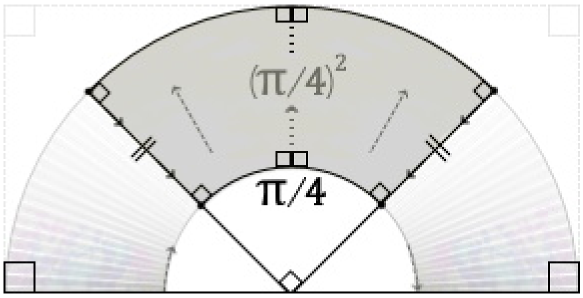

On the Use of an Inverse Square to Solve for π *with Exactitude*

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Abstraction:

By way of *challenging* Archimedes' *basic underlying assumptions,* this paper *falsifies* any & all *assumed* numerical integrity(s) in & of the hitherto endorsed approximation of π as 3.14159... This paper further hypothesizes the *unsolved status* of the *Riemann Hypothesis* "problem" to be owing to a *numerically deficient circle constant* following a discovery of a ' ***Blunder of Millennia*** '.

Admittedly, since *at least* the time of Archimedes, mathematicians have been iteratively & exhaustively *approximating* the circumference of a circle by way of use of *polygons* (of ever-increasing sides n*)*:

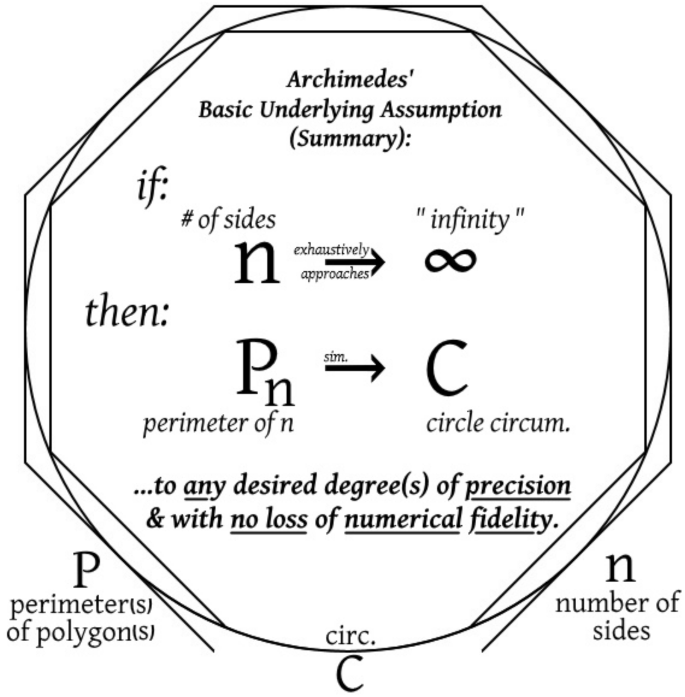


Figure 1.

...all while/as *assuming* such a comparison was & is ever a *valid* one *in the first place*.

Specifically, mathematicians have *long* assumed if & as these polygons' sides ever-approach " infinity ", their perimeters practically describe a circumference of a circle to *any desired degree(s) of precision* & with *no loss* of *numerical fidelity* whatsoever.

Has this *assumption* ever been *questioned* by mathematicians? If not, *why not?* For should it be *false:* Archimedes' method would result in a *mere* ongoing *belief* the numeric approximation & decimation 3.14159265358979...*etc.* if even carried out to *trillions* of decimal places... is *inerrantly* describing a one-and-only *curvature constant* as *it* observes a constant radius of precisely 0.5000...*etc.* with 0.000...*etc.* margin of error. *This*, instead of it otherwise describing two *relatively unlike* polygons' *mere approximation* of one (& whose *fidelity* both *begins* & *ends* with the very same *lack* of fidelity *Archimedes himself* began with.)

Importantly, as a *matter* of *principle:* if such comparison(s) of a polygon(s) to a circle happens to be *false one(s)...* it would so follow that any/all upper- and lower-bounds calculated *using them* (incl. Archimedes') would *likewise* be... *false* ones.

As it happens, according to the recent use of an inverse square to *precisely* calculate pi, Archimedes' approach (& all subsequent polygonal *folly* following it) fundamentally underlies what can only be described as an ongoing "**Blunder of Millennia** . Unlike Archimedes' approach, *this* approach *solves* for pi *with exactitude* (& is herein, to follow.)

First, instead of beginning with Archimedes' initial assumption, we begin with an even more basic one: human beings (*incl.* Archimedes) are *fallible* & tend to *err*. It is therefore necessary (should that be the case here) to *suspend* (if even temporarily, though *entirely*) any & all hitherto taken-to-be-true notions concerning pi as a consequence of Archimedes' approach.

