Relativistic Quantum Information and Metrology

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Thanks to people in my group that contributed

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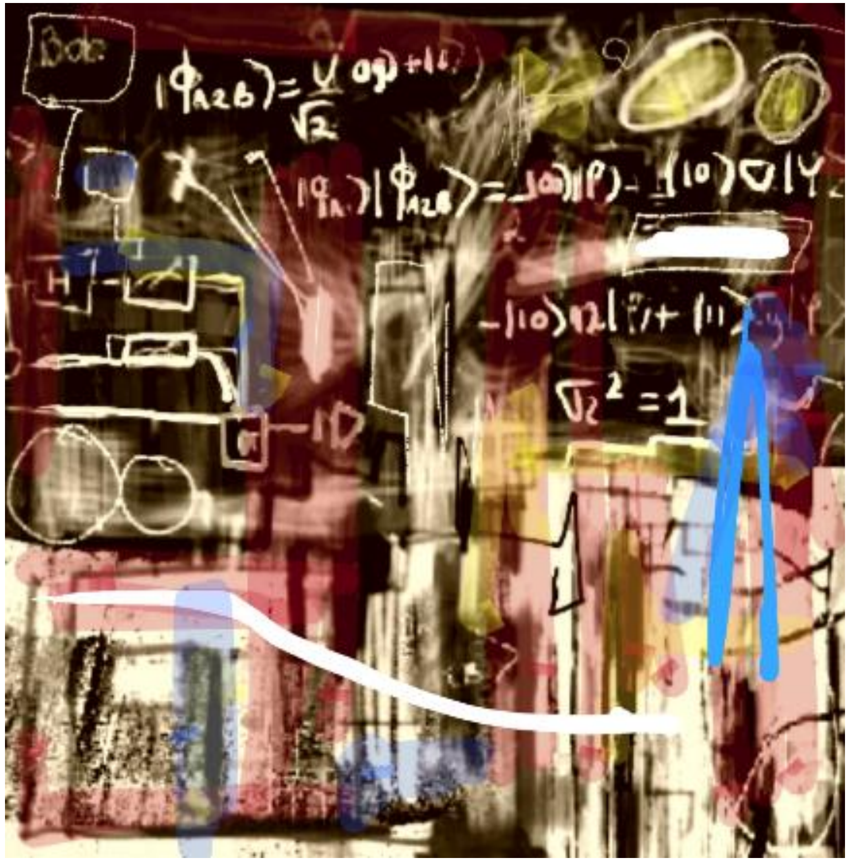
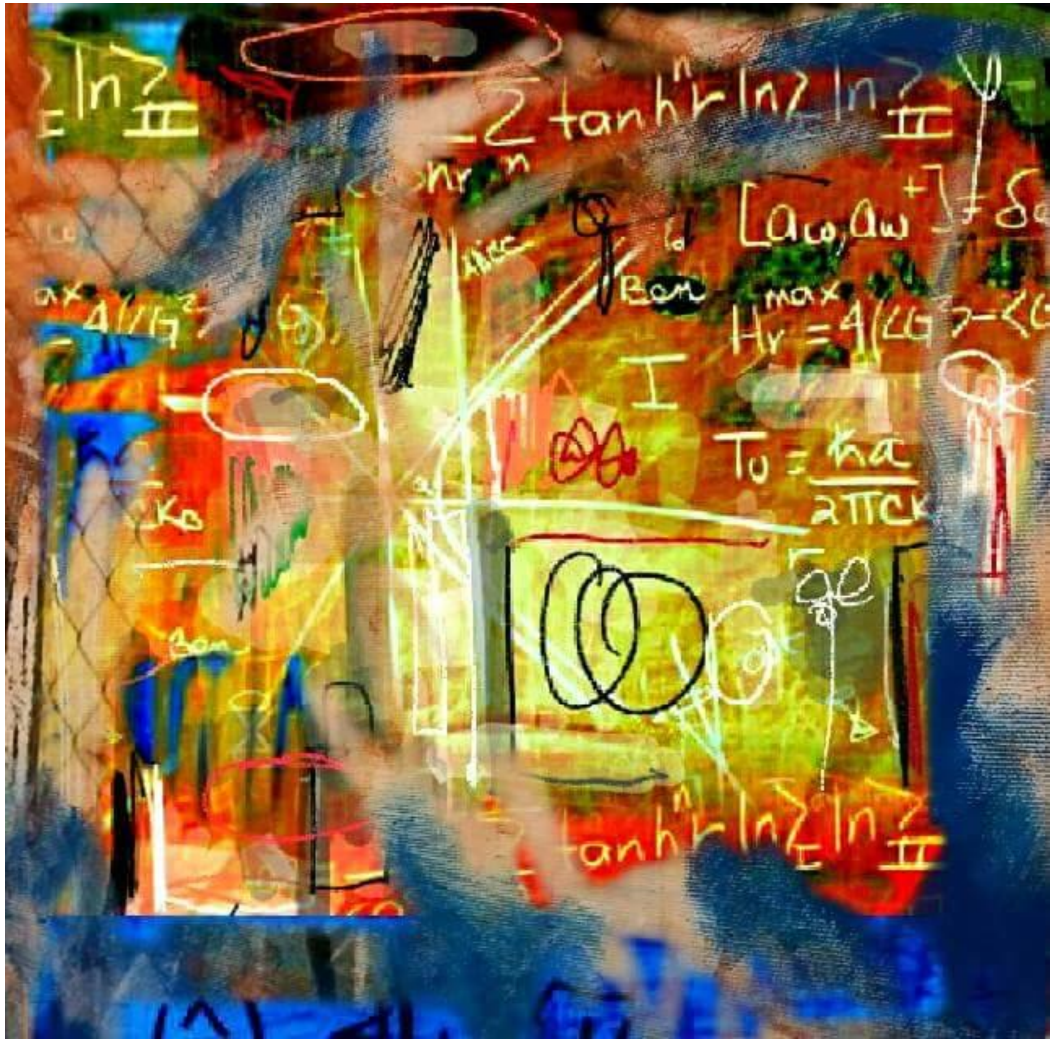
Syllabus for PG course

1. QUANTUM INFORMATION & METROLOGY
2. GENERAL RELATIVITY
3. QUANTUM FIELD THEORY IN CURVED SPACETIME
4. ENTANGLEMENT IN FLAT AND CURVED SPACETIME
5. COVARIANCE MATRIX FORMALISM
6. SYSTEMS FOR RQI
7. RELATIVISTIC QUANTUM METROLOGY

- Motivation
- Technical tools
    - entanglement
    - quantum field theory
    - covariance matrix formalism
    - quantum metrology
    - BECs on spacetime
- Applications
    - Entanglement in flat and curved spacetime
    - Metrology: phononic detector concept for gravity
    - Clocks at the interface of quantum physics and General Relativity



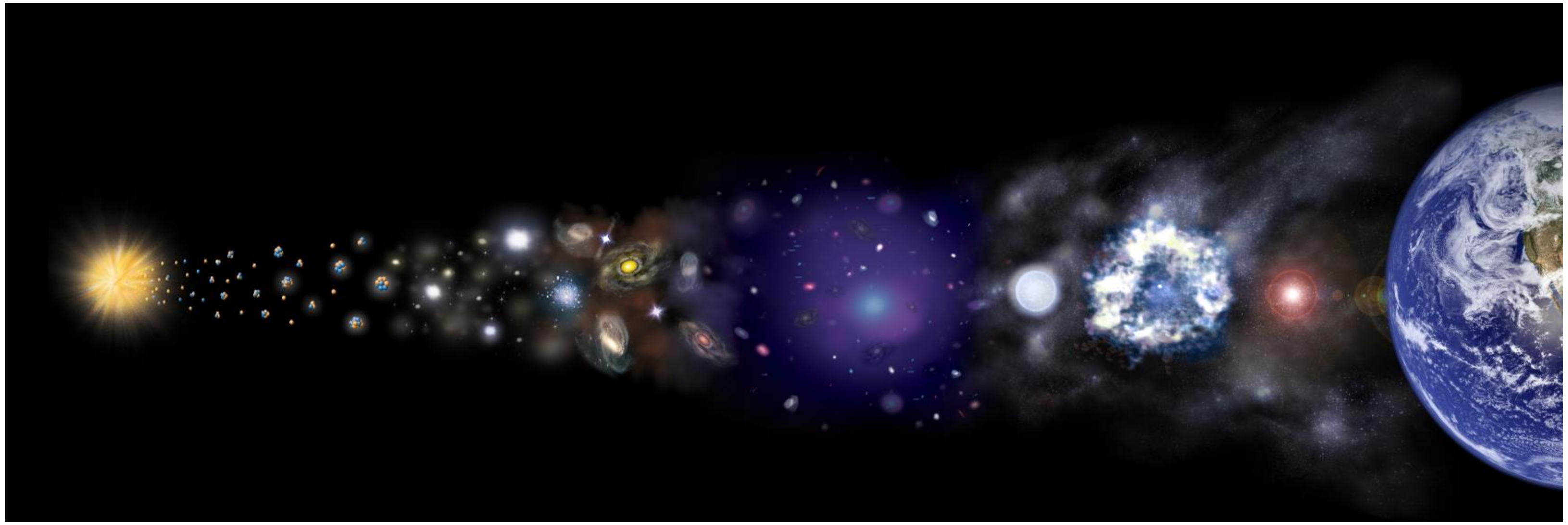
Motivation and background

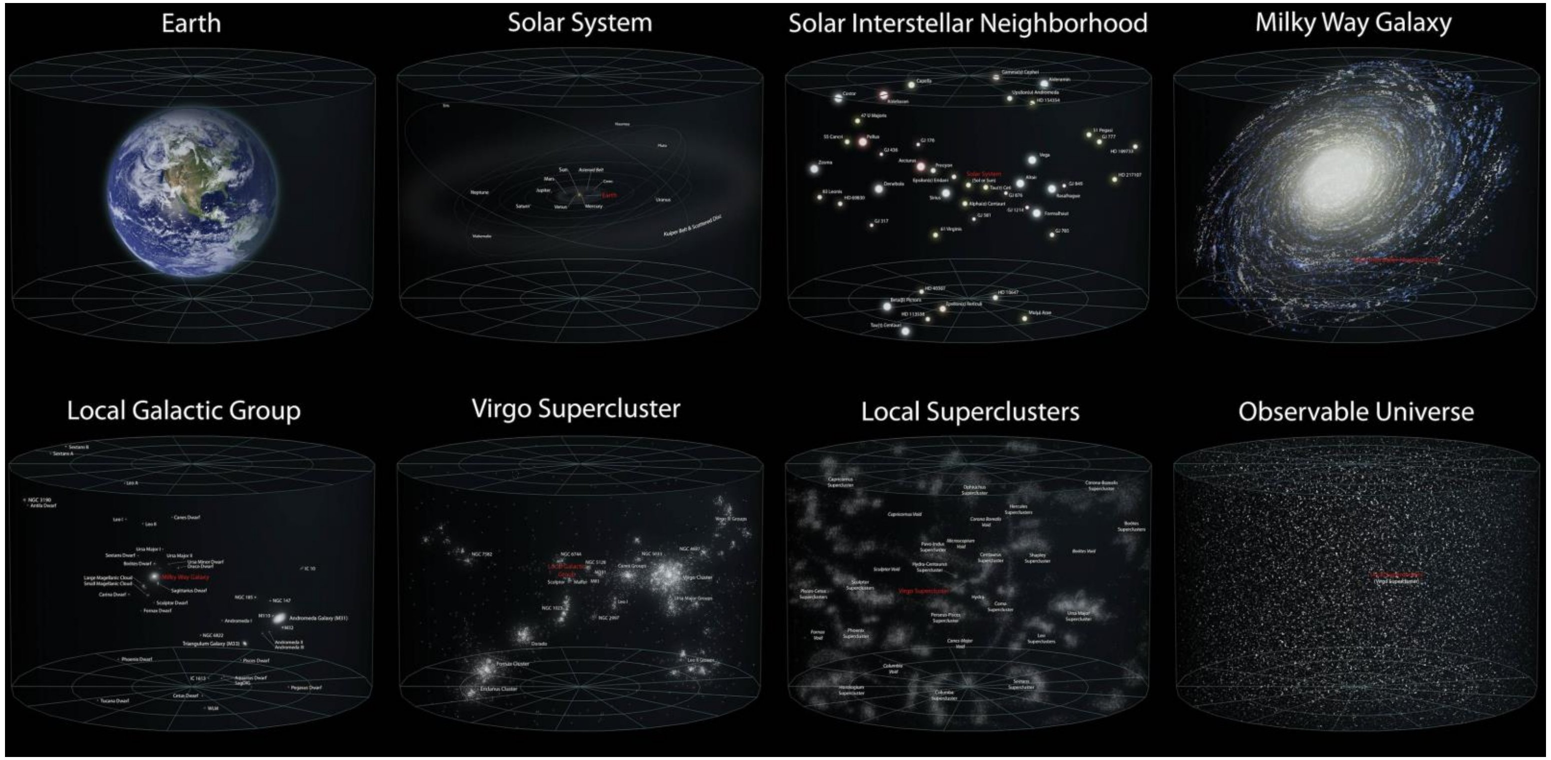


The quantum era reached relativistic regimes

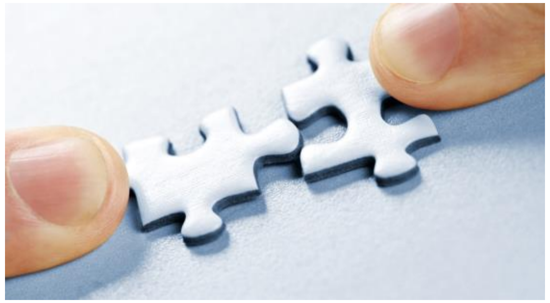
- Practical aspects (necessary corrections)
- Innovation: new technologies
- Fundamental aspects

Quantum Physics
Small scales





General Relativity
Large Scales



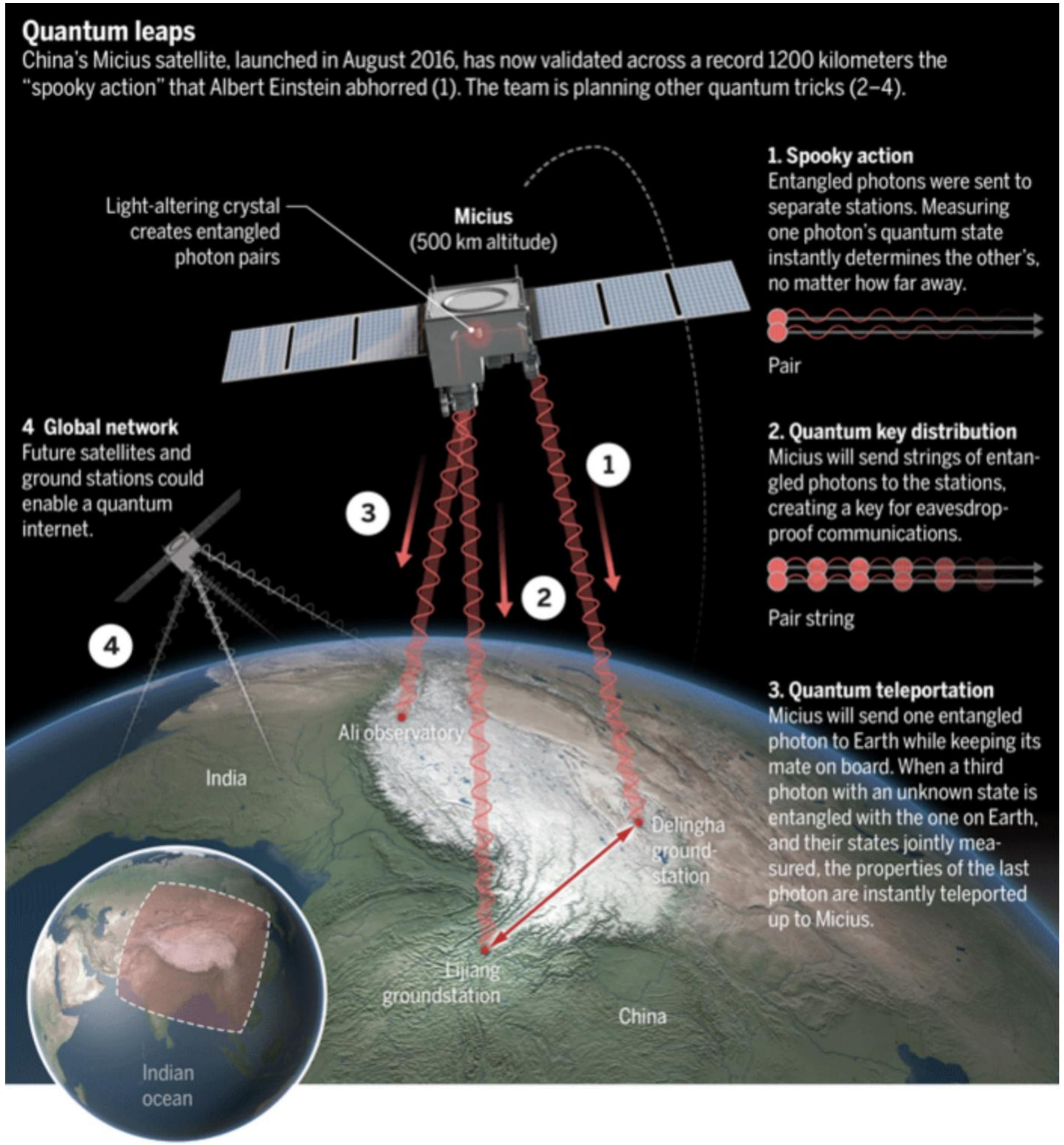
RESEARCH ARTICLE | PHYSICAL SCIENCES

Teleportation of entanglement over 143 km

Thomas Herbst, Thomas Scheidl, Matthias Fink, +3, and Anton Zeilinger Authors Info & Affiliations

Contributed by Anton Zeilinger, August 27, 2015 (sent for review July 14, 2015; reviewed by Franco Nori and Gregor Weihs)

November 2, 2015 | 112 (46) 14202-14205 | https://doi.org/10.1073/pnas.1517007112



CREDITS: (GRAPHIC) C. BICKEL/*SCIENCE*; (DATA) JIAN-WEI PAN

China's quantum satellite demonstrates entanglement at distances of $\sim 10^3$ km

GPS: At these regimes relativity kicks in!

What are the effects of gravity and motion on quantum properties?

Spacetime effects on satellite-based quantum communications

David Edward Bruschi, Timothy C. Ralph, Ivette Fuentes, Thomas Jennewein, and Mohsen Razavi
Phys. Rev. D **90**, 045041 – Published 28 August 2014

Testing the effects of gravity and motion on quantum entanglement in space-based experiments

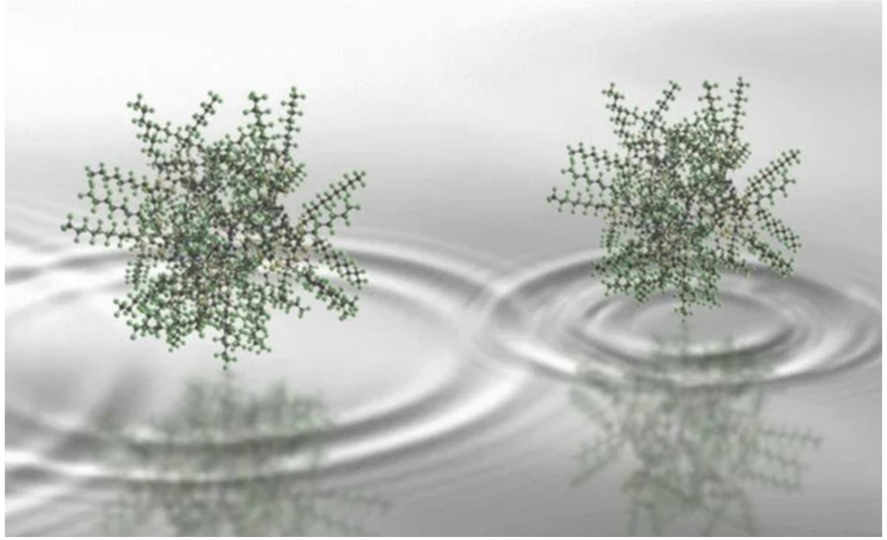
David Edward Bruschi[1], Carlos Sabín[2], Angela White[3,6], Valentina Baccetti[4], Daniel K L Oi[5] and Ivette Fuentes[2]

Published 21 May 2014 • © 2014 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 16, May 2014

How massive can a system be in a quantum superposition?

Molecules: Markus Arndt



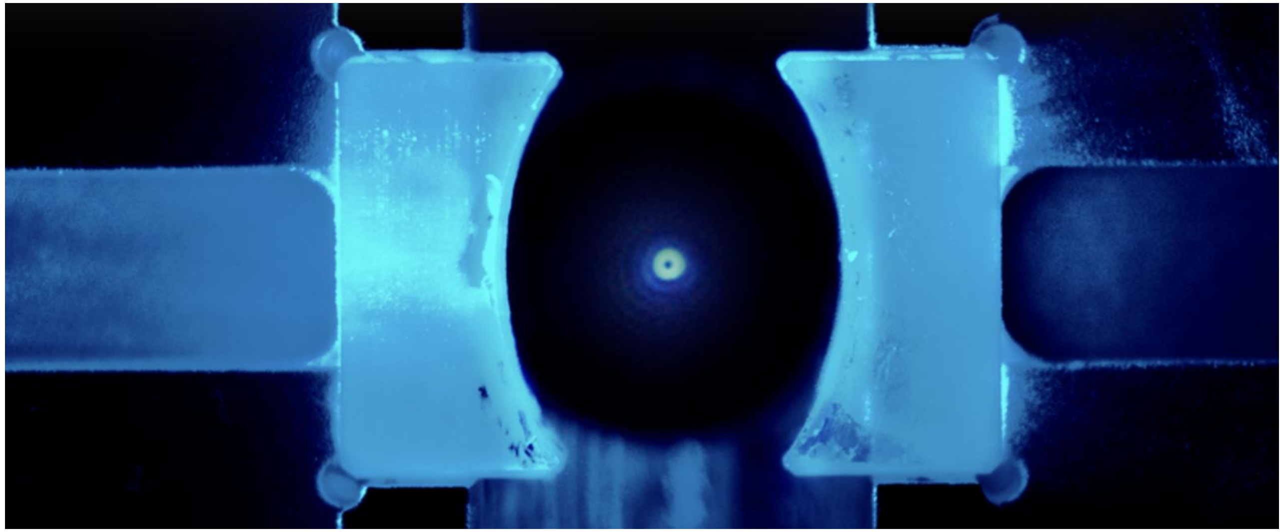
Letter | Published: 23 September 2019

Quantum superposition of molecules beyond 25 kDa

Yaakov Y. Fein, Philipp Geyer, Patrick Zwick, Filip Kiałka, Sebastian Pedalino, Marcel Mayor, Stefan Gerlich & Markus Arndt

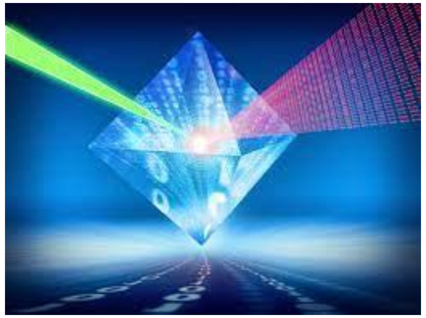
Nature Physics **15**, 1242–1245 (2019) | Cite this article

12k Accesses | **89** Citations | **587** Altmetric | Metrics



Nano-particles: Markus Aspelmeyer ground state 10^8 atomic masses

Diamonds



Bose-Einstein Condensate: 10^{10} atoms

New Journal of Physics
The open access journal at the forefront of physics

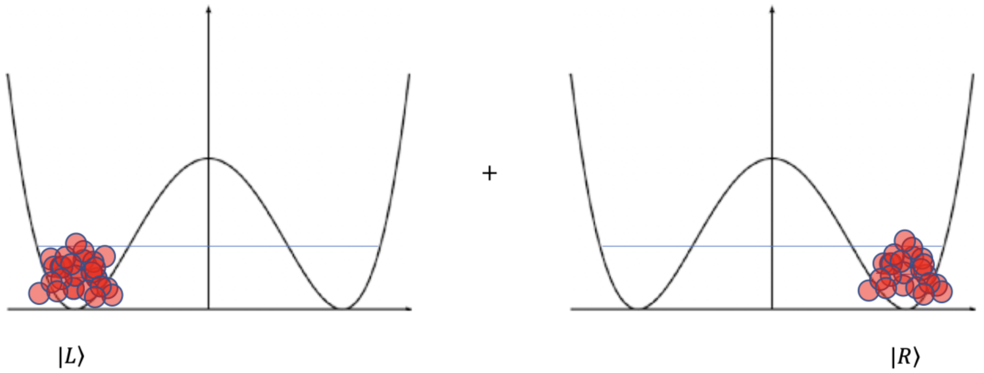
PAPER • OPEN ACCESS

Exploring the unification of quantum theory and general relativity with a Bose–Einstein condensate

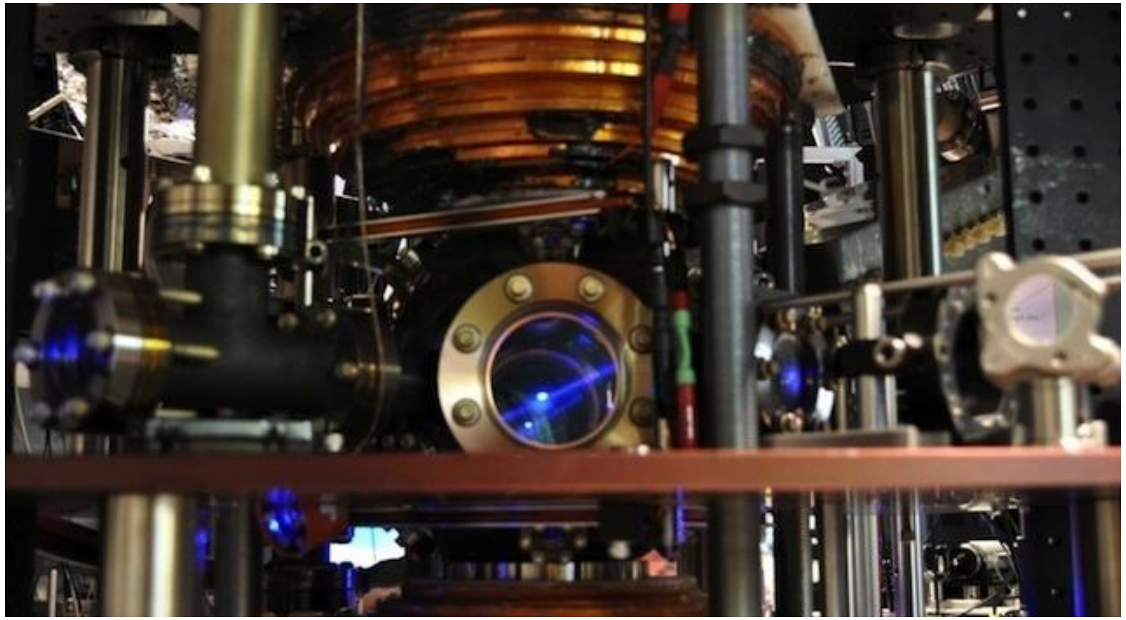
Richard Howl[1], Roger Penrose[2] and Ivette Fuentes[1]

Published 25 April 2019 • © 2019 The Author(s). Published by IOP Publishing Ltd on behalf of the Institute of Physics and Deutsche Physikalische Gesellschaft
New Journal of Physics, Volume 21, April 2019
Citation Richard Howl *et al* 2019 *New J. Phys.* **21** 043047

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N_L 0_R\rangle + |0_L N_R\rangle\right)$$



Massive system
Can reach picoKelvin temperatures and exhibit quantum behavior



Precision of quantum clocks

Article | Published: 16 February 2022

Resolving the gravitational redshift across a millimetre-scale atomic sample

Tobias Bothwell, Colin J. Kennedy, Alexander Aeppli, Dhruv Kedar, John M. Robinson, Eric Oelker, Alexander Staron & Jun Ye

Nature **602**, 420–424 (2022) | Cite this article

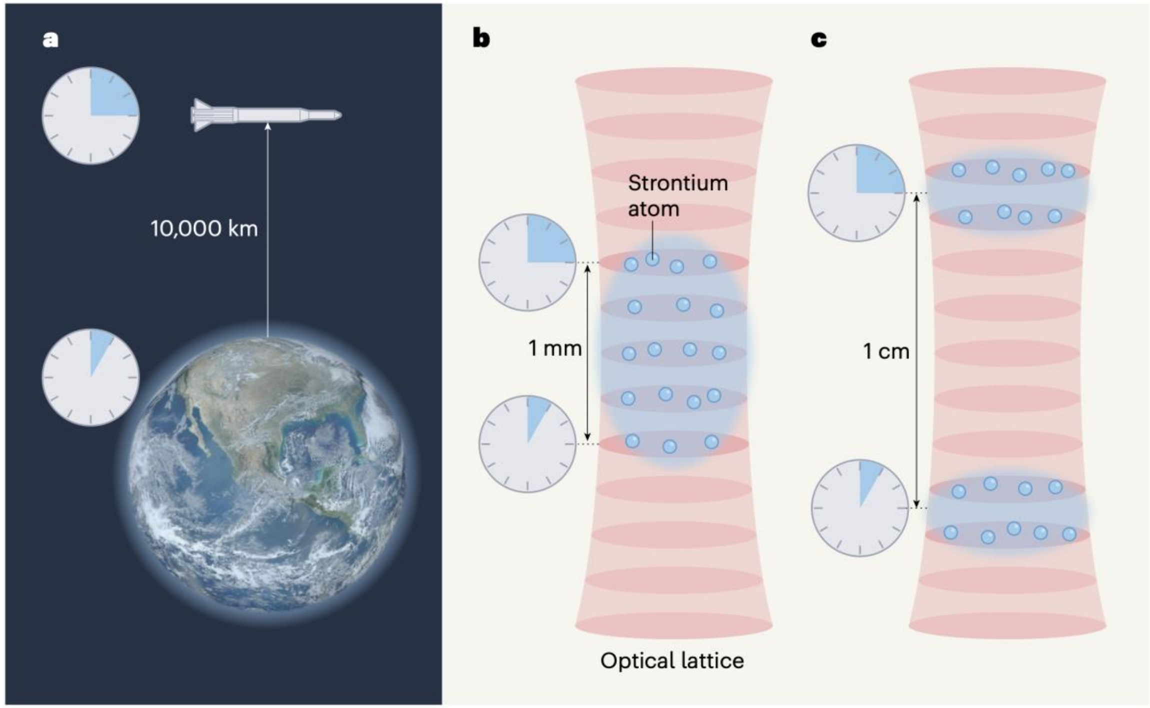
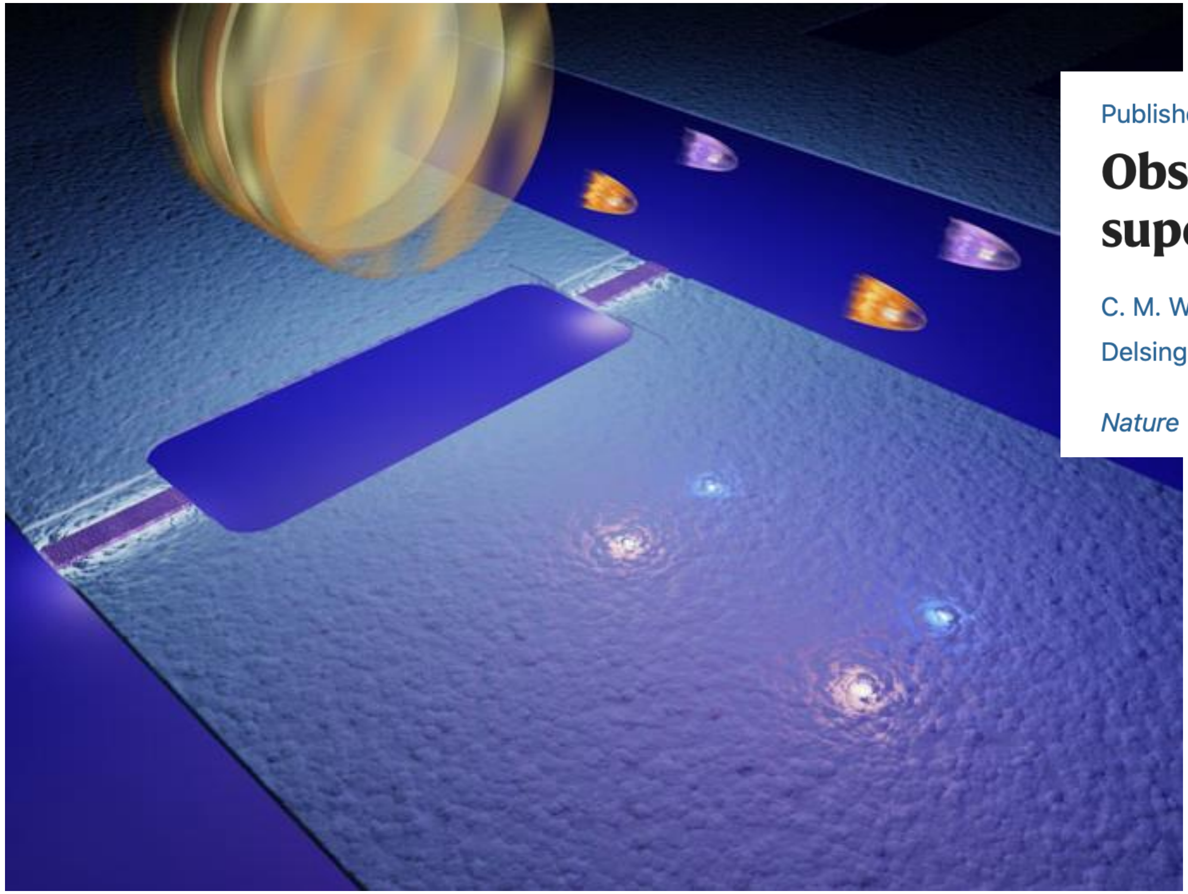


Figure 1 | Measuring time differences in vertically separated clocks. a, The Gravity Probe A experiment[3] measured gravitational redshift (a metric for how gravity changes time) using two clocks separated by a vertical distance of 10,000 kilometres – one was on a spacecraft and the other remained on Earth's surface. The clock on the spacecraft ran faster than the clock on Earth. **b**, Bothwell *et al.*[1] showed that it is possible to measure gravitational redshift even on the submillimetre scale, by probing the timing of electronic transitions in a single cloud of strontium atoms trapped in an optical lattice (formed by the interference pattern of lasers). This required the team to measure an effect that was 20 billion times less pronounced than that detected in the Gravity Probe A experiment. **c**, Zheng *et al.*[2] demonstrated a similar set-up for such measurements using clouds of strontium atoms separated by one centimetre.

