

MKS TAMIL FRIENDS



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10th

Maths - Study material

Chapter 1. Relations and Function

- We publish this study material for request of many teachers and students.
- This study material contains only solution for exercise questions.
- Way to success – 10th Maths guide will be published very shortly.
- Way to success is preparing 10th maths guide with help of expert-experienced teachers to give an assurance for you to score more marks in your public examination.

MKS TAMIL



1. Relations and Functions

Exercise 1.1

1. Find $A \times B$, $A \times A$ and $B \times A$

(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$

$$A \times B = \{2, -2, 3\} \times \{1, -4\}$$

$$= \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$$

$$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$$

$$= \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$$

$$B \times A = \{1, -4\} \times \{2, -2, 3\}$$

$$= \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$$

(ii) $A = B = \{p, q\}$

$$A \times B = \{p, q\} \times \{p, q\}$$

$$= \{(p, p), (p, q), (q, p), (q, q)\}$$

$$A \times A = \{p, q\} \times \{p, q\}$$

$$= \{(p, p), (p, q), (q, p), (q, q)\}$$

$$B \times A = \{p, q\} \times \{p, q\}$$

$$= \{(p, p), (p, q), (q, p), (q, q)\}$$

(iii) $A = \{m, n\}$; $B = \emptyset$

$$A \times B = \{ \}$$

$$A \times A = \{m, n\} \times \{m, n\}$$

$$= \{(m, m), (m, n), (n, m), (n, n)\}$$

$$B \times A = \{ \}$$



2. Let $A = \{1, 2, 3\}$ and $B = \{x | x \text{ is a prime number less than } 10\}$. Find $A \times B$ and $B \times A$

$$A = \{1, 2, 3\}$$

$$B = \{x | x \text{ is a prime number less than } 10\} = \{2, 3, 5, 7\}$$

$$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$$

$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$$

$$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$$

$$= \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$$

3. If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ Find A and B

We know that,

B = set of all first co-ordinates of elements of $B \times A$

$$B = \{-2, 0, 3\}$$

A = set of all second co-ordinates of elements of $B \times A$

$$A = \{3, 4\}$$

4. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$, show that $A \times A = (B \times B) \cap (C \times C)$

$$A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$$

$$\text{LHS: } A \times A = \{5, 6\} \times \{5, 6\}$$

$$= \{(5, 5), (5, 6), (6, 5), (6, 6)\} \dots\dots\dots(1)$$

RHS:

$$B \times B = \{4,5,6\} \times \{4,5,6\}$$

$$= \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$$

$$C \times C = \{5,6,7\} \times \{5,6,7\}$$

$$= \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\}$$

$$(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\} \dots \dots \dots (2)$$

From (1) and (2), $A \times A = (B \times B) \cap (C \times C)$

5. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

$$A \cap C = \{1,2,3\} \cap \{2,3,5\} = \{3\}$$

$$B \cap D = \{2,3,5\} \cap \{1,3,5\} = \{3,5\}$$

LHS:

$$(A \cap C) \times (B \cap D) = \{3\} \times \{3,5\}$$

$$= \{(3,3), (3,5)\} \dots \dots \dots (1)$$

RHS:

$$A \times B = \{1,2,3\} \times \{2,3,5\}$$

$$= \{(1,2), (1,3), (1,5), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5)\}$$

$$C \times D = \{3,4\} \times \{1,3,5\}$$

$$= \{(3,1), (3,3), (3,5), (4,1), (4,3), (4,5)\}$$

$$(A \times B) \cap (C \times D) = \{(3,3), (3,5)\} \dots \dots \dots (2)$$

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true.

6. Let $A = \{x \in \mathbb{W} | x < 2\}$, $B = \{x \in \mathbb{N} | 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$$A = \{x \in \mathbb{W} | x < 2\} = \{0,1\}, \quad B = \{x \in \mathbb{N} | 1 < x \leq 4\} = \{2,3,4\}, \quad C = \{3,5\}$$

LHS:

$$B \cup C = \{2,3,4\} \cup \{3,5\} = \{2,3,4,5\}$$

$$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$$

$$= \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\} \dots \dots \dots (1)$$

RHS:

$$A \times B = \{0,1\} \times \{2,3,4\} = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$$

$$A \times C = \{0,1\} \times \{3,5\} = \{(0,3), (0,5), (1,3), (1,5)\}$$

$$(A \times B) \cup (A \times C) = \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\} \cup \{(0,3), (0,5), (1,3), (1,5)\}$$

$$= \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\} \dots \dots \dots (2)$$

From (1) and (2),

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$\text{LHS: } B \cap C = \{2,3,4\} \cap \{3,5\} = \{3\}$$

$$A \times (B \cap C) = \{0,1\} \times \{3\}$$

$$= \{(0,3), (1,3)\} \dots \dots \dots (1)$$

$$\begin{aligned}
 \text{RHS: } A \times B &= \{0,1\} \times \{2,3,4\} \\
 &= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\} \\
 A \times C &= \{0,1\} \times \{3,5\} \\
 &= \{(0,3), (0,5), (1,3), (1,5)\} \\
 (A \times B) \cap (A \times C) &= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\} \cap \{(0,3), (0,5), (1,3), (1,5)\} \\
 &= \{(0,3), (1,3)\} \dots\dots\dots(2)
 \end{aligned}$$

From (1) and (2),

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$(iii) (A \cup B) \times C = (A \times C) \cup (B \times C)$$

$$\begin{aligned}
 \text{LHS: } A \cup B &= \{0,1\} \cup \{2,3,4\} = \{0,1,2,3,4\} \\
 (A \cup B) \times C &= \{0,1,2,3,4\} \times \{3,5\} \\
 &= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\} \dots\dots\dots(1)
 \end{aligned}$$

RHS:

$$\begin{aligned}
 A \times C &= \{0,1\} \times \{3,5\} = \{(0,3), (0,5), (1,3), (1,5)\} \\
 B \times C &= \{2,3,4\} \times \{3,5\} = \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\} \\
 (A \times C) \cup (B \times C) &= \{(0,3), (0,5), (1,3), (1,5)\} \cup \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\} \\
 &= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\} \dots\dots\dots(2)
 \end{aligned}$$

From (1) and (2), $(A \cup B) \times C = (A \times C) \cup (B \times C)$

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number, Verify that

$$\begin{aligned}
 A &= \text{The set of all natural numbers less than 8} = \{1,2,3,4,5,6,7\} \\
 B &= \text{The set of all prime numbers less than 8} = \{2,3,5,7\} \\
 C &= \text{The set of even prime number} = \{2\}
 \end{aligned}$$

$$(i) (A \cap B) \times C = (A \times C) \cap (B \times C)$$

$$\begin{aligned}
 \text{LHS: } A \cap B &= \{1,2,3,4,5,6,7\} \cap \{2,3,5,7\} = \{2,3,5,7\} \\
 (A \cap B) \times C &= \{2,3,5,7\} \times \{2\} = \{(2,2), (3,2), (5,2), (7,2)\} \dots\dots\dots(1)
 \end{aligned}$$