Specifically, these notions include (though are by no means limited to):

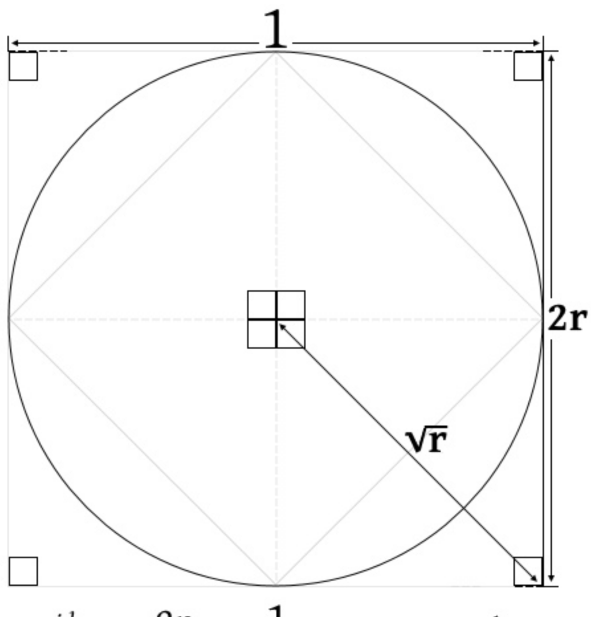
i. the circumference of a circle whose radius is 0.5000... is (approx.) 3.14159...
ii. the numerical value of pi is (therefore) approximately 3.14159..., and
iii. pi is "transcendental" as was "proven" by von Lindemann *et. al.* in 1882.

These "proven" notions could *otherwise* be proven *false* should it ever be the case any *real* circumference of any *real* circle is *verifiably not* 3.14159... *If* so, it would follow this number could *not* possibly *be* pi. *And*, if *that* is so, it would follow Lindemann *et. al.* did *not really* prove pi is transcendental, but rather only proved the approximated decimation of 3.14159... *is,* and this part of the result could remain undisputed. *But,* if pi were *another number entirely*, such as an *algebraic* ratio expressible in an exact form(s)... in *that* case, pi would *not* be transcendental.

It is important to be *conscious* of any (& *ideally* all) conditions under which an *assumption(s)*, *belief(s)* and/or *conclusion(s)* hitherto-taken-to-be *' true '* by *any* corpus of *consensus* can actually be proven to be *' false '* for: the recognition of a *false basic underlying assumption(s)* is (as) a catalytic *spark* preceding any & all possible *explosion(s)* of scientific *discovery* & *progress.*

We begin our own process of discovery by *questioning* Archimedes' use of polygons, asking: "*...on what <u>foundation</u> (if any)* did Archimedes *assume* by iteratively & exhaustively increasing inscribed & circumscribed polygons' sides, their perimeters should reconcile the circumference of a circle without loss of *numerical fidelity*? *Moreover*, concerning a calculation of pi: is there *not* a *better way(s)?"*

We continue by allowing the possibility Archimedes *made a mistake(s)* owing to one or more of his assumptions, so we *reject* Archimedes' approach *entirely* & try a new one by both inscribing & circumscribing a circle of known radius with a *perfect square(s)* and comparing them on a <u>foundation</u> of *four common right angles*. In doing so, unlike Archimedes' approach, *this* approach satisfies the condition of a *like-for-like* comparison: both circle *and* square commonly observe one anothers' four commonly contained right angles *viz.*



$$\therefore \ \frac{\textit{side}}{\textit{diag.}} = \frac{2r}{2\sqrt{r}} = \frac{1}{\sqrt{2}} = \sqrt{r} \quad \textit{obs.} \quad r = \frac{1}{2}$$

∴ the ratio of a squared unit side/diagonal is EQUAL to a RECIPROCAL of the diagonal of the unit (itself).

Figure 2.

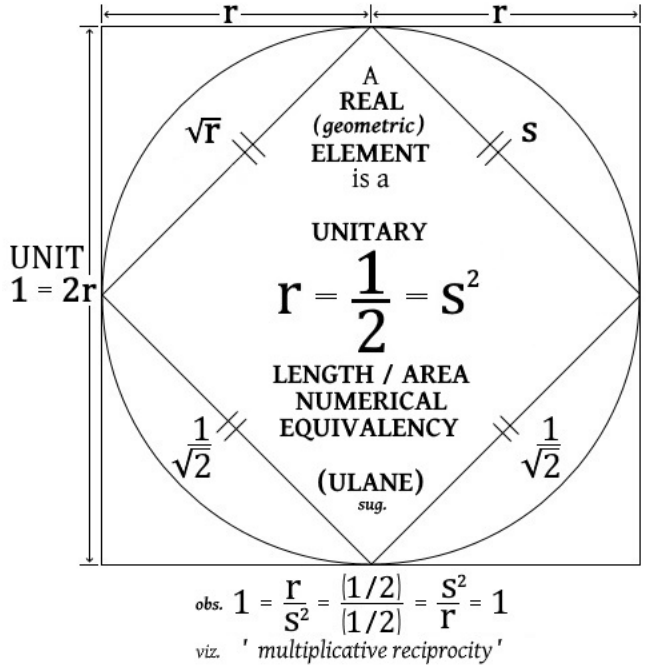


Figure 3.