Relativistic effects in quantum fields



Published: 16 November 2011

Observation of the dynamical Casimir effect in a superconducting circuit

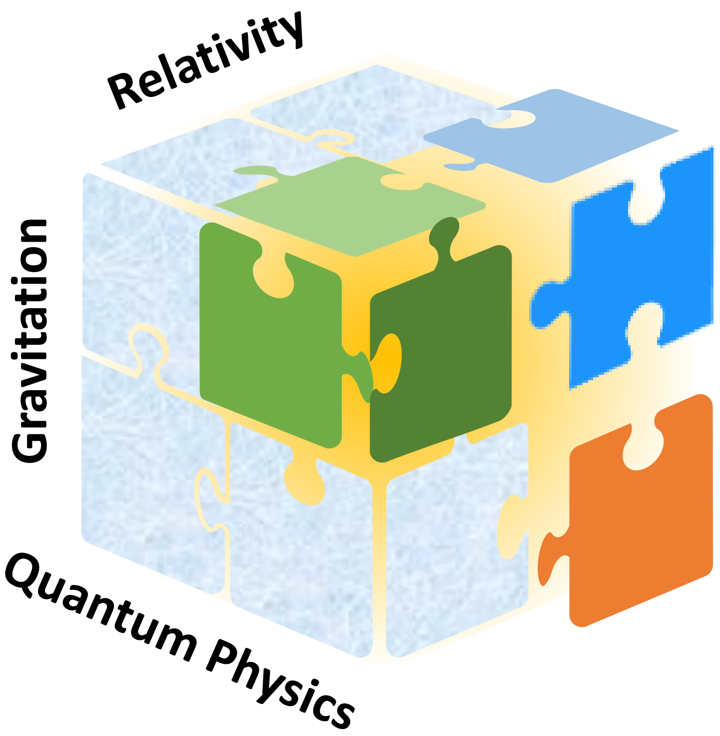
C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori & P. Delsing

Nature **479**, 376–379 (2011) | Cite this article

Quantum effects in time dilation

Testing QFT: particle creation by a moving boundary

Quantum physics and General Relativity are incompatible



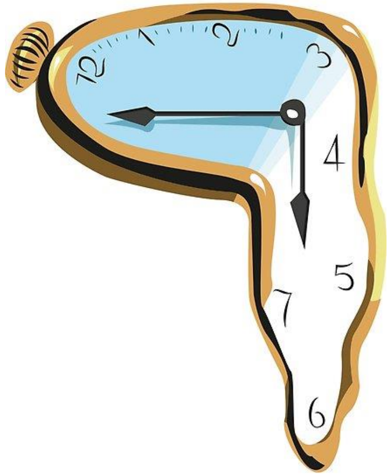
Quantize gravity or "gravitize" quantum theory?

Quantum theory

Time is absolute

Space and time are different notions

Particles can be in a superposition of positions at once.



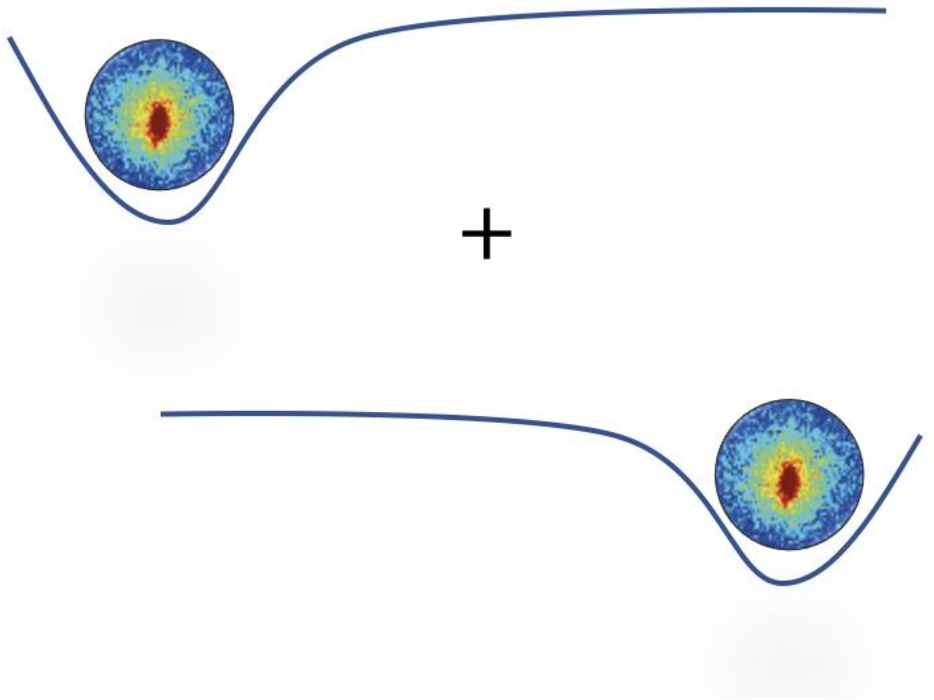
Relativity

Time is observer dependent

Space and time belong together: spacetime

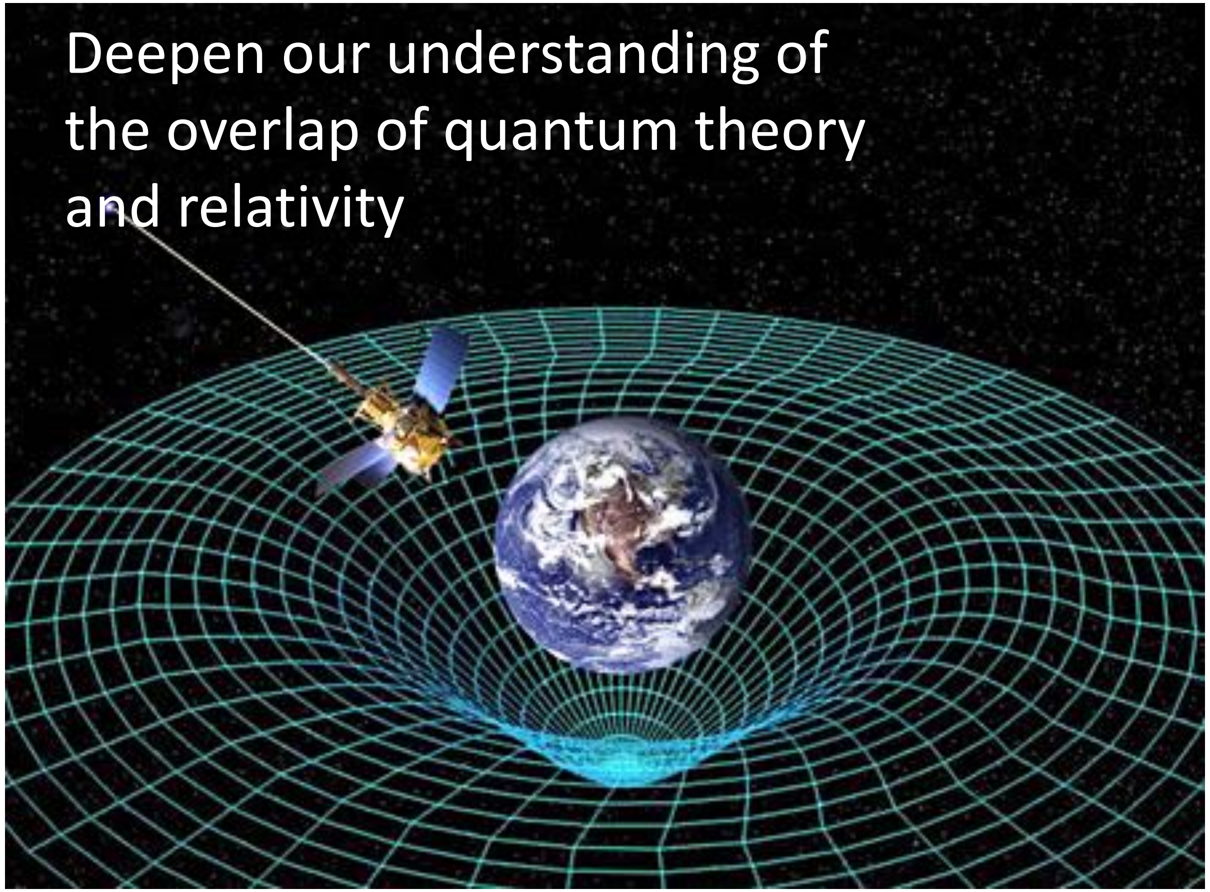
Time flows at different rates in different points in space

Quantize gravity or “gravitize” quantum theory?



Can spacetime be in a superposition of different configurations?

Future relativistic quantum technologies



How relativity affects quantum technologies?

Can relativistic effects help?

New generation of Gravimeters, sensors, clocks

Technical tools

ENTANGLEMENT

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

entangled pair

Quantifying entanglement

PURE STATES:

Schmidt basis

$$|\Phi\rangle_{AB} = \sum_{ij} \omega_{ij} |i\rangle_A \otimes |j\rangle_B \Rightarrow |\Phi\rangle_{AB} = \sum_n \omega_n |n\rangle_A \otimes |n\rangle_B$$

Measure of entanglement: use density matrix $\rho_{AB} = |\Phi\rangle\langle\Phi|_{AB}$

Reduced density matrix (subsystem A) $\rho_A = \mathbf{Tr}_B(\rho_{AB})$

Entanglement between A and B $S(\rho_A) = S(\rho_B)$

von Neumann entropy $S(\rho) = -\mathbf{Tr}(\rho \log_2(\rho))$

MIXED STATES ➡ no analogue to Schmidt decomposition

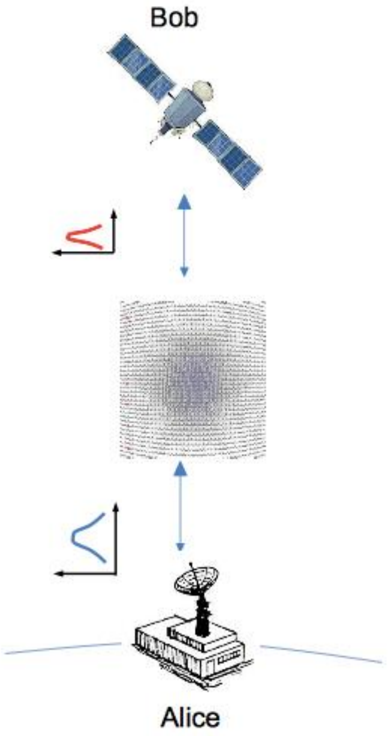
(entropy no longer quantifies entanglement)

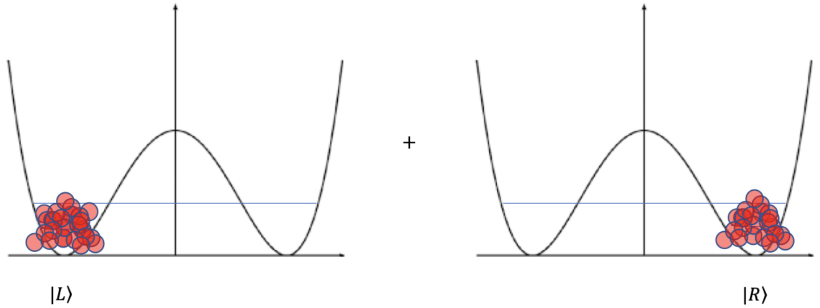
but necessary condition for separability (no negative eigenvalues) suggest to use

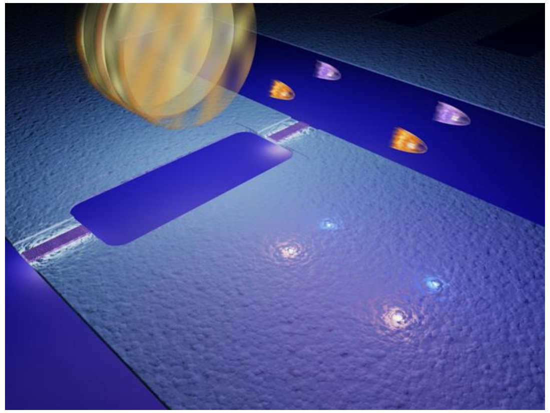
negativity = sum of negative eigenvalues of ρ_{AB}^{PT}

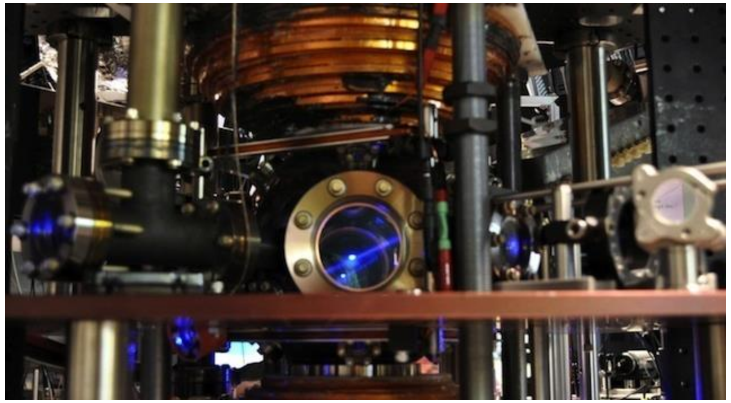
Entanglement and Relativity

How do we quantify entanglement?









Quantum field theory in curved spacetime

Classical spacetime + quantum fields

Combines QT with GR at low energies scales reachable by cutting-edge experiments

Theoretical predictions

Particle creation by spacetime dynamics

Hawking radiation

Davies-Fulling-Unruh effect

PARTICLES FROM FIELDS

Quantum field theory on curved spacetime

↓

Quantum field fundamental • Particles derived notion (if at all)

QUANTUM FIELD THEORY

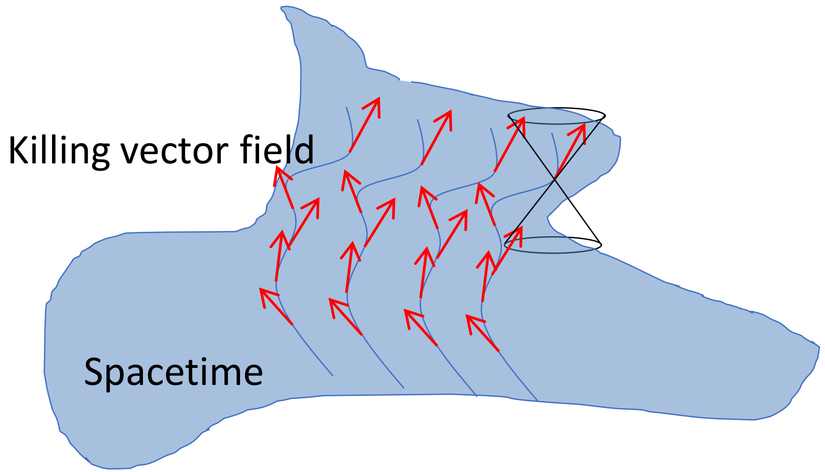
linear field equation $(\Box + m^2)\Phi = 0$

↓

vectorspace of solutions $\mathrm{span}\{\tilde{\Phi}_k\}$

$\tilde{\Phi} \in \mathcal{H} \quad (\phi, \psi) = i \int_\Sigma d\Sigma^\mu [\psi^* \partial_\mu \phi - \phi \partial_\mu \psi^*]$

Inner product not positive definite!



PARTICLE INTERPRETATION

requires classification of modes into positive ↔ negative frequency ω

$$i\partial_T \tilde{\Phi} = \omega \tilde{\Phi} \quad \begin{cases} \tilde{\Phi}^* & \text{pos.} \\ \Updownarrow & \\ \tilde{\Phi} & \text{neg.} \end{cases}$$

↑ timelike Killing vector field

$$\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus \mathcal{H} \circ \mathcal{H} \oplus \ldots \oplus \mathcal{H}^n$$

boson Fock space

$$\phi(x) = \int dk \left[a_k^+ \tilde{\Phi}_k + a_k^- \tilde{\Phi}_k^* \right]$$

↑ creation ↑ annihilation

Massless scalar fields in curved spacetime

field equation for massless fields:
Klein Gordon

$$\Box\Phi = 0$$

determinant of the metric

$$\Box = (\sqrt{-g})^{-1}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$$

metric

When the spacetime admits a time-like Killing vector field

solutions

$$\Phi = \int \phi_n a_n + \phi_n^* a_n^\dagger$$

creation and annihilation operators

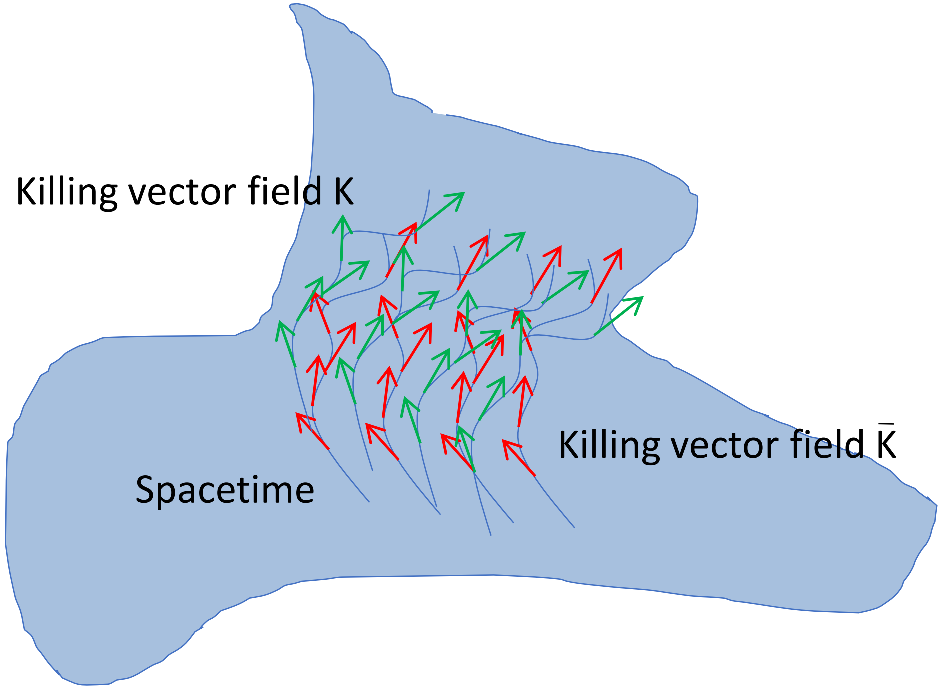
$\hat{a}_n|0\rangle = 0$ vacuum state

$|m_n\rangle = \hat{a}_n^{\dagger\, m}|0\rangle$ Particle states

KILLING OBSERVERS

INSIGHTS

- particles present ill-defined subsystems!
- particles well-defined only for killing observers
- particle interpretation may change with change of Killing vector field



KILLING OBSERVERS

different timelike Killing vectors K and $\bar{K}$

$\Rightarrow$ different splits of basis in pos/neg

$$\{u_p, u_p^*\} \longrightarrow \{\bar{u}_p, \bar{u}_p^*\}$$

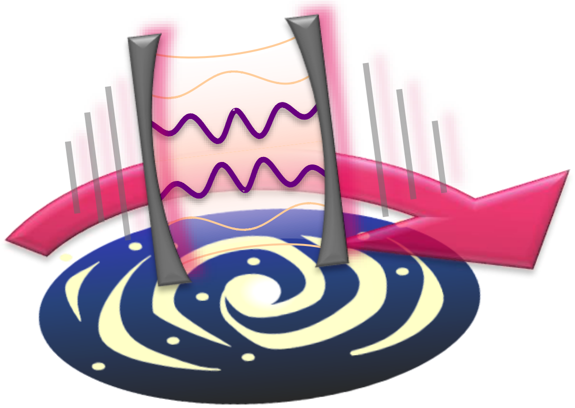
Bogoliubov transformation

$$\bar{a}_p = \int_{q\in\mathcal{P}} [\alpha_{pq}^* a_q - \beta_{pq}^* a_q^\dagger],$$

Squeezed states

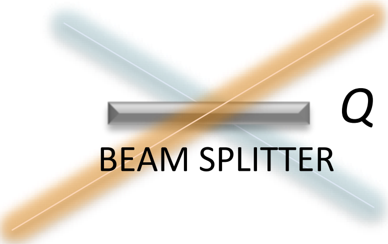
$$|0\rangle = e^{\sum_{i\neq j} \gamma_{ij} a_i^\dagger a_j^\dagger - \gamma_{ij}^* a_i a_j} |\bar{0}\rangle$$

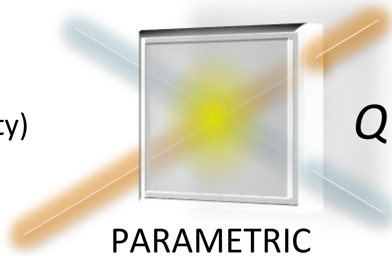
Bogoliubov transformations



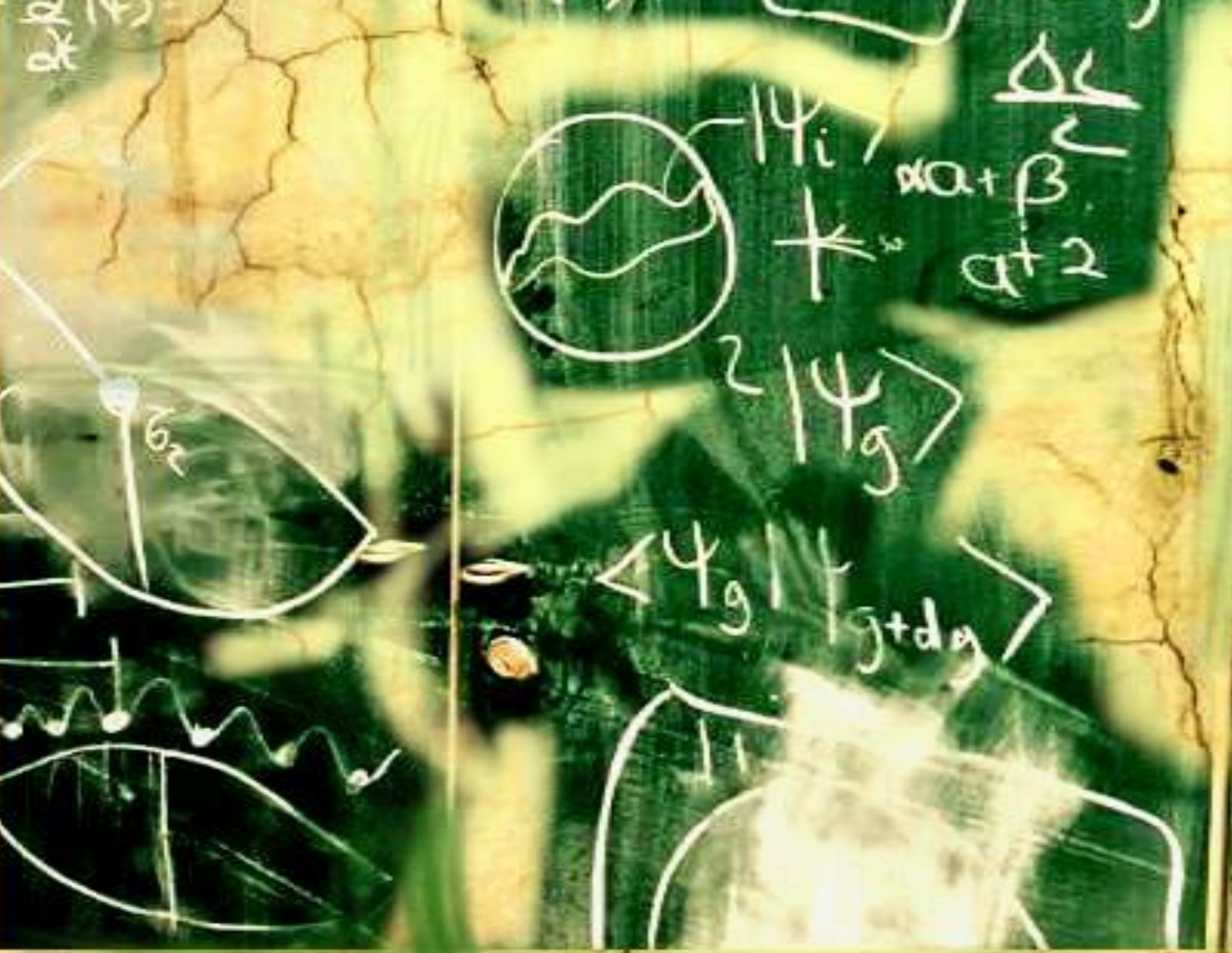
Bogoliubov transformation

- Realizes a linear transformation of the modes: $\tilde{a}_m = \sum_n \left(\alpha_{mn} a_n + \beta^*_{mn} a^\dagger_n \right)$
- *Alphas*: passive terms (beam-splitter like)
- *Betas*: active terms (two-mode squeezers)





Examples: change of observer, space-time dynamics, moving cavity



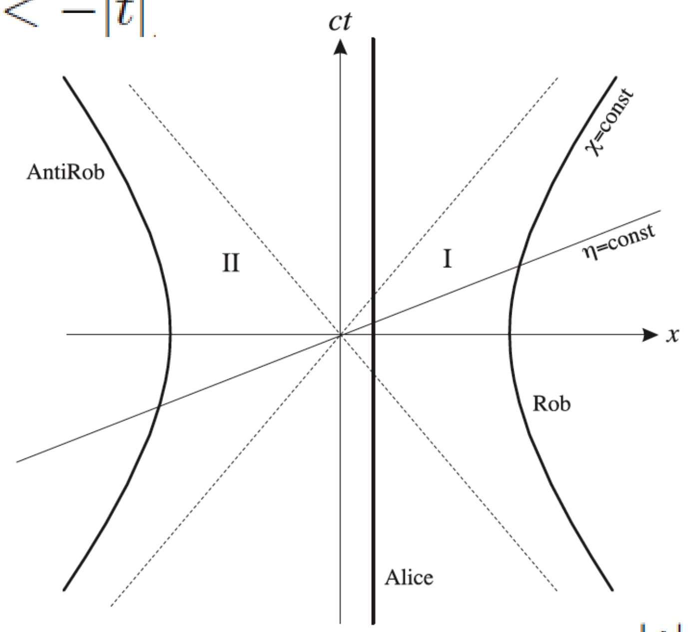
Entanglement in flat and curved spacetime

Ivette Fuentes- University of Southampton

Minkowski and Rindler coordinates

Minkowski coordinates (x,t): inertial observers
Rindler coordinates (χ,η): accelerated observers

