

Cosmic Censorship

- a talk on 

Stach Kuijpers

05/23/2018

Student seminar

A more general Black Hole

$$G = c = 1$$

Schwarzschild metric

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$r_s = 2M$$

Kerr-Newman metric

$$d\tau^2 = - \left(\frac{dr^2}{\Delta} + d\theta^2 \right) \rho^2 + (-a \sin^2 \theta d\phi + dt)^2 \frac{\Delta}{\rho^2} - \left((r^2 + a^2) d\phi - a dt \right)^2 \frac{\sin^2 \theta}{\rho^2}$$

$$r_s = 2M$$

$$r_Q^2 = \frac{Q^2}{4\pi\epsilon_0}$$

$$a = \frac{J}{M}$$

$$\rho^2 = r^2 + a^2 \cos^2 \theta$$

$$\Delta = r^2 - r_s r + a^2 + r_Q^2$$

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Singular when $g_{tt} = 0$ or $g_{rr} \rightarrow \infty$

1 horizon at $r = 2M$

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$g_{rr} \rightarrow \infty$ for $\Delta = 0$

2 event horizons

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$$\Delta = r^2 - r_s r + a^2 + r_Q^2$$

$$g_{tt} = 0 \text{ for } \Delta - a^2 \sin^2 \theta = 0$$

2 "ergospheres"

A more general Black Hole

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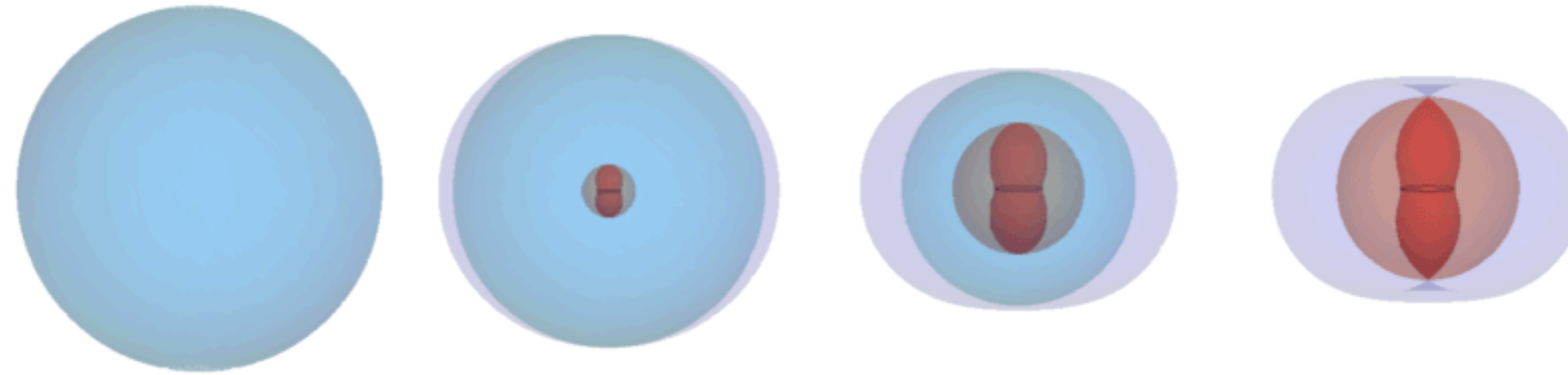
$$\rho^2 = r^2 + a^2 \cos^2 \theta$$