RHS:

$$\begin{aligned}
 A \times C &= \{1,2,3,4,5,6,7\} \times \{2\} \\
 &= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\} \\
 B \times C &= \{2,3,5,7\} \times \{2\} \\
 &= \{(2,2), (3,2), (5,2), (7,2)\} \dots\dots\dots(2)
 \end{aligned}$$

From (1) and (2), $(A \cap B) \times C = (A \times C) \cap (B \times C)$

$$(ii) A \times (B - C) = (A \times B) - (A \times C)$$

LHS:

$$\begin{aligned}
 B - C &= \{2,3,5,7\} - \{2\} = \{3,5,7\} \\
 A \times (B - C) &= \{1,2,3,4,5,6,7\} \times \{3,5,7\} \\
 &= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), \\
 &\quad (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\} \dots\dots\dots(1)
 \end{aligned}$$

RHS:

$$A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$$

$$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5), (6,7), (7,2), (7,3), (7,5), (7,7)\}$$

$$A \times C = \{1,2,3,4,5,6,7\} \times \{2\} = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$$

$$(A \times B) - (A \times C)$$

$$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5), (6,7), (7,2), (7,3), (7,5), (7,7)\}$$

$$- \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$$

$$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\} \dots (2)$$

From (1) and (2)

$$A \times (B - C) = (A \times B) - (A \times C)$$

Exercise 1.2

1. Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relation from A to B ?

(i) $R_1 = \{(2,1), (7,1)\}$

$$A \times B = \{1,2,3,7\} \times \{3,0,-1,7\}$$

$$= \{(1,3), (1,0), (1,-1), (1,7), (2,3), (2,0), (2,-1), (2,7), (3,3), (3,0), (3,-1), (3,7), (7,3), (7,0), (7,-1), (7,7)\}$$

We know that,

$$(2,1) \text{ and } (7,1) \in R_1 \text{ but } (2,1), (7,1) \notin A \times B$$

So, R_1 is not a relation from A to B

(ii) $R_2 = \{(-1,1)\}$

$$\text{Here } (-1,1) \in R_2 \text{ but } (-1,1) \notin A \times B$$

So R_2 is not a relation from A to B

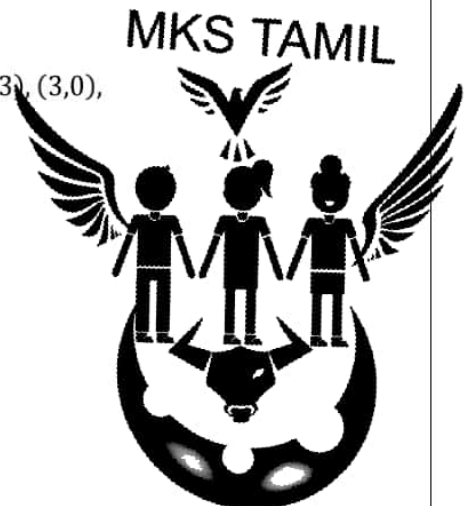
(iii) $R_3 = \{(2,-1), (7,7), (1,3)\}$

$$\text{Here } R_3 \subseteq A \times B$$

Hence R_3 is a relation from A to B

(iv) $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

$$\text{Here } (0,3) \text{ and } (0,7) \in R_4 \text{ but } (0,3) \text{ and } (0,7) \notin (A \times B)$$

So, R_4 is not a relation from A to B 

2. Let $A = \{1, 2, 3, 4, \dots, 45\}$ and R be the relation defined as "is square of" on A . Write R as a subset of $A \times A$. Also, find the domain and range of R .

$$\text{Given } A = \{1,2,3,4, \dots, 45\}$$

$$A \times A = \{(1,1), (1,2), (1,3), (1,4) \dots (45,45)\}$$

Then, R be the relation defined as is "square of" on A .

$$\text{Hence, } R = \{(1,1), (2,4), (3,9), (4,16), (5,25), (6,36)\}$$

$$\text{So } R \subseteq A \times A$$

$$\text{The domain of } R = \{1,2,3,4,5,6\}$$

$$\text{The range of } R = \{1,4,9,16,25,36\}$$

3. A Relation R is given by the set $\{(x, y)/y = x + 3, x \in \{1, 2, 3, 4, 5\}\}$. Determine its domain and range

$$R = \{(x, y)/y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$$

$$\text{Here domain } (x) = \{0, 1, 2, 3, 4, 5\}$$

$$\text{Co-domain } (y) = x + 3$$

$$y_0 = 0 + 3 = 3$$

$$y_1 = 1 + 3 = 4$$

$$y_2 = 2 + 3 = 5$$

$$y_3 = 3 + 3 = 6$$

$$y_4 = 4 + 3 = 7$$

$$y_5 = 5 + 3 = 8$$

$$R = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$$

$$\text{Domain} = \{0, 1, 2, 3, 4, 5\}$$

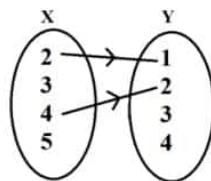
$$\text{Range} = \{3, 4, 5, 6, 7, 8\}$$



4. Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y)/x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(a) An arrow diagram

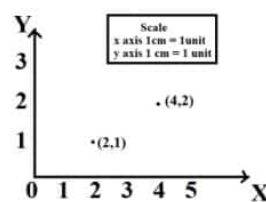


$$\text{Given, } x = 2y$$

$$\text{If } y = 1 \Rightarrow x = 2$$

$$\text{If } y = 2 \Rightarrow x = 4$$

(b) a graph



(c) a set in roster form

$$R = \{(2, 1), (4, 2)\}$$

(ii) $\{(x, y)/y = x + 3, x, y \text{ are natural numbers } < 10\}$

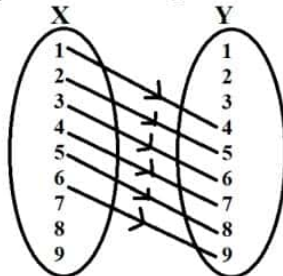
Given, x, y are natural numbers < 10

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

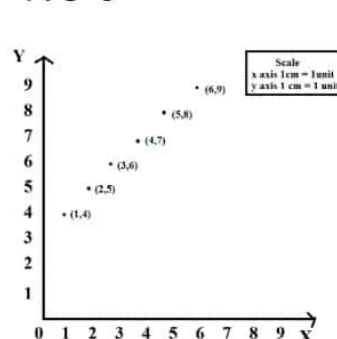
$$y = x + 3$$

$$\text{Here } R(1) = 4, R(2) = 5, R(3) = 6, R(4) = 7, R(5) = 8, R(6) = 9$$

(a) An arrow diagram



(b) graph



(c) Roster Form

$$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$$

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide Rs.10,000, Rs.25,000, Rs.50,000, Rs.1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 where Executive officers and if the relation R is defined by xRy , where x is the salary given to person y , express the relation R through an ordered pair and an arrow diagram.