$x < -|t|$



$|t| < x$

$$\eta = \operatorname{atanh}\left(\frac{t}{x}\right), \quad \chi = \sqrt{x^2 - t^2},$$

$$0 < \chi < \infty \text{ and } -\infty < \eta < \infty$$

$$\chi = 1/a,$$

proper acceleration

<u>Timelike killing observers</u>

(a) inertial observer

(b) uniformly accelerated observers

Example: Klein-Gordon equation in flat spacetime

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$ wave equation

In 1+1 dimensions $(\partial_t^2 - \partial_x^2)\phi = 0.$ c=1

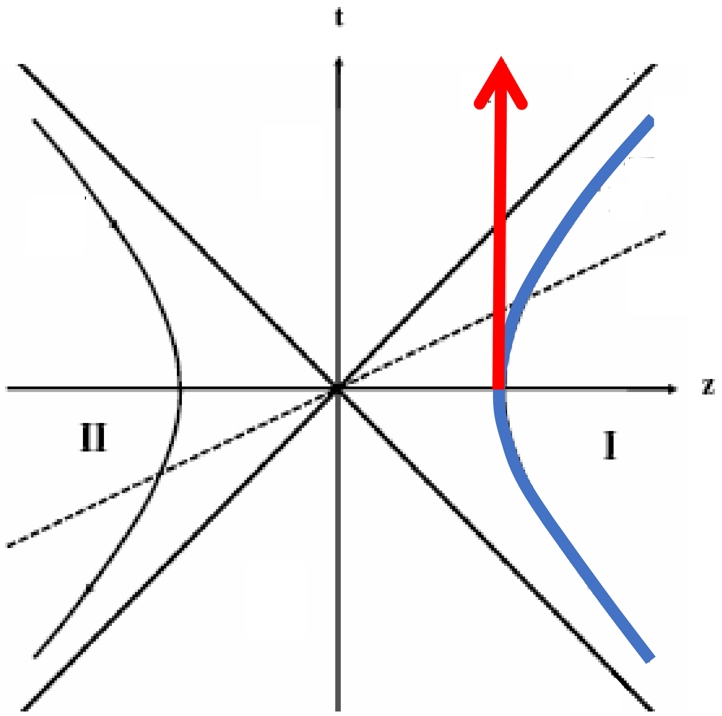
The solutions to this equation are plane waves

$$u_k = \frac{1}{\sqrt{2\pi\omega}} e^{i(kx-\omega t)}$$
$$u_k^* = \frac{1}{\sqrt{2\pi\omega}} e^{-i(kx-\omega t)}$$

with $\omega = |k|$ and $-\infty < k < \infty.$

EXAMPLE: UNRUH EFFECT

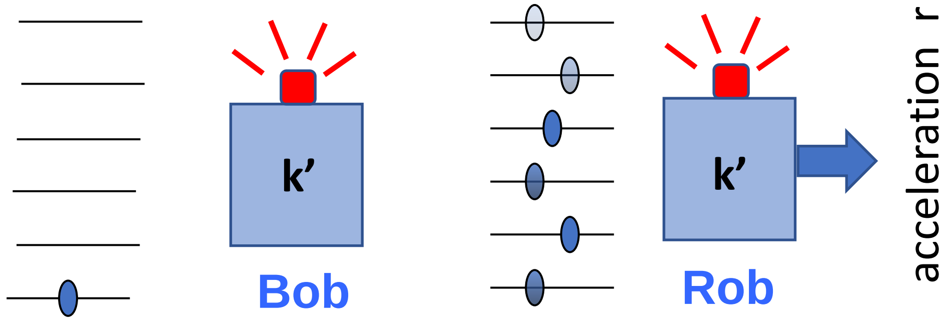
Minkowski spacetime in 1+1 dimensions (flat spacetime = no gravity!)



$$|0_k\rangle^{\mathcal{M}} \sim \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n_k\rangle_I |n_k\rangle_{II},$$

$$\cosh r = (1 - e^{-2\pi\Omega})^{-1/2}, \qquad \Omega = |k|c/a$$

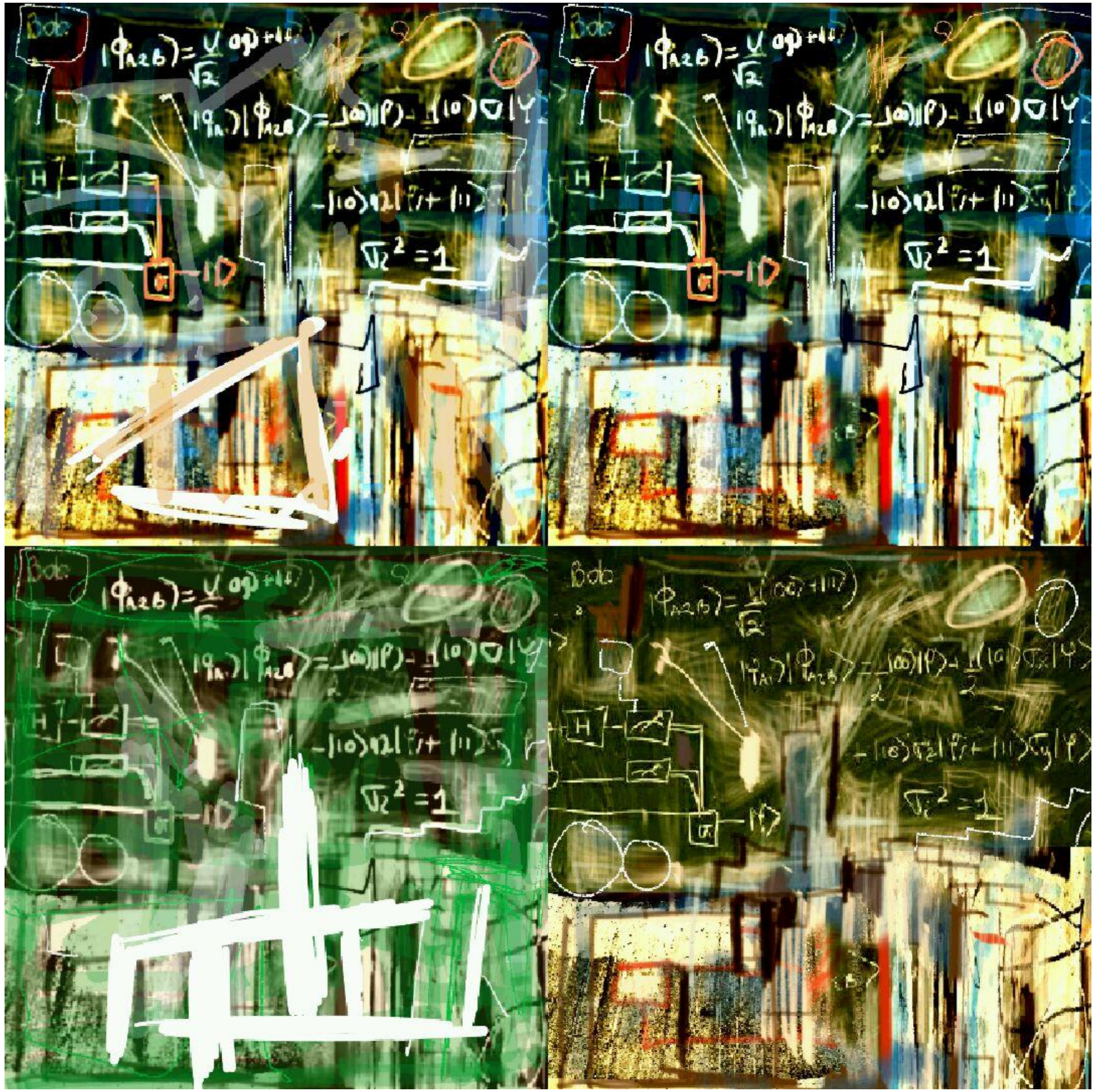
Rob is causally disconnected from region II



trace ⇨ thermal state

Similar effect in black holes: Hawking radiation

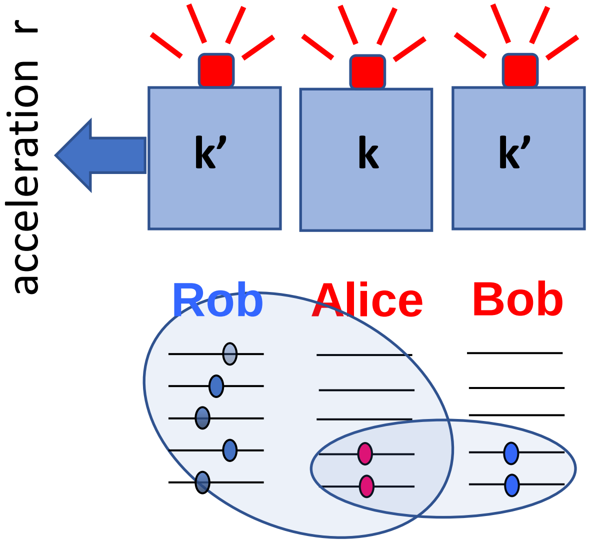
some results

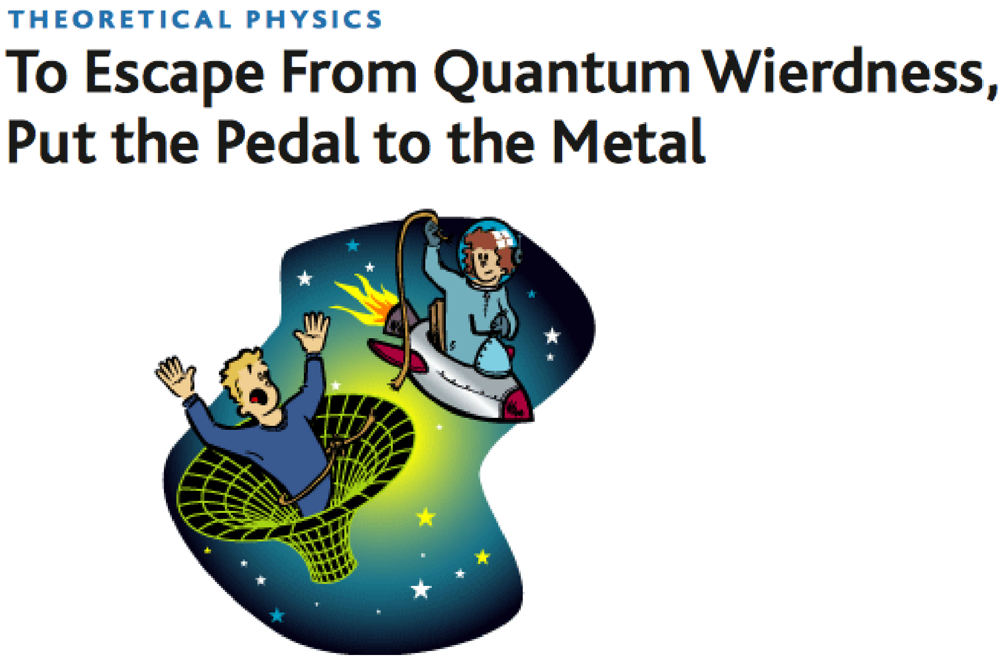


FLAT SPACETIME

Alice and Rob

I. Fuentes-Schuller & R. Mann PRL (2005)





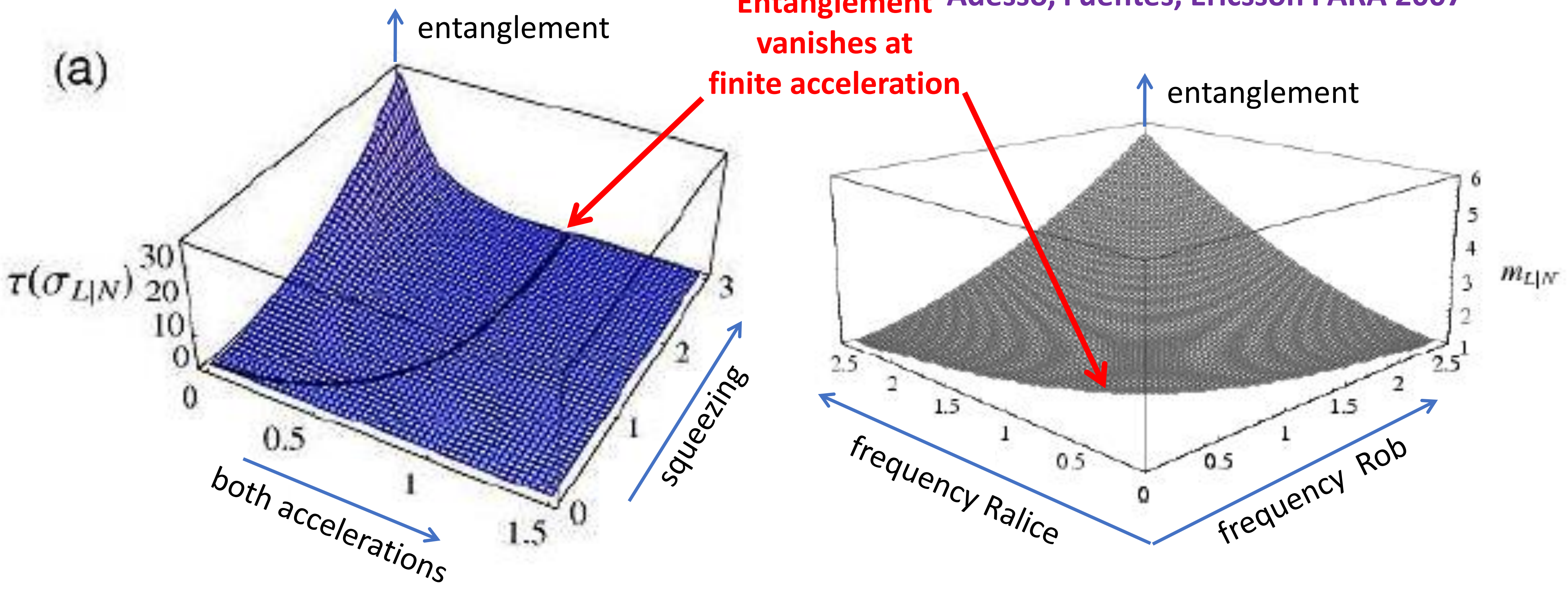
A. Cho, Science (2005)

Entanglement
- observer-dependent
- degrades with acceleration , vanishes for ∞ acceleration

Ralice and Rob

TWO ACCELERATED OBSERVERS (same direction, same acceleration)

Adesso, Fuentes, Ericsson PARA 2007



fixed detection frequencies k , k'

fixed acceleration & squeezing

Entanglement very fragile (gravity in the lab!) but high frequencies help

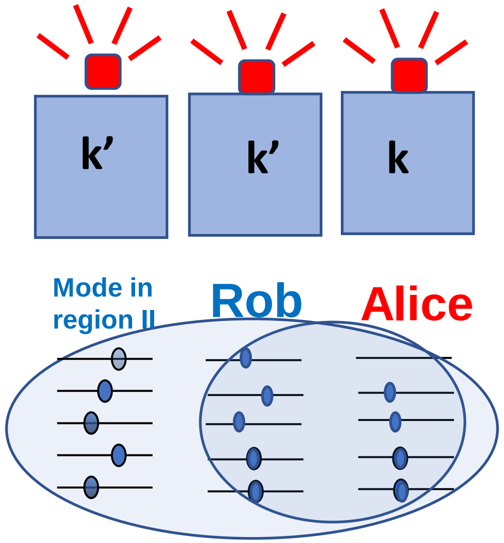
Entanglement sharing

Where did the lost entanglement between Alice and Bob go?

$$|0_k\rangle^{\mathcal{M}} \sim \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r |n_k\rangle_I |n_k\rangle_{II},$$

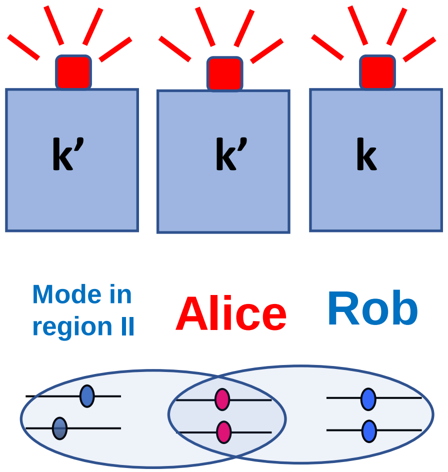
Alsing, Fuentes-S, Mann, Tessier PRA 2006
Adesso, Fuentes-S , Ericsson PRA 2007

Bosonic field



multipartite entanglement!

Fermionic field



Bipartite entanglement between Alice and mode in region II

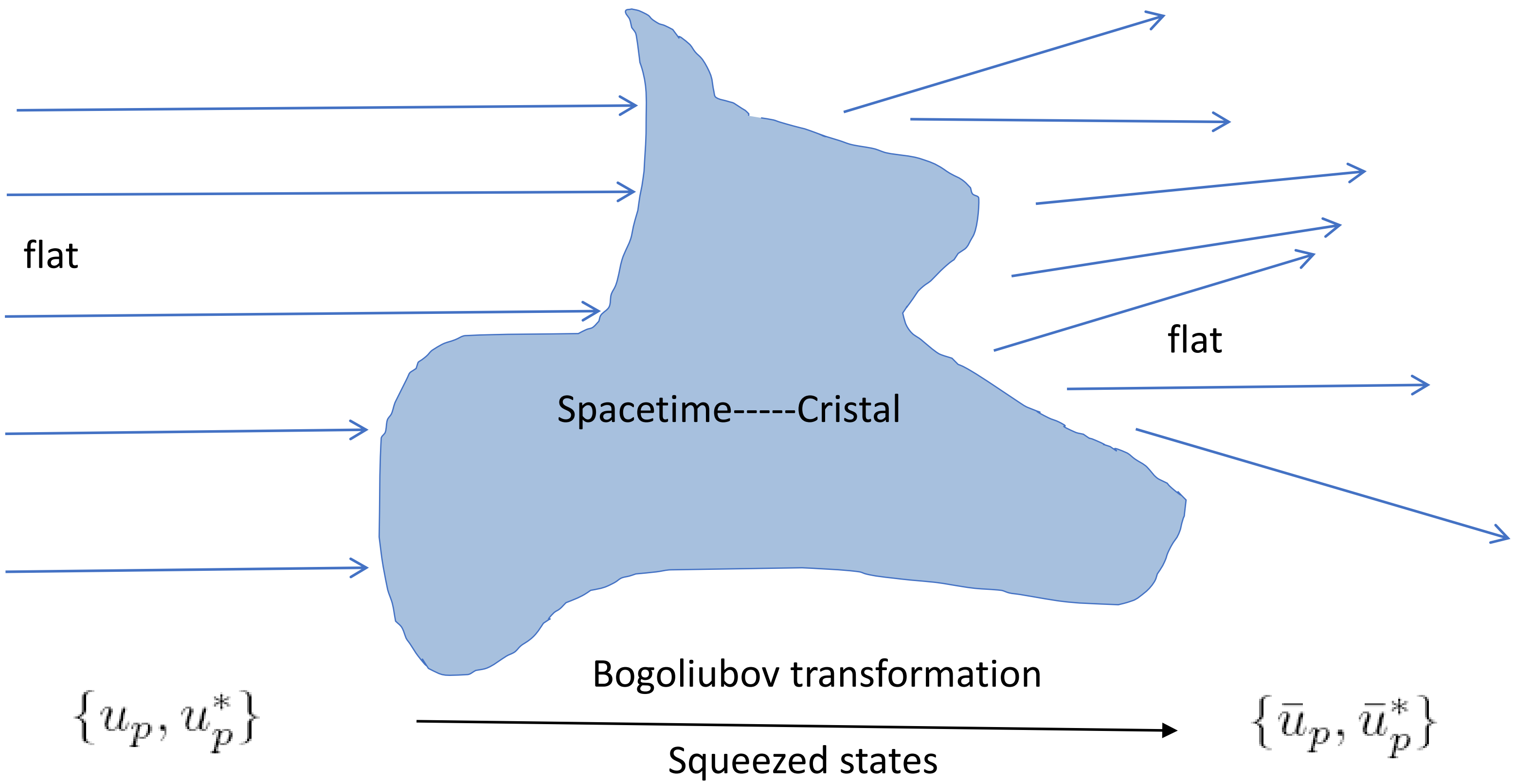
Again important differences between fermions and bosons.

CURVED SPACETIME

SPACETIME AS A CRISTAL

Curve spacetimes generally do not admit timelike killing vector fields…

particular spacetimes with asymptotically flat regions

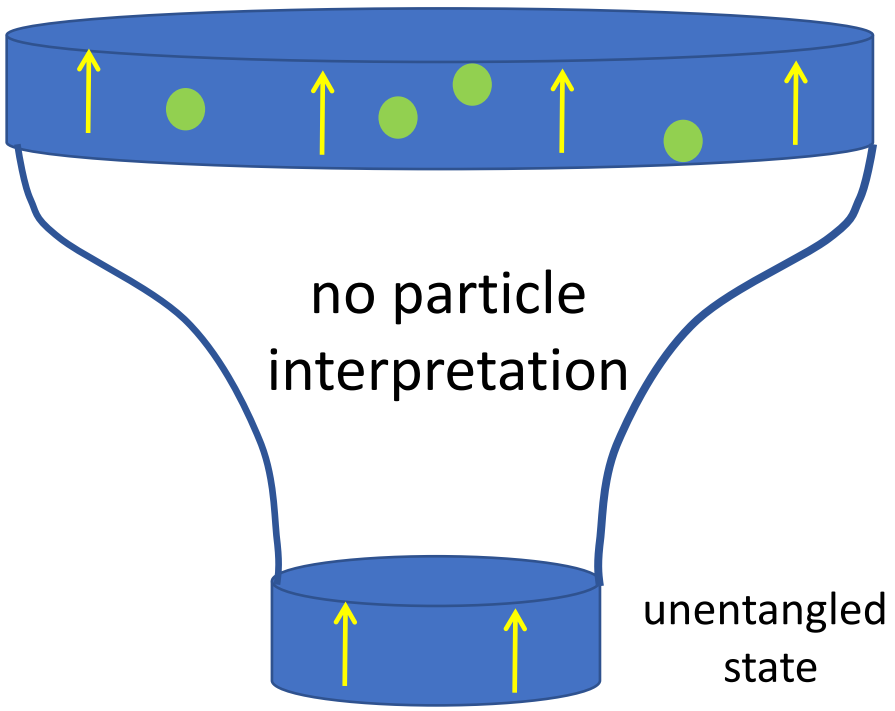


Just like in quantum optics!

COSMOLOGY
and
BLACK HOLES

Entanglement cosmology

Ball, Fuentes-S, Schuller PLA 2006



"History of the universe
encoded in entanglement"

<u>toy model</u>

expansion rate σ

expansion factor ϵ

- calculate entanglement

  asymptotic past $S = 0$

  asymptotic future $S = S(\sigma, \epsilon)$

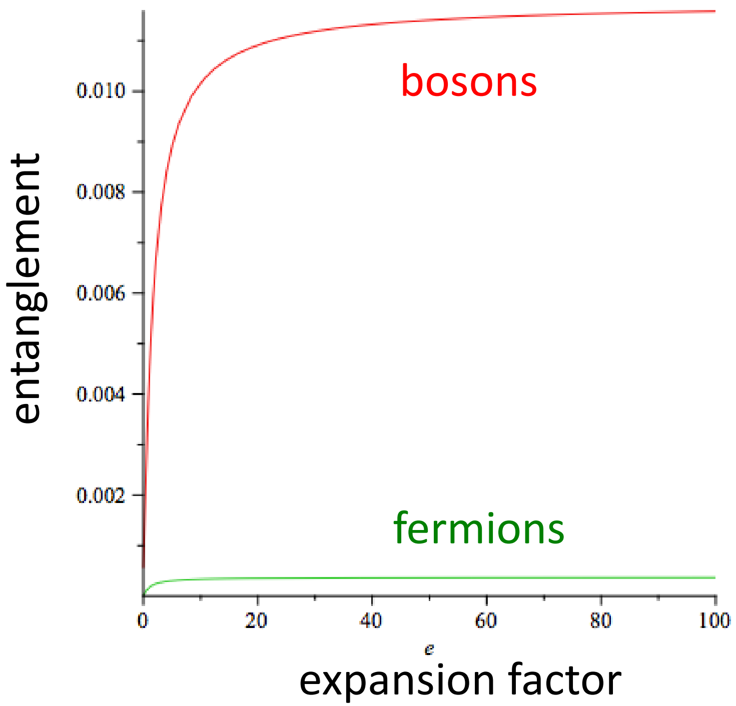
- excitingly, can solve for

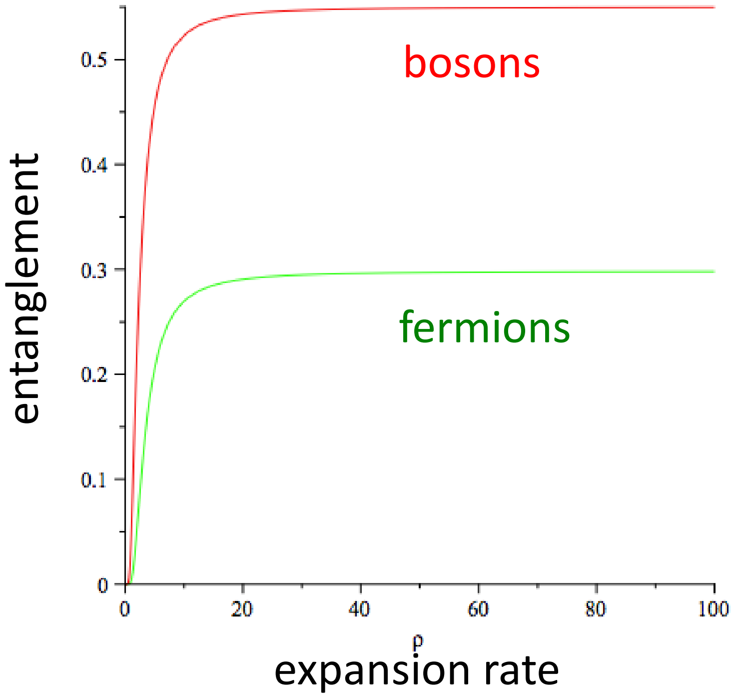
$\sigma = \sigma(S)$ $\epsilon = \epsilon(S)$

Fermionic entanglement cosmology

Fuentes, Mann, Martin-Martinez, Moradi PRD 2010

Fermionic fields in 3+1 dimensions: more realistic model

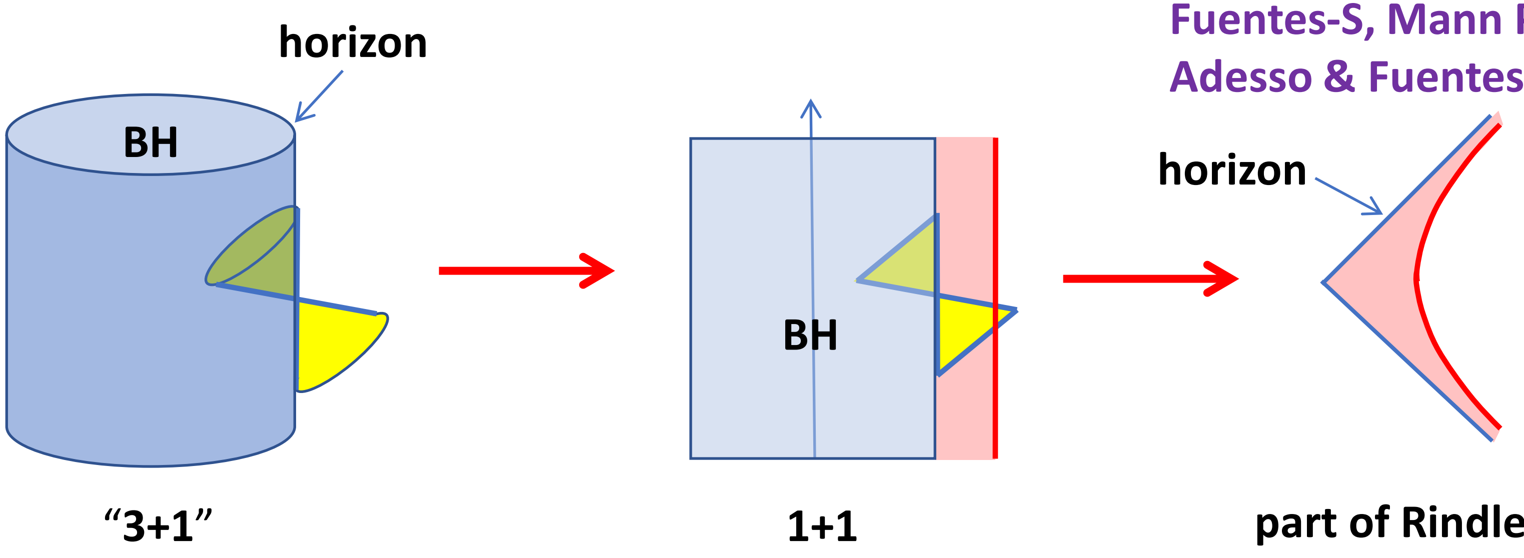


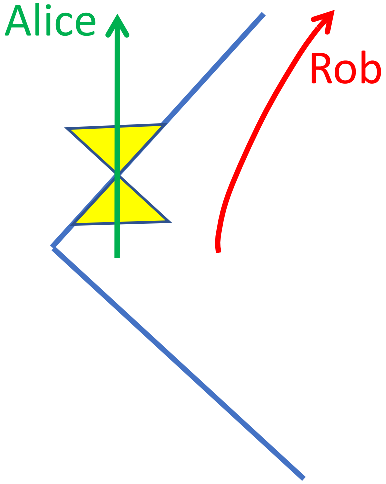


“The Universe entangles less fermionic fields”

Alice falls into a black hole

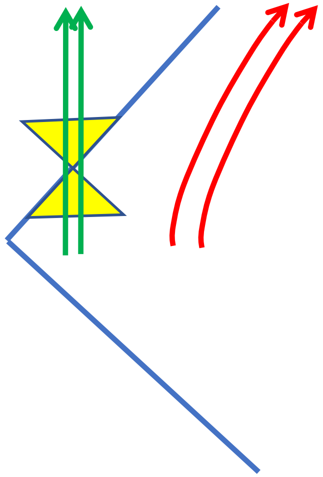
Fuentes-S, Mann PRL 2005
Adesso & Fuentes-S 2007





