The Earth's Surface Accelerates Up (and Out)

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This paper uses the Schwarzchild geometry utilized by the current globe Earth model to show that the surface of the Earth is accelerating upwards.

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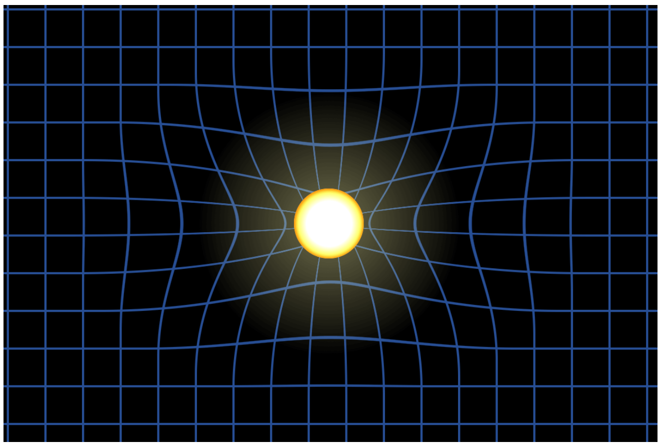

In the currently accepted globe Earth model, and according to Einstein, the Earth's surface accelerates upwards (or outwards). Today, you'll learn how to prove this.

I. THE EQUIVALENCE PRINCIPLE

A common misinterpretation of the equivalence principle is that it allows the entire Earth to be treated as either accelerating or motionless, and that these pictures are equivalent. This isn't true. Both of these views work approximately, but if we believe in the full predictions of Einstein, which hold globally and for the entire Earth, then we are left with only one choice: the Earth's surface, and everything on it, accelerates upwards and outwards.

II. SCHWARZSCHILD

To start our proof, we use the Schwarzchild metric and coordinates, which describe the curved spacetime of a spherical mass like the Earth. The coordinates are:

- coordinate 0: t, time.
- coordinate 1: r, radius or "distance" from the center,
- coordinate 2: θ, latitude,
- coordinate 3: ϕ, longitude.

A point fixed on the surface of the Earth has coordinates

$$(t, R, a, b)$$

where t is coordinate time, R is the radius of the Earth, and a and b are a latitude and longitude. R, a, and b are constant in time.

III. VELOCITY

We can ask, "What's the 4-velocity of our point on the surface of the Earth?"

First of all, what's 4-velocity? It's the 4D version of velocity and describes change in spacetime position with time. Since spacetime combines space and time, velocity is better interpreted as a direction in spacetime. Because of this, 4-velocity always has a unit "size" (magnitude squared) of -1. In other words, the only part of velocity that matters is its direction: the size of every possible 4-velocity is always -1.

We know that our fixed point on the surface of the Earth doesn't move along any of the last 3 coordinates, r, θ, or ϕ. So, its spacetime path and 4-velocity must be directed along the t direction:

$$\mathbf{v} = \langle v^0, 0, 0, 0 \rangle$$

(remember, our coordinates are in the order t, r, θ, ϕ). v^0 is just a symbol for the 0th component of the velocity. We are going to find v^0 using the fact that the velocity has a size of -1.

IV. THE METRIC TENSOR

The Schwarzchild metric tensor tells us the curvature of spacetime around Earth. It looks like[1]

$$\mathbf{g} = \begin{pmatrix} -(1-\frac{s}{r}) & 0 & 0 & 0 \\ 0 & (1-\frac{s}{r})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix},$$

where s is the Schwarzchild radius of the Earth.

The metric tells us the size of 4-vectors. Let's use it with our 4-velocity,

$$\mathbf{v} = \langle v^0, 0, 0, 0 \rangle.$$

We want the size of the 4-velocity to be -1. According to the metric tensor, the size (magnitude squared) of the 4-velocity $\mathbf{v}$ is

$$|\mathbf{v}|^2 = \mathbf{vgv} = -(v^0)^2(1-\frac{s}{R}) = -1$$

(where **vgv** represents matrix multiplication). This solves the mystery of what the symbol v^0 is, so now we can write out our 4-velocity for the surface of the Earth explicitly:

$$\mathbf{v} = \langle (1 - \frac{s}{R})^{-1/2}, 0, 0, 0 \rangle.$$

V. ACCELERATION

The 4-velocity of the Earth's surface has no spatial components. The point only moves through time. It seems like we might be disappointed and see no acceleration then, but let's keep going anyway.

4-acceleration is just like normal acceleration. It's the change in velocity with proper time. But, since we're in curved spacetime, we take a covariant ("absolute") derivative to find this change[2]. So, our acceleration a^μ is (using Einstein notation)

$$a^\mu = \frac{Dv^\mu}{d\tau} = \frac{dv^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} v^\alpha v^\beta.$$

$\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols. They come from the metric and also describe spacetime curvature. The three indices μ, α and β can represent any of the 4 coordinates, and that makes for a total of $4 * 4 * 4 = 64$ combinations or Christoffel symbols. Luckily, we'll only have to look at 4 of them.

Since our velocity only has a time component v^0 which doesn't change, our acceleration becomes

$$a^\mu = \Gamma^\mu_{00} (v^0)^2.$$

The only one of the four Christoffel symbols represented by Γ^μ_{00} that doesn't vanish is the one with $\mu = 1$. It's[3]

$$\Gamma^1_{00} = -\frac{1}{2} g^{11} \frac{\partial g_{00}}{\partial r} = \frac{s}{2r^2}(1 - \frac{s}{r}).$$

Now it's all falling into place (pun intended). Using our result for the velocity component v^0, we arrive at our final calculation of the acceleration on Earth's surface,

$$\textbf{acceleration} = \langle 0, \frac{s}{2R^2}, 0, 0 \rangle.$$

(Our components are in the order t, r, θ, ϕ.)

We can put this in a more recognizable form, too. The Schwarzschild radius of the Earth is $s = \frac{2GM}{c^2}$ where G is the gravitational constant and M the mass of the Earth, and multiplying the acceleration by a factor of c^2 will bring it back into SI units, so we can rewrite it as

$$\textbf{acceleration} \text{ (SI units)} = \langle 0, \frac{GM}{R^2}, 0, 0 \rangle.$$

This form eerily mirrors the form of Newton's gravity.

But now, the acceleration is directed upwards and describes a fixed point on the Earth's surface. We have shown that the acceleration on Earth's surface is directly outwards, in the r direction. Since the r component of acceleration is positive, this means the Earth is accelerating up and out. Nothing falls in (except maybe spacetime itself).

Hopefully you learned something, and if not, hopefully you still had fun. See the sources below for more details.

[1] https://en.wikipedia.org/wiki/Schwarzschild_metric#The_Schwarzschild_metric

[2] https://en.wikipedia.org/wiki/Proper_acceleration#In_curved_spacetime

[3] Page 3. https://web.stanford.edu/~oas/SI/SRGR/notes/SchwarzschildSolution.pdf

[4] Image credit. Accessed Aug 3. https://www.wtamu.edu/~cbaird/sq/images/spacetime_curvature.png