In/at the unit square, an admittedly coined (though otherwise practically important) *numerical agreement(s)* is established: a ***Unitary Length / Area Numerical Equivalency*** **(ULANE)** permitting two offset dimensions r = 1/2 *and* 1/2 = s^2 to be *simultaneously numerically equivalent. Accordingly*: the length of the radius is *numerially equivalent to* the area of the square inscribed therein. This important numerical agreement between two sequentially offset dimensions was/is *overlooked* by Archimedes' approach *entirely.* The equivalency implies the *numerical integrity* of the radius is *bound to* a *real geometric area* of a *real geometric square* the circle *geometrically circumscribes* & thus *areal truncation(s)* of the square cause *linear truncation(s)* of the radius (and/or *vice versa.*)

We otherwise observe an ordinary unit square whose perimeter is 4(2r) = 8r *units* and whose area is $(2r)^2 = 4r^2 = 1$ *units squared* while/as unitarily observing r = 1/2 as the radius of the circle inscribed therein (with the area of the circle relatively $a = \pi r^2 = \pi/4$ units *squared.*) We find the circle circumscribing square $s^2 = 1/2$ whose area is precisely *half* that of the larger unit square.

What we are looking for is a *better way(s)* to calculate the area and/or circumference of the circle *without* a priori *assumption(s), belief(s) and/or conclusion(s)* the number 3.14159... (as arrived at by Archimedean approximation methodologies) is numerically *correct* up-to any & *all* decimal places exhaustively generated *by way of* them.

The *scientific method* (incl. any & all practical applications of it) is *not* engineered to prove any theory true with any *absolutely certainty*, but rather *conversely*: it is meant to *falsify* any & all theories unsupported by *observed* phenomena. We therefore proceed with a *motive* of *real science*: to *explore* & *discover* new *possibilities;* a *new way(s)* of *looking* at things; a new way(s) to *calculate* without the many headaches & traumas induced by trigonometry, calculus, endless iteration(s), approximation(s) and/or invocation(s) of "infinity" *etc.* We *progress* by incessantly & unreservedly *challenging basic underlying assumptions, beliefs & conclusions*.

Recall the radius of the concerned circle is r = 1/2 *units* & find its area a = π/4 *units squared* to be contained by a relative circumference 4a = π *unit length.* These *units* are strictly *numerical:* they primarily *precede any & all* real-world units of measure, as example(s): 1 unit may be one *inch*, one *yard*, one *light-second etc.* The underlying numerical ratios *themselves* imply any & all circles are accordingly *self-referential:* circumferential *length 4a units* surrounds *area a units squared viz.*

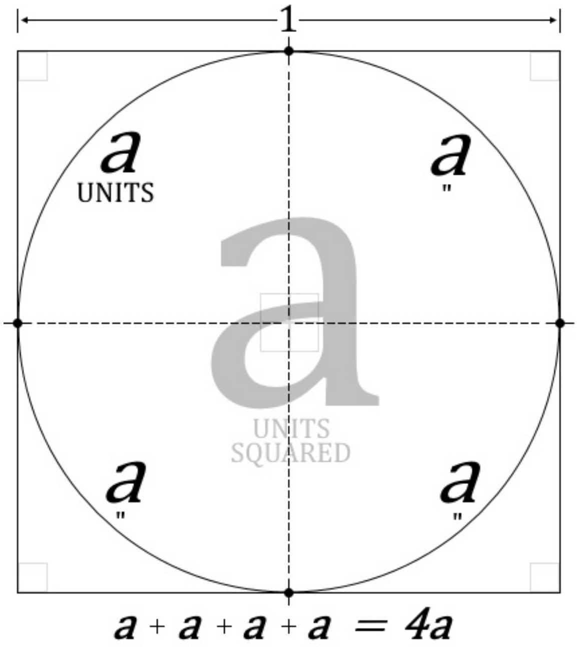


Figure 4.

Instead of inscribing & circumscribing the circle with polygons and risking a blunder of an *unlike* comparison(s), here we have a circular *squared area* a contained by a relative circumferential *length* of 4a. Unlike the polygonal approach, the circumference of the circle *precisely* contains $a = \pi r^2 = \pi/4$ *units squared* (with *units* and units *squared* being *sequentially offset.*)

If not immediately obvious: a *quadrant symmetry* between an axially coincident concentric circle & square naturally occurs & this, too, is catastrophically *overlooked* by Archimedes' approach. This symmetry happens to be an important one as it can & *should* be used to calculate the only possible numerical value of (a) *permitting* such a self-referential relation *in the first place*.

Given the r = 1/2 circle, if we also surround it with *another* squared area equal to $4a = \pi$ *units squared* (thus constructing *π's annulus*), we can then use an *inverse square* operation on the *width* of *π's* annulus to calculate the *only possible* (a) *permitting it* to *simultaneously* be:

i. a circular squared *area* a in *units squared,* while/as
ii. relatively contained by *length* 4a in *units,* while/as
iii. surrounded by an annulus of *area* 4a *units squared,* while/as
iv. *unitarily* observing a radial length r of *precisely* 1/2 = 0.5000...