Entanglement
Classical correlations } degraded for escaping observers

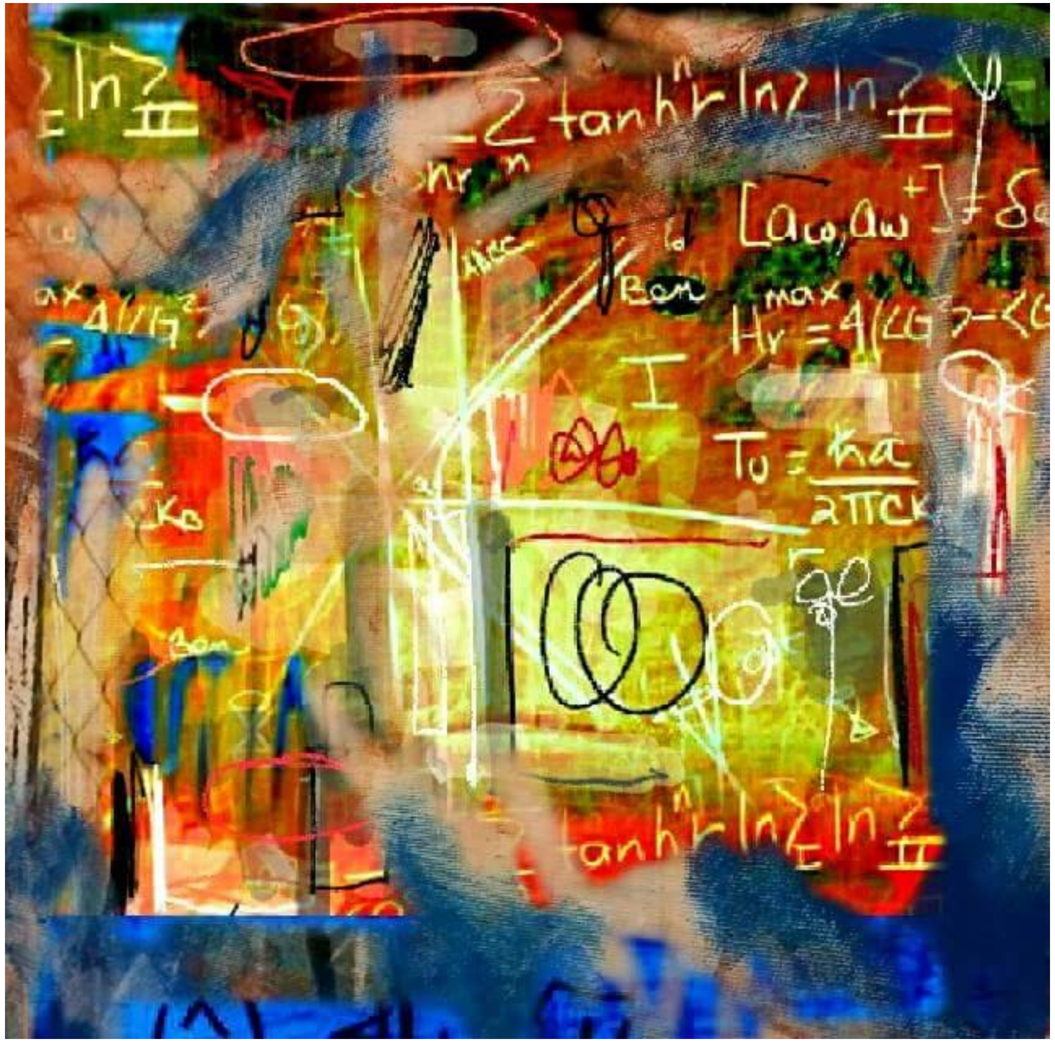
Lost entanglement → multipartite entanglement between modes inside and outside the BH



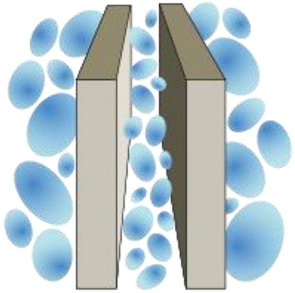
Entanglement in quantum field theory

- Entanglement between global modes
- Entanglement is observer dependent
- Spacetime dynamics create entanglement
- Horizons (black holes) degrade entanglement

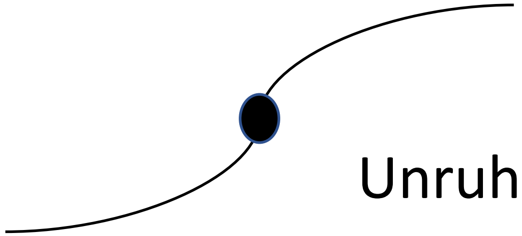
Localized systems for Relativistic Quantum Information



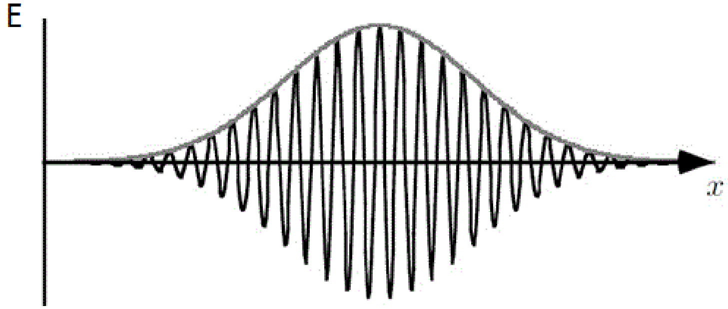
Systems for relativistic quantum information



Localized fields:
Cavities (EM fields)
Trapped BECs



Unruh-DeWitt detectors



Traveling wave-packets

- Entanglement between localized systems
  - cavities
  - detectors
  - localized travelling wave-packets
- gravity effects on quantum properties
- Propose earth-based and space-based experiments

Covariance Matrix Formalism

Convenient for systems consisting of N bosonic modes ⟶ N harmonic oscillators

$$\hat{x}_{(2n-1)} = \frac{1}{\sqrt{2}}(\hat{a}_n + \hat{a}_n^\dagger)$$ generalized position

$$\hat{x}_{(2n)} = \frac{-i}{\sqrt{2}}(\hat{a}_n - \hat{a}_n^\dagger)$$ generalized momentum

Collect in vector form: $\hat{X} = \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_{2n} \end{pmatrix}$

Covariance matrix: $\sigma_{ij} = \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle - 2\, \langle \hat{x}_i \rangle \, \langle \hat{x}_j \rangle$

Displacement vector: $\hat{d} = \begin{pmatrix} \langle \hat{x}_1 \rangle \\ \langle \hat{x}_2 \rangle \\ \vdots \\ \langle \hat{x}_{2n} \rangle \end{pmatrix}$

$\langle \hat{x}_j \rangle$ ⟶ First moments

$\langle \hat{x}_i \hat{x}_j \rangle$ ⟶ Second moments

For Gaussian states the covariance matrix and displacement vector replace the density matrix

Example: vacuum state

$$|0\rangle = |0_1\rangle\,|0_2\rangle \ldots |0_N\rangle = |0\rangle^{\otimes N}$$

Frist moments:

$$\langle \hat{x}_i \rangle = \langle 0|\, \frac{1}{\sqrt{2}} (\hat{a}_i + \hat{a}_i^\dagger)\, |0\rangle = 0$$

Displacement vector: $\hat{d} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

Second moments:

$$\langle \hat{x}_i \hat{x}_j \rangle = \delta_{ij}$$

$$\langle \hat{x}^2 \rangle = \frac{1}{2} \langle 0|\, \left(\hat{a}_i + \hat{a}_i^\dagger\right)^2 |0\rangle = 1/2.$$

Covariance matrix:

$$\sigma = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} = \mathrm{I}$$

Example: single-mode squeezed coherent state

$$|\phi(r,\alpha)\rangle = U(r)D(\alpha)\,|0\rangle \qquad \mathrm{U}(r) = e^{\frac{r}{2}(\hat{a}^2-\hat{a}^{\dagger 2})}$$

$$\mathcal{S}(r,\alpha) = U(r)D(\alpha) \qquad D(\alpha) = e^{\,(\alpha\hat{a}-\alpha^*\hat{a}^\dagger)}$$

Displacement vector: $\hat{d} = \begin{pmatrix}\alpha \\ \alpha^*\end{pmatrix}$

Second moments:

$$\langle\hat{x}^2\rangle = \frac{1}{2}\langle\phi|\,(\hat{a}_i+\hat{a}_i^\dagger)^2|\phi\rangle = \frac{1}{2}\cosh(2r) = \langle\hat{p}^2\rangle$$

$$\langle\hat{x}\hat{p}\rangle = \frac{-i}{2}\langle\phi|(\hat{a}_i+\hat{a}_i^\dagger)(\hat{a}_n-\hat{a}_n^\dagger)|\phi\rangle = -\frac{1}{2}\sinh(2r) = \langle\hat{p}\hat{x}\rangle$$

Covariance matrix: $\sigma = \begin{pmatrix}\cosh(2r) & -\sinh(2r) \\ -\sinh(2r) & \cosh(2r)\end{pmatrix}$

Separable state: $S_a\,(r_a,\alpha_a)\otimes S_b\,(r_b,\alpha_b)|00\rangle$

Example: two-mode squeezed state

$$|\psi_{ab}\rangle = \frac{1}{\cosh(r)} \sum_n tanh^n(r)|n_a\rangle|n_b\rangle$$

Frist moments:

$$\langle\hat{x}_a\rangle = \left\langle\psi_{ab}\right| \frac{1}{\sqrt{2}} (\hat{a}_a + \hat{a}_a^\dagger) \left|\psi_{ab}\right\rangle = 0 = \langle\hat{x}_b\rangle$$

Second moments:

$$\langle\hat{x}_a\hat{x}_a\rangle = \frac{1}{2}\left\langle\psi_{ab}\right|| (\hat{a}_a + \hat{a}_a^\dagger)^2 |\psi_{ab}\rangle = \frac{1}{2}\cosh(2r)$$

Displacement vector: $\hat{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Covariance matrix:

$$\sigma = \begin{pmatrix} \cosh(2r) & 0 & \sinh(2r) & 0 \\ 0 & \cosh(2r) & 0 & -\sinh(2r) \\ \sinh(2r) & 0 & \cosh(2r) & 0 \\ 0 & -\sinh(2r) & 0 & \cosh(2r) \end{pmatrix}$$

State Evolution

$$\sigma_f = S^T \sigma_i S$$

S is the symplectic matrix
It plays an important role in QFT

$$S^T \Omega S = \Omega$$ → symplectic form

$$\Omega = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

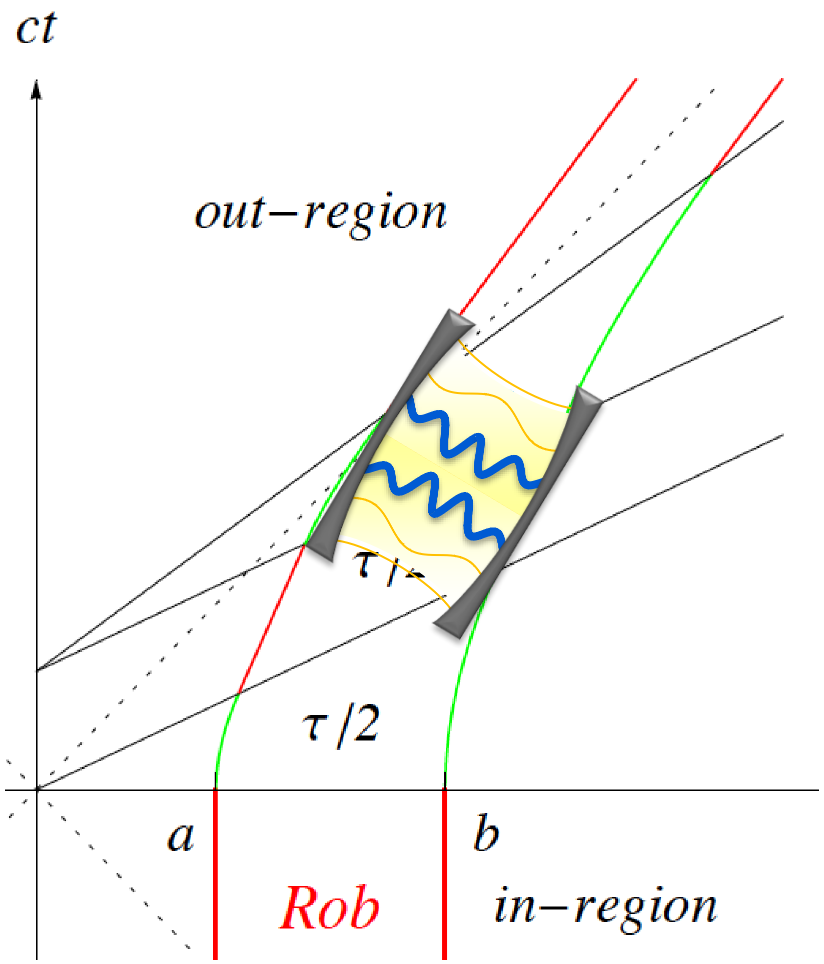
$$2i\Omega_{ij} = [\hat{x}_i, \hat{x}_j]$$

There are computable measures of entanglement if the states are Gaussian

Example:

Negativity $N(\nu_i)$ ν_i is the smallest eigenvalue of $i\Omega\sigma^{PT}$ → Partial transpose

QFT in the symplectic formalism



Articles

Entanglement generation in relativistic quantum fields

Nicolai Friis & Ivette Fuentes

Pages 22-27 | Received 06 Apr 2012, Accepted 10 Jul 2012, Published online: 20 Aug 2012

Download citation https://doi.org/10.1080/09500340.2012.712725

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \dots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \dots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathcal{M}_{mn} = \begin{pmatrix} \Re(\alpha_{mn} - \beta_{mn}) & \Im(\alpha_{mn} + \beta_{mn}) \\ -\Im(\alpha_{mn} - \beta_{mn}) & \Re(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

Inertial cavity

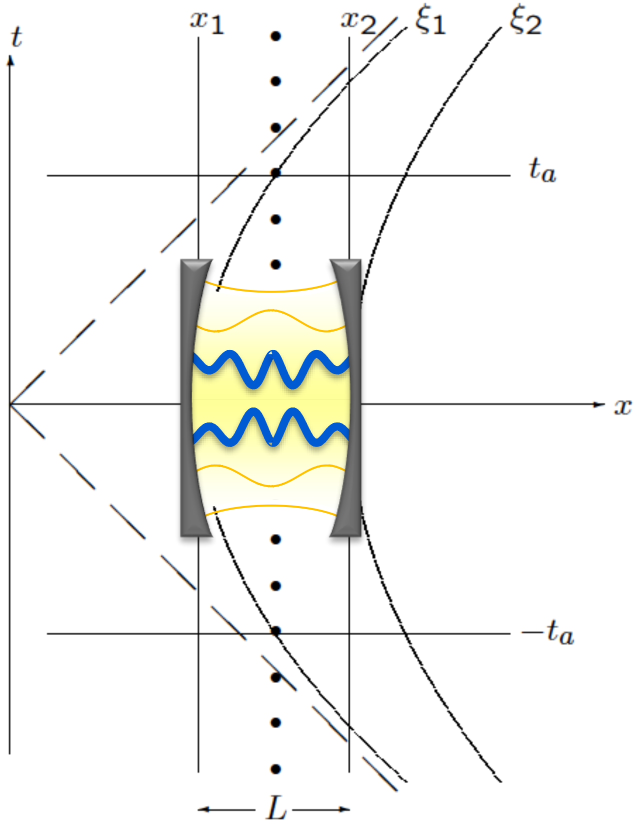
Minkowski coordinates (t, x)

$$\Box\phi(t,x) = 0$$ field equation

solutions: plane waves+ boundary

$$u_k(x,t) = \frac{1}{\sqrt{k\pi}} \sin\left[\frac{k\pi}{L}(x - x_A)\right] e^{-i\omega_k t},$$

$$\omega_k = \frac{1}{L}\sqrt{(k\pi)^2 + m^2},$$ discrete spectrum

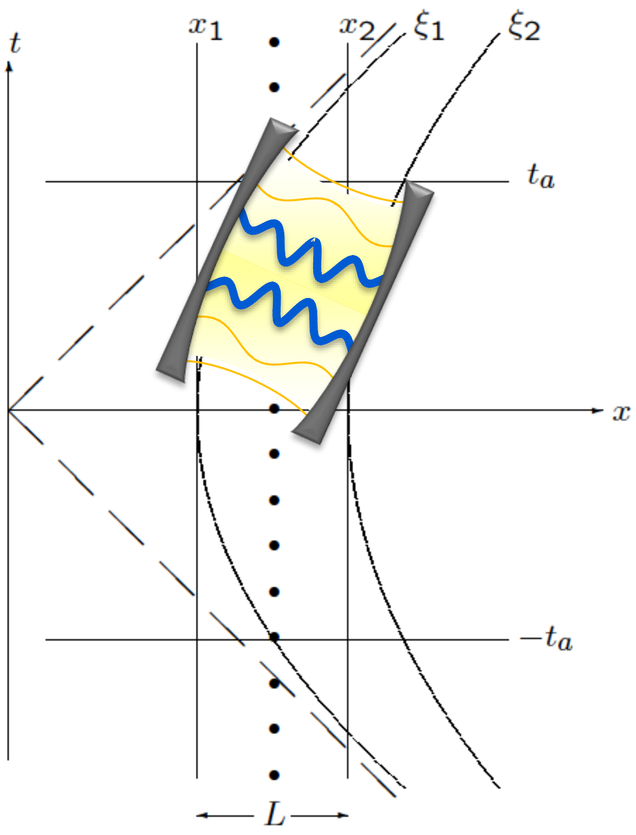


creation and annihilation operators

$$\hat{\phi}(x,t) = \sum_n (u_n(t,x)\hat{a}_n + u_n^*(t,x)\hat{a}_n^\dagger)$$

$\hat{a}_n|0\rangle = 0$ vacuum state $\qquad |m_n\rangle = \hat{a}_n^{\dagger\, m}|0\rangle$ particle states

Uniformly accelerated cavity



Entangling Moving Cavities in Noninertial Frames

T. G. Downes, I. Fuentes, and T. C. Ralph
Phys. Rev. Lett. **106**, 210502 – Published 25 May 2011

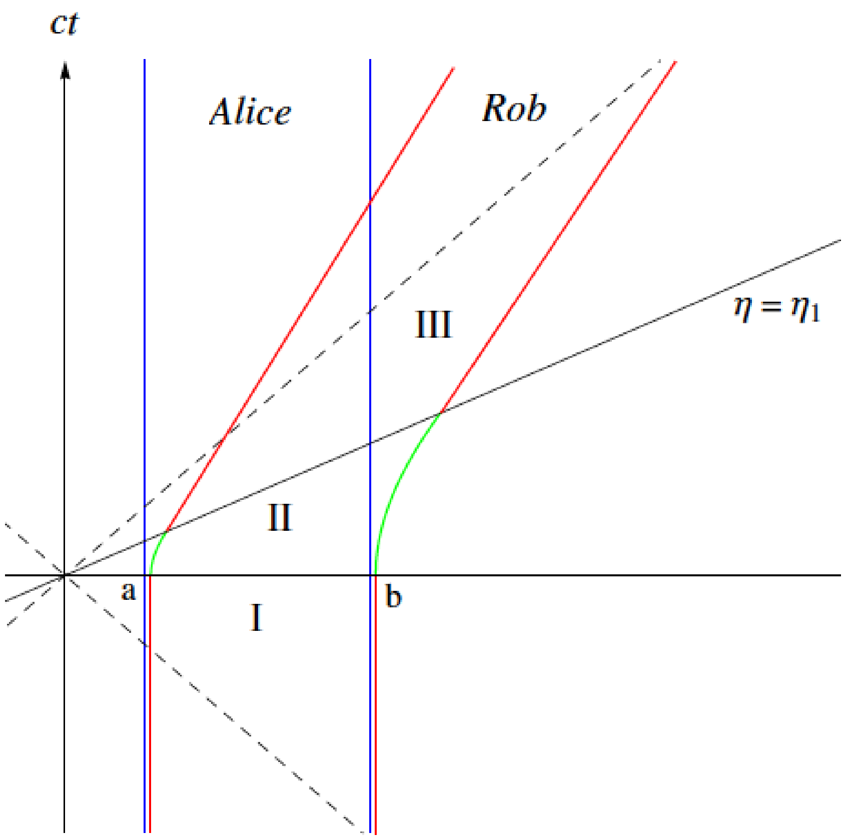
Rindler coordinates

$$\eta = \operatorname{atanh}\left(\frac{t}{x}\right), \quad \chi = \sqrt{x^2 - t^2},$$

The Klein-Gordon equation again is a simple wave equation

$$\Box\, \phi(\eta, \chi) = 0$$

Non-uniform motion



Voyage to Alpha Centauri: Entanglement degradation of cavity modes due to motion

David Edward Bruschi, Ivette Fuentes, and Jorma Louko
Phys. Rev. D **85**, 061701(R) – Published 15 March 2012

Bogoliubov transformations

$$\tilde{a}_m = \sum_n \left(\alpha^*_{mn}\, a_n - \beta^*_{mn}\, a^\dagger_n \right)$$

$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n) \text{ and } \beta_{mn} = -(\tilde{\phi}_n, \phi^*_m)$$

Bogoliubov transformations

$$\begin{pmatrix} \hat{x}_1' \\ \vdots \\ \hat{x}_{2n}' \end{pmatrix} = \begin{pmatrix} \Re(\alpha_{11} - \beta_{11}) & \cdots & \Im(\alpha_{1n} + \beta_{1n}) \\ \vdots & \ddots & \vdots \\ -\Im(\alpha_{n1} + \beta_{n1}) & \cdots & \Re(\alpha_{nn} + \beta_{nn}) \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_{2n} \end{pmatrix}$$

$$\alpha_{nm} = (u_n', u_m) \qquad \beta_{nm} = (u_n', u_m^*)$$

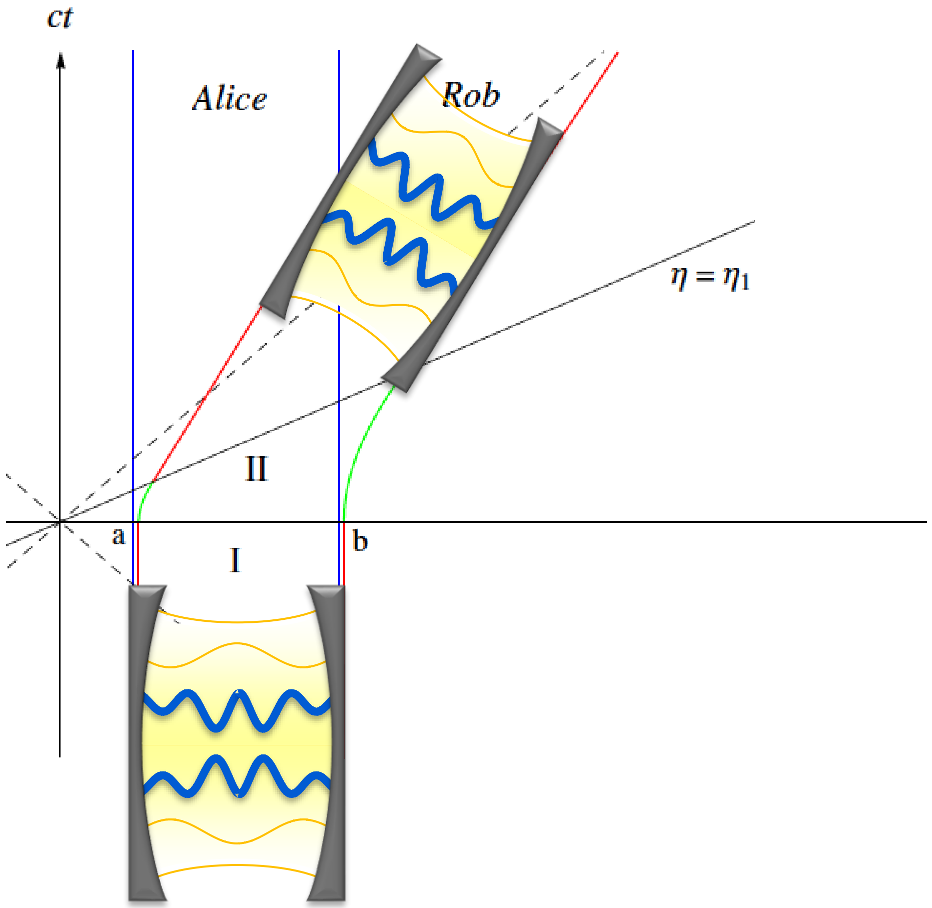
$$(\phi, \psi) = i \int_\Sigma d\Sigma^\mu [\psi^* \partial_\mu \phi - \phi \partial_\mu \psi^*]$$

$$S = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \cdots \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \cdots \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathcal{M}_{mn} = \begin{pmatrix} \Re(\alpha_{mn} - \beta_{mn}) & \Im(\alpha_{mn} + \beta_{mn}) \\ -\Im(\alpha_{mn} - \beta_{mn}) & \Re(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

Can't be computed but…

Approximation...



$$h = aL$$

acceleration → a, length → L

$$\alpha = \alpha^{(0)} + \alpha^{(1)} + \alpha^{(2)} + O(h^3)\,,$$

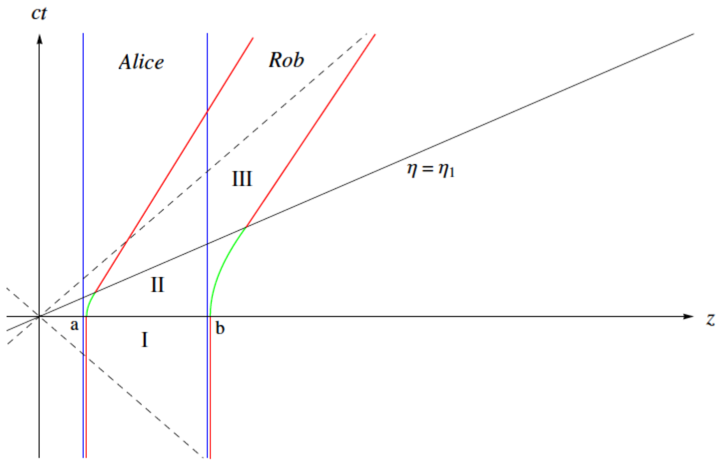
$$\beta = \beta^{(1)} + \beta^{(2)} + O(h^3)\,,$$

$h \ll 1$ computable transformations

During periods of inertial or uniformly accelerated motion

$$T_t = \left(\begin{array}{ccc|ccc} e^{i\omega_1 t} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & e^{i\omega_n t} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & e^{-i\omega_1 t} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & e^{-i\omega_n t} \end{array}\right)$$

$$T_\tau = \left(\begin{array}{ccc|ccc} e^{i\omega_1 \tau} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & e^{i\omega_n \tau} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & e^{-i\omega_1 \tau} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & e^{-i\omega_n \tau} \end{array}\right)$$



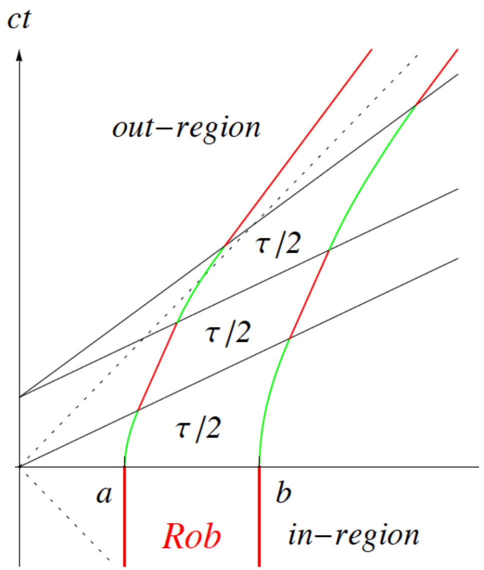
Consider the vacuum as the initial state in region 1

Region I $\quad \sigma_0 = I$

Region II $\quad \sigma_{II} = S^T I S$

Region III $\quad \sigma_{II} = S^{-1T} T_\tau^T S^T S T_\tau S^{-1}$

Basic building block



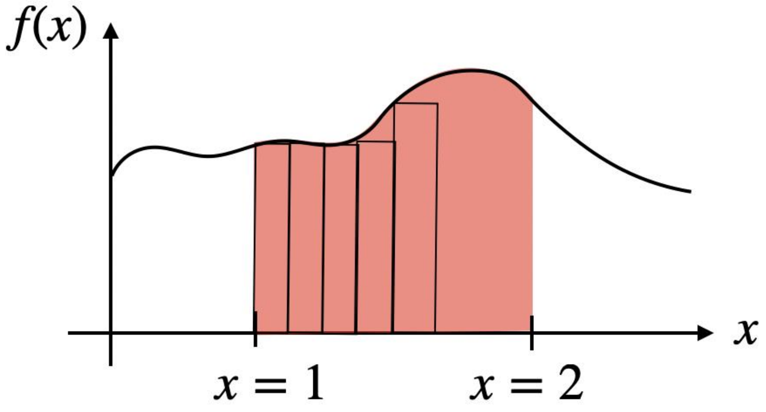
$$B = T_t S T_\tau S^{-1}$$

$$\sigma_B = B^T B$$

General trajectory: grafting $B_F = B_1 \, B_2 \ldots \, B_n$

We can approximate any trajectory

"Integration" method for quantum fields



$$\hat{A}_{mn} = i(\omega_m - \omega_n)\hat{\alpha}_{mn} \int_{\tau_0}^{\tau_f} e^{-i(\omega_m - \omega_n)(\tau - \tau_0)}\, h(\tau)\, d\tau \ ,$$

$$\hat{B}_{mn} = i(\omega_m + \omega_n)\hat{\beta}_{mn} \int_{\tau_0}^{\tau_f} e^{-i(\omega_m + \omega_n)(\tau - \tau_0)}\, h(\tau)\, d\tau \ .$$

Regular Article - Theoretical Physics | Open Access | Published: 31 August 2020

Evolution of confined quantum scalar fields in curved spacetime. Part I

Spacetimes without boundaries or with static boundaries in a synchronous gauge

Luis C. Barbado, Ana L. Báez-Camargo & Ivette Fuentes

The European Physical Journal C **80**, Article number: 796 (2020) | Cite this article

1100 Accesses | **1** Citations | **2** Altmetric | Metrics

Regular Article - Theoretical Physics | Open Access | Published: 29 October 2021

Evolution of confined quantum scalar fields in curved spacetime. Part II

Spacetimes with moving boundaries in any synchronous gauge

Luis C. Barbado, Ana L. Báez-Camargo & Ivette Fuentes

The European Physical Journal C **81**, Article number: 953 (2021) | Cite this article

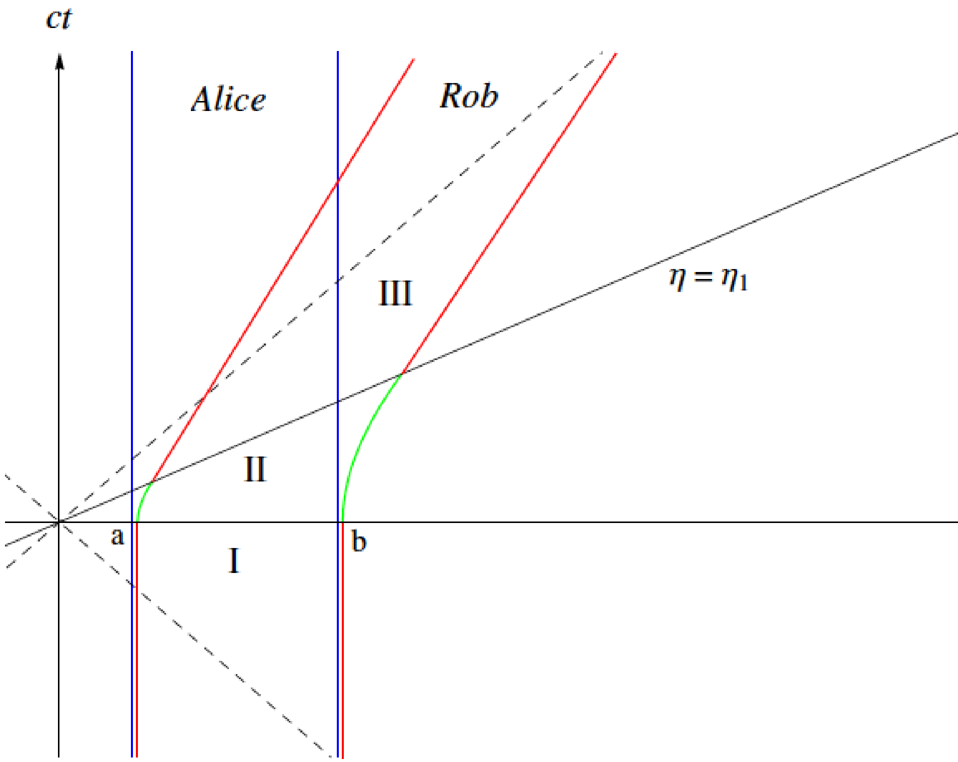
415 Accesses | **1** Altmetric | Metrics