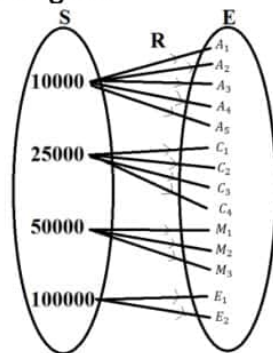
Salaries (S) = {10000, 25000, 50000, 100000}

Employees (E) = { $A_1, A_2, A_3, A_4, A_5, C_1, C_2, C_3, C_4, M_1, M_2, M_3, E_1, E_2$ }

(a) Order of Pairs

$R = \{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5), (25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, M_3), (100000, E_1), (100000, E_2)\}$

(b) An arrow diagram



Exercise 1.3:

1. Let $f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, codomain and range. Is this relation a function?

$$y = f(x) = 2x$$

$$f(1) = 2(1) = 2$$

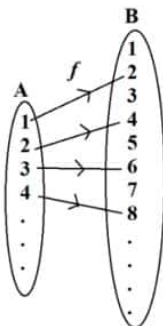
$$f(2) = 2(2) = 4$$

$$f(3) = 2(3) = 6$$

$$f(4) = 2(4) = 8$$

$\vdots$

f be a relation on $\mathbb{N}$



Domain of $f = \{1, 2, 3, 4, \dots\}$

Codomain of $f = \{1, 2, 3, 4, \dots\}$

Range of $f = \{2, 4, 6, 8, \dots\}$

From the arrow diagram of f , for each $x \in A$ there exists only one $y \in B$.

Yes, f is a function.

2. Let $X = \{3, 4, 6, 8\}$. Determine whether the relation $R = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$?

Given $X = \{3, 4, 6, 8\}$

$$Y = \{1, 2, 3, 4, \dots\}$$

$$R = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$$

$$y = f(x) = x^2 + 1$$

$$f(3) = 3^2 + 1 = 10$$

$$f(4) = 4^2 + 1 = 17$$

$$f(6) = 6^2 + 1 = 37$$

$$f(8) = 8^2 + 1 = 65$$

$$R = \{(3, 10), (4, 17), (6, 37), (8, 65)\}$$

We note that each element in the domains of X has a unique image in $\mathbb{N}$.

Yes, it is a function from X to $\mathbb{N}$

3. Given the function $f: x \rightarrow x^2 - 5x + 6$, evaluate

- (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x - 1)$

$$f(x) = x^2 - 5x + 6$$

(i) $f(-1)$

$$\begin{aligned} f(-1) &= (-1)^2 - 5(-1) + 6 \\ &= 1 + 5 + 6 = 12 \end{aligned}$$

(ii) $f(2a)$

$$\begin{aligned} f(2a) &= (2a)^2 - 5(2a) + 6 \\ &= 4a^2 - 10a + 6 \end{aligned}$$

(iii) $f(2)$

$$\begin{aligned} f(2) &= 2^2 - 5(2) + 6 \\ &= 4 - 10 + 6 = 0 \end{aligned}$$

(iv) $f(x - 1)$

$$\begin{aligned} f(x - 1) &= (x - 1)^2 - 5(x - 1) + 6 \\ &= x^2 - 2x + 1 - 5x + 5 + 6 \\ &= x^2 - 7x + 12 \end{aligned}$$

4. A graph representing the function $f(x)$ is given in adjacent figure.

It is clear that $f(9) = 2$

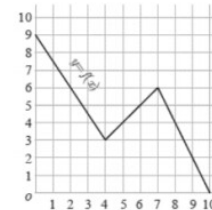
(i) Find the following values of the function

(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$

(ii) for What value of x is $f(x) = 1$

(iii) Describe the following (i) Domain (ii) Range

(iv) What is the image of 6 under f ?



(i) From the figure, (a) $f(0) = 9$ (b) $f(7) = 6$ (c) $f(2) = 6$ (d) $f(10) = 0$

(ii) $f(x) = 1 \Rightarrow x = 9.5$

(iii) (a) Domain of $f = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ or

$$f = \{x/0 \leq x \leq 10, x \in R\}$$

(b) Range of $f = \{0, 1, 3, 4, 5, 6, 7, 8, 9\}$ or

$$f = \{x/0 \leq x \leq 9, x \in R\}$$

(iv) The image of 6 under $f = 5$

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$

Given, $f(x) = 2x + 5, x \neq 0$

$$f(x + 2) = 2(x + 2) + 5 = 2x + 4 + 5 = 2x + 9$$

$$f(2) = 2(2) + 5 = 4 + 5 = 9$$

$$\begin{aligned} \text{Then } \frac{f(x+2)-f(2)}{x} &= \frac{2x+9-9}{x} \\ &= \frac{2x}{x} = 2 \end{aligned}$$

6. A function f is defined by $f(x) = 2x - 3$

(i) find $\frac{f(0)+f(1)}{2}$ (ii) find x such that $f(x) = 0$ (iii) find x such that $f(x) = x$

(iv) find x such that $f(x) = f(1 - x)$

(i) $\frac{f(0)+f(1)}{2}$ $f(0) = 2(0) - 3 = -3$ $f(1) = 2(1) - 3 = -1$ $\frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = -\frac{4}{2} = -2$ (ii) $f(x) = 0$ $2x - 3 = 0$ $2x = 3$ $x = \frac{3}{2}$	(iii) $f(x) = x$ $2x - 3 = x$ $2x - x = 3$ $x = 3$ (iv) $f(x) = f(1 - x)$ $2x - 3 = 2(1 - x) - 3 = 2 - 2x - 3 = -2x - 1$ $2x + 2x = 3 - 1$ $4x = 2$ $x = \frac{1}{2}$
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7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown figure. Express the volume V of the box as a function of x .

Side of the square material = 24 cm

Side of cutting equal = x cm

Square from the corners

Now, it is to be made an open box.

