6.15 MUTUAL INDUCTION

16. *What is meant by mutual induction ? Define the term mutual inductance. Give its units and dimensions.*

Mutual induction. *Mutual induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil.*

As shown in Fig. 6.26, consider two coils *P* and *S* placed close to each other. The coil *P* is connected in series to a battery *B* and a rheostat Rh through a tapping key *K*. The coil *S* is connected to a galvanometer *G*. When a current flows through coil *P*, it produces a magnetic field which produces a magnetic flux through coil *S*. If the current in the coil *P* is varied, the magnetic flux linked with the coil *S* changes which induces an emf and hence a current in it, as is seen from the deflection in the galvanometer. The coil *P* is called the *primary coil* and coil *S*, the *secondary coil*, because it is the former which causes an induced emf in the latter.

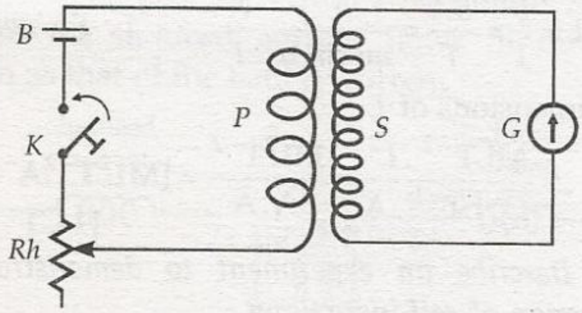

Fig. 6.26 Mutual induction.

Coefficient of mutual induction. At any instant,
Magnetic flux linked with the secondary coil
$\propto$ current in the primary coil

i.e. $\phi \propto I$

or $\phi = MI$ ...(1)

The proportionality constant *M* is called *the mutual inductance* or *coefficient of mutual induction* of the two coils. Any change in the current *I* sets up an induced emf in the secondary coil which is given by

$$\mathcal{E} = -\frac{d\phi}{dt} = -M.\frac{dI}{dt} \quad ...(2)$$

If in equation (1), $I = 1$, then $\phi = M$

Thus the mutual inductance of two coils is numerically equal to the magnetic flux linked with one coil when a unit current passes through the other coil.

Again, from equation (2), if

$$\frac{dI}{dt} = 1, \text{ then } \mathcal{E} = -M$$

The mutual inductance of two coils may be defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.

Mutual inductance of two long solenoids. As shown in Fig. 6.27, consider two long co-axial solenoids S_1 and S_2, with S_2 wound over S_1.

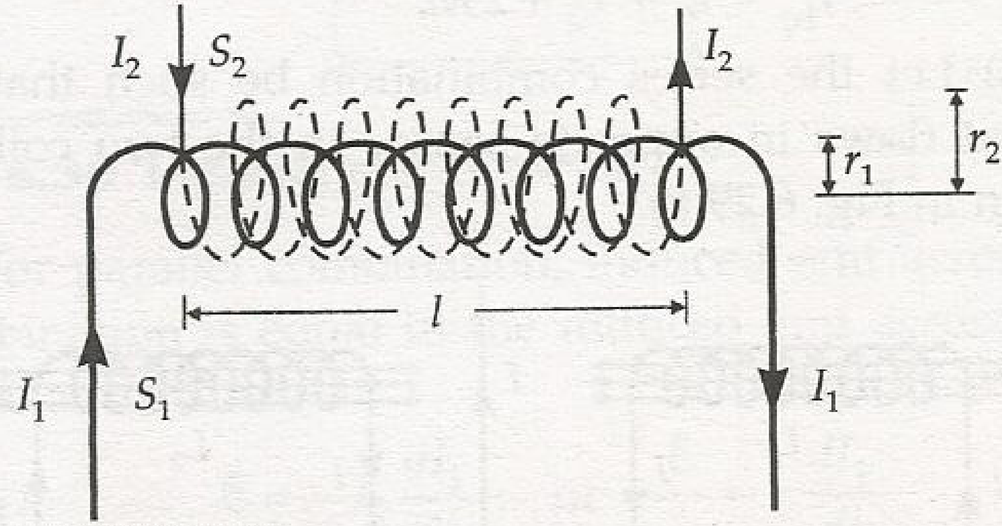

Fig. 6.27 Two long coaxial solenoids of same length *l*.