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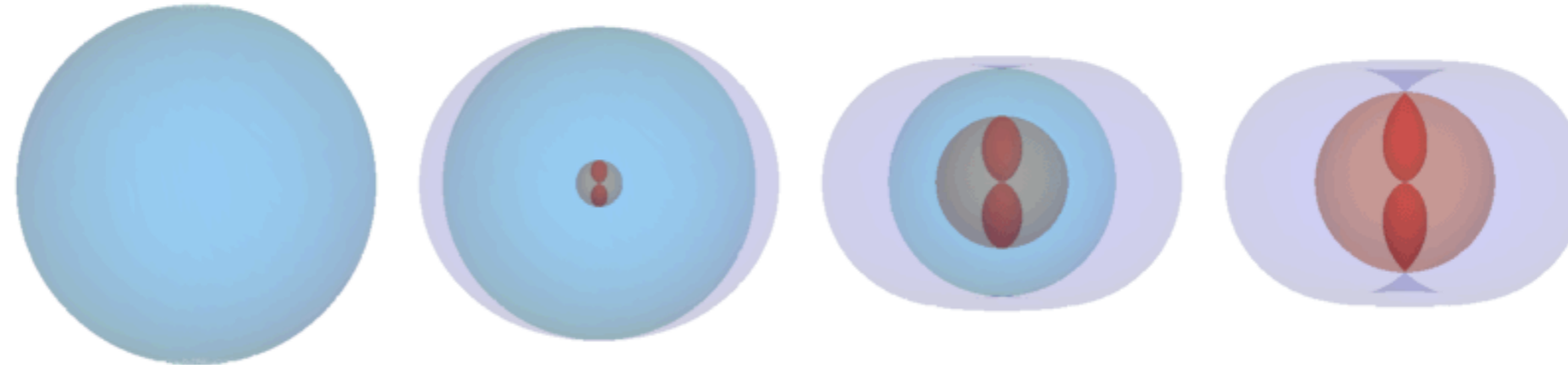
Up to 4 singular surfaces

Surfaces of a Kerr-Newman Black hole

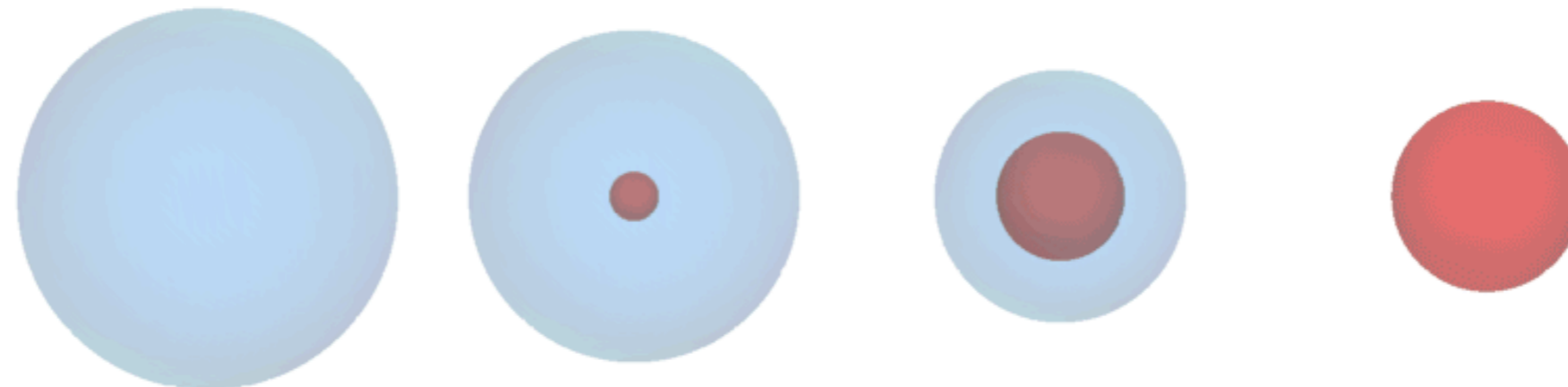
Rotating and charged
(Kerr-Newman)



Rotating, uncharged
(Kerr)



