HANDWRITTEN NOTES OF

(INDUSTRIAL ENGINEERING)

BY

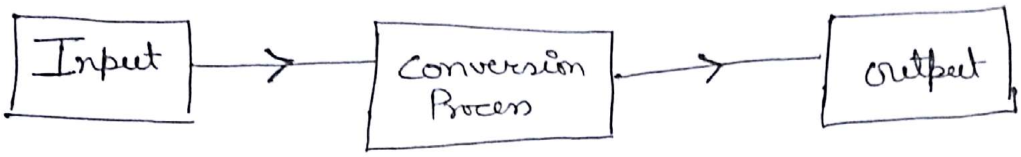
ENGGBUZZ.COM

Industrial Engineering

1. Introduction & Break even Analysis ✓
2. Inventory ** ✓
3. Sequencing ✓ → N Job on 1 m/c, 2 m/c, n m/c
4. PERT – CPM ** ✓
5. Forecasting ** ✓
6. Line Balancing ✓
7. Queuing ✓
8. Work Study
9. SQC not imp → Quality Control
10. Linear programming (graphical, Simplex) **
    (Transportation, Assignment)
11. MRP & VE (Value engineering)

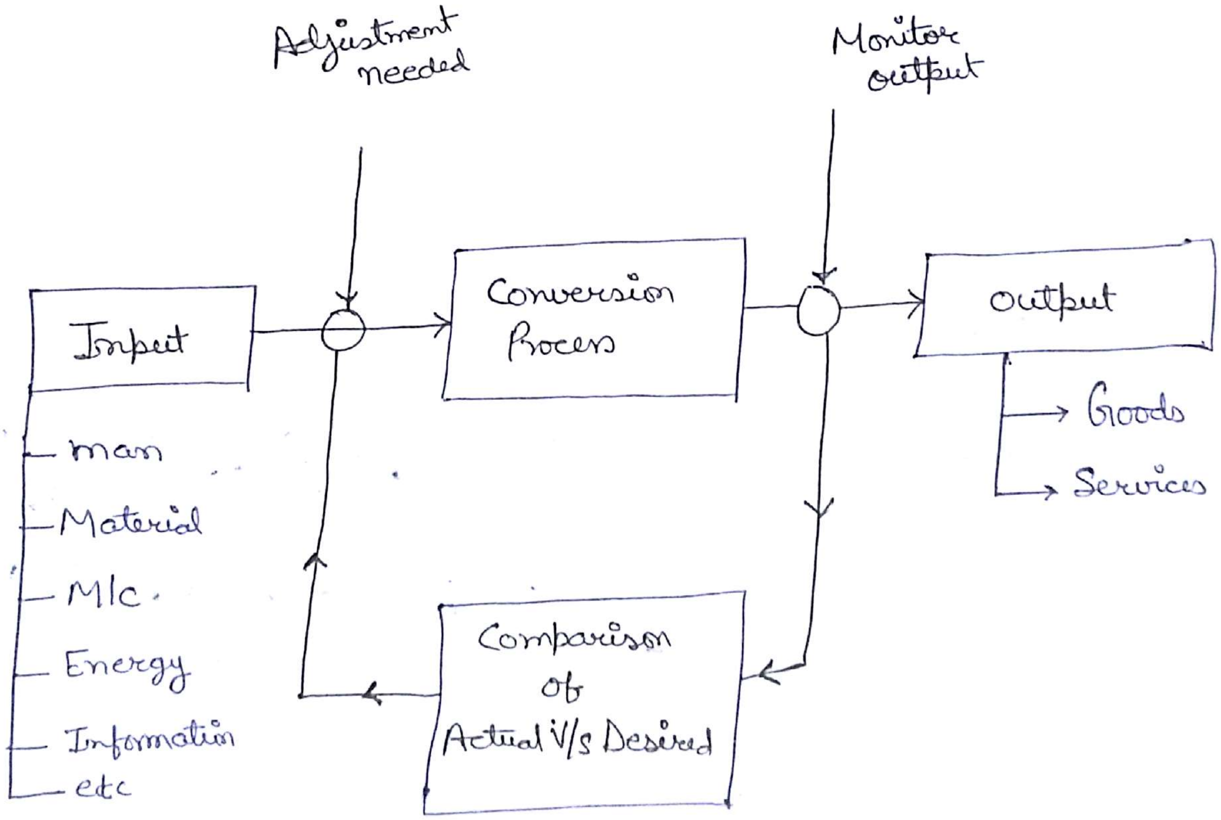
Production:

It is the Step by Step value addition process of Converting One form of material into another form to increase the utility of the product for the user.



Production System:-

It is an organized process of Converting raw material into final product with a feed back loop



Productivity :-

$$\boxed{\text{Productivity} = \frac{\text{Output}}{\text{Input}}}$$

It is a Quantitative ratio b/w What We produced and What We use as resources to produce them.

Every organization always Want to increase productivity by applying new techniques and methods.

Industrial Engineer :-

Industrial Engineer is Concerned With design, installation and improvement of production system.

Its Objective is to eliminate unproductive Operations from the Production System in order to increase productivity.

Production Manager :

Production Manager is Concerned With planning, Controlling and directing, & day to day Working of the production System.

Its Objective is to ~~pro~~ produce "goods & Services" of right Quality and Quantity at pre-determined time and Cost.

i) Cost in Production :

1) Prime or Direct Cost = [Direct Material + Direct Labour + Direct Expenses]

2) Factory overhead or Factory Expenses = [Indirect Material + Indirect Labour + Indirect Expenses]

↓

Indirect Materials → Cutting fluid, Lubricants, Greese, Cotton, Jute Stationary items etc.

\+

Indirect Labour → Watchmen, Supervisor, higher Officers etc.

\+

Indirect Expenses → Rent, Land, Telephone bills, facility development, Electricity bills etc. Canteen,

3) Factory Cost :-

Factory Cost = Prime Cost + Factory overhead

4) Total Cost :-

Total Cost = Factory Cost + Marketting, Advertisment, Transportation etc.

5) Selling Cost :-

Selling Cost = Total Cost + Profit

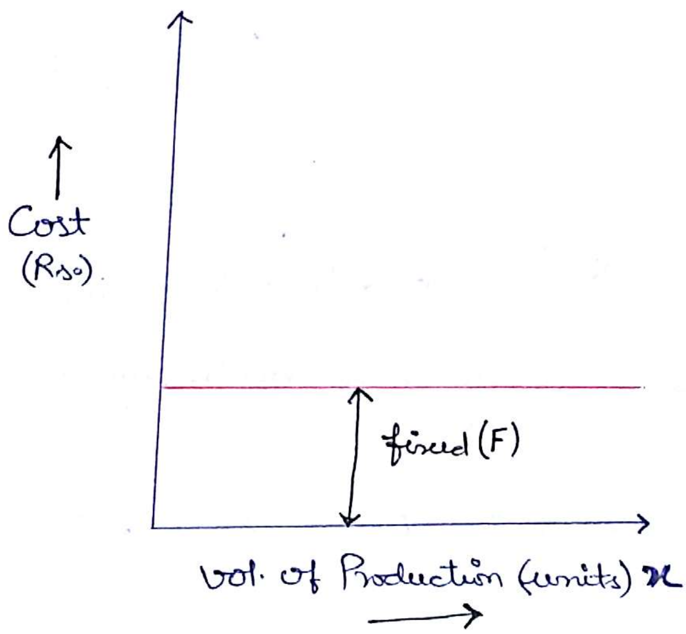
Break Even Analysis (BEA)

i) Total Cost
ii) Selling Cost
iii) Volume of Production

It is a important tool in the hand of Production Manager to analyse the potential profit and loss possible in the future.

i) Total Cost :

It indicates the expenditure make in order to produce Certain number of units and it consist of fixed and variable cost.

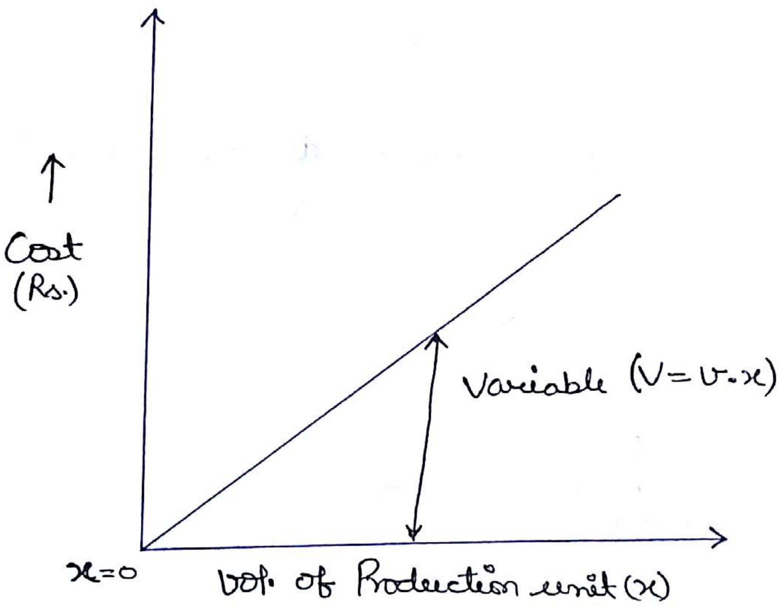


a) Fixed Cost :

→ These Cost remains fix or Constant, irrespective of volume of Production.

→ It include Cost of m/c, Salary of Watchmen, higher Officers Rent of building, Advertisement Cost, Set up cost, Insurance Cost interest of loan taken etc.

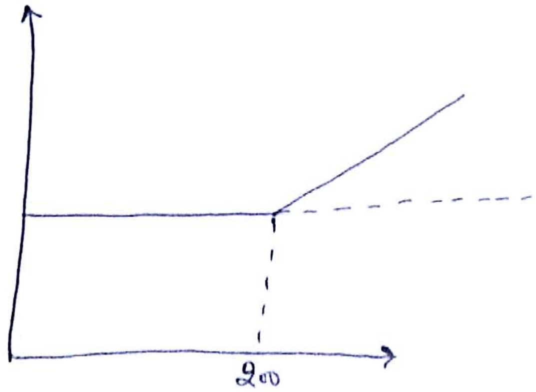
b) Variable Cost ($V = v \cdot x$)



→ This Cost increases directly and propotionally with the volume of Production.

→ It includes Direct Material, Direct Labour, and running Cost.

Semi Variable Cost → Some part fixed then Variable



$F \rightarrow$ Fixed Cost in Rupees [Rs.]

$x \rightarrow$ No of unit produced in order to earn Profit of [P Rs.]

$v \rightarrow$ Variable Cost per unit [Rs./unit]

$V \rightarrow$ Total Variable Cost in Rupees [Rs.]

[$V = v \cdot x$]

s or $S \rightarrow$ Selling Cost Per unit [Rs./unit]

$S \rightarrow$ Total Sale or Revenue

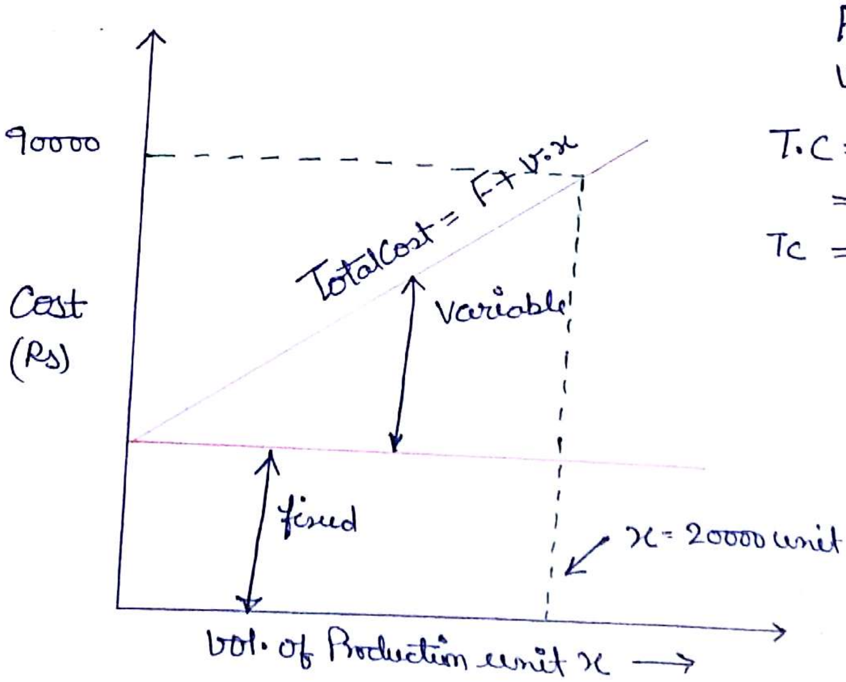
$\boxed{S = s x}$

Revenue → return After

Total Cost :-

$$\text{Total Cost} = F + v \cdot x$$

total Cost = Fixed Cost + Variable cost



$F = 30000$

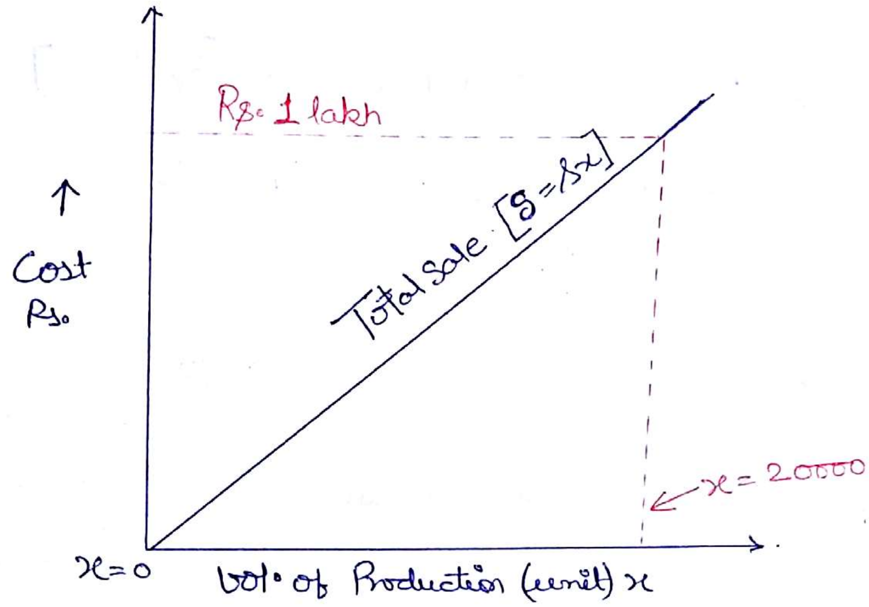
$v = \text{Rs } 3/\text{unit}$

$T.C = F + v.x$

$= 30000 + 3 \times 20000$

$Tc = 90000 \text{ Rs.}$

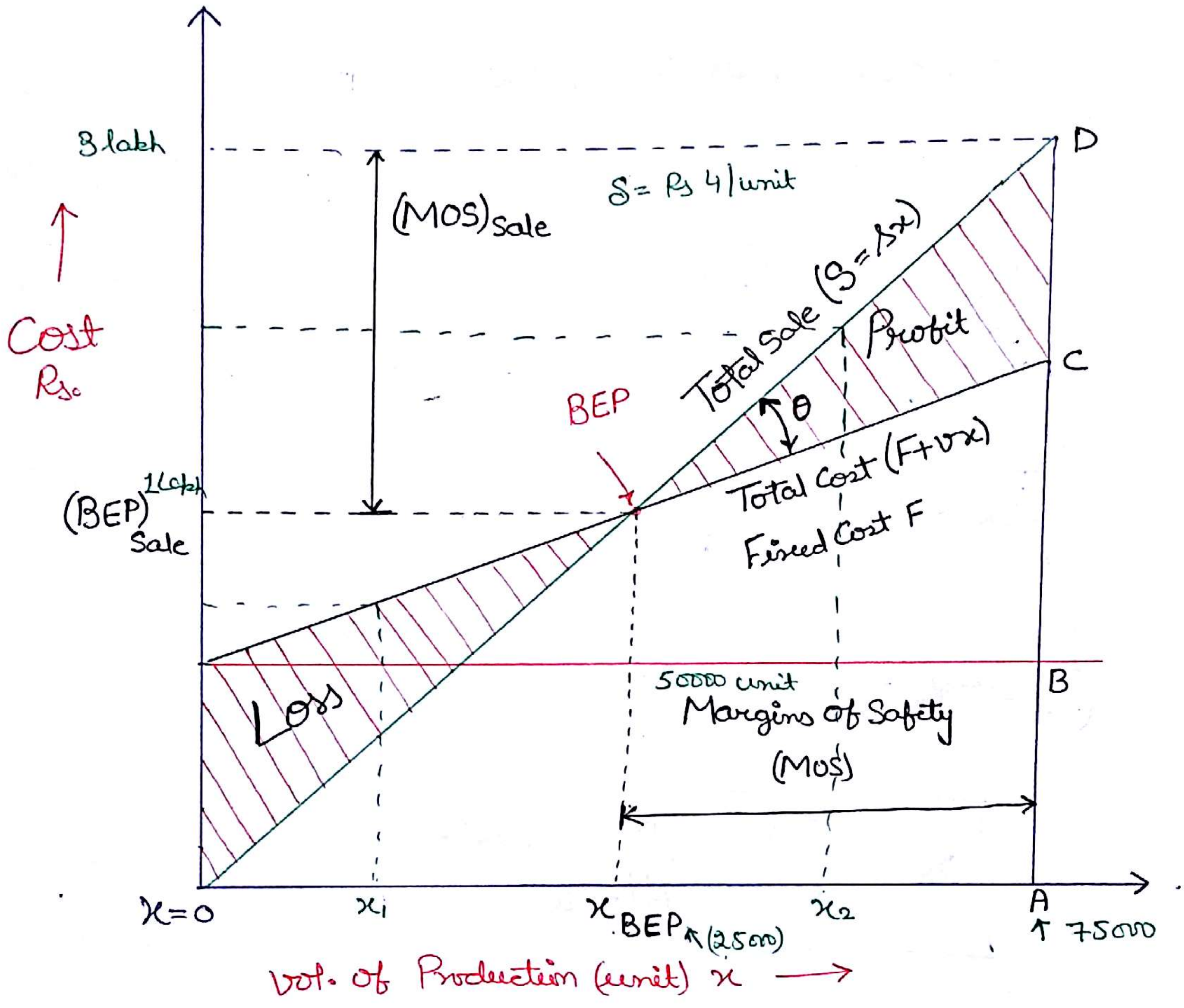
Total Sales or Revenue: $\boxed{S = sx}$



→ It indicates the return obtained by selling of quantity produced.

→ It is directly proportional to the volume of production.

Break Even Chart :-



Break Even point :-

→ It is the volume of production, where total cost equal to total Sale & an organization neither earn profit nor suffer from loss.

→ It is Also Known as (No profit - No loss point).

Total Sale = Total Cost + Profit $\boxed{S = F + V + P}$

Total Sale = $S = sx$

Total Cost = $F + V = F + vx$

Profit = P

$$\boxed{S = F + V + P}$$ Imp

derive by this

$\boxed{\begin{array}{l} S = sx \\ V = vx \end{array}}$

or

$$\boxed{sx = F + vx + P}$$

$$(s - v)\cancel{x} = F + P$$

$$\boxed{x = \frac{F + P}{s - v}} \frac{Rs}{Rs/unit} \text{ unit}$$

At BEP ; $P = 0$

$$\boxed{x = \frac{F}{s - v}} \text{ unit}$$

$$\boxed{(BEP)_{sale} = x_{BEP} \cdot s = \frac{F}{(s - v)} s} \text{ Rs.}$$

$S = sx$

Terms Related to Break Even point

i) Angle of Incidence (θ) :-

It is the angle at which, total sale line cuts the total cost line, Larger this angle (θ) better the Working Conditions will be.

ii) Contribution Margin : (CM) → Marginal Profit or Gross Margin

CM = Total Sale − Total Variable Cost → [Contribution]

Changes in Profit

$$\boxed{CM = S - V = (s-v)x}$$

$$\boxed{\text{Contribution} = (s-v)}$$

$$\boxed{\begin{aligned} S &= sx \\ V &= vx \end{aligned}}$$

$$\boxed{\begin{aligned} v &= 5 \text{ Rs unit} \\ s &= 8 \text{ Rs unit} \end{aligned}}$$

$$\boxed{S = F + V + P}$$

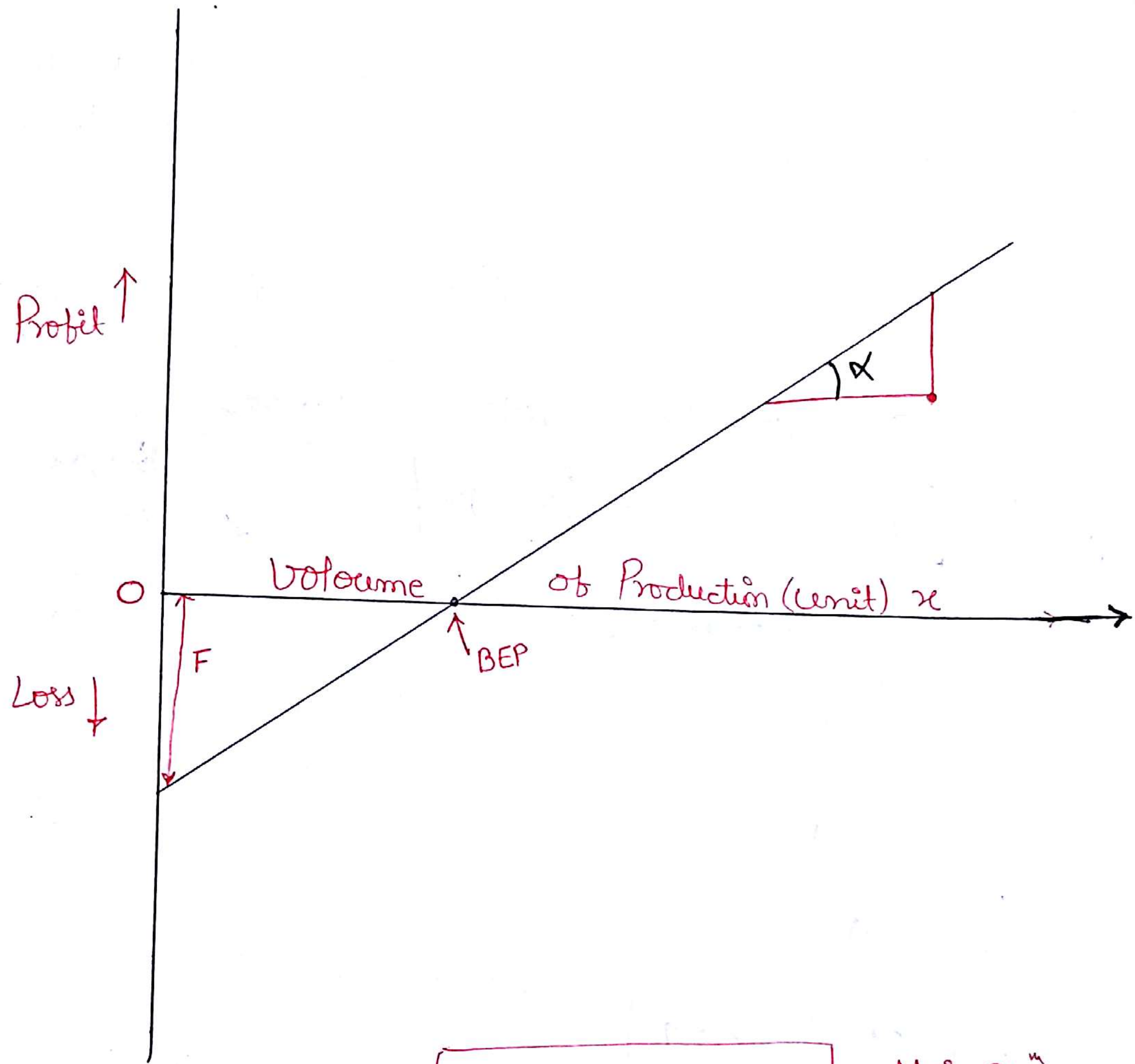
CM → $S - V = F + P$

$$\boxed{CM = F + P\uparrow = (s-v)x\uparrow}$$ → Change in Profit → units ∝ Profit

$$\boxed{P = CM - F}$$ → Adjust Profit or net profit

→ Marginal Profit or Gross Margin

iii) Profit volume Graph :-



$S = F + V + P$ → Main eqn

$Sx = F + vx + P$

$\boxed{P = (S - v)x - F}$

Slope $= (S - v)$

at $x = 0$ $\boxed{P = -F}$

at BEP; $\boxed{x_{BEP} = \frac{F}{(S - v)}}$

iv) Profit volume Ratio :- $(P/V)_{Ratio}$

[1 Company diff Product.]

→ It is the term use to represent profitability related to Sales and it is used mainly when we deal in multi product.

→ This Ratio Always remains Constant for a particular product.

$$(P/V)_{Ratio} = \frac{CM}{S} = \frac{S-V}{S} = \frac{s-v}{s}$$

fixed

$$\boxed{(P/V)_{Ratio} = \frac{F+P\uparrow}{S\uparrow} = \frac{s-v}{s}}$$

→ Constant

$$\boxed{\frac{F+P_1}{S_1} = \frac{F+P_2}{S_2}}$$

$$\boxed{(P/V)_{Ratio} = \frac{\Delta P}{\Delta S}}$$

e.g. 0.22 0.18 to↓ 0.31 ↓ to↑

NOTE:
If there is option of increasing the sale, highest $(P/V)_{Ratio}$ Should be Prefreed and if there is option of decreasing the sale lowest $(P/V)_{Ratio}$ Should be prefreed.

v) Margin of Safety : (MoS)

It is the difference b/w output at full Capacity, compared to output at Break even point.

$$(MOS)_{Sale} = (Sale)_x - (Sale)_{BEP}$$

$$\boxed{(MOS)_{sale} = S_x - S_{BEP}}$$ Rs.

$$(MOS)_{Sale} = Sx - Sx_{BEP}$$

$$= S\left[x - \frac{F}{(S-v)}\right]$$

$$(MOS)_{Sale} = S \cdot \left[\frac{(S-v)x - F}{S-V}\right]$$

$$(MOS)_{Sale} = \frac{P}{\frac{S-V}{S}}$$

$$\boxed{(MOS)_{Sale} = \frac{P}{(P/V)_{Ratio}}}$$

% Wise Margin of Safety :-

$$(MOS)\% = \left[\frac{S_x - S_{BEP}}{S_x}\right] \times 100$$

→ Change in Break even point when;

i) $F \uparrow \longrightarrow x_{BEP} \uparrow$

$\Downarrow$

$$\boxed{x_{BEP} = \frac{F}{s-v}}$$

ii) $v \uparrow \longrightarrow x_{BEP} \uparrow$

iii) $s \uparrow \longrightarrow x_{BEP} \downarrow$

Qn) A product Can be produced by 4 Process as given below in order to produce 100 unit.

Which process should be prefered?

for x=5

	Process	F(Rs)	v (Rs/unit)	
40				
45	I	30	2	→ 230
30	II	40	1	→ 140
35	III	10	4	→ 410
	IV	20	3	→ 320

$x = 100$

$$T.C = F + vx$$

Qn) A company requires a product for which they have three options

I – Purchase at the rate of Rs 10/unit

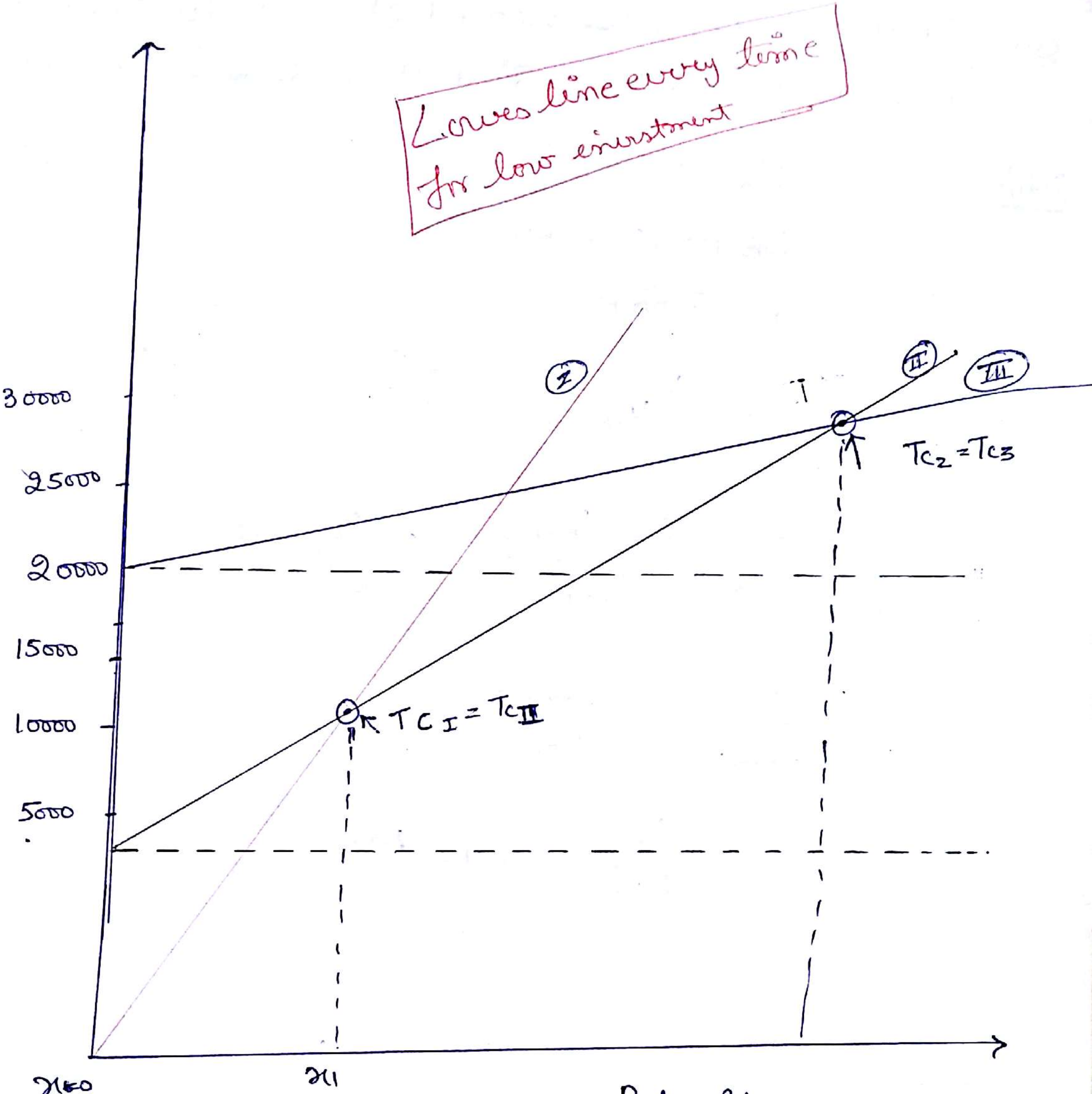
II – Produced by Semi-Auto m/c

$F = 3400$ Rs. $\quad v = $ Rs. 6/unit

III – Produced by fully Auto-m/c

$F = $ Rs $20200 \quad v = $ Rs 3/unit

find the decision Rule?



~~Betw line~~

lowest point / line alway Prefreed.

$Tc_{I} = Tc_{II}$

$x = 850$ ← x_1

$Tc_{II} = Tc_{III}$

$x = 5600$ ← x_2

Qn) Actual Sales is Rs. 30000. BEP Sales Rs. 15000 and fixed Cost is Rs. 6000. Find the profit ~~and~~ at then Actual Sales?

Soln)

$S_x = 30000$ Rs.

$S_{BEP} = 15000$ Rs.

$F = 6000$ Rs.

$$(P/V)_{Ratio} = \frac{F+P}{S}$$

$$\frac{F + \cancel{P_{BEP}}^{0}}{S_{BEP}} = \frac{F + P_x}{S_x}$$

$$\frac{6000}{\cancel{15000}} = \frac{6000 + P_x}{\cancelto{2}{30000}}$$

$$\boxed{P_x = 6000 \text{ Rs.}} \text{ Ans.}$$

Inventory

→ Inventory can be termed as stock on hand at a given point of time, which may be have for the purpose of later use or sale.

→ It has an economic value and it may include, Raw material, work in process inventory, semi finished or subassembly, and final product.

→ In Inventory Control, our aim is to manage inventory in such a manner, that day to day working runs smoothly but at the minimum of the cost.

Inventory Cost :-

Inventory Cost

- Purchase Cost
- Ordering or Setup cost
- Holding or Carrying Cost
- Shortage or Stockout Cost

i) Purchase Cost :-

It is the cost of purchasing inventory item and it depends up on quantity or bulk purchased.

$$\boxed{P.C = \text{No of units} \times \text{Cost per unit}}$$

ii) Ordering or Setup Cost :-

A) Ordering Cost :-

When the inventory is purchased from outside, the cost Associated with bringing inventory within the production System is termed as ordering cost.

It includes, Cost of tender, processing cost, paper work, Communication inspection Cost, transportation Cost etc.

e.g Maruti → Tyre → Not Manufacture TyRe. So Order of tyre is Placed. Tender are Published → Quotations Mach inspection team → Communication → order Transport cost

B) Setup Cost :

→ When the inventory items are produced internally, the cost associated with bringing Shut down production System again into Starting position is termed as set up cost.

→ It includes maintenance cost, Schedule chart, prepration cost Cost associated with bringing raw material, arrangement of Worker, tools, equipment etc.

e.g Maruti → nut bolt
↓
Manufacture – 15-day → enough material to use 3 month
3 month plant off → After 3 Month plant Start in b/w
Maintenance, bill, rent, labour are included in Set up Cost

$$\boxed{\text{Ordering Cost} = \text{No. of order} \times \text{Cost per order}}$$

$$\boxed{\text{Setup Cost} = \text{No. of Setup} \times \text{Cost per Setup}}$$

iii) Holding or Carrying Cost :-

→ It is the Cost associated with Storing, Keeping, & maintaining inventory within the production System.

→ It include Storage cost, handelling cost, damage & depriciation Cost, insurance Cost, interest of loans etc.

→ This cost depends up on the Quantity and period for which inventory is stored.

Inventory Cost is given by;

$$\boxed{\begin{array}{c} \text{Handelling cost} \\ \text{or} \\ \text{Carrying cost} \end{array} = \underset{Q_{Avg}}{\begin{array}{c}\text{Average inventory} \\ \text{for a period}\end{array}} \times \begin{array}{c}\text{Holding Cost per} \\ \text{unit time}\end{array}}$$

iv) Shortage or Stockout Cost :-

→ Shortage simply means ~~Shortage~~ absence of inventory & the loss associated with not serving the customer is termed as shortage or stockout cost.

→ It include Potential profit loss, fast transportation cost & discount etc.

~~Shortage~~

$$\boxed{\text{Shortage Cost} = \underset{\text{Short}}{\text{No. of unit}} \times \text{Shortage cost Per unit}}$$

e.g In Production Plant → let inventory is zero or stock out
Production stop

in Retail → Sell stop due to stockout

in Both the Cases → losses occur
Profit end zero

Now let Showroom of Mercedeez Benz.

if owner have profit of 5 lak by Soldin one car

but he did not take risk to hold inventor

bcz inventory cost is high, then rent of that too costly.

$\boxed{\text{Offers} \rightarrow \text{given}}$

Inventory Classification :-

i) Transit or pipe line Inventory :-

Inventory Cannot provide service while in transportation, and Such inventory is Called transit or pipe line inventory.

ii) Buffer or Safety Stock :-

$d' = 15$ unit/day $> d = 10$ unit/day $> d'' = 6$ unit per day

$LT' = 9$ day $> LT = 6$ day $> LT'' = 4$ day

LT → lead time
Order place करने के
in Hand आने का time

$\boxed{ROL = 60 \text{ unit}}$
⇓
Reorder level

→ It is minimum amount of inventory Kept through out the year and is used ~~through~~ only during adverse Condition to prevent Stock out.

→ It is held for protecting against the fluctuation in the demand rate and the lead time.

→ It is never required under normal Working Condition and used only during Adverse Condition to prevent Stock out

* Lead time (LT) :-

→ It is the time gap b/w Placing an order & inventory on hand, So that it can be used or Consume.

iii) Seasonal Inventory :-

The demand for these items changes with Seasonal Variation.

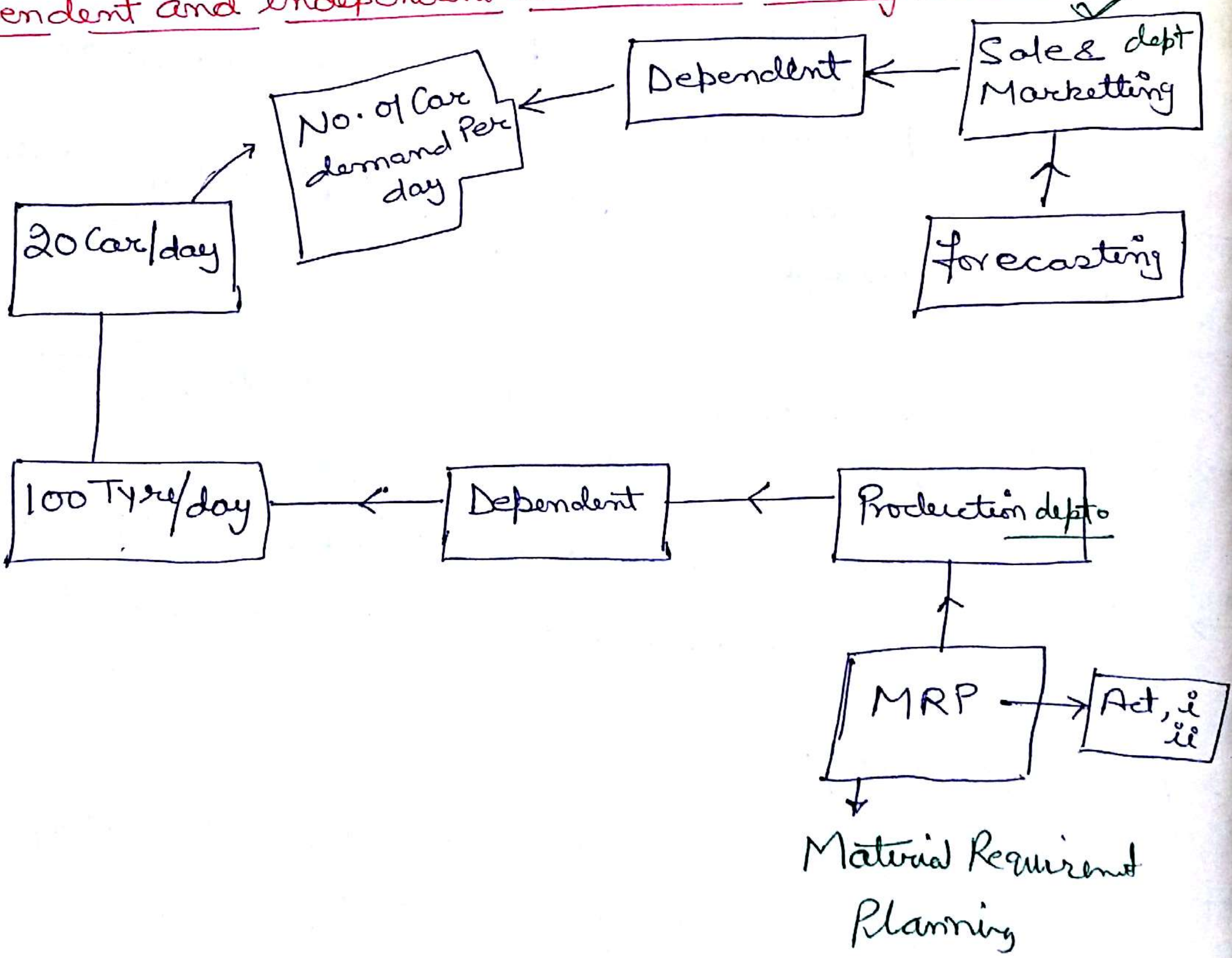
iv) Anticipation Inventory :- (Demand changes with Some Reason)
(by inventory Analysis)**

Petrol, cotton → infected by ↑ in govt tax Policy

These inventory items are build up to meet Anticipated demand in future like, Big selling forecast, government policy change Price hike, Strike, Shut down etc.

Characterstics of Inventory Model :-

i) Dependent and independent demand Inventory items :-



i) Dependent :-

The demand for these items is directly realated or linked to demand of any other item, usually of a higher level of which it becomes a part.

ii) Independent :

The demand for these items is not directly related or linked to any other items. It is difficult to compute and is projected with the help of forecasting.

ii Inventory Review Systems :-

(Inventory filling)

Review System

Q- System
or
Fixed order System
or
Reorder level System
or
Two bin System

P- System
or
Fixed Period System
or
Periodic Review System

Quality/Persiable item

Combination

SS System

d= 10/day

LT= 5 day

$\boxed{\text{RoL= 50 unit}}$

Q= 500 unit/order

450	50

indicating stock (RoL)

Q system Work (fast)

$\boxed{\text{RoL = 50 unit}}$

P system Work (slow)

$\boxed{\text{RoP = 45 day}}$

d= 500 unit

$\boxed{\text{next 15 day}}$

no indication of end of stock

→ Fixed Order System :-

→ In this System, as inventory decreases to reorder level of fresh order for fixed Quantity is placed at that point.

→ In this System size of order is fixed but the time of order is variable.

→ Fixed Period System :-

→ In this System, inventory level is reviewed after a fixed period of time and a fresh order for variable Quantity is placed at that point.

→ In this System, size of order is variable, but the time of placing order is fixed.

iii) Deterministic & Probabilistic Inventory Model :-

a) Deterministic Inventory Model : Demand & LT fixed

→ In these model, demand rate and Lead time remains fixed and Costant and therefore We need not to Carry Safety stock

b) Probabilistic Inventory Model : Demand & LT Not fixed

→ These Models represents the real World Condition, Where, there is uncertainity of demand rate and lead time.

→ In these Models We need to Carray Safety Stock, to prevent Stock out during adverse Condition.

$\boxed{\text{Maruti} \rightarrow \text{Tyre}}$

Production Plant → Manufacturing

Notations for Inventory Model :

D → Annual or Yearly demand of inventory → D (unit/year)

Q → Quantity to be ordered at each order Point → Q (unit/order)

N → No. of order Placed in a Year → N (order/year)

$\boxed{N = \frac{D}{Q}}$ order/year

if D = 54000 unit/year
Q = 9000 unit/order
$N = \frac{D}{Q} = \frac{54000}{9000} = 9$

T → Time length of one inventory cycle or time gap b/w two Successive order.