Let l = length of each solenoid

r_1, r_2 = radii of the two solenoids

$A = \pi r_1^2$

= area of cross-section of inner solenoid S_1

N_1, N_2 = number of turns in the two solenoids

First we pass a time varying current I_2 through S_2. The magnet field set up inside S_2 due to I_2 is

$$B_2 = \mu_0 n_2 I_2$$

where $n_2 = N_2 / l$ = the number of turns per unit length of S_2.

Total magnetic flux linked with the inner solenoid S_1 is

$$\phi_1 = B_2 A N_1 = \mu_0 n_2 I_2 \cdot A N_1$$

$\therefore$ Mutual inductance of coil 1 with respect to coil 2 is

$$M_{12} = \frac{\phi_1}{I_2} = \mu_0 n_2 A N_1 = \frac{\mu_0 N_1 N_2 A}{l}$$

We now consider the flux linked with the outer solenoid S_2 due to the current I_1 in the inner solenoid S_1. The field B_1 due to I_1 is constant inside S_1 but zero in the annular region between the two solenoids. Hence

$$B_1 = \mu_0 n_1 I_1$$

where $n_1 = N_1 / l =$ the number of turns per unit length of S_1.

Total flux linked with the outer solenoid S_2 is

$$\phi_2 = B_1 A N_2 = \mu_0 n_1 I_1 \cdot A N_2 = \frac{\mu_0 N_1 N_2 A I_1}{l}$$

$\therefore$ Mutual inductance of coil 2 with respect to coil 1 is

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Clearly $M_{12} = M_{21} = M$ (say)

$$\therefore \quad M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l = \mu_0 n_1 n_2 \pi r_1^2 l$$

Thus, the mutual inductance of two coils is the property of their combination. It does not matter which one of them functions as the primary or the secondary coil. This fact is known as ***reciprocity theorem***.

Self-induction. When a current flows in a coil, it gives rise to a magnetic flux through the coil itself. As the strength of current changes, the linked magnetic flux changes and an opposing emf is induced in the coil. This emf is called ***self-induced emf*** or ***back emf*** and the phenomenon is known as ***self-induction***.

Self-induction is the phenomenon of production of induced emf in a coil when a changing current passes through it.

Fig. 6.23(*a*) shows a battery and a tapping switch connected in series to a coil. As the switch is closed, the current increases and hence the magnetic flux through the coil increases from zero to a maximum value and the induced current flows in the opposite direction of the battery current. In Fig. 6.23(*b*), as the tapping switch is opened, the current and hence the magnetic flux through the coil decreases from a maximum value to zero and the induced current flows in the same direction as that of the battery current.

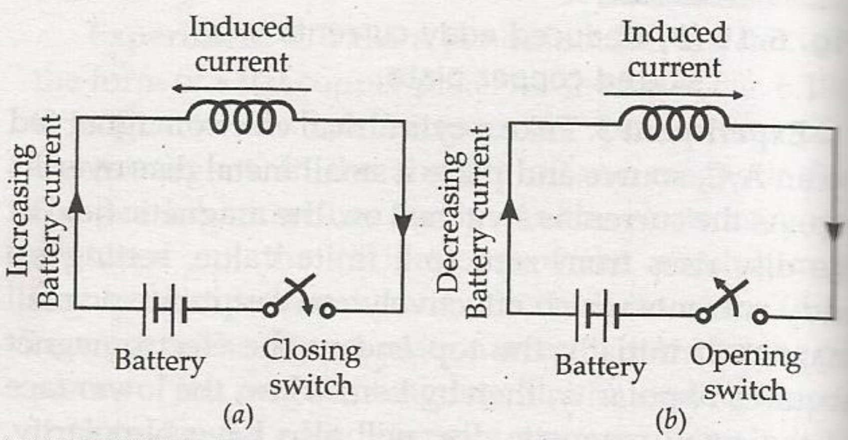

Fig. 6.23 Induced current in a coil when the circuit is (*a*) closed and (*b*) opened.

Coefficient of self-induction. At any instant, the magnetic flux ϕ linked with a coil is proportional to the current I through it, *i.e.*,

$$\phi \propto I$$

or
$$\phi = LI \qquad ...(1)$$

where L is a constant for the given coil and is called *self-inductance* or, more often, simply *inductance*. It is also called *coefficient of self-induction* of the coil. Any change in current sets up an induced emf in the coil given by

$$\mathcal{E} = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \qquad ...(2)$$

If in equation (1), $I = 1$, then $\phi = L$

Thus the self-inductance of a coil is numerically equal to the magnetic flux linked with the coil when a unit current flows through it.