We may recall the area equation of an annulus $\pi(R^2 - r^2)$ wherein R is the ' major ' of two radii (hence *majora*) and r is the other ' minor ' (hence *minora)*. While/as we do not yet have a *numerical* value for π, we *do* know it must be $4a = \pi$ while/as observing r = 1/2 & we know the latter is the *minora* of the annulus:

$$4a(R^2 - r^2) = 4a$$
$$4a(R^2 - (1/2)^2) = 4a$$

To surround the r = 1/2 circle with an *annulus* containing 4a *units squared*, a second circle must uniformly expand from the first until capuring this 4a *between* the two circles *viz.*

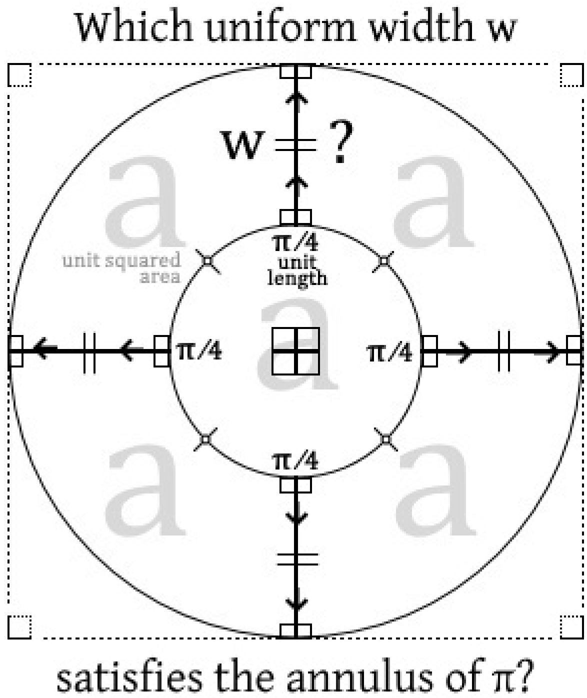


Figure 5.

What we do not *yet* know is the radial length of the other: the m*ajora* R, to contain 4a between it & the *minora*. We therefore substitute in the *knowns* and calculate the *unknown.*

Given:
$4a(R^2 - r^2)$
obs. $r = 1/2$

Equate to 4a:
$4a(R^2 - r^2) = 4a$

Substitute *r* with the known (1/2):
$4a(R^2 - (1/2)^2) = 4a$

Solve for the unknown (R):
$(R^2 - (1/2)^2) = 4a / 4a = 1$
$R^2 - 1/4 = 1$
$R^2 = (1 + 1/4) = 5/4$
$R = \sqrt{(1 + 1/4)}$
$R = \sqrt{5}/2$

Verify:
$4a\,((\sqrt{5}/2)^2 - (1/2)^2) = 4a$
$4a\,(5/4 - 1/4) = 4a$
$\therefore 4a = 4a$

∴ an annulus'
majora R = **√5/2** & minora r = **1/2**
contains an area of *precisely* π = 4a *units squared*
whose radii *sum* to the (so-called) *' golden ratio ' phi*
(thus *disc.* a *geometric relationship* between *phi* and *pi.*)

Archimedes' approach *catastrophically* overlooks an *extremely important* geometric relationship between the so-called golden ratio *phi* and *pi*. Suffice it to say: there is an entire landscape of scientifically *unexplored terrain* concerning such a communion of constants.

The circle equations of the Major and Minor radii circles $R = \sqrt{5}/2$ and $r = 1/2$ are *resp.*:

$$\text{Majora} = x^2 + y^2 = 5/4$$
$$\text{Minora} = x^2 + y^2 = 1/4$$
$$\therefore\ \text{Majora} - \text{Minora} = 1$$

Each circle equation is numerically equal to the *square* of R and r resp. The difference between them is predictably *equal* to a difference of *sequential squares* whose *principle* (p) contains *both* the concerned minora & majora lengths:

$$p = (r \pm R)$$
$$p = (1/2 \pm \sqrt{5}/2)$$
$$\textit{obs.}\ \ p^2 - p = 1.000\ldots$$

We plot these two circles concentrically to compare their *relative areas*:

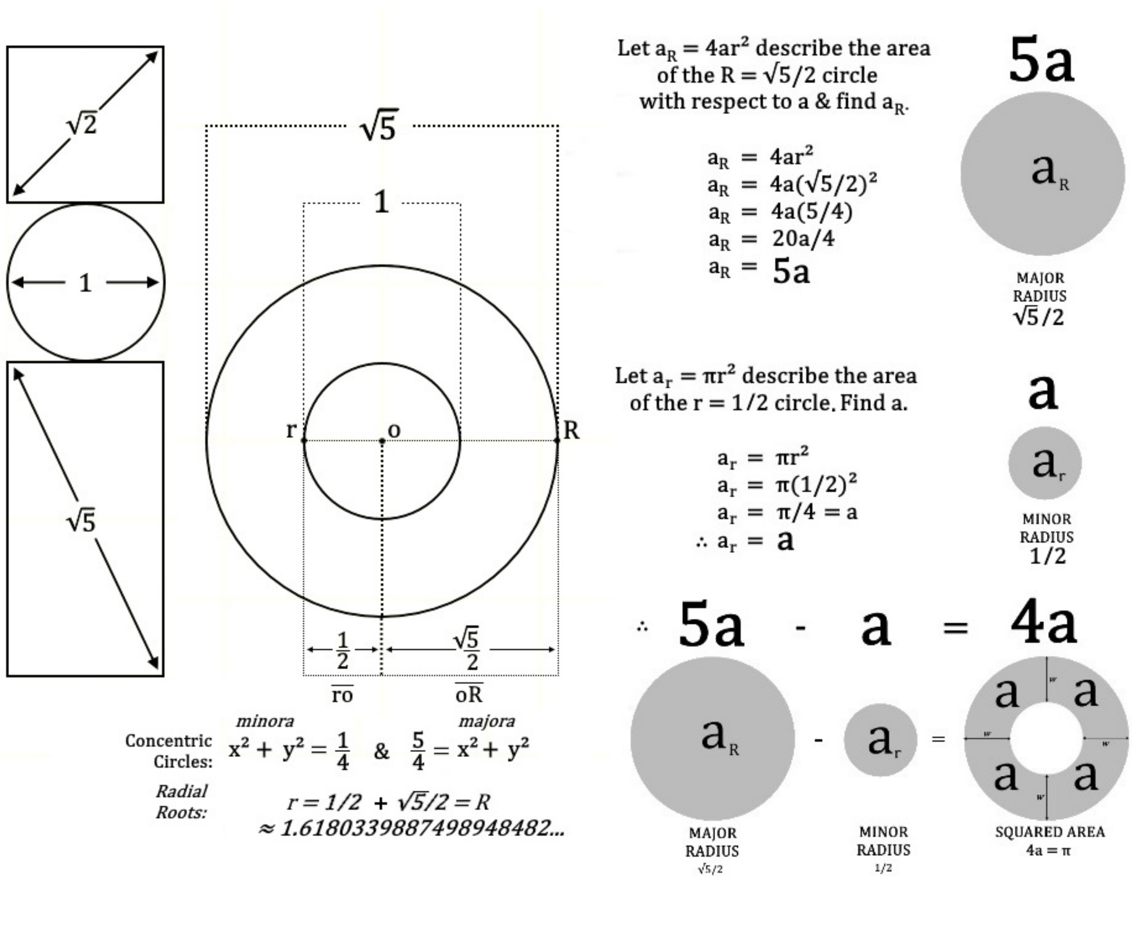
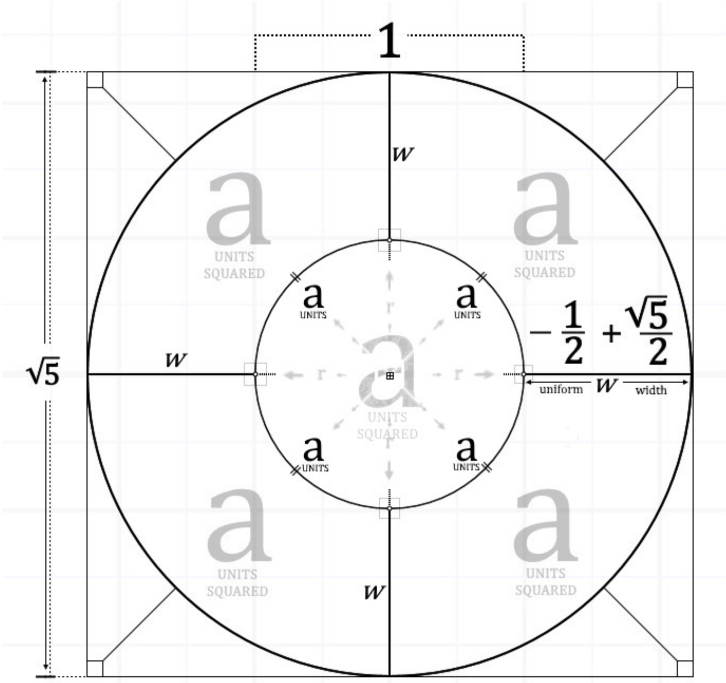


Figure 6.

If one begins with an $R = \sqrt{5}/2$ circle of area (5a) and removes a circular squared area (a) from its center (5a - a), one is left with a squared area *exactly* equal to π = (4a) albeit in the form of an *annulus*. *Significantly*, the area of this π annulus is *equivalent to* the area of an ordinary *unit circle* as described by $\pi r^2 = \pi = 4a$ *obs.* $r = 1.000\ldots$

We are now equipped to answer an important question *viz.*

While/as circumferential length 4a surrounds a units squared, for which *uniform* width w is the 4a = π annulus *satisfied?*



$$4a\left(\left(\frac{\sqrt{5}}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)$$
$$= 4a\left(\frac{5}{4}-\frac{1}{4}\right)$$
$$= 4a\left(\frac{4}{4}\right)$$
$$= 4a$$

∴ the uniform width of pi's annulus capturing 4a units squared while/as surrounding a is:

$$-\frac{1}{2}+\frac{\sqrt{5}}{2}=w$$

≈ 0.61803398....

and is *equal* to a ***reciprocal*** of the (so-called) '***golden ratio*** '.