Entanglement

Computable measures for Gaussian states

Separable initial states of two modes $S_a(s) \otimes S_b(s)|00\rangle$

Negativity $N(\nu_i)$ ν_i is the smallest eigenvalue of $i\Omega\sigma^{PT}$



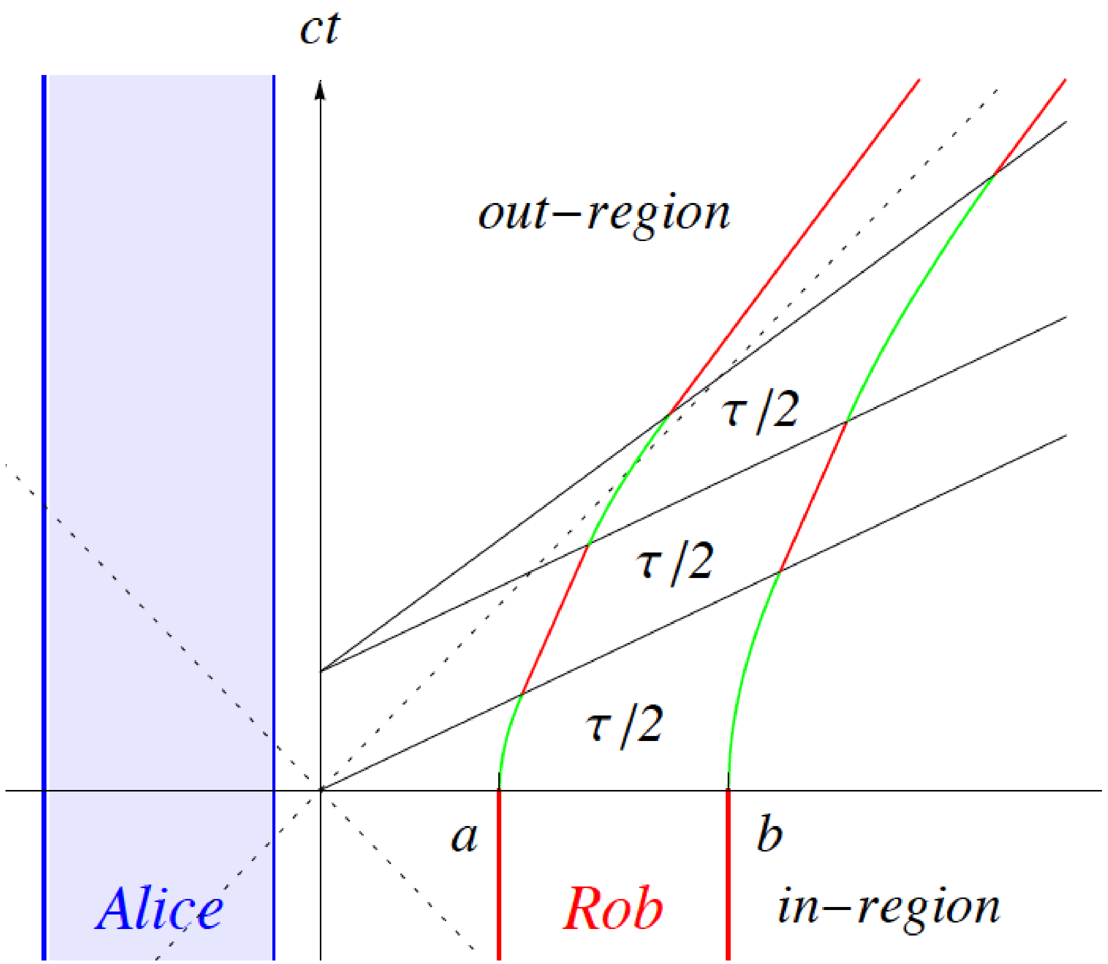
$$\alpha = \alpha^{(0)} + \alpha^{(1)} + \alpha^{(2)} + O(h^3)\,,$$
$$\beta = \beta^{(1)} + \beta^{(2)} + O(h^3)\,,$$

$$\mathcal{N} = (\Re(e^{i\omega_a\tau}\beta_{ab})^2 + (\Im(e^{i\omega_a\tau}\beta_{ab})\cosh(s) - \Im(e^{i\omega_a\tau}\alpha_{ab})\sinh(s))^{1/2}$$

Friis and Fuentes JMO (invited) 2012

motion and gravity create entanglement

Friis, Bruschi, Louko, Fuentes PRD (R) 2012



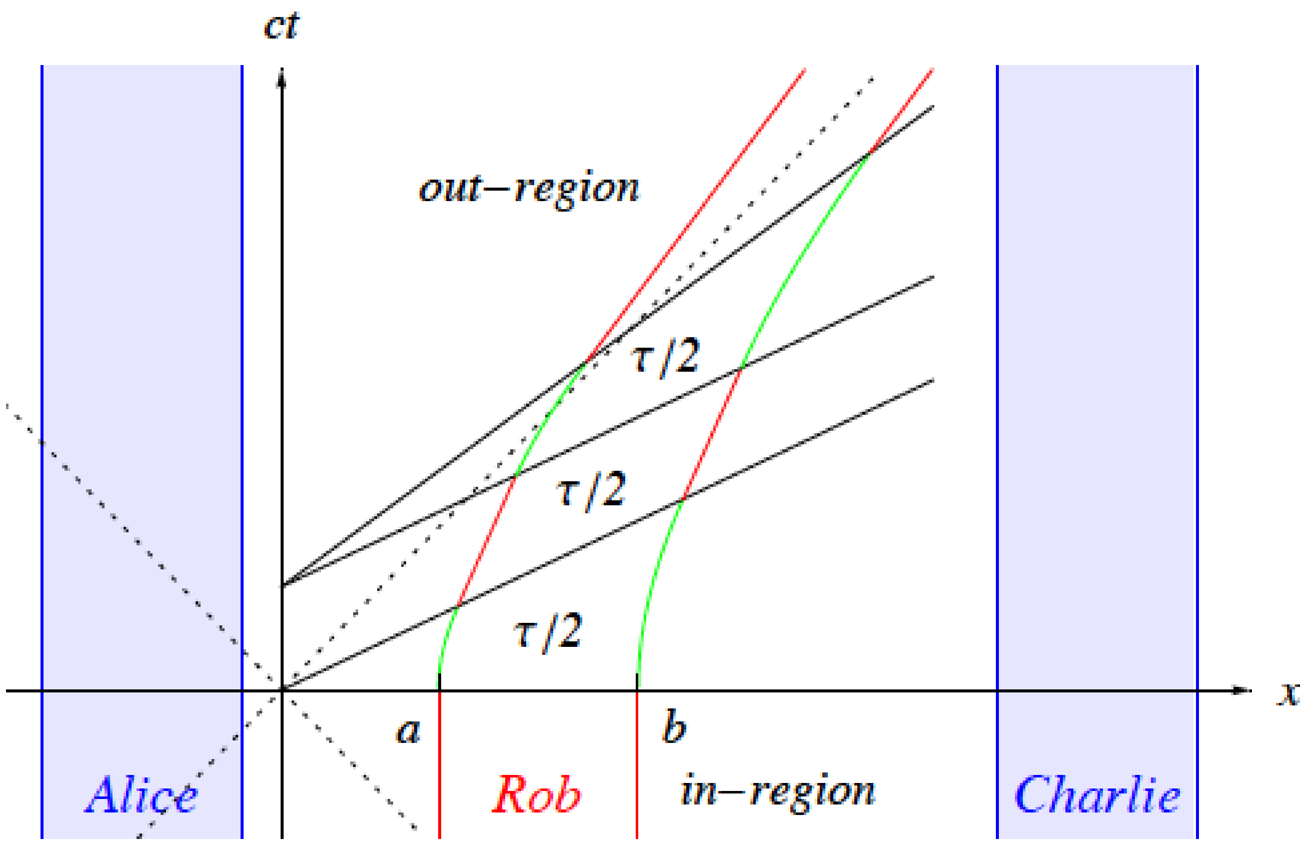
results

non-uniform motion creates entanglement

gravity creates entanglement

multipartite case

Friis, Huber, Fuentes, Bruschi PRD 2012



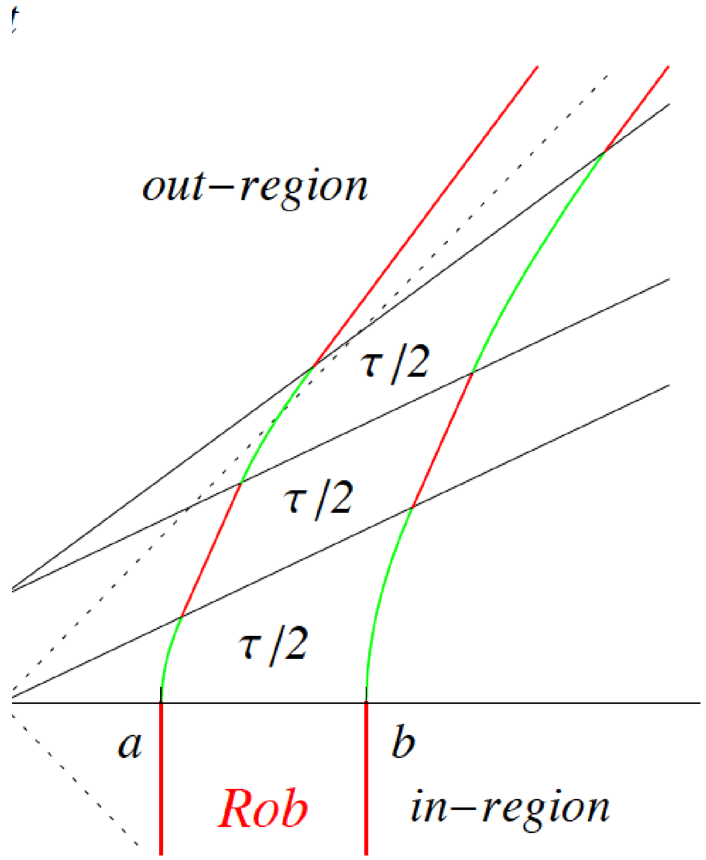
resonance:

$$|\beta^{(1)}_{k\,k'}|\,|\beta^{(1)}_{k'\,k''}|$$

Entanglement: genuine multipartite entanglement created

quantum gates

Friis, Huber, Fuentes, Bruschi PRD 2012
Bruschi, Lee, Dragan, Fuentes, Louko PRL 2012
Bruschi, Louko, Faccio & Fuentes NJP 2013
Bruschi, Sabin, Kok, Johansson, Delsing & Fuentes SR 2016

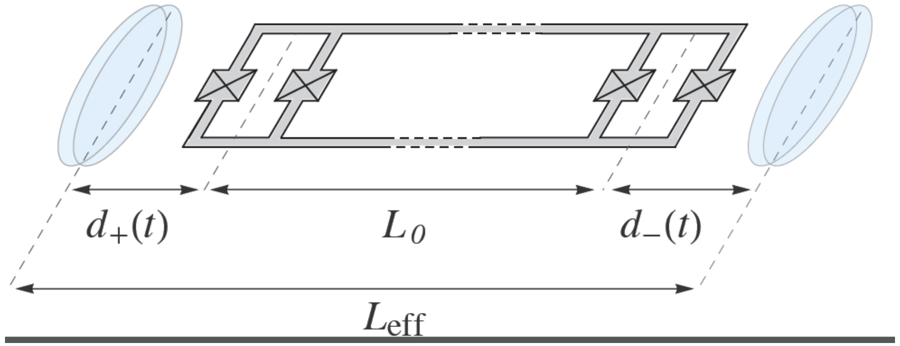


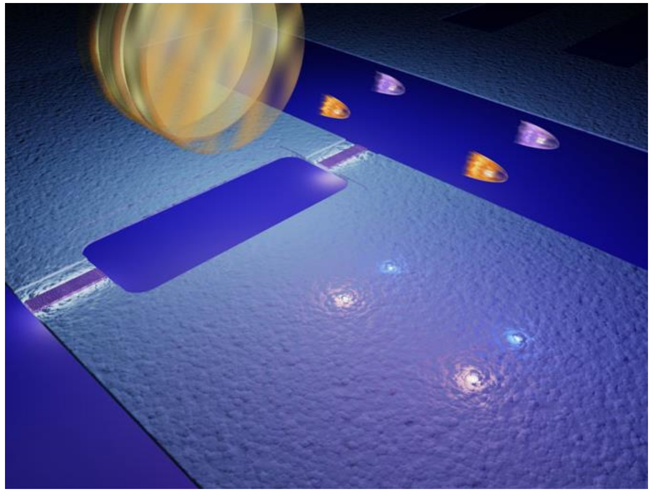
the relativistic motion of quantum systems can be used to produce quantum gates

two-mode squeezer
beam splitter

multi-qubit gates:
Dicke states
Multi-mode squeezer
Cluster states

Cavity moving at relativistic speeds using superconducting circuits



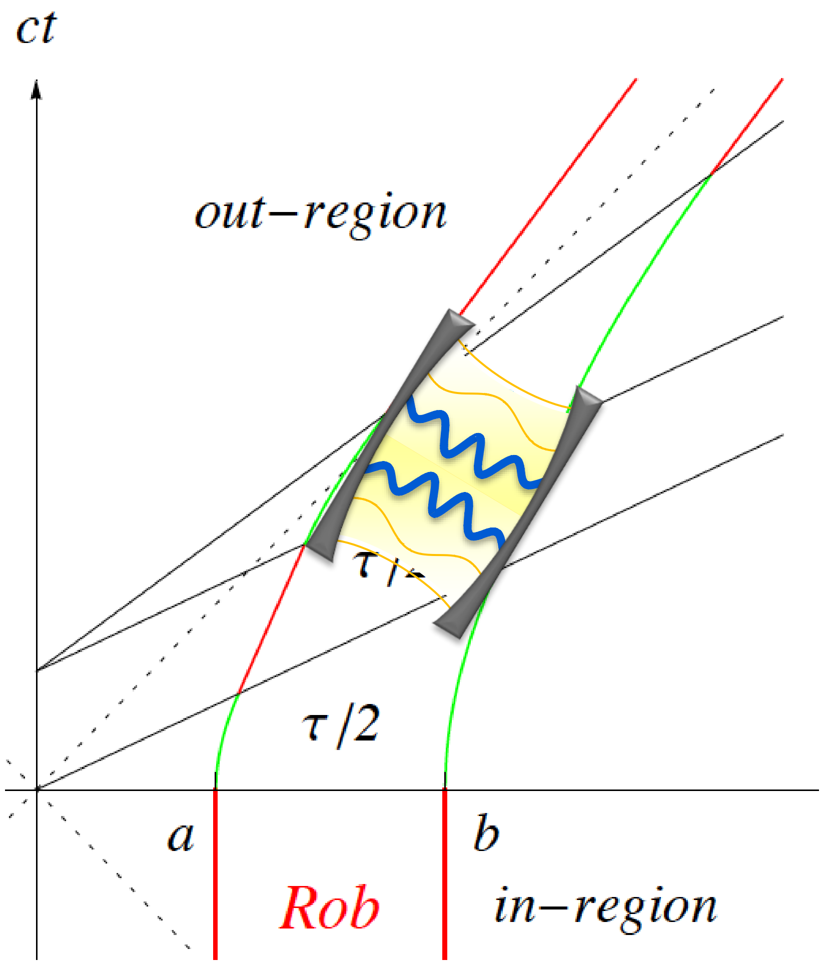


Generating Multimode Entangled Microwaves with a Superconducting Parametric Cavity

C. W. Sandbo Chang, M. Simoen, José Aumentado, Carlos Sabín, P. Forn-Díaz, A. M. Vadiraj, Fernando Quijandría, G. Johansson, I. Fuentes, and C. M. Wilson

Phys. Rev. Applied **10**, 044019 – Published 8 October 2018

Cavities moving in curved spacetime



PAPER • OPEN ACCESS

Dynamical Casimir effect in curved spacetime

Maximilian P E Lock[1,2] and Ivette Fuentes[4,2,3]

Published 7 July 2017 • © 2017 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

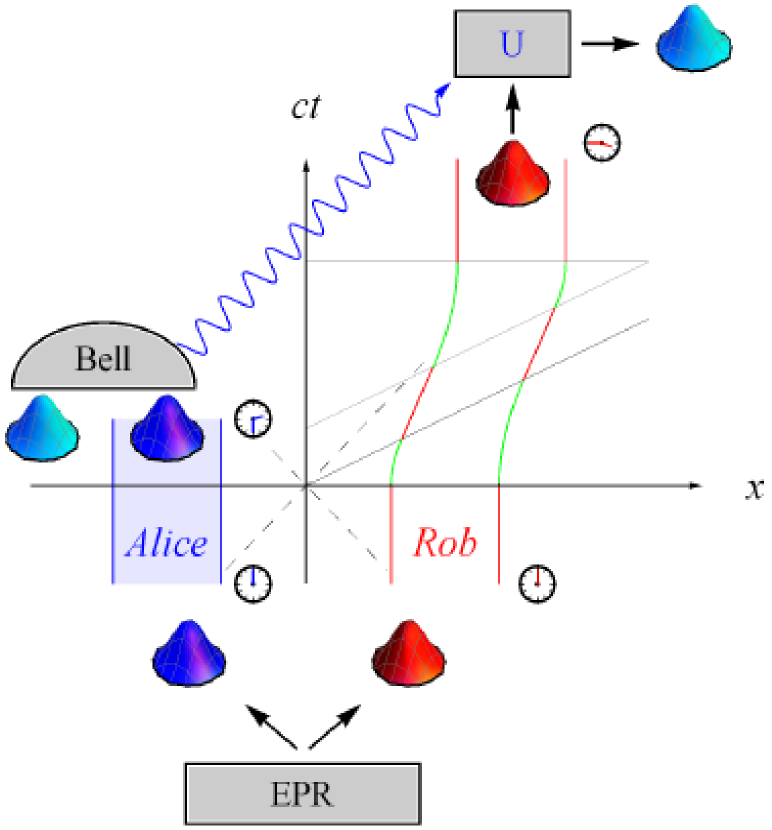
New Journal of Physics, Volume 19, July 2017

Citation Maximilian P E Lock and Ivette Fuentes 2017 *New J. Phys.* **19** 073005

We find corrections due to curvature effects

Teleporation with an accelerated partner

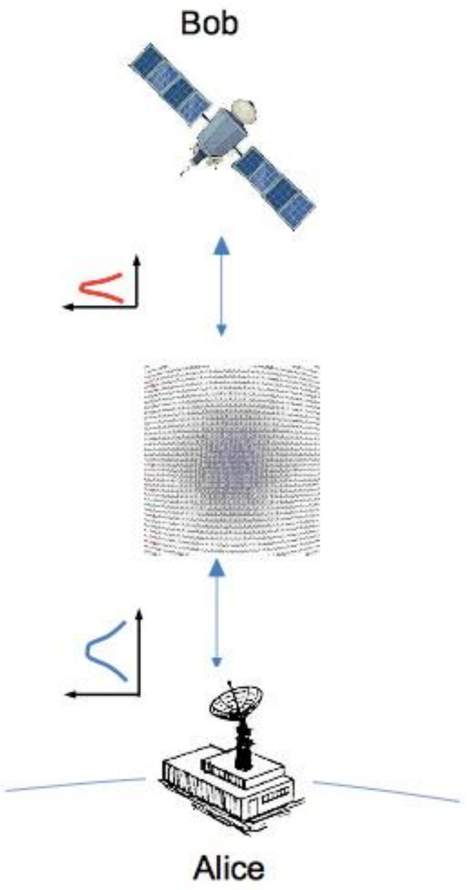
N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson, and I. Fuentes
Phys. Rev. Lett. **110**, 113602 – Published 12 March 2013



The fidelity of teleportation is affected by motion
It is possible to correct by local rotations
and trip planning

Spacetime effects on satellite-based quantum communications

David Edward Bruschi, Timothy C. Ralph, Ivette Fuentes, Thomas Jennewein, and Mohsen Razavi
Phys. Rev. D **90**, 045041 – Published 28 August 2014



Observable effects in satellite-based quantum cryptography

$$\Phi = \int_0^{+\infty} d\omega \left[\phi_\omega^{(u)} a_\omega + \phi_\omega^{(v)} b_\omega + \text{h.c.}\right]$$

$$a_{\omega_0}(t) = \int_0^{+\infty} d\omega\, e^{-i\omega t} F_{\omega_0}(\omega)\, a_\omega.$$

Traveling wave-packet

$$S = \begin{pmatrix} \Theta \mathbb{1}_2 & \sqrt{1-\Theta^2}\mathbb{1}_2 \\ -\sqrt{1-\Theta^2}\mathbb{1}_2 & \Theta \mathbb{1}_2 \end{pmatrix}$$

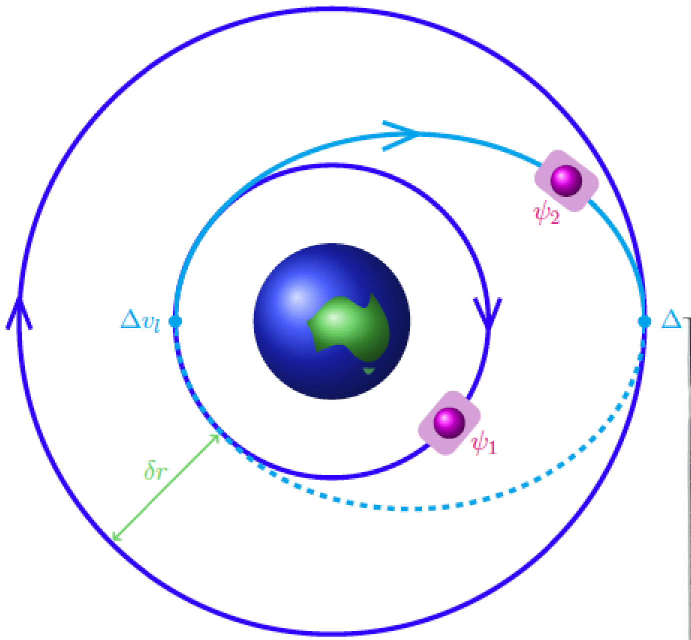
Spacetime acts as a beam-splitter

Testing the effects of gravity and motion on quantum entanglement in space-based experiments

David Edward Bruschi[1], Carlos Sabín[2], Angela White[3,6], Valentina Baccetti[4], Daniel K L Oi[5] and Ivette Fuentes[2]

Published 21 May 2014 • © 2014 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 16, May 2014



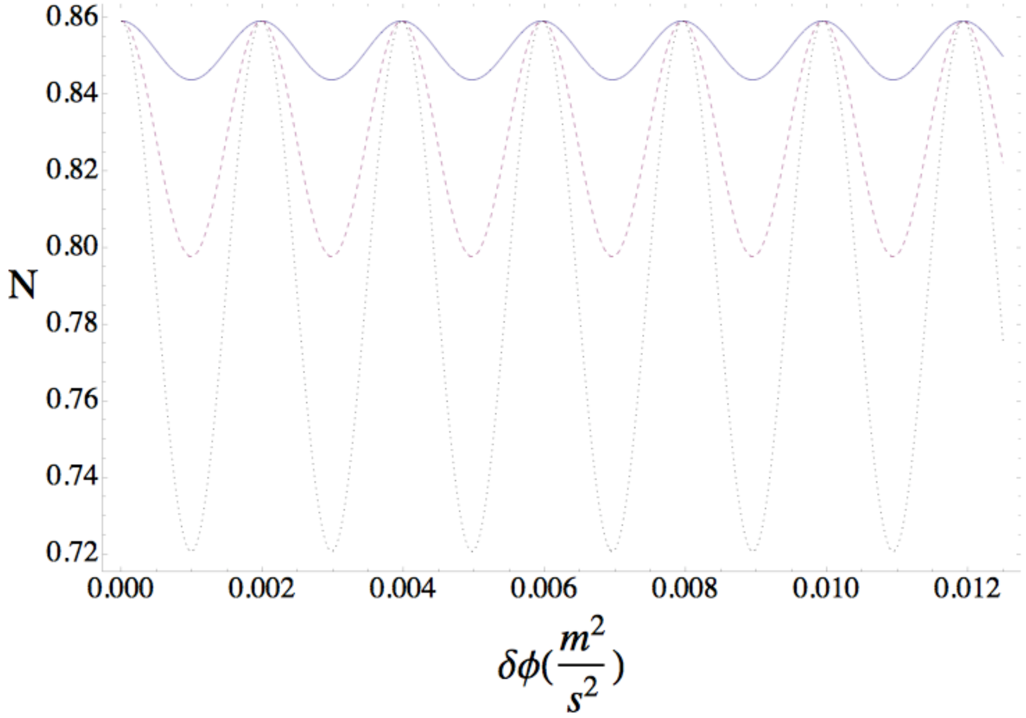


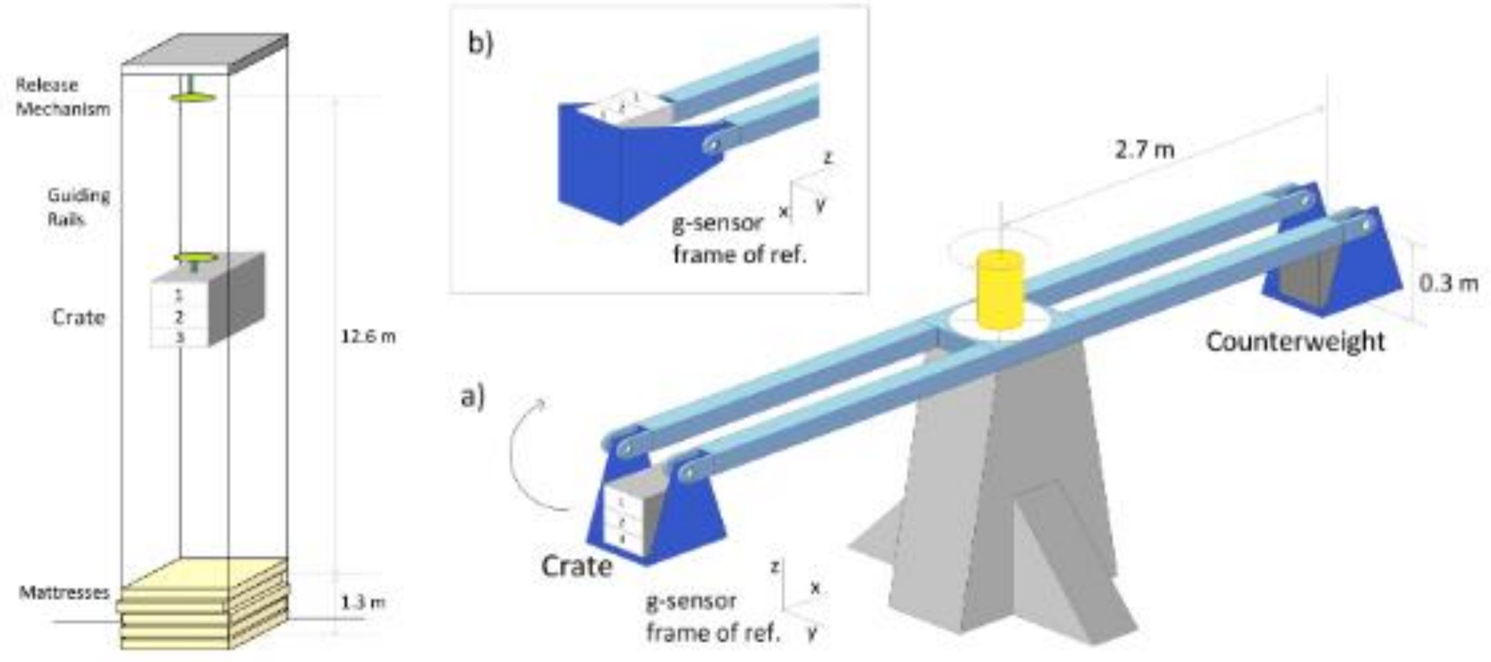
Figure 2. Negativity N vs. difference in gravitational field strength between initial and final orbits $\delta\phi$, after the first change in velocity Δv_l. The acceleration of the satellite is $a = 10^{-3}\,\mathrm{m/s^2}$ (solid, blue), $a = 2\cdot 10^{-3}\,\mathrm{m/s^2}$ (red,dashed), $a = 3\cdot 10^{-3}\,\mathrm{m/s^2}$ (black, dotted) while $L = 100\,\mu\mathrm{m}$, $c_s = 1\,\mathrm{mm/s}$, giving rise to $h^2 \simeq 0.05$ and $\Omega_1 = 2\pi \times 50$ Hz. The initial squeezing is $r = 1/2$.