Again from equation (2), if $\frac{dI}{dt} = 1$, then $\mathcal{E} = -L$

Thus the self-inductance of a coil may be defined as the induced emf set up in the coil due to a unit rate of change of current through it.

Units of self-inductance. From equation (2),

$$L = \frac{\mathcal{E}}{dI/dt}$$

$$\therefore \text{ SI unit of } L = \frac{1\text{V}}{1\text{ As}^{-1}} = 1\text{ VsA}^{-1} = 1\text{ henry (H)}$$

The *self-inductance of a coil is said to be one* **henry** *if an induced emf of one volt is set up in it when the current in it changes at the rate of one ampere per second.*

From equation (1), one may note that self-inductance is the ratio of magnetic flux and current. So its SI unit is *weber per ampere*. Hence

$$1\text{ henry (H)} = 1\text{ VsA}^{-1} = 1\text{ WbA}^{-1}$$

Dimensions of self-inductance. We know that

$$L = \frac{\phi}{I} = \frac{BA}{I} = \frac{F}{qv \sin\theta} \cdot \frac{A}{I} \qquad [\because F = qvB \sin\theta]$$

$\therefore$ Dimensions of L

$$= \frac{\text{MLT}^{-2} \cdot \text{L}^2}{\text{C} \cdot \text{LT}^{-1} \cdot \text{A}} = \frac{\text{ML}^2\text{T}^{-2}}{\text{A} \cdot \text{A}} = [\text{ML}^2\text{T}^{-2}\text{A}^{-2}].$$

$$[1\text{ CT}^{-1} = 1\text{ A}]$$

6.13 SELF-INDUCTANCE OF A LONG SOLENOID

14. *Derive an expression for the self-inductance of a long solenoid. State the factors on which the self-inductance of a coil depends.*

Self-inductance of a long solenoid. Consider a long solenoid of length l and radius r with $r << l$ and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by

$$B = \mu_0 nI$$

Magnetic flux linked with each turn

$$= BA = \mu_0 nIA$$

where $A = \pi r^2$ = the cross-sectional area of the solenoid.

∴ Magnetic flux linked with the entire solenoid is

ϕ = Flux linked with each turn × total number of turns

$$= \mu_0 nIA \times nl = \mu_0 n^2 lAI$$

But $\phi = LI$

∴ Self-inductance of the long solenoid is

$$L = \mu_0 n^2 lA.$$

If N is the total number of turns in the solenoid, then $n = N/l$ and so

$$L = \frac{\mu_0 N^2 A}{l}.$$

If the coil is wound over a material of high relative magnetic permeability μ_r (*e.g.*, soft iron), then

$$L = \mu_r \mu_0 n^2 lA = \frac{\mu_r \mu_0 N^2 A}{l}.$$

6.10 METHODS OF GENERATING INDUCED EMF

10. *Discuss the various methods of generating induced emf.*

Methods of generating induced emf. An induced emf can be produced by changing the magnetic flux linked with a circuit. The magnetic flux,

$$\phi = BA\cos\theta$$

can be changed by one of the following methods :

1. Changing the magnetic field B,
2. Changing the area A of the coil, and
3. Changing the relative orientation θ of B and A.

1. Induced emf by changing the magnetic field B. We have already learnt in section 6.3 how an induced emf is set up in a coil on changing the magnetic flux through it by (*i*) moving a magnet towards a stationary coil, (*ii*) moving a coil towards a stationary magnet and (*iii*) varying current in the neighbouring coil.

2. Induced emf by changing the area of the coil. Consider a conductor CD of length l moving with a velocity v towards right on U-shaped conducting rails situated in a magnetic field B, as shown in Fig 6.17. The field is uniform and points into the plane of the paper. As the conductor slides, the area of the circuit changes from $ABCD$ to $ABC'D'$ in time dt.

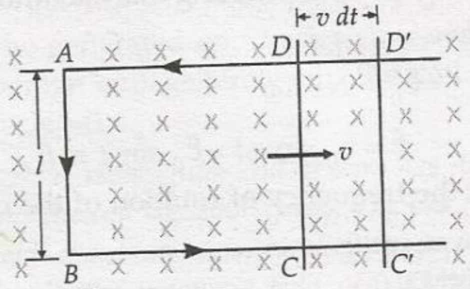

Fig. 6.17 Induced emf by changing area of the loop.