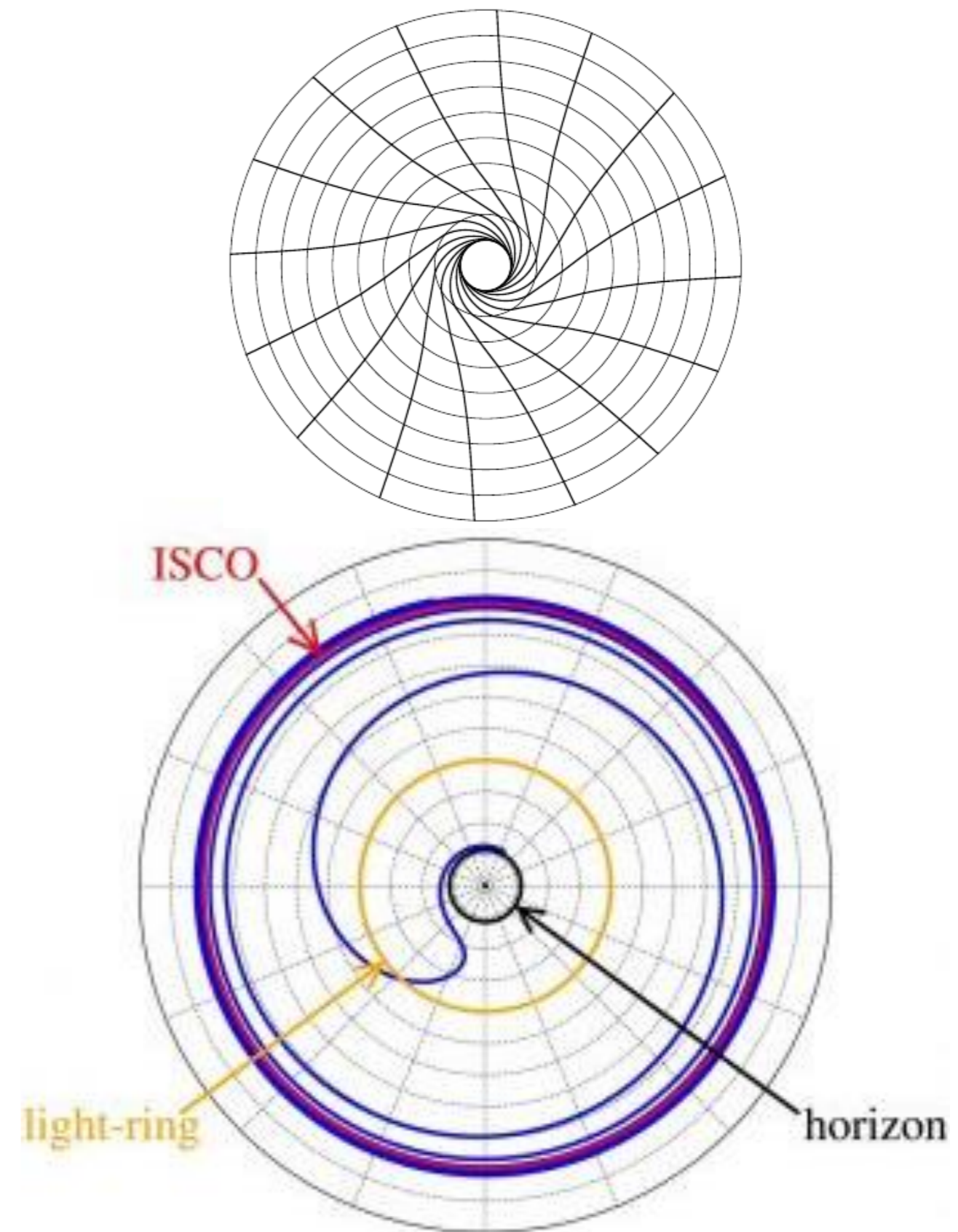
Non-rotating, charged
(Reissner-Nordström)



More critical →

What is this ergosphere?

- GR breaks down past inner event horizon
→ Inner ergosphere non-physical
- Outer ergosphere
→ surface where local frame velocity equals c
- From greek *ergon* for “work”
→ energy can be extracted from ergosphere
→ more on that later!

