture of an electron.

Bohr's quantisation condition of angular momentum. Consider the motion of an electron in a circular orbit of radius r around the nucleus of the atom. According to de Broglie hypothesis, this electron is also associated with wave character. Hence a circular orbit can be taken to be a stationary energy state only if it contains an integral number of de-Broglie wavelengths, *i.e.*, we must have

$$2\pi r = n\lambda$$

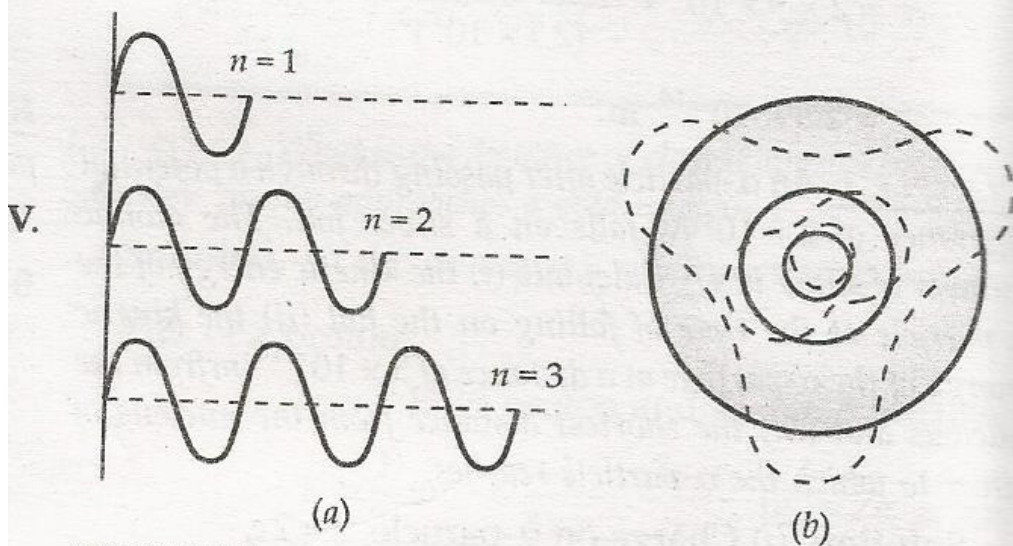


Fig. 12.8 de Broglie waves and hydrogen atom
 (a) Only a certain number of (de Broglie) wavelengths would fit in the electron orbits, and
 (b) discrete standing waves (characteristic frequencies or wavelength) correspond to the discrete orbits in Bohr's theory.

But de Broglie wavelength, $\lambda = \frac{h}{mv}$

$$\therefore 2\pi r = \frac{nh}{mv}$$

The angular momentum L of the electron must be

$$L = mvr = \frac{nh}{2\pi}, \quad n = 1, 2, 3, \dots$$

This is the famous **Bohr's quantisation condition for angular momentum**. Thus only those circular orbits can be the allowed stationary states of an electron in which its angular momentum is an integral multiple of $h/2\pi$.

Bohr's Theory of hydrogen atom : Radii of permitted orbits. According to Bohr's theory, a hydrogen atom consists of a nucleus with a positive charge Ze , and a single electron of charge $-e$, which revolves around it in a circular orbit of radius r . Here Z is the atomic number and for hydrogen $Z=1$. The electrostatic force of attraction between the nucleus and the electron is

$$F = \frac{kZe \cdot e}{r^2} = \frac{kZe^2}{r^2}$$

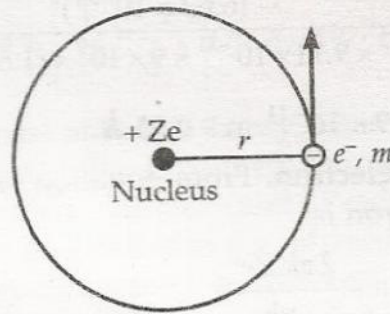


Fig. 12.9 Bohr's model of hydrogen atom.

To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic attraction. Therefore,

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

or $mv^2 = \frac{kZe^2}{r}$... (1)

or $r = \frac{kZe^2}{mv^2}$... (2)

where m is the mass of the electron and v , its speed in an orbit of radius r .

Bohr's quantisation condition for angular momentum is

$$L = mvr = \frac{nh}{2\pi}$$

or $r = \frac{nh}{2\pi mv}$... (3)

From equations (2) and (3), we get

$$\frac{kZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

or $v = \frac{2\pi kZe^2}{nh}$... (4)

Substituting this value of v in equation (3), we get

$$r = \frac{nh}{2\pi m} \cdot \frac{nh}{2\pi kZe^2}$$

or $r = \frac{n^2 h^2}{4\pi^2 mkZe^2}$... (5)

Clearly, the radii of the permitted orbits are proportional to n^2 and increase in the ratio of 1 : 4 : 9 : 16.... The parameter n is called the *principle quantum number*.