Figure 7.

As we've *discovered,* the *reciprocal* of the so-called *golden ratio* is the only *uniform* width *satisfying* pi's annulus. Given the surface area of a sphere is $4\pi r^2$, the presence of the so-called golden ratio *phi* within *pi* is nothing short of *extremely significant* (for implying 3D information is *collapsable* down to 2D and/or conversely 2D information is *expandable* up to 3D *by way of*.)

The question was/is important because four discrete circumferential lengths each a = π/4 units are radially squared outwards, thus allowing 4a = π unit *length* <u>and</u> unit *squared area* to <u>both</u> be plotted and observed *<u>simultaneously</u>.*

Unlike Archimedes' polygonal approach, here we've inscribed and circumscribed the circle with *squared areas* for the purpose of reversing 4a *units squared* back down to 4a *units*. With each of four lengths π/4 uniformly squared *outwards* summarily capturing an area of π = 4a units *squared*, we not only *practically circumvent* Archimedes' initial assumption(s) entirely, we do so while/as diligently *observing* & *preserving* geometric integrity.

By equating *four uniform widths* of π's annulus to *four arithmetic squares* of *length* a:

$$4(\sqrt{w})^2 = 4w = 4a^2$$
$$\therefore w = a^2 \therefore \sqrt{w} = a$$

we thereby take the ***root of the width*** of π's annulus using an inverse square such to pose & answer one final *crucial* question *viz.*

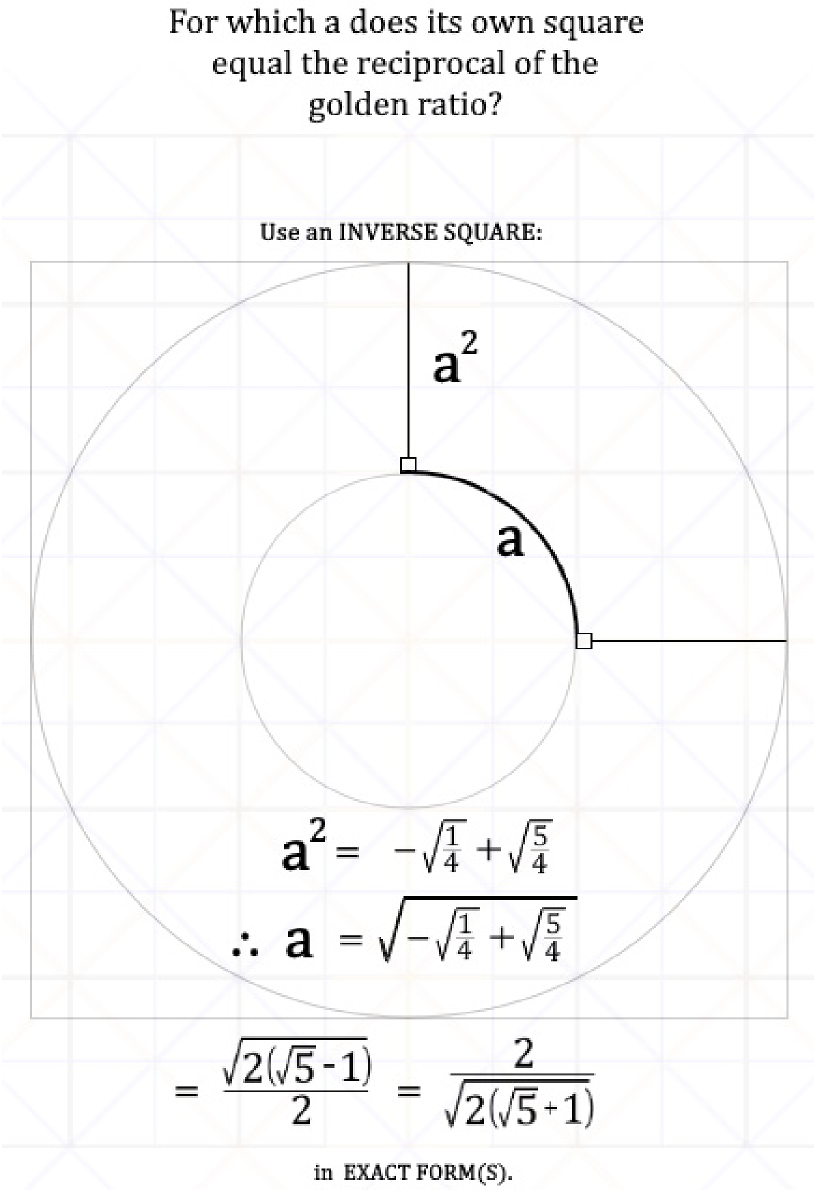


Figure 8.