Side of the open box = $24 - 2x$ cm

Height = x cm

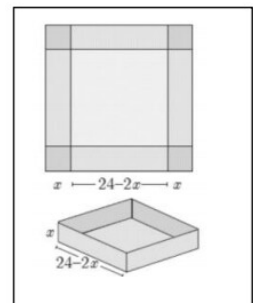
Volume of the open box $V = lbh$ cubic units.

$$= (24 - 2x)(24 - 2x)(x)$$

$$= (24 - 2x)^2(x)$$

$$= (576 - 96x + 4x^2)x$$

$$= 4x^3 - 96x^2 + 576x$$



8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$

Given, $f(x) = 3 - 2x$

To find x , $f(x^2) = (f(x))^2$

$$3 - 2x^2 = (3 - 2x)^2$$

$$3 - 2x^2 = 9 - 12x + 4x^2$$

$$4x^2 + 2x^2 - 12x + 6 = 0$$

$$6x^2 - 12x + 6 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0$$

$$x = 1$$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Given, speed (S) = 500 km/hr

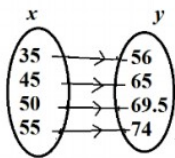
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$S = \frac{d}{t}$$

$$d = 500t$$

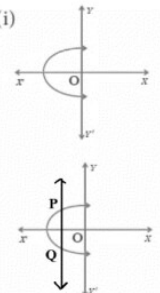
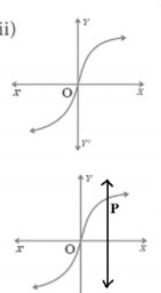
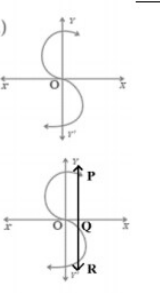
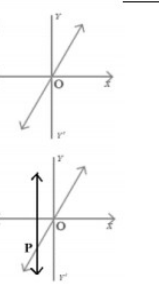
10. The data in the adjacent table depicts the length of a woman's forearm and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forearm length (x) as $y = ax + b$, where a, b are constants. (i) Check if this relation is a function. (ii) Find a and b (iii) Find the height of a woman whose forearm length is 40cm (iv) Find the length of forearm of a woman if her height is 53.3 inches.

Given $y = ax + b$

(i) Arrow diagram  Each element in x is associated with a unique element in y Yes, this relation is a function (ii) find a and b From the table $35a + b = 56$(1) $45a + b = 65$(2) (-) (-) (-) (-) $\underline{-10a \quad \quad = -9}$ $a = \frac{9}{10} = 0.9$ $a = 0.9$ substitute in (1) $35(0.9) + b = 56$ $31.5 + b = 56$ $b = 56 - 31.5 = 24.5$ $a = 0.9$ and $b = 24.5$	(iii) Length = 40cm, $a = 0.9, b = 24.5$ $y = ax + b$ $= (0.9)(40) + 24.5$ $= 60.5$ The height of a woman whose forearm length is 40 cm = 60.5 inches (iv) Height = 53.3 inches $y = ax + b$ $53.3 = (0.9)x + 24.5$ $= 0.9x + 24.5$ $0.9x = 53.3 - 24.5$ $= 28.8$ $x = \frac{28.8}{0.9}$ $x = 32$ cm The length of forearm of a woman = 32 cm
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Exercise 1.4:

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.

(i)  Using vertical line test, the curve does not represent a function as the vertical line meets the curve in two points P and Q	(ii)  The curve represent a function as the vertical line meets the curve in at most one point.	(iii)  The curve does not represent a function as the vertical line meets the curve in three points P, Q and R	(iv)  The graph represent a function as the vertical line meets the line is at most one point P
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2. Let $f: A \rightarrow B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent by (i) set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a graph

Given, $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$

Now $f(x) = \frac{x}{2} - 1$

Thus, $f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$

$f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$

$f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$

$f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$

$f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$

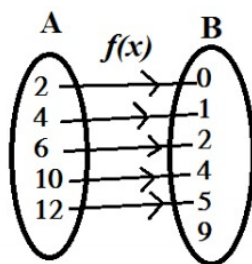
(i) set of ordered pairs

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

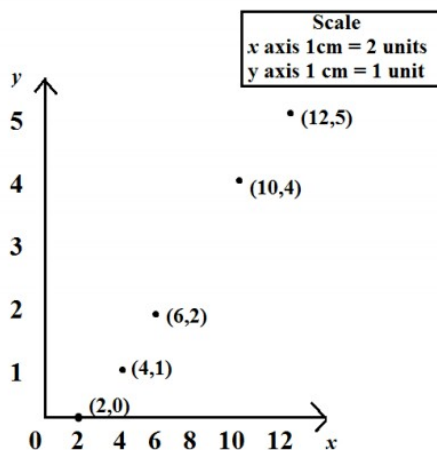
(ii) Table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) An arrow diagram



(iv) Graph



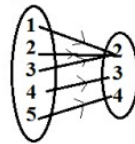
3. Represent the function

$f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$

through (i) an arrow diagram (ii) a table form (iii) a graph

$f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$

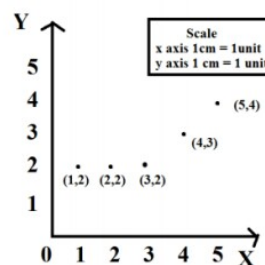
(i) an arrow diagram



(ii) a table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) a graph



4. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x - 1$ is one-one but not onto

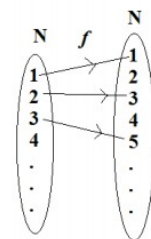
The function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x - 1$

If $x = 1$, $f(1) = 2(1) - 1 = 1$

If $x = 2$, $f(2) = 2(2) - 1 = 3$

If $x = 3$, $f(3) = 2(3) - 1 = 5$

Arrow diagram



Then f is a function from N to N and for different elements in N , there are different images in N . Hence f one-one function.

But the even numbers in the co-domain do not have any pre-images of the domain. Hence f is not onto, So f is one-one but not onto function.

5. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one - one function

The function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by

$$f(m) = m^2 + m + 3$$

$$m = 1, f(1) = (1)^2 + 1 + 3 = 1 + 1 + 3 = 5$$

$$m = 2, f(2) = (2)^2 + 2 + 3 = 4 + 2 + 3 = 9$$

$$m = 3, f(3) = (3)^2 + 3 + 3 = 9 + 3 + 3 = 15$$

$$m = 4, f(4) = (4)^2 + 4 + 3 = 16 + 4 + 3 = 23$$

Since different elements of N have different images in the codomain the function of f is one-one function.

6. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbb{N}$. Let $f: A \rightarrow B$ be defined by $f(x) = x^3$ then,
(i) find the range of f (ii) identify the type of function

Now $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, \dots\}$

Given $f: A \rightarrow B$ and $f(x) = x^3$

$$f(1) = 1^3 = 1, f(3) = 3^3 = 27$$

$$f(2) = 2^3 = 8, f(4) = 4^3 = 64$$

$$(i) \text{ Range of } f = \{1, 8, 27, 64\}$$

(ii) Since distinct elements in A are mapped into distinct images in B , it is a **one-one function**. $2 \in B$ is not the image of any element of A . So, it is **Into function**.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

- (i) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 1$

$$f(x) = 2x + 1$$

$$f(0) = 2(0) + 1 = 1$$

$$f(1) = 2(1) + 1 = 3$$

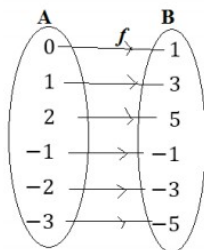
$$f(2) = 2(2) + 1 = 5$$

$$f(-1) = 2(-1) + 1 = -1$$

$$f(-2) = 2(-2) + 1 = -3$$

$$f(-3) = 2(-3) + 1 = -5$$

$$\text{Range of } f = \{1, 3, 5, -1, -3, -5\}$$



As distinct elements of A have distinct images in B and every elements in B has a pre-image in A . The function is **bijective**.

- (ii) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x^2$

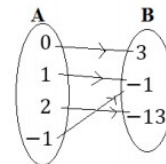
$$f(x) = 3 - 4x^2$$

$$f(0) = 3 - 4(0)^2 = 3$$

$$f(1) = 3 - 4(1)^2 = -1$$

$$f(2) = 3 - 4(2)^2 = -13$$

$$f(-1) = 3 - 4(1)^2 = -1$$



Thus two distinct elements 1 and -1 in A have same image -1 in B . Hence f is not a one-one function. But every elements in B has a pre-image in A . Hence f is a onto function.

Therefore f is a not one-one but onto.

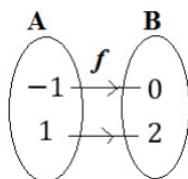
Hence f is **not bijective**.

8. Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f: A \rightarrow B$ defined by $f(x) = ax + b$ is an onto function? Find a and b .

Given $A = \{-1, 1\}$ and $B = \{0, 2\}$

Then $f: A \rightarrow B$ defined by

$f(x) = ax + b$ is an onto function.



[Range of f = co-domain]

That is, $f(-1) = 0$

$$f(1) = 2$$

$$\text{Now } -a + b = 0 \dots\dots\dots(1)$$

$$a + b = 2 \dots\dots\dots(2)$$

$$(1)+(2) \quad \frac{2b = 2}{b = 1}$$

Substitute $b = 1$ in (2)

$$a + 1 = 2$$

$$a = 2 - 1 = 1$$

Thus, $a = 1$ and $b = 1$

9. If the function f is defined by $f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$ find the values of

(i) $f(3)$ (ii) $f(0)$ (iii) $f(-1.5)$ (iv) $f(2) + f(-2)$

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases} \quad \begin{matrix} \text{where } x = 2, 3, 4 \\ \text{where } x = 0 \\ \text{where } x = -2 \end{matrix}$$

(i) $f(3)$ When $x = 3$, $f(x) = x + 2$ $f(3) = 3 + 2 = 5$	(ii) $f(0)$ When $x = 0$ $f(x) = 2$ $f(0) = 2$
(iii) $f(-1.5)$ When $x = -1.5$ $f(x) = x - 1$ $f(-1.5) = -1.5 - 1$ $= -2.5$	(iv) $f(2) + f(-2)$ When $x = 2$, $f(x) = x + 2$ $f(2) = 2 + 2 = 4$ When $x = -2$, $f(x) = x - 1$ $f(-2) = -2 - 1 = -3$ $f(2) + f(-2) = 4 + (-3) = 1$

10. A function $f: [-5, 9] \rightarrow \mathbb{R}$ is defined as follows: $f(x) = \begin{cases} 6x+1 & \text{if } -5 \leq x < 2 \\ 5x^2-1 & \text{if } 2 \leq x < 6 \\ 3x-4 & \text{if } 6 \leq x \leq 9 \end{cases}$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ (iii) $2f(4) + f(8)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \leq x < 2 \\ 5x^2-1 & \text{if } 2 \leq x < 6 \\ 3x-4 & \text{if } 6 \leq x \leq 9 \end{cases} \quad \begin{matrix} \text{; Where } x = -5, -4, -3, -2, -1, 0, 1 \\ \text{; Where } x = 2, 3, 4, 5 \\ \text{; Where } x = 6, 7, 8, 9 \end{matrix}$$

(i) $f(-3) + f(2)$ When $x = -3$ $f(x) = 6x + 1$ $f(-3) = 6(-3) + 1 = -18 + 1 = -17$ When $x = 2$ $f(x) = 5x^2 - 1$ $f(2) = 5(2)^2 - 1 = 20 - 1 = 19$ $f(-3) + f(2) = -17 + 19 = 2$	(ii) $f(7) - f(1)$ When $x = 7$ $f(x) = 3x - 4$ $f(7) = 3(7) - 4 = 21 - 4 = 17$ When $x = 1$ $f(x) = 6x + 1$ $f(1) = 6(1) + 1 = 6 + 1 = 7$ $\therefore f(7) - f(1) = 17 - 7 = 10$
(iii) $2f(4) + f(8)$ When $x = 4$, $f(x) = 5x^2 - 1$ $f(4) = 5(4)^2 - 1$ $= 80 - 1$ $= 79$ When $x = 8$, $f(x) = 3x - 4$ $f(8) = 3(8) - 4$ $= 24 - 4 = 20$ $2f(4) + f(8) = 2(79) + 20$ $= 158 + 20$ $= 178$	(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$ When $x = -2$, $f(x) = 6x + 1$ $f(-2) = 6(-2) + 1$ $= -12 + 1 = -11$ When $x = 6$, $f(x) = 3x - 4$ $f(6) = 3(6) - 4$ $= 18 - 4 = 14$ When $x = 4$, $f(x) = 5x^2 - 1$ $f(4) = 5(4)^2 - 1 = 80 - 1 = 79$ $\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)}$ $= \frac{-22-14}{79-11} = \frac{-36}{68} = -\frac{9}{17}$

11. The distance S an object travels under the influence of gravity in the time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Check if the function $S(t)$ is one-one.

Given $S(t) = \frac{1}{2}gt^2 + at + b$ (a, b constants)

Now take $t = 1, 2, 3, \dots$ seconds

$$t = 1, \quad S(1) = \frac{1}{2}g(1)^2 + a(1) + b \\ = \frac{1}{2}g + a + b$$

$$t = 2, \quad S(2) = \frac{1}{2}g(2)^2 + a(2) + b \\ = 2g + 2a + b$$

$$t = 3, \quad S(3) = \frac{1}{2}g(3)^2 + a(3) + b \\ = 4.5g + 3a + b$$



Since distinct elements of A have distinct image in B . Yes, it is an one-one function.