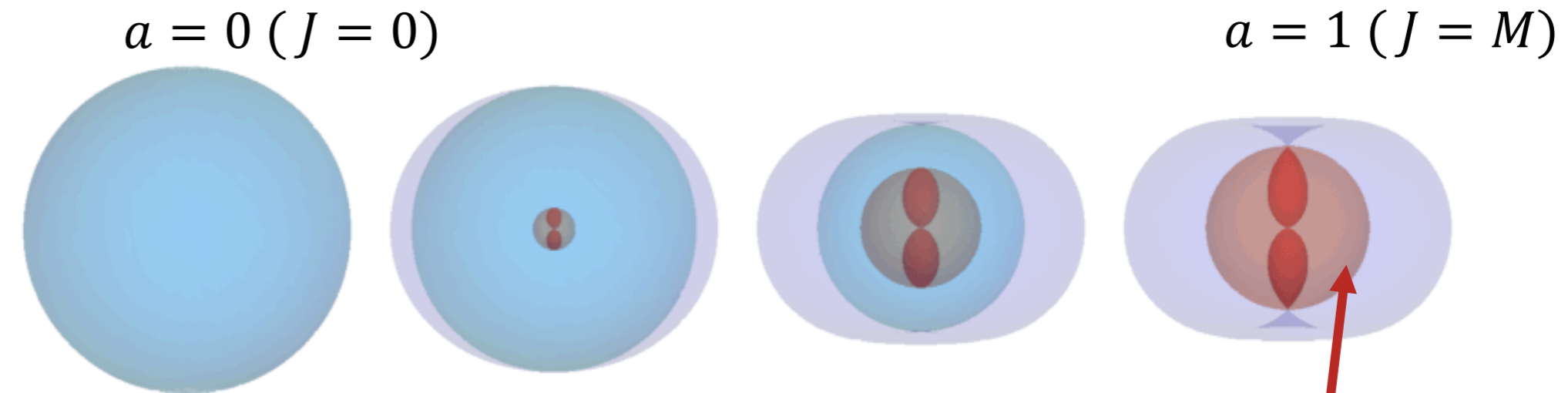


- 1) Dafermos, Luk, The interior of dynamical vacuum black holes I: The C^0 -stability of the Kerr Cauchy horizon. *submitted 2017*.

https://www.aei.mpg.de/1783615/Source_Modeling

Supercriticality and nakedness

Rotating, uncharged
(Kerr)



Both event horizons overlap

- $a^2 + Q^2 = 1$: GR breaks down at outer horizon
- $a^2 + Q^2 > 1$: naked singularity remains
 - problem: GR's determinism breaks down
 - conjecture: Naked singularities do not exist, i.e.
all singularities are hidden behind a null surface (Penrose, 1969)

Gedankenexperiments

- Wald (1974):
Over-spinning a supercritical black hole, $a^2 + Q^2 = 1$

→ Plunge particles with
 1. angular momentum
 2. internal spin
 3. charge



Interstellar (2014)

- 1) Wald, Gedanken experiments to destroy a black hole. *Annals of Physics* **1974**, 82(2), 548-556.
- 2) Matsas, da Silva, Overspinning a nearly extreme charged black hole via a quantum tunneling process. *Phys. Rev. Lett.* **2007**, 99(18), 181301.
- 3) Hod, Weak Cosmic Censorship: As Strong as Ever. *Phys. Rev. Lett.* **2008**, 100, 121101.

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→ Plunge particles with

1. angular momentum
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→ None of the particles get captured

“[E]lectrostatic, centrifugal, and spin-spin repulsion have all conspired”



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Gedankenexperiments

- Matsas, da Silva (2007):
Consider massless scalar particles

→ and find a violation!
- Hod (2008):
Takes higher order backreactions into account

→ cosmic censorship save once more



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- 1) Wald, Gedanken experiments to destroy a black hole. *Annals of Physics* **1974**, 82(2), 548-556.
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Simulations – black hole mergers

