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An Explanatory Note on the Basel II IRB Risk Weight Functions

Why mathematics matters in Banking…

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Introduction UBS

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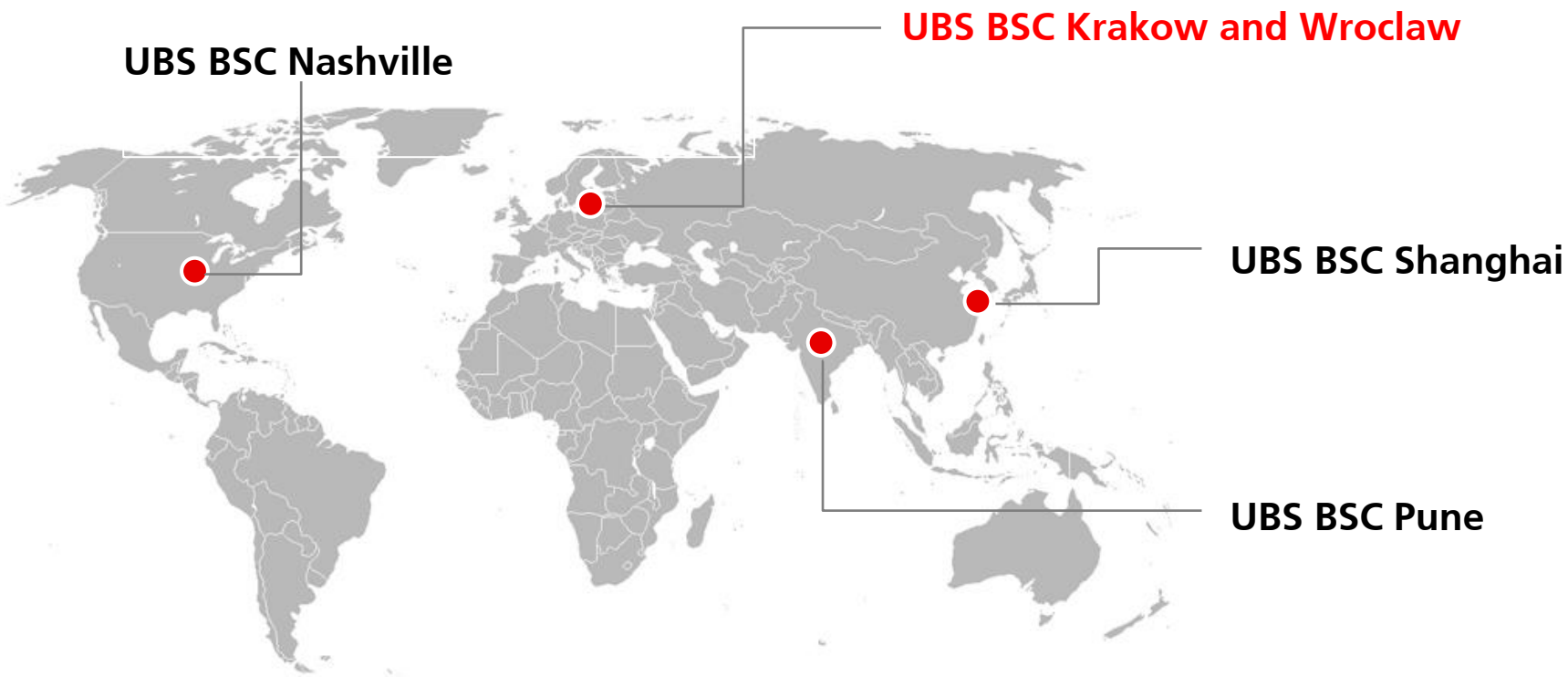

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Section 2

Capital requirements for Banks

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First idea – Leverage ratio

Leverage ratio := Assets / Equity

Leverage ratio Bank A = Leverage ratio Bank B

Asset composition:

50% Treasuries, 25% highly rated loans, 15% in branches and buildings, and 10% in cash

→ conservative assets structure.

Asset composition:

50% in subprime loans, 29% in risky derivatives, 20% in branches, and 1% in cash.

→ risky assets structure.

Using the assets-to-shareholder equity approach is not correctly reflecting the assets compositions and the risks involved **→ the leverage ratio does not describe the full picture…**

→ main idea: re-scale the bank's assets by considering the underlying risk; see next page…

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Second idea – risk based approach

Capital Adequacy Ratio (CAR) := Equity / RWAs

CAR Bank A >> CAR Bank B

where RWAs stands for **R**isk **W**eights **A**sset**s**.