The increase in flux,

$$d\phi = B \times \text{change in area}$$
$$= B \times \text{area } CDD'C' = B \cdot l \cdot v dt$$

This sets up induced emf in the loop of magnitude,

$$|\varepsilon| = \frac{d\phi}{dt} = Blv$$

According to Fleming's right hand rule, the induced current flows in the anticlockwise direction.

3. Induced emf by changing relative orientation of the coil and the magnetic field : Theory of AC generator. Consider a coil $PQRS$ free to rotate in a uniform magnetic field $\vec{B}$. The axis of rotation of the coil is perpendicular to the field $\vec{B}$. The flux through the coil, when its normal makes an angle θ with the field, is given by

$\phi = BA \cos\theta$ where A is the face area of the coil.

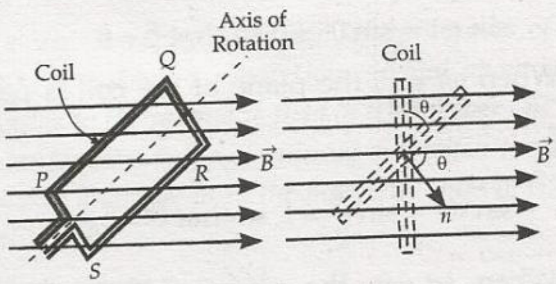

Fig. 6.18 (*a*) Rotating coil in a magnetic field.

If the coil rotates with an angular velocity ω and turns through an angle θ in time t, then

$$\theta = \omega t \quad \therefore \quad \phi = BA \cos \omega t$$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA \cos \omega t) = BA\omega \sin \omega t$$

If the coil has N turns, then the total induced emf will be

$$\varepsilon = NBA\,\omega \sin \omega t$$

Thus the induced emf varies sinusoidally with time t. The value of induced emf is maximum when $\sin \omega t = 1$ or $\omega t = 90°$, *i.e.*, when the plane of the coil is *parallel* to the field $\vec{B}$. Denoting this maximum value by $\mathcal{E}_0$, we have

$$\mathcal{E}_0 = NBA\omega$$

$$\therefore \qquad \mathcal{E} = \mathcal{E}_0 \sin \omega t = \mathcal{E}_0 \sin 2\pi ft$$

where f is the frequency of rotation of the coil.

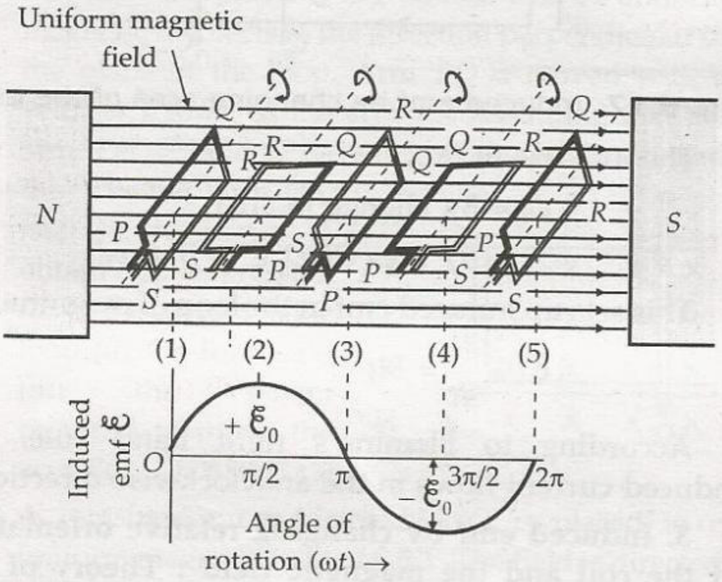

Fig. 6.18 (*b*) Induced emf in a rotating coil.

Fig 6.18(*b*) shows how the induced emf $\mathcal{E}$ between the two terminals of the coil varies with time. We consider the following special cases :

1. When $\omega t = 0°$, the plane of the coil is perpendicular to $\vec{B}$,

$$\sin \omega t = \sin 0° = 0 \text{ so that } \mathcal{E} = 0$$

2. When $\omega t = \frac{\pi}{2}$, the plane of the coil is parallel to field $\vec{B}$,

$$\sin \omega t = \sin \frac{\pi}{2} = 1, \text{ so that } \mathcal{E} = \mathcal{E}_0$$