If :

$(r + r\sqrt{5}) = p$
for *principle* p
and $(-r + r\sqrt{5}/2) = 1/p$
while/as *both* obs. $r = 1/2$,

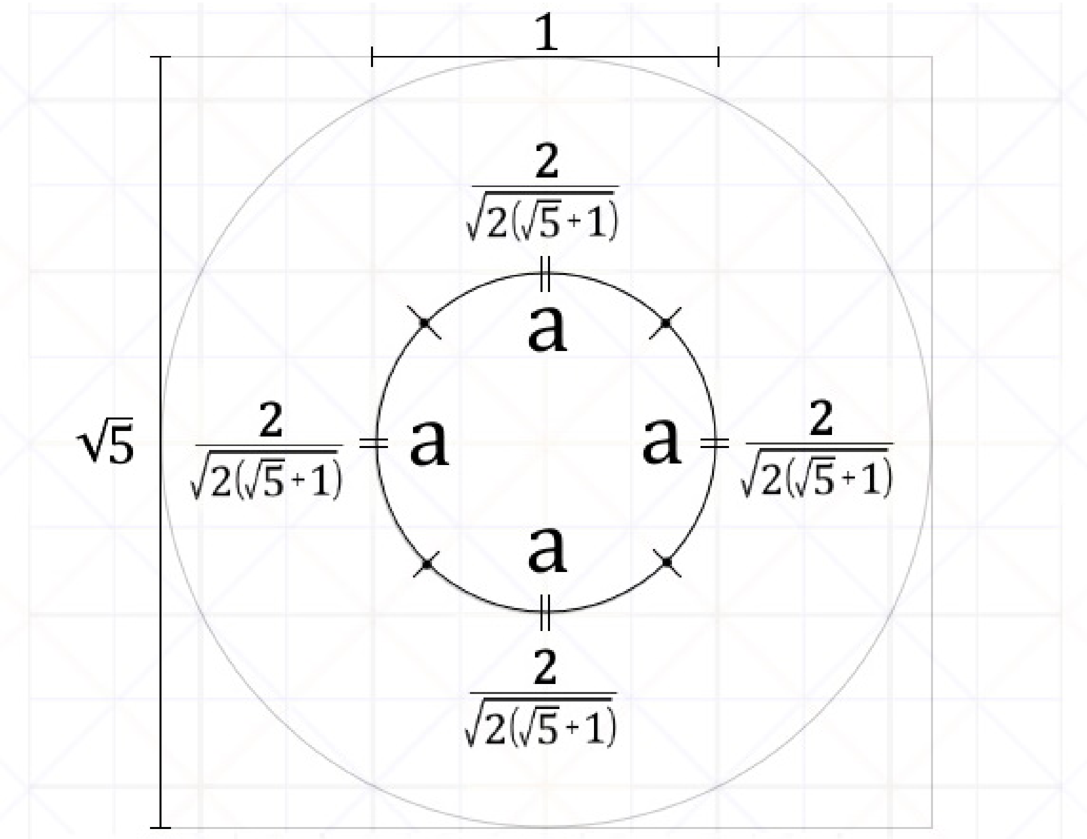
then *simply*:

$a^2 = 1/p$
$\mathbf{p = 1/a^2}$

satisfying an ***inverse square*** ∴

$a^2 = 1/p = (\pi/4)^2$
$a = 1/\sqrt{p} = \pi/4$
$\therefore\ \mathbf{4a = 4/\sqrt{p} = \pi}$

viz.



$$4a = \frac{8}{\sqrt{2(\sqrt{5}+1)}} = \pi$$

$$= 4\sqrt{-\sqrt{\tfrac{1}{4}} + \sqrt{\tfrac{5}{4}}}$$

$$= 2\sqrt{2(\sqrt{5}-1)}$$

$$= \sqrt{8\sqrt{5}-8}$$

INVERSE SQUARE : 3.144605511029693144...

ARCHIMEDES' APPROXIMATION : 3.141592653589793238...

dif. *'Blunder of Millennia'* : 0.003012857439899905...

Figure 9.

$\therefore$ ***π = 8ra = √(8√5-8)***

obs. $r = 1/2 = 0.5000...$

for: $a = \sqrt{p}/p$

obs. $p = (r + r\sqrt{5})$

$\therefore$ ***π = 4(√p/p) = 4/√p***

& is a root of polynomial:

$x^4 + 16x^2 - 256$

$\therefore$ π is ***not*** *transcendental (!)*

Archimedes' over 2000-year-old "approximation" method & result for pi can be *falsified* by comparing the square of its own quarter ($\approx \pi/4)^2$ to the pi annulus' *actual* geometric width of $w = (-1/2 + \sqrt{5}/2)$:

A Radial Square of (Approx.) Pi's Quarter
Falsification of π = 3.14159... Method:

Given the pi annulus' observable width $w = (-1/2 + \sqrt{5}/2) \approx 0.618...$

equate $w = (0.7853981633974483...\text{etc.})^2$ and *compare* the *latter* to the *former:*

$w \approx 0.61685027... \neq w$

$w \approx 0.61803398... \neq w$

∴ *$\pi \neq 3.14159...$ for failing to satisfy the π annulus' uniform width of $(-1/2 + \sqrt{5}/2)$.*

Further, we should predictably expect to find (& rather *immediately*) some enigmatic unsolved problem at the very root of mathematics concerning circle geometry. As it so happens, we *do:* in/as the notoriously *unsolved status* of the Riemann Hypothesis so-called "*problem*".