Regulators require Banks to hold a minimum CAR.

How to derive adequate risk weights? Which are the risk factors that should determine the risk weights?

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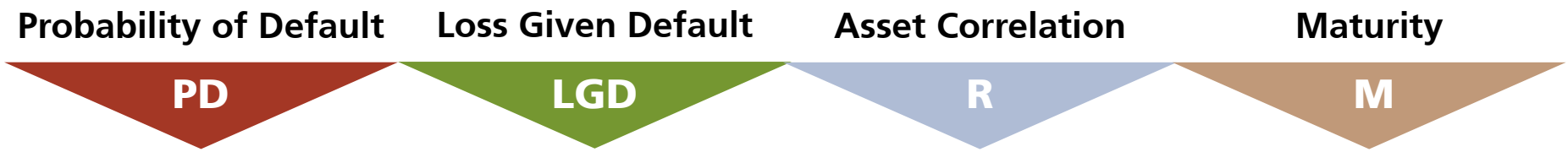
Risk metrics and the "complex" formula

- From the previous page, risk weights seem to be the key figure in order to correctly scale the Bank's asset side by considering its exposure to risk.
- From an intuitive point of view, at least the following risk metrics should influence the risk weights:



The Basel Committee has derived following mathematical formula for the risk weights:

$$\textbf{Risk weights} = [LGD * N\left[(1-R)^{-0.5} * N^{-1}(PD) + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right] - LGD * PD](1-1.5 \times b(PD))^{-1} \times (1 + (M-2.5) * b(PD))$$

where:

- N(x) is the cumulative distribution function of the normal distribution
- b(x) is a univariate function.

Intuition behind it - expected vs. unexpected loss

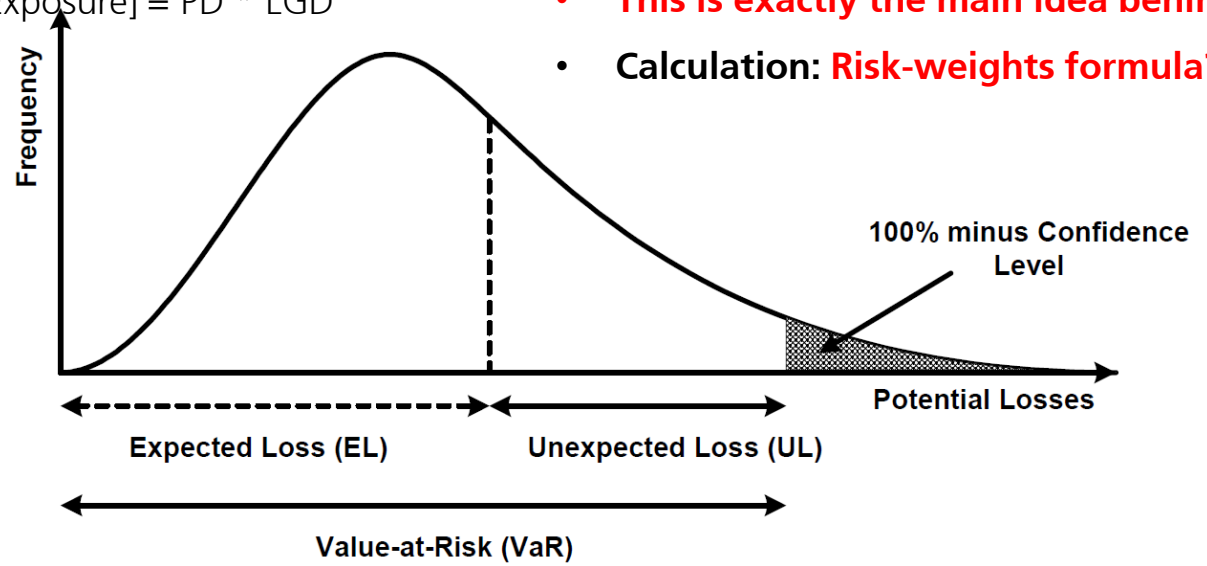
- This minimum capital requirement, described by a minimum CAR, protects the bank from losses and ultimately protects taxpayers from potential expensive bailouts.
- Based on mathematical models, Banks derive:

Expected Loss: Forecast of the average level of credit losses a bank can reasonably expect to experience.