T = [year/order]

$\boxed{T = \frac{1}{N}}$ ** $\boxed{T \cdot N = 1}$ **

N = 4 order/year
T = 3 month/year
$T = \frac{3 \times 4}{4}$ month/order
$\boxed{T = \frac{1}{4} \text{ yr/order}}$

C → Cost of Purchasing one unit of Inventory. [Rs./unit]

C_o → Cost of Placing one order. C_o - [Rs./order]

C_h → Cost of Holding one unit in inventory for one Complete Year. C_h → [Rs./unit/year]

Deterministic Model

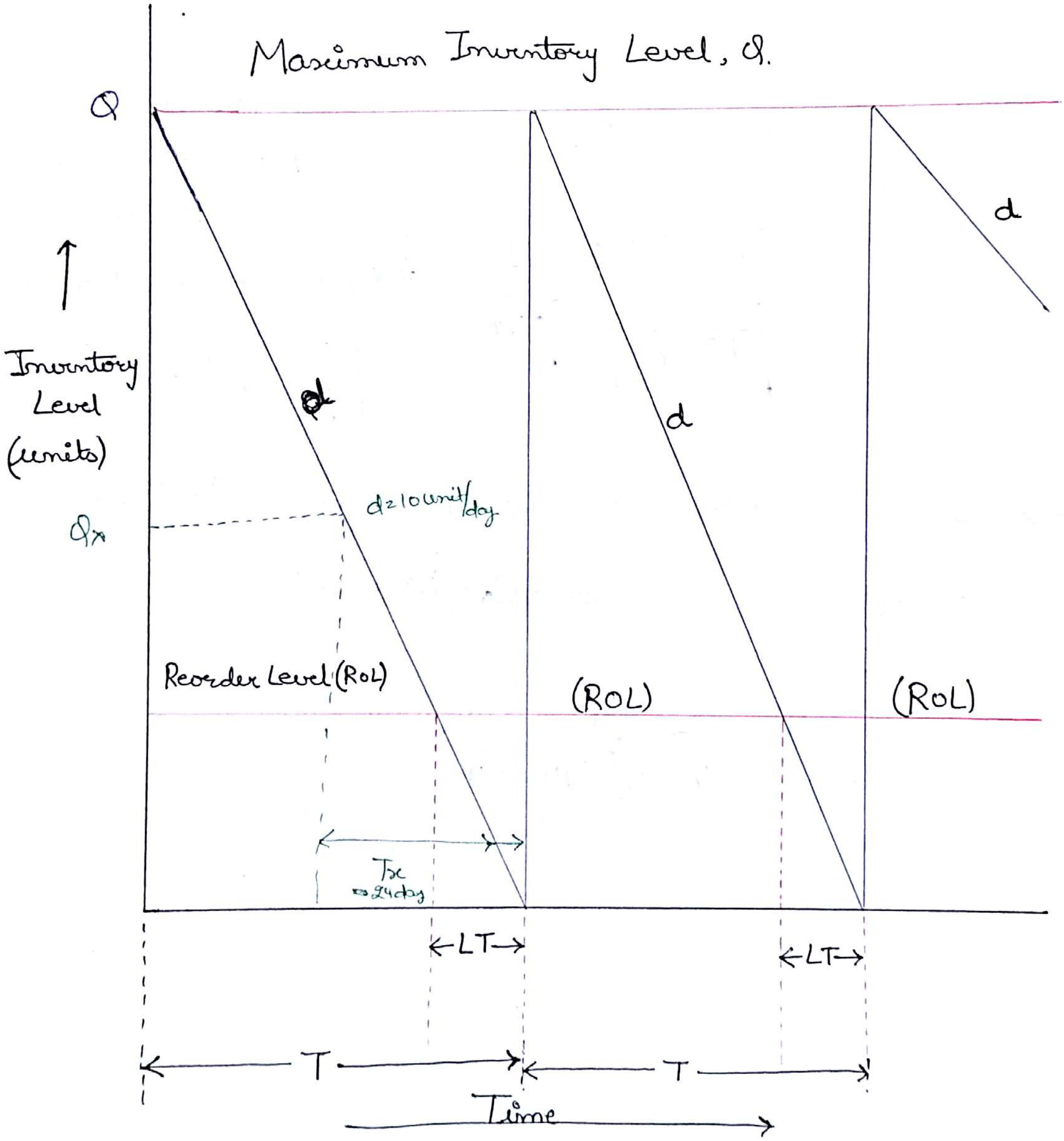
1) Economic Order Quantity [EOQ]

or

Harris - Wilson

or

Infinite Rate Of Replenishment



i) $\rightarrow$ $\boxed{Q_x = T_x \cdot d}$

ii) $\rightarrow$ $\boxed{Q = T \cdot d}$

iii) $\rightarrow$ $\boxed{ROL = (L \cdot T) d}$

iv) $\rightarrow$ $\boxed{d = \frac{Q}{T} = \frac{Q_x}{T_x} = \frac{ROL}{L \cdot T}}$

Total Cost (Total Annual Cost) :- (TAC) or (TC)

$\boxed{TC \text{ or } TAC}$

Total Cost / total Annual Cost = Purchasing cost (P.C) + ordering cost (O.C) + Holding Cost (H.C)

$\boxed{\text{Purchasing Cost} = D \cdot C}$

$\boxed{\text{Ordering Cost} = N \cdot C_o}$ $\Rightarrow$ $\boxed{O \cdot C = \frac{D}{Q} \cdot C_o}$

Holding Cost for Period :

$\boxed{T = \frac{Q}{2} \cdot C_h \cdot T}$

~~25~~, 20, 15, 10, 5, 0

C_h = Rs 2 /unit/day

H.C = Rs. 100

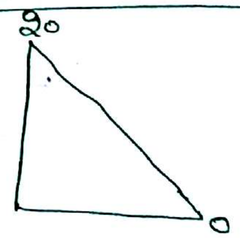
$Q_{Avg} = \frac{50}{5} = 10$ unit

H.C = 10 × C_h × no. of day

= 10 × 2 × 5

H.C = 100 Rs

freeze Main



$Q_{Avg} = \frac{20+0}{2} = 10$

Annual Holding Cost = $\frac{Q}{2} \times C_h$ (T.N) → i
(AHC)

$$\boxed{A \cdot H \cdot C = \frac{Q}{2} \cdot C_h}$$

Total Annual Cost :

$$\boxed{TAC = D \cdot C + \frac{D}{Q\uparrow} \cdot C_o \downarrow + \frac{Q\uparrow}{2} C_h \uparrow}$$

Constant

Q → Variable

D = 25000 unit/year

Total Variable Cost = O.C + HC

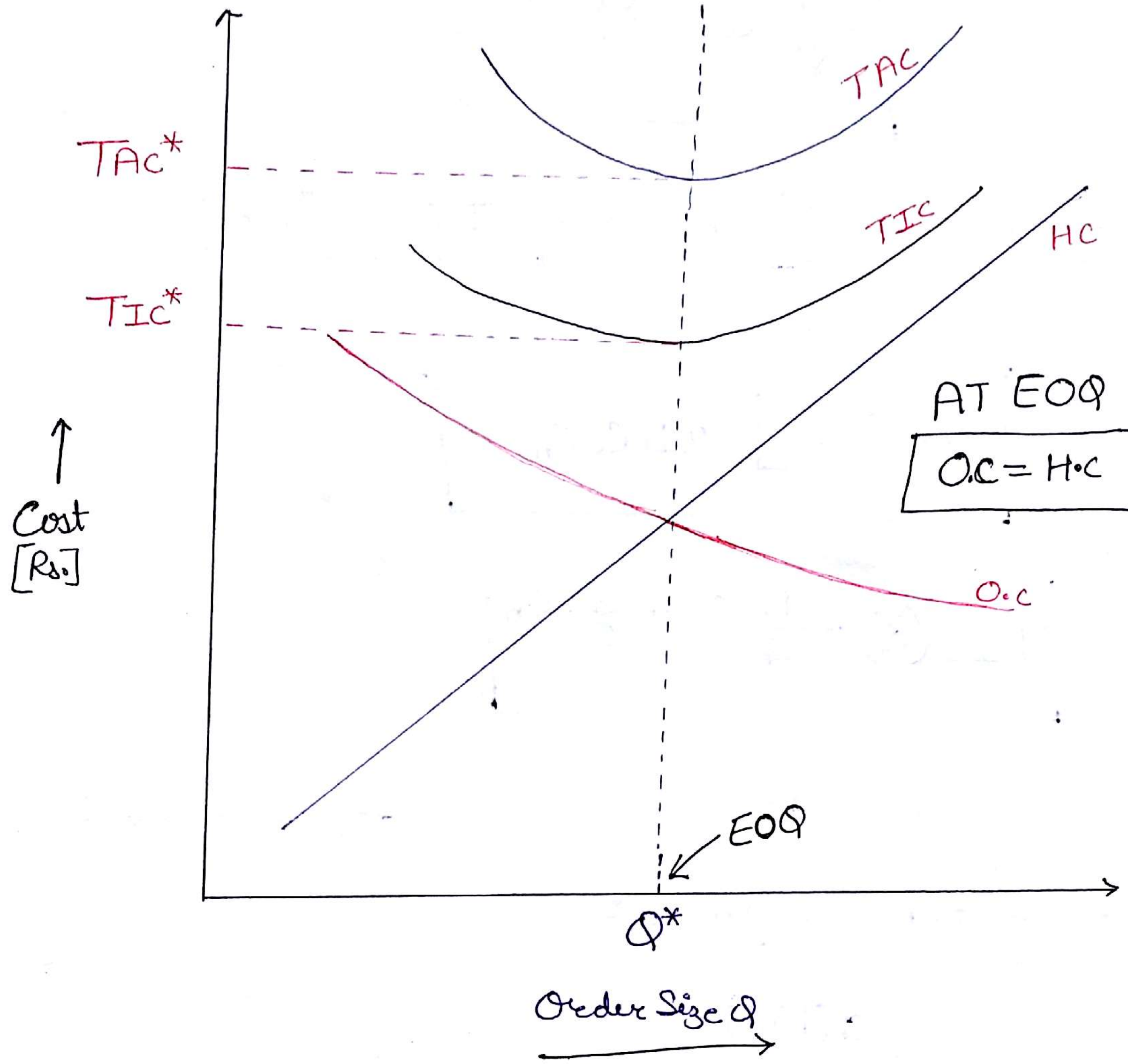
or

Total Inventory Cost

$$\boxed{TIC = \frac{D}{Q} \cdot C_o + \frac{Q}{2} C_h}$$

$$\boxed{TAC = TIC + D \cdot C}$$

→ Purchasing cost



→ The ordering Quantity [Q*] at which holding cost become equal to ordering cost and the total inventory cost is minimum is Known as Economic order Quantity [EOQ]

At EOQ

Ordering Cost = Holding cost

$$\boxed{\frac{D}{Q^*} C_o = \frac{Q^*}{2} \cdot C_h} \qquad \boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}}}$$

Total inventory Cost;

$$TIC = \frac{D}{Q^*} \cdot C_o + \frac{Q^*}{2} \cdot C_h$$

but as

$$\boxed{\frac{D}{Q^*} C_o = \frac{Q^*}{2} \cdot C_h}$$

$$TIC^* = \cancel{2} \frac{Q^*}{\cancel{2}} \cdot C_h \Rightarrow \boxed{TIC = Q^* \cdot C_h}$$

$$\boxed{TIC^* = \sqrt{2 D C_o C_h}}$$

only for EOQ

$$\boxed{TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} \cdot C_h}$$

for EOQ
or
Non EOQ

⇓
generalized equation

TIC → min @ EOQ

for TIC to be minimum ;

$$\frac{d(TIC)}{dQ} = 0$$

$$\frac{C_h}{2} - \frac{D}{Q^{*2}} \cdot C_o = 0$$

Again differentiating above

$$0 - \frac{(-2) D \cdot C_o}{Q^{*3}}$$

$$\frac{+2 D \cdot C_o}{Q^{*3}}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

Model Sensitivity (M.S)

or

Robustness

→ It is the term used to represent, Sensitivity of inventory model for different order size compared to EOQ.

Model Sensitivity is Given by;

$$\boxed{M.S = \frac{TIC(Q)}{TIC(Q^*)}} \quad \text{——— (1)}$$

$$TIC(Q^*) = \frac{D}{Q^*} \cdot C_o + \frac{Q^*}{2} \cdot C_h$$

but, as;

$$\frac{D}{Q^*} \cdot C_o = \frac{Q^*}{2} \cdot C_h$$

$$\boxed{TIC(Q^*) = 2 \cdot \frac{D}{Q^*} \cdot C_o} \quad \text{——— (a)}$$

$$TIC(Q) = \frac{D}{Q} \cdot C_o + \frac{Q}{2} C_h$$

Now let

$$\boxed{Q = KQ^*}$$

$$TIC(Q) = \frac{D}{KQ^*} \cdot C_o + K\frac{Q^*}{2} \cdot C_h$$

$$\boxed{TIC(Q) = \frac{D}{Q^*} \cdot C_o \left[\frac{1}{K} + K\right]} \text{ — (b)}$$

Putting value of a, & b in (1)

**
***
Objective

$$\boxed{M \cdot S = \frac{1}{2}\left[\frac{1}{K} + K\right]}$$

→ $\boxed{M \cdot S \geq 1}$ and is equal to 1, when $K = 1$ i.e $\boxed{Q = Q^*}$

examples

1) 50% more than EOQ. $\boxed{a_0 = K = 1}$

$K = 1.5$

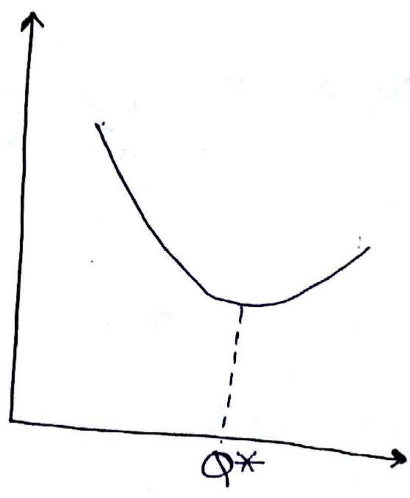
$$M \cdot S = \frac{1}{2}\left[\frac{1}{1.5} + 1.5\right] = 1.0833$$

2) 50% less than EOQ.

$K = 0.5$

$$M \cdot S = \frac{1}{2}\left[\frac{1}{0.5} + 0.5\right] = 1.25$$

EOQ curve originally behaving like this



NOTE : Very important Point

1) When Holding Cost is given $\boxed{C_h}$ in terms of interest or %

it always correspond to unit price of inventory and the interest

rate should be always Yearly.

$$\boxed{C_h = i\% \text{ of } C}$$

Yearly

$C =$ Rs. 50/unit

$i\% = 1.5\%$/month

$\downarrow \times 12$

$i\% = 18\%$/year

$$\boxed{C_h = 0.18 \times 50 = 9 \text{ Rs./unit/year}}$$

Qn) Total Inventory Cost at the Order Size of 400 units & 900 units are Equal. Then determine EOQ i.e Q^*

Soln)

$$TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

$$TIC(400) = TIC(900)$$

$$\frac{D}{400} \cdot C_o + \frac{400}{2} C_h = \frac{D}{900} C_o + \frac{900}{2} \cdot C_h$$

$$D \cdot C_o \left[\frac{1}{400} - \frac{1}{900}\right] = C_h (450 - 200)$$

$$\Rightarrow \frac{D \cdot C_o \times \overset{2}{\cancel{500}}}{400 \times 900} = C_h \times \cancel{250}$$

$$\frac{2DC_o}{C_h} = 400 \times 900$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{400 \times 900} = 600$$

$$Q^* = 600$$

if $TIC(Q_1) = TIC(Q_2)$

then $\boxed{Q^* = \sqrt{Q_1 \cdot Q_2}}$

Qn) Determine EOQ Value when annual demand is worth Rs. 50000, ordering Cost is 2% of order value & Holding Cost is 10% of unit price.

Soln)

$D = 50000 \qquad C = 50000$

$O.C\ (C_o) = 2\%$ order value $= Q^* C = 2$

$H.C\ (C_h) = 10\%$ of unit price

Ordering Cost $= 0.2 \times Q^* . C$

$H.C = 0.1\,C$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

$$Q^* = \sqrt{2 \times \left(\frac{50000}{C}\right) \frac{0.02\, Q^* . \cancel{C}}{0.1 \times \cancel{C}}}$$

$$\cancel{(Q^*)^2} = \frac{100000 \times 0.02\, \cancel{Q^*}}{C \times 0.1} = \cancel{Q^{*2}} = \frac{20000}{C} \cancel{Q^*},$$

$$\boxed{Q^* . C = Rs.\ 20000}$$ Ans

Qn) For a Production System

$D = 18000$ unit/year

$C =$ Rs. 8/unit

$C_o =$ Rs. 240/order

$C_h = 12\%$ of C

Lead time (LT) = 10 days

300 working days per year

then determine?

1) Q^*
2) N^* → No. of order Placed in year
3) T^*
4) TIC^*
5) ROL
6) No. of days of Stock at Reorder point
7) Amount of Saving with EOQ against earlier Practice of 4 order in a year.
8) Increase in total cost Associated with ordering Cost.
   a) 25% more than EOQ
   (b) 40% less than EOQ.

Soln)

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\cancel{\frac{2 \times 18000 \times 240}{1.7280}}} = \sqrt{\frac{2 \times 18000 \times 240}{0.96}}$$

$Q = 3000$ unit/order

② $N^* = \frac{D}{Q^*} = 6$ order/unit

③ $T^* = \frac{1}{N^*} = \frac{1}{6}$ year/order $= \frac{1}{6} \times 300$

$T^* = 50$ day/order

(4) $TIC^* = \sqrt{2DC_oC_h}$

$= 2880$

(5) $(ROL) = LT \times d$

$d = \dfrac{D}{\text{Working Cost}}$

$d = \dfrac{18000}{300} = 60$ unit/day

$ROL = 10 \times 60 = 600$ Unit

(6) Lead time $(LT) = 10$ days

(7) $N = 4$ Order/year

$Q = \dfrac{18000}{4} = 4500$ unit/order

$TIC = N \cdot C_o + \dfrac{Q}{2} \cdot C_H$

$= 4 \times 240 + \dfrac{4500}{2} \times (0.96)$

$TIC =$ Rs 3120

Saving $= 3120 - 2880$

$=$ Rs. 240

(8)(a) @ 25% more than EOQ

$Q = 1.25 Q^* = 3750$ unit/year

@ $TIC = \dfrac{18000}{3750} \times 240 + \dfrac{3750}{2} \times 0.96 =$ Rs. 2952

Increase $= 2952 - 2880 =$ Rs $72 \uparrow$

8(b) @ 40% less than EOQ

$K = 0.6$

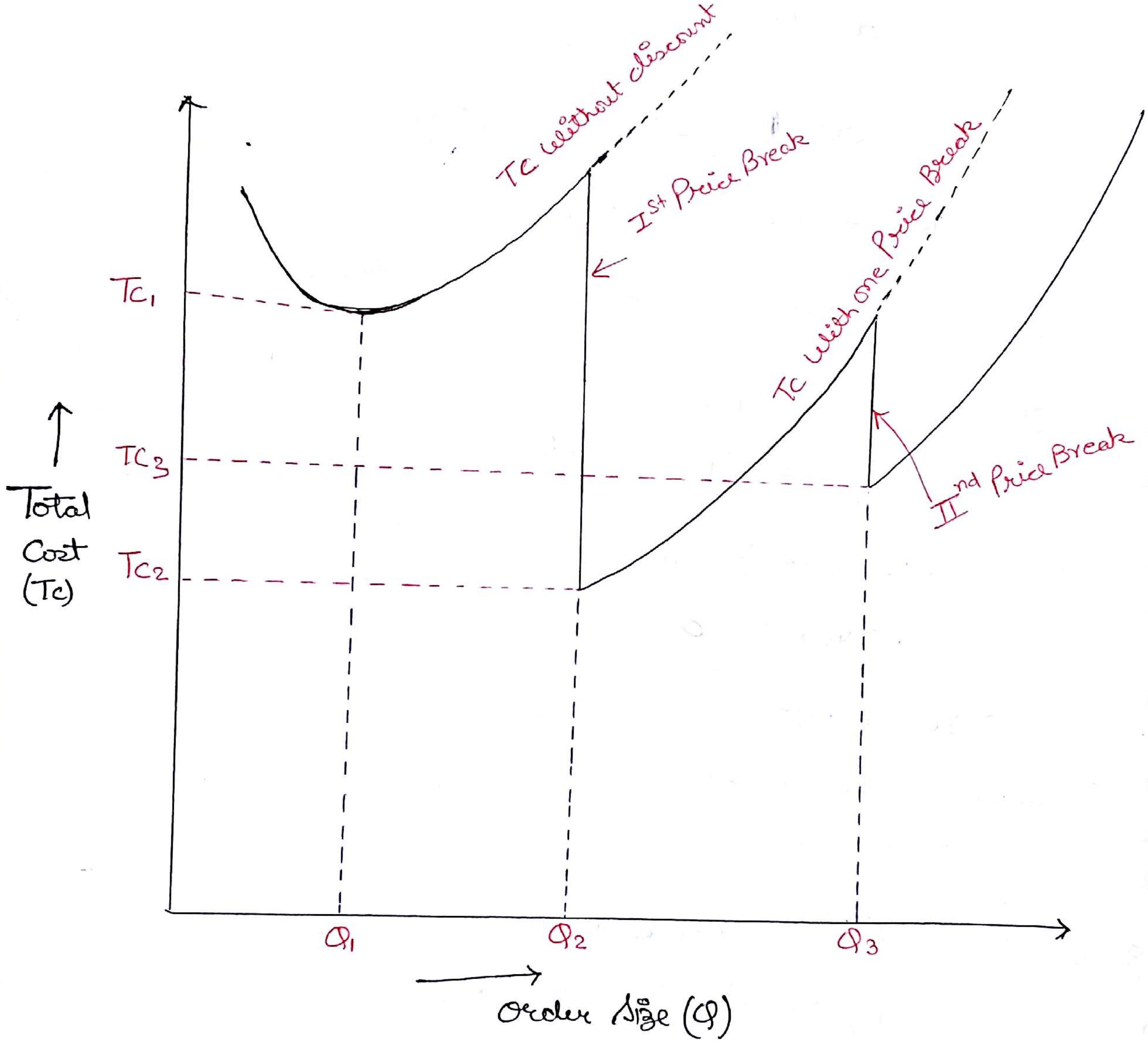
$M.S = \frac{1}{2}\left(\frac{1}{0.6} + 0.6\right] = 1.13 = \dfrac{TIC(Q)}{2880} \Rightarrow TIC = 3264$ Rs.

Increase $=$ Rs. $384 \uparrow$

ii)

EOQ With Price Break
or
Quantity Discount

[IInd Model of deterministic inventory Model]



$$TC = DC + \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

D = 20000 unit/year

C = Rs 10/unit

if Q ≥ 1000 . C Rs 9/unit

if Q ≥ 2000 C Rs 8.5/unit

In Some Conditions, discount is Offered on unit price of inventory for large Quantity purchase. These discount take the form of price break. As discount is always offered on unit Price of inventory. So in order to determine the best order Size, We need to Consider purchasing cost along with ordering & Holding Cost.

i) In these problems first We Compute,
feasible EOQ

ii) then
Total Cost is computed at @ EOQ

iii) and
the next higher order Size having price break.

Whenever the total cost comes out to be minimum gives the best order Size.

Qn) In a Production System, Annual demand is 8000 unit Ordering Cost is Rs. 1800 and holding Cost is 10% of unit price of inventory. Items can be purchased in a lot as given below. Determine the Best order Size. (IES 2008)

Lot Size	unit price
1 — 999 →	220
1000 — 1499 →	200
1500 — 1999 →	190
2000 — Above →	185

more
...

Sol(n) We Know that

$$EOQ \text{ i.e } Q^* = \sqrt{\frac{2 C_0 D}{C_h}}$$

$C_h = 10\%$ of C

Starting from the lowest price and searching feasible EOQ.

$C =$ Rs. 185/unit

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{(185 \times 0.1)}}$$

$Q^* = 1247.7$ unit/order

It is not feasible as $C =$ Rs. 185 is for $Q \geq 2000$

Proceeding to next,

~~C = Rs. 185~~ higher unit price - $C =$ Rs. 190/unit

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{190 \times 0.1}}$$

$Q^* = 1231.17$ unit/order.

Again not feasible

$C =$ Rs. 200/unit

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{200 \times 0.1}}$$

$Q^* = 1200$ unit/order

It is feasible as for $C =$ Rs. 200 Q must be b/w 1000 to 1499.

Now We Compute total cost at feasible EOQ

i.e $Q^* = 1200$

and the next higher price break point of

$Q = 1500$ & $Q = 2000$

$$Tc(Q) = D \cdot C + \frac{D}{Q} C_o + \frac{Q}{2} \cdot C_h$$

$$Tc(1200) = 8000 \times 200 + \frac{8000}{1200} \times 1800 + \frac{1200}{2} \times (200 \times 0.1)$$

$Tc @ (1200) = Rs. 1624000$

$$Tc(1500) = 8000 \times 190 + \frac{8000}{1500} \times 1800 + \frac{1500}{2} \times (190 \times 0.1)$$

$Tc @ (1500) = 1543850$ Rs.

$$Tc(2000) = 8000 \times 185 + \frac{8000}{2000} \times 1800 + \frac{2000}{2} \times (185 \times 0.1)$$

$Tc @ 2000 = Rs. 1505700$

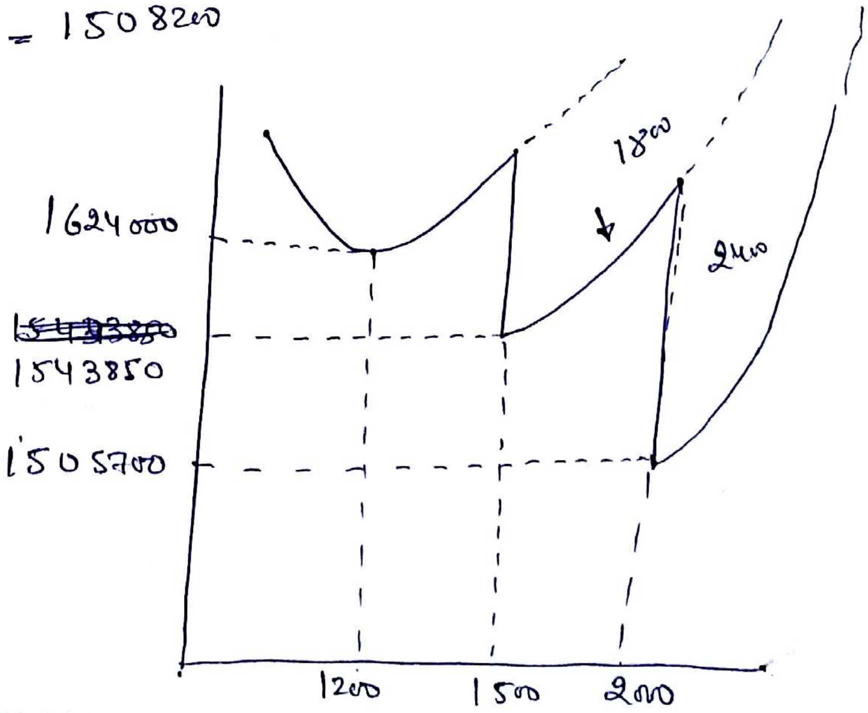
total Cost is min @ 2000, so order is best to place.

$$Tc(1800) = 8000 \times 190 + \frac{8000}{1800} \times 1800 + \frac{1800}{2}(190 \times 0.1)$$

$$= 1520000 + 8000 + \cancel{17100}\ 9000$$

$Tc(1800) = 1545100$

$2400 = 1508200$



Qn) $D = 2000$ unit/yr.

$C = $ Rs. 1 unit

$C_o = $ Rs. 10/order

$C_h = $ Rs 0.16 unit/yr.

Qn) find Q^*, TIC

Qn) if $Q = 1000$, 5% discount on C

if $Q = 2000$ 7% discount on C

Determine best order size?

iii) **Production Model**

or

Build up Model

$p = 100$ unit/day ↑

$d = 20$ unit/day ↓

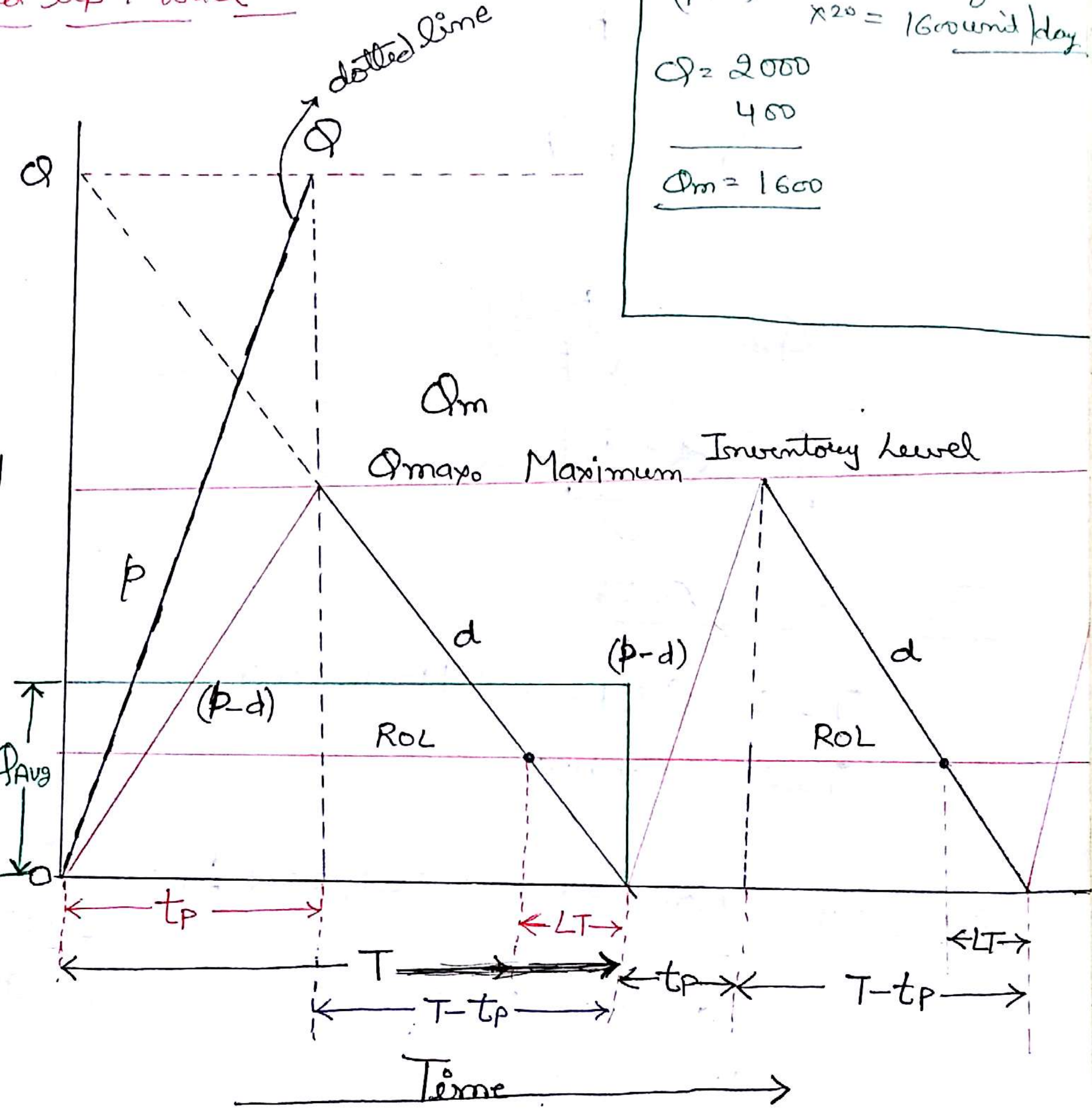
$(p-d) = 80$ unit/day
$\times 20 = 1600$ unit/day

$Q = 2000$
400

$Q_m = 1600$

own Production

→ Setup cost included



→ This Model is similar to first Model "EOQ". The only difference that inventory build up is gradual rather than instaneous.

p → Production or build up rate

d → Demand or Consumption rate

t_p → Production or Manufacturing Cycle time

own Production
OQ → amount of Quantity Manufg
but same time Consume also
and QM only we get after t_p.

$$Q = t_p \cdot p$$

$$t_p = \frac{Q}{p}$$

$$Q_m = t_p (p-d)$$

$$Q_m = Q\left(\frac{p-d}{p}\right)$$

$$\boxed{Q_m << Q}$$

Total inventory Cost;

TIC = Setup cost + Holding cost

TIC = S·C + H·C

Setup Cost = No. of Set up × Cost / setup → e.g.

$$\boxed{S \cdot C = \frac{D}{Q} \cdot C_o}$$

D = 48000 unit/year

Q = 6000 unit/year

$\frac{D}{Q}$ = Setup

Holding Cost :

$$\boxed{H \cdot C = Q_{Avg} \times C_h}$$

→ Q_{Avg} by triangle using / Assuming rectangle as both Δ have same Base.

□ Area = Δ area.

$$Q_{Avg} \times T = \frac{1}{2} \times T \times Q_m$$

$$Q_{Avg} = \frac{Q_m}{2}$$

$$Q_{Avg} = \frac{Q}{2}\left(\frac{p-d}{p}\right)$$

$$H \cdot C = \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)$$

$$\boxed{TIC = \frac{D}{Q} C_o + \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)}$$

total Inventory Cost.

Variable in equ^n is Q

For TIC, to be minimum, differentiate above w.r.to Q.

i.e; ~~do~~

$$\frac{d(TIC)}{d(Q)} = 0$$

$$\frac{C_h}{2} \cdot \left[\frac{p-d}{p}\right] - \frac{D}{Q^{*2}} \cdot C_o = 0$$

$$\boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}\left(\frac{p}{p-d}\right)}}$$

$$\boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}} \boxed{\sqrt{\left(\frac{p}{p-d}\right)}}}$$

Production factor $= > 1$

to first Model

$p \rightarrow \infty$

$$\sqrt{\frac{1}{1 - \frac{d}{p}}} \quad \frac{d}{p} \rightarrow 0, \; p \rightarrow \infty$$

<u>At EOQ;</u>

$\boxed{S \cdot C = H \cdot C}$ → total Inventory cost min at this Point. **

$$TIC^* = \sqrt{2DC_oC_h}\sqrt{\frac{p-d}{p}} \rightarrow <1$$

Qn) A Company uses 12000 units of Component X in a year. Component $\boxed{X}$ is made in 30 Batches of 400 units on a m/c that makes 8 units per hour. The Company operates for 2000hr/year and its cost Rs. 60 to set up the m/c.

C_h = Rs. 10 unit/year

find out wether the existing production plan is optimum and if not, suggest a new plan.

Find the Amount of Saving Possible with the new plan.

Also determine production cycle time, max. inventory level & cycle time, corresponding to optimum condition.

Soln

$D = 12000$ unit/year

2000 hr/year

$C_o = S.C = Rs.\ 60$

$C_h = Rs.\ 10$ unit/year

$p = 8$ unit/hr

$d = \frac{12000}{2000} = 6$ unit/hr.

$$\sqrt{\frac{2DC_o}{C_h}\left(\frac{p}{p-d}\right)}$$

1) $Q^* = 758.94$ unit/~~year~~ setup

Can't be in fraction so have to Round off it.

$N^* = \frac{D}{Q^*} = \frac{12000}{758.94} = 15.81$ setup/year

Alway round off corrosponding to no. of setup N^*

a) $N = 16$ $\quad Q = 750$

b) $N = 15$ $\quad Q = 800$

@ (a) N = 16 Q = 750

$$TIC = N \cdot C_o + \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)$$

$$= 16 \cdot 60 + \frac{750}{2} \cdot 10 \cdot \frac{2}{8}$$

$$\boxed{TIC = Rs.\ 1897.5}$$

@ b) N = 15 Q = 800

$$TIC = 15 \times 60 + \frac{800}{2} \times 10 \times \frac{2}{8}$$

$$\boxed{TIC = Rs.\ 1990}$$

Best Order Size, is; N = 16, batches and ~~Q=750~~ Q = 750 unit/set up

Current Policy;

N = 30 Q = 400

$$TIC = 30 \times 60 + \frac{400}{2} \times 10 \times \frac{2}{8}$$

$$\boxed{TIC = Rs.\ 2300}$$

$$\boxed{\text{Saving} = 402.5\ Rs.}$$

Now Production Cycle time; t_p

$$t_p = \frac{Q}{P}$$

$$t_p = \frac{Q}{P} = \frac{750}{8} = 93.75\ hr/setup$$

Maximum inventory Level; Q_m - is

$$Q_m = Q\left(\frac{p-d}{p}\right) = 187.5\ units/setup$$

Cycle time;

$$T = \frac{Q}{d} = \frac{750}{6} = 125\ hr./setup$$

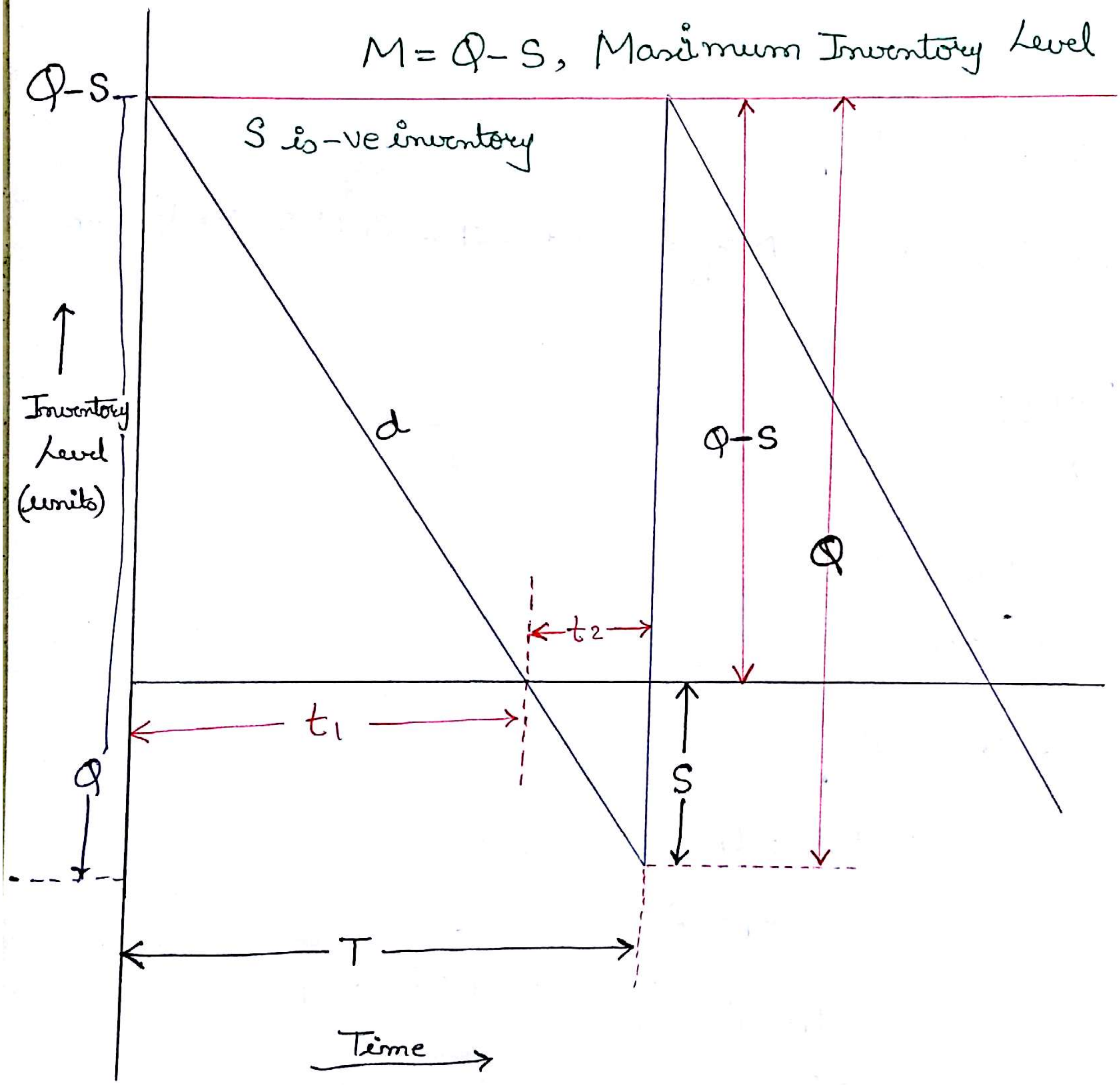
iv) Shortage Model

or

Stock Out Model

or

Back Order Model



→ This model is Similar to, first model EOQ, the only difference that Shortages are allowed.

→ Planned Shortage or backorder is the Condition when Customer Places an order and find that inventory is out of Stock, then he wait for next Shipment to make his order fullfill.

Notations:

$S \rightarrow$ NO. of units short / Backordered.

$C_b \rightarrow$ Backorder or Shortage Cost per unit back ordered / year.

$C_b =$ Rs./unit/year (Absence Of Inventory Per Year)

Total Inventory Cost;

$$TIC = O.C + H.C + \text{Shortage Cost } (S.C)$$

$$\boxed{O.C = \frac{D}{Q} \cdot C_o}$$

$$\boxed{H.C = \frac{(Q-S)^2}{2Q} C_h}$$

$$\boxed{SC = \frac{S^2}{2Q} \cdot C_b}$$

Holding Cost (H·C) for Period T ;

$$\boxed{(H\cdot C) = \left(\frac{Q-S}{2}\right) t_1 \cdot C_h}$$

$$(Q-S) = t_1 \cdot d \qquad \Rightarrow d = \frac{Q-S}{t_1}$$

$$Q = T \cdot d \qquad d = \frac{Q}{T}$$

$$\frac{Q-S}{t_1} = \frac{Q}{T} \qquad \frac{Q-S}{Q} = \frac{t_1}{T}$$

$$\frac{t_1}{T} = \frac{Q-S}{Q}$$

$$\boxed{t_1 = \left(\frac{Q-S}{Q}\right) T}$$

$$H\cdot C = \left(\frac{Q-S}{2}\right)\left(\frac{Q-S}{Q}\right) \cdot T \cdot C_h$$

$$H\cdot C = \frac{(Q-S)^2}{2Q} \cdot C_h T$$

Annual H·C ;

$$\boxed{\text{Annual } H\cdot C = \frac{(Q-S)^2}{2Q} \cdot C_h (T \cdot N) \rightarrow 1\text{year}}$$

Shortage Cost for Period T ;

$$S\cdot C = \frac{S}{2} \cdot t_2 \cdot C_b$$

$$S = t_2 \cdot d$$

$$Q = T \cdot d$$

$$\frac{t_2}{T} = \frac{S}{Q} \qquad \boxed{t_2 = \left(\frac{S}{Q}\right) T}$$

~~Shortage Cost $(S.C) = \frac{S}{2}\left(\frac{S}{Q}\right)T$~~

Shortage Cost $= (S.C) = \frac{S}{2}\left(\frac{S}{Q}\right)T \cdot C_b$

$$\boxed{S.C = \frac{S^2}{2Q} \cdot C_b \cdot T}$$

Annual Shortage Cost ;

***

$$\boxed{A.S.C = \frac{S^2}{2Q} \cdot C_b \cdot (T \cdot N)} \rightarrow \text{for 1 year}$$

Total inventory Cost (TIC) :-

**

$$\boxed{TIC = \frac{D}{Q} C_o + \frac{(Q-S)^2}{2Q} C_h + \frac{S^2}{2Q} \cdot C_b}$$

↓ on solving

**

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}} \sqrt{\frac{C_b + C_h}{C_b}}} \rightarrow \text{Cost factor} > 1$$

for I model EOQ

$C_b \rightarrow \infty$

$$\sqrt{1 + \frac{C_h}{C_b \rightarrow \infty} \rightarrow 0}$$

at EOQ ;

At EOQ;

Ordering Cost = Holding cost + Shortage Cost

$$\boxed{O \cdot C = H \cdot C + S \cdot C}$$

Best order size;

$$\boxed{TIC^* = \sqrt{2 D C_o C_h} \sqrt{\frac{C_b}{C_b + C_h}}} \quad < 1$$

Optimum number of ~~Ba~~ units Backorder or Short;

$$\boxed{(Q^* - S^*) C_h = S^* \times C_b}$$

$$\frac{Q^* - S^*}{S^*} = \frac{C_b}{C_h}$$

adding 1 both Side;

$$\frac{Q^*}{S^*} = \frac{C_b + C_h}{C_h}$$

***

$$\boxed{S^* = Q^* \left(\frac{C_h}{C_b + C_h} \right)}$$

Maximum Inventory Level;

$$M^* = Q^* - S^*$$

$$\boxed{M^* = Q^* \frac{C_b}{C_b + C_h}}$$

Qn) A Dealer Supplies following information; ~~ann~~

annual demand = 10000 units

Ordering cost = Rs 10/order

Inventory Caring Cost = 20% of C ~~unit price~~

unit price = Rs 20/unit

The dealer is Considering Possibility of backordering & he had ~~estimat~~ estimated that the annual cost of backordering per unit will be 25% of unit price. then Determine;

1) optimum number of units. He should buy.
2) Quantity to be backordered.
3) Max. inventory level.
4) Would you recommend to allow backordering, if so, the annual Cost Saving by adopting the policy of Backordering.

Soln)

$D = 10000$ unit

~~$C_o = 100000$ Rs.~~

~~$C_h = 20000$ Rs.~~ $\quad C_h = 4$

~~$C_o = 200000$ Rs.~~

$C_b = 5$ Rs/~~order~~unit

~~$C_b = 5\%$~~

$C =$ Rs. 20/unit

$C_o =$ Rs. 10/order

$C_h =$ Rs 4

1) $Q^* = 300$ unit/order

② $S^* = 183$

③ $M^* = 167$

4) (a) Without Backordering

$$TIC^* = \sqrt{2DC_oC_h} = Rs.\ 894.98$$ best

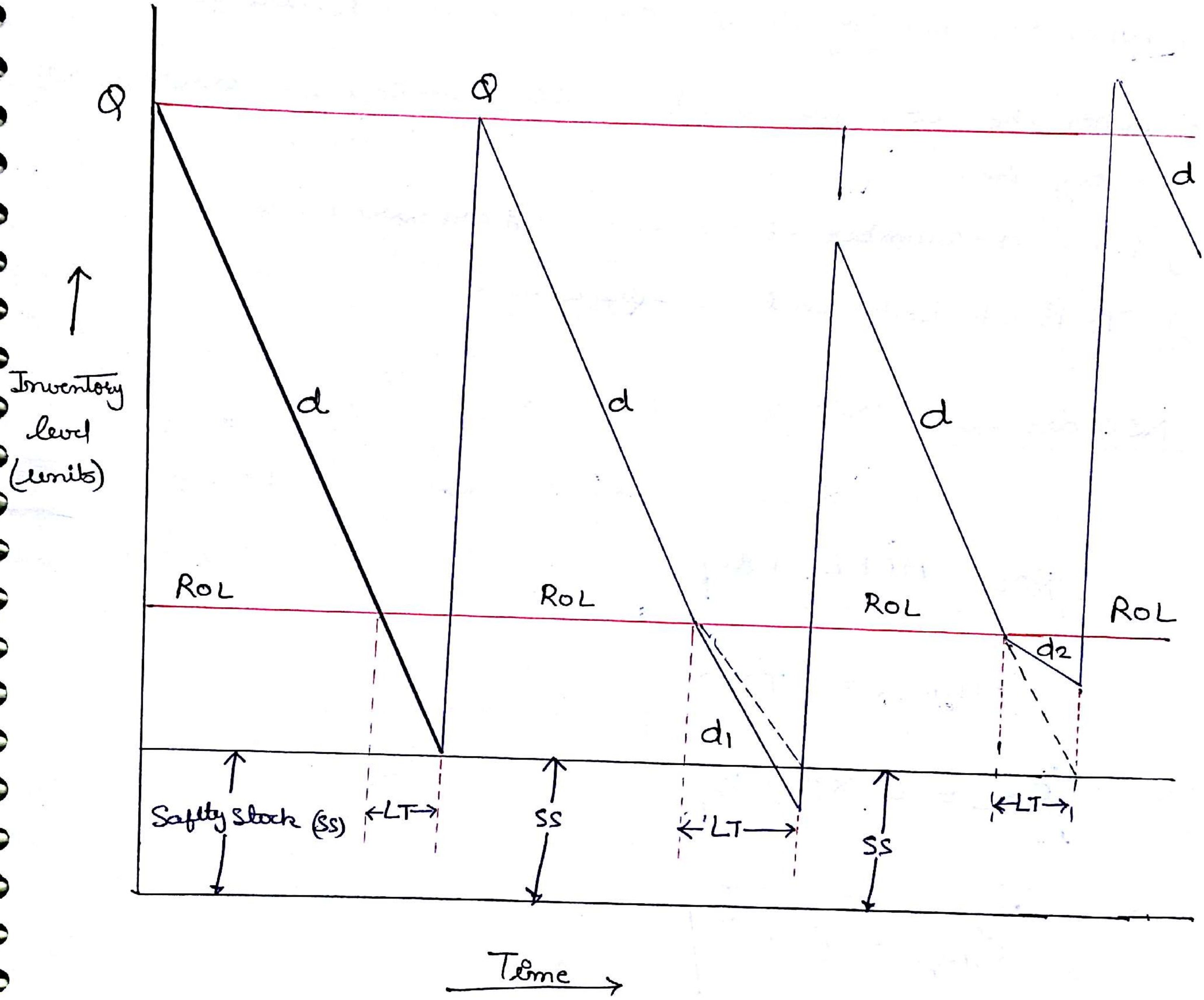
(b) With Backorder

$$TIC^* = \sqrt{2DC_oC_h} \cdot \sqrt{\frac{C_b}{C_b+C_h}} = Rs.\ 666.67$$

$\boxed{\text{Saving} = Rs.\ 227.76}$

Probablistic Model

Lead time & Demand changes & how to Kept Safety Stock



Factors encouraging higher Safety Stock :-

1) When the demand rate and lead time variations are more, and fluctuating.

2) When the inventory Holding cost is less is not of more concern.

3) When the loss due to absence of inventory i.e. Shortage cost is very high.

4) When the number of orders in a year are more.

5) To Provide better Customer Satisfaction.

Reorder Level (ROL) :

(ROL) = Average Demand during Lead time (LT) + Safety stock

$$\boxed{ROL = ADDLT + SS}$$

$$ADDLT = LT \times d$$

$$\boxed{ROL = LT \times d + SS}$$

$$\boxed{Q_{Avg} = \frac{Q}{2} + SS}$$

i) Demand Profit Model
or
Static Inventory Model

for Perishable items & time Bounded & (demand uncertain)

→ In this Model, demand is uncertain and decision is based on single order i.e reordering is not Permitted.

→ This model is applied for perishable item, like vegetables, fruits, flowers etc. or for those items which becomes outdated very fast

~~ϕ~~ →

D → Demand

S → Supply

p → Profit/unit

l → loss/unit

1) if $D > S$

$\Rightarrow$ $\boxed{(D-S)\cdot p}$ → Potential loss.

2) if $S > D$

$\Rightarrow$ $\boxed{(S-D)\cdot l}$

where p;

$$\boxed{p = S_p - C + C_b}$$

$$\boxed{l = C - C_s + C_h}$$

p → Potential profit or Loss/unit. for not meeting the demand.

S_p → Selling [price/unit].

C → Purchasing cost/unit.

C_b → backorder or Shortage cost or good will loss.

l → unsold item loss/unit.

C_s → Salvage or Scrap Value/unit.

C_h → holding Cost/unit.

In this model, in order to maximise our profit, we select the ordering Quantity "S" in such a manner

Such that;

$$\boxed{P(S-1) < \frac{p}{p+l} \le P(S)} \quad **$$

Where,

$P(S-1) \rightarrow$ Cumulative probability of the demand for ~~(S-1)~~ (S-1) unit.

$P(S) \rightarrow$ Cummmulative probability of the demand for (S) unit

---

if $\frac{p}{p+l} = 0.47$

Demand	Probo	Cumm Prob	
1	0.06	0.06	
2	0.10	0.16	
3	0.14	0.30	
4	0.11	0.41	(S-1) P
5	0.09	0.50	P(S)
6	0.07	0.57	

if, $S_p = 0$, $C = 0$, $C_s = 0$

then,

$$P(S-1) < \frac{C_b}{C_b + C_h} \leq P(S)$$

The thing which is not given in Question is taken as Zero

$p = \cancel{Sp}^{0} - \cancel{C}^{0} + C_b$

$l = \cancel{C}^{0} - \cancel{C_s}^{0} + C_h$

Qn) A Shopkeeper purchase a Seasonal product at the beggining of Season and Cannot reorder. The item Cost is Rs. 40 and Sell it at Rs. 75 each. For any item that cannot meet on demand He had estimated the goodwill loss of Rs. 20. Any item unsold will have a Salvage value of Rs. 25 and Holding Cost during the Period is 25% of Purchasing Cost. find the optimum stock to maximize the profit.