12. The function ' t ' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find

(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C) = 212$

(v) the temperature when the Celsius value is equal to the Fahrenheit value

The function t is defined by, $t(C) = F$, where $F = \frac{9}{5}C + 32$

(i) $t(0) = \frac{9}{5}(0) + 32 = 32^\circ F$

(ii) $t(28) = \frac{9}{5}(28) + 32 \\ = 9(5.6) + 32 \\ = 50.4 + 32 \\ = 82.4^\circ F$

(iii) $t(-10) = \frac{9}{5}(-10) + 32 \\ = -18 + 32 \\ = 14^\circ F$

(iv) when $t(C) = 212$

$$\frac{9}{5}C + 32 = 212$$

$$\frac{9}{5}C = 212 - 32 = 180$$

$$C = \frac{180 \times 5}{9} = 100^\circ C$$

(v) we know that

$$t(C) = F \text{ where } F = \frac{9}{5}C + 32$$

$$t(F) = C \text{ where } C = \frac{5}{9}F + 32$$

If the temperatures are same then two ' t 's in the formula should represent the same temperature. So then we multiply each side by $\left(-\frac{5}{4}\right)$

$$t = \frac{9}{5}t + 32^\circ$$

$$t - \frac{9}{5}t = 32^\circ$$

Multiply each side by $\left(-\frac{5}{4}\right)$

$$-\frac{5}{4}\left(t - \frac{9}{5}t\right) = 32^\circ \times \left(-\frac{5}{4}\right)$$

$$-\frac{5}{4}t + \frac{9}{4}t = -40^\circ$$

$$\frac{-5t + 9t}{4} = -40^\circ$$

$$\frac{4t}{4} = -40^\circ$$

$$t = -40^\circ$$

Exercise 1.5**1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$**

(i) $f(x) = x - 6, g(x) = x^2$ $f \circ g = f(g(x))$ $= f(x^2)$ $= x^2 - 6 \dots\dots\dots(1)$ $g \circ f(x) = g(f(x))$ $= g(x - 6)$ $= (x - 6)^2 \dots\dots\dots(2)$ From (1) and (2) $f \circ g \neq g \circ f$	(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$ $f \circ g(x) = f(g(x))$ $= f(2x^2 - 1)$ $= \frac{2}{2x^2 - 1} \dots\dots\dots(1)$ $g \circ f(x) = g(f(x)) = g\left(\frac{2}{x}\right)$ $= 2\left(\frac{2}{x}\right)^2 - 1 = \frac{8}{x^2} - 1 \dots\dots\dots(2)$ From (1) and (2) We see that $f \circ g \neq g \circ f$
(iii) $f(x) = \frac{x+6}{3}, g(x) = 3 - x$ $f \circ g(x) = f(g(x))$ $= f(3 - x)$ $= \frac{3 - x + 6}{3} = \frac{9 - x}{3} \dots\dots\dots(1)$ $g \circ f(x) = g(f(x))$ $= g\left(\frac{x+6}{3}\right)$ $= 3 - \left(\frac{x+6}{3}\right) = \frac{9 - x - 6}{3} = \frac{3 - x}{3} \dots\dots\dots(2)$ From (1) and (2) we get that, $f \circ g \neq g \circ f$	(iv) $f(x) = 3 + x, g(x) = x - 4$ $f \circ g(x) = f(g(x))$ $= f(x - 4)$ $= 3 + x - 4$ $= x - 1 \dots\dots\dots(1)$ $g \circ f(x) = g(f(x))$ $= g(3 + x)$ $= 3 + x - 4$ $= x - 1 \dots\dots\dots(2)$ From (1) and (2) we get that, $f \circ g = g \circ f$
(v) $f(x) = 4x^2 - 1, g(x) = 1 + x$ $f \circ g(x) = f(g(x))$ $= f(1 + x)$ $= 4(1 + x)^2 - 1$ $= 4 + 8x + 4x^2 - 1$ $= 4x^2 + 8x + 3 \dots\dots\dots(1)$	$g \circ f(x) = g(f(x))$ $= g(4x^2 - 1)$ $= 1 + 4x^2 - 1$ $= 4x^2 \dots\dots\dots(2)$ From (1) and (2) We see that $f \circ g \neq g \circ f$

2. Find the value of k , such that $f \circ g = g \circ f$

(i) $f(x) = 3x + 2, g(x) = 6x - k$ $f \circ g(x) = f(g(x))$ $= f(6x - k)$ $= 3(6x - k) + 2 = 18x - 3k + 2$ $g \circ f(x) = g(f(x))$ $= g(3x + 2)$ $= 6(3x + 2) - k = 18x + 12 - k$ Given that, $f \circ g = g \circ f$ $18x - 3k + 2 = 18x + 12 - k$ $18x - 18x - 3k + k = 12 - 2$ $-2k = 10$ $k = -5$	(ii) $f(x) = 2x - k, g(x) = 4x + 5$ $f \circ g(x) = f(g(x))$ $= f(4x + 5)$ $= 2(4x + 5) - k = 8x + 10 - k$ $g \circ f(x) = g(f(x))$ $= g(2x - k)$ $= 4(2x - k) + 5 = 8x - 4k + 5$ Given that $f \circ g = g \circ f$ $8x + 10 - k = 8x - 4k + 5$ $8x - 8x - k + 4k = 5 - 10$ $3k = -5$ $k = -\frac{5}{3}$
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3. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$, show that

$$f \circ g = g \circ f$$

$$\text{Given } f(x) = 2x - 1, g(x) = \frac{x+1}{2}$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) - 1 \\ &= x + 1 - 1 = x \dots\dots\dots(1) \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(2x - 1) = \frac{2x-1+1}{2} = \frac{2x}{2} = x \dots\dots\dots(2) \end{aligned}$$