- Given that this is **expected**, this is the cost of doing business and therefore it is covered within the pricing (charged to the client) and provisions calculations.
- **Calculation:** EL[% of Exposure] = PD * LGD

Unexpected Loss: Losses above expected levels whose time and severity is impossible to know in advance.

- Given that this is **unexpected**, Banks need to ensure to have enough capital for absorbing these losses at any point in time → capital requirements.
- **This is exactly the main idea behind RWAs.**
- **Calculation: Risk-weights formula?**



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Section 3

Derivation of the Formula for Risk Weighted Assets

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Economic Foundations of the Risk Weight Formula

Risk weights = ? ✓ ✓

$$\left[LGD * N\left[(1-R)^{-0.5} * N^{-1}(PD) + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right] - LGD * PD\right] * f(PD, M)$$

Looks like a conditional Expected Loss in %

This is the well-known Expected Loss in %

Scaling factor, the so called *Maturity adjustment*.

Summarizing:

The risk weights formula describes the unexpected loss in %; this is derived as the difference between the conditional EL and the EL. A scaling factor is needed because long-term credits are riskier than short-term credits. As a consequence, the capital requirement should increase with maturity.

Modelling Assumptions behind RWAs

$$\left[LGD * N\left[(1-R)^{-0.5} * N^{-1}(PD) + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right] - LGD * PD\right] * f(PD, M)$$

Where is this coming from? This must be a "kind of" conditional PD…

Recall the one-factor Merton:

- A firm defaults when the value of its assets V_i falls below a certain level given by the default barrier K_i.
- The asset value of a firm is decomposed into a common/systematic factor f and an idiosyncratic noise component ξ:

$$V_i = \sqrt{\mathrm{R}} f + \sqrt{1-R}\xi_i$$

where

- f is a common factor in the economy that affects equally all the companies and is N(0,1) distributed.
- ξ_i is an idiosyncratic factor that only affects company "i" and is also N(0,1) distributed.
- R is the asset correlation, i.e. the correlation between asset value V_i and V_j $\quad \forall\, i \neq j$.

Some mathematics of the RWAs formula

In the one factor model, default occurs when $V_i \leq K_i$. If PD is the default probability, then

$$PD_i = P(V_i \leq K_i) = N(K_i) \rightarrow K_i = N^{-1}(PD_i)$$

Therefore an appropriate default threshold K_i can be determined by applying the inverse of the normal distribution to the average PD_i.

Conditional on the common factor $f = y$, it can be shown that:

- the firms' values V_i as well as the defaults are independent,
- the conditional probability of default of firm i reads:

$$PD_i(y) := P(V_i \leq K_i | f = y) = P\left(\sqrt{\mathrm{R}} f + \sqrt{1 - R}\xi_i \leq K_i | f = y\right)$$

$$= \boldsymbol{N\left((1-R)^{-0.5} * K_i - \left(\frac{R}{1-R}\right)^{0.5} * y\right)}$$

Final derivation of the RWAs formula

Given that in the RWA formula we are looking for the unexpected loss in a severe / stress market condition, we set the value of the systematic factor at a very conservative value. The Basel Committee sets its value at 0.01%:

$$y = N^{-1}(0.001) = -N^{-1}(0.999)$$

The PD conditional on this conservative value of the systematic factor reads then

$$PD_i(-N^{-1}(0.999)) = N\left((1-R)^{-0.5} * K_i + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right)$$

Recalling that $K_i = N^{-1}(PD_i)$, we get exactly the last component of the RWAs formula previously discussed:

Risk weights =

$$\left[LGD * N\left[(1-R)^{-0.5} * N^{-1}(PD) + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right] - LGD * PD\right] * f(PD, M)$$