Demand	Probability	Cumulative prob.
1	0.08	0.08
2	0.07	0.15
3	0.10	0.25
4	0.12	0.37
5	0.09	0.46
6	0.13	0.59 P(S-1)
7	0.11	0.70 P(S)
8	0.14	0.84
9	0.10	0.94
10	0.06	1

$S_p = 75$

$C = 40$

$C_b = 20$

$C_s = 25$

$C_h = 10$

$p = 75 - 40 + 20$

$p = 55$

$l = 40 - 25 + 10$

$l = 25$

$$\frac{p}{p+l} = \frac{55}{80} = 0.6875$$

Qn) find the shortest cost range when the holding cost is Rs. 3 & demand & Probability distribution is as given below with Optimum stock level of 7 unit

	Demand	Probability	Cummulative Probability Cumm. Pr
	1	0.05	0.05
	2	0.08	0.13
	3	0.09	0.22
	4	0.15	0.37
	5	0.11	0.48
(S-1) →	6	0.12	0.60 → P(S-1)
S →	7	0.04	0.64 → P(S)
	8	0.16	0.8
	9	0.11	0.91
	10	0.09	1

$C_h = 3$

$p = Sp - C + C_b$ $\qquad$ $l = C - C_S + C_h$

$p =$

$$P(S-1) < \frac{C_b}{C_b+C_h} \le P(S)$$

$$0.6 < \frac{C_b}{C_b+3} \le 0.64$$

~~0.6~~ $\boxed{Rs.\ 4.62 < C_b < Rs.\ 5.33}$ Ans

ii) Service Level Model :-

→ This model is preferred, where the different cost factor involve with inventory are not known adjactly.

→ It is based up on probability theory. and the amount of safety stock is kept according to the level of service, management wants to achieve.

Service Level is given by;

$$\text{Service Level} = \left[\frac{\text{No. of unit supplied without delay}}{\text{Total no. of units demanded}}\right]_{LT}$$

$S \rightarrow 0 \text{ to } 1$

$S \rightarrow 0 \text{ to } 100\%$

95% Service Level is the Standard Value, and it means that 95% of the Customer demand on an average is fullfilled during Lead time and only 5% of the Customers order on an average is Rejected due to Stock out during Lead time.

When the demand during lead time may be Approximated by a normal distribution with certain average $[\bar{x} \text{ or } \mu]$ and Standard deviation (σ)

and Reorder level is given by,

$$\boxed{ROL = \bar{x} + Z\cdot\sigma}$$

$Z\cdot\sigma$ = Safety Stock

$\bar{X}$ = Average demand during Lead time

$$\boxed{\bar{X} = LT\cdot d}$$

σ = Standard deviation for the demand variation during Lead time

Z = Standard normal Variante, whose value depends up on service level required

NOT, stockout

Z	Service Level (%)
0.84	80%
1.28	90%
1.645	95%
2.33	99%

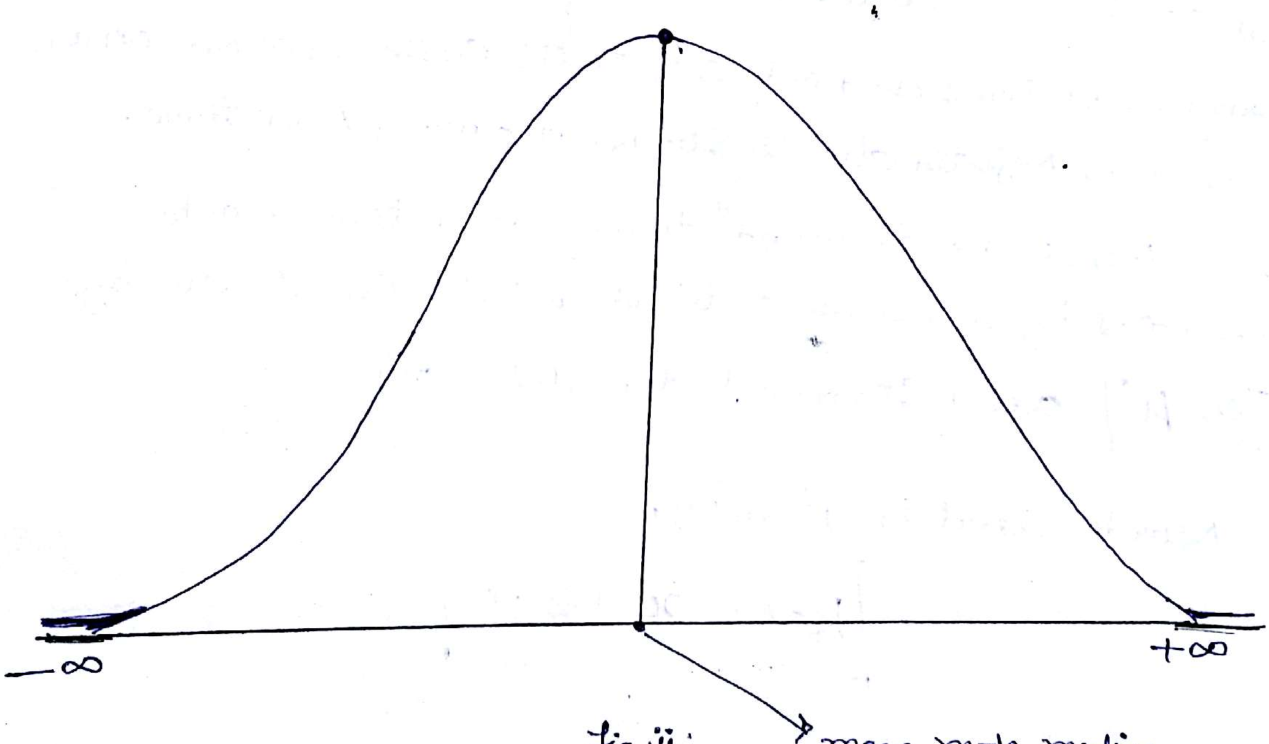


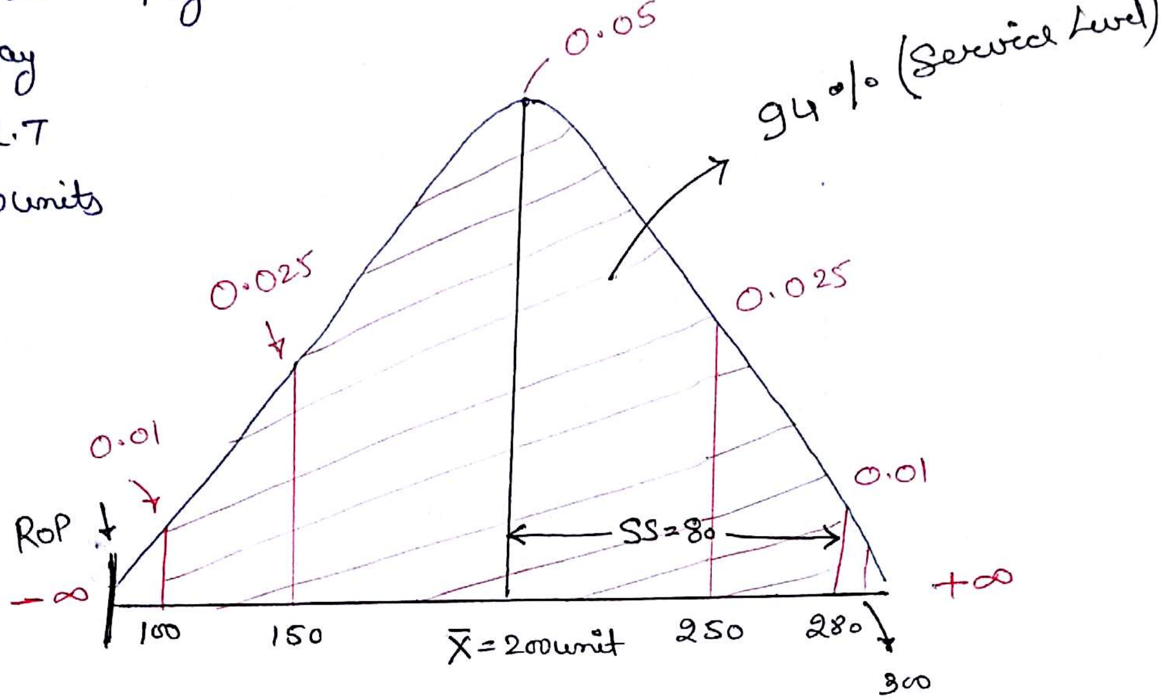
fig iii

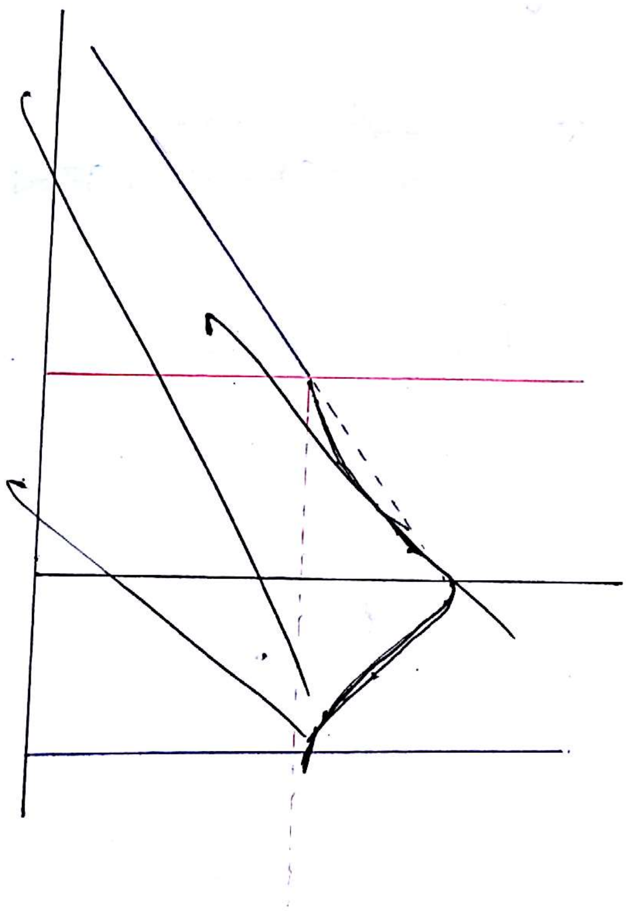
Let D = 20 unit/day

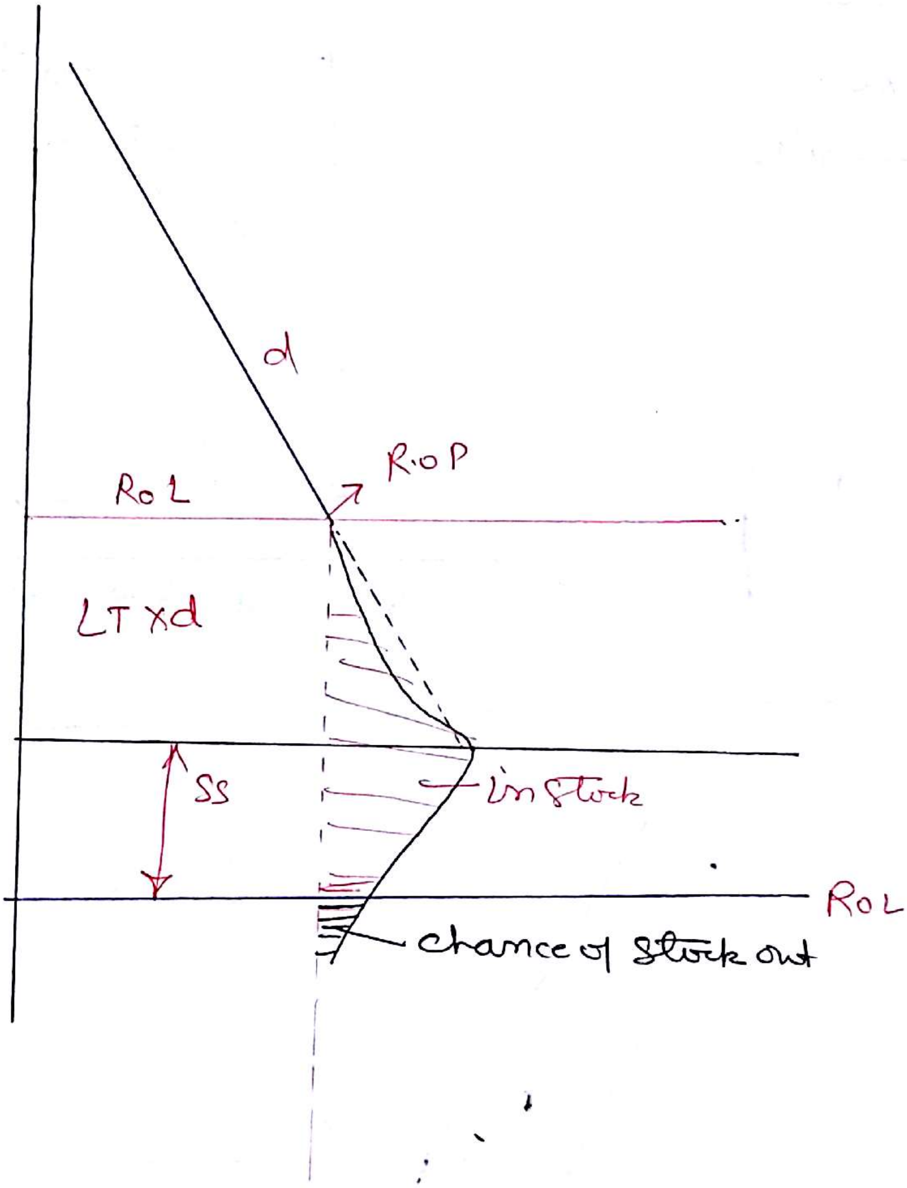
LT = 10 day

$\bar{X} = D \times L.T$

$\bar{X} = 200$ units






d
R.O.P
RoL
LT x d
SS
in stock
RoL
chance of stock out

σ = Variation in demand. $\boxed{\sigma \uparrow \rightarrow SS \uparrow}$

x_1, x_2, x_3

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3}$$

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2}{3}}$$

100	
20	

$\mu = 60$

60	
60	

$\mu = 60$

80	
40	

$\mu = 60$

70	
50	

$\mu = 60$

Safety stock :

$$\boxed{SS = \underset{1.645}{Z}\sigma}$$

One Cycle consist of 2 Parts,

1 half = σ_1

2nd half = σ_2

then σ for Complete Cycle

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

$$\boxed{\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}}$$

Qn) Average weekley demand is of 800 unit and Weekley Standard deviation is of 100 unit. Holding Cost is Rs. 0.2 per unit per week i.e Rs. 0.2/unit/week and unit price of inventory is Rs. 40. Lead time is of 4 week, then for 95% Service level, determine

i) Safety Stock

ii) Reorder Level

iii) Annual Cost of ~~reordering~~ (maintaing)

$$\bar{X} = \frac{800+100}{2} = \frac{900}{2} = 450$$

$$\boxed{\bar{x} = 450}$$

$$\sigma = \sqrt{(800-450)^2 + (100-450)^2}$$

$$= \sqrt{(350)^2 + (350)^2}$$

$$= \sqrt{}$$

SS =

$$\boxed{\sigma = 494.97}$$

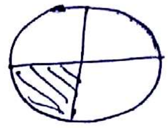
σ of 4 week = 1979

Sol^n)

$d = 800$ unit/week

$\sigma = 100$ unit /week

$LT = 4$ week



As LT is of 4 week and σ is given into weekly

So, converting σ corresponding to lead time

$\sigma'^2 = \sigma_1^2 + \sigma^2 + \sigma^2 + \sigma^2$

$\sigma'^2 = 4\sigma^2$

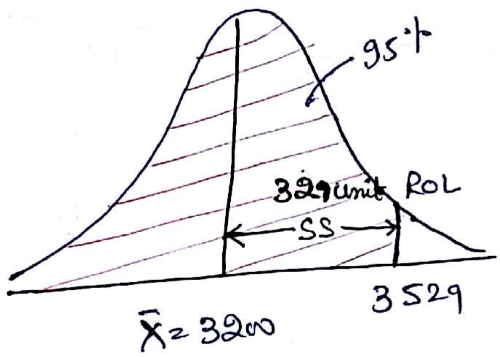
$\sigma' = 2\sigma$

$\sigma' = 2 \times 100 = 200$ unit

Safety Stock $= Z \cdot \sigma'$

$SS = 1.645 \times 200$

$\boxed{SS = 329 \text{ units}}$



② Reorder level

~~SS = Zσ'~~

~~SS = 1.6σ~~

$RoL = LT \times d + SS$

$= 4 \times 800 + 329$

$= 3529$ unit

③ Annual cost of maintaining

$C_h =$ Rs. 0.2/unit/week × 52 week/year

$C_h =$ Rs. 10.4 unit/year

$= SS \cdot C_h$

$= 329 \times 10.4 = \boxed{\text{Rs } 3421.6}$

Inventory Control and Classification :-

1) ABC Control :

Always Better Control

	Usage %	Items %
A —	50-60%	10-20%
B —	30-40%	30-40%
C —	10-20%	50-60%

Item	Item %	Demand (D)	Unit Price C	Usage Value (D·C)	usage %	
1	10%	200	200	40K	$\frac{4000}{\Sigma x} \times 100$	32%
2	10%	70	60	4200	$\frac{4200}{\Sigma x} \times 100$	20%
3	10%	300	500	1.5L	$\frac{1.5L}{\Sigma x} \times 100$	18
4						
5						
6						
7						
8						
9						
10				(Σx)	100%	

A

B

6

4
3
1

C

usage 100%

Uso

1) ABC – Control :- (D.C) (usage) Shows Part of units in Annual budget

→ In ABC Control, inventory items are classified into A, B, C, categories, depending upon their usage value.

→ For A category items → inventory is kept almost nil. Frequent Review is Done.

→ For C category items → Large amount of inventory is kept.
• Reviewd after a long period.

→ Most preferred.

(Pareto Law 80-20 Law)

2) VED, Vital, Essential, Desirable :-

Inventory items are classified on the basis of importance of inventory item for the production system.

3) HML High, Medium, Low :- (C) → (Rs./unit)

Inventories are classified on the basis of unit price of inventory items.

4) SDE → Scare, Difficult, Easy → Availability

Inventories are classified, on the basis of availability of inventory item. e.g (Thermal Power plant)

Sequencing

In Sequencing, our aim is to find a order in which different jobs are to be processed on different machines So that the idle time is minimized & utilization is optimised.

It is essential for smooth flow of material.

And effective utilization of manpower & machine.

Rules or Assumptions in Sequencing :

1) If Nothing is mentioned, the processing order for machine remains fixed or constant.

2) One job on one machine at a time.

3) Once a job is Started, it must be fully completed.

4) Time taken by the job, from one machine to another is negligible.

5) Irrespective of order, the processing time for the job remains Constant.

N - Job on one Machine :-

Terms:

1) Job flow time:-

It is the time from some starting point untill that particular job is completed

2) Make Span Time (MST):

It is the time from when processing begin. on the first job in the set untill the last job is completed.

3) Tardiness or Lateness :

It is the amount of time by which a job is delayed beyond its due date.

4) Average no. of Jobs in System :-

It is the term use to represent, average no. of jobs present all the time within the system, untill one set of job is completed.

→ It is the Ratio of total job flow time over make span time

Sequencing Rules for N-Jobs on Machines:

Lowest is preferred.

1) Shortest Processing Time (SPT):-

In this rule, jobs are processed in increasing order of their processing time. i.e min. time first & max. time at last.

Sequencing	Job	Processing time	Due Date / Expected time	EDD
2	1	8	45	2
1	2	6	52	3
4	3	10	62	5
5	4	15	58	4
3	5	9	38	1

ii) Earliest Due date:- [EDD]

In this Rule jobs are sequenced in increasing order of their due date

Job	Processing time	Due date	EDD
1	8	45	2
2	6	52	3
3	10	62	5
4	15	58	4
5	9	38	1

iii) Critical Ratio Rule (CR):

$$\text{Critical Ratio (CR)} = \frac{\text{Due Date}}{\text{Processing Time}}$$

Jobs are Sequenced in increasing order of Critical Ratio.

iv) Slack Time Remaining Rule (STR):

$$\text{Slack time} = \text{Due date} - \text{Processing Time}$$

Pn) Set of jobs are to be processed on a single m/c, obtained a Sequence using SPT & EDD Rule. Also determine makespan time, job flow time for each job. Average job flow time per job, Average tardiness per job, Average number of jobs in system & no. of tardi jobs.

SPT	Job	Processing Time	Due date	EDD
2	A	8	71	4
6	B	15	64	3
3	C	10	82	6
1	D	6	59	2
8	E	19	88	7
5	F	14	42	1
4	G	12	91	8
7	H	17	76	5

Job	P.T	DD	Job flow time	Tardiness
D	6	59	0+6=6	0
A	8	71	6+8=14	0
C	10	82	14+10=24	0
G	12	91	24+12=36	0
F	14	42	36+14=50	8
B	15	64	50+15=65	1
H	17	76	65+17=82	6
E	19	88	82+19=101	13
	101		ΣJ.F.T = 378	

101 ↓ Processing time || Make span time

MST = 101

2) D-6, A-14, C-24, G=36

(4) Average Job flow time =

i.e. = SPT

(3) Average Job flow time = $\frac{\text{Total Job flow time}}{\text{No. of Job}} = \frac{378}{8} = 47.25$

(4) Average Tardiness Per Job

$$\text{Average Tardiness / Job} = \frac{\text{Total tardiness}}{\text{No. of Job}} = \frac{28}{8} = 3.5$$

(5) Average no. of Job in System :-

SPT

$$\text{Average no. of Job in System} = \frac{\text{Total Job flow time}}{\text{Processing time (Make Span time)}} = \frac{378}{101} = 3.74$$

(6) No. of Tardiness Job = 4, FBHE

Job	PT	DD	Job flow time	Tardiness
F	14	42	0+14 = 14	0
D	6	59	14+6 = 20	0
B	15	64	20+15 = 35	0
A	8	71	35+8 = 43	0
H	17	76	43+17 = 60	0
C	10	82	60+10 = 70	0
E	19	88	70+19 = 89	1
G	12	91	89+12 = 101	10
	$\Sigma = 101$		$\Sigma 432$	

① MST = 101

② F – 14, D – 20, B – 35, A – 43, H – 60, C – 70, E – 89 G – 101

③ $\frac{432}{8} = 54$ Average Job flow time

EDD

④ Average Tardiness = $\frac{\text{Total Tardiness}}{\text{No. of Job}} = \frac{11}{8} = 1.375$

⑤ Average no of job in system =

$$\frac{432}{101} = 4.277$$

⑥ No. of Retarded Job ② 2 jobs

Qn). Four job are to be processed on a Single m/c as per data given below

i) using EDD rule, find the no. of jobs delayed

ii) using SPT Rule, find the total tardiness

SPT	Job	P.T	DD	EDD
2	1	4	6	— 1
3	2	7	9	— 2
1	3	2	19	— 4
4	4	8	17	— 3

SPT

Job	P. T.	DD.	Job flow time	Retardation
3	2	19	0+2=2	0
1	4	6	4+2=6	0
2	7	9	6+7=13	4
4	8	17	13+8=21	4
	$\Sigma=21$ (M.ST)		$\Sigma=42$	

Job	P.T	D D	Job flow time	Tardiness
1	4	6	0+4=4	0
2	7	9	4+7=13	9
4	8	17	13+8=21	4
3	2	19	21+2=24	5
	Σ21		Σ=62	

N-Jobs on two m/c

M/c	A	B
1	Ai	Bi
2		
3		
4		
⋮		
N		

Q)

m/c	A	B
1	6	9
2	10	8
3	6	9

$\boxed{1\ 3\ 2}$

or

$\boxed{3\ 1\ 2}$

Q)

mc	A	B
1	8	10
2	6	9
3	6	11

$\boxed{2\ \ 3\ \ 1}$

Q)

1	7	8
2	6	6
3	8	9

$\boxed{2\ 1\ 3}$

or

$\boxed{1\ 3\ 2}$

Q)

1	8	10
2	9	6
3	11	6

N – Job on two m/c Problems;

are solved by Johnson's Rule. Only Applicable for 2 M/c

<u>and the Steps involved are;</u>

1) Find out the min. in Ai and Bi.

2) If the min. is for Particular job on m/c A then perform that Job at the Start.

3) If the min. is for the particular job on m/c B then perform that job in the last.

4) Strike off the job which is assigned so that it can't be considered again.

5) Continue the similar manner untill all job are assigned.

Qn) find the Optimum Sequence for the following set of job to be processed on the machine. Also find the expected time, idle time for each machine and their % utilization.
Also prepare $\boxed{\text{Grantt chart}}$ for both the machine

Job		m/c I		m/c II
A	—	~~7~~	—	~~3~~
B	—	~~4~~	—	~~8~~
C	—	~~2~~	—	~~6~~
D	—	5	—	6
E	→	~~9~~	—	~~4~~
F	→	~~8~~	—	~~1~~
		$\Sigma M/c\,I = 35$		$\Sigma m/c\,II = 28$

C B D   E A F

$\boxed{C - B - D - E - A\ F}$ Sequence

Job	m/c I		m/c II	
	In	out	In	out
C	0	2	2	8
B	2	6	8	16
D	6	11	16	22
E	11	20	22	26
A	20	27	27	30
F	27	35 (1)	35	36 → MST

M.S.T

both out
out → Max m/c
Check for Max

m/c II idle time (8)

MST = 36 $\boxed{m/c\ I = 1}$

$\boxed{\text{Idle time} = \text{MST} - \text{Working time}}$

m/c I Idle time → 36 − 35 = 1 min

m/c II idle time → 36 − 28 = 8 min

$$\boxed{\%\ \text{utilization} = \frac{\text{Working time}}{\text{MST}} \times 100}$$

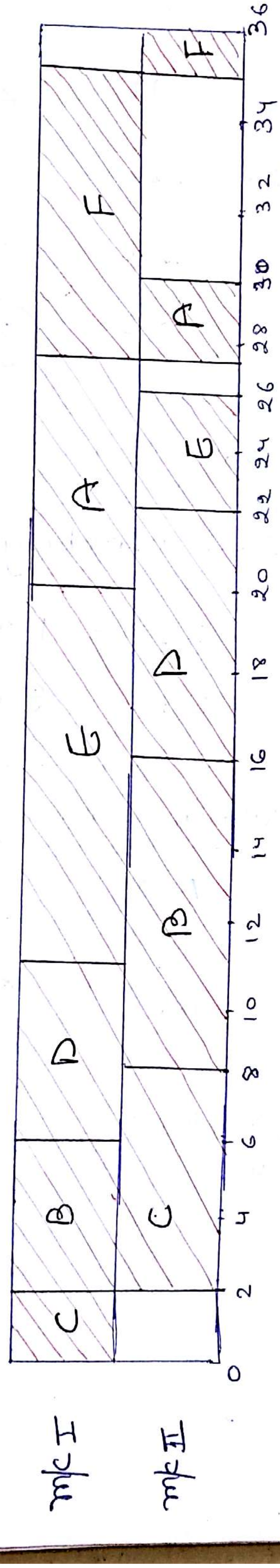
Job flow time for B = 16

$$m/c\ I = \frac{35}{36} \times 100\ \%$$

$$m/c\ II = \frac{28}{36} \times 100\ \%$$

Grantt. Chart:-

v.imp



Qn) find the optimum sequence for the following set of job. also find make span time (MST) and idle time for each m/c.

Job	M/c I	M/c II
A	3	7
B	6	9
C	5	8
D	10	5
E	7	4
F	9	10
G	8	4
H	6	9
I	5	12
	Σ59	68

A C I B H F D G E

(1) m/c (I) idle time =

71 − 59 = 12 min

m/c II 71 − 68 = 3 min

Job	M/c I In	M/c I out	M/c II In	M/c II out
A	0	3	3	10
C	3	8	10	18
I	8	13	18	30
B	13	19	30	39
H	19	25	39	48
F	25	34	48	58
D	34	44	58	63
G	44	52	63	67
E	52	59	67	71

71 — MST

MST = 76

idle time =

m/c I

N- Job on 3 m/c

m/c	A	B	C
Job	A_i	B_i	C_i
1			
2			
3			
4			
⋮			
N			

Condition

① most min $A_i \geq$ Most Max. B_i

② Most min $C_i \geq$ Most Max. B_i

$$X_i = A_i + B_i$$

$$Y_i = B_i + C_i$$

Now
Apply Johnson Rule to find Sequence.

PERT-CPM

Gomb

Project :-

It is a group or Combination of inter-related activities that must be executed in certain fixe order before the entire task is completed.

Activities are inter related in a logical sequence in the sense that some activity can only be started when all the activities earlier to it are completed.

Event :-

Event denotes the point of time or the accomplishment occuring at a moment and is normally used to denote the starting and the end point of an activity.

It neither consume any time nor resources for its complition.

Activity :

It is a recognizable part of a project, which consume time & Resources for its Completion ~~&~~ it may involve Physical or mental work.

When all the activities are executed then only project gets Completed

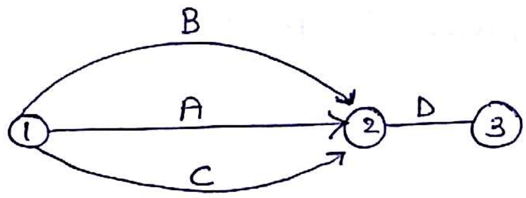
Network Diagram :-

It is the graphical representation of a logical Sequence in which different activities are interrelated to each other ~~by Go~~ while Completing a project.

Rules for Network Construction :-

1) An activity Can only be started, when all activities earlier to it are Completed.

2) No two or more activities may have the same head & tail events



NOT Possible X

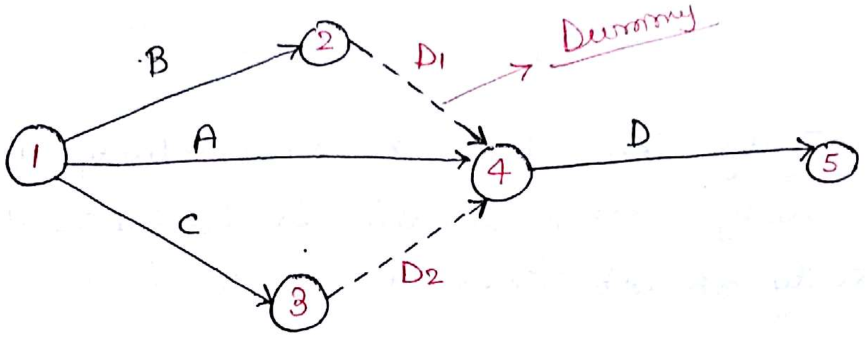
A → 1 → 2
B → 1 − 2
C → 1 − 2

- In above Condition, to represent the same logic, We need to use dummy activity. Represent by dotted line.

Dummy Activity :

An activity, which is used to show the logic, dependency or relationship of one activity over the other, but doesn't Consume any time or resources for its Completion is termed as dummy activity.

It is Represented by dotted line arrow.

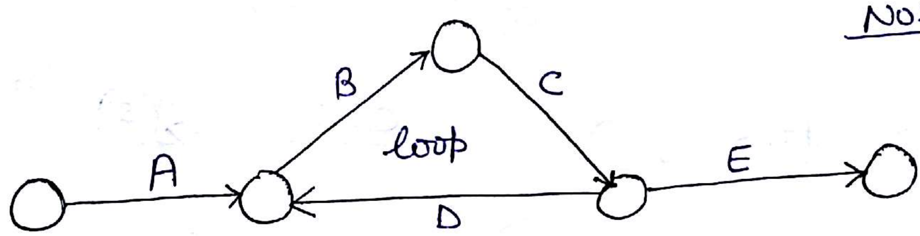


3) The Length and direction of arrow is incidative only time flow from left to right on the network diagram.

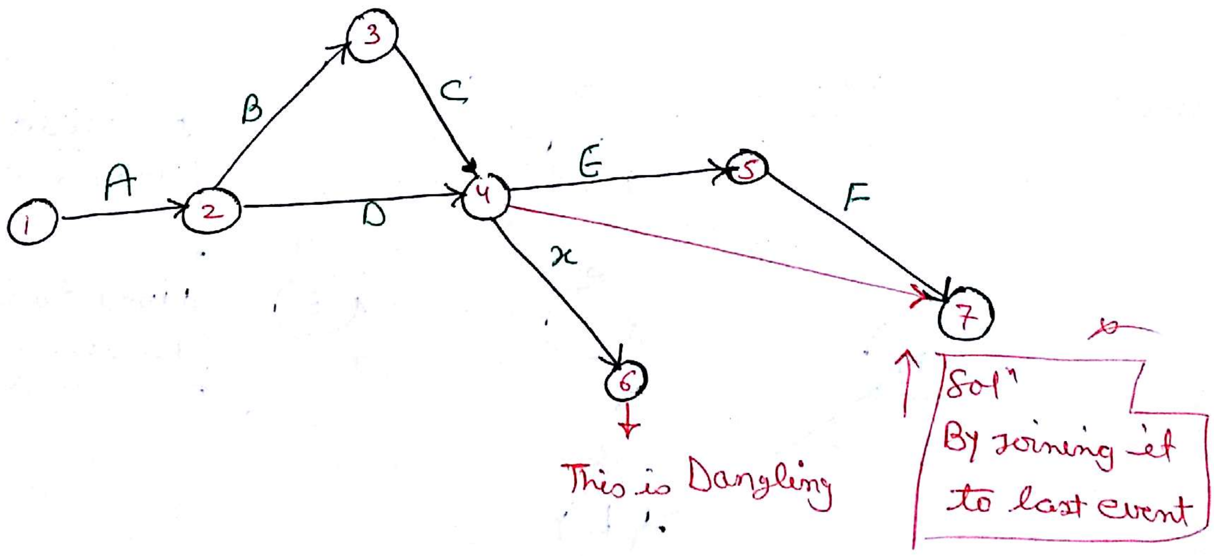
4) Dummy Activity Should only be used, when it is very necessary but there is no restriction on the number of dummy activity used.

5) There Should be no looping and Dangling on the network diagram

No Solution



Looping



Dangelling :

When activity other than the final activity, doesn't have any Sucessive activity then situation is called as Dangeling. Such activity should be connected to the last event of the network diagram.

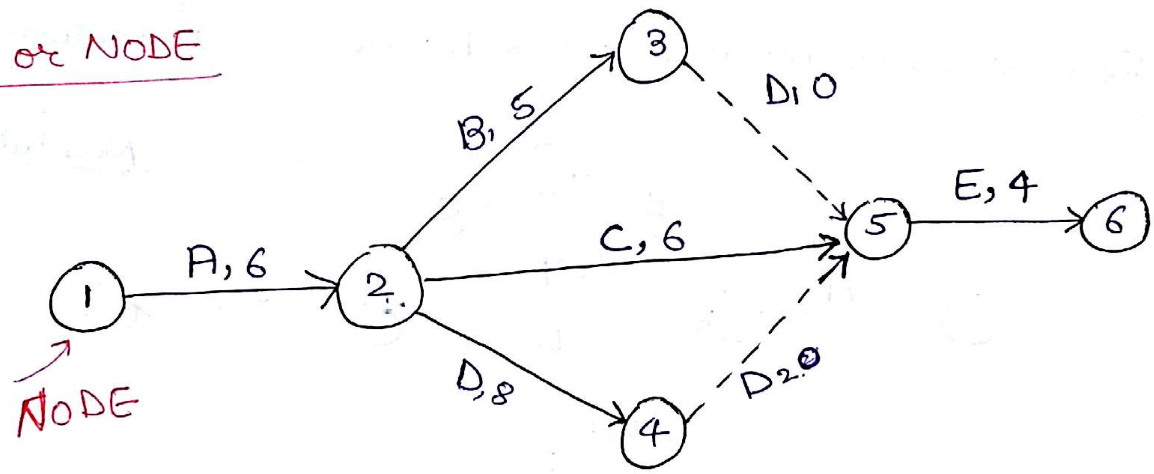
Types Of Network Diagram :

i) Event or NODE (EON)

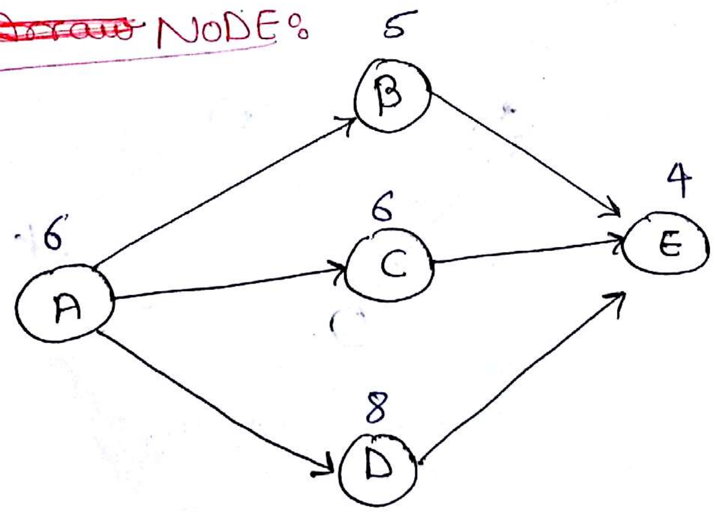
or

Activity on Arrow (A or A)

Event or NODE



2) Activity on ~~Arrow~~ NODE :



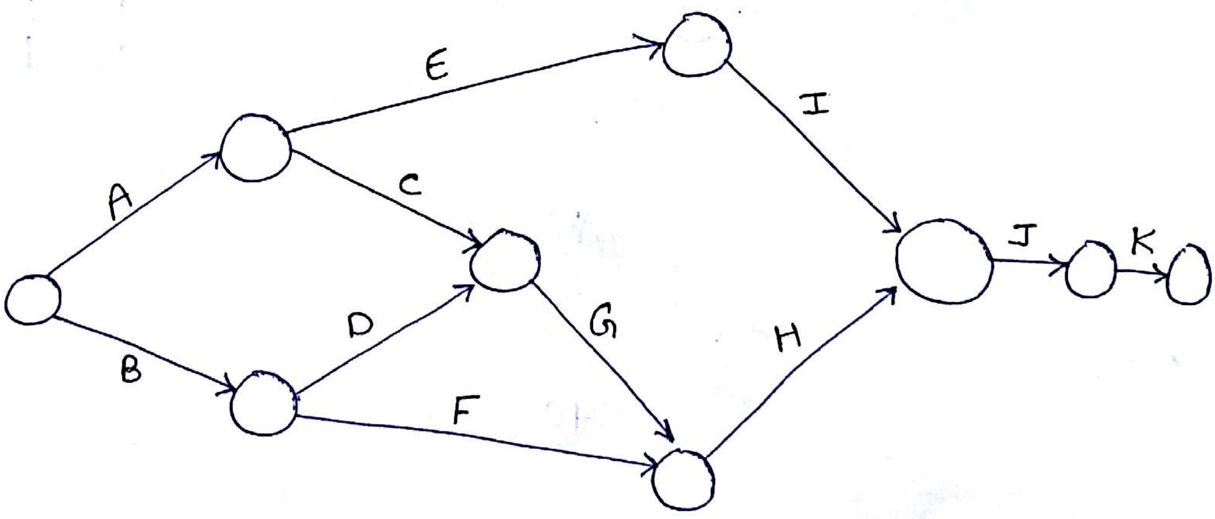
NOT used in PERT & CPM

Used in Line Balancing

Activity on Node, diagram, doesn't require dummy activity and it is considered to be simple and easy, irrespective of these advantages, Event on Node diagram is more popular in PERT & CPM.

Qn) Draw the Network diagram for the following set of Activities

Activities	Precedence
A	–
B	–
C	A
D	B
E	A
F	B
G	C, D
H	G, F
I	E
J	H, I
K	J



Fulkerson's Rule :→ [For Numbering Of Events]

① Which has NO incomming

② then neglect outgoing from above & Mark them No.

③ then neglect all outgoing from above and then mark.

Qn) Draw the Network diagram for the following Conditions.

i) A & D ~~Satrt~~ start at same ~~point~~ time

ii) F follows A

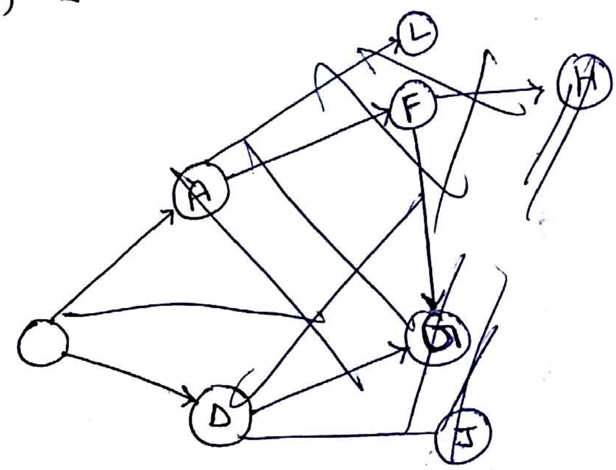
iii) ~~K~~ follows A but proceeds L → K After A but K before L

iv) G follows D but preceds J

v) G follows ~~E~~ F but preeceeds H

vi) M follows H but preeceeds L

vii) J and L terminate at the same time



✓A —

✓D —

✓F — A

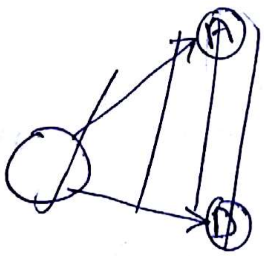
K — A — L

G — D — J

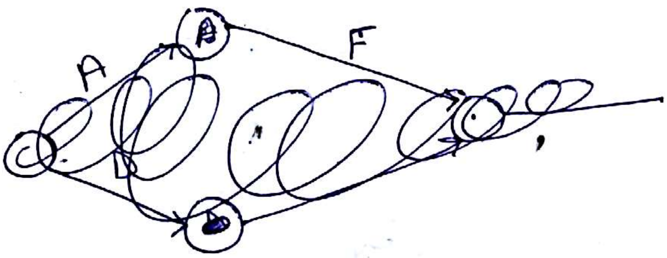
G — F — H

M — H — L

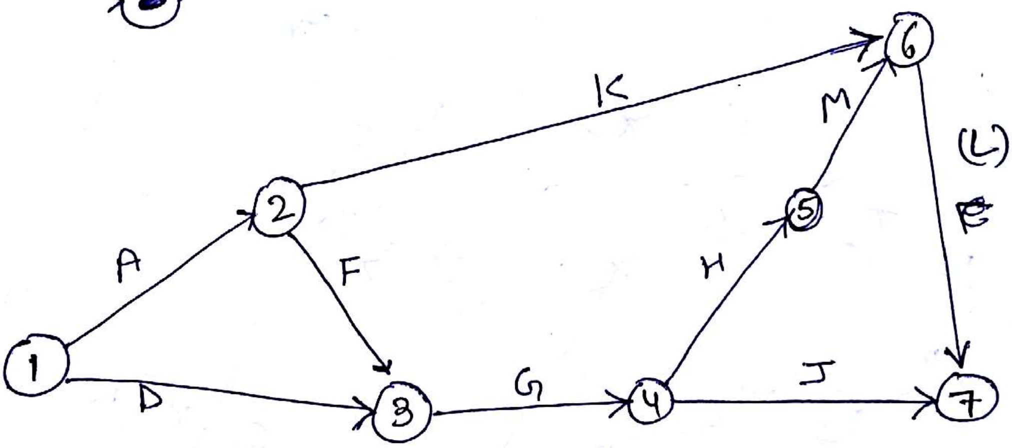
J — L



Activity	Precedence
A	–
D	–
F	A
G	D, F
H	G
J	G
K	A
L	K, M
M	H

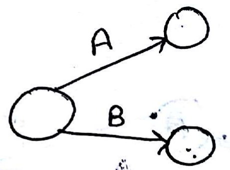


Sol^n

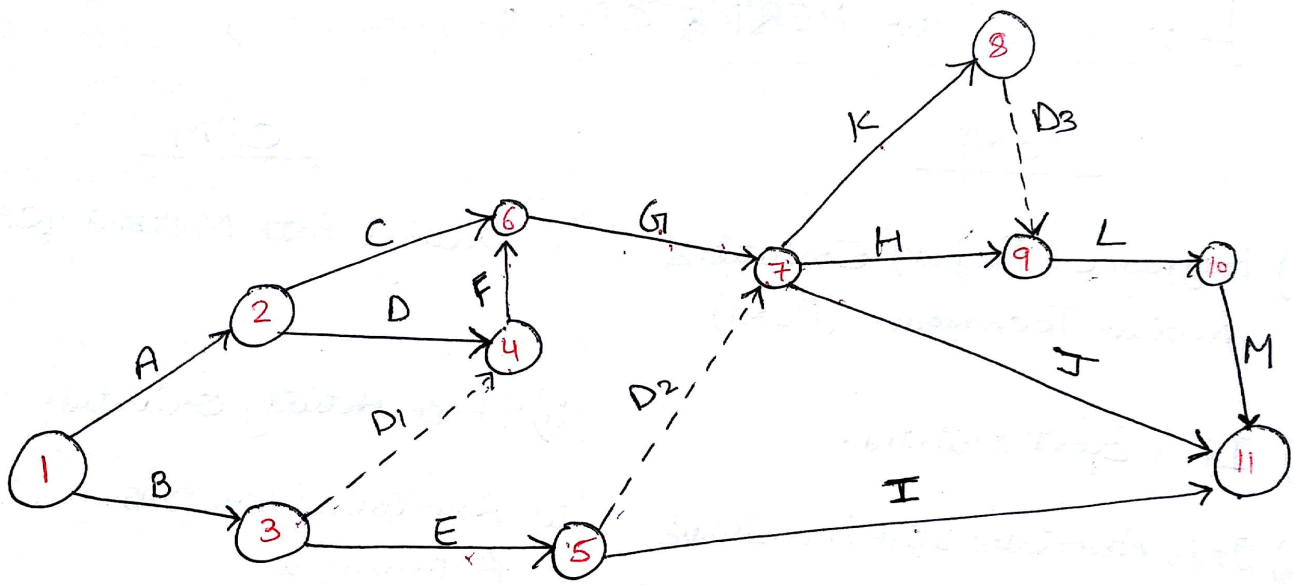


Q(m)

Activity	Procedence
A	—
B	—
C	A
D	A
E	B
F	B, D
G	C, F
H	G, E
I	E
J	G, E
K	G, E
L	H, K
M	L



Soln



Difference b/w PERT & CPM :-

PERT	CPM
1) Programe (Project) Evalution & Review Technique. (PERT)	2) Critical Path Method (CPM)
i) It is Event oriented.	i) It it Activity Oriented.
ii) It is Associated With Probabilistic Activities.	ii) Associated With Deterministic Activities.
iii) It is based up on three ③ time estimate to Complete an Activity.	iii) Based upon single time to Complete an activity.
IV) It is used Where time required to Complete Various activities is not Certain.	IV) used for Repetetive Job, Where one as prior experience Of Handelling Similar project.
v) It usually doesn't Consider Cost analysis.	v) It gives importance to Cost analysis. & Crashing is done to minimize the Cost of CPM Project
Application	Application
VI) It is used Mainly for Research and development project.	VI) used mainly for Construction Project

PERT

It is used for uncertain project and is based on three time estimate to complete an activity.

