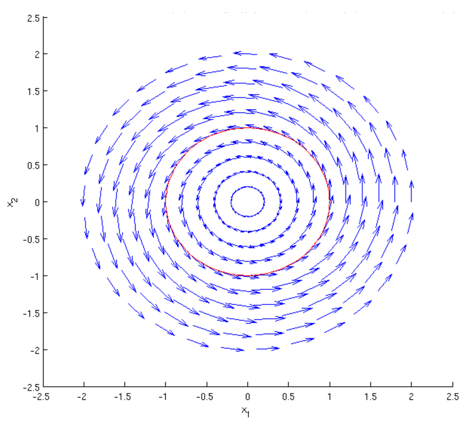
Exploration of the Validity of the Ergodic Theorem by Instability of Limit Cycles

Jason von Juterczenka, Schülerforschungszentrum Nordhessen

My Question: Is the ergodic theorem completely fulfilled in reality?

- Chaotic systems, (systems with a sensitive dependence on the initial conditions), subject to different laws than linear systems
- Ergodic theorem: In a six-dimensional phase space the trajectory of a chaotic system comes as close as you want to any energetically accessible point if t → ∞

Model	Reality
frictionless	friction between my pendulums
No air resistance	Air resistance at the pendulum´s rods
Six-dimensional	Three-dimensional
Undefined period	max. 2 days
Infinite accuracy	Optical evaluation
No centrifugal force	Centrifugal force increasing with frequency
Exactly constant environmental conditions	Small deviations



$$\overline{A} = \langle A \rangle$$

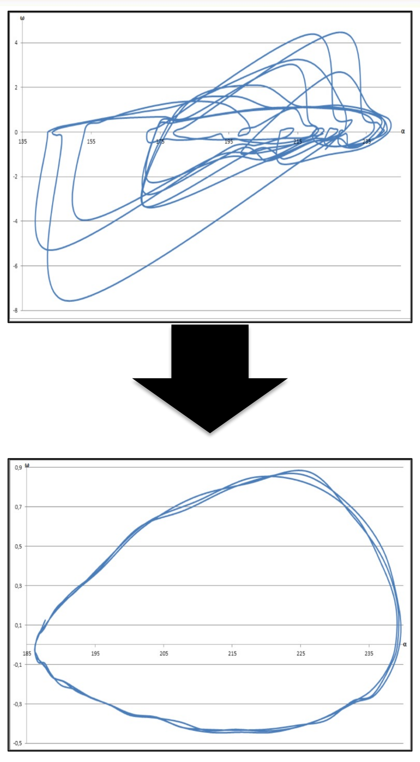
- Can friction and air resistance inhibit chaotic behavior?

The influence of friction

- As I did measurements with a higher friction and increased the excitation frequency there was no chaos sometimes
- Higher friction influences the chaotic behavior in two different ways:

1.It leads to a faster transition to a periodic state.

2.It shitfts the chaos entry frequency backwards.

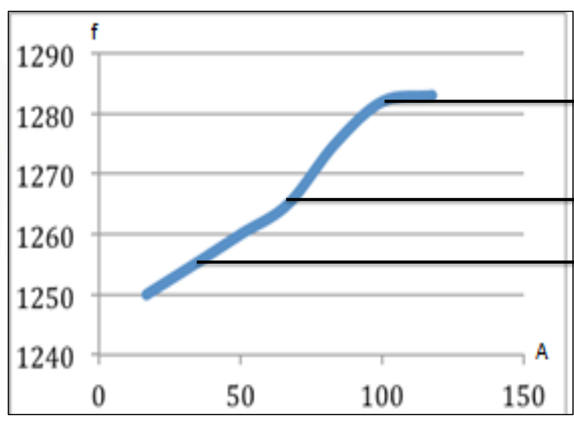


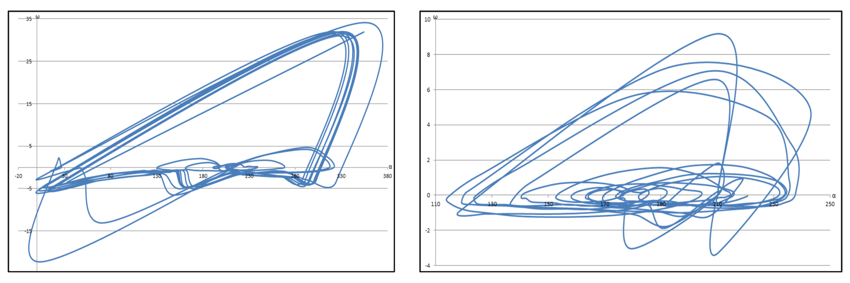
My results

- The chaotic behavior of the driven double pendulum is frequency-dependent. ✓ **proven**
- Internal and external friction shift the chaos entry frequency backwards **and** lead to a faster transition to a periodic state. ✓ **proven**
- At high frequencies, the friction loses influence. ✓ **proven**
- The Ergodic theorem is only partially fulfilled in realitiy. ✓ **proven**
- Limit cycles shouldn´t be stable. ? **supposed**

Friction at high frequencies

- Influence of the friction does not remain constant
- Zone of frequencies which is very sensitive to changes in the friction
- At high excitation frequencies, chaos entry frequency approaches a limit value