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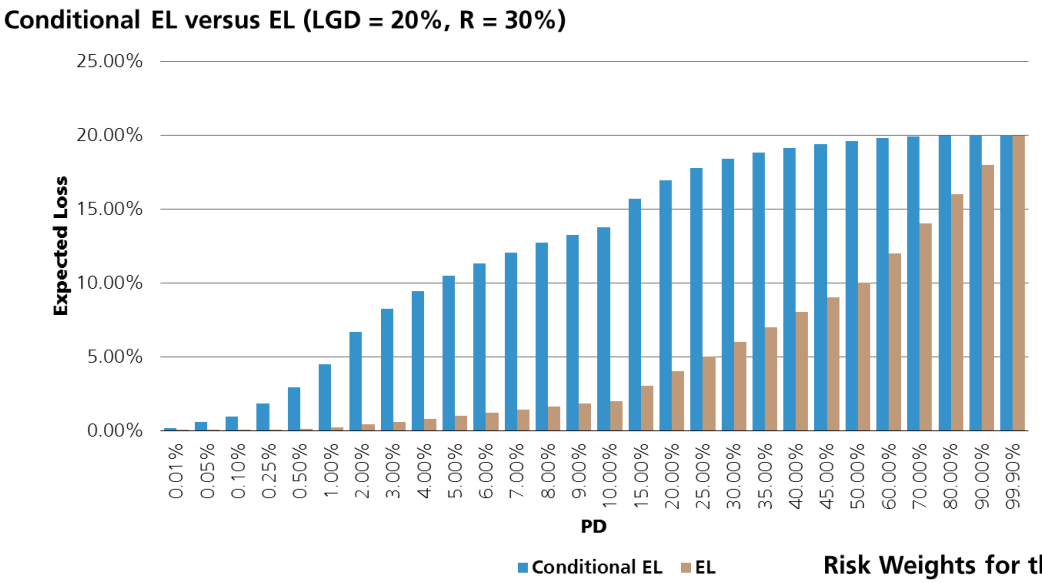
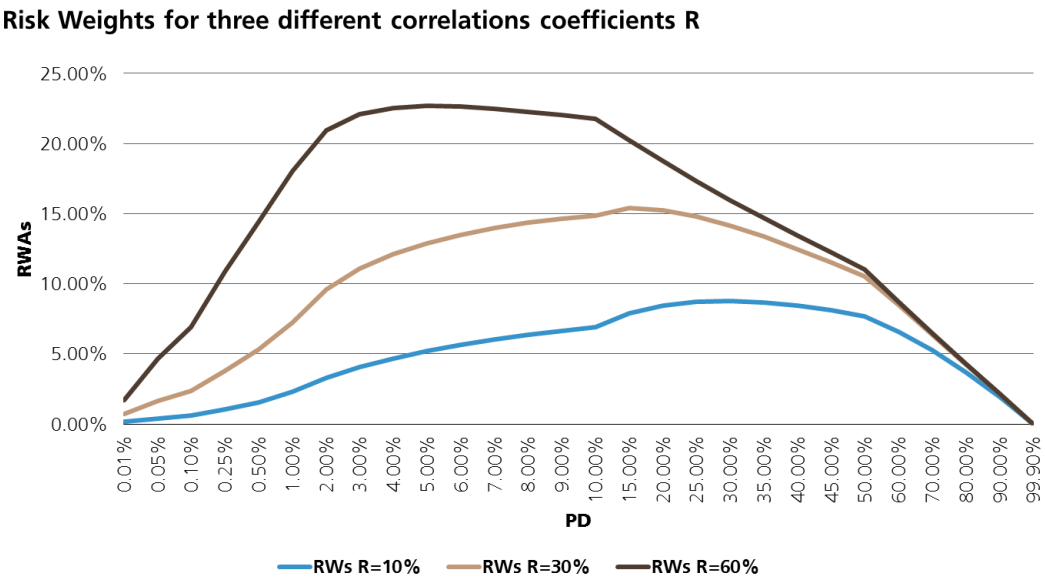
Sensitivity of the RWAs formula

- Unexpected Loss = Conditional EL - EL





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Section 4

Concluding remarks

Conclusion

1. One of the most dramatic changes to the banking industry since the last financial crisis is the rollout of new capital requirements for banks.
2. This capital protects the bank from losses and ultimately protects taxpayers from potential expensive bailouts.
3. There are several financial ratios that describe how well-capitalized a Bank is, e.g. the **Leverage ratio := Assets / Equity**. This concept does not sufficiently reflect the riskiness of the Assets and might give Banks a wrong incentive on how to structure the asset side of their Balance Sheet.
4. In order to correctly take the risk of the different assets into account, the Basel Committee requires Banks to have a **Capital Adequacy Ratio (CAR)** := Capital / RWAs above a pre-defined level.
5. RWAs are derived based on a mathematical formula, the starting point being a one-factor model:

$$\textbf{Risk weights} = [LGD * N\left[(1-R)^{-0.5} * N^{-1}(PD) + \left(\frac{R}{1-R}\right)^{0.5} * N^{-1}(0.999)\right] - LGD * PD](1-1.5 \times b(PD))^{-1} \times (1 + (M-2.5) * b(PD))$$

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Section 5

Q&A

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Questions

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