# MATHEMATICS LEVEL IIC <br> Time- 60 minutes 

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding oval on the answer sheet.

Notes: (1) A calculator will be necessary for answering some (but not all) of the questions in this test. For each question you will have to decide whether or not you should use a calculator. The calculator you use must be at least a scientific calculator; programmable calculators and calculators that can display graphs are permitted.
(2) For some questions in this test you may have to decide whether youi calculator should be in the radian mode or the degree mode.
(3) Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.
(4) Unless otherwise specified, the domain of any function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
(5) Reference information that may be useful in answering the questions in this test can be found on the page preceding question 1.

USE THIS SPACE FOR SCRATCHWORK.

1. If $\sqrt[5]{4-3 x}=3$, what is the value of $x$ ?
(A) -82.33
(B) -79.67
(C) -1.75
(D) 0.92
(E) 82.33
2. If $f(a, b)=\frac{a+b}{2}$, which of the following is equal to $f(4,8)$ ?
(A) $f(0,6)$
(B) $f(1,6)$
(C) $f(2,4)$
(D) $f(2,16)$
(E) $f(3,9)$
3. $\frac{6!}{3!5!}=$
(A) 60
(B) 24
(C) 6
(D) 1
(E) $\frac{1}{60}$
4. The graph of which of the following equations has a slope of $\frac{1}{2}$ ?
(A) $y=\frac{1}{2}$
(B) $y=2 x$
(C) $y=2 x+1$
(D) $y=x+\frac{1}{2}$
(E) $y=\frac{x}{2}+1$
5. If $f(x)=x+\sqrt{x}$ and $g(x)=f(f(x))$, then $g(1.7)=$
(A) 1.7
(B) 3.0
(C) 4.7
(D) 6.9
(E) 9.0

## MATHEMATICS LEVEL IIC TEST—Continued

6. For all $m \neq 0, \frac{1-\frac{1}{m}}{\frac{1}{m}}=$
(A) 1
(B) $m-1$
(C) $\frac{m-1}{m}$
(D) $\frac{1-m}{m}$
(E) $m-\frac{1}{m}$
7. The graph of $y=b x-1$ has points in the first quadrant if and only if
(A) $b \neq 0$
(B) $b<-1$
(C) $-1<b<1$
(D) $0<b<1$
(E) $b>0$
8. If $\tan x=5$, then $\frac{\tan x}{\cot x}=$
(A) $\frac{1}{5}$
(B) 1
(C) 5
(D) 10
(E) 25
9. If $\frac{a+b c}{w e+f}=g$ and if $e f g \neq 0$, which of the following is equal to $w$ ?
(A) $\frac{a+\dot{b} c-f g}{e g}$
(B) $\frac{a+b c-g}{e}$
(C) $\frac{a-b c+f g}{e g}$
(D) $\frac{a+b c-f}{e g}$
(E) $\frac{a+b c-e g}{f g}$
10. If the probability of a certain event occurring is $\frac{4}{9}$, what is the probability of this event not occurring?
(A) $\frac{4}{13}$
(B) $\frac{4}{9}$
(C) $\frac{5}{9}$
(D) $\frac{9}{13}$
(E) $\frac{9}{4}$
11. If $x^{4}-19=19$ and $x \geq 0$, then $x=$
(A) 0
(B) 2.08
(C) 2.48
(D) 4.36
(E) 6.16

## MATHEMATICS LEVEL IIC TEST—Continued

12. In Figure 1, if $\theta=38^{\circ}$, what is the value of $t$ ?
(A) 0.15
(B) 0.20
(C) 2.46
(D) 3.13
(E) 3.15

USE THIS SPACE FOR SCRATCHWORK.


Figure 1
Note: Figure not drawn to scale.
13. Joe has a test average of 87 in mathematics. If his test average makes up 70 percent of his overall grade and the final exam makes up the remaining 30 percent, what must be his final exam score to give him an overall grade of exactly 90 ?
(A) 91
(B) 93
(C) 95
(D) 97
(E) 99
14. An operation is defined on pairs of integers by
$(a, b) \nabla(c, e)=(a-c, b-e)$. If
$[(1,2) \nabla(-3,6)] \nabla(x, y)=(1,1)$, then $(x, y)=$
(A) $(5,-5)$
(B) $(3,-7)$
(C) $(3,-5)$
(D) $(3,5)$
(E) $(-3,5)$

## MATHEMATICS LEVEL IIC TEST-Continued

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15. If $\cos t=\frac{5}{6}$. what is the value of $\cos 2 t$ ?
(A) 0.72
(B) 0.39
(C) 0.28
(D) 0.15
(E) -0.83
16. Which of the following is a zero of $f(x)=2 x^{2}-3 x-1$ ?
(A) -1.00
(B) 0.28
(C) 0.50
(D) 1.78
(E) 3.56
17. What is the number of digits in the number obtained by multiplying $12,121,212$ by 3,579 ?
(A) 4
(B) 9
(C) 10
(D) 11
(E) 12