In 1859, Bernhard Riemann hypothesized the real part of *all* " non-trivial " zeros of his (importantly: π-dependent) zeta function is 1/2 = 0.5000... The "*problem*" is: neither he nor anyone else *since* could conclusively *prove* it. *Why not? What was/is the impasse?* In the same way it is sometimes more effective to address a question with another question, in this case: it is perhaps more effective to address Riemann's unproven hypothesis with *another hypothesis* (albeit one *tacitly informative* of the real underlying "*problem*".)

A "Blunder of Millennia"
Hypothesis

The unsolved status of the Riemann Hypothesis problem is owing to an *unrecognized inexactitude* in & of the hitherto endorsed numerical approximation of the circle constant:

$\pi \approx 3.141592653589793238...$ *etc.*

According to the use of an inverse square, $\pi \neq 3.14159...$
but rather $\pi = \sqrt{(8\sqrt{5}-8)}$ *with exactitude*
whose approximation is rather:

$\pi \approx 3.144605511029693144...$ *etc.*

while/as strictly observing a
unitary length / area numerical equivalency

$$r = 1/2 = s^2$$

If true, in hindsight: the real *"problem"* would have *begun* in the very same *unsolvable* state it is in *now,* for the real *root* would be owing to a hitherto *unrecognized inexactitude* in & of the circle constant *pi itself.* If so, *clearly*: prior to the construction of his analysis, Bernhard Riemann would have *failed* to rigorously *verify* (or in this case: *falsify*) the geometric *integrity* of *3.14159...* before ever allowing it into his zeta function. Riemann would have *assumed* the number is *without blemish* & so would have carried on with it *accordingly*. From Archimedes, to Riemann, to present: *all* would have commonly operated on a single unrecognized *false basic underlying assumption* (of a species one would have hoped to have realized *before* one *expired*.)

As for what the *real culprit* would be: a mere *matter of principle* - under no circumstance can *any* one prove *anything* ' *true* ' so long as one *begins* with a *false assumption(s)*. For example, such a question as "*...for which s does ζ(s) = 0?* " *naively assumes* there are *no circumstances* under which there are *none.* One such circumstance w*ould be*: the very integrity of the ζ function *itself* is *upset* due to a *deficient circle constant* (the integrity of which the function primarily *relies* on.) In such a case, above *all* else: one must first-and-foremost ***fix the circle constant*** (at which point the <u>real</u> underlying "problem" would *certainly* <u>cease to exist</u>.)

The *real lesson* in waiting *would be*: since at least the time of Archimedes over 2000 years ago, the *mere possibility* of this problem ever arising would have ceased to exist the moment any *one single* mathematican(s), scientist(s), physicist(s) etc. invested the necessary time & energy to scrupulously ***check their answer(s)*** for the circle constant by comparing them to an *actual* inverse square-based ' *reality* ' (such to satisfy the ' ***real*** ' part in & of *the* ' ***real element*** '.)

Ultimately, in the end (or rather: *since the beginning*) our commonly suffered & prevailing ' ***crisis in science*** ' would immediately be *clarified* by only *one* corollary equality predicting only *one* culprit *categorical ignorance* of only *one* **universal constancy** of *one itself (!) For if*:

$$p = (r + r\sqrt{5})$$

while/as

$$1/p = (-r + r\sqrt{5})$$

obs. $r = 1/2 = 0.5000...$

and if:

$$\pi \neq 3.14159...$$

but *rather:*

$$\pi = \sqrt{(8\sqrt{5}-8)}$$

(with exactitude)

$$\approx 3.1446055...$$

it *follows:*

$$r = 1/2 = s^2$$
$$4r = 2 = 4s^2$$
$$4w = 4/p = 4a^2$$
$$w = 1/p = a^2$$
$$\sqrt{w} = 1/\sqrt{p} = a$$

$$\therefore \; 4\sqrt{w} = 4/\sqrt{p} = 4a = \pi$$
$$\therefore \; p = 16 / \pi^2$$
$$\therefore \; \pi^2 = 16 / p$$
$$\therefore \; 16 = p\pi^2$$
$$\therefore \; \mathbf{1} = \mathbf{p(\pi/4)^2}$$

one
is a *product* of a *golden ratio* & *square* of *pi's quarter*
(albeit *admittedly*: only a *precise* pi's quarter.)

Finally, *should* the preceding *one* be
the one-and-only *unitarily correct one,*
concerning it:

res ipsa loquitur

...the thing speaks for itself.