The configurations of two equal Kerr sources.—The family of two equal Kerr sources kept apart by a massless strut is described by the metric [8]

$$\begin{aligned}
 ds^2 &= f^{-1}[e^{2\gamma}(d\rho^2 + dz^2) + \rho^2 d\varphi^2] - f(dt - \omega d\varphi)^2, \\
 f &= \frac{A\bar{A} - B\bar{B}}{(A+B)(\bar{A}+\bar{B})}, \quad e^{2\gamma} = \frac{A\bar{A} - B\bar{B}}{16\lambda_0\bar{\lambda}_0 R_1 R_2 R_3 R_4}, \\
 \omega &= \omega_0 - \frac{2\text{Im}[G(\bar{A} + \bar{B})]}{A\bar{A} - B\bar{B}}, \\
 A &= (R_1 - R_2)(R_3 - R_4) - 4\sigma^2(R_1 - R_3)(R_2 - R_4), \\
 B &= 2s\sigma[(1 - 2\sigma)(R_1 - R_4) - (1 + 2\sigma)(R_2 - R_3)], \\
 G &= -zB + s\sigma[2R_1 R_3 - 2R_2 R_4 - 4\sigma(R_1 R_2 - R_3 R_4) \\
 &\quad - s(1 - 4\sigma^2)(R_1 - R_2 - R_3 + R_4)], \quad (1)
 \end{aligned}$$

where the functions R_i are defined by the expressions

$$\begin{aligned}
 R_i &= X_i \sqrt{\rho^2 + (z - \alpha_i)^2}, \\
 X_1 &= -1/X_4 = \phi(\mu + \sqrt{\mu^2 - 1}), \\
 X_2 &= -1/X_3 = \phi(\mu - \sqrt{\mu^2 - 1}), \\
 \alpha_1 &= -\alpha_4 = s \left(\frac{1}{2} + \sigma \right), \quad \alpha_2 = -\alpha_3 = s \left(\frac{1}{2} - \sigma \right), \\
 \sigma &= -\frac{i\sqrt{\mu^2 - 1}}{\nu}, \\
 \mu &= \frac{(\phi^2 + 1)[\phi(\nu^2 - 4) + i\nu(\phi^2 - 1)]}{2[(\phi^2 + 1)^2 - i\nu\phi(\phi^2 - 1)]}, \quad (2)
 \end{aligned}$$

