Algebra / Teaching Skills Teaching Skills Test

Q.1: Prove that the value of the expression $(\mathbf{a}-\mathbf{1})(\mathbf{a}-\mathbf{3})(\mathbf{a}-\mathbf{4})(\mathbf{a}-\mathbf{6})+\mathbf{10}$ is positive for every value of $\boldsymbol{a} \in \boldsymbol{R}$.

Answer: If we put value greater than 6 in the expression $(\mathrm{a}-1)(\mathrm{a}-3)(\mathrm{a}-4)(\mathrm{a}-6)+10$ then the resulted values are clearly positive.

If we put the values $a = 1{,}3{,}4{,}6$ then the resulted value is 10 explained below

$$for\ a = 1, \qquad (1-1)(1-3)(1-4)(1-6)+10$$

$$(0)(1-3)(1-4)(1-6)+10$$

As we can see at the start of the expression 0 is in multiplication in first term , So the first term becomes zero.

$$0+10=10$$

Similarly for $a = 3, 4, 6$ the $(1-3)$, $(1-4)$, and $(1-6)$ portions in first term becomes zero, As result the first term becomes zero and we left with 10.

Now let's check the expression for $a = 2{,}5$

$$for\ a = 2, \qquad (2-1)(2-3)(2-4)(2-6)+10$$

$$(1)(-1)(-2)(-4)+10$$

As there are three minus (-) signs in product in first term, so we will take minus before the first term.

$$-8+10$$

Or

$$10-8=2$$

$$for\ a = 5, \qquad (5-1)(5-3)(5-4)(5-6)+10$$

$$(4)(2)(1)(-1)+10$$

$$-8+10$$

Or

$$10-8=2$$

Now if we put $a = 0, -1, -2, -3$ .... In the given expression $(\mathrm{a}-1)(\mathrm{a}-3)(\mathrm{a}-4)(\mathrm{a}-6)+10$, the resulted valued are positive because the product in the first term become positive.

$$for\ \ a = 0, \quad we\ get\ \ 72$$

$$for\ \ a = -1, \quad we\ get\ \ 10 \quad and\ so\ on$$

So we concluded that the given expression is positive for every value a $\in$ R.

Q.2: What is the equation of the parabola with a vertex at $(2, 1)$) and focus at $(2, 3)$. Explain how to graph the parabola?

Answer: The given vertex is (2,1)) and focus is $(h, k) = (2,3)$

From the points of vertex and focus we can clearly say that the axis of symmetry is a vertical line x=2. The axis of symmetry is parallel to y-axis and perpendicular to x-axis. The vertex lies above the focus, so the parabola will open up in upward direction from the focus. As the parabola is of vertical type then we will use the equation given below.

$$(x - h)^2 = 4a(y - k) \qquad (i)$$

$$a = 3 - 1 = 2 \ because\, x\ coordinates\ same$$

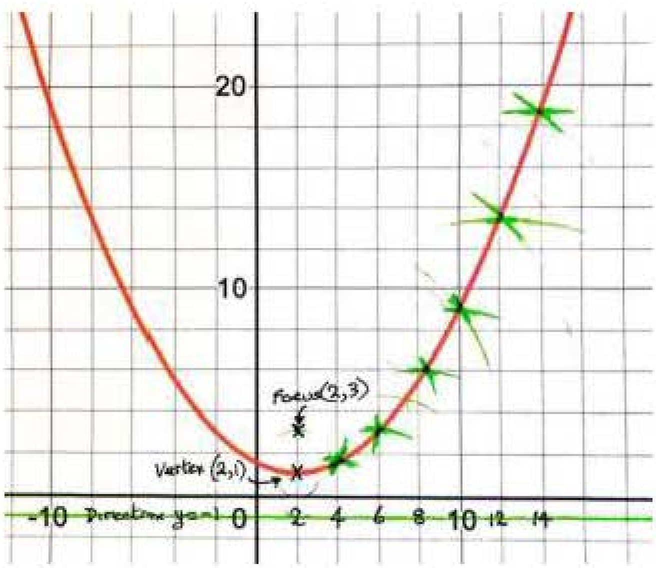
Pus $a = 2, h = 2, k = 3$ in equation (i)

$$(x - 2)^2 = 4(2)(y - 3)$$

$$(x - 2)^2 = 8(y - 3)$$

$$x - 2 = \pm\sqrt{8(y - 3)}$$

$$x = \pm\sqrt{8(y - 3)} + 2$$



Q.3: Determine all natural numbers divisible by 8 whose sum of digits is less than 10 and the product of digits is equal to 12.

Answer: We can write 12 in the form of factors or single digit multiplication taking into consideration the provided conditions. The sum of digits is less than 10 and product is equal to 12.

$$1\text{x}2\text{x}6 = 12$$

$$2\text{x}2\text{x}3 = 12$$

$$1\text{x}2\text{x}2\text{x}3 = 12$$

$$1\text{x}1\text{x}2\text{x}2\text{x}3 = 12$$

It means we have to find the permutation of 126, 223, 1223, and 11223 which are divisible by 8, product of digits is equal to 12 and the sum of digits is less than 10. As we know from basic mathematical rule that these number must end with even number and the sum of last 3 digits must be divisible by 8. The permeations of these numbers are given below in table.