These are :-

1) Optimistic time (to or a)
2) Pessimistic time (tp or b)
3) Most likely time (tm or m)

1) Optimistic time [to or a] :

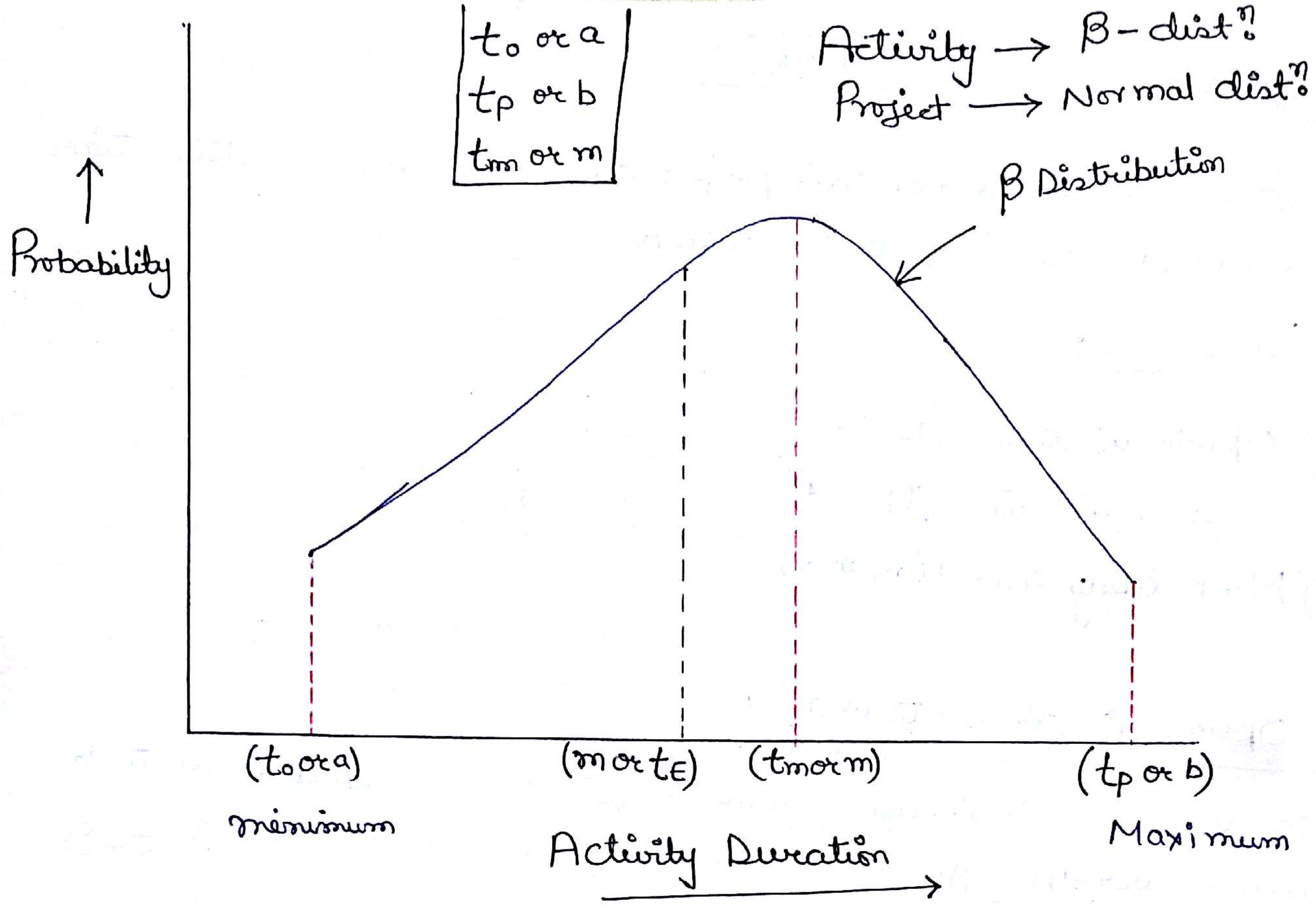
It denotes the minimum time required to complete an activity when everything goes according to the plan. Atleast 5 day. ☺

2) Pessimistic time (tp or b) :-

It denotes the maximum time required to complete an activity, when everything goes against the plan. Max 5 day

3) Most likely time (tm or m) :-

It is the time required to complete an activity, when executed under normal working conditions.



The fundamental assumption in PERT is, the three time estimate ~~from~~ form the end point of the distribution curve & activity is assumed to followed β-Distribution.

It is also assumed, that the probability of completing an activity i.e time a, & b are equal, and the probability of completing activity i.e m time, is 4 times of a, b

Average or expected time to complete an activity is given by;

$$\boxed{\mu \text{ or } t_E = \left[\frac{a+4m+b}{6}\right] = \frac{t_o+4t_m+t_p}{6}}$$

Standard deviation "σ"

$$\sigma = \left[\frac{b-a}{6}\right] = \left[\frac{t_p - t_o}{6}\right]$$

Variance :-

[uncertainity]

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

NOTE:

1) Varience gives the measure of uncertainity of Activity Completion.

→ Higher the value of varience, larger the uncertainity will be.

Varience $\propto$ uncertainity

Critical Path :

→ It is the max. time Consuming path, from the first event to last event in a network diagram.

→ The time taken along critical path is termed as expected Project Completion time i.e t_E.

→ The activities along the critical path are termed as critical activities & they are represented by double line Arrow.

Probability of Completing Project Within Schedule Time :-

→ If T_E, is expected project Completion time, σ

→ σ is standard deviation along critical path, then probability of Completing project within the schedule time T_S is given by;

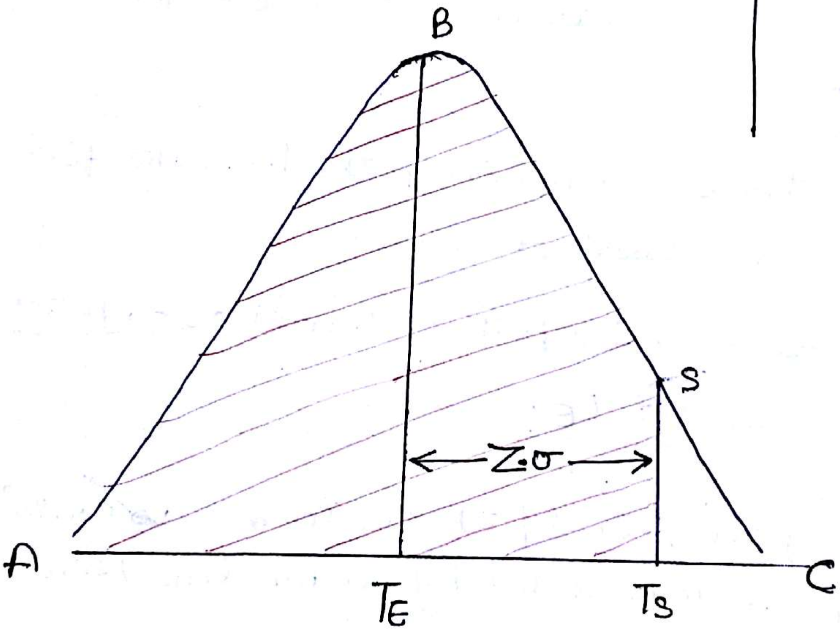
$$\boxed{Z = \frac{T_S - T_E}{\sigma}} \quad **$$

Where,

Z = is standard normal variant.

$$T_S = T_E + \sigma Z$$

$$\boxed{Z = \frac{T_S - T_E}{\sigma}}$$



Probability %.

$$\boxed{P(T_S) = \frac{\text{Area } ABS}{\text{Area } ABC}}$$

$$\sigma = \sqrt{\text{Sum of Varience along Critical Path}}$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_5^2 + \sigma_6^2 + \sigma_8^2}$$

<u>Critical Path :-</u> $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8$

Q

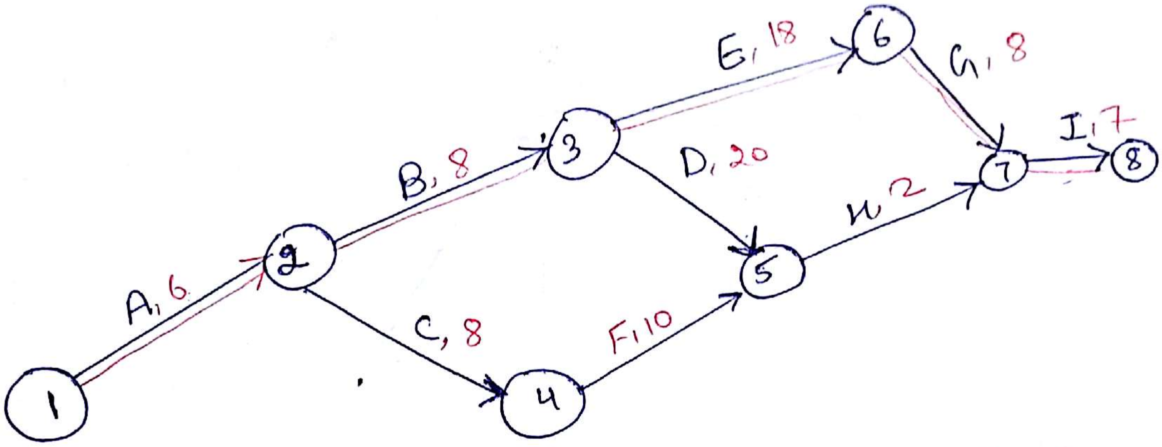
Qn) For the following Set of activities, draw the network diagram and determine;

i) Critical Path & expected project Completion time.

ii) Determine the Probability Of Completing the project in 50 days.

iii) If a, Company makes an agreement to Complete the project in 50 days, failing which, they would pay rupees 1000/- per day as fine. Find the probability, that the fine may be paid but not exceeding 5000 Rs.

Activity	Precedence	Time days a	m	b	$t_E = \frac{a+4m+b}{6}$	$\sigma = \frac{(b-a)}{6}$
A	–	4	6	8	6	0.66
B	A	5	7	15	8	1.66
C	A	4	8	12	8	1.33
D	B	15	20	25	20	1.66
E	B	10	18	26	18	2.66
F	C	8	9	16	10	1.33
G	E	4	8	12	8	1.33
H	D, F	1	2	3	2	0.33
I	G H	6	7	8	7	0.33
					$\Sigma t_E = 87$	



⟹ Critical Path

$$\boxed{1-2-3-6-7-8 = 47}$$

$1-2-4-5-7-8 = 33$

$1-2-3-5-7-8 =$

1) Critical Path;

A − B − E − G − I

$T_E = 47$ days.

2) $T_S = 50$ days , $T_E = 47$ days

$$\sigma = \sqrt{(0.66)^2 + (1.66)^2 + (2.66)^2 + (1.33)^2 + (0.33)^2}$$

$$\boxed{\sigma = 3.48 \text{ days}}$$

3) $Z = \dfrac{T_S - T_E}{\sigma} = \dfrac{3}{3.48}$

$Z = 0.8608$

↓ Probability

80.45

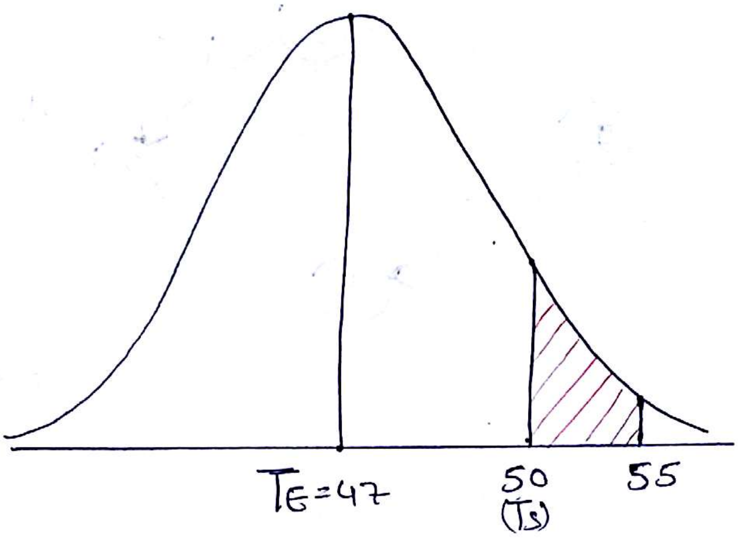
① Mode ~~top~~ 2 time

Select SD①

Shift + 3

P 1

direct Z



$$Z_1 = \frac{55-47}{3.496} = 2.288 \xrightarrow{\text{Prob}} 98.89\%$$

$$Z_2 = \frac{50-47}{3.496} = 0.8581 \xrightarrow{\text{Prob}} 80.45\%$$

$\boxed{18.44\%}$

Chance of Completion

Critical Path :-

The procedure of finding critical path is similar. Both in PERT & CPM.

It Consist of two phases;

1) Forward Pass Computation
2) Backward Pass Computation

1) Forward Pass Computation :

In this we compute, the time by which, event is expected to be completed at the earliest.

$$i \xrightarrow{t_E^{ij}} j$$

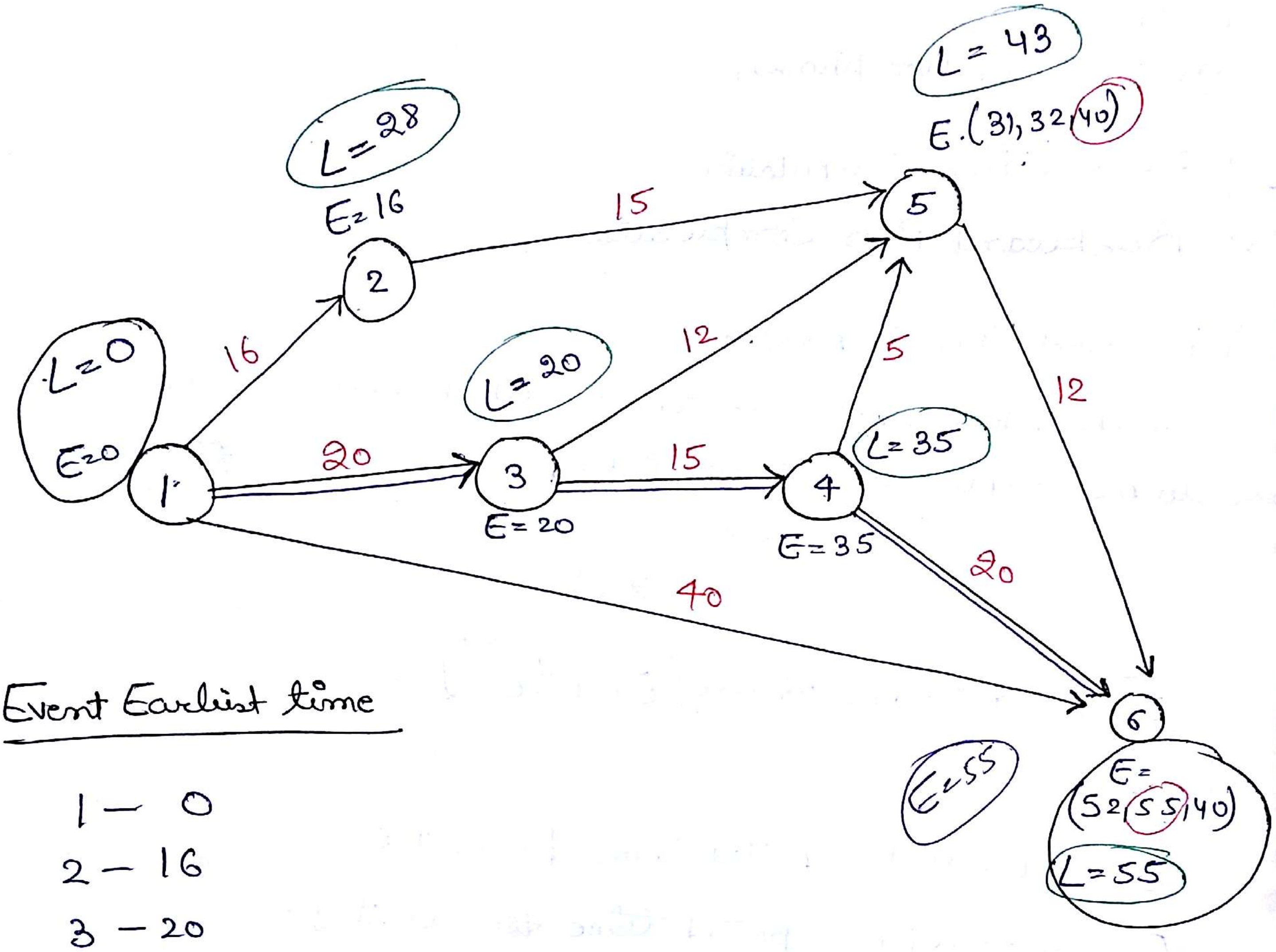
$$E_j = \text{Maximum of all } \left[E_i + t_E^{ij} \right]$$

where,

$E_i \rightarrow$ Earliest expected time for event i.

$E_j \rightarrow$ Earliest expected time for event j.

$t_E^{ij} \rightarrow$ Expected time for activity ij



Event Earliest time

1 — 0

2 — 16

3 — 20

4 — 35

5 — Max [31, 32, 40] 40

6 — Max. [52, 55, 40] 55

ii) Back Ward Pass Computation :

In this We Compute the time by which an event must be Completed at the latest

$$\underset{L_i}{(i)} \xrightarrow{t_E^{ij}} \underset{L_j}{(j)}$$

$$L_i = \text{Minimum of } [L_j - t_E^{ij}]$$

Where

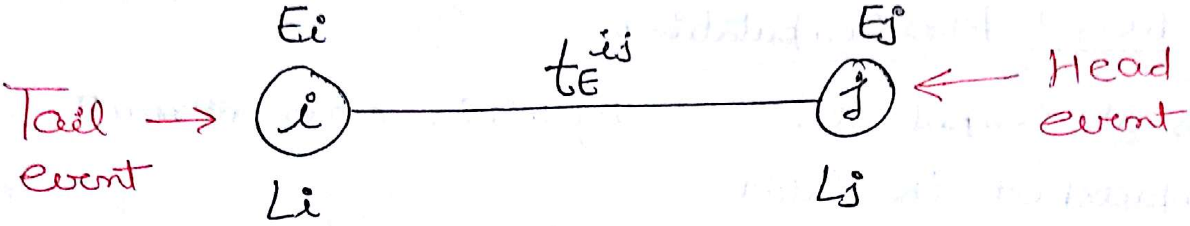
$L_i \rightarrow$ is the Latest allowable time for event (i).

$L_j \rightarrow$ Latest allowable time for event (j).

$t_E^{ij} \rightarrow$ Expected time for activity ij.

$\boxed{\text{Outgoing event min}}$

event	Latest time
6	— 55
5	— 43
4	— min [38, 35] 35
3	— min [31, 20] 20
2	— 28
1	— min (12, 0, 15) 0



For any activity to be critical, the following ③ conditions must be satisfy :-

i) Head event Slack = 0 $\quad [L_j - E_j] = 0$

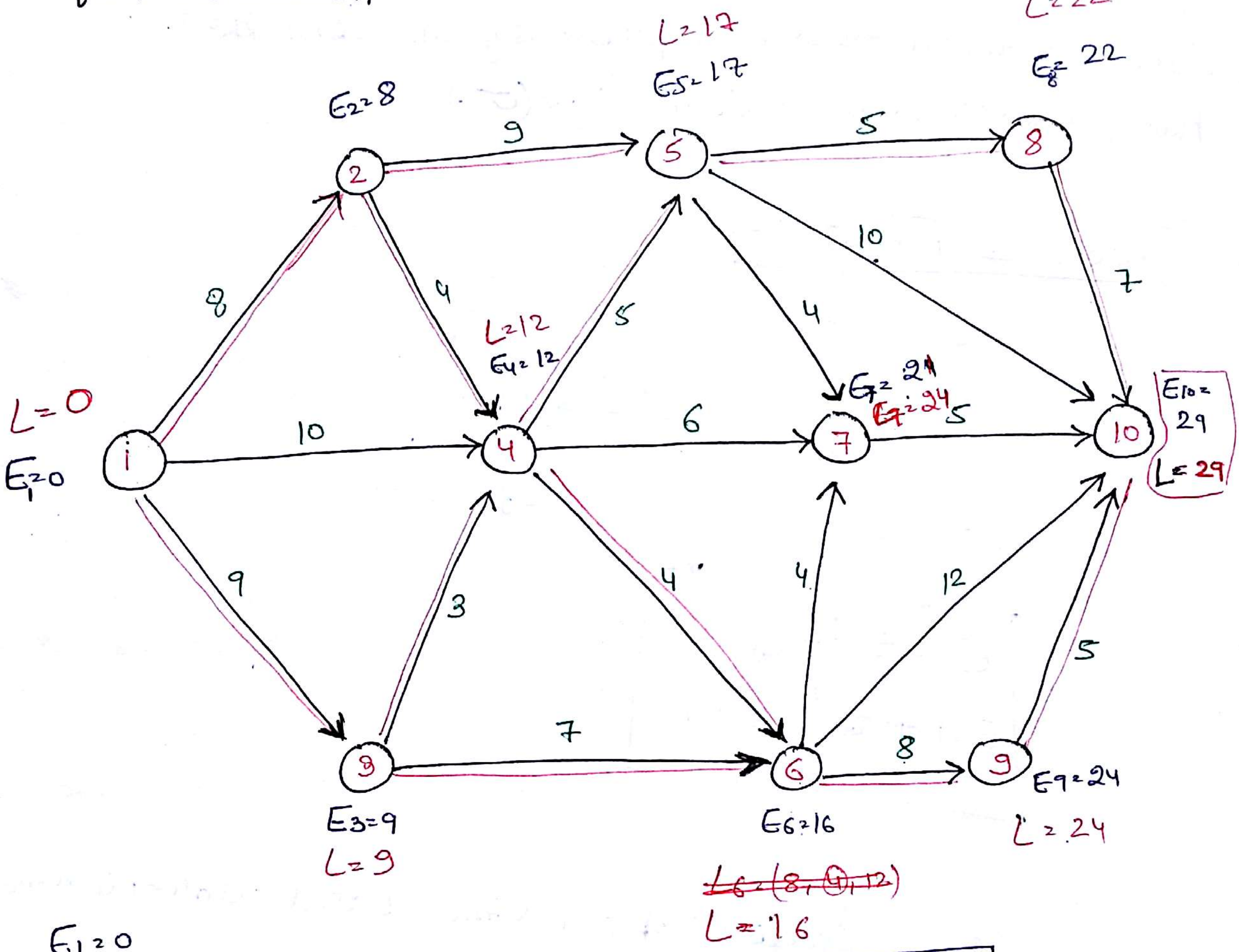
ii) Tail event Slack = 0 $\quad [L_i - E_i] = 0$

iii) There is

$$(L_j - L_i) = (E_j - E_i) = t_E^{ij}$$

"Critical Path is termed as critical, bcz, if any activity on this path is delayed by certain amount of time, the whole project is delayed by the same amount of time."

Q) For the network diagram shown below, find the Critical path & expected project Completion time.



$E_1=0$

$E_2=8$

$E_3=9$

$E_4=(12,10,12)$ $\underline{12}$

$E_5=[17,17]$ 17

$E_6=16$

$E_7=(18,16,21)$ $\underline{21}$

$E_8=22$

$E_9=24$

$E_{10}=$

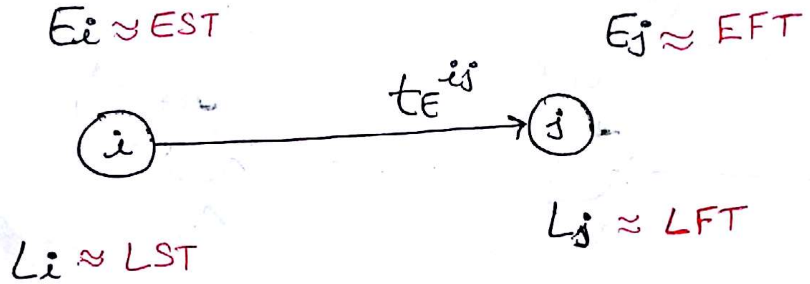
1) 1-2-5-8-10
2) 1-2-4-5-8-10
3) 1-2-4-6-9-10
4) 1-3-6-9-10
5) 1-3-4-6-9-10
6) 1-3-4-5-8-10

$$\boxed{Z=\frac{T_S-T_E}{\sigma}}$$ Same

NOTE:

In case of PERT, if there are more than one critical path then in order to determine probability, we select the path having max. standard deviation i.e (σ)

Slack & Float :-



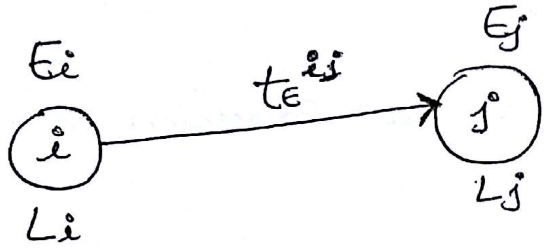
$$\boxed{\begin{aligned} E_i + t_E^{ij} &= E_j \\ EST + t_E^{ij} &= EFT \end{aligned}}$$

The terms like Earliest Expected time, Latest allowable time and Slack correspond to event in PERT.

While the terms like, Earliest Start time, Earliest finish time, Latest Start time, Latest finish time and float corresponds to activities in "CPM".

$$\boxed{\begin{aligned} &\text{Slack} \rightarrow \text{event} \\ &\text{Float} \rightarrow \text{Activities} \end{aligned}}$$

1) Slack or Event Float :-



$$\boxed{S_i = L_i - E_i}$$

$$\boxed{S_j = L_j - E_j}$$

→ Slack denotes the amount of time by which the particular event can be delayed without ~~delaying~~ delaying the project schedule.

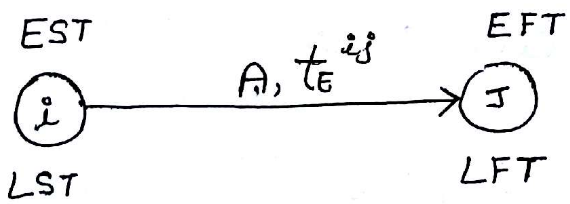
2) Flot Event :

i) Total Float (TF)

ii) Free Float

iii) Independent Float

i) Total Float (TF) :



$$\boxed{TF = LFT - EFT = LST - EST}$$

→ It denotes the amount of time by which, an activity can be delayed, without delaying the project schedule.

→ If total float value is

i) +ve → Resources are surplus & can be allocated for other activities.

ii) −ve → Resources are not sufficient and activity may ~~cost~~ not complete on time.

iii) 0 → Resources are just sufficient to complete activity on time.

ii) Free Float (FF) :-

→ It is that part of total Float, which can be used without affecting the float of successive or succeeding activity.

→ It is extra time by which, an activity can be delayed, so that the succeeding activity can be started at their EST i.e earliest start time.

$$\boxed{FF = TF - \text{Head Event Slack}}$$

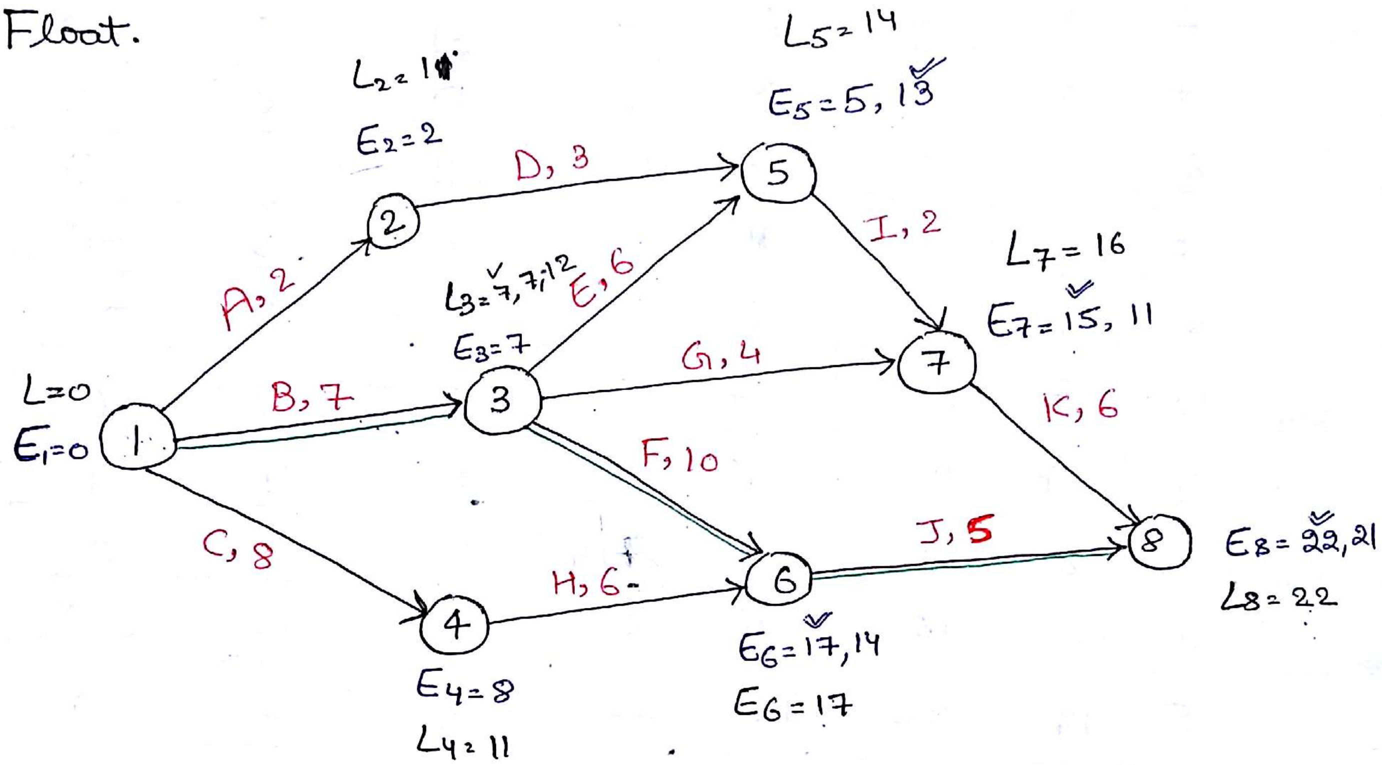
iii) Independent Float (IF) :-

→ It is the Amount of time, which can be used without affecting either the head or tail events.

$$\boxed{IF = FF - \text{Tail Event Slack}}$$

$$\boxed{TF \geq FF \geq IF} \text{ Always}$$

Q3) For the Network diagram shown below, find the Critical Path and t_E i.e expected project completion time, Draw a table showing the details of each Activity along with total & Free; & Independent Float.



$E_1 = 0$	$L_1 = 0$
$E_2 = 2$	$L_2 = 11$
$E_3 = 7$	$L_3 = 7$
$E_4 = 8$	$L_4 = 11$
$E_5 = 13$	$L_5 = 14$
$E_6 = 17$	$L_6 = 17$
$E_7 = 15$	$L_7 = 16$
$E_8 = 22$	$L_8 = 22$

Activity i-J	t_E^{ij}	Earliest		Latest		Float		
		EST	EFT	LST	LFT	Total	Free	Indep
A, 1-2	2	0	2	9	11	9	0	0
B, 1-3	7	0	7	0	7	0	0	0
C, 1-4	8	0	8	3	11	3	0	0
D, 2-5	3	2	5	~~11~~	14	9	8	-1^* (0)
E, 3-5	6	7	13	8	14	1	0	0
F, 3-6	10	7	17	7	17	0	0	0
G, 3-7	4	7	11	12	16	5	4	4
H, 4-6	6	8	14	11	~~17~~ 17	3	3	0
I, 5,7	2	13	15	14	16	1	0	-1^* (0)
J, 6-8	5	17	22	17	22	0	0	0
K, 7-8	6	15	21	16	22	1	1	0

For Activity (any) i-J

$$\boxed{TF = L_j - (E_i + t_E^{ij})}$$

for D $= 14 - (2+3) = 9$

For Free Float;

$$\boxed{FF = TF - \text{Head Event Slack}}$$

$=$

$$\boxed{FF = E_j - [E_i + t_E^{ij}]}$$

direct Method from Network diagram

for D $= 13 - (2+3)$

For Total Float

$$\boxed{IF = FF - \text{Tail event Slack}}$$

at D = $8 - 9 = -1^{*}$ (0) — Both for i.e. es and Specify ~~...~~ that (−1) have no sense so 0 is taken

forgate

For activity i−j

$$\boxed{IF = E_j - [L_i + t_E^{ij}]}$$

at D = $13 - (11 + 3) = -1^{*}$ (0)

EST = EFT − T_E

LST = LFT − T_E

Total Float = LST − EST

Free float = TF − HES — Head diff

Independent float = FF − TES — Tail diff

Crashing or time-Cost Model :-

CRASHING
or
Time - Cost

→ It is an extension of Critical Path Method, that Consider a Compromise b/w the time and the Cost required, to Complete a project.

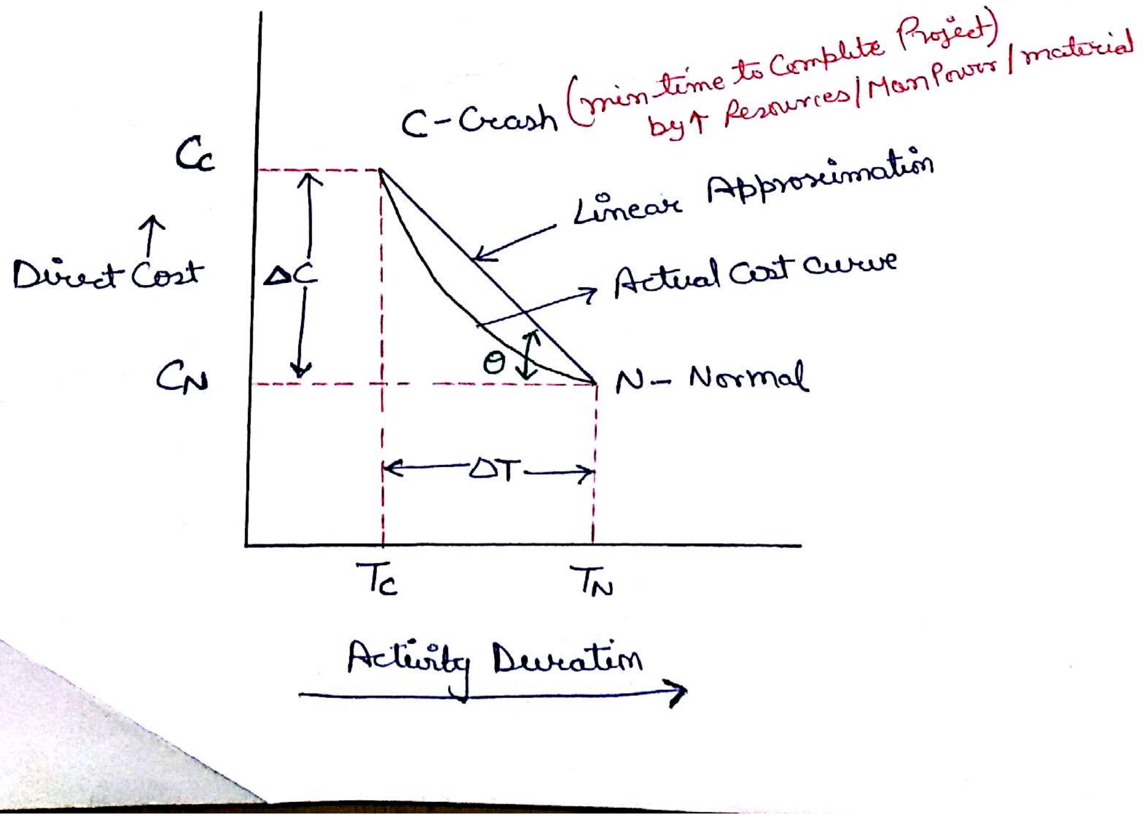
Total Cost of Any Project Consist of direct & indirect cost involve in its Complition.

i) Direct Cost
ii) Indirect Cost

i) Direct Cost:

→ It is the Cost directly involve in the execution of an activity.

→ It include direct material, direct labour, Cost of m/c, equipment etc.



→ Crash time is the minimum Activity duration, to which an activity can be compressed by increasing the Resources & hence by increasing the direct Cost, the slope of the line, gives amount of increase in the direct Cost per unit time for Crashing an activity.

$C_c \rightarrow$ Cost at Crash Pt.

$$\boxed{\text{Cost time Slope} :- \frac{\Delta C}{\Delta T} = \frac{C_c - C_N}{T_N - T_c}}$$

Example

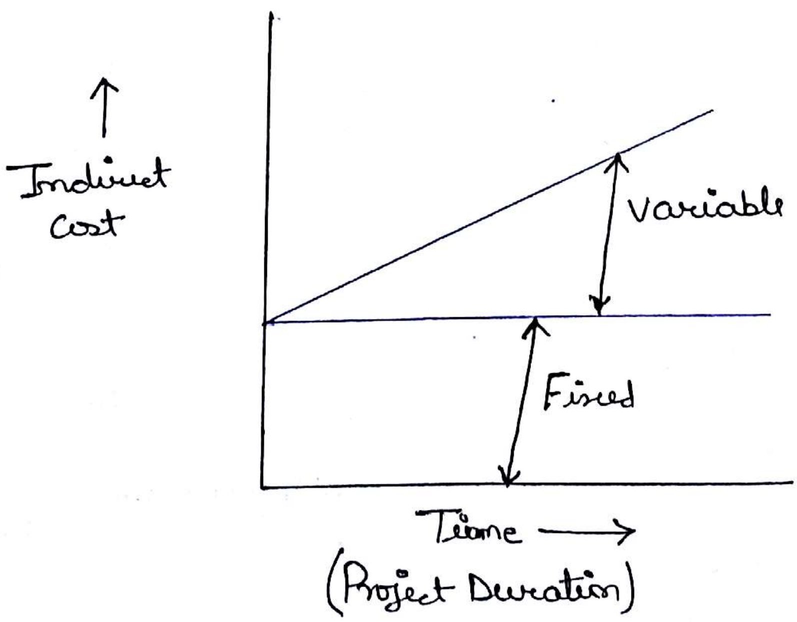
Normal		Crash Cond^n	
10 days	Rs 8000	6 days	Rs. 14000

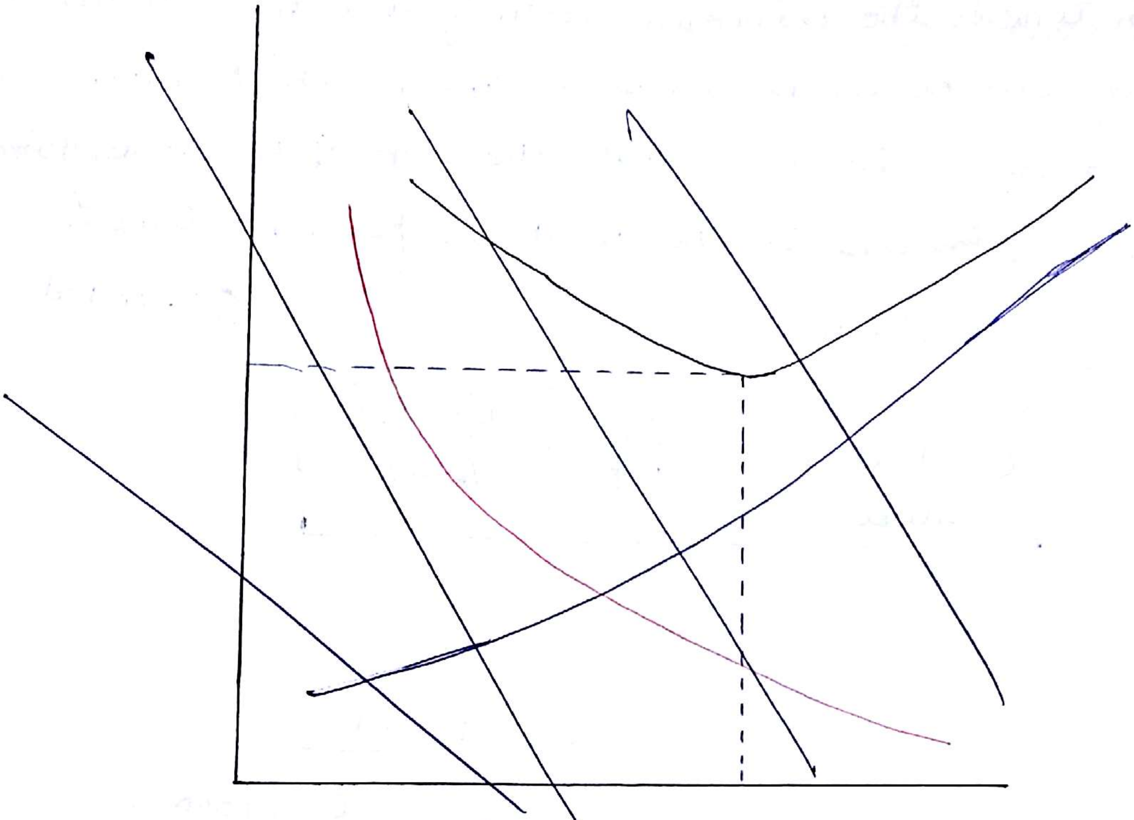
$$\Delta C = \frac{Rs. 6000}{4\ day} = \boxed{1500/day}$$

ii) Indirect Cost :

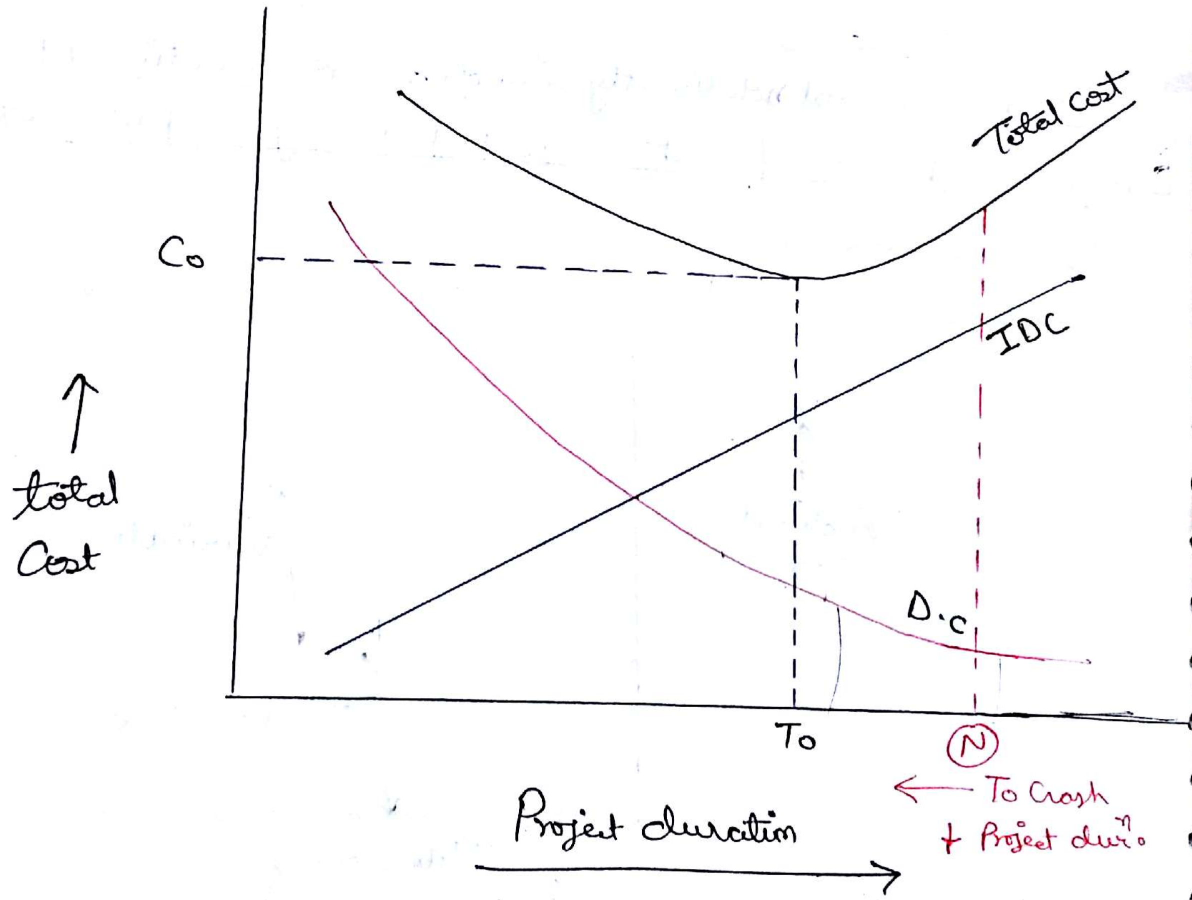
It is the cost not directly involved in execution of Project, ~~that~~ but is compulsory for the safe and timely completion of the project.

i) Fixed Indirect cost
ii) Variable Indirect cost





Crashing (Hit & trial Method)



The Objective of crashing ~~an~~ network, is to determine optimum project duration corresponding to minimum cost of projects, and the steps involved are;

i) In the critical path, select the critical activity having minimum cost slope.

ii) Reduce the duration of this activity by one time unit.

iii) Revise the network diagram by adjusting the time and the cost of crashed activity. Again find critical path, project duration & total cost of project.

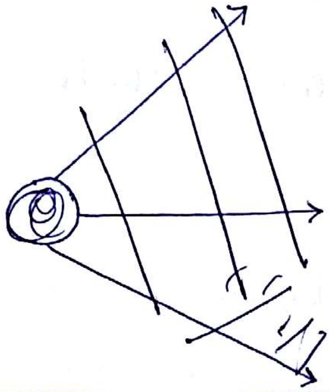
IV) If the optimum project duration is obtained then stop, then otherwise repeats the steps from I(i).

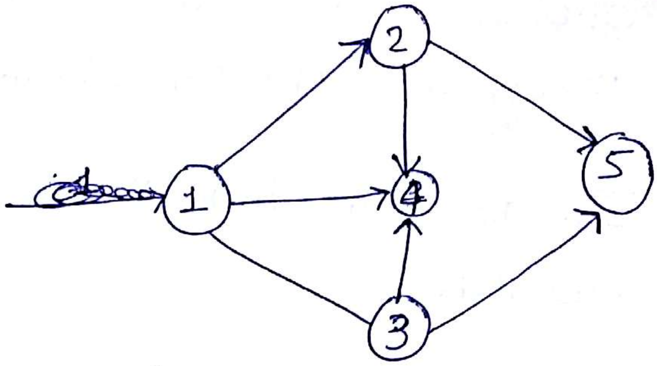
Qn) Draw the network Diagram and crash the network to optimum project duration corresponding to minimum cost of project. It is Given that the indirect cost is Rs. 900 per day.

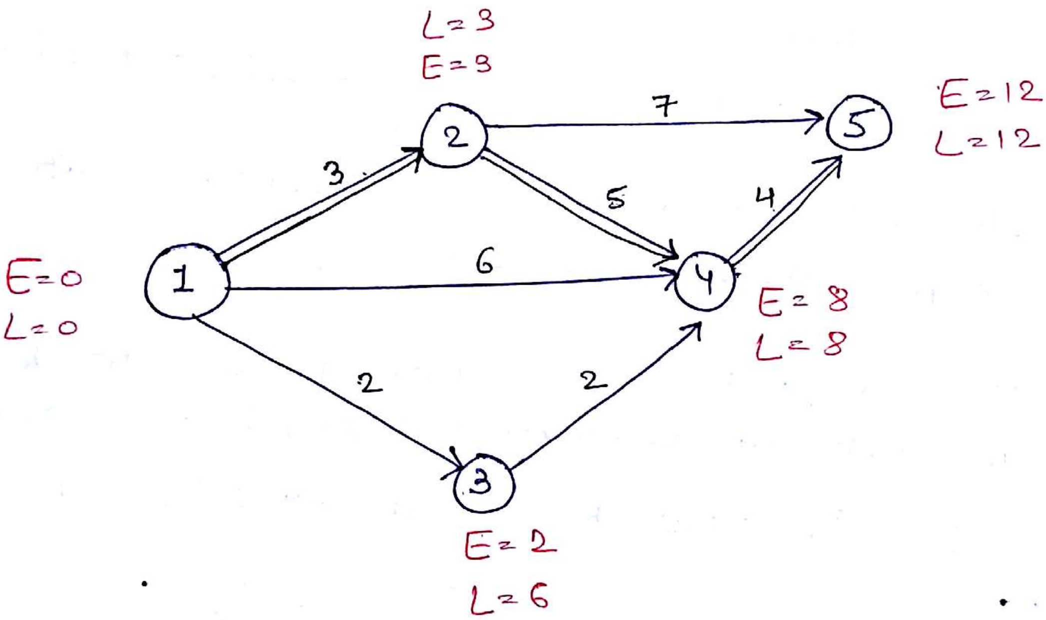
Activity i-j	Normal Time day	Normal C_N Cost ₹	Crash Time day	Crash C_C Cost ₹	$\Delta C/\Delta T$
1-2	3	500	2	1000	500 ✓
1-3	2	750	1	1500	750
1-4	6	1400	4	2600	600
2-4	5	1000	3	1800	400 ✓
2-5	7	1150	6	1450	300
3-4	2	800	2	800	—
4-5	4	1000	2	2400	700 ✓
		6600			

$$\frac{C_C - C_N}{T_N - T_C}$$

$$\frac{1000-500}{3-2}$$







$T_E = 12$ days

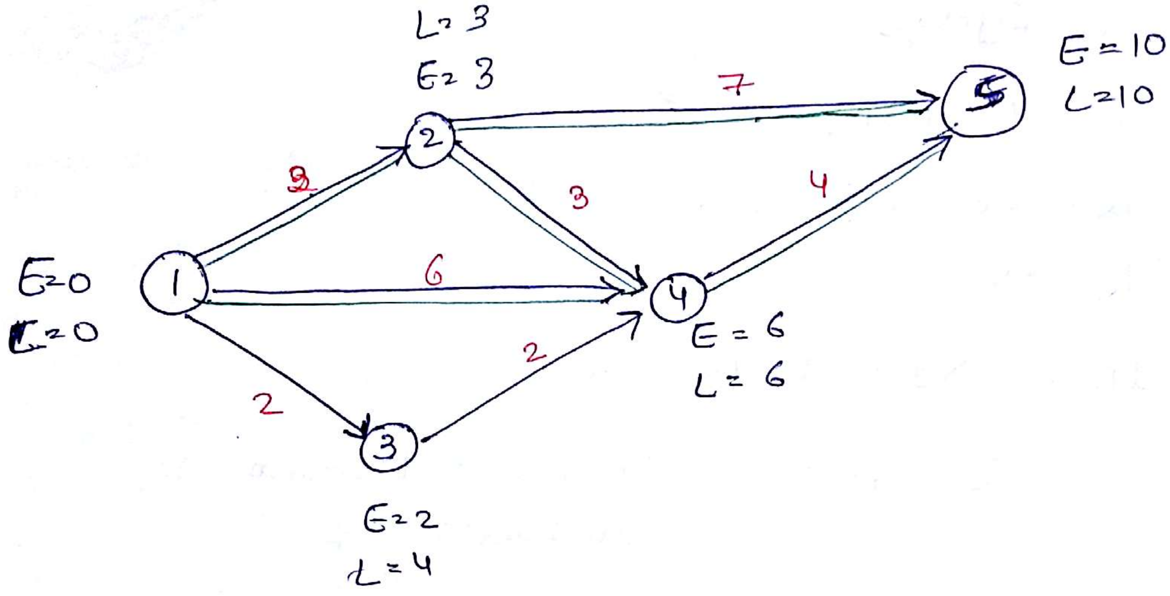
$TC = DC + IDC$

$DC =$ Rs. 6600

$IDC = 12 \times 900 = 10800$

$TC = 17400$

Now, Crashing min. of Cost of activity along critical Path. It is activity (2-4), and Crashing it by two days & Revised Network diagram & Cost of Project is as given below.