and the constants λ_0 and ω_0 have the form

$$\begin{aligned}
 \lambda_0 &= \frac{1}{\nu^2} (1 - \phi^2)[(1 + \phi^2)^2 - \nu^2 \phi^2], \\
 \omega_0 &= \frac{2is(1 - \phi^4 - 2i\mu\nu\phi^2)}{(1 + \phi^2)^2 - \nu^2 \phi^2}. \quad (3)
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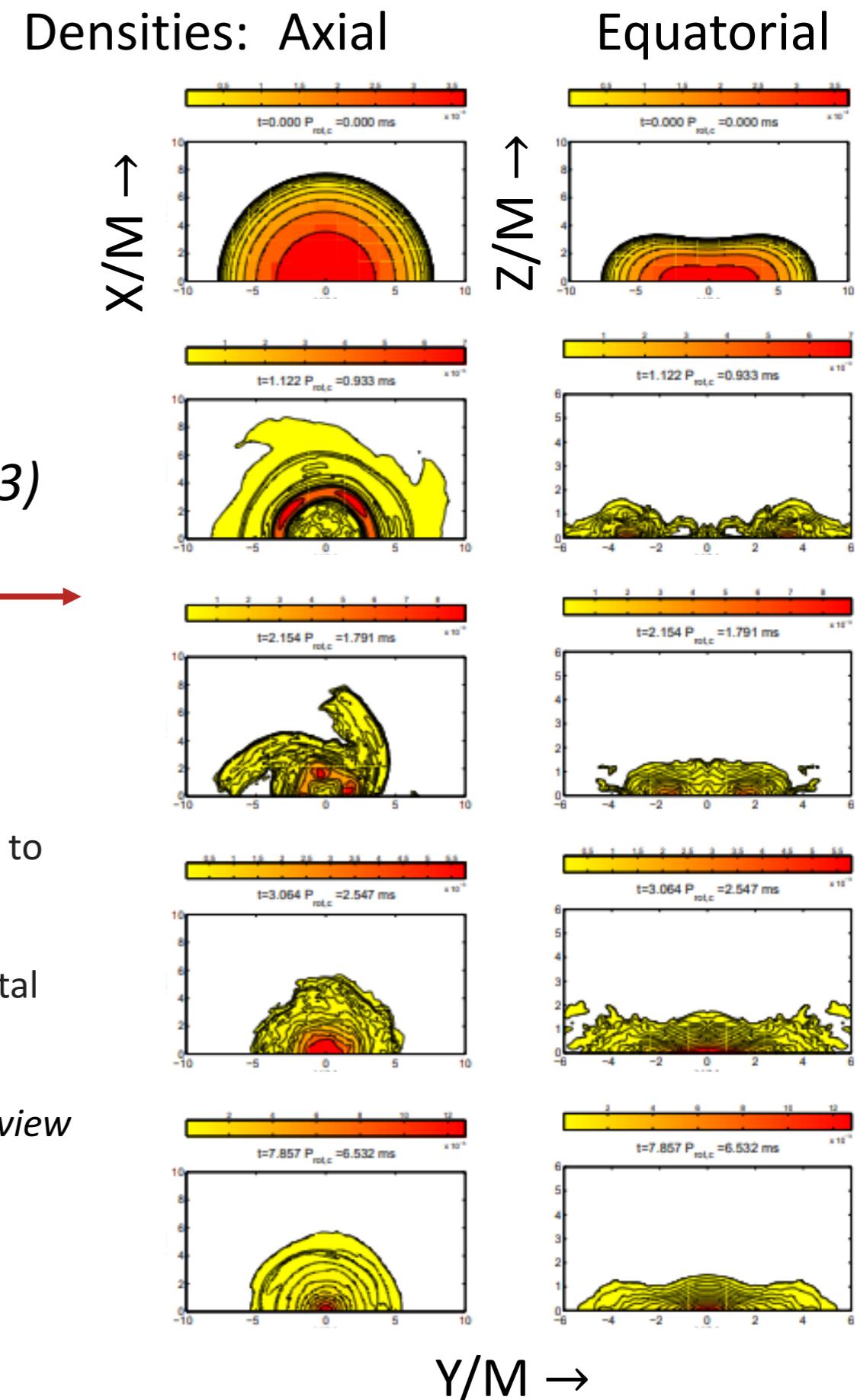
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Simulations – black hole mergers

- Spin-spin interaction on cosmic scale (1,2)
- “In all cases the collision results in a single BH plus gravitational radiation, i.e. there is no sign of any violation of cosmic censorship.” (3)
- supra-Kerr neutron star progenitors yield Kerr BH’s (4) →