Number	Even Permutations	Divisible by 8 (required numbers)
126	126, 162, 216, 612	216
223	232, 322	232
1223	1232, 1322, 2132, 2312, 3122, 3212	1232, 2312
11223	11232, 11322, 12132, 12312, 13122, 13212, 21132, 21312, 23112, 31122, 31212, 32112.	11232, 12312, 21312, 23112, 32112.

So the enquired numbers are: 216, 232, 1232, 2312, 11232, 12312, 21312, 23112, and 32112.

Verification:

Numbers	Sum must be less than 10	Product is equal to 12
216	2+1+6 =9	2x1x6=12
232	2+3+2 = 7	2x3x2=12
1232	1+2+3+2 =8	1x2x3x2=12
2312	2+3+1+2 =8	2x3x1x2=12
11232	1+1+2+3+2 =9	1x1x2x3x2=12
12312	1+2+3+1+2=9	1x2x3x1x2=12
21312	2+1+3+1+2=9	2x1x3x1x2=12
23112	2+3+1+1+2=9	2x3x1x1x2=12
32112	3+2+1+1+2=9	3x2x1x1x2=12

Q.4: if $\log_2 3$ =a and $\log_7 2$ = b what is $\log_6 28$?

Answer: Given that $log_2 3 = a\,, log_7 2 = b\ then\ find\ log_6 28$

We will consider $log_6 28$

$$log_6 28 = log_6(4x7)$$

Now apply the log product rule

$$log_6 28 = log_6 4 + log_6 7$$

$$log_6 28 = log_6 2^2 + log_6 7$$

Now applying log power rule to the first term on Right hand side

$$log_6 28 = 2log_6 2 + log_6 7$$

The right hand side can also be written as

$$log_6 28 = \frac{2}{log_2 6} + \frac{1}{log_7 6}$$

$$log_6 28 = \frac{2}{log_2(2x3)} + \frac{1}{log_7(2x3)}$$

Again apply log product rules

$$log_6 28 = \frac{2}{log_2 2 + log_2 3} + \frac{1}{log_7 2 + log_7 3}$$

As $log_2 2 = 1$ then

$$log_6 28 = \frac{2}{1 + log_2 3} + \frac{1}{log_7 2 + log_7 3}$$

Now substitute the given value we got

$$log_6 28 = \frac{2}{1+a} + \frac{1}{b+ab}$$

$$log_6 28 = \frac{2}{1+a} + \frac{1}{b(1+a)}$$

$$log_6 28 = \frac{1}{1+\mathrm{a}}\left(2 + \frac{1}{b}\right)$$

$$log_6 28 = \frac{1}{1+\mathrm{a}}\left(\frac{2\mathrm{b}+1}{b}\right)$$

$$\boxed{log_6 28 = \frac{2\mathrm{b}+1}{\mathrm{b}\,(1+\mathrm{a})}}$$

$$(a)(b) = (log_2 3)\,(log_7 2)$$

$$(a)(b) = (log_2 3)\left(\frac{1}{\log_2 7}\right)$$

$$(a)(b) = \left(\frac{\log_2 3}{\log_2 7}\right)$$

Now apply base change rule $\log_b 2 = \frac{\log_a 2}{\log_a b}$

$$(a)(b) = \log_7 3$$

Q.5: Determine all natural numbers $x, y, and\ z$ such that $x < y < z$, and $xyz + xy + xz + yz + x + y + z + 1 = 2020$.

Answer: Given that $xyz + xy + xz + yz + x + y + z + 1 = 2020$.

Rearrange this equation

$$(xyz + yz) + (xy + y) + (xz + z) + (x + 1) = 2020.$$

Now taken common factor from each group

$$yz(x + 1) + y(x + 1) + z(x + 1) + (x + 1) = 2020.$$

Now we can see that $(x + 1)$ is a common factor

$$(x + 1)(yz + y + z + 1) = 2020.$$

Now we can rearrange this equation as

$$(x + 1)[(yz + y) + (z + 1)] = 2020.$$

Or

$$(x + 1)[y(z + 1) + (z + 1)] = 2020.$$

$$(x + 1)[y(z + 1) + (z + 1)] = 2020.$$

Or

$$(x + 1)(z + 1)(y + 1) = 2020 \quad (i)$$

Now decompose 2020 on right hand side of equation (i) to its factors. So we can write

$$(x + 1)(z + 1)(y + 1) = (2)(10)(101)$$

Comparing the corresponding components on both sides, we got

$$x + 1 = 2 => x = 2 - 1 = 1$$

$$y + 1 = 10 => y = 10 - 1 = 9$$

$$z + 1 = 101 => z = 101 - 1 = 100$$

So the required natural numbers are $x = 1, y = 9, z = 100$ that satisfy the condition $x<y<z$. Putting these values back into equation (i) satisfy this equation. Which means these find out values are correct.