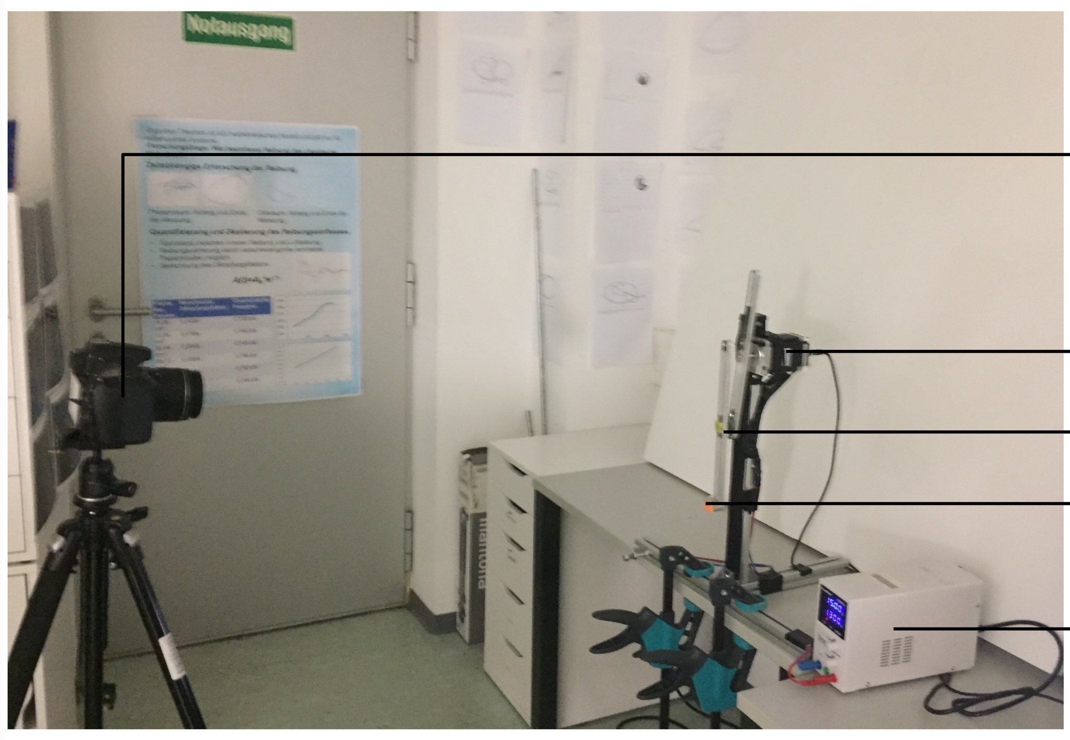




My measurement method

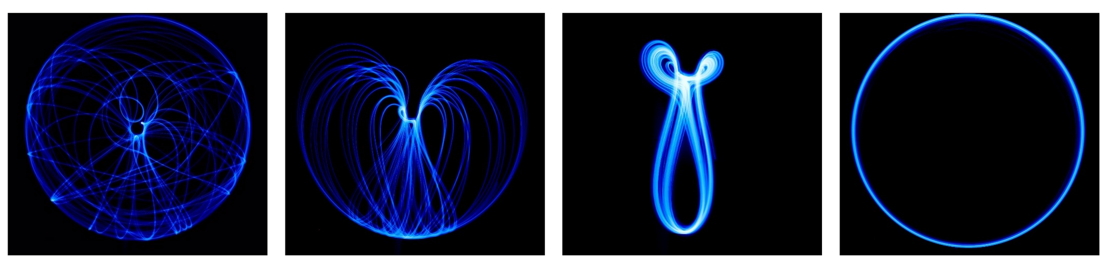
- With laminated paper strips, I increased the air Resistance of the pendulum and thus the friction.
- Deflected it and calculated the damping factor of the pendulum based on the steepness of the resulting sine curve

$$A(t) = A_0 * e^{-r*t}$$



Limit cycle thesis

- Since a periodic state is reached at high frequencies despite minimal frictional influence, the system under investigation is a limit cycle
- For limit cycles, adjacent trajectories of the starting point may diverge or converge
- What happens depends on the distance of the starting point from the limit cycle
- Distance lies on x- and y-axis, is therefore an energy volume in phase space
- The further development of the system is sensitively dependent on the initial conditions
- Individual limit value for each starting point from which the trajectory converges
- Catchment area as sphere would contain complementary energy values
- Therefore probably rather “instable points" around the limit cycle
- Slightly different boundary cycle would hit unstable points after any time and decay
- If the limit cycle did not deviate, it would be subject to feedback and would also decay
- Limit cycles are inevitably unstable



Poincaré (already proven)

Own thesis

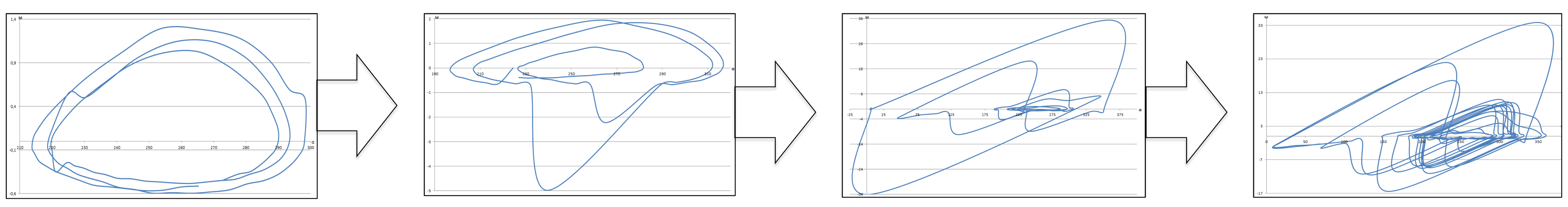
Bifurcation in the Ergodic Theorem

1.Completely periodic behavior

2.Periods running in parallel

3.Phase transitions at irregular intervals

4.Formation of angles characteristic of phase transitions



[redacted]