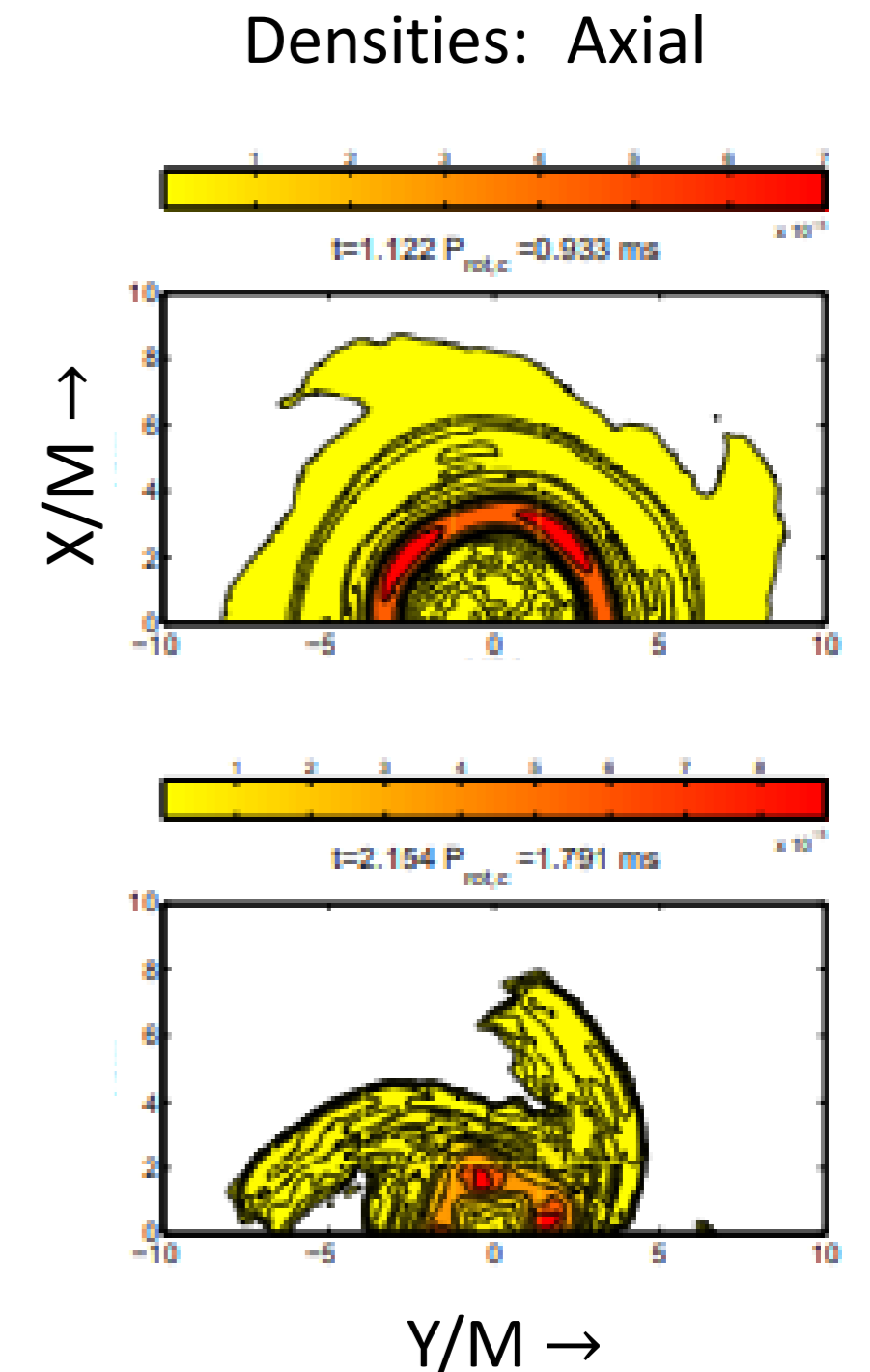
- 1) Manko, Ruiz, “Black hole-naked singularity” dualism and the repulsion of two Kerr black holes due to spin-spin interaction. *arXiv preprint* **2018**, arXiv:1803.03301.
- 2) Campanelli, Zlochower, Loustó, Gravitational radiation from spinning-black-hole binaries: The orbital hang up. *Phys. Rev. D* **2006**, 74, 041501.
- 3) Sperhake, Cardoso, Pretorius, Berti, Gonzalez, High-energy collision of two black holes. *Physical review letters* **2008**, 101(16), 161101.
- 4) Giacomazzo, Rezzolla, Stergioulas, Collapse of differentially rotating neutron stars and cosmic censorship. *Physical Review D* **2011**, 84(2), 024022.



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Experimental searches

- Square Kilometer array (1)
→ Looks for over-spinning BH pulsars
- Gravitational wave detectors (2)
→ Can measure orbital hang up

- 1) Kramer, Backer, Cordes, Lazio, Stappers, Johnston, Strong-field tests of gravity using pulsars and black holes. *New Astronomy Reviews* **2004**, 48(11), 993-1002.
- 2) Dreyer, Kelly, Krishnan, Finn, Garrison, Lopez-Aleman, Black-hole spectroscopy: testing general relativity through gravitational-wave observations. *Classical and Quantum Gravity* **2004**, 21(4), 787.