The radius of the innermost orbit of the hydrogen atom, called *Bohr's radius* can be determined by putting $Z = 1$ and $n = 1$ in equation (5) and it is denoted by r_0 .

$$\begin{aligned}\therefore r_0 &= \frac{h^2}{4\pi^2 m k e^2} \\ &= \frac{(6.63 \times 10^{-34})^2}{4\pi^2 \times 9.11 \times 10^{-31} \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2} \text{ m} \\ &= 5.29 \times 10^{-11} \text{ m} \approx 0.53 \text{ \AA}.\end{aligned}$$

Speed of electron. From equation (4), the orbital speed of electron is

$$v = \frac{2\pi k Z e^2}{nh}$$

For hydrogen, $Z = 1$, therefore,

$$v = \frac{2\pi k e^2}{nh} = \left(\frac{2\pi k e^2}{ch} \right) \frac{c}{n}$$

or
$$v = \alpha \cdot \frac{c}{n}$$

The quantity $\alpha = \frac{2\pi k e^2}{ch}$, is a dimensionless constant and is called *fine structure constant*. Its value is

$$\alpha = \frac{2\pi \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^8 \times 6.63 \times 10^{-34}} = \frac{1}{137}$$

$$\therefore v = \frac{1}{137} \cdot \frac{c}{n}$$

For first orbit ($n = 1$),
$$v = \frac{c}{137}$$

Thus the speed of the electron in the innermost orbit is 1/137 of the speed of light in vacuum, while the speed in the outer orbits varies inversely with n .

Energy of the electron. It includes the electron's kinetic energy and the electrostatic potential energy of the two charges.

Kinetic energy of the electron in n th orbit is

$$\text{K.E.} = \frac{1}{2} m v^2 = \frac{k Z e^2}{2r} \quad [\text{Using equation (1)}]$$

Potential energy of the electron in n th orbit is

$$\text{P.E.} = k \frac{q_1 q_2}{r} = \frac{k Z e \cdot (-e)}{r} = -\frac{k Z e^2}{r}$$

Hence total energy of the electron in n th orbit is

$$E_n = \text{K.E.} + \text{P.E.}$$

$$\begin{aligned}
 &= \frac{kZe^2}{2r} - \frac{kZe^2}{r} = -\frac{kZe^2}{2r} \\
 &= -\frac{kZe^2}{2} \cdot \frac{4\pi^2 mkZe^2}{n^2 h^2}
 \end{aligned}$$

[Using equation (5)]

or

$$E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2 h^2} \quad \dots(6)$$

The negative value of the total energy indicates that the electron is bound to the nucleus by means of electrostatic attraction and some work is required to be done to pull it away from the nucleus.

12.9 ▼ SPECTRAL SERIES OF HYDROGEN ATOM

9. On the basis of Bohr's theory, explain the origin of the various spectral series of hydrogen atom.

Spectral series of hydrogen atom. From Bohr's theory, the energy of an electron in the n th orbit of hydrogen atom is given by

$$E_n = -\frac{2\pi^2 m k^2 Z^2 e^4}{h^2} \cdot \frac{1}{n^2}$$

According to Bohr's frequency condition, whenever an electron makes a transition from a higher energy level n_2 to a lower energy level n_1 , the difference of

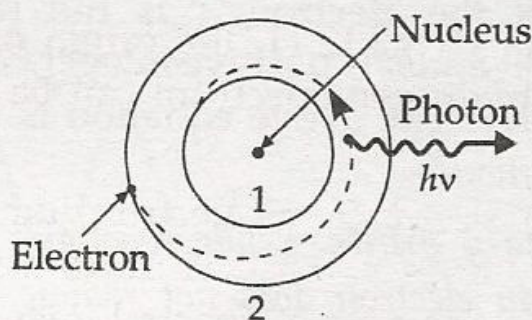


Fig. 12.10 Emission of a spectral line by a hydrogen atom.

energy appears in the form of a photon. The frequency ν of the emitted photon is given by

$$h\nu = E_{n_2} - E_{n_1}$$

or

$$h\nu = \frac{2\pi^2 m k^2 e^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

or

$$\nu = \frac{2\pi^2 m k^2 e^4}{h^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

As $c = \nu\lambda$, therefore **wave number** $\bar{\nu}$, which is the reciprocal of wavelength λ , is given by

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = \frac{2\pi^2 m k^2 e^4}{ch^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

or

$$\bar{\nu} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

where $R = \frac{2\pi^2 m k^2 e^4}{ch^3}$, is the *Rydberg constant* and its value is $1.0973 \times 10^7 \text{ m}^{-1}$. The above equation is the **Rydberg formula** for the spectrum of hydrogen atom. This formula indicates that the radiation emitted by the excited hydrogen atom consists of certain specific wavelengths or frequencies, the values of which depend on quantum numbers n_1 and n_2 .