T_E = 10 days

TC = DC + IDC

DC = Rs. 6600 + 800 (400 × 2)

Critical Path = 10 × Cost.

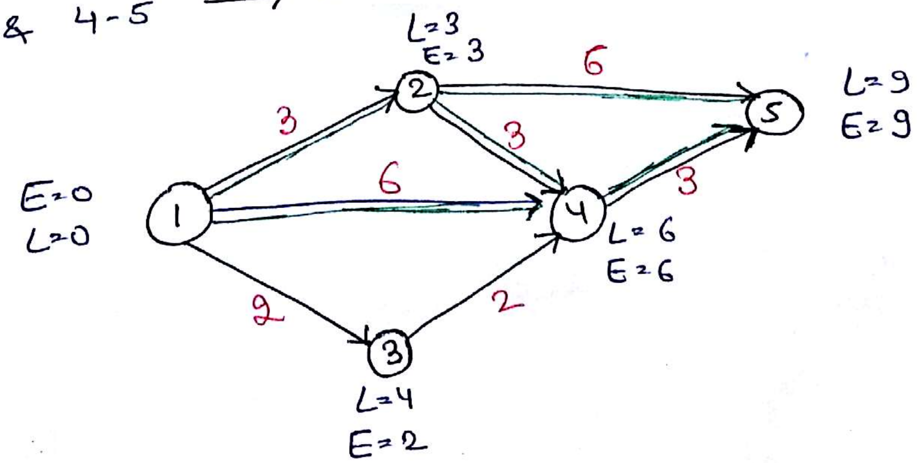
IDC = 10 × 900 = Rs. 9000

$\boxed{TC = 16400 \text{ Rs.}}$

on crashing (2-4)

Now the Network has three critical Path & now crashing on one activity, Project duration doesn't change. So we need to Crash, atleast two activities simultaneously, which gives three options. out of these we select an option for which the Σ of cost slope ∝/∂t is minimum.

1) 1 – 2 & 1 – 4 → 500 + 600 = 1100
2) 2 – 5 & 4 – 5 → 300 + 700 = 1000 ✓ Here crashing done by 1 day
3) 1 – 2 & 4 – 5 → 500 + 700 = 1200



$T_E = 9$ days

$TC = DC + IDC$ ist crashing

$TC = 6600 + (400 \times 2) + 1000$ — 2^{nd} crashing

$TC = 8400$

$IDC = 9 \times 900 =$ Rs. 8100

$TC = 16500$ Here cost increases. So we have to stop.

So $\boxed{\begin{array}{l} T_o = 10 \text{ days} \\ C_o = \text{Rs. } 16400 \end{array}}$

Objective

1st crash

DC ↑ Rs. 800

IDC ↓ Rs. 1800

Tc ↓ Rs. 1000

2nd Crash

DC ↑ Rs. 1000

IDC ↓ Rs. 900

TC ↑ Rs. 100

Fore Casting

Forecasting Can be termed as prediction of future Sales and Demand of a particular product.

It is the Projection Based upon past data and the ~~P~~Art of human Judgement. The Survival of any organization depends upon How well they can project, the demand in future.

Need / Benifits of Forecasting :

i) It helps in determining, the volume of production & Production rate.

ii) It forms the basis for, production budget, labour budget, material budget etc.

iii) It is essential for product design & development.

iv) It suggest the need for plant expansion.

v) It helps in establishing price policy.

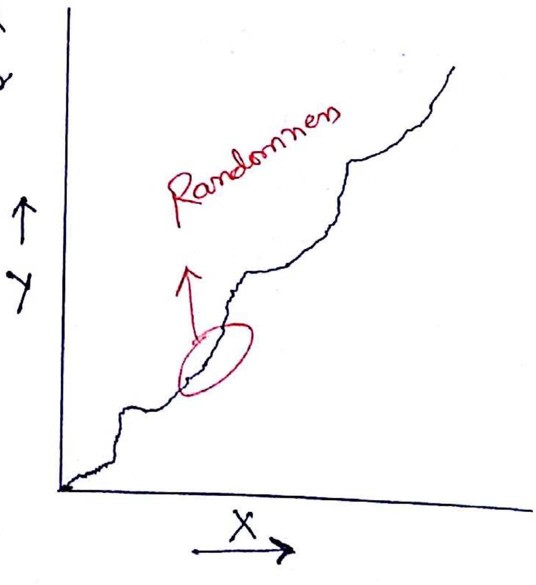
vi) It helps in deciding the extent of marketting, Advertising & distribution required.

Types of Demand Variation :-

→ Causal Method

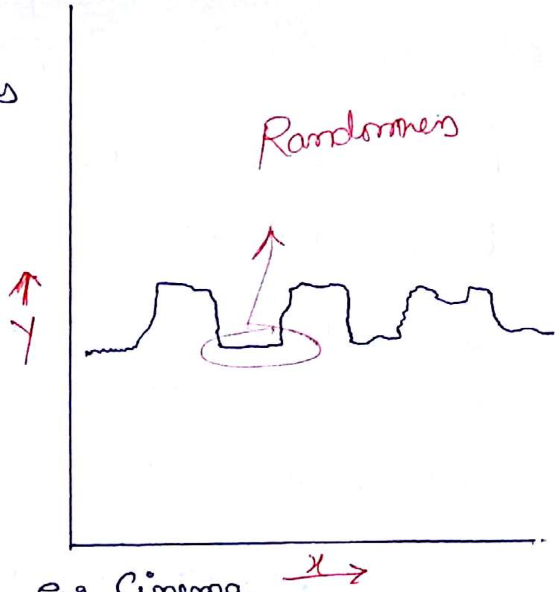
i) Trend Variation (T) :

→ It shows the long term, upward or downward movement in the demand pattern of Particular product.



ii) Seasonal Variation (S) :

It shows a short term, regular variations repeated after a short period of time may be weekley or daily.

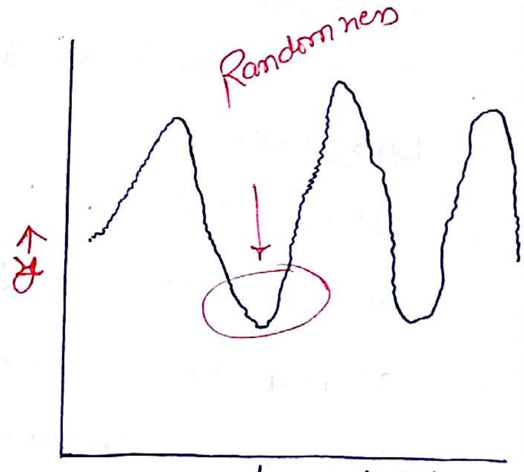


e.g Cinema, ↑ on Week End

iii) Cyclic Variation (C) :

→ used. time Series data

It shows the long term, wave like demand variation, normally for more than a year.



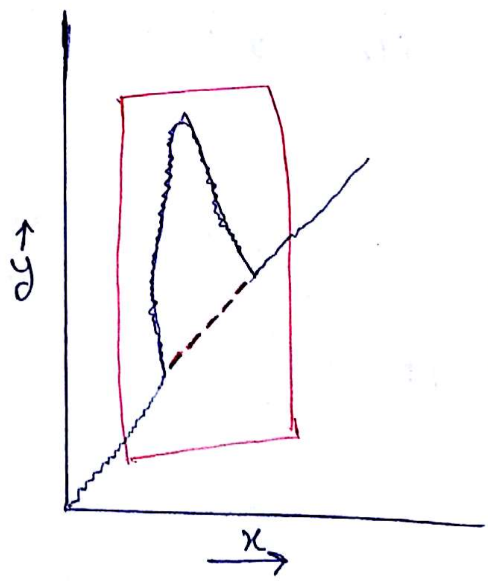
ice cream / cold drink

iv) Irregular Variation (I) :-

These variation's are caused due to unusual circumstances, which are not reflective of normal behaviour.

like, govt. Policy change, Price hike, Strike, Shut down etc.

These are not consider under Forecast.



Chang in govt Policy, Excess demand

Type Of Fore Cast

Fore Cast

- Qualitative or Subjective
  - Judgement
    - Opinion Survey
    - Market trial
    - Market Research
    - Delfi Technique
- Quantitative or Objective
  - Time Series
    - Past Average
    - Simple moving Average (SMA)
    - Weighted Moving Average (WMA)
    - Exponential Smoothing
  - Casual or Econometric
    - Correlation
    - Regression

Long (2-5 year)
Short (1-3 year)
Mid (3-12 month, 2 years)

→ Judgemental Forecasting :

→ This Method is based upon art of human judgement i.e how well a human being can predict, the demand of Product in future.

→ This Method Does't require past data or Sales figure.

i) Opinion Survey :

In this method, opinions are collected from the Customer, retailer, & distributor, regarding the demand pattern of Product. These information are used while forecasting.

ii) Market trial :- (Applied for Low cost material)

→ This is method is applicable for new product & in that case, Product is introduce b/w limited population in the form of free Sample.

→ The response from the limited population is used to project the demand ~~of~~ from bigger population.

→ It is Applied for low cost Consumable.

examples : Toothpaste, Chocolate, Cold drink, cosmetics item etc.

iii) Market Research :-

→ In this method, the work or Survey is assigned to an external marketting agencies and the purpose of research is to Collect information, regarding the demand of a product.

→ The details about various factor, which influences the demand like Customer income, customer Occupation, location. Quantity Quality etc. are related to get the forecast.

iv) Delphi Technique :-

→ In this Method a pannel of experts are asked sequences Question, in which the respond one Question is used to produced next Question.

→ It is step by step procedure, in which the information available to some expert is made available to other and final forecast is obtained. by the common opinion of all the experts.

Quantitative Forecasting :-

i) Time Series :-

→ In this Method, Past data are arranged in cronological order as dependent variable and time as independent variable.

→ Based upon these past data, we need to project the demand in future.

Year	2008	2009	2010
Demand	210	360	310

i) Past average Method :-

In this Method, forecaste is given by Average or mean of the actual demand data, for the previous period

ii) Simple moving Average or Rolling average :

n = no. of Period of SMA

1^{st} forecast = $(n+1)^{th}$ Period

$n = 3$

1^{st} fore cast = 4^{th}

→ This method used passed data & Calculate rolling average for constant period. fresh average in computed at the end of each period by adding the actual demand data for the most recent period & deleting the data for older period.

mean of last three

Year	Dem
2008	240
2009	310
2010	360
2011	330
2012	300
2013	350
2014	390
2015	430

mean

depend Mn.

In this method, as data changes from Period to period then it is termed as moving average method.

iii) Weighted moving Average (WMA) :-

$$SMA = (n=4) = F_{2012} = \frac{330+360+310+240}{4}$$

$$SMA\ (n=4),\ F_{2012} = 0.25 \times 330 + 0.25 \times 360 + 0.25 \times 310 + 0.25 \times 240$$

but Now for,

Weighted moving average,

$$WMA\ (n=4),\ F_{2012} = 0.4 \times \underset{\uparrow\ 2008}{330} + 0.3 \times 360 + 0.3 \times 310 + 0.3 \times \underset{\uparrow\ 2008}{240}$$

→ This method gives unequal Weight to each demand data; in such a manner, that summision of all weights always equal to one.

→ The most recent data is given the heightest weight and the weight assigned to oldest data will be the least.

Method to find Weight;

Sum of Digit method;

n = no. of period of WMA

i) find the sum of n - natural number,

$$\boxed{\sum n = \frac{n \cdot (n+1)}{2}}$$

2) Arrange them in decreasing order of Weight as:

$$\frac{n}{\Sigma n}, \frac{n-1}{\Sigma n}, \frac{n-2}{\Sigma n} \text{ ------ } \frac{1}{\Sigma n}$$

$n = 4$ $\quad \Sigma n = 10$

$\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$

$n = 5$ $\quad \Sigma n = 15$

$\frac{5}{15}, \frac{4}{15}, \frac{3}{15}, \frac{2}{15}, \frac{1}{15}$

Qn) For the given data, generate the forecast for each of the time period using Simple moving average for $n=3$ periods and Weighted moving average for $n=4$ periods. Also find the forecast for 9th, 10th, 11th Period

$n=9$
$n=10$
$n=11$

1	340
2	460
3	520
4	
5	
6	
7	
8	
9	
10	
11	

Period	Demand	SMA n=3	WMA n=4
1	340		
2	460		
3	520		
4	400	440	
5	310	460	442
6	430	410	394
7	610	380	397
8	580	450	475
9		540	532
10		540	532
11		540	532

As we dont have
actual demand
so 10^{th} & 12 will
be same as 11

v) Exponential Smoothing Process :- **

→ This method requires only the current demand & forecasted value for the current period to gives next forecast.

→ This method is a modified form of Weighted moving average, which gives weight to all the previous data, but the weights assigned are ~~exponentio~~ in exponentially decreasing order.

The most recent data is given the heighest Weight & Weight assigned to older data decreases exponentially

General form of exponential Smoothing :-

$$F_t = \alpha D_{t-1} + \alpha(1-\alpha) D_{t-2} + \alpha(1-\alpha)^2 D_{t-3} + \alpha(1-\alpha)^3 D_{t-4} + \cdots\cdots + \infty$$

$\boxed{\alpha \text{ must lie b/w } 0 \text{ to } 1}$ Very fast decreasing

$$F_t = \alpha D_{t-1} + (1-\alpha)\left[\alpha D_{t-2} + \alpha(1-\alpha) D_{t-3} + \alpha(1-\alpha)^2 D_{t-4} + \cdots \infty\right]$$

$$\boxed{F_t = \alpha D_{t-1} + (1-\alpha) F_{t-1}} \quad \text{— } 1$$

OR

$$\boxed{F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})} \quad \text{— } 2$$

Error :-

$$\text{Error} = e_i = \Delta i = D_i - F_i$$

$$\boxed{F_t = F_{t-1} + \alpha e_{t-1}}$$

Where,

$\alpha \rightarrow$ is Known as Smoothing Constant and it is equivalent to n period of moving Average and is given by

α for production in Plants

$\boxed{\alpha = 0.1 - 0.2}$

Smoothing Constant $\boxed{\alpha = \frac{2}{n+1}}$

Data are more — using exponentially Smoothing
but less data — use SMA or WA Weighted Averag.

NOTE :

if for the initiall period, forecasting value is not given then,

i) take the actual demand for the first period, equal to Forecast i.e $T_1 = F_1$ and proceed.

ii take the Average of the actual demand data, as the forecast for the first period and proceed

Qn) The Sale of Car in a Show room in 4 Conservative month 70, 68, 82, 95 respectively with smoothing Constant of 0.4. Find the forcast for the next month.

Month	Sale
1	70
2	68
3	82
4	95

D_i

$70 + 0.4$

Month	Sale	F_i	e_i
1	70	70	0
2	68	70	-2
3	82	69.2	12.8
4	95	74.32	20.68

82.59 ≈ 83 unit

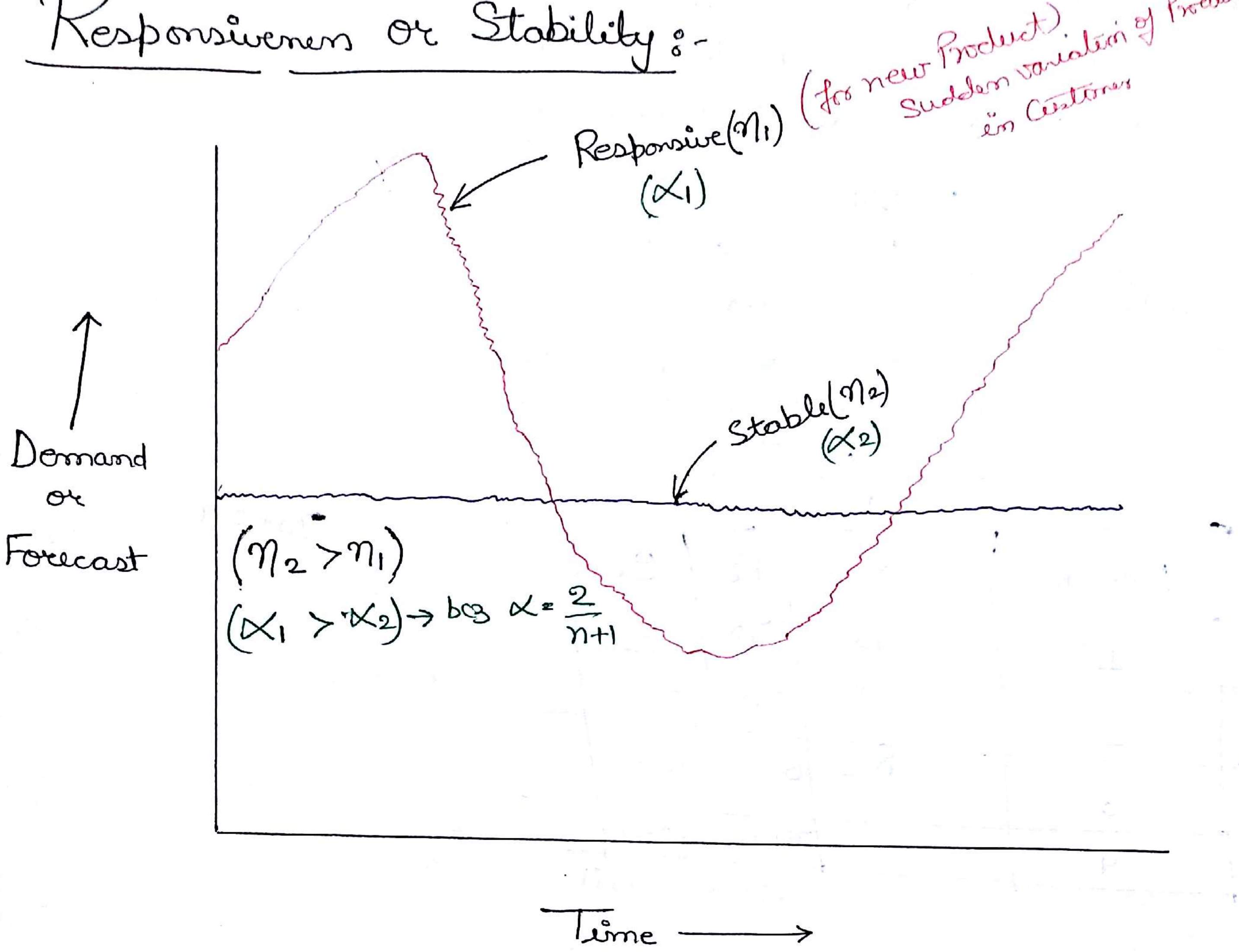
Month = 4

$= \alpha + (1-\alpha) + \alpha(1-\alpha)^2 + \alpha(1-\alpha)^3$

$= 0.4(1 + 0.6 + 0.6^2 + 0.6^3)$

$\boxed{= 0.8704}$

Responsiveness or Stability :-



Responsive :-

Responsiveness indicates that the forecast have fluctuating or swigging Pattern. it is prefered for new product & for that no. of Period is Kept small.

Stability :

→ Stability means that, forecast pattern is flat, smooth, or as less fluctuation.

→ It is preferred for old existing product & for that no. of Period should be large.

As $\boxed{\alpha = \frac{2}{n+1}}$ $\boxed{F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})}$

① if $\alpha = 0$, $n \to \infty$ (limit of Stability)

$$\boxed{F_t = F_{t-1}}$$

② if $\alpha = 1$ & $n \to 1$ (limit of Responsiveness)

$$\boxed{F_t = D_{t-1}}$$

(Actual demand is now forcast here)

(1 Pair going average)

Objective

$$0 \longleftarrow \alpha \longrightarrow 1$$

(Stable) (Responsive)

Forecast Error :-

$$e_i = D_i - F_i$$

When error is studied for long duration, it becomes helpful to find a particular pattern or trend, which may regulate our future production. The most commonly used method to find forecast error are;

1) Mean Absolute deviation :

$$\boxed{MAD = \frac{\sum_{i=1}^{n} |D_i - F_i|}{n}}$$

It indicates the average magnitude of error, in every period without considering Sign

* example

S.NO	Di	Fi	ei
1	160	150	+10
2	150	180	-30
3	180	160	+20

$\Sigma e_i = 0$

$M.AD = \frac{\Sigma e_i}{\text{total No.}}$

$MAD = \frac{60}{3} = 20$

2) Mean Forecast Error (MFE) or Bias :- Support dir^n tell

→ It measures the forecast error, with regard to direction and shows any tendency of over ~~and~~ or under forecast.

→ +ve bias indicate under estimated forecasting.

→ -ve bias indicate over estimated forecasting

Running Sum Forecast Error :-

$$RSFE = \sum_{i=1}^{n} (D_i - F_i)$$

$$\boxed{Bias = \frac{RSFE}{n}}$$

3) Mean Square Error (MSE) :-

$$\boxed{MSE = \frac{\sum_{i=1}^{n} (D_i - F_i)^2}{n}}$$

σ → standard deviation of forecast error

$$\boxed{\sigma = \sqrt{MSE}}$$

→ Mean Square error is used to Compute standard deviation for forecast error, which is utilized to plot Control chart for forecast error....

4) Mean Absolute Percentage Error (MAPE) :-

~~MAD~~

$$\boxed{MAPE = \frac{\sum_{i=1}^{n} \left| \frac{D_i - F_i}{D_i} \times 100 \right|}{n}}$$

→ It is the average of % error Compare to actual demand.

→ It is used to put error in prospective, bcz, there is difference b/w 50 out of 100 and 50 out 1000.

50

30% to 40%

↓ ↓

±15% ±20%

V) Tracking System (TS) :-

$$\boxed{TS = \frac{RSFE}{MAD}}$$

± 4 or ± 5

→ It tells how well the forecast is predicting the actual Value. The value of 0 would be idle.

→ But ± 4 or ± 5 is acceptable range.

Qn) The demand for luxury car has been shown below. the expert forecasted sale of 100 Car for the month of march The smoothing Constant 0.15. Find the forecast for the month Of August. Also find $\boxed{MAD}$, $\boxed{MSE}$, $\boxed{MAPE}$ & $\boxed{Bias}$

$F_t = F_{t-1} + \alpha e_{t-1}$

Month	Demand	Forecast	error	ei
March	150	100	50+	
April	200	107.5	92.5	
May	100	121.375	-21.375	
June	50	118.168	-68.168	
July	150	107.54	42.06	
Aug		114.25	$\Sigma ei = 95.01$	

$\Sigma|ei| = 274.6$ $\Sigma ei = 17928.45$

1)

$\frac{ei}{Di} \times 100$

$\Sigma \frac{ei}{Di} \times 100 = 265.34$

1) MAD : $\frac{\Sigma|e_i|}{n} = 54.8$

2) MSE : $\frac{\Sigma e_i^2}{n} = 3585.6$

3) MAPE : $\frac{\Sigma \left|\frac{e_i}{D_i} \times 100\right|}{n} = 53.06$

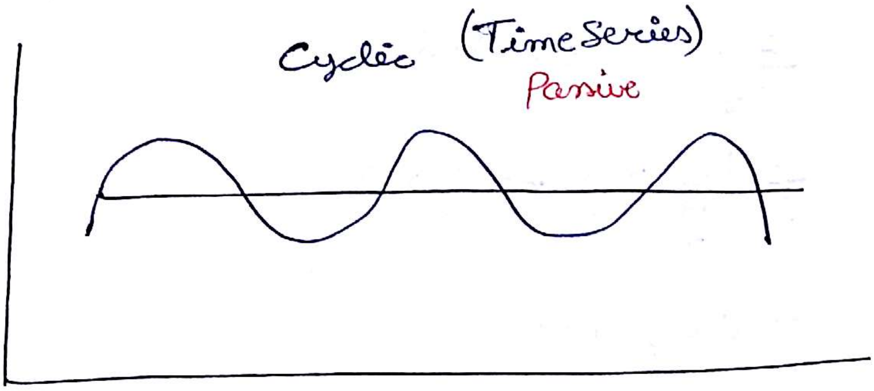
4) Bias : $\frac{\Sigma e_i}{n} = 19.0$

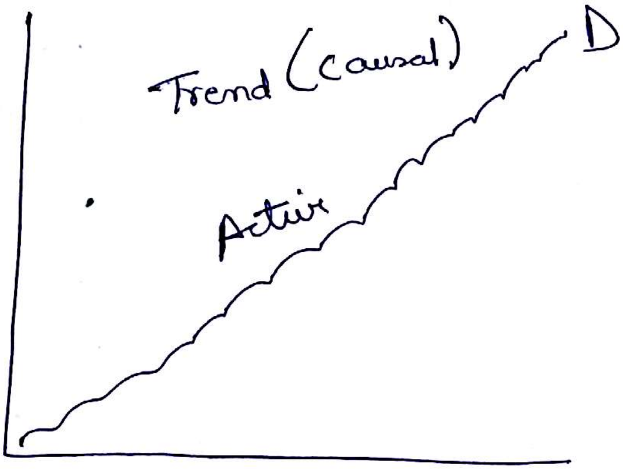
For this time series (Passive)

for this Casual (Active)

SNO.	DCyclic	DTrend
1	80	60
2	130	90
3	180	110
4	220	150
5	190	180
6	150	240
7	120	270
8	180	320
9	210	380
10		

Causal or Econometric Method :-





In, this method. Forecaster will try to establish cause & effect ~~relation~~ relation b/w demand of product & any other variable. the demand is dependent.

The objective is to establish a relation such that changes in one variable becomes useful for the predication for other.

i) Co-relation Analysis :-

- Degree of closeness b/w two Variable

$$\boxed{r = +1 \text{ to } -1}$$

x & y	
$r = +0.61$	$r = -0.48$
$x = \uparrow 100$	$x \uparrow 100$
$y = \uparrow 61$	$y \downarrow 48$

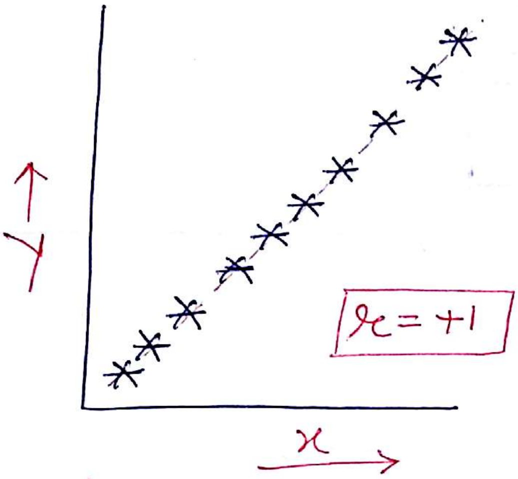
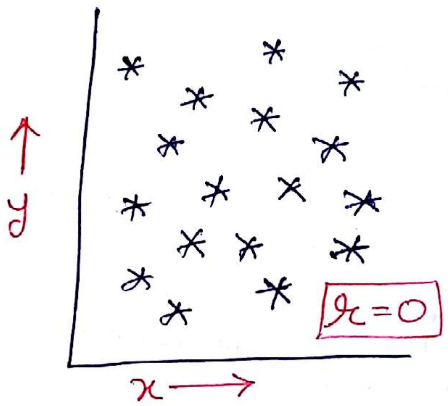
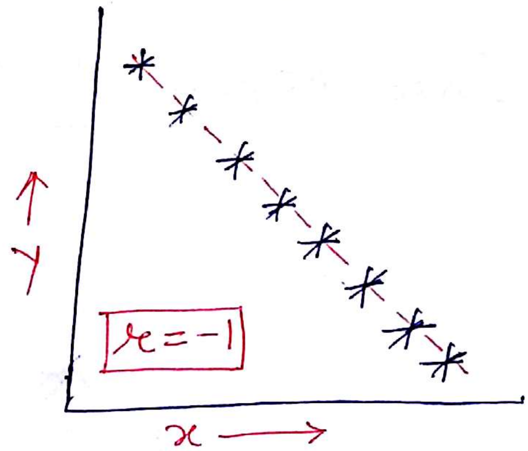


figure 1)





→ It indicates the degree of closeness b/w two and its values ranges from +1 to −1.

→ It is an indicator of extent to which knowledge of one Variable becomes useful for the ~~prediction~~ prediction of other.

→ The co-relation cofficient b/w two Variable x & y is given by

$$r = \frac{\sum (x-\bar{x}) \cdot (y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \cdot \sum (y-\bar{y})^2}}$$

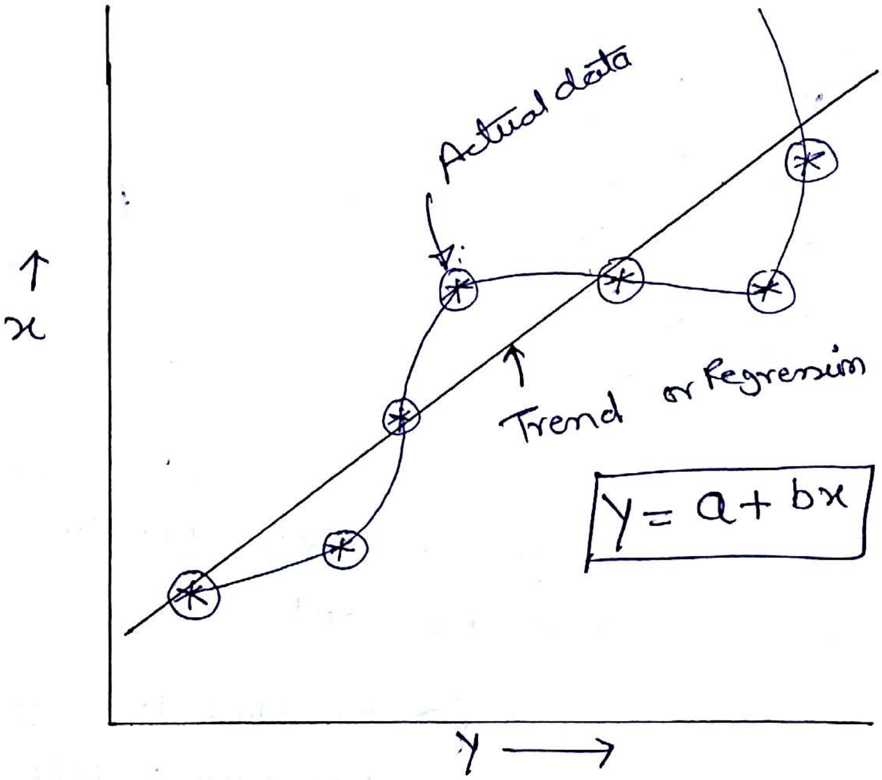
Where

$\bar{X}$ & $\bar{Y}$ are average value of independent x & Y

$$\boxed{r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}}}$$

Very imp (C)

Linear Regression Analysis :-



→ It is the mathematical Technique of obtaining line of best fit b/w the dependent variable which is usually, demand of product and any other variable on which demand is dependent.

→ In Regression Analysis, The relationship b/w some independent Variable x & dependent Variable Y is given by a straight line.

$$\boxed{Y = a + bx} \quad \text{——} \ (1)$$

Where a is intercept on Y axis.

and b is slope of line.

n = no. of Periods of Data

Taking Σ both side of eqn (1)

$$\boxed{\Sigma Y = a \cdot n + b \Sigma x} \quad \text{——} \ (2)$$

Now multiply eqn (1) by x

$$x \cdot y = ax + bx^2$$

Taking Σ on both side

$$\boxed{\Sigma x \cdot y = a \cdot \Sigma x + b \Sigma x^2} \quad \text{——} \ (3)$$

eqn (3) multiply by n on both side & Subtract to ~~eqn (2) × x~~

eqn (2) × Σx

$$n \cdot \Sigma x \cdot y = \cancel{a \cdot n \Sigma x} + b \cdot n \Sigma x^2$$

$$\Sigma x \cdot \Sigma y = \cancel{a \cdot n \Sigma x} + b (\Sigma x)^2$$

$$n \cdot \Sigma x \cdot y - \Sigma x \cdot \Sigma y = b \left[n \Sigma x^2 - (\Sigma x)^2 \right]$$

$$\boxed{b = \frac{n\sum x\cdot y - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}}$$

from eqn ②

$$\boxed{a = \frac{\sum y - b\cdot\sum x}{n}}$$

$$\boxed{y = a + b\cdot x}$$ — Trend or Regression

y → Dependent

x → Independent

Least Square Method :-

When the independent Variable x is uniform or Linear as it such a form that it Can be modified to make $\sum x = 0$ then the Calculation become very simple and the method is Called least square Method.

$$\boxed{\sum x = 0}$$

$$\boxed{b = \frac{\sum x\cdot y}{\sum x^2}}$$

$$\boxed{a = \frac{\sum y}{n}}$$

i) $n = \text{odd}$

Year	Demand	x
2007		-2
2008		-1
2009		0
2010		$+1$
2011		$+2$
2012		$+3$

ii) $n = \text{even}$

Year	Demand	x	x
2007		-2.5	-5
2008		-1.5	-3
2009		-0.5	-1
2010		$+0.5$	$+1$
2011		$+1.5$	$+3$
2012		$+2.5$	$+5$
		$\Sigma x = 0$	$\Sigma x = 0$

Qn) A Car manufacturer has recently held road side car exhibition for new model of Car. The no. of Salesman employed at each exhibition and the no. of Car Booked, it is as given below. find linear regression eqn

& estimate the number of Car Booked if 10 Salesman are employed in an exhibition.

No. of Salesman	No. of Car booked
5	8
8	160
	148
6	156
8	
9	168
3	102
5	142
4	98
6	152
6	142

x — independent
y — dependent

No. of Salesman x	No. of Carbook y	Trend val.	x^2	$x \cdot y$
5	132	128.75	25	600
8	160	162.5	64	1280
6	148	140	36	
8	156	162.5	64	
9	168	173.75	81	
3	102	106.25	9	
5	142	128.75	25	
4	98	117.5	16	
6	152	140	36	
6	142	140	36	

$\Sigma x = 60$ $\quad \Sigma y = 1400$ $\quad \Sigma x^2 = 392$ $\quad \Sigma x \cdot y = 8760$

$n = 10$

$$\Sigma y = a \cdot n + b \Sigma x \quad \text{— (1)}$$

$$\Sigma y = 10 \cdot a + b \Sigma x \Rightarrow \boxed{1400 = 10 \cdot a + 60 \cdot b}$$

$$\Sigma x \cdot \Sigma y = a \Sigma x + b \Sigma x^2 \quad \text{— (2)}$$

$$\boxed{8760 = 60 \cdot a + 392 \cdot b}$$

$a = 72.5$ $\quad b = 11.25$

$$\boxed{Y = 72.5 + 11.25x}$$

for $\boxed{\begin{array}{l} x = 10 \\ Y = 185 \end{array}}$

Qn) The Sales of an Automobile Company is Rs. is as given below. Forecast the demand for next two Year. using Least Square Method.

Year	Sales(Cr)
2005	30
2006	33
2007	37
2008	39
2009	42
2010	46
2011	48
2012	50
2013	55
2014	58

Soln by Least Square Method

Year	Sale (cr)	x	x	x^2	$x.y$
2005	30	-4.5	-9		
2006	33	-3.5	-7		
2007	37	-2.5	-5		
2008	39	-1.5	-3		
2009	42	-0.5	-1		
2010	46	+0.5	+1		
2011	48	+1.5	+3		
2012	50	+2.5	+5		
2013	55	+3.5	+7		
2014	58	+4.5	9		
	$\Sigma y = 438$	$\Sigma x = 0$	$\Sigma y = 0$	$\Sigma x^2 = 330$	$\Sigma x.y = 502$

$$\boxed{b = \frac{\sum x \cdot y}{\sum x^2}} \quad 1.52$$

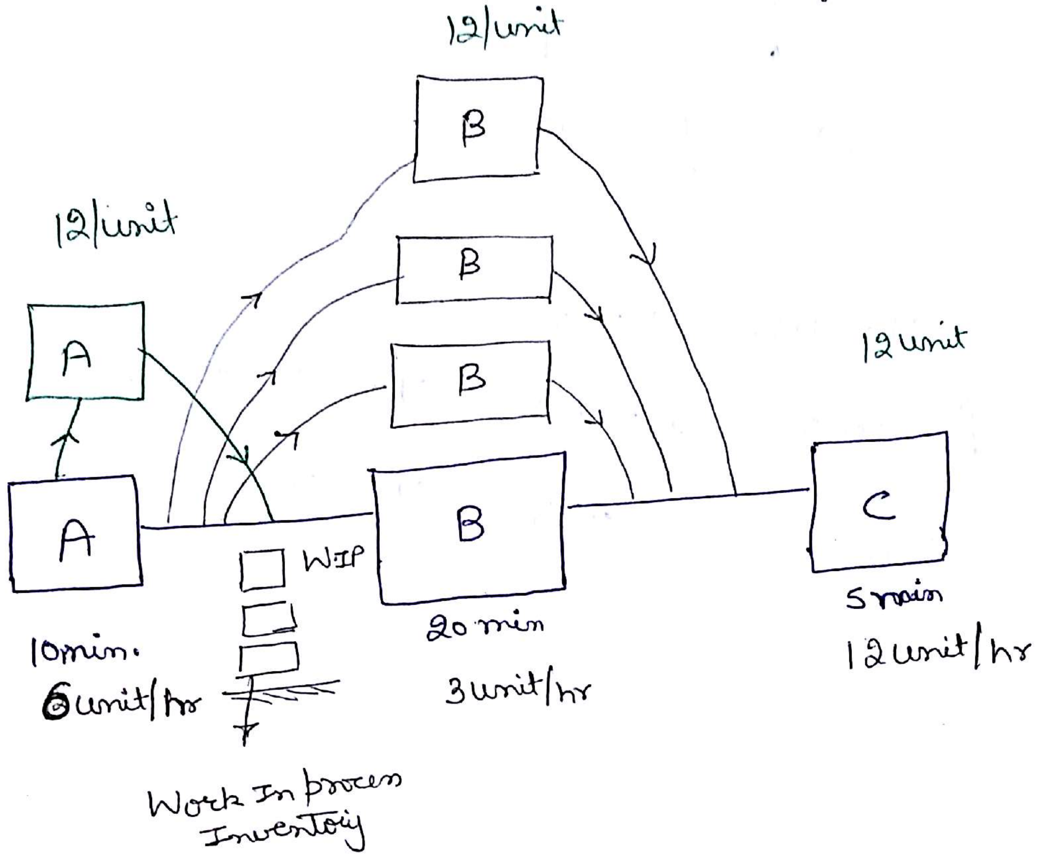
$$a = \frac{\sum y}{n} = 43.8$$

$$y = 43.8 + 1.52x$$

for 2015, $x = +11$. $y = 60.53$ Cr.

2016 $x = +13$ $y = 63.57$ Cr

Assembly Line Balancing



25% – Blue
50% – B+B
100% A+B+B+B

$$\eta = \frac{\text{Actual output}}{\text{Installed output}}$$

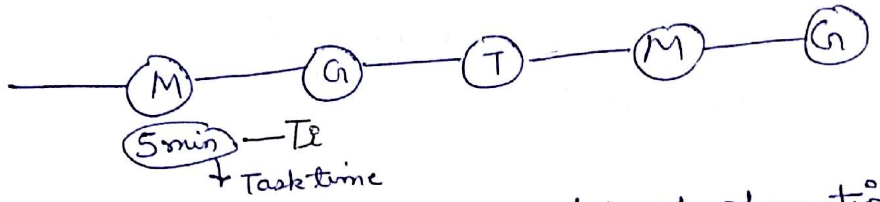
→ The Aim of Assembly line balancing is to group different facilities & equipment into different Work Station in Such a manner that utilization is optimised and idle time is minimized

Advantages :-

i) Decrease in Work in process inventory.
ii) Reduction in material handelling.
iii) Effective utilization of manpower & m/c.
iv) uniform rate of Production.
v) Easy Production Control

Terms Associated With Assembly balancing Line

1) Work Element :-



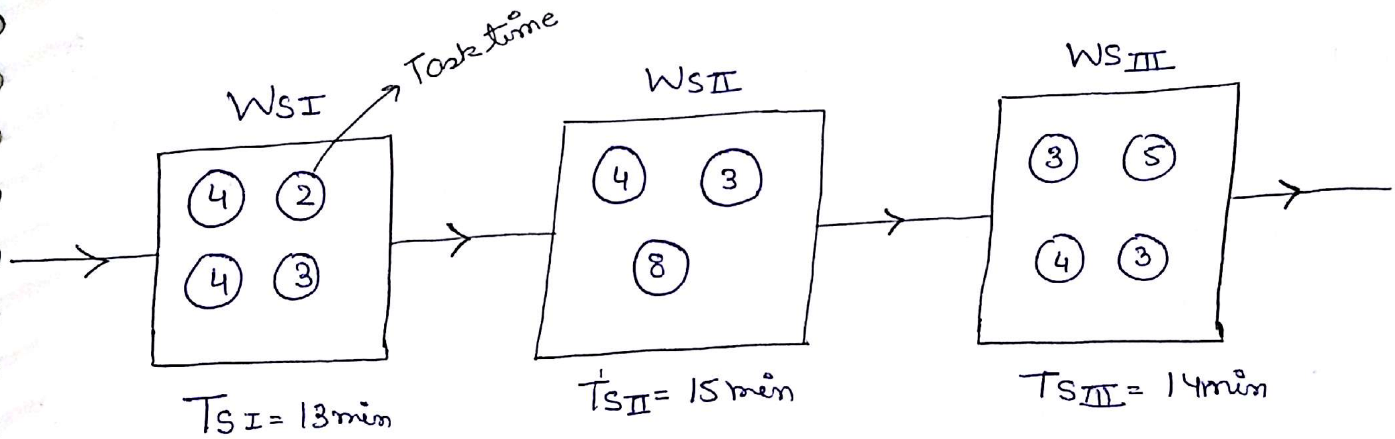
Every Job is Completed by a set of operation and each operation which is performed on the job is called Work element

2) Task Time : (Ti)

It is the Standard time required to Complete Work element.

3) Work Station :

It is the specific location on the assembly line, where given amount of Work elements are Completed Within a fixe period of time.



4) Station Time :- [Tsi]

It is the time required to Complete Work element, assigned in a Work station.

5) Total Work Content : (TWC) :-

- It is the time required to complete one set of job.

It is given by ;

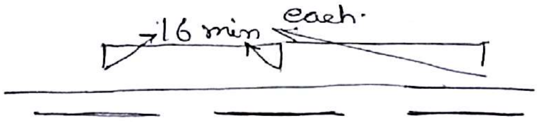
either the Summission of all the Station time

or the Σ of the all the element task time.

$$\boxed{T.WC = \sum T_{si} = \sum T_i = 42\,min}$$

6) Cycle Time (T_c) :- $\boxed{T_c \geq (T_{si})_{max}}$

- It is the amount of time for which a job is to be assembled remains in a work station.
- It is the time gap b/w two sucessive product, comming out from the assembly line.



7) Balance Delay (BD) :-

It is the ratio of total idle time of the job ~~of the~~ on the assembly line, to the total time spent by the job on the assembly line.

Balance delay is given by ;

$$\boxed{B.D = \left[\frac{n\,T_c - TWC}{n \cdot T_c}\right] \times 100}$$

$n = 3$, $T_c = 16\,min$ $TWC = 42\,min$

$n \rightarrow$ no of Work station

$T_c \rightarrow$ Cycle time

$TWC \rightarrow$ Total Work Content

$$BD\,\% = \frac{3 \times 16 - 42}{3 \times 16} = \frac{6}{48} \times 100$$

this Value show effectiveness & is as small as possible required

8) Line Efficiency: (η_L)

$$\boxed{\eta_L = \frac{TWC}{n \cdot T_c} \times 100}$$

$$\boxed{\eta_L = 100 - BD\%}$$

9) Smoothness Index: (SI)

It is the term use to represent load distribution b/w the different Workstation, Compare to a station, consuming maximum time.

$$SI = \sqrt{\sum_{i=1}^{n} (\text{Max. Station time} - \text{Station time})^2}$$

$$SI = \sqrt{\sum_{i=1}^{n} \left((T_{si})_{Max} - T_{si}\right)^2}$$

example:

$$SI = \sqrt{(15-13)^2 + (15-15)^2 + (15-14)^2}$$

10) Minimum number of Work Station required :-

$$\boxed{\eta_{min} = \frac{TWC}{T_c}}$$

Example:→ $\boxed{\eta_{min} = \frac{42}{16} = 2.625 \approx 3}$

Methods of Line Balancing :-

** If Ask about Method then used but rather than using Method I only.

1) Largest Candidate Rule.
2) Rank Positioned Weighted Method (RPWM).
or
Helgeson & Burnie

1) Largest Candidate Rule :-

The steps involved are :-

i) List all the element in the decreasing order of their task time.

ii) To Assign an element, in a Workstation, Start from the begining of list, moving downward, searching first feasible element, which can be placed in a work station.

- A feasible element is one, that satisfy the precedence requirement and when that element is placed in a Workstation, the total time of Workstation should not exceed the cycle time.

iii) Strike of the element which is assigned in work station so that it can not be consider again

iv) Continue in the similar manner, untill all the element will assign to different workstation.

Qn) For the following set of element, draw the precedence dia. Balance the line & determine the Balay Balance delay line efficiency & Smoothness index & take the Cycle time as 1 min.