Effects of gravity and motion on entanglement

Experimental test of photonic entanglement in accelerated reference frames

Matthias Fink, Ana Rodriguez-Aramendia, Johannes Handsteiner, Abdul Ziarkash, Fabian Steinlechner, Thomas Scheidl, Ivette Fuentes, Jacques Pienaar, Timothy C. Ralph & Rupert Ursin

Nature Communications **8**, Article number: 15304 (2017) | Cite this article

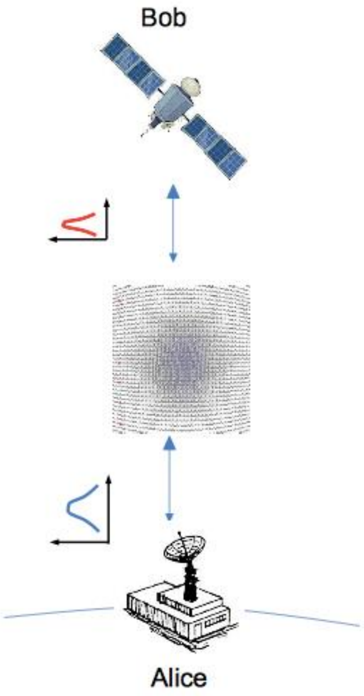


entanglement under uniform acceleration is conserved

Future experiments: non-uniform acceleration and Satellite-based experiments

Quantum estimation of the Schwarzschild spacetime parameters of the Earth

David Edward Bruschi, Animesh Datta, Rupert Ursin, Timothy C. Ralph, and Ivette Fuentes
Phys. Rev. D **90**, 124001 – Published 1 December 2014



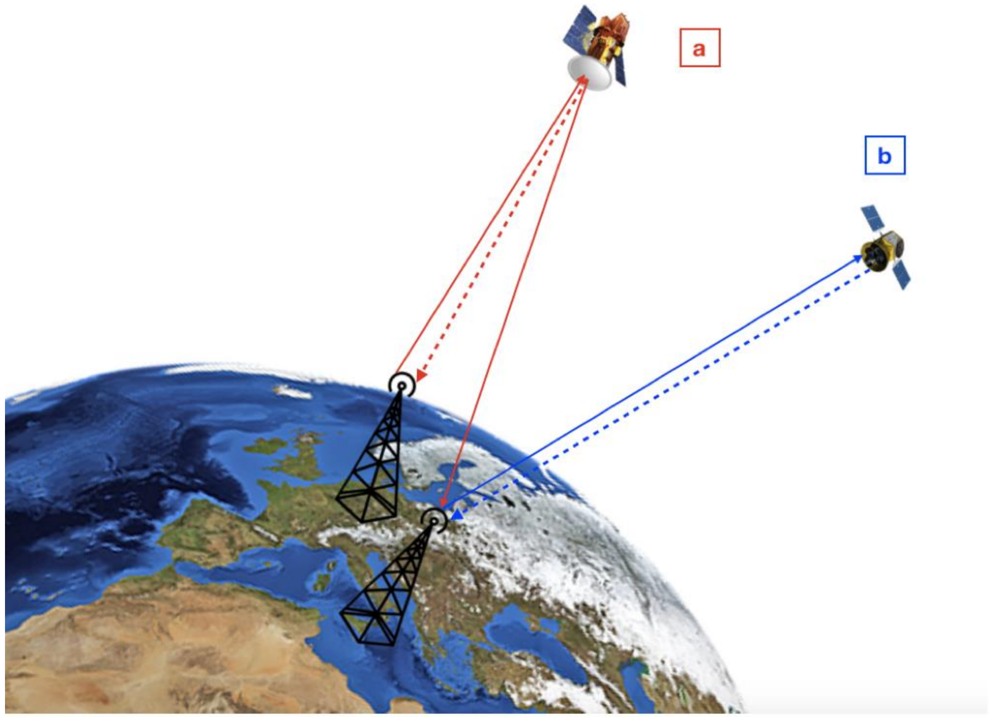
$$\boldsymbol{g} = \mathrm{diag}\left(-\left(1-\frac{2M}{r}\right), \frac{1}{1-\frac{2M}{r}}, r^2, r^2\sin^2\theta\right)$$

Estimating the Schwarzschild radius of the Earth
Using traveling wave-packets

$$\frac{\Delta r_S}{r_S} \geq \frac{8\,\sigma\,r_A\,(r_A+L)}{\sqrt{N(\Omega_1^2+\Omega_2^2)}\,r_S\,L\,\sinh r} \sim 5.8\times 10^{-7}$$

Quantum-metrology estimation of spacetime parameters of the Earth outperforming classical precision

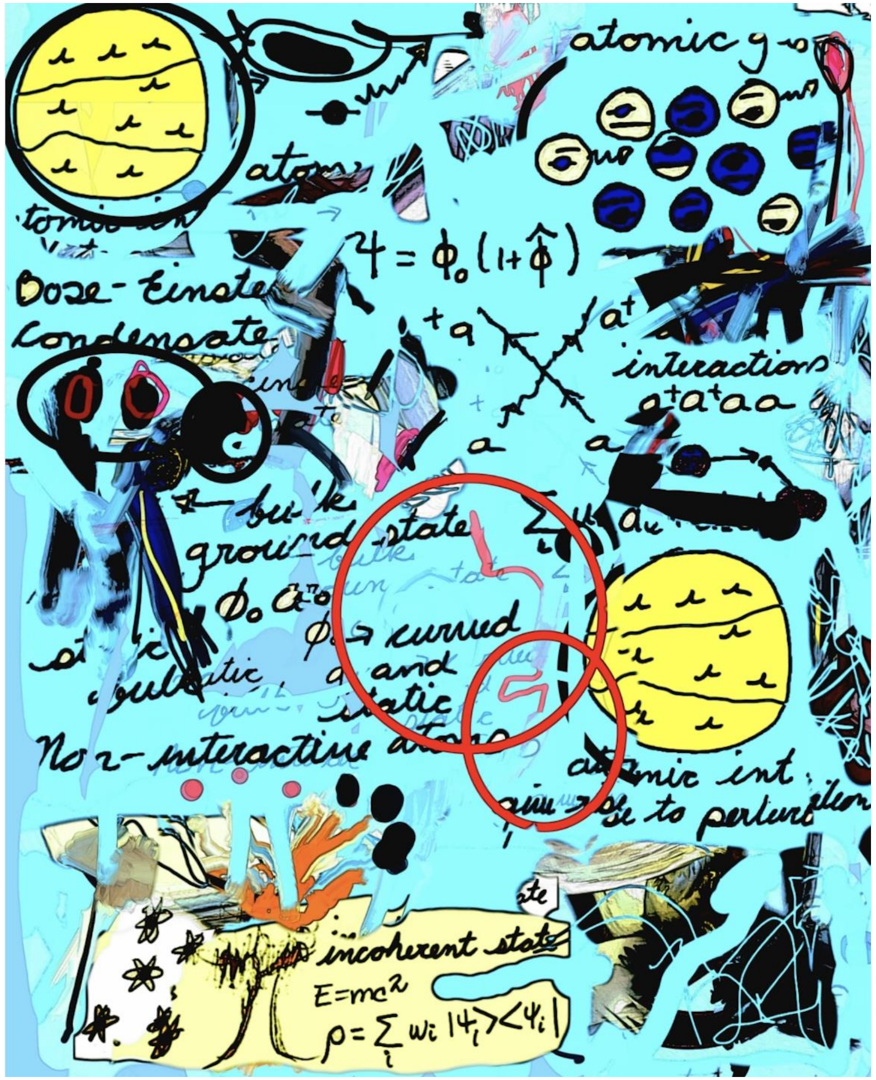
Jan Kohlrus, David Edward Bruschi, and Ivette Fuentes
Phys. Rev. A **99**, 032350 – Published 29 March 2019



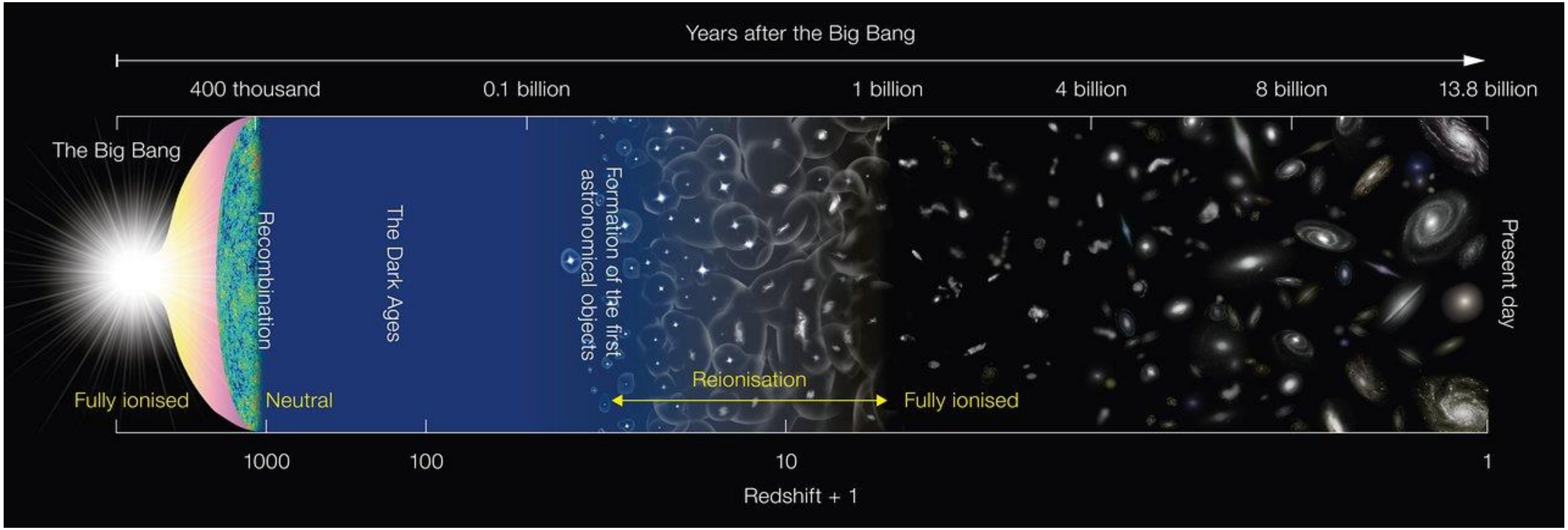
Orbit	Light's trajectory	$\Delta r_S/r_S$	$\Delta R_E/R_E$	$\Delta h/h$
LEO	Lab1 ⇆ Lab2	8.50×10^{-10}	1.12×10^{-9}	3.56×10^{-9}
	Lab1 → Lab1	8.87×10^{-10}	1.17×10^{-9}	3.71×10^{-9}
	Lab2 → Lab2	8.15×10^{-10}	1.07×10^{-9}	3.41×10^{-9}
	Sat → Lab1	1.77×10^{-9}	2.33×10^{-9}	7.43×10^{-9}
	Sat → Lab2	1.63×10^{-9}	2.14×10^{-9}	6.83×10^{-9}
VLEO	Lab1 → Lab1	6.06×10^{-10}	6.30×10^{-10}	1.57×10^{-8}
	Sat → Lab 1	1.21×10^{-9}	1.26×10^{-9}	3.15×10^{-8}

TABLE II: Precision bounds obtained through the quantum metrology scheme described above, for the different possible configurations of the reflecting and downlink schemes. Results for uplinks are very similar to those in downlinks.

Relativistic Quantum Metrology



Many fundamental questions unanswered



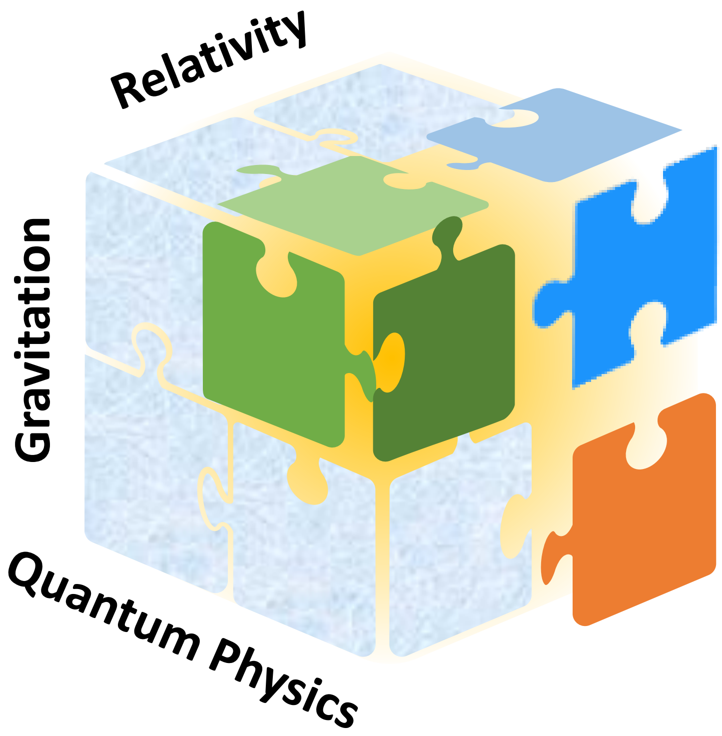
What is the nature of dark matter?

Is dark energy driving the accelerated expansion of the Universe?

What physics dominated the Universe at early times?

Does the equivalence principle holds for quantum systems?

Underpinning our difficulty to find answers



Quantum physics and General Relativity
are incompatible

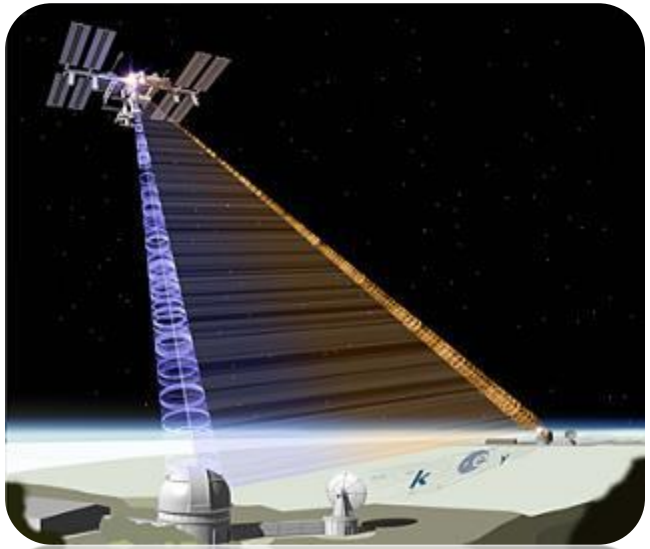
Technology and instruments must be developed first

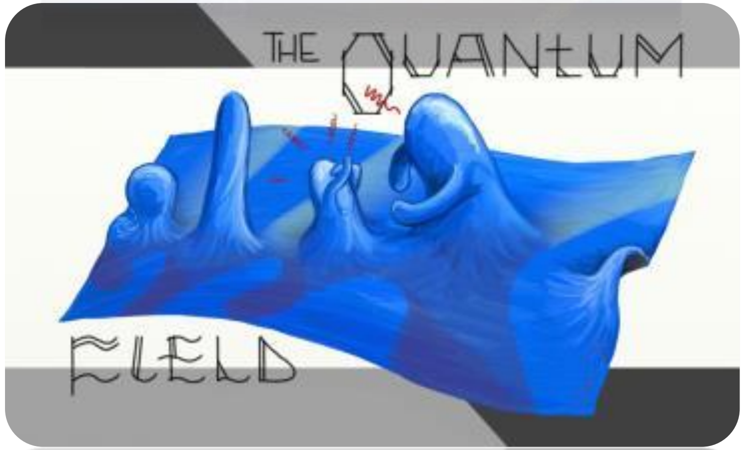
The interplay of theory and experiment

Main aim: Develop new instruments
window to scales that have not been explored before
New Physics!

Quantum technologies can help us put the pieces of the puzzle together

Quantum metrology



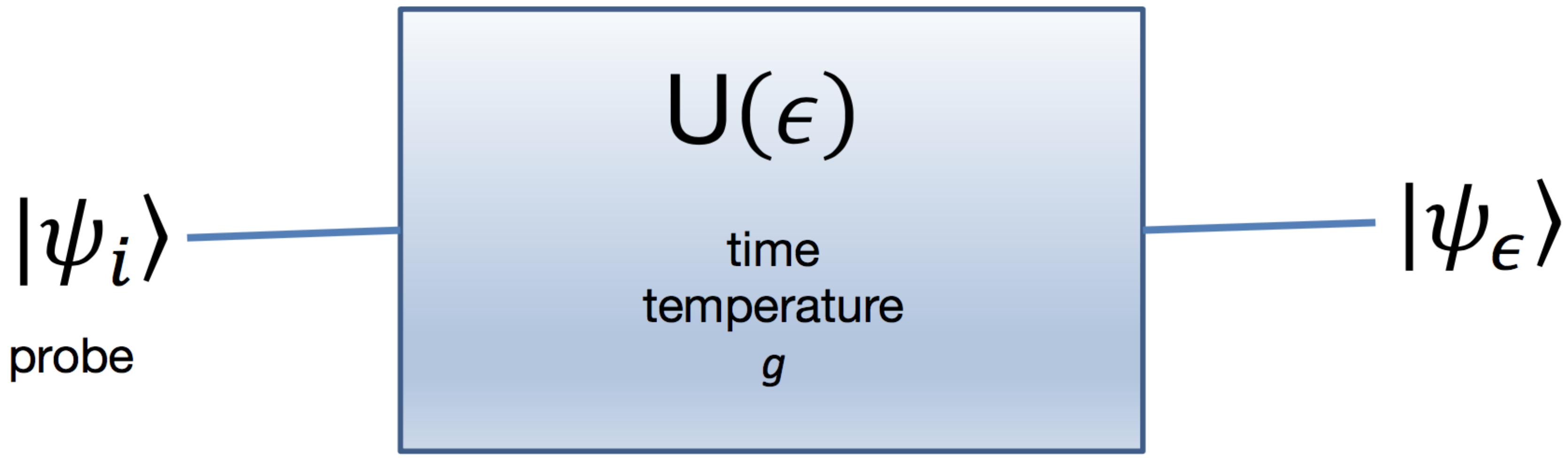


- Enables ultrasensitive devices for measuring fields, frequencies, time
- Quantum clocks and sensors are being sent to space… relativity cannot be ignored

Used to measure gravitational parameters…

gravitational field strengths
accelerations

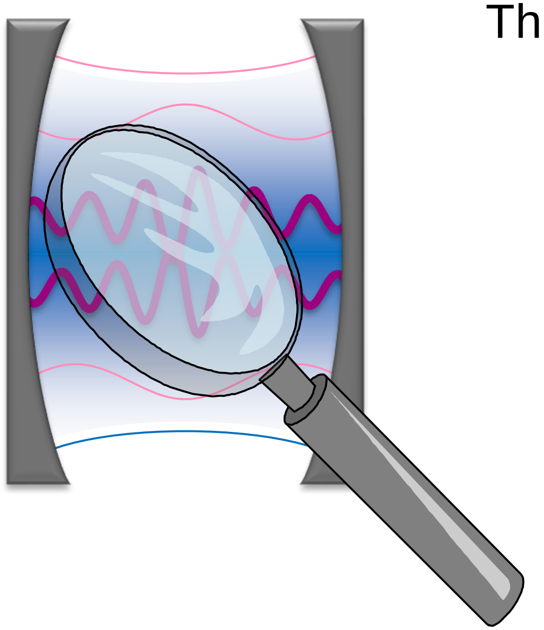
Quantum Metrology



$$\langle \psi_\epsilon | \psi_{\epsilon+d\epsilon} \rangle \ll 1$$

Exploit quantum properties of the probe state to estimate with high precision parameters in the theory (Hamiltonian)

Quantum Metrology



There is an optimal measurement such that

parameter

$$\langle (\Delta\hat{\epsilon})^2 \rangle \geq \frac{1}{M H_\epsilon}$$

Error

Quantum Fisher information

M number of measurements

$$H_\epsilon = \frac{8\left(1 - \sqrt{\mathcal{F}(\sigma_\epsilon, \sigma_{\epsilon+d\epsilon})}\right)}{d\epsilon^2}$$

Fidelity

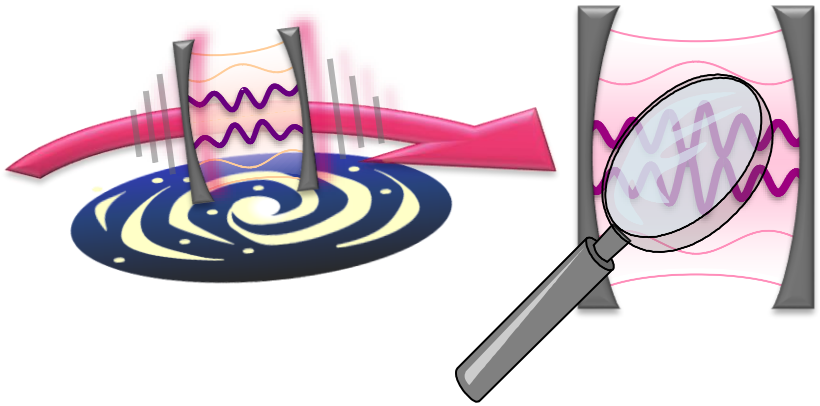
state σ_ϵ

$\langle \psi_\epsilon | \psi_{\epsilon+d\epsilon} \rangle \ll 1$

General framework for RQM

Ahmadi, Bruschi, Sabin, Adesso, Fuentes, Nature Sci. Rep. 2014
Ahmadi, Bruschi, Fuentes PRD 2014

Fisher information in QFT:
Analytical formulas in terms of
general Bogoliubov coefficients



Single-mode
Two-mode channels

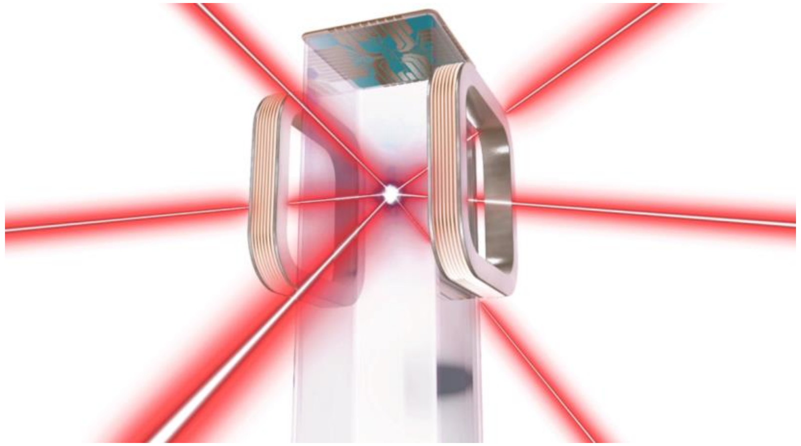
for small parameters

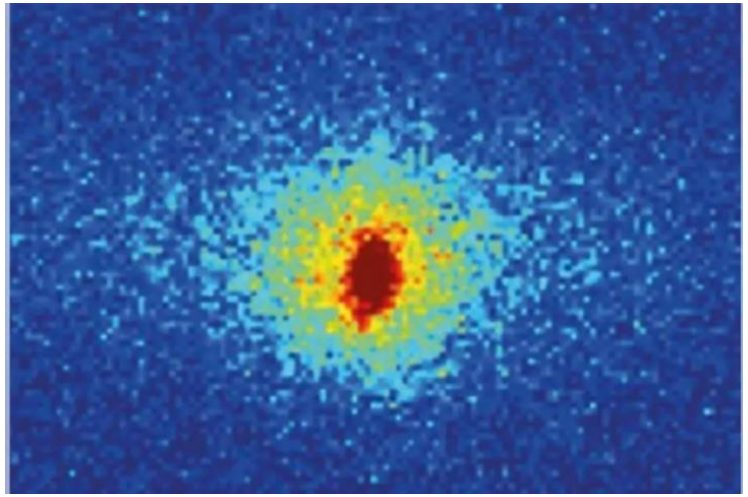
$$
\begin{aligned}
H &= \epsilon^{-2}\Re\Big[4\cosh r(f_\alpha^n + f_\beta^n + f_\alpha^m + f_\beta^m) \\
&+ 4\cosh^2 r(2|\beta_{nm}(t)|^2 - f_\alpha^n + f_\beta^n - f_\alpha^m + f_\beta^m) \\
&- 4\sinh^2 r(-f_\alpha^n + f_\beta^n - f_\alpha^m + f_\beta^m + 2\beta_{nm}(t)^2 - 2\alpha_{nm}(t)^2) \\
&+ 4\sinh r\,\Re[\mathcal{G}^{\alpha\beta}_{nm} + \mathcal{G}^{\alpha\beta}_{nm}] - 4\cosh^4 r|\beta_{nm}(t)|^2 \\
&- \frac{1}{2}\sinh^2 2r(2|\alpha_{nm}(t)|^2 - 3|\beta_{nm}(t)|^2 - \beta_{nm}(t)^2\Big].
\end{aligned}
$$

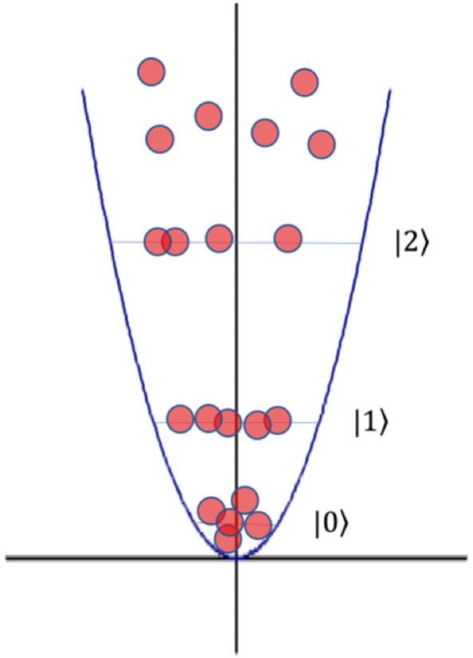
$$
\begin{aligned}
f_\alpha^i &= \frac{1}{2}\sum_{n\neq k,k'}|\alpha_{ni}|^2 \\
f_\beta^i &= \frac{1}{2}\sum_{n\neq k,k'}|\beta_{ni}|^2 \\
\mathcal{G}^{\alpha\beta}_{ij} &= \sum_{n\neq k,k'}\alpha_{ni}\beta^*_{nj}
\end{aligned}
$$

.

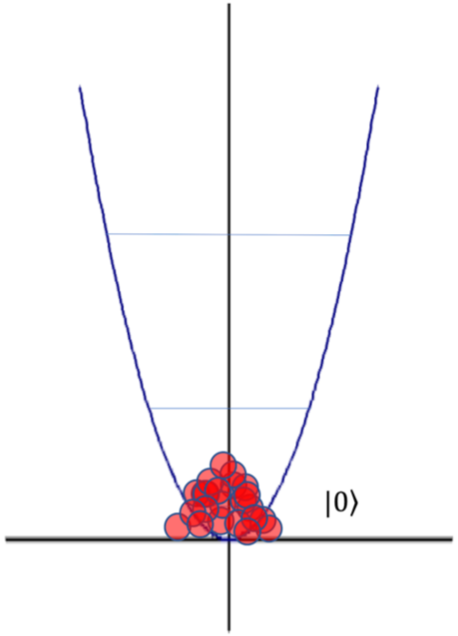
Bose-Einstein Condensate (BEC)





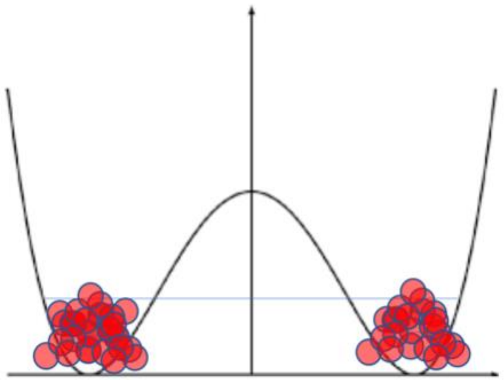


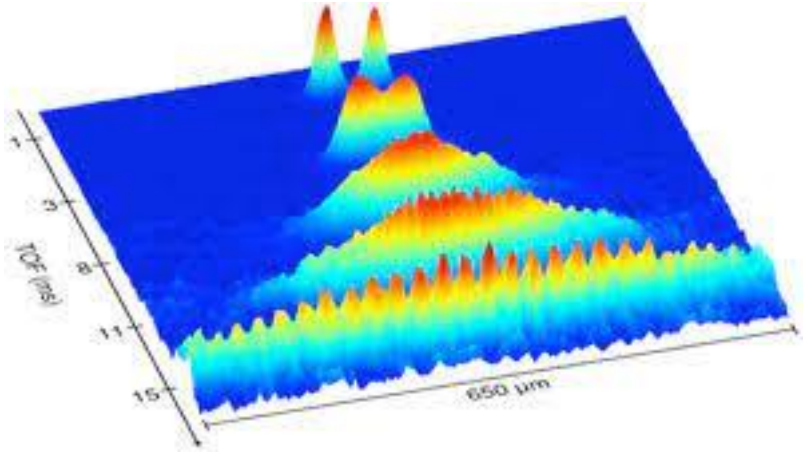
Cold atoms in a thermal distribution



Atoms in a BEC form a lump

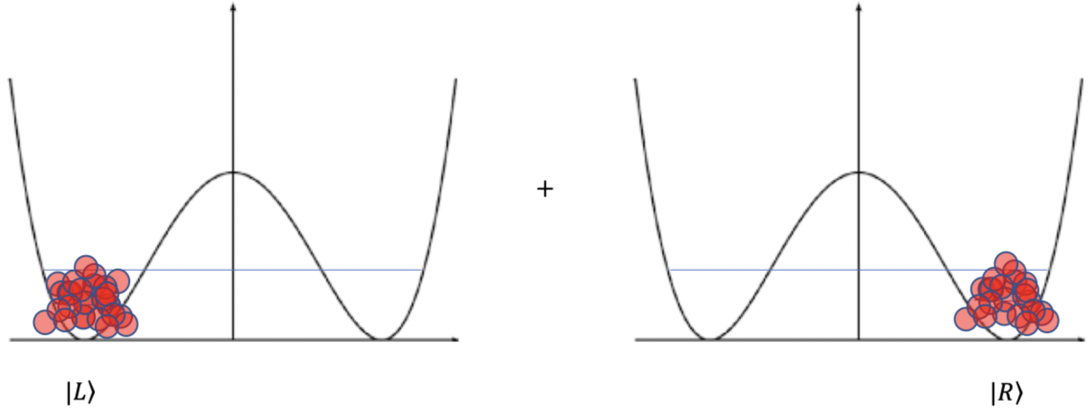
Spatial superpositions in a double-well potential





Schrödinger Cat state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|N_L 0_R\rangle + |0_L N_R\rangle)$$



Test Penrose's gravitational induced collapse

Decoherence time must be longer than the collapse time

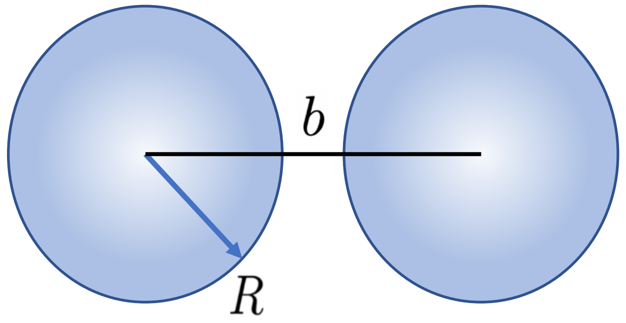
$$\tau \sim \frac{\hbar}{E_G} \qquad E_G = \frac{13Gm^2N^2}{14R}.$$

Non-uniform spherical just touching

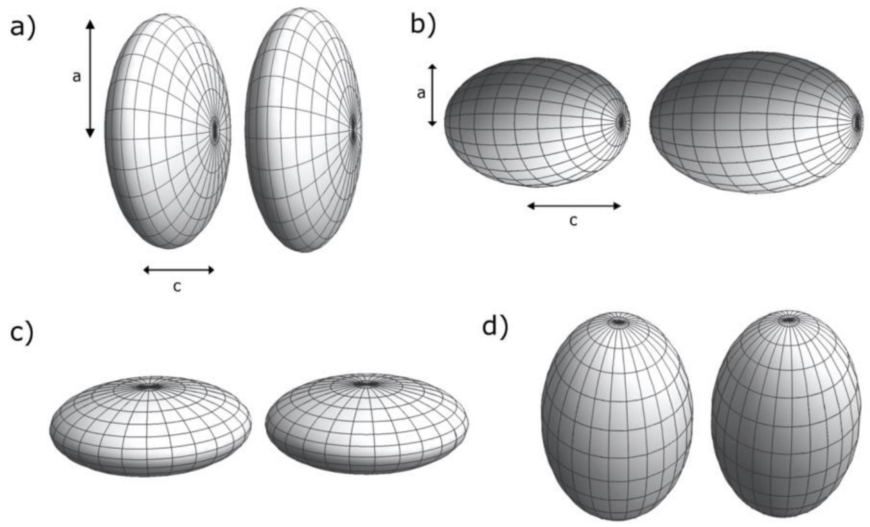
Gravitational self-energy

For a uniform sphere

$$E_G = \begin{cases} \frac{6GM^2}{5R}\left(\frac{5}{3}\lambda^2 - \frac{5}{4}\lambda^3 + \frac{1}{6}\lambda^5\right) & \text{if } 0 \leq \lambda \leq 1, \\ \frac{6GM^2}{5R}\left(1 - \frac{5}{12\lambda}\right) & \text{if } \lambda \geq 1, \end{cases}$$



where $\lambda := b/(2R)$



Gravitational collapse depends strongly in the mass geometry: signature

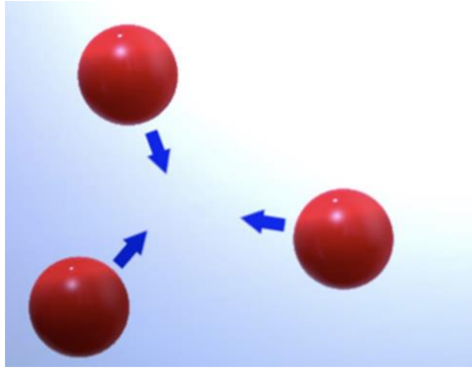
a) and b) can collapse at shorter times depending on the ellipticity

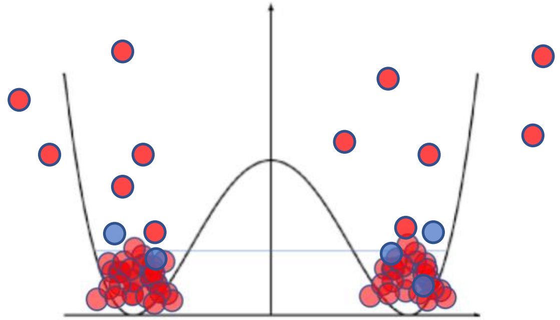
Howl, Penrose & Fuentes, NJP 2019

Decoherence and noise

Howl, Penrose & Fuentes, NJP 2019

- Three-body recombination
- Two-body losses
- Thermal cloud interactions
- Foreign atom interactions
- Decoherence from the trapping potential





Advantage: atom-atom interactions can be controlled in a BEC

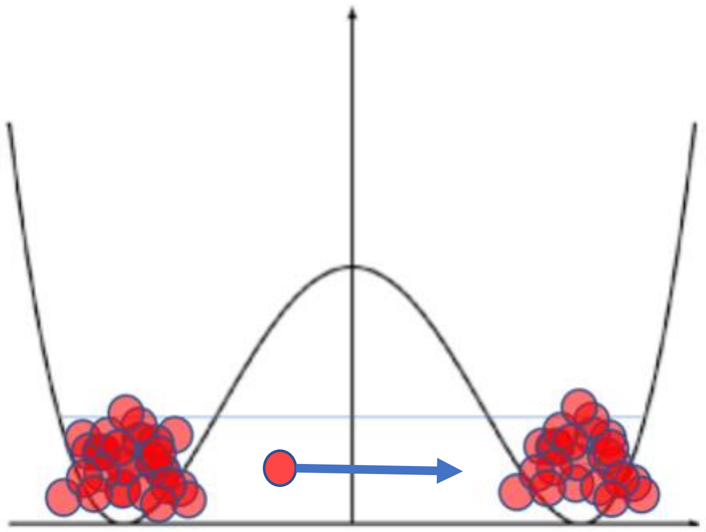
It would be convenient to supressed them after the superposition has been created.