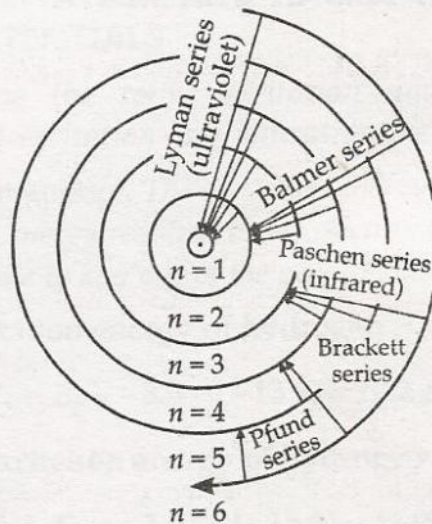


Fig. 12.11 Spectral series of hydrogen atom.

As shown in Fig. 12.11, the origin of the various series in the hydrogen spectrum can be explained as follows :

(i) **Lyman series.** If an electron jumps from any higher energy level $n_2 = 2, 3, 4, \dots$ to a lower energy level $n_1 = 1$, we get a set of spectral lines called *Lyman series* which belong to the *ultraviolet region* of the electromagnetic spectrum. This series is given by

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n_2^2} \right], \quad n_2 = 2, 3, 4, \dots$$

(ii) **Balmer series.** The spectral series corresponding to the transitions $n_2 = 3, 4, 5, \dots$ to $n_1 = 2$, lies in the *visible region* and is called *Balmer series*. For this series.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right], \quad n_2 = 3, 4, 5, \dots$$

(iii) **Paschen series.** If $n_2 = 4, 5, 6, \dots$ and $n_1 = 3$, we get a spectral series in the *infrared region* which is called *Paschen series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{3^2} - \frac{1}{n_2^2} \right], \quad n_2 = 4, 5, 6, \dots$$

(iv) **Brackett series.** If $n_2 = 5, 6, 7, \dots$ and $n_1 = 4$, we get a spectral series in the *infrared region* which is called *Brackett series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{4^2} - \frac{1}{n_2^2} \right], \quad n_2 = 5, 6, 7, \dots$$

(v) **Pfund series.** If $n_2 = 6, 7, 8, \dots$ and $n_1 = 5$, we get a spectral series in the *infrared region* which is called *Pfund series*.

$$\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{5^2} - \frac{1}{n_2^2} \right], \quad n_2 = 6, 7, 8, \dots$$

The greatness of Bohr's theory lies in the fact that it not only successfully explained the already known series of Lyman, Balmer and Paschen but also predicted two new series in the infrared region which were later on discovered by *Brackett* (in 1922) and *Pfund* in (1924). *Neil Bohr* was awarded the 1922 Nobel prize in physics for this work.

12.10 ▼ ENERGY LEVEL DIAGRAM FOR HYDROGEN

10. What is the energy level diagram for an atom? Calculate the energies of the various energy levels of a hydrogen atom and draw an energy level diagram for it.

Energy level diagram. It is a diagram in which the energies of the different stationary states of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale. Such a diagram illustrates more clearly the known facts about the stationary states and the emission or absorption of various spectral lines.

Energy levels of hydrogen. From Bohr's theory, the total energy of an electron in n th orbit is given by

$$E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2 h^2}$$

For hydrogen $Z = 1$, therefore,

$$E_n = -\frac{2\pi^2 mk^2 e^4}{h^2} \cdot \frac{1}{n^2}$$

Energy of the electron in the first orbit ($n = 1$) is given by

$$E_1 = -\frac{2 \times (3.14)^2 \times 9.11 \times 10^{-31} \times (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4}{(6.63 \times 10^{-34})^2} \cdot \frac{1}{1^2}$$

$$= -21.76 \times 10^{-19} \text{ J} = -\frac{21.76 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= -13.6 \text{ eV}$$