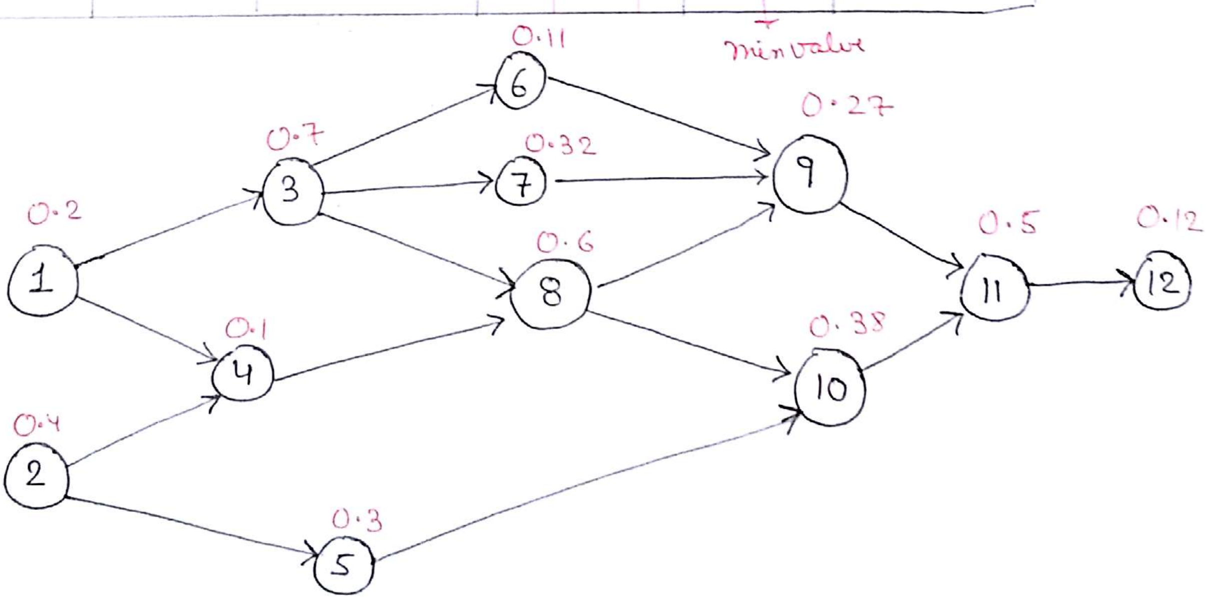
Arranging in decreasing order

Max. value

① ②

Element	Ti (min)	Precedence		Element		Ti (min)	Precedence
1	0.2	–		1	3	0.7	1
2	0.4	–		2	8	0.6	3,4
3	0.7	1		3	11	0.5	9,10
4	0.1	1, 2		4	2	0.4	–
5	0.3	2		5	10	0.38	5,8
6	0.11	3	New → ALTERATION BY LAW	6	7	0.32	3
7	0.32	3		7	5	0.3	2
8	0.6	3,4		8	9	0.27	6,7,8
9	0.27	6, 7, 8		9	1	0.2	–
10	0.38	5,8		10	12	0.12	11
11	0.5	9,10		11	6	0.11	3
12	0.12	11		12	4	0.1	1,2

min value



As Cycle time = 1 min.

WS	Element	T_i	T_{si}	Idle time
I	2	0.4	1.0	0
	5	0.3		
	1	0.2		
	4	0.1		
II	3	0.7	0.81	0.19
	6	0.11		
III	8	0.6	0.98	0.02
	10	0.38		
IV	7	0.32	0.59	0.41
	9	0.27		
V	11	0.5	0.62	0.3
	12	0.12		

$\rightarrow$ T_i & T_{si} always $\leq$ cycle Time

$n = 5$ $T_c = 1\,min$, $TWC = 4\,min$

1) $BD = 20\%$

2) $\eta_L = 80\%$

3) $SI = 0.59$

Qn) Civil Service (2013)

A Company is engaged in Assembly of Wagon on a Conveyor. 500 Wagons are required per day & production time available per day is 420 minute. find the minimum no. of Work station required, Balance delay (B.D) & Line efficiency. Represent the different Workstation on the Network dia

500 Wagon/day

420 min/day

$$\frac{420 \times 60}{500} = 50.4 \text{ sec}$$

$\boxed{50 \text{ sec} = T_c}$

Activity	Ti (Sec)	Precedence
A	45 (2)	—
B	11 (5)	A
C	9 (6)	B
D	50 (1)	—
E	15 (3)	D
F	12 (4)	C -
G	12 (4)	C -
H	12 (4)	E -
I	12 (4)	E
J	8 (7)	F, G, H I
K	9 (6)	-
L		

(2)

Activity	Ti (Sec)	Precedence
~~D~~	~~50~~	~~—~~
~~A~~	~~45~~	~~—~~
~~E~~	~~15~~	~~D~~
~~F~~	~~12~~	~~C~~
~~G~~	~~12~~	~~C~~
~~H~~	~~12~~	~~E~~
~~I~~	12	E
~~B~~	~~11~~	~~A~~
~~C~~	~~9~~	~~B~~
~~K~~	~~9~~	~~—~~
~~J~~	~~8~~	~~F G H I~~

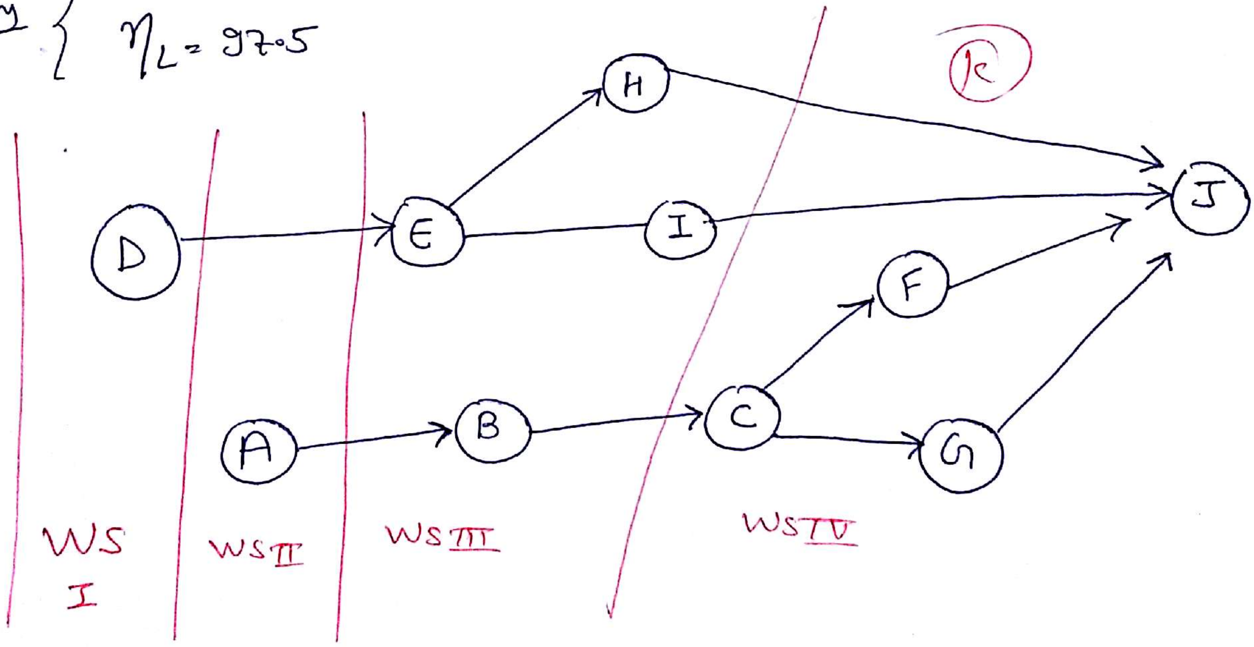
3

$T_c = 50$ sec

Ws	Activity	T_i	T_{si}	Idle time
I	D	50	50	0
II	A	45	45	5
III	E	15		
	H	12	50	0
	I	12		
	B	11		
IV	C	9		
	K	9	50	0
	F	12		
	G	12		
	J	8		

Ans $\left\{ \begin{array}{l} BD = 2.5\% \\ \eta_L = 97.5 \end{array} \right.$

Network dia by using Table ③ relation wd ②



ii) Rank Position Weighted Method (RPWM)

or

Helgeson & Burnie

→ In this method, Arrange all the Work element, in the decreasing order of their position Weight.

→ Position Weight of an element, correspond to the time of the longest path from the begining of element through the remaining Network.

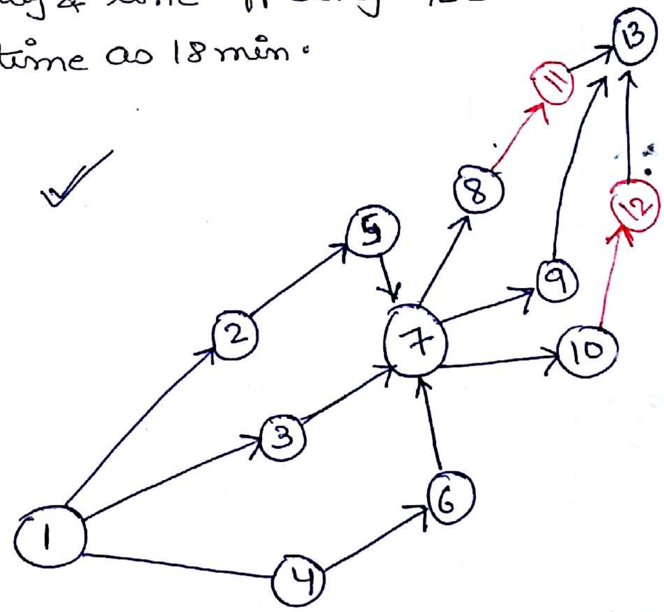
→ Allocation is done Similar to, largest Candidate rule. The only difference in final table, that, all the element should be arranged in decreasing order of their position weight

Qn) Design an Assembly line for the following set of elements by (RPWM). Also Calculate Balance delay & line efficiency η_L & Smoothness index. Take the cycle time as 18 min.

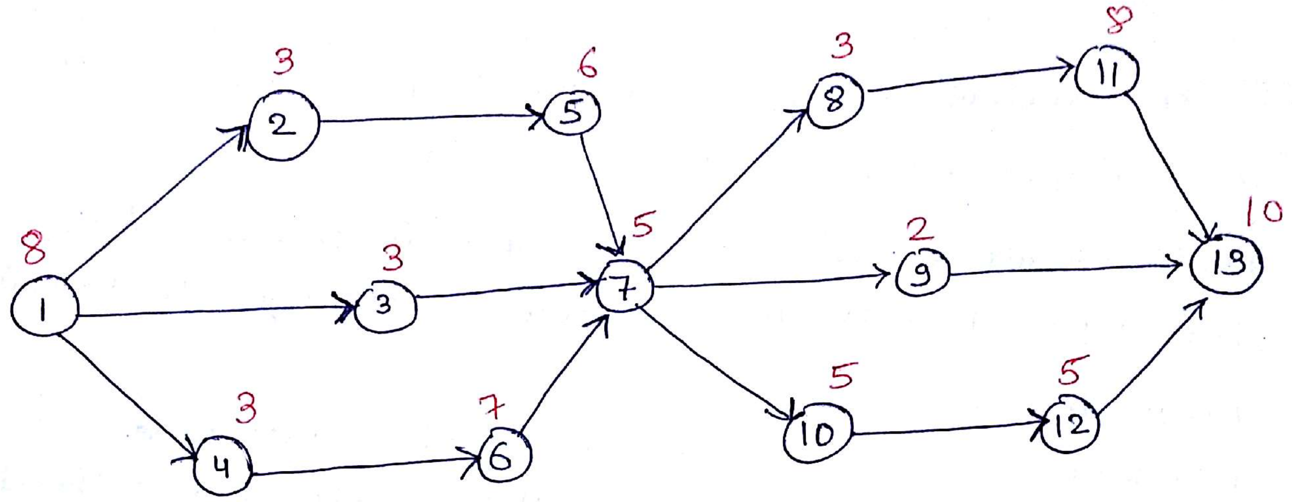
$T_c = 18$ min

Step I

Weight	Element	T_i (min)	Precedence
44	1	8	–
35	2	3	1
29	3	3	1
36	4	3	1
32	5	6	2
33	6	7	4
26	7	5	3, 5, 6
21	8	3	7
12	9	2	7
20	10	5	7
18	11	8	8
15	12	5	10
10	13	10	9, 11, 12



जिसका Weight find करना है। वहां से longest Path की value & Weight every Point Required है।



Step II Make New Table by arranging elements according to their Weight in decreasing order.

W/s	Element	Ti	Post wt.	Precedence	Tsi	Idle Time
1	1	8	44	—	14	4
	4	3	36	1		
	2	3 =14 < 15	35	1		
2	6	7	33	4	16	2
	5	6	32	2		
	3	3	29	1		
3	7	5	26	3, 5, 6	13	5
	8	3	21	7		
	10	5	20	7		
4	11	8	18	8	15	3
	12	5	15	10		
	9	2	12	7		
5	13	10	10	9, 11, 12	10	8

Balance delay:

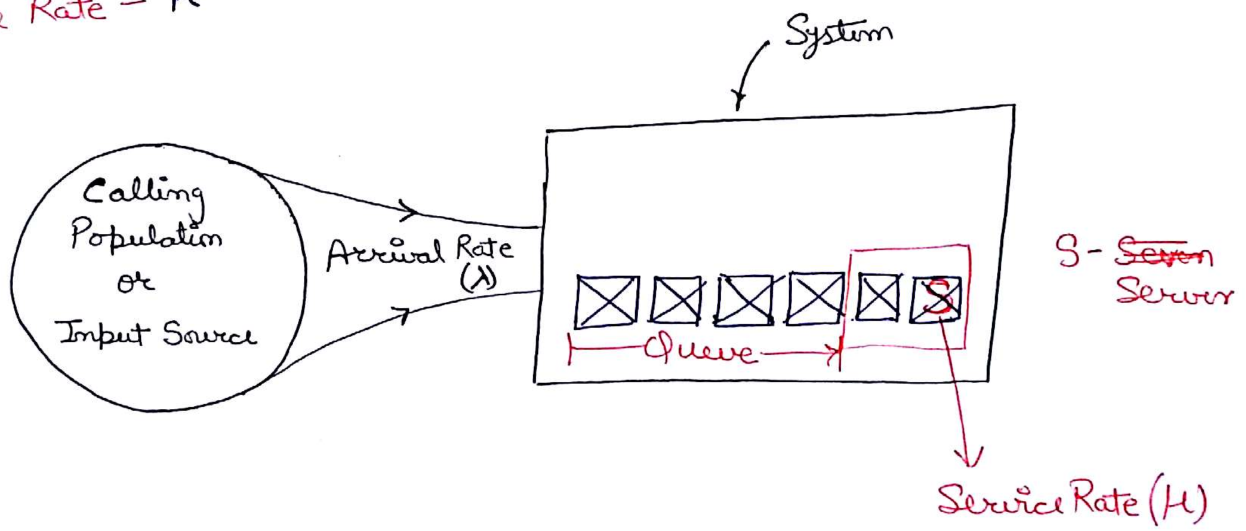
$$\boxed{B.D = \frac{5.15 - TWC}{5.15} \times 100 = 24.44\% \quad \eta_L = 75.56\% \quad SI = 7.07}$$

Queuing Theory
or
Waiting Line theory

- Aim of Queuing theory is achievement of economic balance b/w the Cost of providing Service & Cost associated with the Weight required for that Service.
- It is Applied to service oriented organization like m/c, Repair shop, Production shop, workshop, food chain, ATM etc.

Characterstics of Queuing Model :

Arrival Rate – λ Server – S
Service Rate – μ

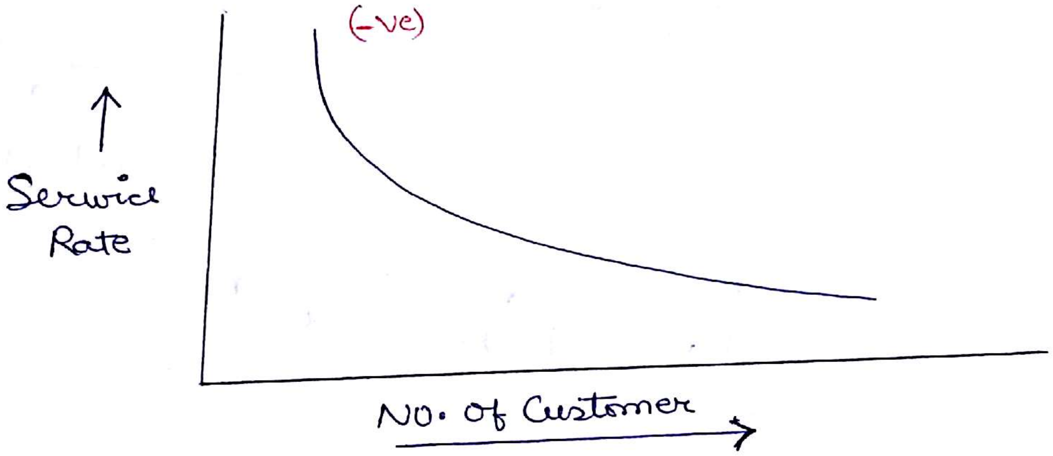


1) Arrival Rate / Arrival Pattern (λ) :

- The no. of Customers arriving per unit time is termed as Arrival rate.
- Customer Arrival is random & therefore it is assumed to follow Poisson's distribution → Prefered bcz. Random

2) Service Rate or Service Pattern :

The no. of Customers serviced per unit time is known as service Rate. and it is assumed to follow exponential distribution.



3) Service Rule / Service Order :-

i) FIFO or FCFS
ii) LIFO or LCFS
iii) SIRO → service in Random order
iv) Priority treatment

- It gives the information about the Queue dicisiplene which means the order by which, Customer are picked up from the waiting line. in order to provide service.

4) System & Calling Population :-

System :

System is a place or facility, where Customer arrive in order to get service & its Capacity may be finite or infinite.

Calling Population

The entire Sample of Customer from which only a few visits the system is termed as Calling population or input Source.

Its Capacity may be finite or infinite

It is infinite → When arrival of few Customer doesn't have any effect on the arrival pattern of future Customer

5) Customer Attitude :-

- Attitude
  - Patient
  - Impatient
    - Jockey
    - Balking
    - Reneging
    - Cheater

Jockey :-

Customer Keep on changing Queue, in hope to get Service faster.

Balking :

Customer Doesn't join the Queue & leaves the system as Queue is very long.

Reneging :

Customer join the Queue for short duration and then leave the system as Queue is moving very slow.

Cheater :

Customer takes illegal means like fighting. Bribing, etc. in hope to get Service faster.

Representation of Queuing Model :-

Queuing Model are represented by $\boxed{\text{Kendall \& Lee.} \quad (a/b/c) : (d/e/f)}$

Where,

a → Probability distribution for arrival pattern.
b → Probability distribution for service pattern.
c → No. of Server within the system.
d → Service rule or service order.
e → Size / capacity of System.
f → Size / capacity of calling population or input Source.

Symbols:

a & b

→ M – Markovian (Poisson)
↳ for arrival Pattern. or exponential Service pattern.

→ E – Erlangian (Gamma)
↳ For arrival / service pattern.

→ D → Deterministic arrival / Service pattern.

→ C → 1, 2, 3, 4 No. of Server

→ d → FIFO, LIFO, SIRO, GD (General Service discipline

→ e & f →

N – finite
∞ – infinite

① → Arrival Rate $= \lambda$ (Poisson)

$\lambda = 15$ Cust./hr.

② → Inter arrival rate $= \frac{1}{\lambda} = \frac{1}{15}$ hr/cust $= 4$ min/cust

(Exponential)

---

③ → Service Rate $= \mu$ (Exponential)

$\mu = 20$ Cust/hr.

④ → Inter-Service Rate Time $= \frac{1}{\mu} = \frac{1}{20}$ hr/cust $= 3$ min/cust

(Poisson)

---

if $\lambda > \mu$

Service Rate → μ

Arrival Rate → λ

- Queue length will Keep on increasing and after the Certain period of time, incomming population will not get service.
- In these Conditions We Can't find Solutions as system ultimately fails.

ii) if $\lambda \leq \mu$:- System Work

$$\boxed{\rho = \frac{\text{Arrival Rate}}{\text{Service Rate}} = \frac{\lambda}{\mu}}$$ ***

if $\mu = 20$ cust./hr.

① if $\lambda = 5$ cust./hr ; $\rho = \frac{5}{20} = 0.25$ → 0.25% Probability of Cust. to Wait

2) if $\lambda = 10$ Cust/hr. ; $\rho = \frac{10}{20} = 0.50$

3) if $\lambda = 15$ cust/hr $\cdot \rho = \frac{15}{20} = 0.75$

4) if $\lambda = 20$ cust/hr $\rho = \frac{20}{20} = 1.00$

→ The Ratio of Arrival to Service Rate indicates the % time Server is busy and is Known as utilization factor, Average utilization System utilization, Channel efficiency and Clearing Ratio.

- It also indicates the probability that the New customer have to Wait.

There are two formula's for ρ Two Formula for $\boxed{\rho = \frac{\lambda}{\mu}}$ ***

i) Probability that the system is idle or Probability Of Zero Customer in the system :-

$$\boxed{P_0 = 1 - \rho} \quad \text{— (1) formula}$$

ii) Probability of having adjactly "n" customer in the System

$$\boxed{P_n = \rho^n \cdot P_0} \quad \text{—— (2) formula}$$

Probability → $P_0 + P_1 + P_2 + P_3 + P_4 + \cdots + = 1$

$P_3 + P_4 + \cdots = 1 - (P_0 + P_1 + P_2)$

Probability $= (1 - P_0 - \rho^1 P_0 + \rho^2 P_0)$

iii) Average no. of Customer in the System.

In this we include both the Customers waiting in the Queue along with those getting Service.

$$\boxed{L_s = \sum_{n=0}^{\infty} n \cdot P_n}$$

"Average No. of Customer in the System"

$P_n \rightarrow$ Probability of N Customer in the System.

Generalised form

$$\boxed{L_s = \frac{\rho}{1-\rho}}$$

is used

IV) Average no. of Customer in the Queue:-

In this we do not include the Customer getting service.

$$\boxed{L_q = \sum_{n=2}^{\infty} (n-1) \cdot P_n}$$

Generalised form $\longrightarrow$

$$\boxed{L_q = \frac{\rho^2}{1-\rho} = L_s - \rho = L_s \cdot \rho}$$

infinite → System Capacity → Stable

Little's Law :-

For a stable System, Average no. of Customer in the System or Queue is given by ;

= Average customer arrival rate × Average Waiting time of the Customer in system or Queue

$$\boxed{L_s = \lambda \cdot W_s} \rightarrow W_s = \frac{L_s}{\lambda}$$

$$\boxed{L_q = \lambda \cdot W_q} \rightarrow W_q = \frac{L_q}{\lambda}$$

$$\boxed{W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu}} \rightarrow$$

Where,

$W_s \rightarrow$ Average Waiting time of the Customer in the System

$W_q \rightarrow$ Average waiting time of the Customer in the Queue.

imp

Average → System

Mean → Queue

Qn) No. of Person arriving at a Service Centre is ~~meet~~ 8 Customer Per hour & the Service provider takes ~~[illegible]~~ 5 min per Customer then determine;

i) L_s & L_q

ii) W_s & W_q

Soln

~~$\lambda = 8$/hr~~ $\lambda = 8$ customer/hour

$\frac{1}{\mu} = 5$ min/customer $\qquad \mu = 12$ customer/hr.

~~$L_s =$~~ $\boxed{\rho = \frac{2}{3}}$ $\qquad \frac{\lambda}{\mu} = \rho$

$L_s = 2 \qquad L_q = 1.33$

$W_s =$

$W_q = \frac{L_q}{\lambda} = \frac{1.33}{8}$.

Qn) A Shopkeeper service 10 customer per hour and the customer arrival is 7 customer per hour. Find the probability of at least three customer waiting in the Queue.

Soln)

$\mu = 10$ customer per hour

$\lambda = 7$ Customer per hour

$\rho = \frac{\lambda}{\mu} = \frac{7}{10}$

$\boxed{\rho = 0.7}$

$P_n = \rho^n - P_0$

Atleast 3 in the Queue

means atleast 4 in in the System.

$P_4 + P_5 = 1 - (P_0 + P_1 + P_2 + P_3)$

$= 1 - (P_0 + \rho^1 P_0 + \rho^2 \cdot P_0 + \rho^3 P_0) = \rho^n = 0.2401$

→ Probability of "n" arrival for the System during period T

arrival → Poisson's Ratio

Generalised form of Poisson's distⁿ =

$$\boxed{P(n,T) = \frac{(\exp)^{-\lambda \cdot T} \cdot (\lambda T)^n}{n!}}$$

units must be Same While Applying

→ Probability that more than T time period is needed to Serve Customer :

Service → exponential distⁿ.

$$\boxed{P = (\exp)^{-\mu T}}$$

→ Probability that the Waiting time in the Queue is greater than T :-

$$\boxed{P(W_q > T) = \rho\,(e - \rho)^{-\frac{T}{W_s}}}$$

→ Probability that the Waiting time in the System greater than T :-

$$\boxed{P(W_s > T) = (\exp)^{-\frac{T}{W_s}}}$$

Work-Study

Method or Motion Study | Time study or Work Measurement

1) Method or Motion Study :-

It is the set of Technique develop to divide a job, into Smaller Part, followed by its rearrangement to make it more effective & productive, and the steps involved are ;

Select → Record → Examine → Develop → Install → Maintain

** Aim is ↓ Higher Productivity

Recording Techniques :-

These are design to Simplify & Standardadise the Recording Work and most Commonly used recording Techniques are;

i) Process Chart : (P.C)

Symbols :-		Activity
○	———	Operation
△	———	Storage
□	———	Inspection
⇒	———	Transportation
D	———	~~Deta~~ Delay

⊡ → Operation & Inspection

a) Outline Process chart :-

- These charts are not much detailed and are use to give a little bit of information about what is going on, within the production System.
- It uses only two symbols i.e operation ○ & inspection □

b) Flow Process Chart :

- i) → Material Type Flow Process chart
- ii) → Man Type Flow Process chart
- iii) → M/c Type Flow Process chart

- These charts are much detailed and record all the activities Sequence wise along with time, distance, additional remark
- All the five Symbols are used in these chart.

c) Two Handed Process chart :-

- It is used to Record, the activities of Left hand related to right hand of an operator & activities of two hands are Synchronised on a Common ~~typ~~ time Scale.
- It is used for Short duration, Repeated Sitting jobs, all the five Symbols are used in these chart.

2) Time Scale Chart :

a) Multiple Activity Chart :-

☐ — Idle

▨ — Working

Time	Man	M/c	Remark
1.5 min	▨		Job loaded
6 min.		▨	m/c Working
0.5 min	▨		Job unloaded

Cycle time = 8 min.

Gantt chart → only for machine

Percentage utilization

$$\text{man} = \frac{2}{8} \times 100 = 25\%$$

$$\text{m/c} = \frac{6}{8} \times 100 = 75\%$$

- It is the Chart in which the activities of more than 1 items are recorded on Common Scale (time) to Show their inter-relationships.
- The Study of these charts makes it possible to rearrange these activities in such a manner that utilization may be Optimized

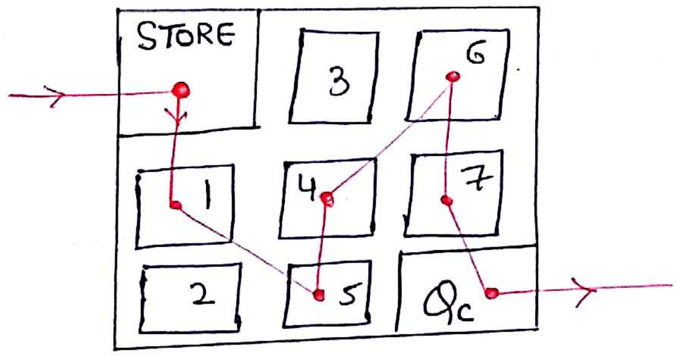
b) Gang Process chart :- F-1 Race Maintainance Group

- It is another type of multiple activity chart.
- Which shows the Relationship carried out by different members of group ~~wla relationed~~ Related to each other while performing a Single Task.

~~(a) Flow Diagram~~

3) Diagram :

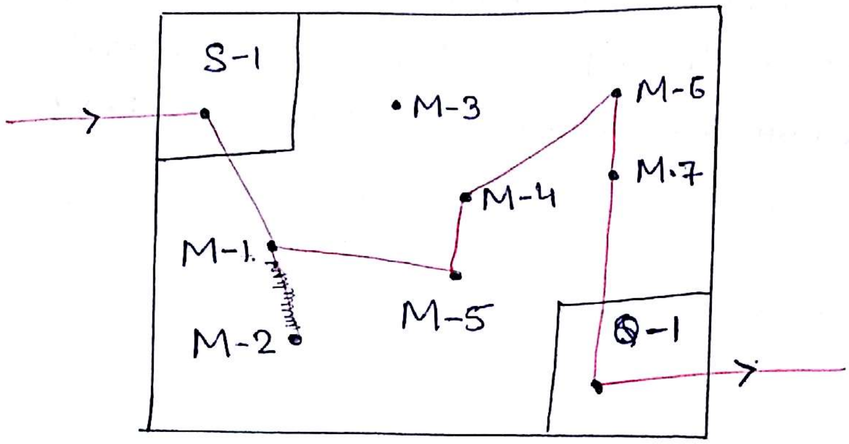
a) Flow Diagram :



- It is the Scaled Plan or model of the Working area, showing the details, about different facilities, equipment along with there number symbol.
- These are used along with flow process chart to give Complete information about what is going on within the production System

b) String Diagram :-

Pins and thread are used



Scale - 1cm = 5m

→ It is the ~~[illegible]~~ scale plan or model, on which a thread is used to trace and measure the path, travel by ~~[illegible]~~ worker or material

→ The aim is to find, a path having min. distance travelled.

c) Travel Chart :

movement From →

movement to ↓

	1	2	3	4	5	6
1			3			
2						
3					2	
4	4					
5		1				
6				5		

Show movement
2-5-3-1-4-6

- Travel Chart is tabular record used for presenting Quantitative data about the movement of Worker or material
- This chart is Always Square, and each Square represents a Work station.

→ Micro Motion Study :- By Gilberth

- It is used to Study those operations which are very fast, very short duration and repeated several number of times.
- To facilated micromotion study, gilberth divided all the basic Hand & eye motion into 17 fundamental motion known as Therblig. One was added later on, Now they are total 18 Therblig.
- Each Therblig has a specific Symbol, notation and Colour for Recording purpose

→ SIMO Chart : Simultaneous motion cycle chart

- It is micro motion form of ~~two process~~ ~~Two~~ Two Handed process chart which is based up on film analysis. It is used for short duration repeated sitting job and time is measured in ~~clock~~ Wink Counter.

- All the 18 Symbols of therblig are used in these chart.

$1 \text{ Wink} = \frac{1}{2000} \text{ min}$

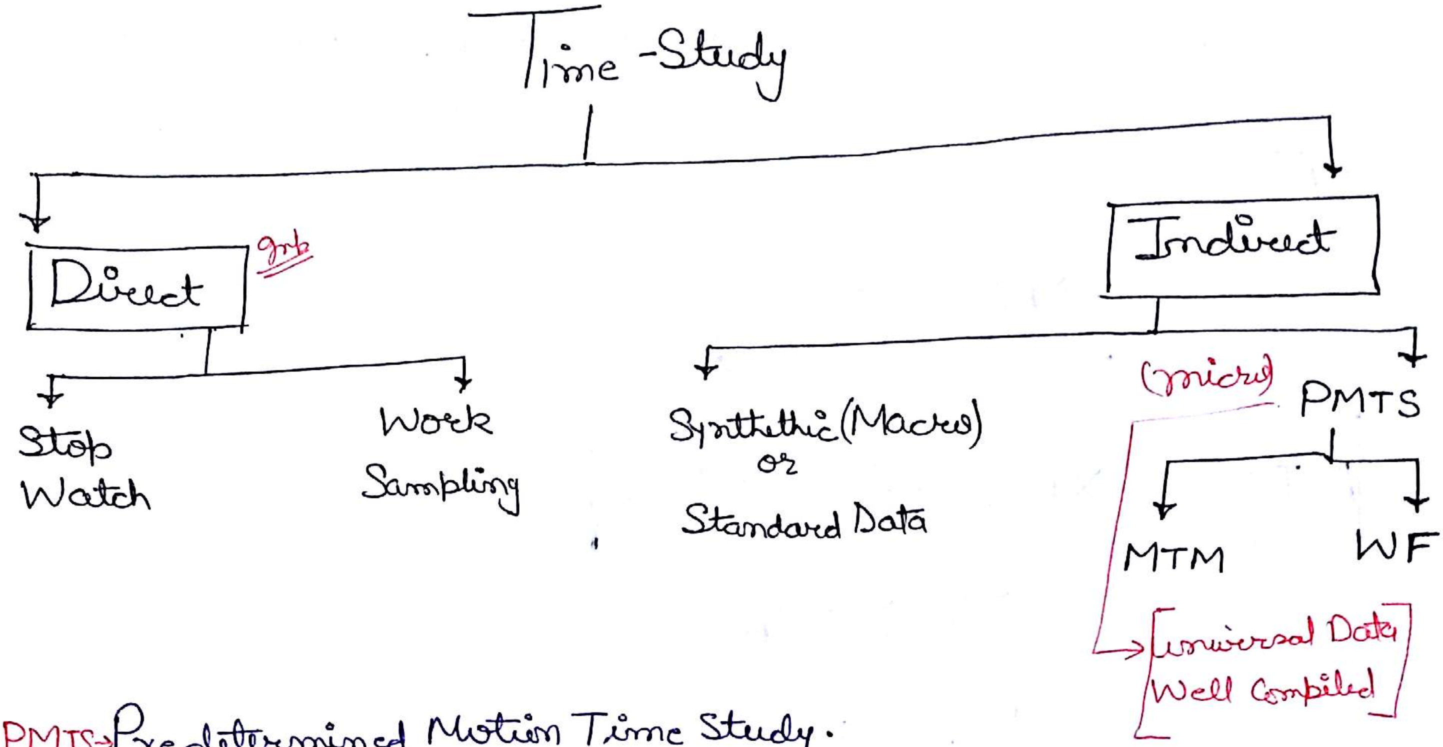
Cycle Graph :- By Gilbert :-

- In cycle graph, continous source of light, bulb is attached to the hand of operator and movement of light is recorded by Camera.

- The Study is performed in a dark room.

Chrono Cycle Graph :- extension of cycle graph

- In this Method, light source is intrupted, so that path appear as the series of dot. The pointed end indicate the dir^n of movement and the distance b/w the dot, ~~tes~~ tells the speed of the moment (movement)



PMTS → Predetermined Motion Time Study.

MTM → Method time measurement.

WF → Workfactor System.

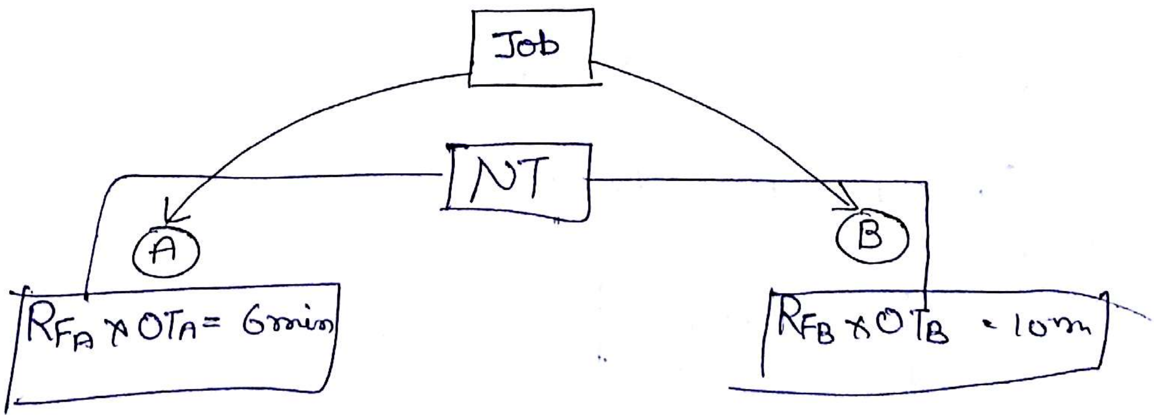
1) Direct :

a) Stop Watch :-

Observed or elemental time :-

→ It is a time measured & observed by an observer using some measuring device like stop watch.

Normal time :-



- It is a time required to complete a job by a normal average worker under normal working condition.

$$NT = OT \times \text{Performance Rating factor}$$

$$\boxed{NT = OT \times RF}$$

$$\boxed{NT = OT_A \times RF_A = OT_B \times RF_B}$$

$$OT_B > OT_A$$

$$RF_A > RF_B$$

NOTE :

Rating factor is applied only to mannual control operation and is never applied for machine element

Standard Time :-

- It is a time required to complete a job, taking all the uncertainity Related to the production system into account.

$$\boxed{ST = NT + \text{Allowances}}$$

$$\text{Allowances} = 20\% \text{ of } (NT)$$

$$\boxed{ST = NT + 0.2\,NT}$$

$$\boxed{ST = 1.2\,NT}$$

Allowances :-

It is the extra time provided to a Qualified Worker above the normal time to work Continously for long duration, few of these are, Rest, Personal, Fatigue, Contingency, Policy.

- Rest Allowance.
- Personal Allowance.
- Fatigue Allowance.
- Contingency Allowance.
- Policy Allowance.

Qn) An operated rated at 125% to 10 min to Complete an Observed job. if total 10% Allowances are required for jobs then find no. of jobs Completed in shift of 8 hour duration.

Soln)

$$NT = 10 \times 1.25 = 12.5 \text{ min}$$

$$ST = NT + 0.1NT = 1.1NT$$

$$\boxed{ST = 13.75 \text{ min}}$$

$$\text{No. of job} = \frac{8 \times 60}{13.75} = 34.9 \approx \boxed{34 \text{ jobs}}$$

Qn) Observed time in minute for four cycle of an operation Consisting of 5 element using Stop Watch is as given below find the standard time per unit, when element ② & ④ are m/c element & for all other element, operater is rated at 120% take the total allowances as 20% of Standard time

Element	Cycle time (min) 1	2	3	4	Avg OT	RF	NT = OT×RF
1	5.05	4.95	5.0	5.0	5.0	120%	6.0
2	7.80	7.75	7.85	7.80	7.8	m/c	7.8
3	5.25	5.35	5.30	5.30	5.30	120%	6.36
4	8.65	8.60	8.55	8.60	8.60	m/c	8.60
5	4.05	3.95	3.95	4.05	4.0	120%	4.8
							33.56

Work Sampling OR Activity Sampling :- [L.H.S Tippet]

50hr → 120unit

↓

400obs.

↓

80 = No work

Production time $= 0.8 \times 50$
$= 40hr.$

$O.T/unit = \dfrac{40 \times 60}{120} = 20min$

- It is a Work measurement Technique, in which large no. of Random observations are made at random interval over specified Period of time over a group of Worker or machine.
- It is based upon probability theory, but higher the no. of observation better the results will be.
- It is the best Technique to determine, allowances required by Worker or operator.

No. of Observation for Defined level Of Confidence :-

$$\boxed{P.L = Z.\sigma_P}$$

$n \rightarrow$ no. of observation

$P \rightarrow$ Propotion or fraction Occurance of Activity

$L =$ Limit Of Accuracy

$Z =$ Standard normal variant whose value depens upon ~~Condt~~ Confidence Level required.

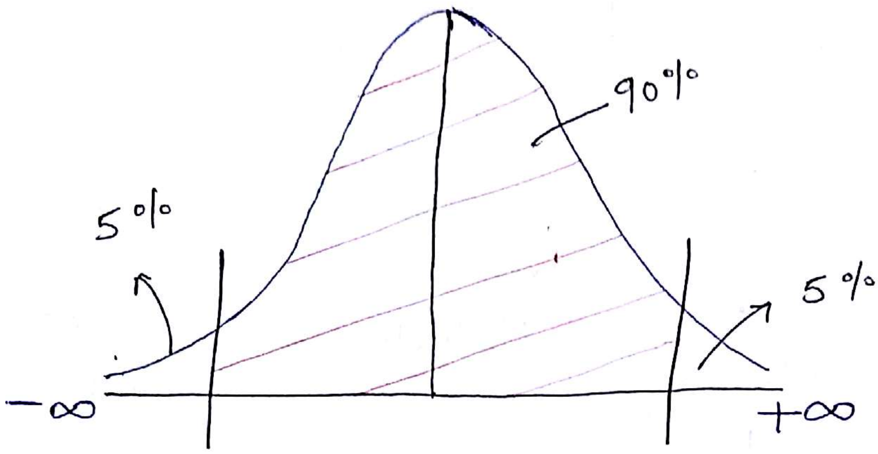
Where,

$$\sigma_P = \sqrt{\frac{P(1-P)}{n}}$$

$$P.L = Z\sqrt{\frac{P(1-P)}{n}}$$

$$\boxed{n = \frac{Z^2}{L^2}\frac{(1-P)}{P}}$$

Z	Confidence Level %
1.96 →	95 %
2 →	95.45 %
3 →	99.74 %



100 Obervation

$$P = \frac{78}{100} = 0.78$$

$$\boxed{\begin{aligned} L &= 5\% = 0.05 \\ L &= \pm 5\% = 0.10 \end{aligned}}$$

Confidence Precision Factor $= K = \frac{Z}{L}$

for objective take $Z = 2$
Ratter than $Z = 1.96$

Safety Stock $= Z \times \sigma$

IInd Method

$$\boxed{P.L = Z \cdot \sigma_P}$$

$$\boxed{L = Z \cdot \sigma_P}$$

$$L = Z\sqrt{\frac{P(1-P)}{n}}$$

$$\boxed{n = \frac{Z^2}{L^2} P(1-P)}$$

for 95% $Z = 2$

~~n = 4P(P~~

$$\boxed{n = \frac{4P(1-P)}{L^2}}$$

Qn) Find the no. of Observation for 14 Activities out of 20 and 95% of Confidence Level & 5% Accuracy

$Z = 2$
$L = 0.05$
$P = 0.7 = \frac{14}{20}$

$n = \frac{2^2}{0.05^2} \frac{(1-0.7)}{0.7}$

1600

$n = 685.714$

$n = \frac{1.96^2}{0.05^2} \frac{(1-0.7)}{0.7}$

$n = 659$ observation

Qn) A Work Sampling Study Was Conducted in the m/c Shop & the data Recorded are total no. of Observation = 2000

no Activity observation = 400

* Ratio b/w mannual to m/c 3:2

Propotion of activities

* Rating factor is 120%
* Total no. of Pieces produced during
* Study is 240 unit
* duration of Study 150 hr.

Calculate Standard time per unit assuming 15% Allowances.

Soln

$P = \frac{1600}{2000} = 0.8$

Total obs^n = 2000
NoWork = 400
Work obs^n = 1600
% time Working = 80%

Production time = 0.8×150
= 120 hr.

O.T/time = $\frac{120 \times 60}{240}$ = 30 min

Observation time

Observation time →

$$(OT)_M = \frac{3}{5} \times 30 = 18 \text{ min}$$

$$(OT)_{m/c} = \frac{2}{5} \times 30 = 12 \text{ min}$$

$$NT = (OT)_{man} \times R.F + (OT)_{m/c}$$

$$\boxed{NT = 33.6 \text{ min}}$$

if Allowances are not Mention in Question then take from Question

$\boxed{\text{Here is } 20\%}$

$$ST = NT + 0.15\, NT$$

$$= 1.15 NT$$

$$\boxed{ST = 38.6 \text{ min}}$$ Ans

Linear Programming

By:-
(George. B. Dantzing)

- Linear programming is used for optimization of our limited resources, when there are no. of Alternate Solution, possible for the problem.

- It is the matthematical Techonique, and the term Linear, is used for the Variable and it simply means that, the relationship b/w the Variable can be represented in the form of straight line.

Requirement Of L.P.

i) Objective function:

It is the function, which we need to Optimize & it should be Cleary identificable & measurable in Quantitative term like Maximization of Profit, sales, minimization of Cost.

ii) Constraint or Condition:-

These are the limited Resources, Within Which, We need to optimize our Objective function.

iii) All the Variables in the objective function & Constraint, should be Linear and non-negative

General Statement of Linear Programming :

$$\text{Max. } Z = C_1x_1 + C_2x_2 + \text{-------} + C_nx_n$$

Constraint

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \text{---------} + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \text{------------} + a_{2n}x_n \leq b_2 \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad \vdots \qquad\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \text{------------} + a_{mn}x_n \leq b_m \end{cases}$$

Nonnegative Condition $\longrightarrow$ $x_1, x_2, \text{-----} x_n \geq 0$

Where,

a_{ij}, b_i, & C_j are Constraint and x_j is Variable

$$i = 1, 2, \text{------} m.$$
$$j = 1, 2, \text{------} m.$$

$a_{ij} \rightarrow$ Technological Coefficient for Substitution

$b_i \longrightarrow$ Resource value

$C_j \longrightarrow$ Profit Coefficient

$x_j \longrightarrow$ Decision or Choice variable

Graphical Method :-

Steps in Graphical Method :-

i) Identified the problem & defined decision variable, objective funⁿ and Constraint.

ii) Draw the Graph, that include all the Constraints and identify the Common feasible region.

iii) Find out the point, within the feasible Region, that optimizes the objective function. This point gives the final Solution.

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Qn) 39

	Prod \ m/c	P	Q	R	Profit
$x_1 \rightarrow$	A	10	6	5	60
$x_2 \rightarrow$	B	7.5	9	13	70
	Max. hr. / week	75	54	65	

Step → The Key decision is to determine, no. of unit produced of A & B in a Week.

- Let these are x_1 & x_2 Respectively.

Step2 → Feasible Alternatives are all the values of x_1 & $x_2 \geq 0$

Step 3 → The objective is to maximize Weekly profit, when the profit Per unit is given. So the objective function.

$$Max. Z = 60x_1 + 70x_2$$

Step 4 → Restriction of max. m/c'n time available for the three weeks in a week.

So the Constraints are.

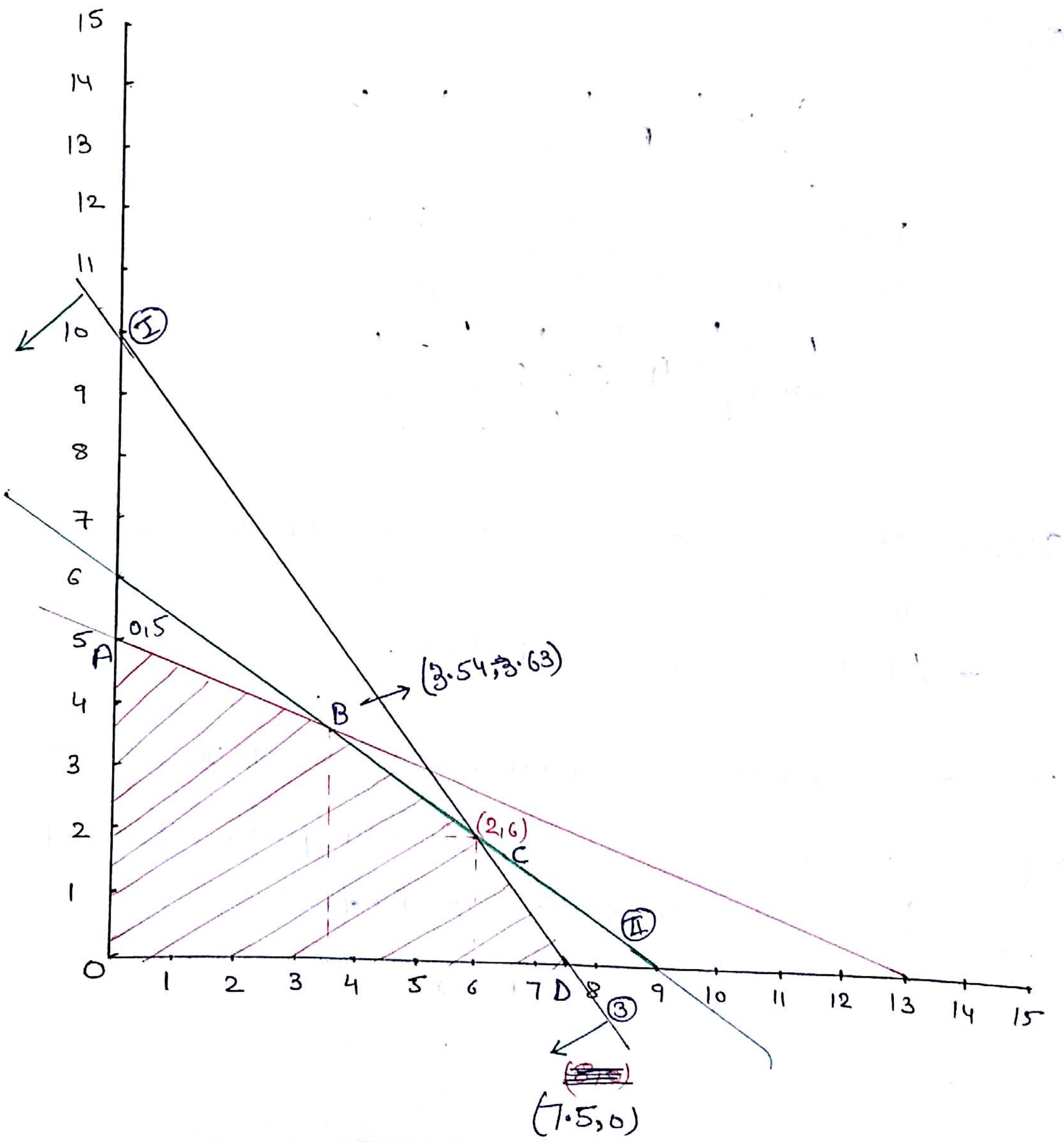
$$P \rightarrow \quad 10x_1 + 7.5x_2 \leq 75 \longrightarrow \frac{x_1}{7.5} + \frac{x_2}{10} = 1$$

$$Q \rightarrow \quad 6x_1 + 9x_2 \leq 54 \longrightarrow \frac{x_1}{9} + \frac{x_2}{6} = 1$$

$$R \rightarrow \quad 5x_1 + 13x_2 \leq 65 \longrightarrow \frac{x_1}{13} + \frac{x_2}{5} = 1$$

$$x_1, x_2 \geq 0$$

Step 5 → All the Constraints are plotted on a graph, to get the feasible Region.



