$$\lim_{x\to\infty} \sqrt{x+1}(\sqrt{x+2}-\sqrt{x+3})$$

$$= \lim_{x\to\infty} \sqrt{x+1}(\sqrt{x+2}-\sqrt{x+3}) \cdot \frac{\sqrt{x+2}+\sqrt{x+3}}{\sqrt{x+2}+\sqrt{x+3}} \qquad \textit{Multiplying conjugate}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x+1}((x+2)-(x+3))}{\sqrt{x+2}+\sqrt{x+3}} \qquad \textit{Using } (a+b)(a-b)=a^2-b^2$$

$$= -\lim_{x\to\infty} \frac{\sqrt{x+1}}{\sqrt{x+2}+\sqrt{x+3}} \qquad \textit{Putting constants out of the limit}$$

$$= -\lim_{x\to\infty} \frac{\sqrt{x+1}}{\sqrt{x+2}+\sqrt{x+3}} \cdot \frac{1/x^2}{1/x^2}$$

$$= -\lim_{x\to\infty} \frac{\sqrt{1+1/x}}{\sqrt{1+2/x}+\sqrt{1+3/x}}$$

$$= -\frac{\sqrt{1+0}}{\sqrt{1+0}+\sqrt{1+0}} \qquad \lim_{n\to\infty} \frac{1}{n} = 0$$

$$= \boxed{-\frac{1}{2}}$$

$$\lim_{x\to 0}(1-3x^2+2x^3)^{1/x}$$

$$= \lim_{x\to 0}[(x-1)(2x^2-x-1)]^{1/x} \qquad x-1 \textit{ is a factor}$$

$$= \lim_{x\to 0}[(x-1)^2(2x+1)]^{1/x} \qquad \textit{Completely factorize}$$

$$= \lim_{x\to 0}[(1-x)^2(1+2x)]^{1/x} \qquad (a-b)^2=(b-a)^2$$

$$= \lim_{x\to 0}(1-x)^{2/x}(1+2x)^{1/x} \qquad \textit{Index laws}$$

$$= \left(\lim_{x\to 0}(1-x)^{2/x}\right)\left(\lim_{x\to 0}(1+2x)^{1/x}\right) \qquad \lim(ab)=\lim(a)\lim(b)$$

$$= \left(\lim_{y\to\infty}\left(1+\frac{-2}{y}\right)^{y}\right)\left(\lim_{z\to\infty}\left(1+\frac{2}{z}\right)^{z}\right) \qquad \textit{Substitute } y=\frac{2}{x}, z=\frac{1}{x}$$

$$= (e^{-2})(e^{2}) \qquad \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^{n} = e^{x}$$

$$= \boxed{1}$$