$\gamma = 1/(8\pi)$ A ^{133}Cs BEC with 4x10^9 atoms and R=1 μm would collapse in approximately 2 seconds

$\gamma = 8\pi$ $2\,\mathrm{s}$ would occur when $N \approx 10^9$ and $R = 0.1\,\mathrm{mm}$ or $N \approx 10^8$ and $R = 1\,\mu\mathrm{m}$.

Other BEC states

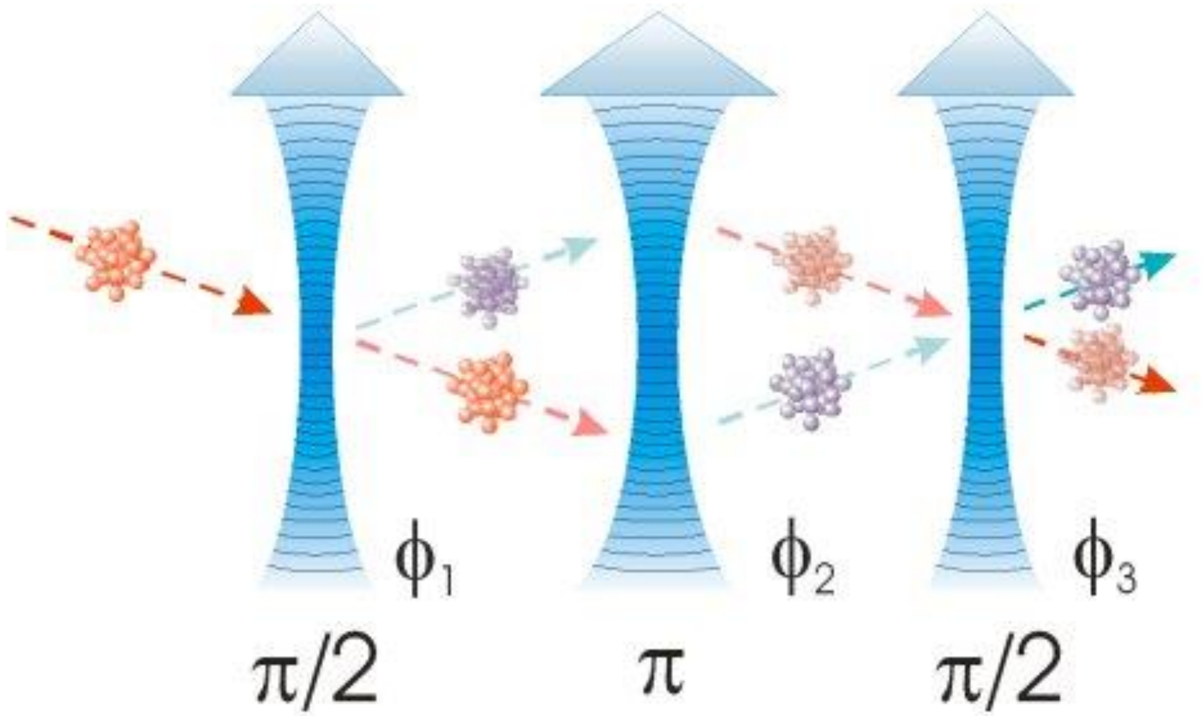
- Work in progress with Penrose & Westbrook



In a BEC atoms are not bound together and can tunnel between left and right wells.

We are studying gravitational self-energy for a variety of BEC quantum states

Atom interferometer: quantum spatial interferometry



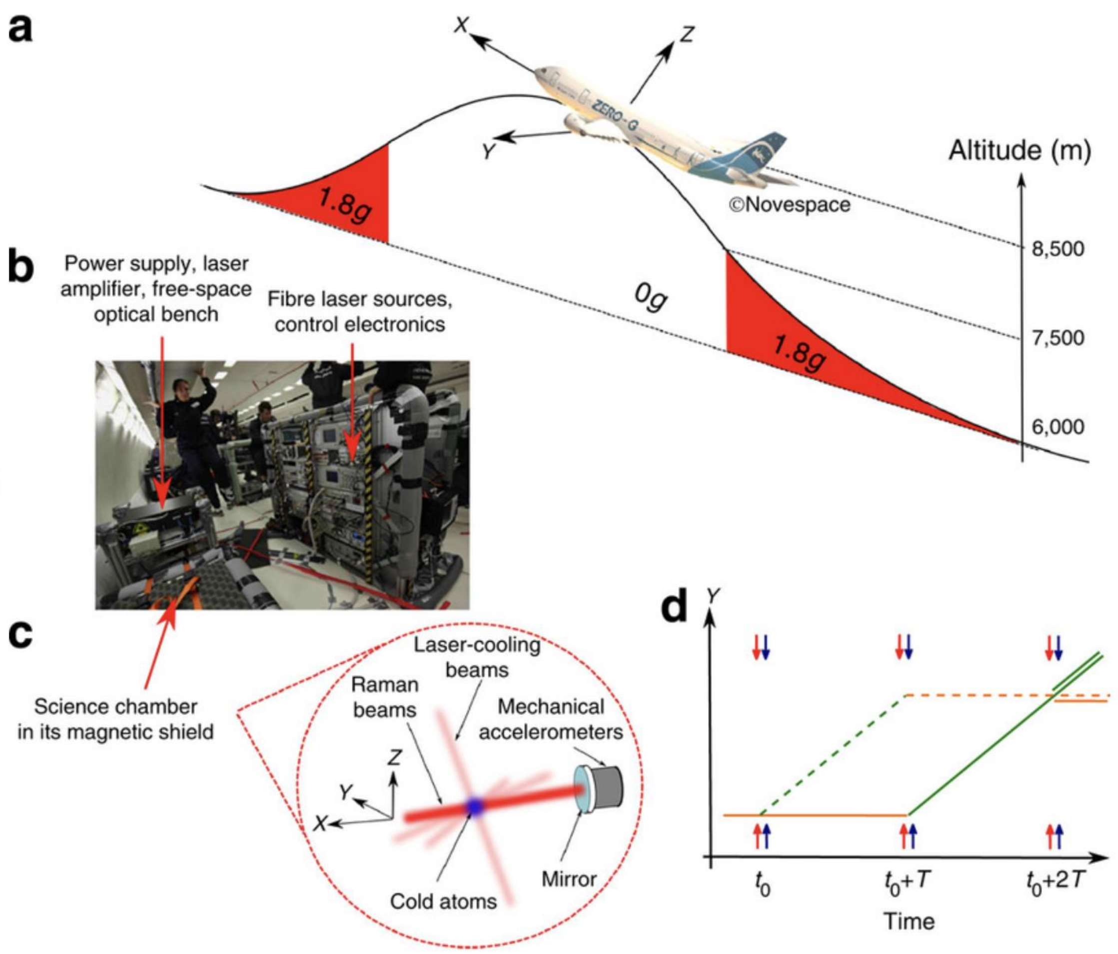
$$\delta a = 1 \Big/ \left(\sqrt{N} k T^2 \right)$$

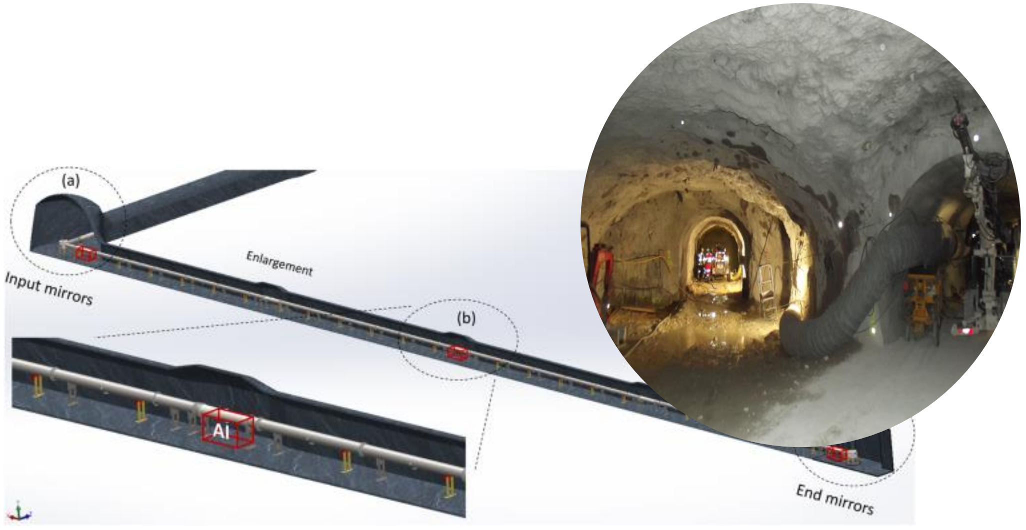
Single particle detector, local

Interferometry in the spatial domain: limited by time of flight

Compatible with Newtonian physics

Gravimeters are going Big






Physics ABOUT BROWSE PRESS COLLECTIONS

Viewpoint: Free-Falling Interferometry

Markus Arndt, Faculty of Physics, VCQ, QuNaBioS, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austr

February 25, 2013 • Physics 6, 23

Atom interferometry in free fall demonstrates fundamental quantum physics and a new level of technology

But…I want one in my phone

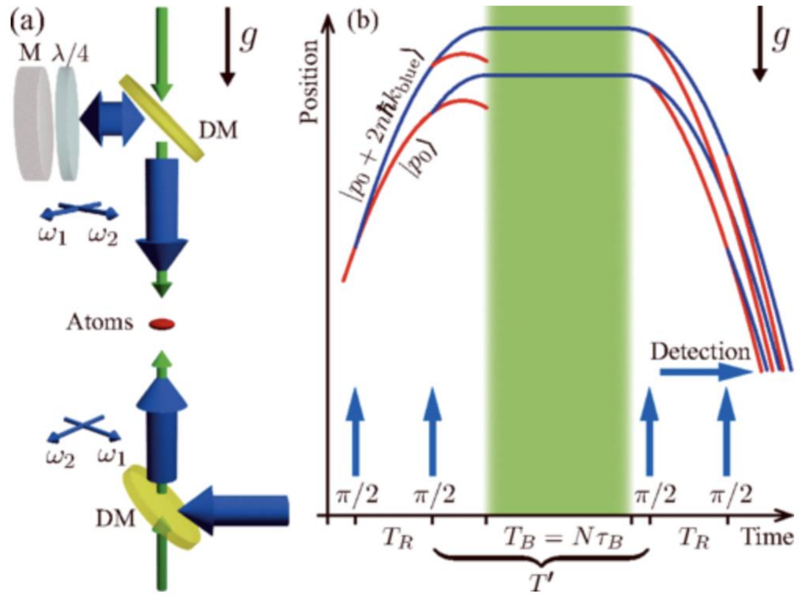
Change of paradigm!

PHYSICAL REVIEW A
covering atomic, molecular, and optical physics and quantum information

Trapped-atom interferometer with ultracold Sr atoms

Xian Zhang, Ruben Pablo del Aguila, Tommaso Mazzoni, Nicola Poli, and Guglielmo M. Tino
Phys. Rev. A **94**, 043608 – Published 4 October 2016

Images



MINIATURIZED ATOM-CHIP GRAVIMETER

Matterwave interferometers based on cold atoms are commonly used as gravimeters. They reach accuracies of up to 10^{-9}g and are nowadays even commercially available.

We have demonstrated a compact quantum gravimeter, which employs an atom chip for the rapid and efficient creation of Bose-Einstein condensates (BEC). At the same time, the atom chip serves for complete state preparation of the atomic cloud and as a retroreflector for the laser beam to create an optical lattice. With the lattice, we split, redirect, and recombine the BEC to form a Mach-Zehnder interferometer and measure the local gravitational acceleration.

To extend the interferometer time and increase the device's sensitivity, we employ the optical lattice for an innovative launch mechanism. In this way, we acquire an intrinsic sensitivity of $\Delta g/g = 10^{-7}$, while keeping all atom-optical operations in a volume of less than a one-centimeter cube.

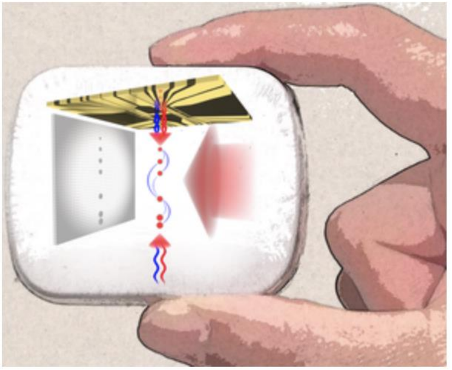


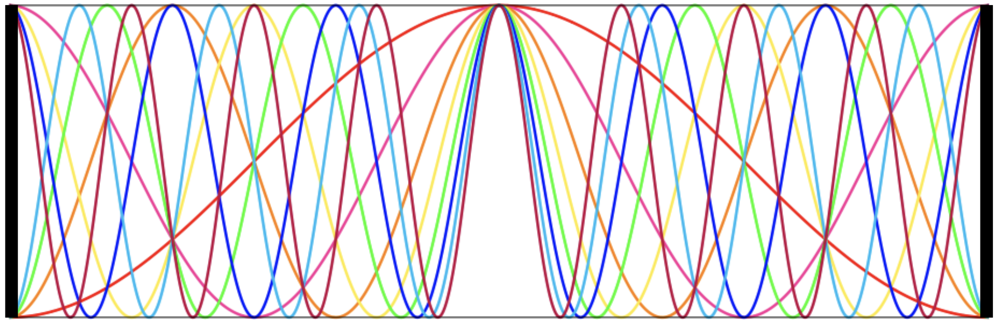
Image by S. Abend and E. Rasel/Leibniz Univ. of Hannover (Physics 9, 131, 2016)

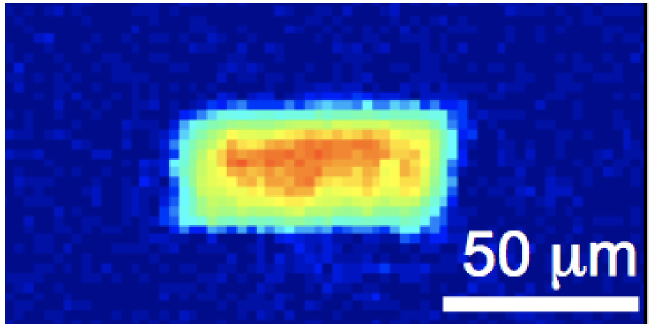
RELATED PUBLICATIONS

S. Abend et. al. *Atom-Chip Fountain Gravimeter* Phys. Rev. Lett. **117**, 203003 (2016)

Quantum frequency interferometry

Howl & Fuentes arXiv:1902.09883





Uses interactions: collective excitations, entanglement between atoms

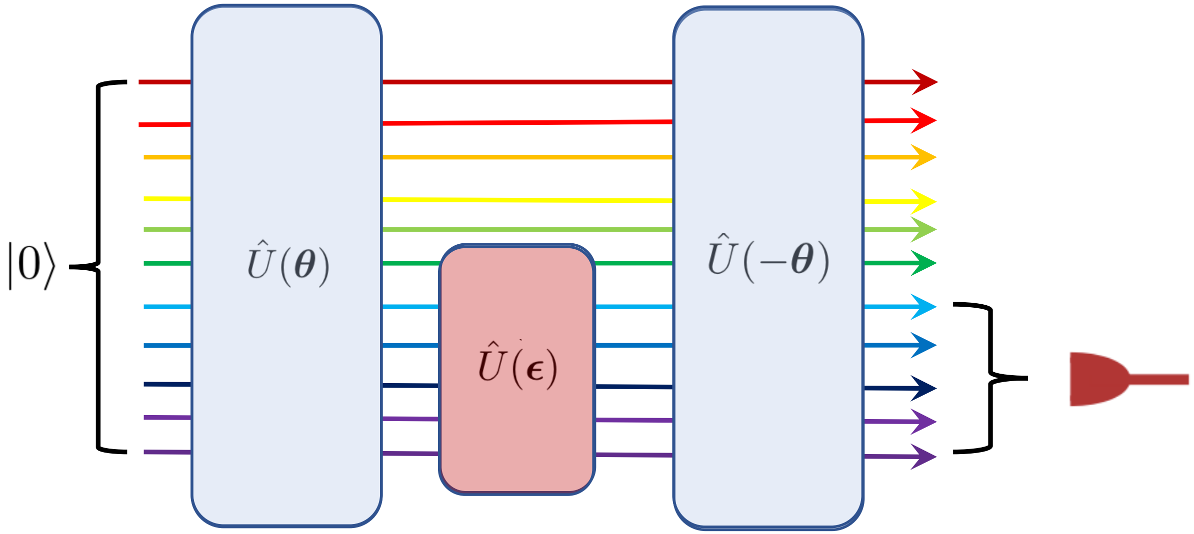
Implementation of frequency modes: phonons in a BEC (massless quantum field)

Interferometry in the frequency (time) domain, non-local

We use parametric amplification produced by the non-linearity introduced by atomic collisions

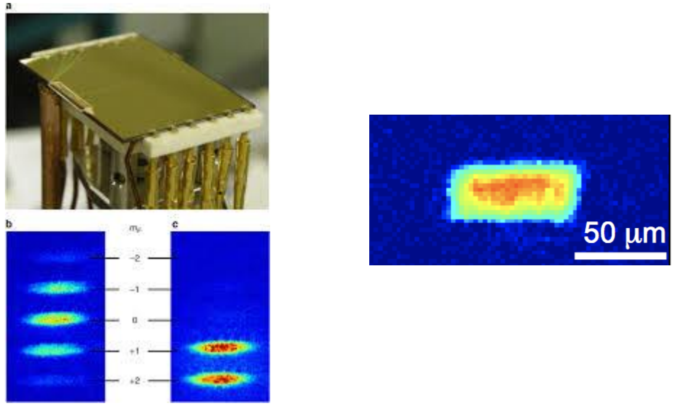
Compatible with General Relativity: underpinned by QFT in curved spacetime

BECs and quantum frequency interferometry



- Detector can be miniaturized
- High sensitivity
- High resilience to noise

Howl & Fuentes arXiv:1902.09883

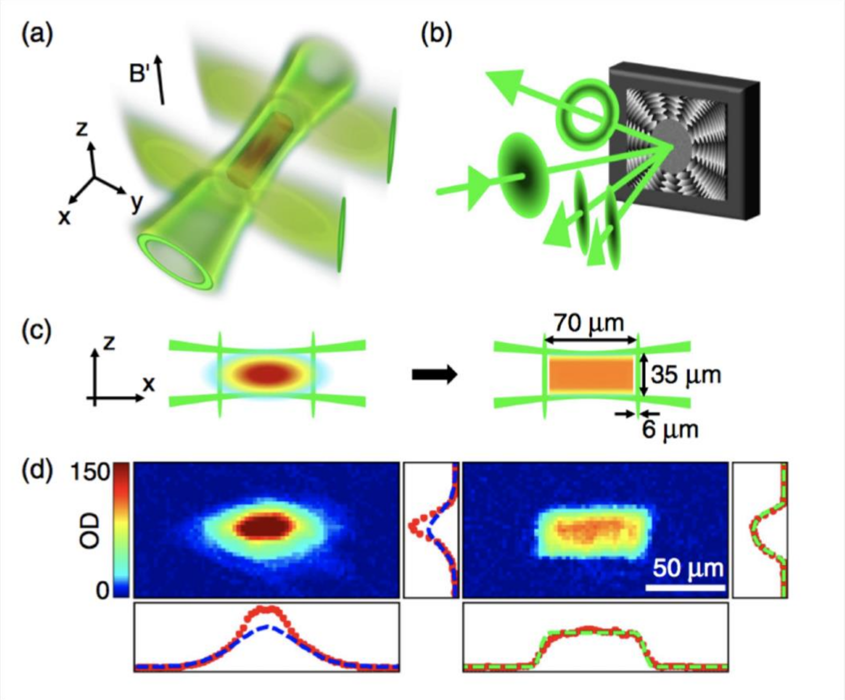


Quantum sensors underpinned by QFTCS

- Continuous source gravitational wave detector
- Quantum relativistic clocks
- Dark energy
- Proper acceleration
- Local gravitational fields (UK patent No.1908538.0)
- Gravitational gradient (UK patent No. 2000112.9)
- Curvature
- Spacetime parameters
- Dark Matter!

Demonstrate particle creation by spacetime dynamics!

Bose Einstein Condensate in a box



mean field (ground state)

$$\widehat{\Phi} = \phi(1 + \widehat{\psi})$$

phonons
density fluctuations due to interactions

quasi-uniform density

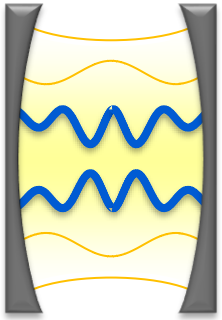
$$\rho = \phi^{\dagger}\phi$$

Gaunt *et. al.* PRL 110 200406 (2013)

BEC in spacetime

A covariant formalism is available

Phonons are a relativistic quantum field



$$\mathcal{L} = -\sqrt{-g}\left\{ g^{\mu\nu}\partial_\mu \hat{\Phi}^\dagger \partial_\nu \hat{\Phi} + \left(\frac{m^2c^2}{\hbar^2} + V\right)\hat{\Phi}^\dagger\hat{\Phi} + U\left(\hat{\Phi}^\dagger\hat{\Phi}, \lambda_i\right)\right\}$$

$$U\left(\hat{\Phi}^\dagger\hat{\Phi}, \lambda_i\right) \approx \frac{\lambda}{2}\hat{\Phi}^\dagger\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi}$$

$$\widehat{\Phi} = \phi(1 + \widehat{\psi})$$

For uniform density

$$\Box\,\psi = \frac{1}{\sqrt{-\mathfrak{g}}}\partial_\mu(\sqrt{-\mathfrak{g}}\,\mathfrak{g}^{\mu\nu}\partial_\mu)\psi$$

effective metric

$$\mathfrak{g}_{ab} = \rho\frac{c}{c_s}\left[g_{ab} + \left(1 - \frac{c_s^2}{c^2}v_a v_b\right)\right]$$

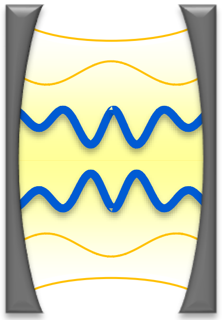
spacetime metric

analogue metric

Demonstrate QFT in CS

Fagnocchi et. al NJP 2010 (quantum field flat spacetime)
Visser & Molina-Paris NJP 2010 (classical field in curved spacetime)
D. E. Bruschi et. al., NJP 16 5:053041, 2014 (quantum field in curved spatime)
D. Hartley. et. al., PRD 89 025011, 2018

BEC in flat spacetime



$$\mathfrak{g}_{ab} = \left(\frac{n_0^2\, c_s^{-1}}{\rho_0 + p_0}\right) \begin{pmatrix} -c_s^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Minkowski space but with speed of sound

$$\tau = (c/c_s)t \quad \Rightarrow \quad ds^2 = -cdt^2 + dx^2$$

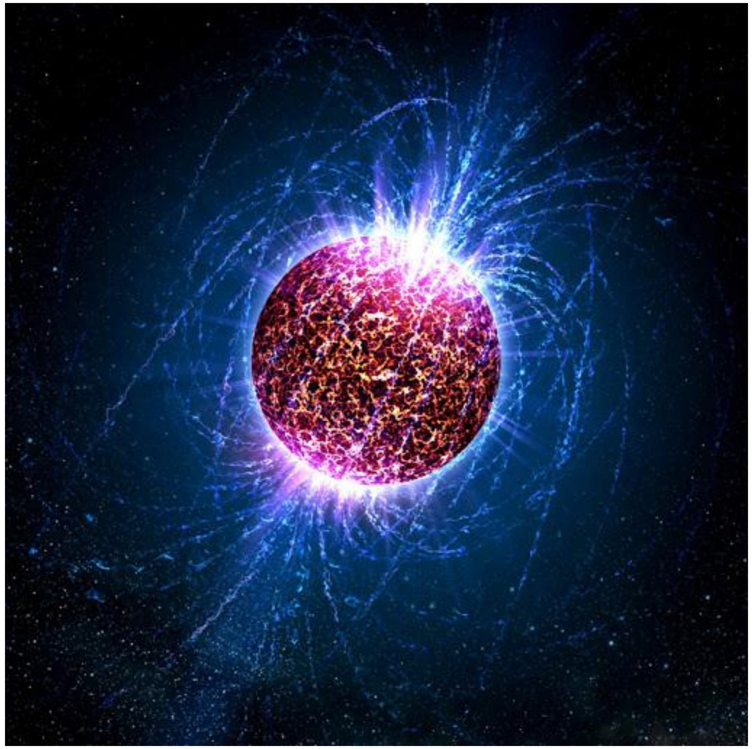
phonons in a cavity-type 1-dimensional trap

$$\omega_n = \frac{n\,\pi\,c_s}{L}$$ spectrum

$$\Box\phi(t,x) = 0$$

$$\phi_n = \frac{1}{\sqrt{n\,\pi}} \sin\frac{n\pi(x - x_L)}{L}\, e^{-i\,\omega_n\,t}$$

Solutions to the K-G equation



Gravitational wave spacetime

$$\mathfrak{g}_{ab} = \left(\frac{n_0^2\, c_s^{-1}}{\rho_0 + p_0}\right) \begin{pmatrix} -c_s^2 & 0 & 0 & 0 \\ 0 & 1 + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & 1 - h_+(t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & -h_+(t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

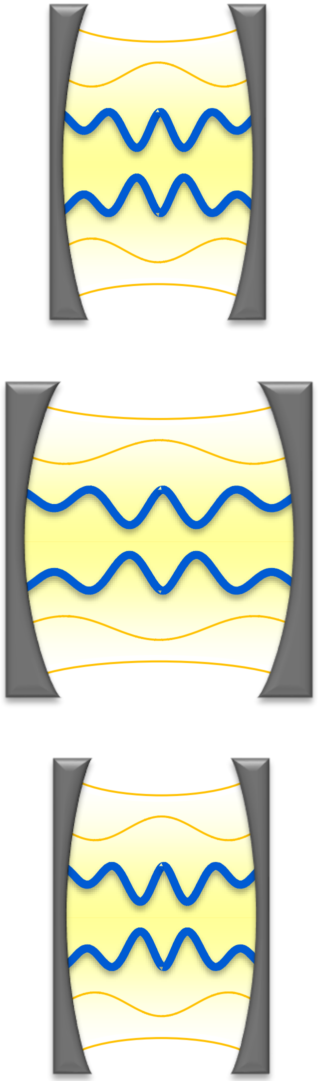
In a one-dimensional trap

$$ds^2 = -c_s^2\, dt^2 + (1 + h_+(t))\, dx^2.$$

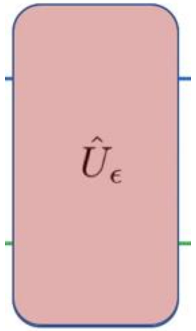
$$h_+(t) = \epsilon \sin \Omega t$$ Continuous sources

$$\omega_n = \frac{n\, \pi\, c_s}{L}$$ Resonance!