- The Shaded Region OABCD is a region of feasible Solⁿ. and any point Within this region Can be our Solution under the given Constraint.

Optimality:

Now We put the values of Corner point of the feasible region in the Objective function. The point Which optimises the Objective function gives the final Solution.

$$Z(O) = 60\times 0 + 70\times 0 = 0$$
$$Z(A) = 60\times 0 + 70\times 5 = 350$$
$$Z(B) = 60\times 3.54 + 70\times 3.63 = 466.5$$
$$\boxed{Z(C) = 60\times 6 + 70\times 2 = 500}$$
$$Z(D) = 60\times 7.5 + 70\times 0 = 450$$

Max. at Z(C) (6, 2) Point

$$\boxed{x_1 = 6,\ x_2 = 2}$$

Rs. 500

Imb

→ Explain why one of the Vertices of the feasible Region become the Optimum Solution point

- One of the Vertices of the feasible Region gives the final Solution bcz the Objective function is the Straight line, With the Constant Slope. and as it move away from the origin, Objective function increases and optimum value Will be at one of the Corner extreme Point.
- Objective function Will be tangent to that point and gives the Optimum Solution.

Binding and Non Binding :-

$x_1 = 6$ $\quad x_2 = 2$

P → $10x_1 + 7.5x_2 \leq 75$ → $75 = 75$ Binding

Q → $6x_1 + 9x_2 \leq 54$ → $54 = 54$ ~~Non~~ Binding

R → $5x_1 + 13x_2 \leq 65$ → $65 \neq 56$ Non Binding

When We put the values of optimum Solution in the Constraint and $\boxed{LHS = RHS}$ the Constraints is termed as Binding

Other wise non-Binding.

Final Solution is Always obtained from the Binding Constraint.

Q) Solve the following LP Problem for minimization.

$$\text{Min } Z = 6x_1 + 4x_2$$

$$3x_1 + 3x_2 \geq 40 \qquad 3x_1 + x_2 \geq 40$$

$$2x_1 + 5x_2 \geq 44$$

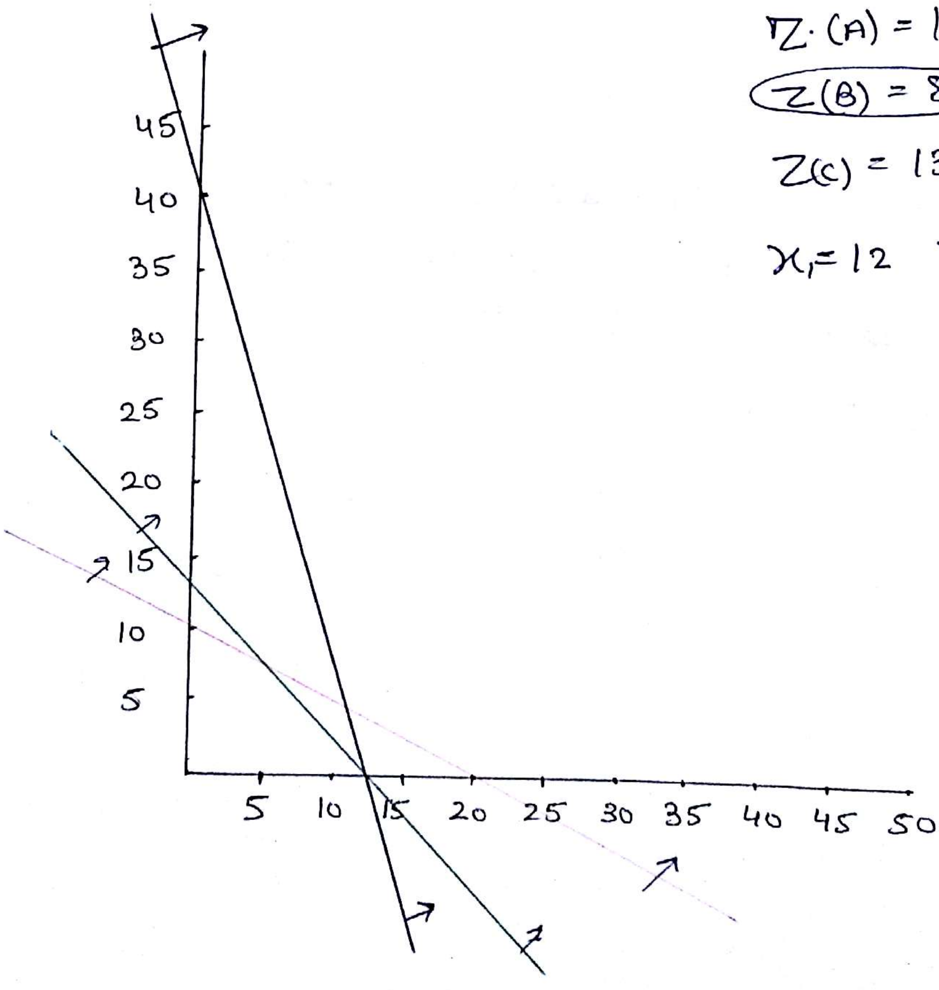
$$x_1, x_2 \geq 0$$

Soln)

$$\frac{x_1}{40/3} + \frac{x_2}{40/3} = 1 \qquad \boxed{\frac{40}{3} = 0.33}$$

$$\frac{x_1}{(40/3)} + \frac{x_2}{40} = 1$$

$$\frac{x_1}{22} + \frac{x_2}{(44/5)} = 1$$



$Z(A) = 160$

$Z(B) = 88$

$Z(C) = 132$

$x_1 = 12 \quad x_2 = 4$

Redundant or Degenerate or Unnecessary Constraint :-

Constraint is doesn't become part of Boundary, making feasible Region is termed has redundant Constraint.

The inclusion & exclusion of Such Constraint doesn't have any effect on the final solution of the problem.

Speacial Cases :-

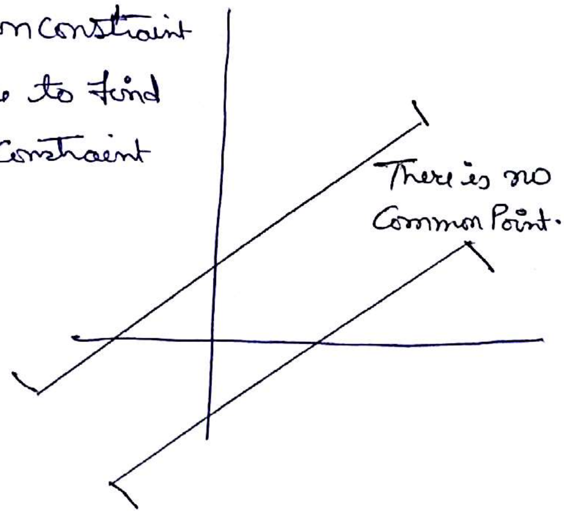
A) Infinite or multi optimal Solution :-

- Infinite no. of Solution means, We get same optimum value of the objective function for different Varying variable.
- We always get a unique solution, When the slope of objective funⁿ is different from Constraint
- infinite no. of Solution is obtained, When slope of objective funⁿ becomes equal to one of the binding Constraint.

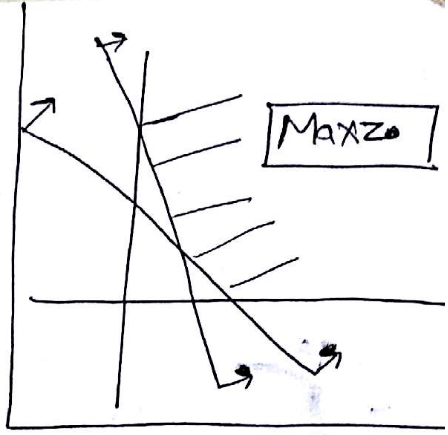
B) No Solution or Infeasibility :-

In Some Condition Constraints may be unconstraint in such a manner, that, it is not possible to find a feasible solⁿ. ~~that~~ Which satisfy all the Constraint

There is no solⁿ ~~of~~ to Such problem.



c) Un Bounded Solution :-



- In some condition, the highest value of objective function, goes up to infinite & it simply means that, common feasible region, is not bounded by the limit on the constraint.

- It is termed as unbounded solution

B

Simplex Method

- It is the Set by Step procedure, in which, We proceed in a Systematic manner from an initial feasible Solution.
- With an improve upon that initial Solution until in Certain no. of Steps may reach at optimal Solution

Standard Form For Simplex :-

i) Resource value :

i) All the Resource value for the given Constraint Should be non-negative.

$$2x_1 - 5x_2 \leq -40$$

$$\boxed{-2x_1 + 5x_2 \geq = 40}$$

ii) All the inequalities of the given Constraint Should be Converted into equalities. inequal Sign into equal Sign

$$4x_1 + x_2 \leq 60$$

$$4x_1 + x_2 + \underset{\substack{\uparrow \\ \text{Slack variable}}}{S_1} = 60$$

$$5x_1 + 2x_2 \geq 85$$

$$5x_1 + 2x - \underset{\substack{\uparrow \\ \text{Surplus variable}}}{S_2} = 85$$

iii) Each of the decision variable for the objective funⁿ. of constraint should be non-negative and Linear.

$$\boxed{x_j \geq 0}$$

$$\text{Max. } Z = c_1 x_1 + c_2 x_2 + \cdots\cdots + c_n x_n$$

Constraint
$$\begin{cases} a_{11} x_1 + a_{12} x_2 + \cdots\cdots + a_{1n} x_n \leq b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots\cdots + a_{2n} x_n \leq b_2 \\ \vdots \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\quad \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \cdots\cdots + a_{mn} x_n \leq b_m \end{cases}$$

$$\boxed{x_j \geq 0}$$

No. of Variable $\longrightarrow \boxed{n > m} \longrightarrow$ No. of equality Constraint

n

$\swarrow \qquad \searrow$

m — Basic variable; $n - m = 0$ — Non-Basic variable

Basic variable

$\downarrow$

if (-ve) then called Basic Solⁿ

$\downarrow$

But if all are +ve i.e. $x_j \geq 0$ Basic feasible Solⁿ

- if there are m, equality Constraint & n is no. of variable and $\boxed{n > m}$ then we need to put $(n-m)$ variable equal to zero known as non Basic variable. and solve the Remaining m → Basic variable to give basic solution.

- (This Step Reduces the number of Alternate Solution, whose max. limit is Given by;

$$^{n}C_{m} = \frac{n!}{m!(n-m)!}$$

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Q. 41)

$$\text{Max. } Z = 40x_1 + 35x_2$$

R.M = $2x_1 + 3x_2 \leq 60$

$4x_1 + 3x_2 \leq 96$

Non-ve Constraint $x_1, x_2 \geq 0$

$$\text{Max. } Z = 40x_1 + 35x_2 + 0S_1 + 0S_2$$

$2x_1 + 3x_2 + S_1 = 60.$

$4x_1 + 3x_2 + S_2 = 96.$

$x_1, x_2, \geq 0$

$n = 2$
$m = 4$

$x_1 = 0$
$x_2 = 0$ Non Basic

Ist feasible Solution,

$x_1 = 0 \quad x_2 = 0$

$S_1 = 60 \quad S_2 = 96$

$Z = 0$ Rs.

Key element

c_i	Basic	x_1	x_2	S_1	S_2	b_i	$\theta_i = b_i/a_{ij}$
0	S_1	2	3.	1	0	60	30
0	S_2	$\boxed{4}$	3	0	1	96	24 ←
	C_j	40	35	0	0		
	$Z_j = \Sigma c_i \cdot a_{ij}$	0	0	0	0		
	$\Delta_j = C_j - Z_j$	40	35	0	0		

↑
Key column

- Calculate Δj value as the difference of Cj & Zj and it is termed as net evaluation Row or net opportunity Cost Row.

- The value of Δj Row indicates the amount of increase or decrease if the objective function that would occur, if one unit represented by the Column head is brought into the current solution.

- A Simplex table indicates the current solution as optimum, when all the values in the Δj row are

All element in Δj are either

i) −ve or zero when LP is for maximization.

ii) +ve or zero when LP is for minimization

- The current problem is maximization, so we select the highest positive value in Δj row and the selected Column is called Key Column. and the variable in the Column as incomming variable

- Now divide the bi values, from corresponding elements of Key Column to get replacement Ratio. In this Column, We select the min. +ve value and the selected row is called Key Row. With the variable in the row as outgoing variable.

- The element at the intersection of Key Column & Key Row is termed as Key element.

Steps

1) Key element is converted into unity by multiplying or dividing the Key row by common multiplying factor.

2) All the element in Key column are made zero except Key element which should be unity or 1.

This is done by adding or substracting proper multiples of Key row from other row.

3) In the New Table, outgoing variable is replaced by incoming variable.

c_i	Basis	x_1	x_2	S_1	S_2	b_i
0	S_1	0	3/2	1	-1/2	12
40	x_1	1	3/4	0	1/4	24

$R_2 \to R_2 \div 4$

$R_1 \to R_1 - 2R_2$

2nd feasible sol^n

$x_1 = 24, \; x_2 = 0$

$S_1 = 12, \; S_2 = 0 \quad Z = Rs\; 960$

Key element

c_i	Basic	x_1	x_2	S_1	S_2	b_i	$\theta_i = \frac{b_i}{a_{ij}}$
0	S_1	0	3/2	1	-1/2	12	8 ←
40	x_1	1	3/4	0	1/4	24	32
	C_j	40	35	0	0		
	$Z_j = \Sigma c_i \cdot a_{ij}$	40	30	0	10		
	$\Delta_j = C_j - Z_j$	0	5 ↑	0	-10		

Ci	Basis	x_1	x_2	S_1	S_2	bi
35	x_2	0	1	2/3	-1/3	8
40	x_1	1	0	-1/2	1/2	18

3rd feasible Solution

$R_1 \to R_1 \times \frac{2}{3}$

$R_2 \to R_2 - \frac{3}{4} R_1$

$x_1 = 18, \; x_2 = 8$

$S_1 = 0 \;, \; S_2 = 0$

$Z = Rs. 1000$

3rd Simplex Table.

Ci	Basis	x_1	x_2	S_1	S_2	bi
35	x_2	0	1	2/3	-1/3	8
40	x_1	1	0	-1/2	1/2	18
	Cj	40	35	0	0	
	$Z_j = \Sigma(c_i \cdot a_{ij})$	40	35	10/3	25/3	
	$\Delta_j = C_j - Z_j$	0	0	-10/3	-25/3	

Big-M Method

$\boxed{\geq \text{ or } =}$

$4x_1 - x_2 \geq 50$

$4x_1 - x_2 - S_1 = 50$

$x_1 = 0,\ x_2 = 0 \quad S_1 = -50$

$$\boxed{\begin{aligned} Max &= -MA_1 \\ Min &= +M \cdot A_1 \end{aligned}}$$

By Adding one Artificial variable

Non-Basic

$4x_1 - x_2 - S_1 + A_1 = 50$ $\quad \boxed{A_1 = 50}$

Non Basic $= x_1, x_2, S_1 = 0$

$5x_1 - 3x_2 = 25$

A_1 — This must not be seen in final Solution &
Aim is to eleminate

It is the Modified form of Simplex Method & is always Required whenever the Constraints are $\boxed{\geq \text{ or } =}$ Type, irrespective of Whether the problem is for maximization or for minimization.

In these Conditions We introduce an Artificial variable ~~both~~ in the Current solution to get an initial Working Matrix. These Artificial variable must not appear in the final Solution and this is ensured by providing an extreemly (-ve) value to their profit Coefficient In the objective function.

$$\text{Maximization} = -MA_1$$

$$\text{Minimization} = +M \cdot A_1$$

Where

M ~~ ~~, is no. higher than any finite number.

Special Cases :

i) Infinite or multi optimum Solution :

non Basic

Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i
x_3							
S_1							
x_1							
~~ ~~ Δj	0		0	0	0		

then Problem of infinite Solution.

When a non Basis variable in the optimum sol^n have zero value for ~~ ~~ Δj row. then Solution is not unique & it indicates that problem has infinite no. of Solutions

ii) unbounded Solution :

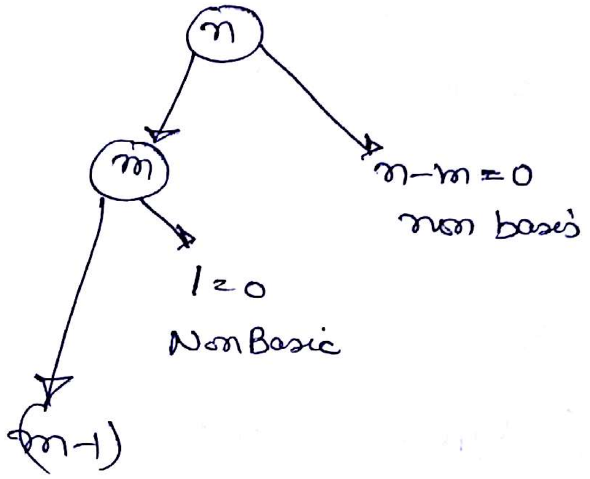
If in a Case all the values in the Replacement Ratio Column (θi) are either (-ve) or infinite ~~then~~ the solution terminate & it indicates that the problem has unbounded Solution.

iii) No Solution / infeasibility :-

When in the final Solution, artificial variable remains in the basis. then there is no feasible sol^n to the problem.

IV) Degenerate Solution :-

- When one or more of the basic variable becomes equal to zero during calculation, then the solution is called degenerate and the condition is known as Degeneracy.
- In a degenerate solution, the no. of Basic variable becomes less than equality constraints.



Duality

$$\begin{array}{ccc} \boxed{\text{Primal}} & \xrightarrow{\text{Transpose}} & \boxed{\text{Dual}} \\ & \searrow \quad \swarrow & \\ & (z) & \end{array}$$

- The initial given problem is termed as primal and the problem Obtained by Transposing row and Column, ~~by~~ but having the Same Optimum value of objective function is termed as Dual

Primal :-

a) Maximization :

$\leq$ type Constraint

b) Minimization :

$\geq$ Type Constraint

$\boxed{\text{Primal}} \longleftrightarrow \boxed{\text{Dual}}$

Max. $\longleftrightarrow$ Min.

$n \longleftrightarrow m$

$m \longleftrightarrow n$

$b_i \longleftrightarrow C_j$

$C_j \longleftrightarrow b_i$

$\leq$ Type Constraint $\longleftrightarrow$ Non-Negative Variable

$=$ Type Constraint $\longleftrightarrow$ Unrestricted Sign Variable (+ or -ve)

Qn) Find the dual for the following LP problem for minimization

$$\text{Min } Z = 4x_1 - 7x_2 + 13x_3$$

$n=3$	$n=5$
$m=5$	$m=3$

$$3x_1 - x_2 + 6x_3 \geq 8$$

$$5x_1 - 2x_3 \leq 7$$

$$4x_2 - 5x_3 \geq 12$$

$$x_1 - 3x_2 \geq 6$$

$$x_1 - 5x_2 + 7x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

Soln)

$$\text{Min } Z = 4x_1 - 7x_2 + 13x_3$$

$$\begin{array}{l|l} y_1 & 3x_1 - x_2 + 6x_3 \geq 8 \\ y_2 & -5x_1 + 2x_3 \geq -7 \\ y_3 & 4x_2 - 5x_3 \geq 12 \\ y_4 & x_1 - 3x_2 \geq 6 \\ y_5 & -x_1 + 5x_2 - 7x_3 \geq -15 \end{array}$$

$$\text{Max. } W = 8y_1 - 7y_2 + 13y_3 + 6y_4 \quad 15y_5$$

$$3y_1 - 5y_2 + y_4 - y_5 \leq 4$$

$$-y_1 + 4y_3 - 3y_4 + 5y_5 \leq -7$$

$$6y_1 + 2y_2 - 5y_3 - 7y_5 \leq 13$$

$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

$$\boxed{2x_1 - 5x_2 = 25}$$ for = to sign.

$2x_1 - 5x_2 \geq 25$

$2x_1 - 5x_2 \leq 25$

Transportation

	D_1	D_2	-----	D_n	Supply
F_1	C_{11} x_{11}	C_{12} x_{12}	-----	C_{1n} x_{1n}	a_1
F_2	C_{21} x_{21}	C_{22} x_{22}	-----	C_{2n} x_{2n}	a_2
⋮	⋮	⋮	⋮	⋮	⋮
F_m	C_{m1} x_{m1}	C_{m2} x_{m2}	-----	C_{mn} x_{mn}	a_m
Demand →					

Aim of Transportation problem is to meet the demand & Supply, in the most effective manner to minimizing total transportation cost.

$$\rightarrow \boxed{\sum_{i=1}^{n} a_i = \sum_{j=1}^{n} b_j}$$

$$\rightarrow \boxed{\text{Min.}\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \cdot x_{ij}}$$

1) Feasible Solution :

A Set of non-negative individual allocation which is satisfy all the given Constraint is Known as feasible Solution

2) Basic Feasible Solution:

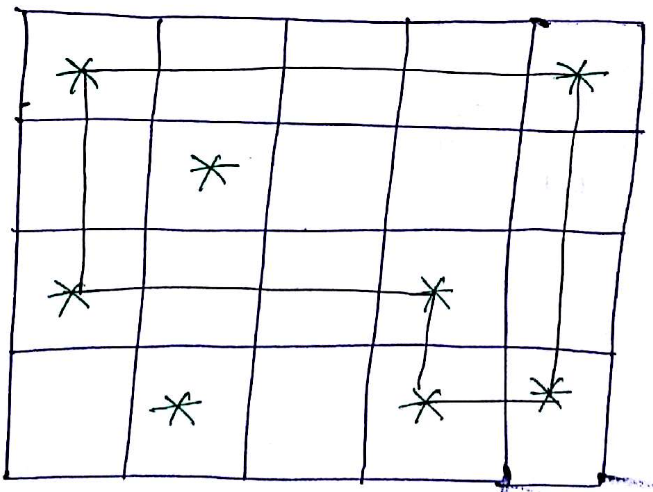
In (mxN) transportation model, if the total no. of Allocation is exactly equal to (m+n-1) then the solution is called Basic feasible Solution. (m+n-1) = No. of Allocation

3) Non- Degenerate basic feasible Solution :

For (mxn) transportation Model, solution is called Non-Degenerate When the following two Conditions are Satisfied :-

i) Total no. of allocation are exactly equal to m+n-1.

ii) These (m+n-1) allocations must be at independent position.



(4x5)

Dependent

- By dependent position, We mean that, it is always impossible to form a close loop, by joining These allocations by the series of Horizontal & Vertical Lines from one allocated Cell to Another.

NOTE :-

→ Optimality test Can only be perform, when initial Sol^n is Non degenerate.

Balanced & Unbalanced Transportation problem :-

- The total Supply from all the factories equal to total Demand from all the destination, problem is, Called balanced.

  other wise Unbalanced.

- If the given problem is unbalanced, then balanced it by adding dummy Source or destination

$$\text{Supply} = \text{Demand}$$

Qn) A Complete trasport product from 3 factories to 4 destination as given in the table, find Optimum allocation to minimize total transportation Cost.

	1	2	3	4	
P_1	20	30	50	17	7
P_2	70	35	40	60	10
P_3	40	12	60	25	18
	5	8	7	15	

	1	2	3	4	
P_1	20	30	50	17	7
P_2	70	35	40	60	10
P_3	40	12	60	25	18
	5	8	7	15	35

① North West Corner Method (Rule):

$\boxed{5}$ 20	30	50	17	7/2/0
70	$\boxed{6}$ 35	$\boxed{4}$ 40	60	10/4/0
40	12	$\boxed{3}$ 60	$\boxed{15}$ 25	18/15/0
5/0	8/6/0	7/3/0	15/0	

$$20 \times 5 + 35 \times 6 + 40 \times 4 + 60 \times 3 + 15 \times 25$$

$$Z = 100 + 210 + 160 + 180 + 375 = 1025$$

No. of Allocation = 5

$$m + n - 1 = 5$$

$$3 + 4 - 1 = 5$$

$$6 \neq 5$$

b) Row Minima :

	1	2	3	4	
P_1	20	30	50	$\boxed{7}$ 17	7/0
P_2	70	$\boxed{8}$ 35	$\boxed{2}$ 40	60	10/2/0
P_3	$\boxed{5}$ 40	12	60	$\boxed{8}$ 25	18/10/5/0
	5/0	8/0	7/5/0	15/8/0	35

No. of Allocation = 5

$$\begin{array}{l} \quad\ 119 \quad + 280 \quad + 80 \quad + 200 \quad + 200 \quad + 300 \\ Z = 17\times7 + 35\times8 + 40\times2 + 40\times5 + 25\times8 \times 60\times5 \end{array}$$

$$Z = 1179$$

3) Column Minima

	1	2	3	4	
P_1	20	30	50	17	7/2/0
P_2	70	35	40	60	10/3/0
P_3	40	12	60	25	18/10/0
	5/0	8/0	7/0	15/13/3/0	

$Z = 940$ Rs.

4) Least Cost Method or Matrix Minima :

	1	2	3	4	
P_1	20	30	50	$\boxed{7}$ 17	7/0
P_2	$\boxed{3}$ 70	35	$\boxed{7}$ 40	60	10/3
P_3	$\boxed{2}$ 40	$\boxed{8}$ 12	60	$\boxed{8}$ 25	18/10/2/0
	5/3	8/0	7/0	15/8/0	

$Z = 7 \times 17 + 3 \times 70 + 7 \times 40 + 2 \times 40 + 8 \times 12 + 8 \times 25$

$\boxed{Z = 985}$

5) Vogel's Approximation Method

or

Unit Cost Penalty Method

[Max. Penalty → Min Cost]

	1	2	3	4				
P_1	$\boxed{5}$ 20	30	50	$\boxed{2}$ 17	7/0 (3)	(13)	(33)	33
P_2	70	35	$\boxed{7}$ 40	$\boxed{3}$ 60	10/3/0 (5)	(5)	(20)	20
P_3	40	$\boxed{8}$ 12	60	$\boxed{10}$ 25	18/10/0 (13)	(13)	(35) ←	
	5/0	8/0	7/0	15/5				
	(20)	(18)	(10)	(8)				
	↑	(18) ↑	(10)	(8)				
			(10)	( )				
			10	43 ↑				

$$Z = 20 \times 5 + 17 \times 2 + 40 \times 7 + 60 \times 3 + 8 \times 12 + 25 \times 10$$

$$Z = 100 + 34 + 280 + 180 + 96 + 250$$

$$Z = 940$$

No. of Allocation :- 6

$$m + n - 1 = 6$$

$$7 - 1 = 6$$

$$\boxed{6 = 6}$$

- In this model we take the difference b/w smallest and 2nd smallest element in each Row & Column & write them below the respective row & Column.

- Then we select the highest individual difference & max. Possible allocation is done in the min. Cost cell in the Selected row or column.

- The row or column whose requirement become zero is Selected out, so that it cannot be considered again

- Continuing in similar manner untill all the allocation are done.

Optimality :

As a total no. of Allocation is adjactly equal to

$$m+n-1 = \text{No. of allocation}$$

and at independent position. so Optimality test Can be performed.

1) Stepping Stone's Method :

	1	2	3	4
P_1	[5] 20	30	50	[2] 17
P_2	70	+3 35	40	[3] −3 60
P_3	40	[8] 3− 12	60	[10] +3 25

(8, 3) ✓

+60	+40	+30	+35
−25	−20	−17	−60
+60	+17	+25	+25
−40	−25	−12	−12
+55	+12	+26	−12

- In this method We allocate one unit in an unlocated empty Cell & Compute the effect on the Cost of matrix.
- It is hit and trial Method, in which Chances of making error are more & therefore not much preferred.

2) Modified Distribution Mean Method (MODI Method)

or

U-V Method

The Steps Involved are;

i) develop Cost Matrise for allocated Cells only.

$\boxed{5}$ 20	30	50	$\boxed{2}$ 17
70	35	$\boxed{7}$ 40	$\boxed{3}$ 60
40	$\boxed{8}$ 12	60	$\boxed{10}$ 25

(Cj)

$u_i \downarrow$ $v_j \rightarrow$

	V_1	V_2	V_3	V_4
U_1 =	20			17
U_2			40	60
U_3		12		25

Let $v_1 = 0$

$U_1 + V_1 = 20$ $\qquad$ $U_1 + V_4 = 17$

$U_2 + V_3 = 40$ $\qquad$ $U_2 + V_4 = 60$

$U_3 + V_2 = 12$ $\qquad$ $U_3 + V_4 = 25$

Computing u_i & v_j value by taking $v_1 = 0$

u_i-

	0	-16	-23	-3
20	20			17
63			40	60
28		12		25

$u_1 = 20$ $v_1 = 0$

$u_2 = 63$ $v_2 = -16$

$u_3 = 28$ $v_3 = -23$

Develop u_i & v_j Matrix for unallocated cells ~~bestide~~ by entering Σ of u_i & v_j value for unallocated cell.

($u_i + v_j$) Matrix for unallocated cell.

	4	-3	
63	47		
28		5	

($u_i + v_j$) Matrix for unallocated cells

Subtract the cell value of ($u_i + v_j$) Matrix for unallocated cell from the original cost Matrix to get cell evaluation Matrix

	26	53	
7	-12		
12		55	

Cell evaluation Matrix

If any of the cell value in the cell evaluation matrix is (−ve) then the current solution is not optimum.

Step → In the cell, evaluation Matrix, identify the cell with the most (−ve) value, mark it & it is called identified cell.

Step → form a loop, such that it start from the identified cell & ~~common of path~~ corner of the should ~~only~~ Already have only Allocation.

Step → Make identified cell as positive and each other cell at the corner of path, alternatively (−ve), (+ve), (−ve) & so on.

Step → Make a new allocation to the identified cell, by entering the smallest allocation on the path, that has been assigned a negative sign. The Basic cell whose allocation becomes 0 leaves the solution.

	26	53	
7	+3 ✓ −12		−3
12	−3	55	+3

↓

[5] 20	30	50	[2] 17
70	[3] 35	[7] 40	60
40	[5] 12	60	[13] 35

New cost table Matrix

$\boxed{Z = 904 \text{ Rs}}$

	$V_1=0$	$V_2=-16$	$V_3=-11$	$V_4=-3$
$u_1=20$	20			17
$u_2=51$		35	40	
$u_3=28$		12		25

$u_1 = 20$ | $V_1 = 0$
$u_2 = 51$ | $V_2 = -16$
$u_3 = 28$ | $V_3 = -11$
| $V_4 = -3$

$u_i + v_j \rightarrow$

	4	9	
51			48
28		17	

Cell evaluation Technique

	26	41	
19.			12
12		43	

There is no (-ve) value in Cell evaluation Matrix.

So Current Solution is optimum.

1) Special Cases :

1) Degeneracy :-

$$\boxed{(m+n-1) \neq \text{No. of Allocation}}$$

When a no. of allocation become less than $(m+n-1)$, then Optimality Can't be performed and Such a Solution is Called Degenerate

2) Maximization Problem :

(I)

30	80	70	20
60	10	90	40
50	100	20	70

Subtract entire matrix by 100

↓ New Matrix

70	[34] 20	[18] 30	80
[26] 40	90	[51] 10	60
50	[28] 0	80	[19] 30

Best Allocate (I) Matrix Cost Always

$Z = 34 \times 80 + 18 \times 70 + 26 \times 60 + 51 \times 90 + 100 \times 0 + 19 \times 70$

- Maximization problems are solved by, converting it into first into minimization

  This is done by selecting highest cost in matrix & subtract the entire matrix by this highest cost.

Qn) Unit Transportation Cost in Rupees is given in the Cost Matrix below, determine the initial feasible solution using Vogel's approximation. & find the optimum distribution possible for the Company.

	D	E	F	G	
A	44	50	40	39	180
B	42	51	54	53	170
C	41	40	42	45	200
	90	100	120	180	550 / 490

A	44	50	40	$\boxed{180}$ 39	0	180/0	(39)	(1)	(1)	(1)
B	$\boxed{90}$ 42	51	$\boxed{20}$ 54	53	$\boxed{60}$ 0	170/110/20/0	(42) ←	(9)	(11)	← (1)
C	41	$\boxed{100}$ 40	$\boxed{100}$ 42	45	0	200/100/0	(40)	(1)	(1)	(3)
	90/0	100/0	120/100/0	180/0	60/0					
	(1)	(10)	(2)	(6)	(0)					
	(1)	(10) ↑	(2)	(6)						
	(1)		(2)	(6)						
			(2)	(6)						
			12	↑						

44	50	40	[180] 39	0	180
[90] 42	51	[20] 54	53	[60] 0	170
41	[100] 40	[100] 42	45	0	200
90	100	120	180	60	

$m+n-1 =$ No of Allocation

$3+5-1 = 6$

$8-1 =$

$\boxed{7 \neq 6}$

$\boxed{Z = 20080 \text{ Rs.}}$

Hence this Solution is Degenerate.

As the total no. of Allocation, is 6, which is less than $(m+n-1)$ equal to 7.. So the Current Solution is degenerate.

44	50	40	[180] 39	[ε] 0
[90] 42	51	[20] 54	53	[60] 0
41	[100] 40	[100] 42	45	0

- Now Allocating infinitely small but (+ve) value $\epsilon \approx 0$ at vacant min. cost cell such that all allocation remains at independent position.
- In the final solution, we put $\epsilon \approx 0$.

	0	10	12	-3	-42
42				39	0
42	42		54		0
30		40	42		

44	50	② (ϵ) 40	39	① (ϵ) 0
42	51	54	53	0
41	40	42	45	0

$\boxed{Z = Rs.\ 20060}$

Assignment

Steps :-

1) Square Matrix $(m = n)$

2) $x_{ij} = 0$ or 1

all $a_i = 1$ & $b_j = 1$

$\square$ – Allocation $(x_{ij} = 1)$

$\times$ – Non-Allocation $(x_{ij} = 0)$

$$\text{Min } Z = \left[\sum_{i=1}^{n} \cdot \sum_{j=1}^{m} C_{ij} \cdot x_{ij}\right]$$

- Assignment problem is a special case of Transportation problem, where matrix must be Square Matrix & in every row & Column, only one allocation is possible.

Qn) Solve the problem.

	A	B	C	D
J1	20	36	31	27
J2	24	34	45	22
J3	22	45	38	18
J4	37	40	35	28

four Technician are required to perform different Job. whose time in hour is given below. Assign the Job to the technician to minimize work time.

Hungarian Method (Flood's ~~~~ Technique)

Steps Involved are;

1) Develop opportunity Cost matrix.

(a) Subtract the smallest element in each Row from every element
Of Corrosponding Row.

				min
0	16	11	7	— 20
2	12	23	0	— 22
4	27	20	0	— 18
9	12	7	0	— 28

(b) Subtract the smallest element in each Column from every element of Corrosponding Column.

$\boxed{0}$	4	4	7
2	$\boxed{0}$	16	$\cancel{0}$
4	15	13	$\boxed{0}$
9	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

→ ~~Opportun~~ Opportunity Cost Matrix

Z = 20 + 34 + 35 + 18 = 107 hours.

(3) Make allocation in Opportunity Cost Matrix.
if the total No. of Allocation is exactly equal to the size of Matrix then the Current solution is Optimum. otherwise perform optimality

Q3) Solve the following Assignment problem for minimization of ~~cost~~ Cost.

20	30	40	50
40	50	60	70
70	80	90	80
30	50	80	40

0	10	20	30
0	10	20	30
0	10	20	10
0	20	30	10

(4×4)

0	0	0	20
0	0	0	20
0	0	0	0
0	10	10	0

$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	20
$\cancel{0}$	0	$\cancel{0}$	20
$\cancel{0}$	$\cancel{0}$	0	$\cancel{0}$
$\cancel{0}$	10	10	0

$20+50+90+40$

$70+40+90$

200

$\cancel{0}$	$\boxed{0}$	0	20
$\cancel{0}$	0	$\boxed{0}$	20
$\cancel{0}$	0	0	$\boxed{0}$
$\boxed{0}$	10	30	$\cancel{0}$

Qn) Solve the following Assignment problem for minimization of Cost.

+ — od

| — No

9	22	58	11	19	27
43	78	72	50	63	48
41	28	91	37	45	33
74	42	27	49	39	32
36	11	57	22	25	18
3	56	53	31	17	28

(4) ✓	✓ 2a			✓	✓ 2b	
$\cancel{0}$	13	49	0	$\cancel{0}$	13	✓
$\cancel{0}$	35	29	5	10	$\boxed{0}$	✓ (3a)
13	$\cancel{0}$	63	7	7	$\cancel{0}$	✓ (1)
47	15	$\boxed{0}$	20	2	$\cancel{0}$	
25	$\boxed{0}$	46	9	4	2	✓ (3b)
0	53	50	26	4	20	✓ (5)

- As the total no. of allocation is 5 Which is less than the Size Of Matrix $\boxed{n=6}$ So the Current Solution is not Optimum.
- Now We proceed to find the minimum no. of lines required to Cover all zero atleast once, and the Steps involved are

1) Mark all Row for which assignment have not been made [3rd Row]

2) Mark all Column which have unassigned Zero, in the marked row i.e 2nd & Sixth Column.

3) Mark All Row, which have assignment, in the marked Column. i.e 2nd & 5th Row

4) Continue step 2 & 3 untill chain of Marking ~~and~~ ends.

5) Draw the minimum no. of lines ~~to~~ ~~to~~ through unmarked row and through Marked Column to cover all zero at least once.

*

6) Select the Smallest element that donot have line through them, Subtracted it from all the uncut element, adding it to every element at, the intersection of two lines & leave the remaining elements of the matrix unchanged. Make Allocation in the New opportunity Cost Matrix.

4	17	49	$\boxed{0}$	~~0~~	17
$\boxed{0}$	35	25	1	6	~~0~~
13	~~0~~	59	3	3	$\boxed{0}$
51	19	$\boxed{0}$	20	2	4
25	$\boxed{0}$	42	5	~~0~~	2
~~0~~	53	46	22	$\boxed{0}$	20

$Z = 11 + 43 + 33 + 27 + 11 + 17$

$Z = 142$ Ans

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Quality Control

- Quality can be termed as, fitness for use, where fitness is defined, by the Customer itself, who is using that product.
- Quality of a product refers to the degree ~~of~~ to which, product satisfy Customer's expectations.
- Quality has no specific meaning, unless it is related to Specific function & performance of product.

Quality Cost

- Failure cost
  - Internal
  - External
- Appraisal cost
- Preventive cost

Failure Cost:

It is a Cost of Producing defective part or product, within an Organization,

i) Internal failure Cost:

- If the defect is detected inside the organization, it is termed as internal failure cost.
- It include cost of rework, material & product loss, Scrap, Down-time Depriciation etc.

ii) External failure Cost :

- If the defect is detected by the Customer, while using that product it is termed as external failure Cost.
- It include Replacement Cost, Return product Cost, Warranty Cost Loss of goodwill, fine, claim etc.

2) Appraisal Cost :-

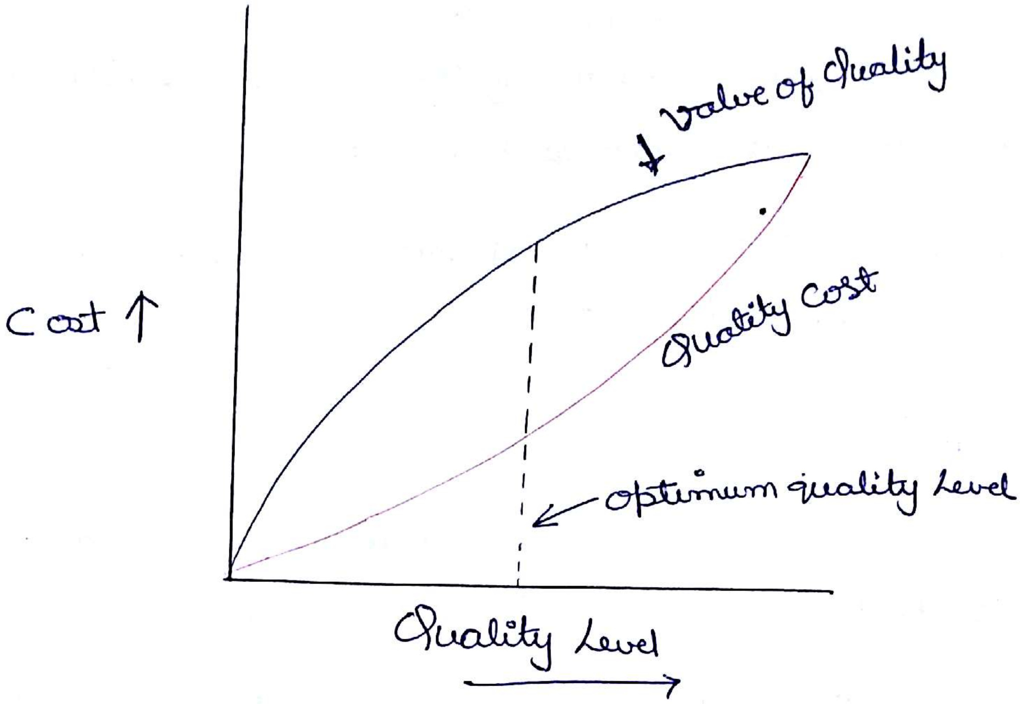
- It is the Cost associated with measuring, evaluating & finding out Defective part within the production System.
- It includes Inspection equipment Cost, Salary of inspector, Intruption of production during Sample Collection, Lab Cost etc.

3) Preventive Cost :

- Whatever the expenditure is made, in order to minimize failure and appraisal Cost, can be termed as preventive Cost.
- It includes. Quality improvement programe, training of Worker Maintenance Cost, Quality programe, on time tool replacement, M/c change etc.

Value of Quality :

- It is the term use to represent the return obtained, directly or indirectly due to good Quality of Product, is termed as value of Quality.
- Good Quality Can earn by good response from Customer, increase in market Share, Firm price policy, higher percentage of Sucessful bids, other benifits to the income of organisation.



Inspection and Quality Control :-

- Inspection Simply means, checking & Sorting out, defective & non defective product,
 Whereas, Quality control is a broaded term which includes no. of steps including inspection and regulate the Quality of future production.

- In Quality Control, if the product is defective, We search for the reason, behind defective part and also include steps to be taken So that, those type of defectives may not be repeated in ~~future~~ future.

Variation :

Variation

- Common or Chance or Random Cause
- Assignable Cause

i) Common Cause Variation :

- These are difficult to trace & difficult to Control even under the best Condition of production.
- These variations are of Lower Magnitude always within the limit. And Defective parts are not produce due to them.

ii) Assignable Cause :

- These variation's are of higher magnitude, Close beyond the limit, and effective parts are produced due to them.
- These are due to Some particular Reasons. like, m/c setting change improper training of operator, defective raw material, tool wear, m/c vibration etc.

A) Type I – Error :-

- When there is no problem within the system, but still we conclude that there is some assignable cause of variation.

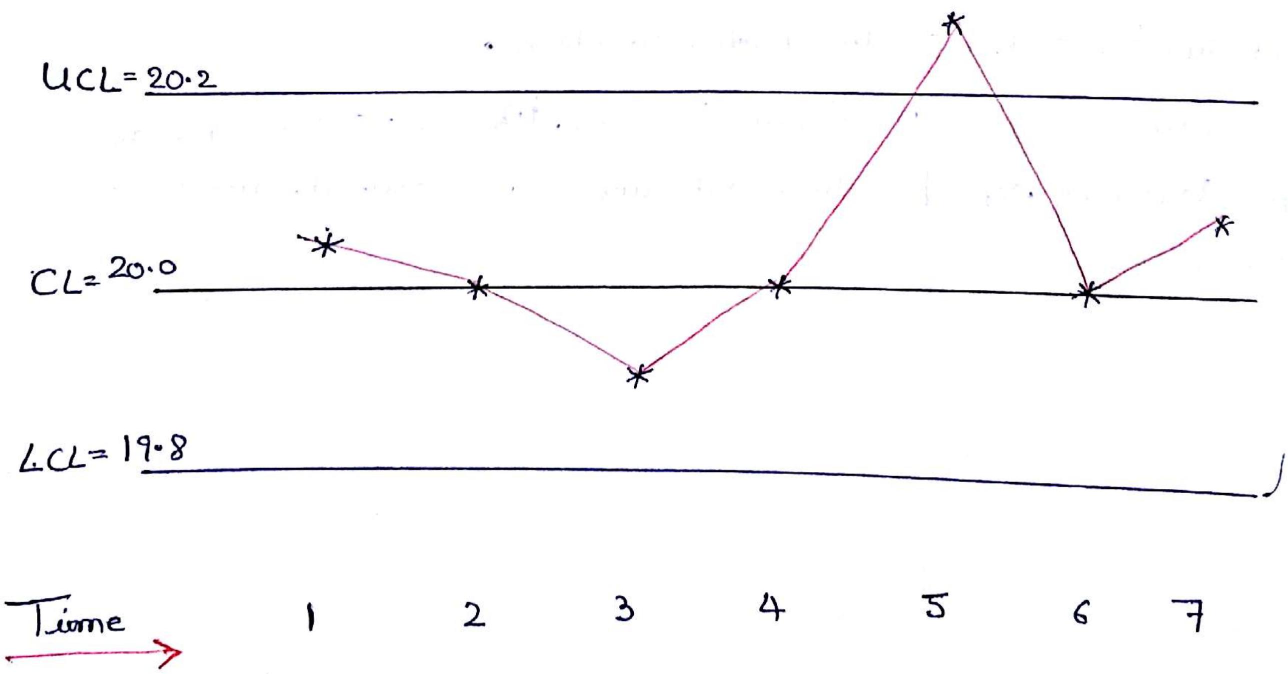
B) Type II – Error :-

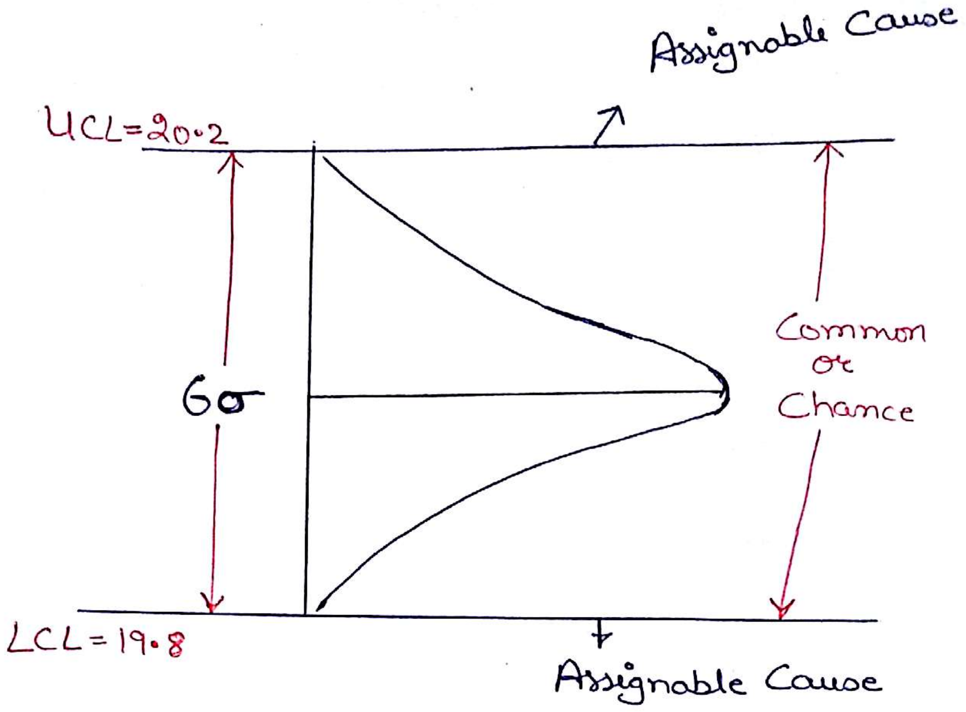
- When there is some problem within the system, but we conclude that, there is no assignable cost of variation.