From (1) and (2), $f \circ g = g \circ f$

4. (i) If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a , if

$$g \circ f(a) = 1$$

$$\text{Given } f(x) = x^2 - 1, g(x) = x - 2$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(x^2 - 1) = x^2 - 1 - 2 = x^2 - 3 \end{aligned}$$

$$\text{Given } g \circ f(a) = 1$$

$$\text{Hence } a^2 - 3 = 1$$

$$a^2 = 1 + 3$$

$$a^2 = 4$$

$$a = \pm 2$$

(ii) Find k , if $f(k) = 2k - 1$ and $f \circ f(k) = 5$

$$\begin{aligned} f \circ f(k) &= f(f(k)) \\ &= f(2k - 1) \\ &= 2(2k - 1) - 1 = 4k - 2 - 1 = 4k - 3 \end{aligned}$$

$$\text{But } f \circ f(k) = 5$$

$$4k - 3 = 5$$

$$4k = 5 + 3 = 8$$

$$k = \frac{8}{4} = 2$$

5. Let $A, B, C \subseteq N$ and a function $f: A \rightarrow B$ be defined by $f(x) = 2x + 1$ and $g: B \rightarrow C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$

$$f: A \rightarrow B \text{ be defined by } f(x) = 2x + 1$$

$$g: B \rightarrow C \text{ be defined by } g(x) = x^2 \text{ and } A, B, C \subseteq N$$

$$f \circ g(x) = f(g(x)) = f(x^2) = 2x^2 + 1$$

$$\text{Range of } f \circ g = \{y/y = 2x^2 + 1, x \in N\}$$

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2$$

$$\text{Range of } g \circ f = \{y/y = (2x + 1)^2, x \in N\}$$

6. Let $f(x) = x^2 - 1$. Find

(i) $f \circ f$

$$\begin{aligned} f \circ f(x) &= f(f(x)) \\ &= f(x^2 - 1) \\ &= (x^2 - 1)^2 - 1 \\ &= x^4 - 2x^2 + 1 - 1 \\ &= x^4 - 2x^2 \end{aligned}$$

(ii) $f \circ f \circ f$

$$\begin{aligned} f \circ f \circ f(x) &= f(f(f(x))) \\ &= f(f(x^2 - 1)) \\ &= f((x^2 - 1)^2 - 1) \\ &= f(x^4 - 2x^2 + 1 - 1) \\ &= f(x^4 - 2x^2) \\ &= [x^4 - 2x^2]^2 - 1 \end{aligned}$$

7. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?

$$f: R \rightarrow R \text{ defined by } f(x) = x^5$$

$$\begin{aligned} f \circ f(x) &= f(f(x)) \\ &= f(x^5) \\ &= (x^5)^5 = x^{25} \end{aligned}$$

$$f \circ f(1) = (1)^{25} = 1$$

$$f \circ f(2) = (2)^{25}$$

$$f \circ f(3) = (3)^{25}$$

Since each elements in f have distinct images, f is one-one

$$g: R \rightarrow R \text{ defined by } g(x) = x^4$$

$$\begin{aligned} g \circ g(x) &= g(g(x)) = g(x^4) \\ &= (x^4)^4 = x^{16} \end{aligned}$$

$$g \circ g(-1) = (-1)^{16} = 1$$

$$g \circ g(1) = (1)^{16} = 1$$

$$g \circ g(2) = (2)^{16}$$

Thus two distinct elements -1 and 1 have same images. Hence g is not one-one

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(x^4) \\ &= (x^4)^5 = x^{20} \end{aligned}$$

$$f \circ g(1) = (1)^{20} = 1$$

$$f \circ g(-1) = (-1)^{20} = 1$$

Thus two distinct elements -1 and 1 have same images. Hence $f \circ g$ is not one-one

8. Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below, show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$ $f \circ g(x) = f(g(x))$ $= f(3x + 1)$ $= 3x + 1 - 1 = 3x$ Then $(f \circ g) \circ h(x) = (f \circ g)(h(x))$ $= (f \circ g)(x^2)$ $= 3x^2 \dots\dots\dots(1)$ $(g \circ h)(x) = g(h(x))$ $= g(x^2)$ $= 3x^2 + 1$ So, $f \circ (g \circ h)x = f(3x^2 + 1)$ $= 3x^2 + 1 - 1$ $= 3x^2 \dots\dots\dots(2)$ From (1) and (2), $(f \circ g) \circ h = f \circ (g \circ h)$	(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$ $f \circ g(x) = f(g(x))$ $= f(2x)$ $= (2x)^2 = 4x^2$ Then $(f \circ g) \circ h(x) = (f \circ g)(h(x))$ $= (f \circ g)(x + 4)$ $= 4(x + 4)^2$ $= 4(x^2 + 8x + 16)$ $= 4x^2 + 32x + 64 \dots\dots\dots(1)$ $(g \circ h)x = g(h(x))$ $= g(x + 4) = 2(x + 4) = 2x + 8$ So, $f \circ (g \circ h)x = f(2x + 8)$ $= (2x + 8)^2$ $= 4x^2 + 32x + 64 \dots\dots\dots(2)$ From (1) and (2) $(f \circ g) \circ h = f \circ (g \circ h)$
(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$ $f \circ g(x) = f(g(x))$ $= f(x^2) = x^2 - 4$ Then $(f \circ g) \circ h(x) = f \circ g(h(x))$ $= f \circ g(3x - 5)$ $= (3x - 5)^2 - 4$ $= 9x^2 - 30x + 25 - 4$ $= 9x^2 - 30x + 21 \dots\dots\dots(1)$	$(g \circ h)x = g(h(x))$ $= g(3x - 5) = (3x - 5)^2$ $= 9x^2 - 30x + 25$ $f \circ (g \circ h)(x) = f(9x^2 - 30x + 25)$ $= 9x^2 - 30x + 25 - 4$ $= 9x^2 - 30x + 21 \dots\dots\dots(2)$ From (1) and (2), $(f \circ g) \circ h = f \circ (g \circ h)$

9. Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$. Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from f to Z $f(x) = mx + c$ can be written $f = \{(x, mx + c) / x \in Z\}$ $f(-1) = 3$ $-m + c = 3 \dots\dots\dots(1)$ $f(0) = -1$ $c = -1 \dots\dots\dots(2)$ Substitute $C = -1$ in (1) $-m + c = 3$ $-m - 1 = 3$ $m = -1 - 3 = -4$ $\therefore f(x) = -4x - 1$	10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit $C(t) = 3t$ is linear. $C(t_1) = t$ $C(t_2) = 2t \text{ where } t = t_1 + t_2$ $C(t) = C(t_1 + t_2) = C(t_1) + C(t_2)$ $t + 2t = 3t$ $\therefore C(t) = 3t \text{ is linear}$
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Exercise 1.6

1. If
- $n(A \times B) = 6$
- and
- $A = \{1, 3\}$
- then
- $n(B)$
- is

(1) 1 (2) 2 (3) 3 (4) 6

$$n(A \times B) = 6, n(A) = 2$$

$$n(A \times B) = n(A) \times n(B)$$

$$n(B) = \frac{n(A \times B)}{n(A)} = \frac{6}{2} = 3$$

- 2.
- $A = \{a, b, q\}, B = \{2, 3\}, C = \{p, q, r, s\}$
- then
- $n[(A \cup C) \times B]$
- is

(1) 8 (2) 20 (3) 12 (4) 16

$$A \cup C = \{a, b, q\} \cup \{p, q, r, s\} = \{a, b, p, q, r, s\}$$

$$n(A \cup C) = 6$$

$$n(B) = 2$$

$$n[(A \cup C) \times B] = n(A \cup C) \times n(B) = 6 \times 2 = 12$$

3. If
- $A = \{1, 2\}, B = \{1, 2, 3, 4\}, C = \{5, 6\}$
- and
- $D = \{5, 6, 7, 8\}$
- then the state which of the following statement is true