3. When $\omega t = \pi$, the plane of the coil is again perpendicular to B,

$$\sin \omega t = \sin \pi = 0, \text{ so that } \mathcal{E} = 0.$$

4. When $\omega t = \frac{3\pi}{2}$ the plane of the coil is again parallel to $\vec{B}$,

$$\sin \omega t = \sin \frac{3\pi}{2} = -1 \text{ so that } \mathcal{E} = -\mathcal{E}_0$$

5. When $\omega t = 2\pi$, the plane of the coil again becomes perpendicular to $\vec{B}$ after completing one rotation,

$$\sin \omega t = \sin 2\pi = 0 \text{ so that } \mathcal{E} = 0$$

As the coil continues to rotate in the same sense, the same cycle of changes repeats again and again. As shown in Fig. 6.18(*b*), the graph between emf $\mathcal{E}$ and time t is a sine curve. Such an emf is called *sinusoidal* or *alternating emf.* Both the magnitude and direction of this emf change regularly with time.

The fact that an induced emf is set up in a coil when rotated in a magnetic field forms the basic principle of a dynamo or a generator.

6.6 MOTIONAL EMF FROM FARADAY'S LAWS

7. *What is motional emf ? Deduce an expression for the emf induced across the ends of a conductor moving in a perpendicular magnetic field.*

Motional emf from Faraday's law : Induced emf by change of area of the coil linked with the magnetic field. *The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf.* As shown in Fig. 6.8, consider a conductor PQ of length l free to move on U-shaped conducting rails situated in a uniform and time independent magnetic field B, directed normally into the plane of paper. The conductor PQ is moved inwards with a speed v. As the conductor slides towards left, the area of the rectangular loop $PQRS$ decreases. This decreases the magnetic flux linked with the closed loop. Hence an emf is set up across the ends of conductor PQ because of which an induced current flows in the circuit along the path $PQRS$. The direction of induced current can by determined by using Fleming's right hand rule, stated in the next section.

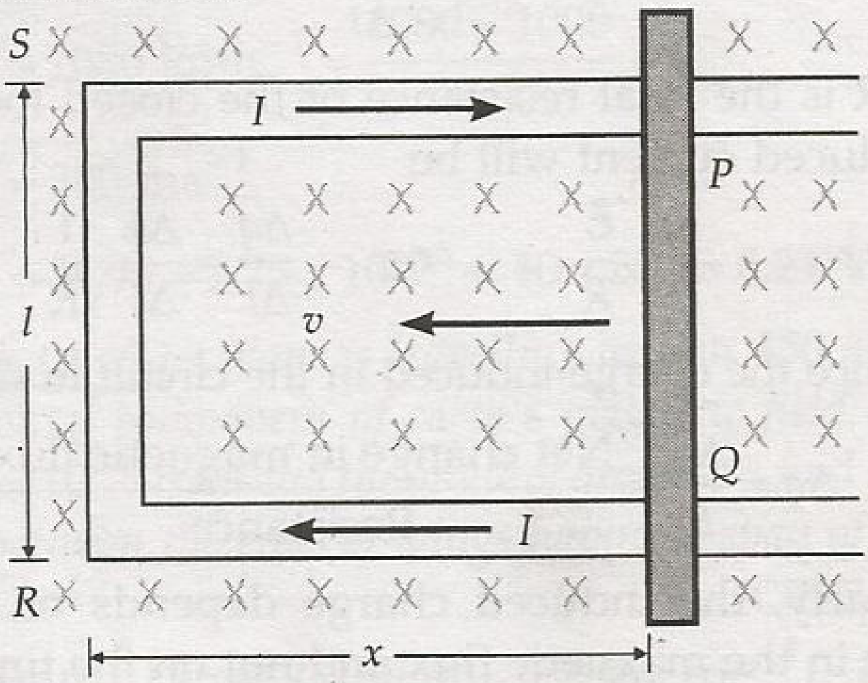

Fig. 6.8 Induced current by changing area of the rectangular loop.