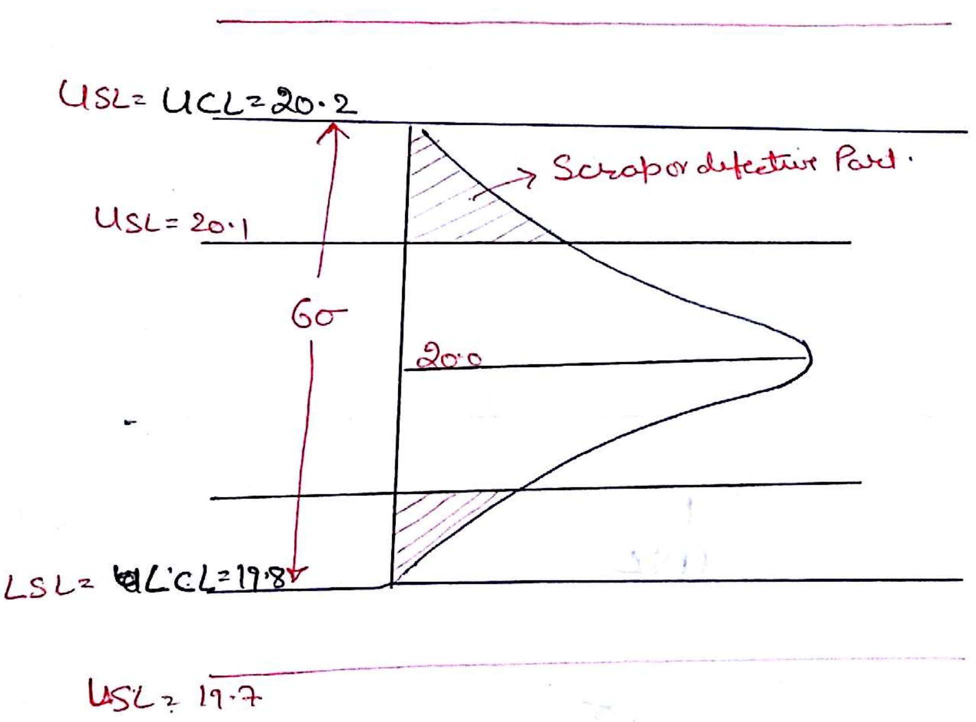
Control Chart :-

- Control Chart is a graph use to study, how a process changes over time, in which observations are plotted in time order.

- Control Chart has the centre line, for the average & upper line for the upper control limit & lower line for the lower control limit.




Assignable Cause
UCL=20.2
6σ
Common or Chance
LCL=19.8
Assignable Cause


USL= UCL=20.2
Scrap or defective Part.
USL= 20.1
6σ
20.0
LSL= LCL=19.8
USL= 19.7

USL - LSL = 30

30 = 2σ　　σ = 15

30 = 4σ　　σ = 7.5

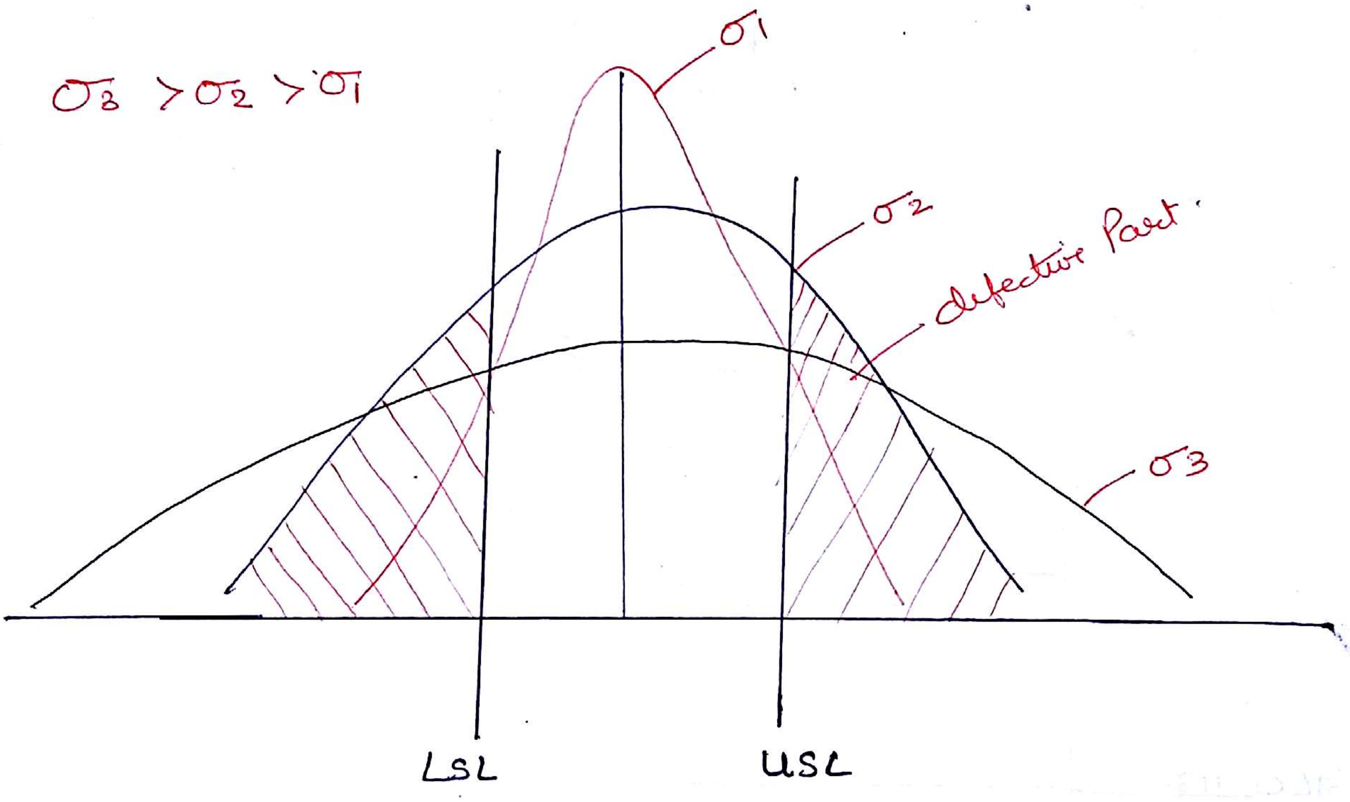
30 = 6σ　　σ = 5.0 → Cost effective

30 = 8σ　　σ = 3.75

30 = 10σ　　σ = 3.0

30 = 12σ　　σ = 2.5 → Quality Best But ↑ Cost of variation

$\sigma_3 > \sigma_2 > \sigma_1$



2σ → 320/1000

4σ → 50/1000

6σ → 3/1000 → Effective cost

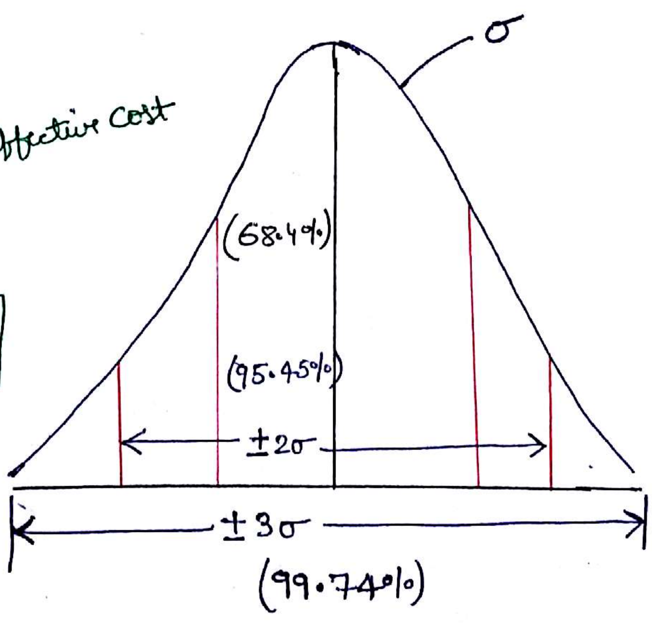
8σ → 0.184/1000

$\boxed{12\sigma \rightarrow 3.2/\text{million}}$

↓

Best Quality

High Cost.



- $\pm 3\sigma$ limit are selected most of the time, for plotting Control Chart, therefore such Charts are Called 3σ Control Chart

Types of Control Chart :

Control Chart

- Variable — FOR; (Measure)
  - Mean or $\overline{X}$ Chart
  - Range or R-Chart
- Attribute — For; Count (Yes or No)
  - P-Chart
  - C-Chart

A) Variable Control Chart :-

- These charts are applied to data, that follows Continous distribution and Can be measured on Continous Scale. for example; time, distance, Weight, Temperature etc.
- These data Continous and they were assumed to follow normal Distribution.

B) Attribute :

- These data are Counted & Cannot have fraction or decimal.
- These data arise while determining the presence or absence of Something like succes or failure, good - Bad, Defective - Non Defective etc.
- This datas are discontinous and therefore assumed to follow Binomial distribution.

$\boxed{n}$ $\boxed{n}$ $\boxed{n}$ $\boxed{n}$ - - - - - - - - $\boxed{n}$

Sample	1	2	3	4	- - - - - - - -	N
Mean	$\overline{X}_1$	$\overline{X}_2$	$\overline{X}_3$	$\overline{X}_4$	- - - - - - - -	$\overline{X}_N$
Range	R_1	R_2	R_3	R_4	- - - - - - - -	R_N

50gm

$\boxed{n \geqslant 4}$

$\boxed{n=8}$

51.2	50.6
48.9	50.1
52.6	49.4
48.3	50.5

Grand Average
or
Average of Sample Mean

$$= \overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \overline{X}_3 + \overline{X}_4 + \cdots \overline{X}_n}{N}$$

Average Range, $\overline{R} = \dfrac{R_1 + R_2 + R_3 + R_4 + \cdots + R_N}{N}$

Mean Chart:

- It shows the centering of the process or in other words it shows the variations in the Average of sample.

Range Chart:

- It monitors the dispersion or variation of process.
- It is the measure of spread of sample

Control limits

1) $\bar{X}$ – chart :-

Centre line (CL) $= \bar{\bar{X}}$

Upper Control limit (UCL) $= \bar{\bar{X}} + 3\sigma_{\bar{X}}$

Lower Control Limit (LCL) $= \bar{\bar{X}} - 3\sigma_{\bar{X}}$

Where

$$\boxed{\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}} \text{ — (c)}$$

$$\boxed{UCL = \bar{\bar{X}} + \frac{3\sigma}{n}}$$

$$\boxed{LCL = \bar{\bar{X}} - \frac{3\sigma}{\sqrt{n}}}$$

Where,

$\sigma_{\bar{X}} \rightarrow$ Standard Deviation for the Sample mean.

$\sigma \rightarrow$ Standard Deviation for the process or universe.

$n \rightarrow$ Sample Size or no. of observation in each Sample.

$\bar{R} = \sigma \cdot d_2 \longrightarrow$ (a)

$$\boxed{\sigma = \frac{\bar{R}}{d_2}}$$

$$UCL = \bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}}$$

$$LCL = \bar{\bar{X}} - \frac{3\bar{R}}{d_2\sqrt{n}}$$

$$UCL = \bar{\bar{X}} + A_2\bar{R}$$

$$LCL = \bar{\bar{X}} - A_2 \cdot \bar{R}$$

Where $\boxed{A_2 = \frac{3}{d_2\sqrt{n}}}$ — (B)

Where; d_2 & A_2 are the Constant factor, whose value depends up on the sample size (n).

Control limit:

2) R- Chart :-

Centre Line (CL) = $\bar{R} \cdot \sigma \cdot d_2$

Upper Control line (UCL) = $\sigma \cdot d_2 + 3\sigma d_3$

Lower Control line (LCL) = $\sigma d_2 - 3\sigma d_3$

Where,

$\bar{R} = \sigma d_2 \longrightarrow \boxed{\sigma = \frac{\bar{R}}{d_2}}$

$$UCL = \bar{R} + \frac{3\bar{R} \cdot d_3}{d_2} \quad -(1)$$

$$LCL = \bar{R} - \frac{3\bar{R} \cdot d_3}{d_2} \quad -(1a)$$

$$UCL = \bar{R}\left[1 + \frac{3d_3}{d_2}\right] \quad -(2)$$

$$LCL = \bar{R}\left[1 - \frac{3d_3}{d_2}\right] \quad -(2a)$$

$$UCL = D_4 \cdot \bar{R} \quad -3$$

$$LCL = D_3 \cdot \bar{R} \quad -(3a)$$

Where, $\boxed{D_4 = \left[1 + \frac{3d_3}{d_2}\right]}$

& $\boxed{D_3 = 1 - \frac{3d_3}{d_2}}$

& for $n < 7$, $D_3 = 0$

Where d_2, d_3, D_3, D_4 are the Constant factor, whose value depends only on the sample size (n).

Qn) The following data give reading for ten sample, of size 8 each, in the production of certain Component. Draw the Control Chart for the mean & Range and point out which sample if any are out of Range.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean	5.4	5.1	5.4	4.9	5.2	4.7	5.1	5.0	5.0	5.2
Range	0.4	0.7	0.7	0.8	0.9	0.5	0.6	0.6	0.7	0.6

for $n=8$, $d_2 = 2.847$, $D_3 = 0.136$, $D_4 = 1.864$

~~for $n=10$, $d_2 = 3.114$ $D_3 = 0.287$, $D_4 = 2.113$~~

Soln $\bar{\bar{X}} = 5.1$ $\quad \bar{R} = 0.65$

$\bar{R} = \sigma d_2 \Rightarrow \sigma = \frac{0.65}{2.847} = 0.2283$ $\boxed{\sigma = 0.2283}$

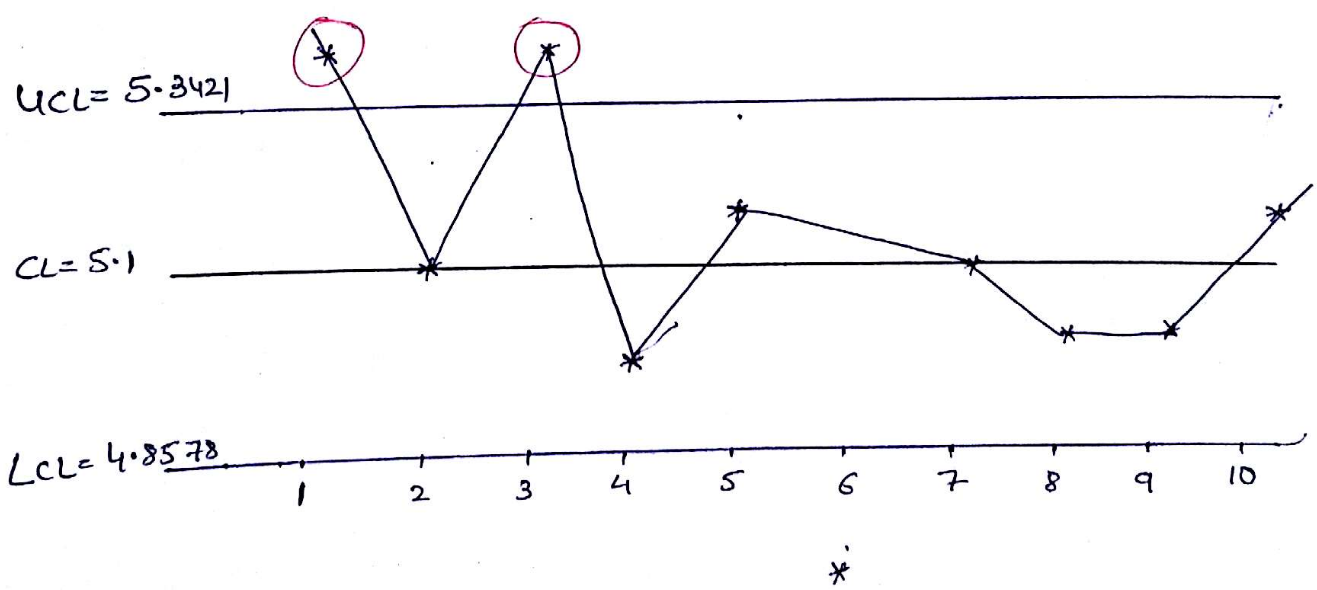
1) $\bar{X}$-Chart

$CL = \bar{\bar{X}} = 5.1$

$UCL = \bar{\bar{X}} + \frac{3\sigma}{\sqrt{n}} = 5.1 + \frac{3 \times 0.2283}{\sqrt{8}} = 5.3421$

$LCL = \bar{\bar{X}} - \frac{3\sigma}{\sqrt{n}} = 4.8578$

$\bar{X}$- Chart:

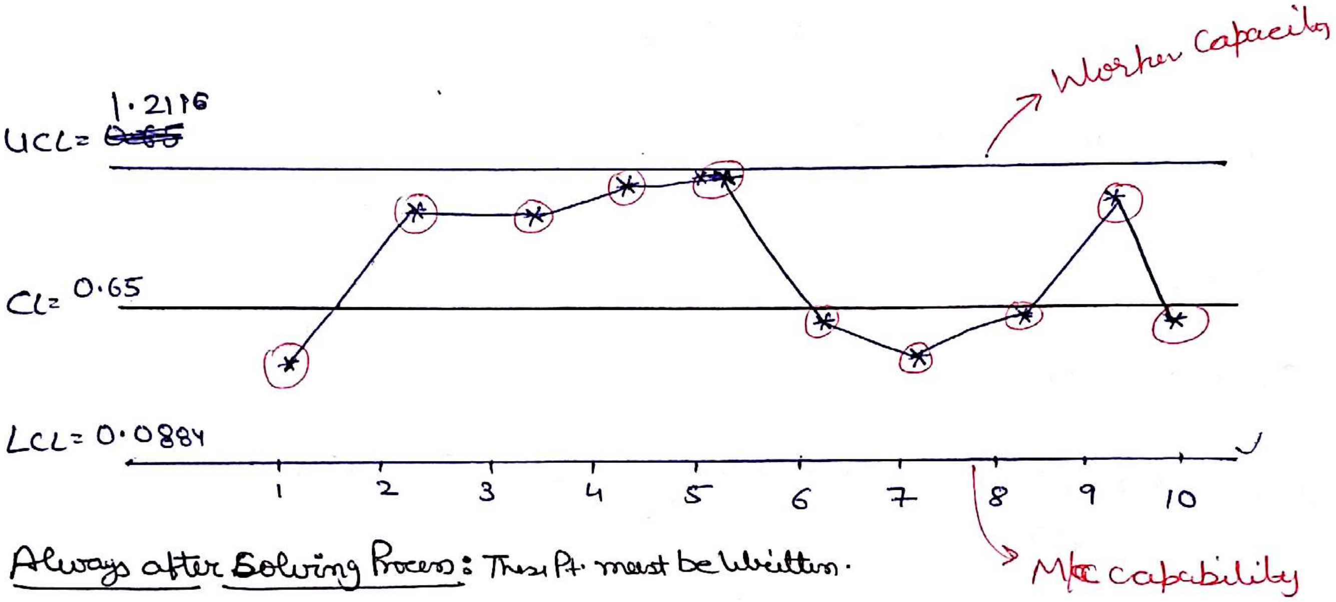


2) R- Chart

$$CL = \bar{R} = 0.65$$

$$UCL = D_4 \cdot \bar{R} = 1.2116$$

$$LCL = D_3 \cdot \bar{R} = 0.0884$$



Always after Solving Process: This Pt. must be Written.

The process is not under control, and it may be due to any one of the assignable reasons like m/c setting change, improper training of operator, defective Raw material, tool Wear, m/c vibration etc.

:- Attribute Control Chart :-

i) P- Chart (Proportion or Fraction Defective Chart) :-

Sample Size used to Change (Discrete)

Sample No.	Sample Size	No Of Defective	~~Por to~~ Proportion defective
1	$n_1 \to 50$	$d_1 \to 3$	$P_1 = \frac{d_1}{n_1}$
2	$n_2 \to 25$	$d_2 \to 2$	$P_2 = \frac{d_2}{n_2}$
3	$n_3 \to 100$	$d_3 \to 5$	$P_3 = \frac{d_3}{n_3}$
4	n_4	d_4	$P_4 = \frac{d_4}{n_4}$
⋮	⋮	⋮	⋮
N	n_N	d_N	$P_N = \frac{d_N}{n_N}$

$P_i = \frac{d_i}{n_i}$

- Average Proportion defective $= \bar{P} = \frac{P_1 + P_2 + P_3 + P_4 + \cdots + P_N}{N}$

- Average Sample Size $= \bar{n} = \frac{n_1 + n_2 + n_3 + n_4 + \cdots + n_N}{N}$

Control Limit :

- $CL = \bar{P}$
- $UCL = \bar{P} + 3 \cdot \sigma_{\bar{P}} = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{\bar{n}}}$

$\sigma_{\bar{P}} = \sqrt{\frac{\bar{P} \cdot (1-\bar{P})}{\bar{n}}}$

0.06

0.02 $LCL = \bar{P} - 3 \cdot \sigma_{\bar{P}} = \bar{P} - 3\sqrt{\frac{\bar{P} \cdot (1-\bar{P})}{\bar{n}}}$

$\to 0$

- P- Charts are used, Where We Can Compute the total Sample Size & no. of ~~effective~~ defective .. ~~if the~~
- It is prefered for the Condition, Where Sample size is variable

Special Cases of P- Chart :-

i) nP- Chart (Number of Defective Chart) (Sample Size remain Constant throughout)

$$\boxed{n_1 = n_2 = n_3 = n_4 = \cdots = n_N = \bar{n} = n}$$

nP- Chart limit

$$UCL = n\bar{P} + 3\sqrt{n\bar{P}\cdot(1-\bar{P})}$$

$$LCL = n\bar{P} - 3\sqrt{n\bar{P}\cdot(1-\bar{P})}$$

$$CL = n\cdot\bar{P}$$

$$UCL = n\bar{P} + 3\sqrt{n\cdot\bar{P}\cdot(1-\bar{P})} \quad \text{— (a)}$$

$$LCL = n\cdot\bar{P} - 3\sqrt{n\cdot\bar{P}(1-\bar{P})} \quad \text{— (b)}$$

- It is prefered for the Condition Where sample size remain Constant throughout.

ii) C- Chart (Count of Defect Chart) :- ~~Binomial~~ Poisson's Distribution

- C- Charts are used Where We Can Compute only the number of defect, but Cannot Compute the proportion defect.
- Defect is Random, therefore it is assumed to follow poisson's distribution. + Random

For Poisson's distribution = $\boxed{\text{Varience} = \text{mean}}$

$$\sigma^2 = \bar{C} \longrightarrow \boxed{\sigma = \sqrt{\bar{C}}}$$

Control limit:

$$CL = \bar{C}$$

$$UCL = \bar{C} + 3\cdot\sqrt{\bar{C}}$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

Where,

$\bar{C} \rightarrow$ Average no. of defect

Q2) A manufacturer find from his experiment that on an average 1 out of 10 item produced by a m/c is defective

On a particular day he select the lot of 100 item's Randomly & finds that 18 of them are defective. Is the process under Control.

Soln)

$$\bar{P} = \frac{1}{10} = 0.1$$

$$n = 100 \rightarrow d = 18$$

using → nP- chart

$$CL = n\bar{P} = 10 \text{ unit}$$

$$UCL = n\bar{P} + 3\sqrt{n\bar{P}(1-\bar{P})}$$

$$= 10 + 3\sqrt{100 \times 0.1 \times 0.9}$$

$$\boxed{UCL = 19 \text{ unit}}$$

3) Acceptance Sampling →(Low Cost Product)

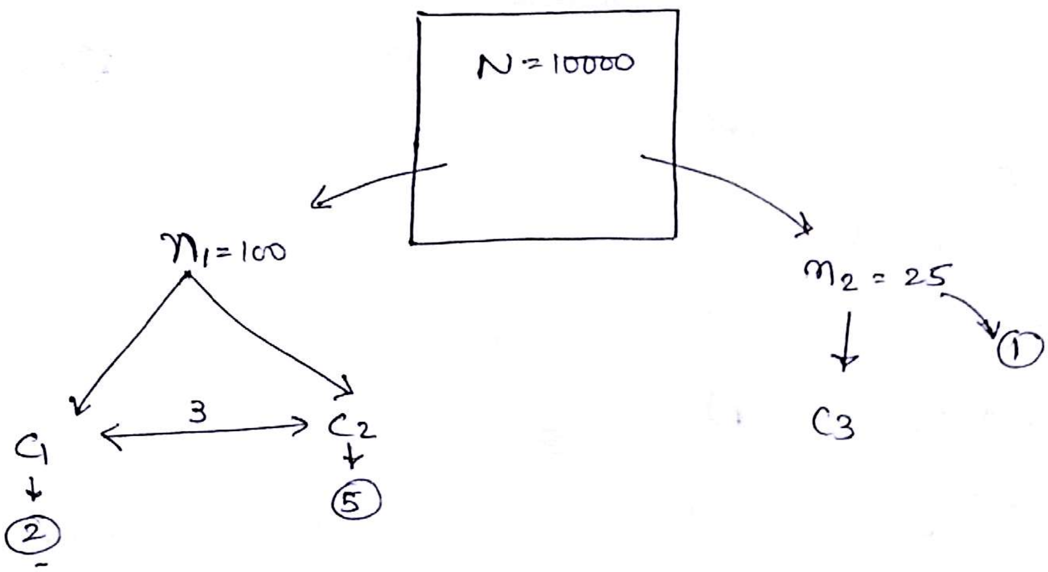
- It is the Method of inspection Where Sample of goods, is randomly inspected in Order to decide Whether to accept or reject the entire LOT.
- It is used Where inspecting every items is either feasibly not Possible or Would be very expensive.
- It is the only Method of inspection, Where testing is done through destructing pattern.

Sampling Plan :-

i) Single Sampling Plan:

Sample is taken once and if no. of defective are equal to or less than ~~and~~ acceptance No. entire lot is accepted, otherwise Rejected.

ii) Double Sampling Plan:

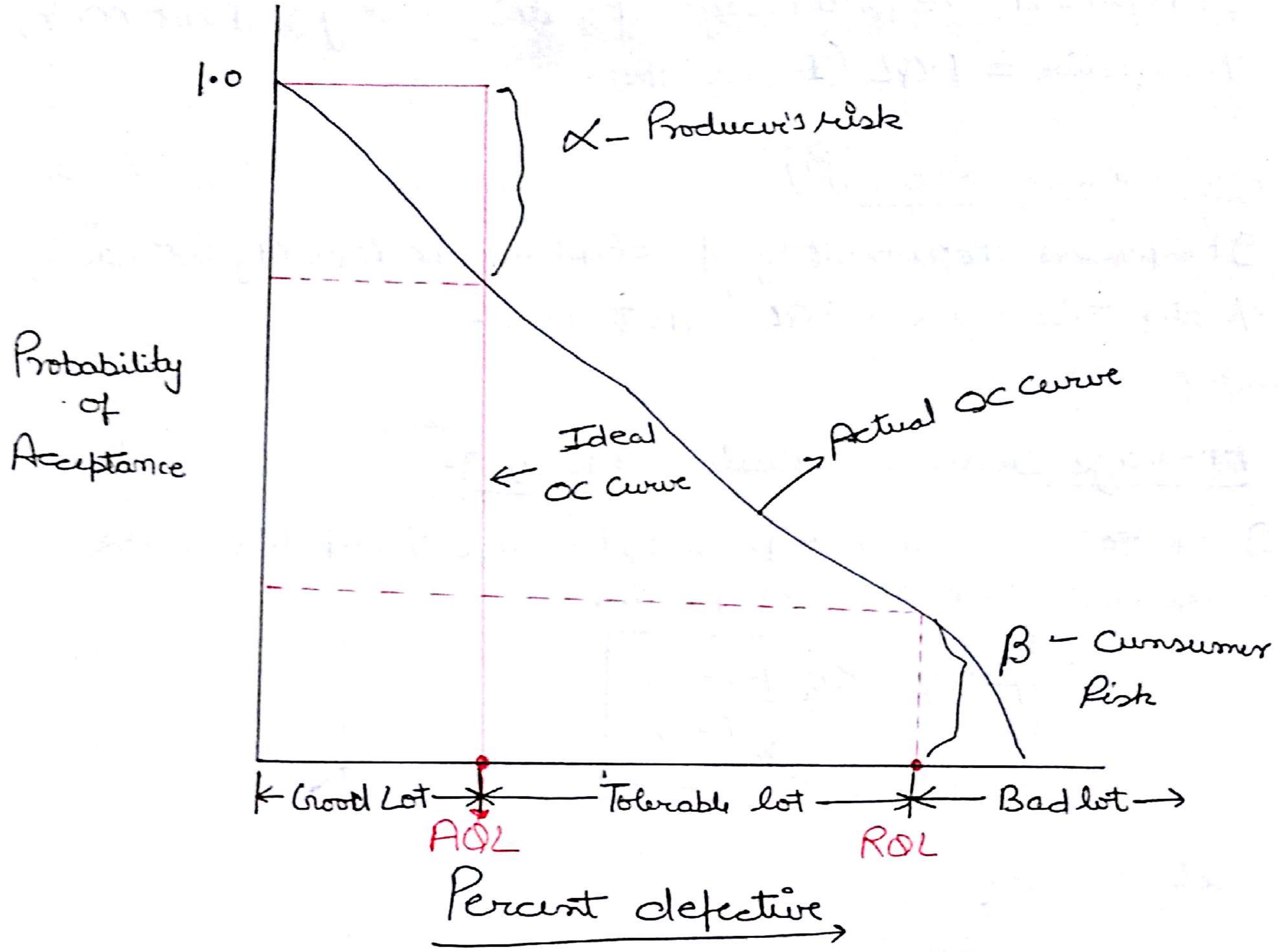


- A sample of ~~N_1 units~~ n_1 units is taken randomly, if the total no. of defective are C_1 or less the entire lot is accepted.
  if it is C_2 or more, then entire lot is rejected and
  if no. of defective are b/w C_1 & C_2, then another sample of n_2 items is taken randomley.
- If total no. of defective from the two sample together is C_3 or less the entire lot is accepted other wise rejected.

NOTE:

- As the no. of sampling plan increases, chances of making errors at average no. of unit inspected decreases, but operating difficulty increases.

Operating Characteristic Curve



OC Curve is the graph b/w probability of Acceptance against the ~~fraction~~ percent deffective in a lot & term associated with OC Curve are,

i) Acceptable Quality Level [AQL] :

- There is small % defective, which Consumer's do not problem in accepting.
- AQL, represent that Level.

ii) Rejectable Quality Level (RQL] :

- Lot torenance % defective (LTPD), Consumers normally tolerate a few more defective above AQL, but then comes a limit beyond which, they do not accept, any more defectives. RQL Represent that level.

iii) Producer's risk : (α)

It Represent the probability of Rejecting a very good lot having % defective = AQL (I error Type).

iv) Consumer's Risk : (β)

It Represent the probability of accepting a bad Quality lot, having % defective equal to RQL (Type II error).

* Average outgoing Quality (AOQ) :-

It is the term use to represent, Average % defective in the outgoing product, after inspection.

$$\boxed{AOQ = P.P_a\left(\frac{N-n}{N}\right)}$$

if $N \gg n$

$$\boxed{AOQ = P.P_a}$$

Where,

P = % deffective

P_a = Probability of acceptance

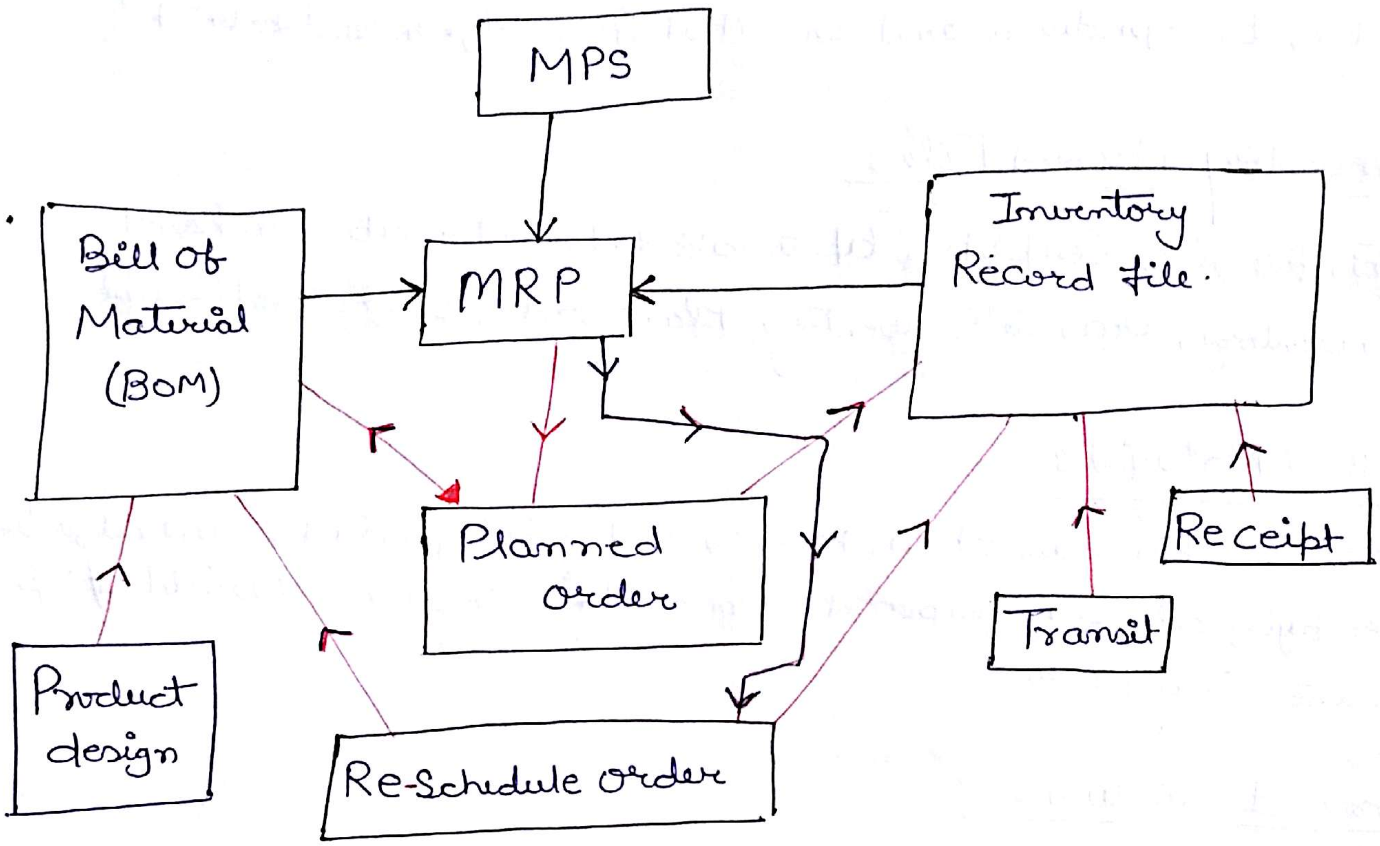
N = Lot Size

n = Sample Size

MRP

Material Requirement PLanning

Structure of MRP



- MRP, is a method of Working out ~~sa~~production plan in the multi stage production system, that produces many product, and their sub assembly. require raw material and
- It is used So that all the things needed should be available within the production System at appropriate time and production Can be Carried out Without any delay.
- Today MRP is a Computer based information system, for production, Schedulling and purchasing of dependent demand item.

Master Production Schedule (MPS) :

- It is the Complete time table of our Schedule production, in future.
- It gives information about, what product is to be produced, when it is to be produced and in what Quantity. ~~to be producted~~

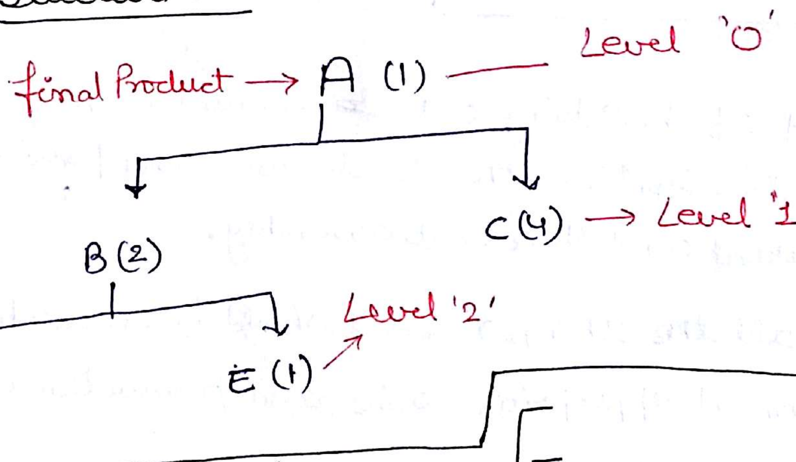
Inventory Record File :

This file gives Complete & up-todate information about on hand inventory, transit inventory, Plant Order, & Scheduled Reciept.

Bill Of Material :

It gives information about, How to each final product is manufactured Specifying all sub Component item & their sequence of Build ~~#~~ up in the final product.

Product Structure :



**

$$\boxed{\text{Net Requirement} = \text{Gross Requirement} - \left[\text{Inventory on Hand} + \text{Schedule Reciept}\right]}$$

Qn) find the net Requirement for which we should place an order in order to produce 400 unit of Product X. When the inventory on hand & the Schedule Receipt is as given below.

X (1) 400

A(1) 400 − 80 = 320

B(2) 800
= 400 × 2 − 210 − 190
= 400

C(1) = 400 − 90
= 310

D(3)
= 320 × 3 − 220
= 560

E(2)
= 320 × 2 − 300
= 340

F(2)
= 310 × 2 = 620

G (1) 310
310 − 110

H (4)
= 200 × 4 − 330 − 270
= 200

Sol Inventory on Hand

A − 80, B − 210, C − 90

D − 200, G − 110, H − 330

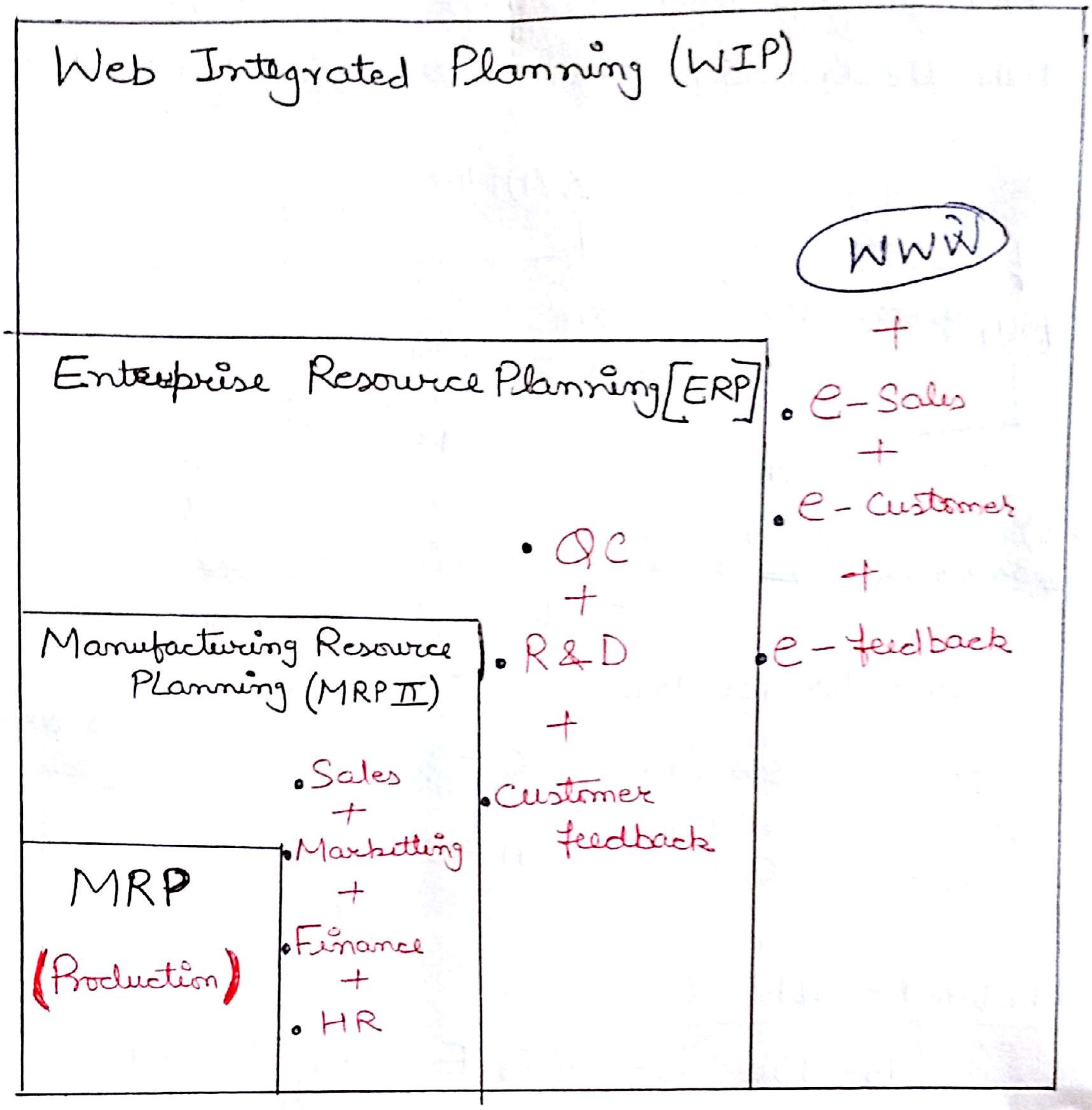
Schedule Receipt

B − 190; D − 180, E − 300, H − 270

A − 320, B − 410, C = 310, D =

Advantages of MRP :

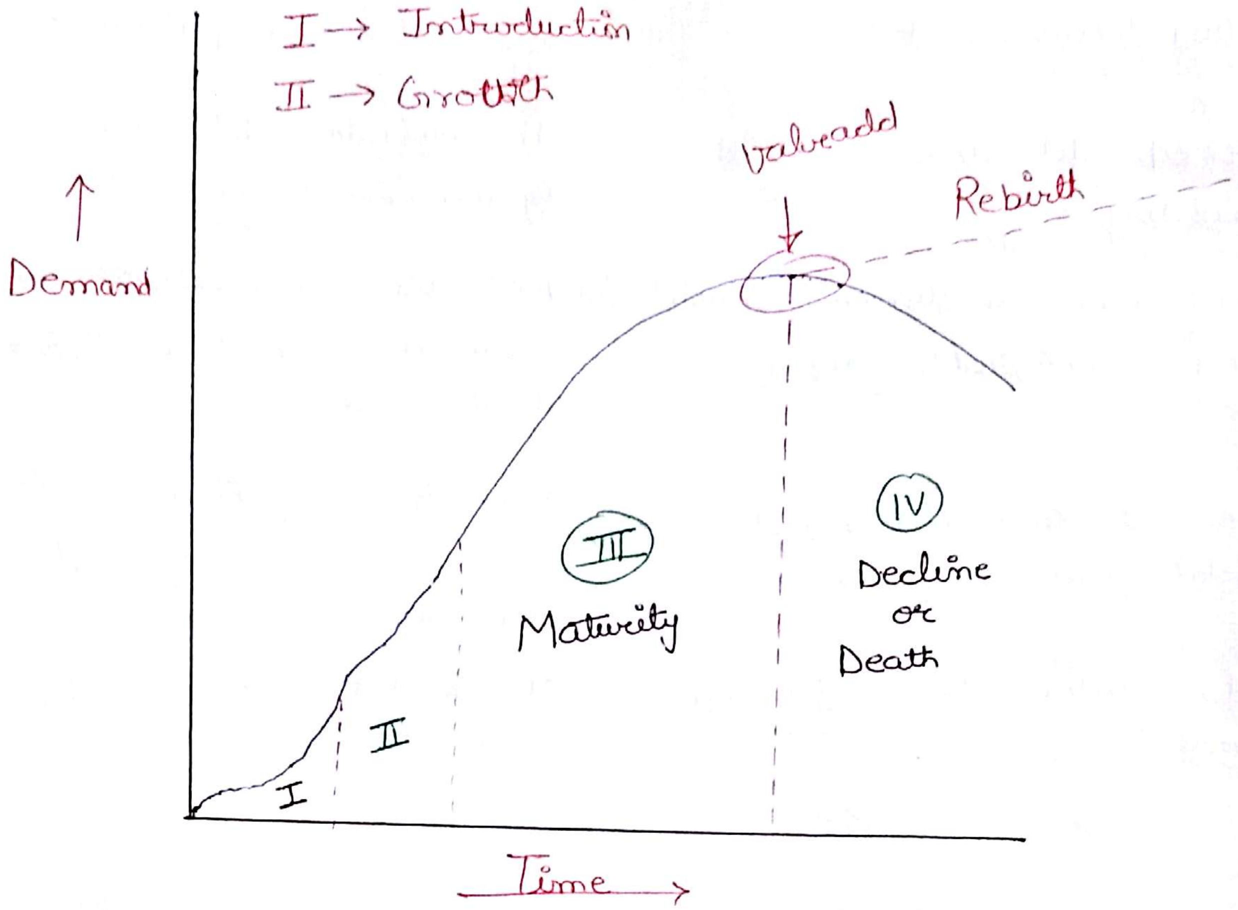
1) It help us to know when and how much to order.
2) It helps in inventory reduction.
3) It help to avoid delay in production.
4) It helps to give timely information to the marketing department about the expected delivery time.


Web Integrated Planning (WIP)
WWW
Enterprise Resource Planning [ERP]
e-Sales
+
e-Customer
+
e-feedback
QC
+
R & D
+
Customer feedback
Manufacturing Resource Planning (MRP II)
Sales
+
Marketting
+
Finance
+
HR
MRP
(Production)

MRP PUSH (Material Resource Planning)	JIT PULL (Just in Time)
1) It is the Push System, where Product is produced to meet the future requirement	2) It is from PULL System, where materials are provided only when there is demand of Product.
2) It Keep Safety Stock along with inventory	2) It eleminate Safety Stock & Keep Very less inventory.
3) It Can Handle dynamic Situation where demand Suddenly changes.	3) Not able to Handle dynamic Condition as incapable of large & Sudden variation.
4) No Need to maintain good Relation with the vendor.	4) Need to maintain good Relation with the vendor, to get timely information.
5) It is suited for batch or job type production	5) It is Suited for mass flow production

Value Engineering :-

Product Life Cycle :-



$$\boxed{\text{Value} = \frac{\text{Function or Performace or Quality}}{\text{Cost}}}$$

Value is Mainly of four types;

1) Cost value :-

- It is a Cost of Manufacturing a product & it is the sum of Raw material, labour, tool, & other overhead cost.

2) Use value :

It is the amount of cost included into the product to perform its particular function.

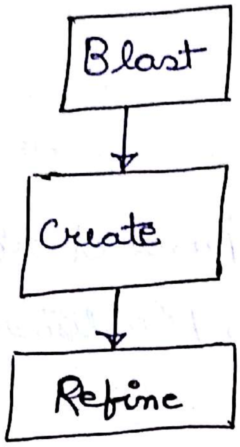
3) Esteem value :

- It is a amount of Cost, included into a product to make it more attractive and appealing.

4) Exchange value :-

- All the properties and features of a product, which makes it possible to trade or exchange of product with any other product is termed as exchange value.

Steps in value anylysis :



1) Blast :

- Select the product, for which we want to increase value & Collect all the information regarding the function & cost of the product.

2) Create :

- Develop New Alternate function at lesser Cost and Critically examine all those alternate.

3) Refine :

- Select the best Alternate which increases the value of product & ~~steps it~~. install it

Advantages :

1) Reduction in the no. of useless part.
2) Reduction in the amount of Scrap.
3) Overall Cost Reduction.
4) Better Customer Satisfaction.

Value Analysis :

- Value analysis is a Applied to existing product with a view to improve its value.

Value engineering :

- Value engineering is applied to product at design stage before reaching into the hands of Customer.