$$[\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

Hence we can write,

$$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

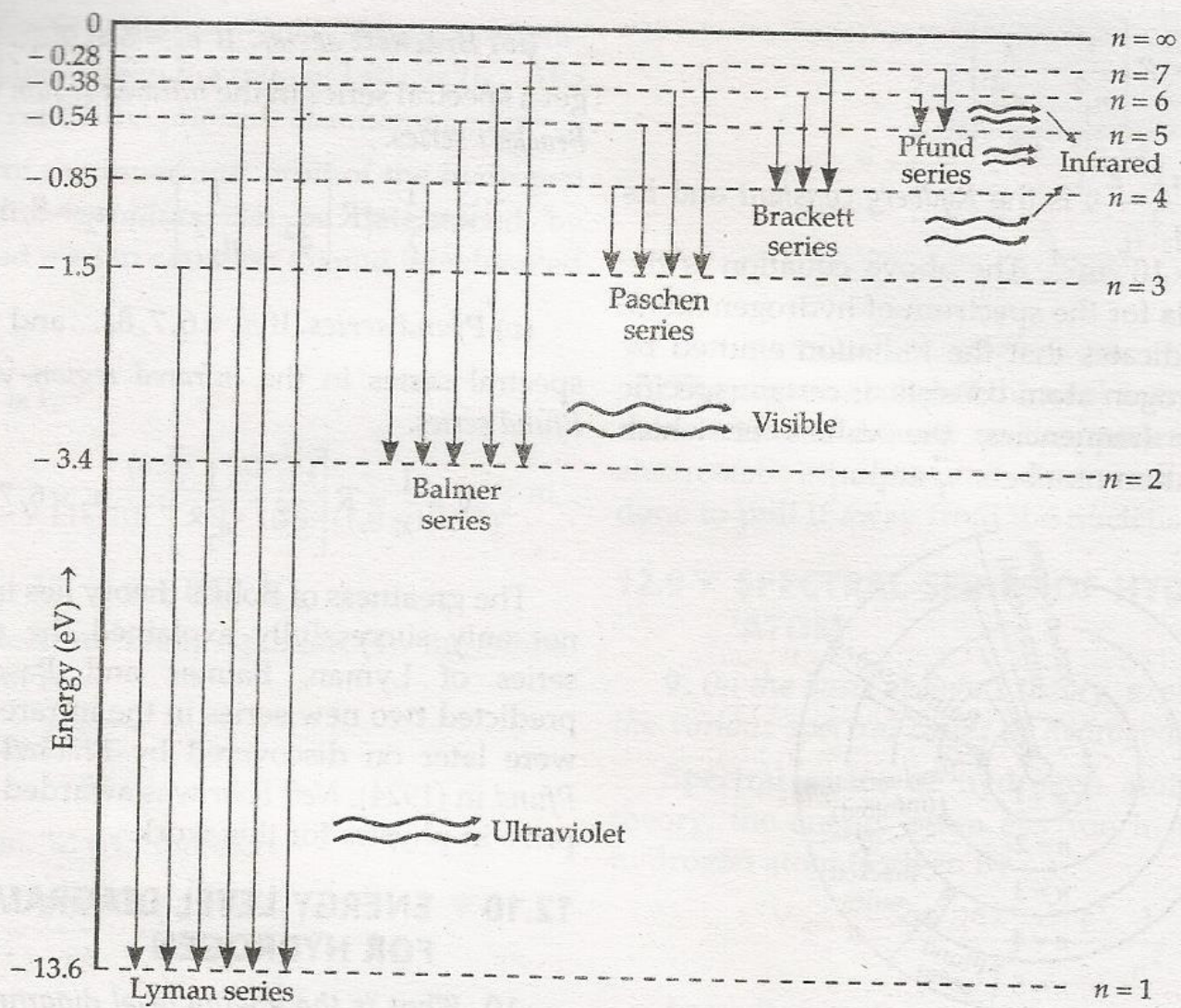


Fig. 12.12 Energy level diagram of hydrogen atom and its various spectral series.

Setting $n = 2, 3, 4$, etc., we get the energies for the higher orbits as follows :

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV} ;$$

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV} ;$$

$$E_4 = \frac{-13.6}{4^2} = -0.85 \text{ eV}$$

$$E_5 = \frac{-13.6}{5^2} = -0.54 \text{ eV}$$

$$E_6 = \frac{-13.6}{6^2} = -0.37 \text{ eV}$$

Clearly, an electron can have only certain definite values of energy while revolving in different orbits. This is called *energy quantisation*.

The energy state corresponding to $n = 1$, has the lowest energy equal to -13.6 eV . This state or orbit is called the *ground or normal state* of the atom. Normally the electron in the hydrogen atom remains in the ground state. When the hydrogen atom receives energy from outside by processes such as electron collisions, the electron may make transition to some higher energy state like E_2, E_3, E_4 , etc. which are called the *excited states*.

Figure 12.12 shows the energy level diagram of the hydrogen atom in which the energies of the various permitted orbits have been represented by parallel horizontal lines according to some energy scale. The principal quantum number n labels the energy states in the ascending order of energy.