Suppose a length x of the loop lies inside the magnetic field at any instant of time t. Then the magnetic flux linked with the rectangular loop $PQRS$ is

$$\phi = BA = Blx$$

According to Faraday's law of electromagnetic induction, the induced emf is

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

or $$\mathcal{E} = Blv$$

where $dx/dt = -v$, because the velocity v is in the decreasing direction of x. The induced emf Blv is called ***motional emf*** because this *emf is induced due to the motion of a conductor in a magnetic field.*

Motional emf from Lorentz force. A conductor has a large number of free electrons. When it moves through a magnetic field, a Lorentz force acting on the free electrons can set up a current. Fig. 6.10 shows a rectangular conductor in which arm *PQ* is free to move. It is placed in a uniform magnetic field *B*, directed normally into the plane of paper. As the arm *PQ* is moved towards left with a speed *v*, the free

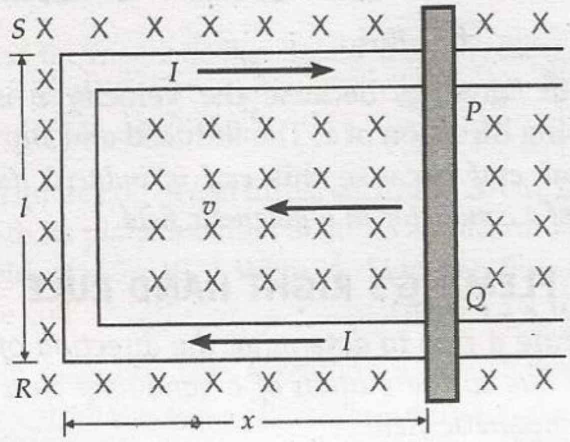

Fig. 6.10 Motional emf.

electrons of *PQ* also move with the same speed towards left. The electrons experience a magnetic Lorentz force, $F_m = qvB$. According to Fleming's left hand rule, this force acts in the direction *QP* and hence the free electrons will move towards *P*. A negative charge accumulates at *P* and a positive charge at *Q*. An electric field *E* is set up in the conductor from *Q* to *P*. This field exerts a force, $F_e = qE$ on the free electrons. The accumulation of charges at the two ends continues till these two forces balance each other, *i.e.*,

$$F_m = F_e$$

or $$qvB = qE \quad \text{or} \quad vB = E$$

The potential difference between the ends *Q* and *P* is

$$V = El = vBl$$

Clearly, it is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf,

$$\varepsilon = Blv$$

As this emf is produced due to the motion of a conductor, so it is called a ***motional emf***.

Current induced in the loop. Let *R* be the resistance of the movable arm *PQ* of the rectangular loop *PQRS* shown in Fig. 6.10. Suppose the total resis- tance of the remaining arms *QR*, *RS* and *SP* is negligible compared to *R*. Then the current in the loop will be

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$$

Force on the movable arm. The conductor *PQ* of length *l* and carrying current *I* experiences force *F* in the perpendicular magnetic field *B*.

The force is given by

$$F = IlB \sin 90° = \left(\frac{Blv}{R}\right) lB = \frac{B^2 l^2 v}{R}$$

This force (due to induced current) acts in the outward direction opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.

Power delivered by the external force. The power supplied by the external force to maintain the motion of the movable arm is

$$P = Fv = \frac{B^2 l^2 v^2}{R}$$

Power dissipated as Joule loss. The power dissipated in the loop as Joule heating loss is

$$P_J = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}.$$

6.4 LAWS OF ELECTROMAGNETIC INDUCTION

4. *State the laws of electromagnetic induction. Express these laws mathematically.*

Laws of electromagnetic induction. There are two types of laws which govern the phenomenon of electromagnetic induction :

A. Faraday's laws which give us the magnitude of induced emf.

B. Lenz's law which gives us the direction of induced emf.

A. Faraday's laws of electromagnetic induction : These can be stated as follows :

First law. *Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.*

Second law. *The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. Mathematically,*

$$|\mathcal{E}| = \frac{d\phi}{dt}$$

B. Lenz's law : *This law states that the direction of induced current is such that it opposes the cause which produces it, i.e., it opposes the change in magnetic flux.*

Mathematical form of the laws of electromagnetic induction : Expression for induced emf. According to the Faraday's flux rule, Magnitude of induced emf

= Rate of change of magnetic flux

or $$|\mathcal{E}| = \frac{d\phi}{dt}$$

Taking into account Lenz's rule for the direction of induced emf, Faraday's law takes the form :

$$\mathcal{E} = -\frac{d\phi}{dt}$$

The negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux.

If the coil consists of N tightly wound turns, then the emfs developed in all these turns will be equal and in the same direction and hence get added up. Total induced emf will be

$$\mathcal{E} = -N\frac{d\phi}{dt}$$

If the flux changes from ϕ_1 to ϕ_2 in time t, then the average induced emf will be

$$\mathcal{E} = -N\frac{\phi_2 - \phi_1}{t}$$

If ϕ is in webers and t in seconds, then $\mathcal{E}$ will be in volts.