(1) $(A \times C) \subset (B \times D)$ (2) $(B \times D) \subset (A \times C)$ (3) $(A \times B) \subset (A \times D)$ (4) $(D \times A) \subset (B \times A)$

$$A \times C = \{1, 2\} \times \{5, 6\} = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$$

$$= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

$$A \times B = \{1, 2\} \times \{1, 2, 3, 4\}$$

$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$$

$$A \times D = \{1, 2\} \times \{5, 6, 7, 8\}$$

$$= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8)\}$$

We observe that the Cartesian products $(A \times C) \subset (B \times D)$ is true

4. If there are 1024 relations from a set
- $A = \{1, 2, 3, 4, 5\}$
- to a set
- B
- , then the number of element in
- B
- is

(1) 3 (2) 2 (3) 4 (4) 8

$$\text{Number of relations} = 2^{mn}$$

$$2^{mn} = 1024$$

$$2^{5n} = 1024$$

$$2^{5n} = 2^{10}$$

$$5n = 10$$

$$n = \frac{10}{5} = 2$$

5. The range of the relations
- $R = \{(x, x^2) | x \text{ is a prime number less than } 13\}$
- is

(1) $\{2, 3, 5, 7\}$ (2) $\{2, 3, 5, 7, 11\}$ (3) **$\{4, 9, 25, 49, 121\}$** (4) $\{1, 4, 9, 25, 49, 121\}$

$$\text{Given, } R = \{(x, x^2) | x \text{ is a prime number less than } 13\}$$

$$f(x) = x^2$$

$$f(2) = 2^2 = 4$$

$$f(3) = 3^2 = 9$$

$$f(5) = 5^2 = 25$$

$$f(7) = 7^2 = 49$$

$$f(11) = 11^2 = 121$$

$$\text{Range of } R = \{4, 9, 25, 49, 121\}$$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is

(1) $(2, -2)$ (2) $(5, 1)$ (3) $(2, 3)$ **(4) $(3, -2)$**

Given, $(a + 2, 4) = (5, 2a + b)$

Now $a + 2 = 5$

$a = 5 - 2 = 3$

Then, $2a + b = 4$

$2(3) + b = 4$

$6 + b = 4$

$b = 4 - 6 = -2$

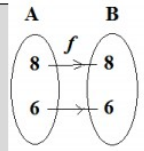
$(a, b) = (3, -2)$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

(1) m^n (2) n^m **(3) $2^{mn} - 1$** (4) 2^{mn}

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b respectively.

(1) $(8, 6)$ (2) $(8, 8)$ (3) $(6, 8)$ (4) $(6, 6)$

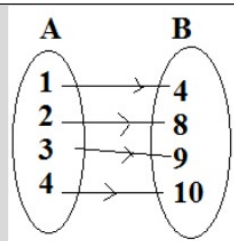


$f(x) = x, x \in A$

$\therefore (a, b) = (8, 6)$

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \rightarrow B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a

(1) Many-one function (2) Identity function
(3) One-to-one function (4) Into function



Distinct elements of A have distinct images in B . It is one -to- one function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is

(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$ **(3) $\frac{2}{9x^2}$** (4) $\frac{1}{6x^2}$

$$f \circ g = f(g(x)) = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2 = 2\left(\frac{1}{9x^2}\right) = \frac{2}{9x^2}$$

11. If $f: A \rightarrow B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to
 (1) 7 (2) 49 (3) 1 (4) 14

If f is bijective if f is both a one-one and an onto function. $n(A) = 7$

12. Let f and g be two functions given by

$$f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$$

$$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\} \text{ then the range of } f \circ g \text{ is}$$

- (1) $\{0,2,3,4,5\}$ (2) $\{-4,1,0,2,7\}$ (3) $\{1,2,3,4,5\}$ (4) $\{0,1,2\}$

x	$f(x)$	x	$g(x)$
0	1	0	2
2	0	1	0
3	-4	2	4
4	2	-4	2
5	7	7	0

- (i) $f \circ g(x) = f(g(x)) = f(g(0)) = f(2) = 0$
 (ii) $f \circ g(x) = f(g(x)) = f(g(1)) = f(0) = 2$
 (iii) $f \circ g(x) = f(g(x)) = f(g(2)) = f(4) = 2$
 (iv) $f \circ g(x) = f(g(x)) = f(g(-4)) = f(2) = 0$
 (v) $f \circ g(x) = f(g(x)) = f(g(7)) = f(0) = 1$

Range of $f \circ g = \{0,1,2\}$

13. Let $f(x) = \sqrt{1+x^2}$ then

- (1) $f(xy) = f(x) \cdot f(y)$ (2) $f(xy) \geq f(x) \cdot f(y)$
 (3) $f(xy) \leq f(x) \cdot f(y)$ (4) None of these

Put $x = 1, y = -1$

$$f(x) = \sqrt{1+x^2}$$

$$f(x) = f(1) = \sqrt{1+1^2} = \sqrt{2}$$

$$f(y) = f(-1) = \sqrt{1+(-1)^2} = \sqrt{2}$$

$$xy = (1)(-1) = -1$$

$$f(xy) = f(-1) = \sqrt{1+(-1)^2} = \sqrt{2}$$

$$f(xy) \leq f(x) \cdot f(y)$$

14. If $g = \{(1,1), (2,3), (3,5), (4,7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
 (1) $(-1,2)$ (2) $(2,-1)$ (3) $(-1,-2)$ (4) $(1,2)$

$$f = \{(x, \alpha x + \beta) / x \in R\}$$

$$\text{If } x = 1, f(x) = 1$$

$$\text{That is } \alpha + \beta = 1 \dots\dots(1)$$

$$\text{If } x = 2, f(x) = 3$$

That is $2\alpha + \beta = 3$(2)

From (1) and (2)

$$\alpha + \beta = 1$$

$$2\alpha + \beta = 3$$

$$\alpha = 2$$

Substitute $\alpha = 2$ in (1)

$$\alpha + \beta = 1$$

$$2 + \beta = 1$$

$$\beta = 1 - 2 = -1$$

$$(\alpha, \beta) = (2, -1)$$

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is

(1) linear

(2) cubic

(3) reciprocal

(4) quadratic

$$f(x) = (x + 1)^3 - (x - 1)^3$$

$$= (x^3 + 3x^2 + 3x + 1) - (x^3 - 3x^2 + 3x - 1)$$

$$= x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1$$

$$= 6x^2 + 2$$

Hence it is quadratic

MKS TAMIL