# MATHEMATICS LEVEL IIG TEST—Continued 

18. If $\log _{x} 3=9$, then $x=$
(A) 0.50
(B) 1.13
(C) 1.22
(D) 2.00
(E) 2.08
19. Which of the following is a point at which the ellipse $\frac{x^{2}}{5}+\frac{y^{2}}{15}=1$ intersects the $x$-axis ?
(A) $(2.2,0)$
(B) $(3.9,0)$
(C) $(4.5,0)$
(D) $(5.0,0)$
(E) $(15.0 .0)$
20. The function $f$ is given by $f(x)=x-[x]$, where $[x]$ is defined to be the greatest integer that is less than or equal to $x$. If $1 \leq x<2$, then $f$ is also given by $f(x)=$
(A) $x-2$
(B) $x-1$
(C) $x$
(D) $x+1$
(E) $x+2$

## MATHEMATICS LEVEL IIC TEST—Continued

## USE THIS SPACE FOR SCRATCHWORK.

21. In Figure 2, $r \sin \theta=$
(A) $x$
(B) $y$
(C) $\frac{x}{y}$
(D) $\frac{y}{x}$
(E) $x+y$


Figure 2
22. What is the remainder when $2 x^{4}-3 x^{2}-x+3$ is divided by $x+1$ ?
(A) -3
(B) -1
(C) 1
(D) 2
(E) 3
23. In Figure 3, what is the length of segment $A C$ ?
(A) 4.47
(B) 5.00
(C) 5.39
(D) 6.23
(E) 9.00


Figure 3

## MATHEMATICS LEVEL IIC TEST-Continued

24. What is a value of $\cos (\arcsin 0.90)$ ?
(A) 0.44
(B) 0.58
(C) 0.67
(D) 0.71
(E) 0.90
25. What is the area of a triangle whose vertices are $(\sqrt{2}, 0)$, $(2, \sqrt{10})$, and $(5,0)$ ?
(A) 3.59
(B) 5.67
(C) 7.91
(D) 11.18
(E) 11.34
26. If $x=\sqrt{t}-1$ and $y=t^{2}$, what is $y$ in terms of $x$ ?
(A) $(x+1)^{4}$
(B) $(x-1)^{4}$
(C) $(x+1)^{2}$
(D) $(x-1)^{2}$
(E) $x^{2}+1$

## MATHEMATIGS LEVEL IIC TEST—Continued

USE THIS SPACE FOR SCRATCHWORK.
27. What is the maximum value of $f(x)=4-(x-1)^{2}$ ?
(A) 1
(B) 3
(C) 4
(D) 5
(E) 16
28. If a certain product now worth $\$ 450$ increases in value at the rate of 8 percent per year, how much will it be worth 6 years from now?
(A) $\$ 630$
(B) $\$ 661$
(C) $\$ 666$
(D) $\$ 714$
(E) $\$ 771$
29. The lst term of an arithmetic sequence is 3 and the 5th term is 17 . What is the 150 th term of the sequence?
(A) 420.2
(B) 521.5
(C) 524.5
(D) 528.0
(E) 698.3

## MATHEMATICS LEVEL IIC TEST—Continued

30. The cosine of an angle is one-half the sine of the same angle. What is the tangent of this angle?
(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) It cannot be determined from the information given.
31. The graph in Figure 4 could be a portion of the graph of which of the following functions?
I. $f(x)=x^{3}+a x^{2}+b x+c$
II. $g(x)=x^{5}+a x^{3}+b x+c$
III. $h(x)=x^{7}+a x^{6}+b x^{5}+c x^{4}+d x^{3}+e x^{2}+f x+g$
(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III


Figure 4
32. A right circular cylinder has radius 3 and height 3 . If $A$ and $B$ are two points on its surface, what is the maximum straight-line distance between $A$ and $B$ ?
(A) $3 \sqrt{6}$
(B) $3 \sqrt{5}$
(C) 6
(D) $3 \sqrt{3}$
(E) $3 \sqrt{2}$
33. What is the degree measure of the smallest positive angle $\theta$ for which $6 \sin ^{2} \theta-\sin \theta-2=0$ ?
(A) $9.6^{\circ}$
(B) $19.5^{\circ}$
(C) $30^{\circ}$
(D) $41.8^{\circ}$
(E) $90^{\circ}$
34. The graph of $x^{2}-y^{2}-2 x-4 y-4=0$ is a hyperbola centered at
(A) $(-1,-2)$
(B) $(-1,2)$
(C) $(1,-2)$
(D) $(1,2)$
(E) $(2,1)$
35. Which of the following could be a portion of the graph of $f(x)=\frac{e^{x}+e^{-x}}{2}$ ?
(A)

(B)

(C)

(D)

(E)


## MATHEMATICS LEVEL IIC TEST—Continued

36. If $\frac{p}{r}$ is an integer, which of the following must also be an integer?
(A) $p-r$
(B) $p+2 r$
(C) $\frac{r}{p}$
(D) $p r$
(E) $\frac{2 p}{r}$
37. A function $f$ has the property that whenever $x_{2}>x_{1}$, then $f\left(x_{2}\right) \geq f\left(x_{1}\right)$. Which of the following could be the graph of $f$ ?
(A)