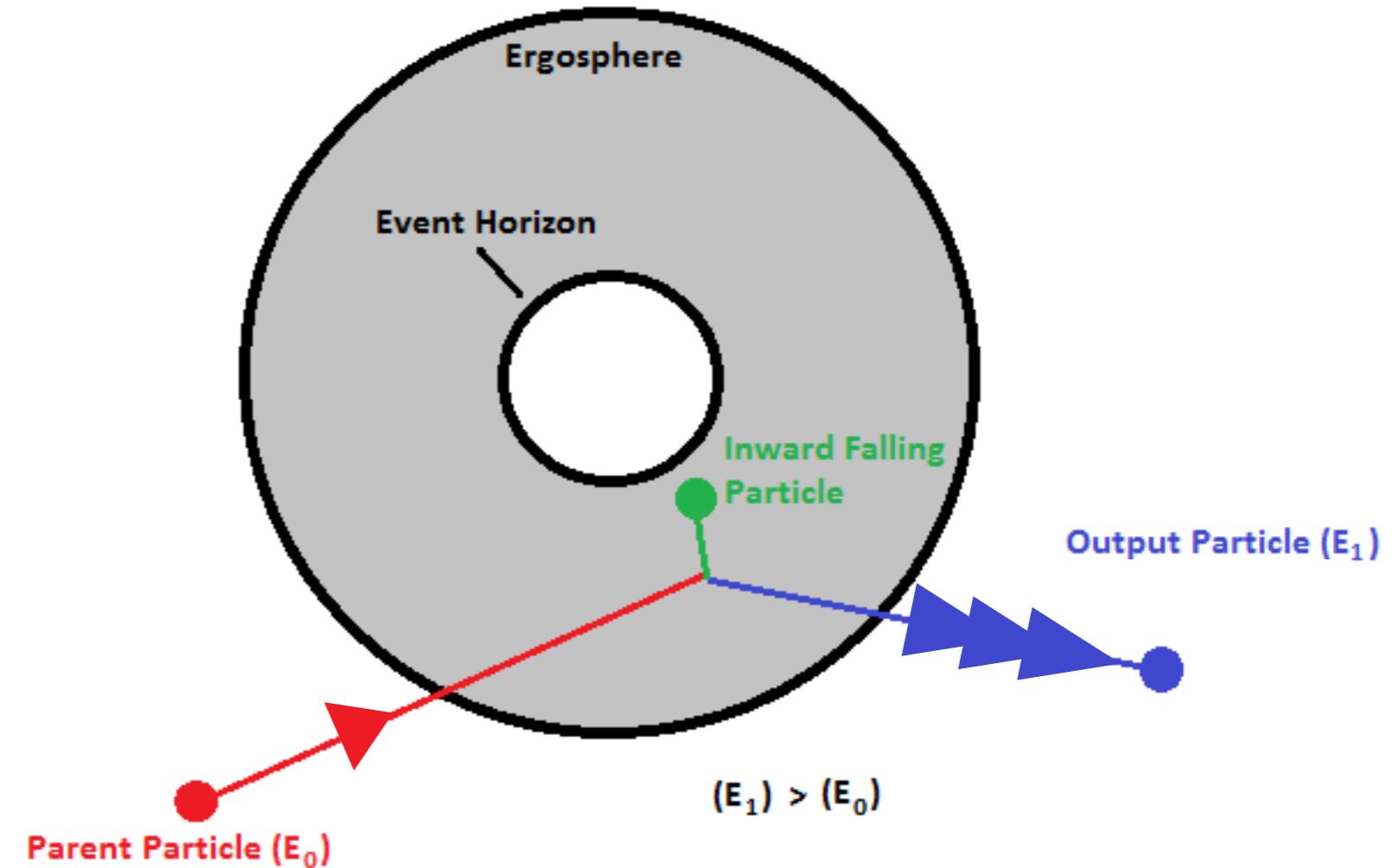


What did we learn?

So we did not crack GR...

→ What can we do?

- Penrose process



- Black hole bomb <https://youtu.be/ulCdoCfw-bY?t=323> (1:35 min)

1) Press, Teukolsky, Floating orbits, superradiant scattering and the black-hole bomb. *Nature* **1972**, 238(5361), 211.

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