(B)

(C)

(D)

(E)


## MATHEMATICS LEVEL IIC TEST-Continued

38. The two circles $x^{2}+y^{2}=1$ and
$(x-\sqrt{2})^{2}+(y \cdots \sqrt{2})^{2}=1$ are tangent to each other.
What are the coordinates of the point of tangency?
(A) $(0,0.71)$
(B) $(0.5,0.5)$
(C) $(0.71,0)$
(D) $(0.71,0.71)$
(E) $(1.41,1.41)$
39. What is $\lim _{x \rightarrow-2} \frac{2 x^{2}+3 x-2}{x^{2}-4}$ ?
(A) 1.25
(B) 1.0
(C) 0.5
(D) 0
(E) The limit does not exist.
40. A function $f$ is an even function if, for all values of $x$ in the domain, $f(-x)=f(x)$. Which of the following is an even function?
(A) $f(x)=2^{x}$
(B) $f(x)=x^{2}+x$
(C) $f(x)=x$
(D) $f(x)=\sin x$
(E) $f(x)=\cos x$

## MATHEMATICS LEVEL IIC TEST-Continued

41. Two cars start from the same point $P$ and travel along separate straight highways. If these two highways originate at $P$, forming an angle of $80^{\circ}$, how many miles apart are the two cars after each has traveled 110 miles?
(A) 86
(B) 141
(C) 156
(D) 191
(E) 220
42. The shaded portion in Figure 5 shows the graph of
(A) $\left(y-\frac{1}{2} x\right)(y+x) \geq 0$
(B) $(y-2 x)(y+x) \leq 0$
(C) $(y-2 x)(y+x) \geq 0$
(D) $(y+2 x)(y-x) \leq 0$
(E) $(y+2 x)(y-x) \geq 0$


Figure 5
43. If $f(n)=\frac{1}{e^{n}}$, what is the least integer $n$ such that $f(n)<0.0001 ?$
(A) 9
(B) 10
(C) 11
(D) 12
(E) 13

## MATHEMATICS LEVEL IIC TEST—Continued

USE THIS SPACE FOR SCRATCHWORK.
44. In right $\triangle A B C$ in Figure $6, \frac{\sin A+\cos B}{\cos B}$ is equal to which of the following?
(A) 2
(B) $\frac{a+c}{c}$
(C) $\frac{2 a}{b}$
(D) $\frac{2 b}{c}$
(E) $\frac{2 a}{c}$


Figure 6
45. What is the volume, in cubic centimeters, of a rectangular solid that has faces with areas 2,4 , and 8 square centimeters?
(A) 128
(B) 64
(C) 32
(D) 16
(E) 8
46. For every positive number $t$, a function $f_{t}$ is defined by

$$
f_{t}(x)=\left\{\begin{array}{cl}
1, & x<0 \\
1-\left(\frac{1}{t}\right) x, & 0 \leq x \leq t \\
0, & x>t
\end{array}\right.
$$

If $t>5$, then $f_{t}(2)=$
(A) 0
(B) 1
(C) $\frac{5-t}{t}$
(D) $\frac{t+5}{t}$
(E) $\frac{t-2}{t}$
47. In Figure 7, $A B C D E$ is áregular pentagon with side of length 2 . What is the $x$-coordinate of $D$ ?
(A) 2.62
(B) 3.62
(C) 3.73
(D) 3.90
(E) 4.90


Figure 7

## MATHEMATICS LEVEL IIC TEST—Continued

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48. If $f$ is the function with domain $[0,12]$ and range $[0,1]$ whose graph is the line segment shown in Figure 8, what is $f^{-1}(0.4)$ ?
(A) 30
(B) 4.8
(C) 2.5
(D) 0.25
(E) 0.033
49. What does $|3+5 i|$ equal ?
(A) 0.80
(B) 1.67
(C) 3.97
(D) 5.83
(E) 8.00
50. A committee of 3 mathematics majors and 4 history majors is to be chosen from a group of 20 mathematics majors and 16 history majors, respectively. How many different committees can be formed?
(A) 12
(B) 320
(C) 2,960
(D) $2,074,800$
(E) $2.86 \times 10^{15}$


Figure 8

| (6/93) |  |
| :---: | :---: |
| 1. B | 31. D |
| 2. E | 32. B |
| 3. D | 33. D |
| 4. E | 34. C |
| 5. C | 35. A |
| 6. B | 36. E |
| 7. E | 37. C |
| 8. E | 38. D |
| 9. A | 39. A |
| 10. C | 40. E |
| 11. C | 41. B |
| 12. C | 42. C |
| 13. D | 43. B |
| 14. C | 44. A |
| 15. B | 45. E |
| 16. D | 46. E |
| 17. D | 47. B |
| 18. B | 48. B |
| 19. A | 49. D |
| 20. B | 50. D |
| 21. B |  |
| 22. E |  |
| 23. C |  |
| 24. A |  |
| 25. B |  |
| 26. A |  |
| 27. C |  |
| 28. D |  |
| 29. C |  |
| 30. D |  |