Field transformations



The dynamics of spacetime produces a Bogoliubov transformation on the field modes



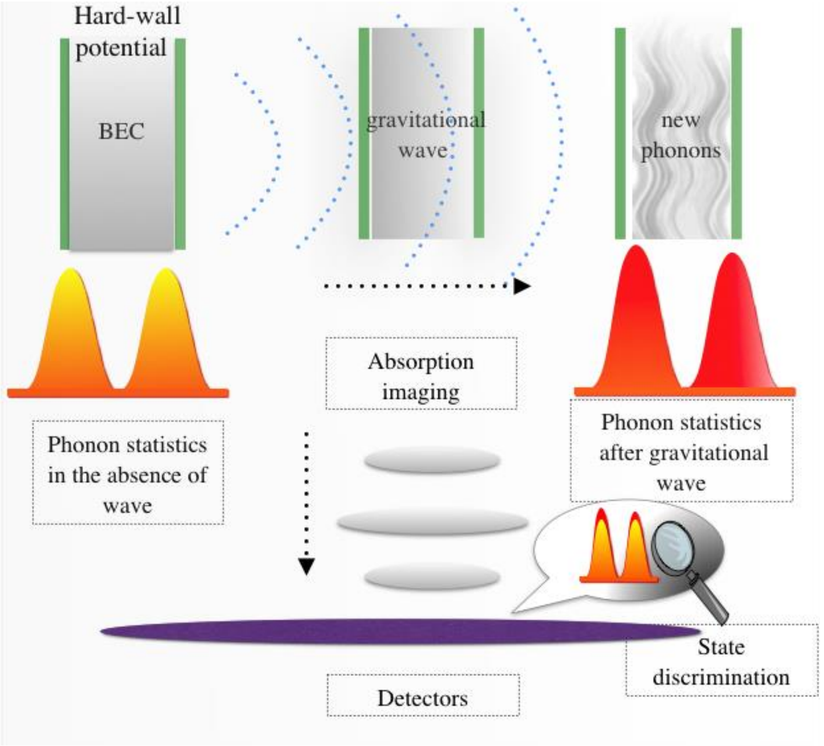
$$\tilde{a}_m = \sum_n \left(\alpha^*_{mn} \, a_n - \beta^*_{mn} \, a^\dagger_n \right)$$

$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n) \text{ and } \beta_{mn} = -(\tilde{\phi}_n, \phi^*_m)$$

Bogoliubov transformation for monocromatic gravitational wave

$$\beta_{jk}(t) = -\frac{\epsilon}{2}\sqrt{\frac{n}{m}}\,\omega_m \, t \, [-x_L + (-1)^{m+n}(L + x_L)]\delta_{jm}\,\delta_{kn} + \mathcal{O}(\epsilon^2)$$
$$\alpha_{jk}(t) = 0 + \mathcal{O}(\epsilon^2),$$

Application: gravitational wave detector



Phonon creation by gravitational waves

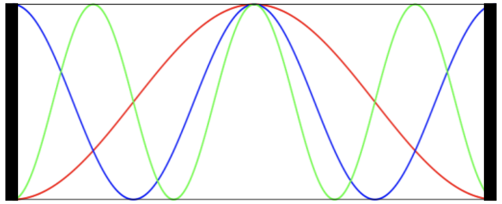
Carlos Sabín[1], David Edward Bruschi[2], Mehdi Ahmadi[1] and Ivette Fuentes[1]

Published 7 August 2014 •

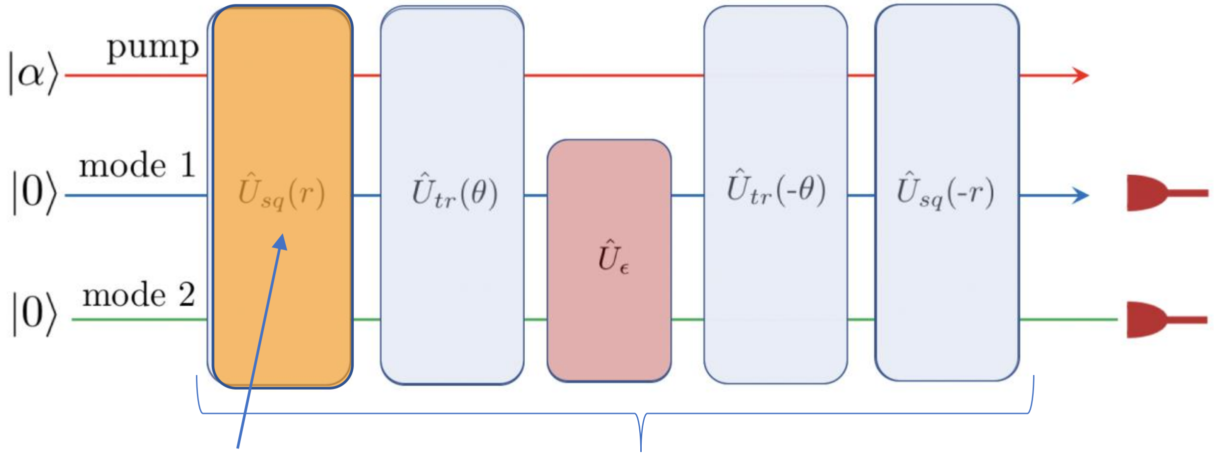
New Journal of Physics, Volume 16, August 2014

Requires high phonon numbers $\sim\sqrt{N_0}$
and long phonon lifetimes ~10s

Three mode application

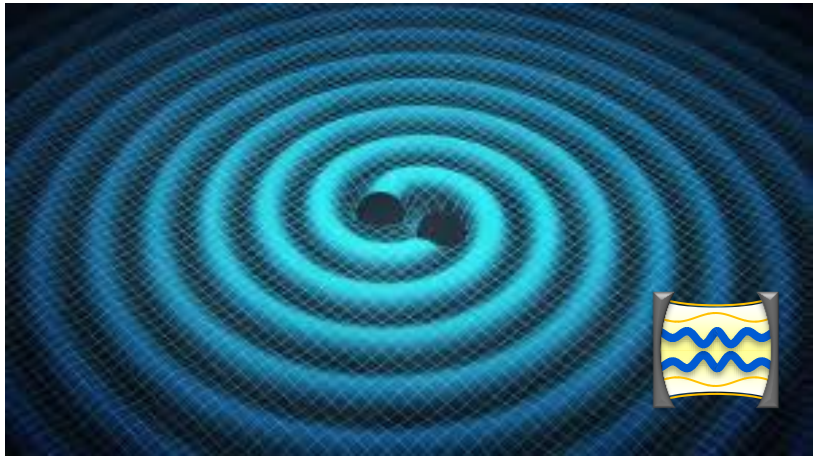


Circuit representation

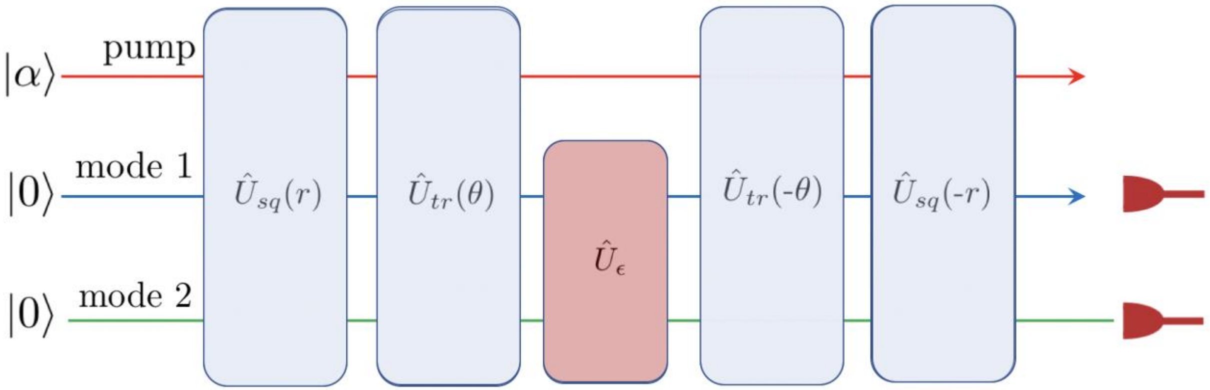


Sabin et al 2014 Howl & Fuentes arXiv:1902.09883

Improves the sensitivity by several orders of magnitude.
Squeezing can be much smaller than assumed previously and the system can suffer from short phononic lifetimes.



Interferometric transformations



Two-mode squeezing operation

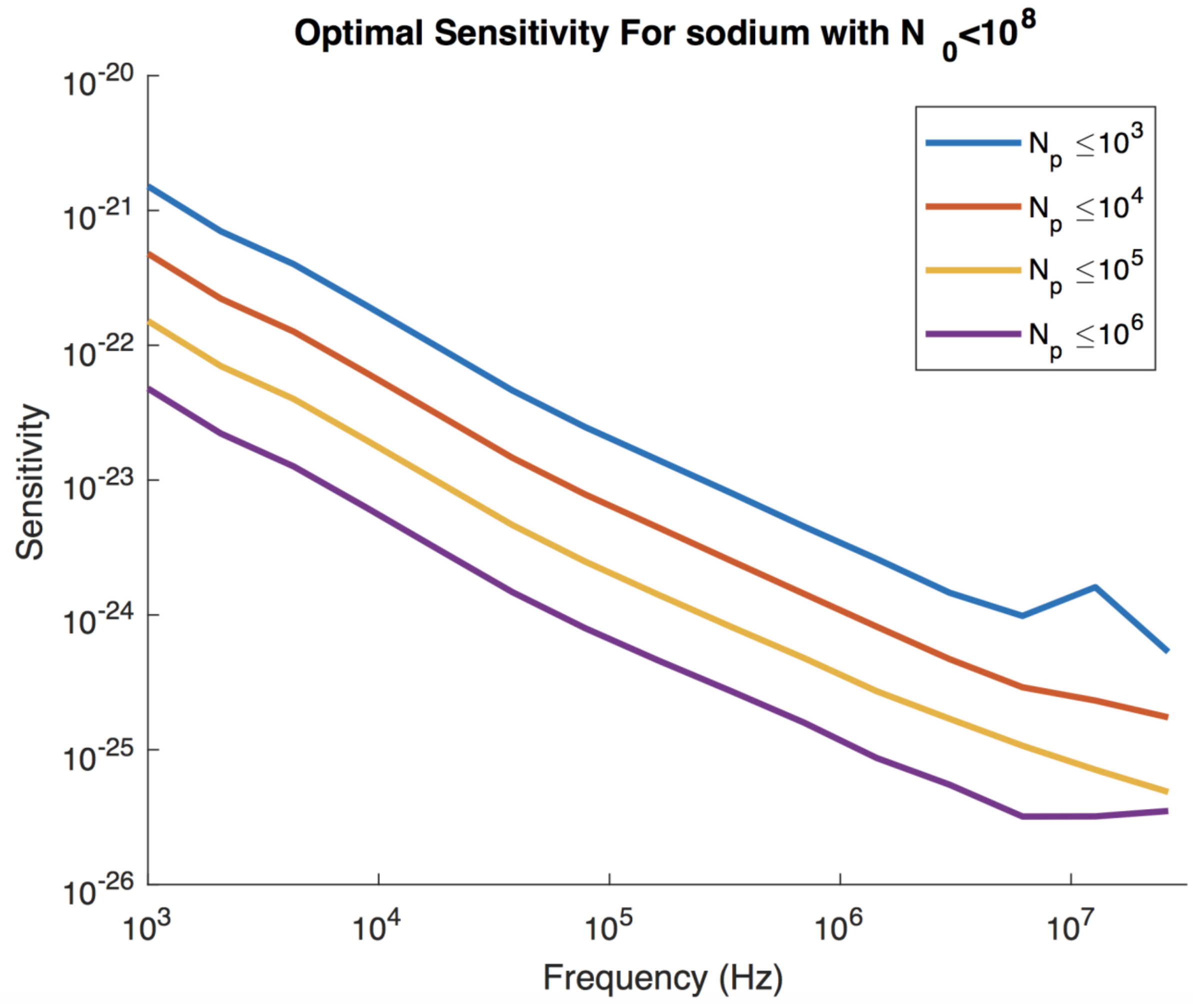
$$\boldsymbol{S}_s = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cosh r\mathbf{1} & \sinh r(\cos\vartheta_{sq}\boldsymbol{\sigma_z} + \sin\vartheta_{sq}\boldsymbol{\sigma_x}) \\ \mathbf{0} & \sinh r(\cos\vartheta_{sq}\boldsymbol{\sigma_z} + \sin\vartheta_{sq}\boldsymbol{\sigma_x}) & \cosh r\mathbf{1} \end{pmatrix},$$

The tritter transformation is

$$\boldsymbol{S}_{tr} = \begin{pmatrix} \cos\theta\mathbf{1} & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & \cos^2(\frac{\theta}{2})\mathbf{1} & -\sin^2(\frac{\theta}{2})\mathbf{1} \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & -\sin^2(\frac{\theta}{2})\mathbf{1} & \cos^2(\frac{\theta}{2})\mathbf{1} \end{pmatrix}.$$

Howl & Fuentes arXiv:1902.09883

Detector sensitivity



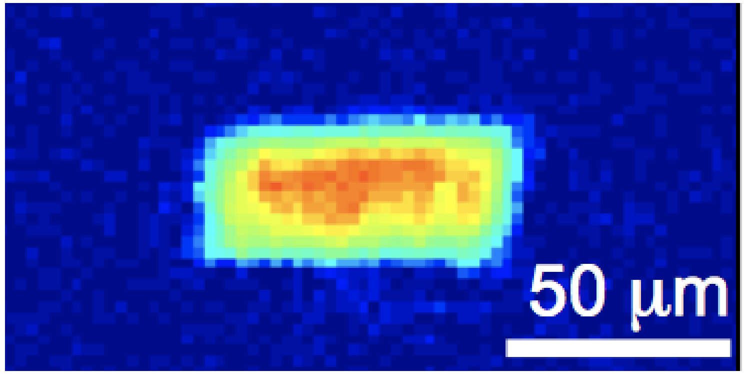
$$\Delta\epsilon \geq \frac{1}{\sqrt{MF_Q}}$$

$$\Delta\epsilon \geq \frac{m}{\sqrt{2\pi}\hbar}\frac{\alpha^3}{\theta N_0^2\sqrt{N_p\tau t}}\sqrt{\frac{L^7}{a^3}\frac{\sqrt{nl}\,(l-n)^2}{(l^2+n^2)}}$$

m atomic mass
θ tritter angle
N_0 number of atoms in the ground state
N_p number of phonons in the two-mode squeezed state
$\alpha = \sqrt{A}/L$, A area, L length
a scattering length
$M = \tau/t$ number of measurements
τ integration time
t interaction time (lifetime of the phonons)
$\Omega = \omega_n + \omega_l$ the frequency of the gw
$\omega_j = j\pi c_s/L$ phonon frequency
n, l mode numbers

Howl & Fuentes arXiv:1902.09883

Constraints



	$(\gamma^{La}_{k_BT\ll\mu})^{-1}$ (1.56)	$(\gamma^{La}_{k_BT\gg\mu})^{-1}$ (1.56)	$(\gamma^{Be,0})^{-1}$ (1.55)	$t_{1/2}$ (3.3)
$t \approx$	$\frac{640}{3\pi}\frac{\hbar^7\beta^4 L n_0^3 a_s^2}{l m^3}$	$\frac{8}{\pi}\frac{\beta L}{a_s l}$	$\frac{640}{3\pi^6}\frac{m L^5 n_0}{\hbar l^5}$	$\frac{1}{50}\frac{m}{\hbar n_0^2 a_s^4}$
$r \propto$	$\frac{\hbar^8\beta^4 L^2 n_0^{9/2} a_s^{7/2}}{l^2 m^4}$	$\frac{\hbar^2 L^2 \beta n_0^{3/2} a_s^{1/2}}{l^2 m}$	$\frac{L^6 n_0^{5/2} a_s^{3/2}}{l^6}$	$\frac{L}{l n_0^{1/2} a_s^{5/2}}$
$\beta\omega_l$	$2\sqrt{\pi}\frac{\hbar l\sqrt{n_0 a_s}}{L m}$		weak interactions	$\lvert a_s\rvert n_0^{1/3} \ll 1$
phonon regime	$\sqrt{\frac{\pi l^2}{L^2 n_0 a_s}} \ll 1$		ultracold regime	$\frac{1}{4\pi}\frac{m}{\hbar^2\beta\, n_0\, a_s} \ll 1$

Table 8.2: Approximate values for the limitations to the phonon and condensate life times and resulting proportionalities of the maximum two-mode squeeze factor from SECTION 8.1 in the case where the respective damping or particle losses become dominant.
The two bottom rows show a measure $\beta\omega_l$ for the thermal occupation of the initial state, which should be minimal, and restrictions from the diluteness condition and the assumptions of the phononic and the ultracold regime.
To see where one could optimize and which boundaries will be encountered, all quantities above are given in terms of the parameters characterizing an individual experiment.

Quantum decoherence of phonons in Bose–Einstein condensates

Richard Howl[1], Carlos Sabín[2], Lucia Hackermüller[3] and Ivette Fuentes[1,4]

Published 29 November 2017 • © 2017 IOP Publishing Ltd

Journal of Physics B: Atomic, Molecular and Optical Physics, Volume 51, Number 1

Citation Richard Howl *et al* 2018 *J. Phys. B: At. Mol. Opt. Phys.* **51** 015303

P. Juschitz, Two-mode Phonon Squeezing in Bose-Einstein Condensates for Gravitational Wave Detection arXiv:2101.05051

Detector sensitivity

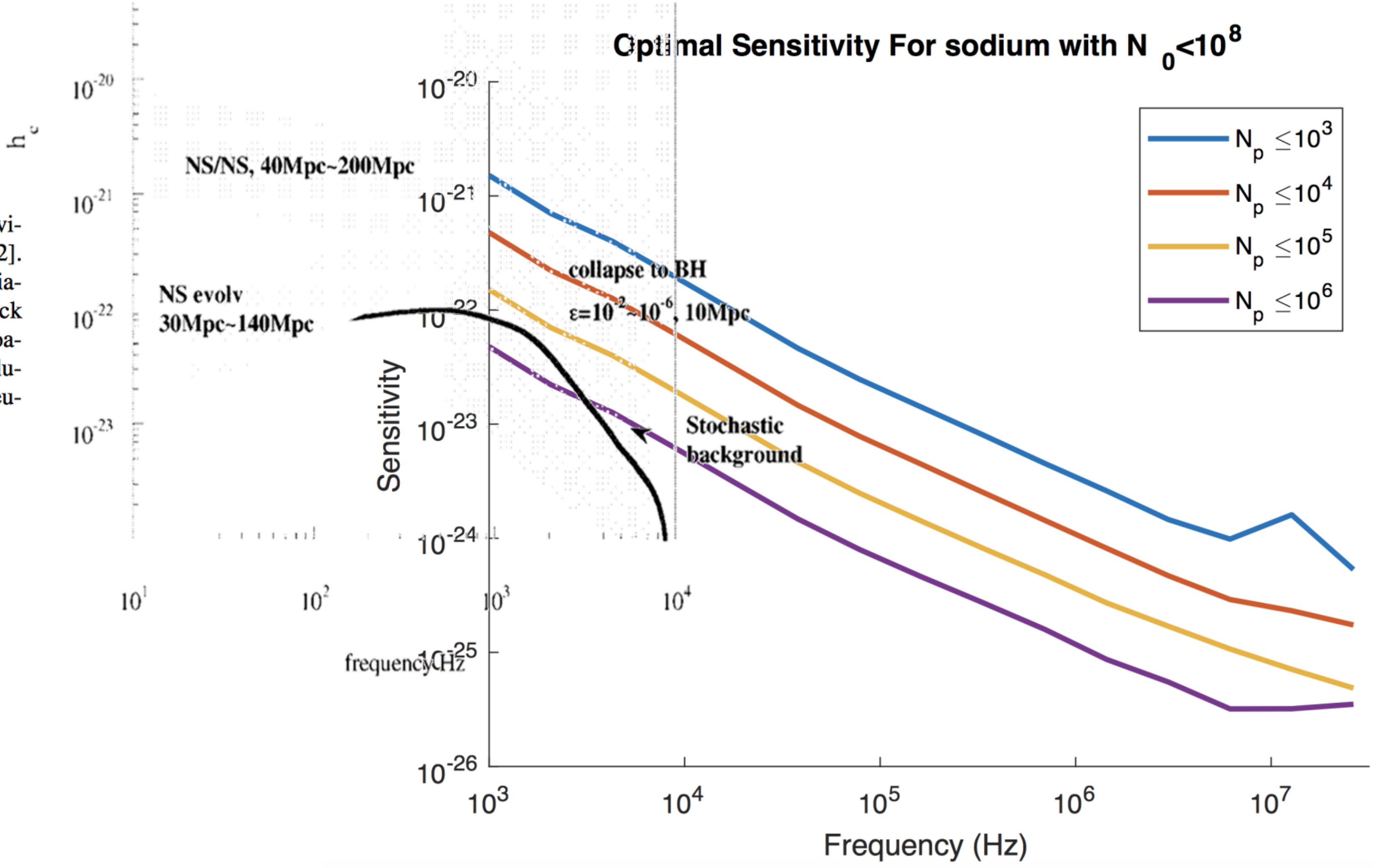
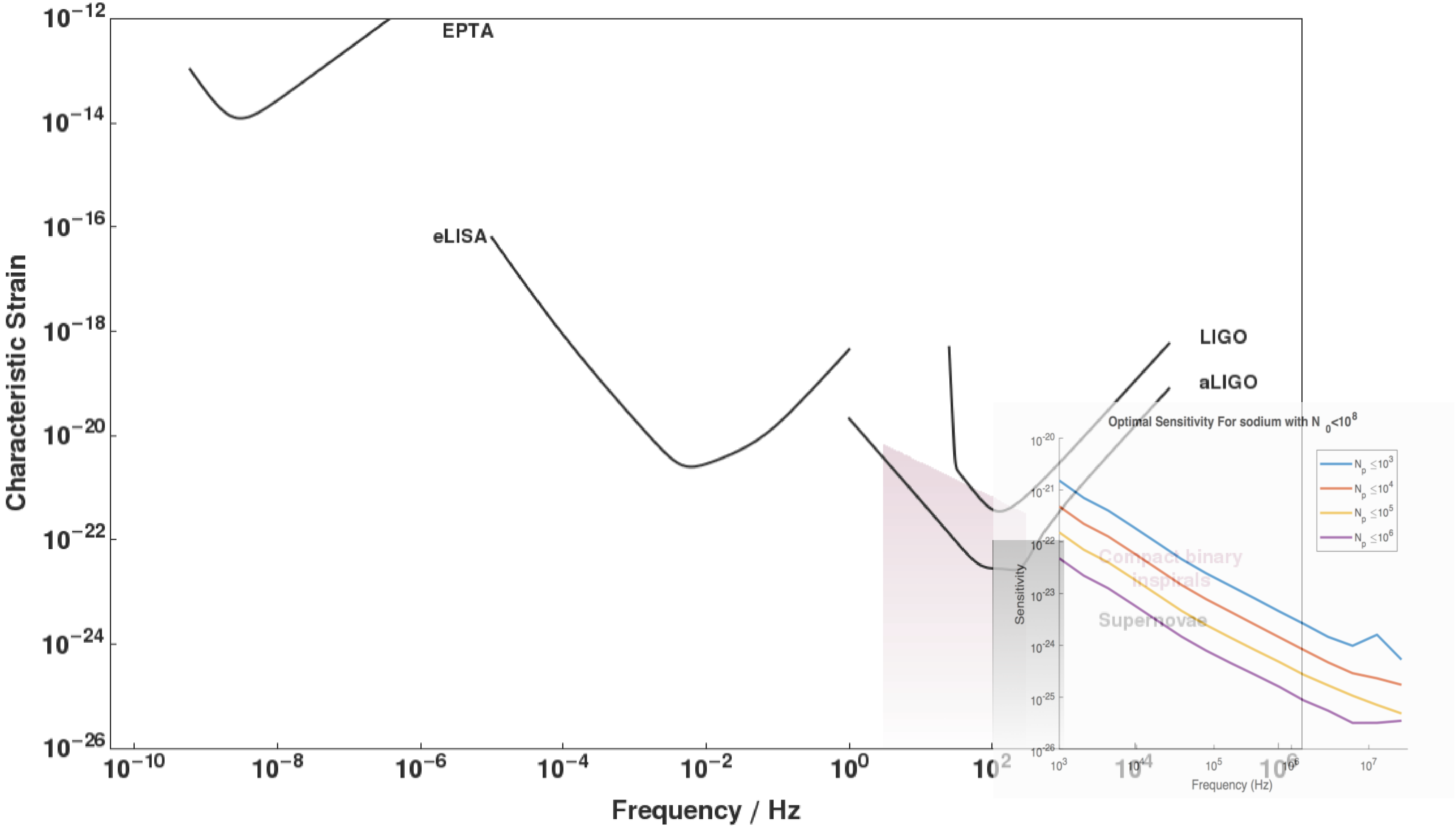


Figure 8. Spectrum of gravitational wave sources [18, 22]. In this figure, the abbreviations are: BH, collapse to black hole; NS/NS, neutron star coalescence; NS evol, secular evolution of a nonaxisymmetric neutron star.

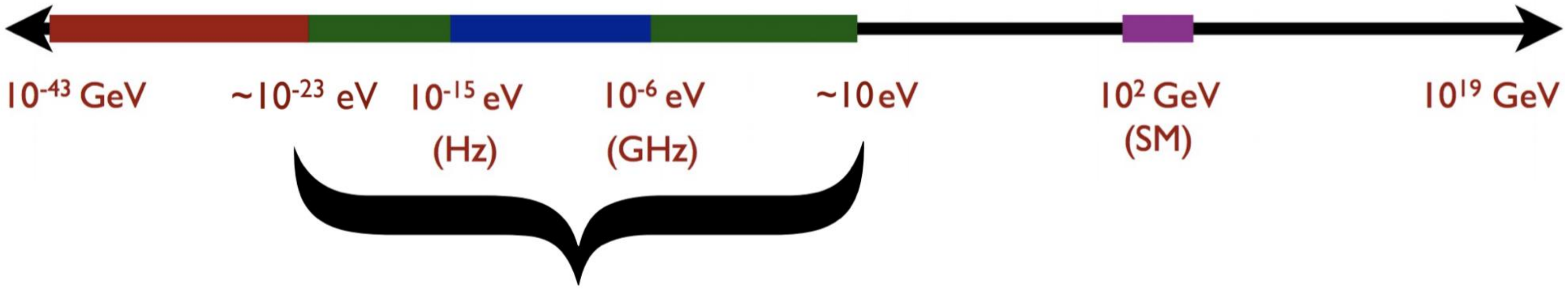
Search for yet unknow sources



Know sources: 10 kHz

Exotic sources:
primordial black holes
boson stars
Early Universe Cosmology
Phase transitions
Preheating after inflation,
Cosmic strings
Dark matter
Ultralight 10^{-8}- 10^{14} Hz
Decay: Penrose CCC

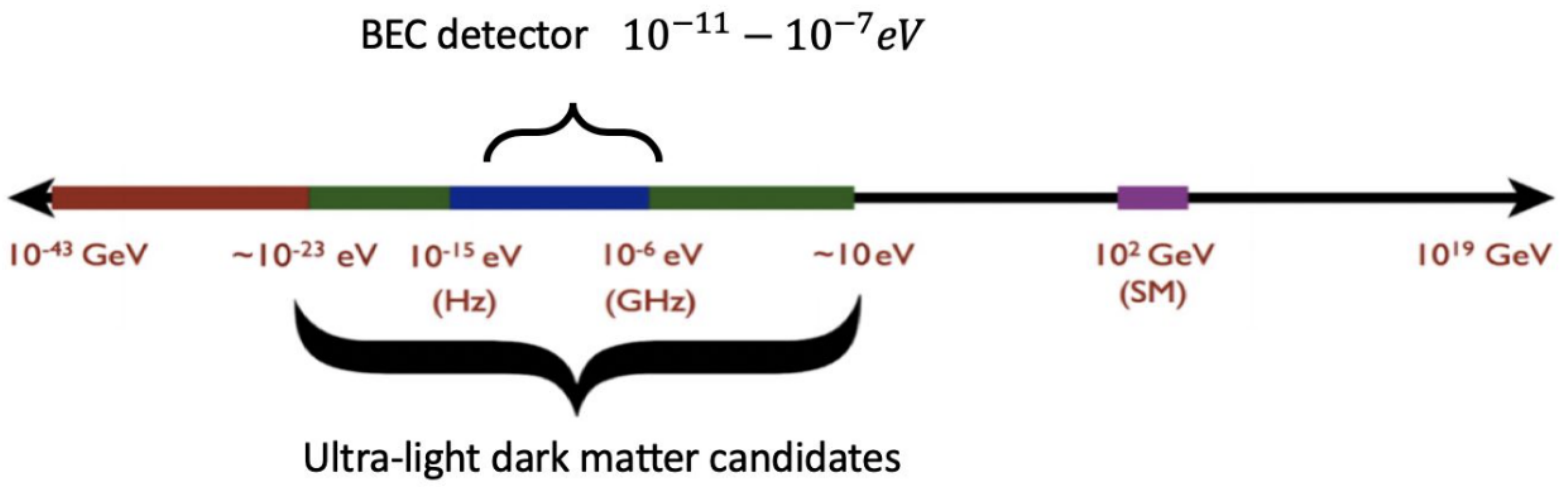
Ultra-Light Dark Matter
(bosonic)



→ search for *coherent effects of the entire field*, not single hard particle scatterings

Generic Candidates: Light pseudo-Nambu-Goldstones (axions and "axion like particles" — ALPs); Massive hidden vector bosons (aka "dark photons"); Light scalars (moduli/dilatons…)

Slide by John March-Russel


BEC detector $10^{-11} - 10^{-7} eV$
10^{-43} GeV
$\sim 10^{-23}$ eV
10^{-15} eV
(Hz)
10^{-6} eV
(GHz)
~10 eV
10^{2} GeV
(SM)
10^{19} GeV
Ultra-light dark matter candidates

Different principle

Interferometer arm length L

$$\frac{\Delta L}{L}$$

Resonance

$$\Omega = \omega_n + \omega_m$$

wave

quantum excitations

Quantum Weber Bar

Temperature

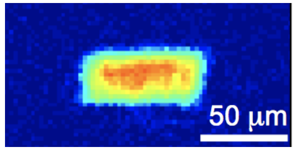
Weber bar

$T \sim 4\ K$

BEC

$T \sim 5x10^{-10} K$

Initial quantum states
Squeezing
Parametric amplification



How can it work if its so small?

$$\omega \sim \frac{C_s}{L}$$

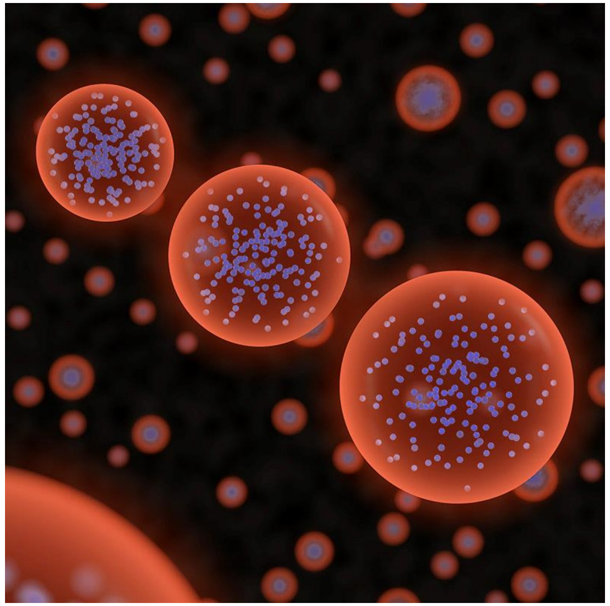
$$\omega \sim \frac{C}{L}$$

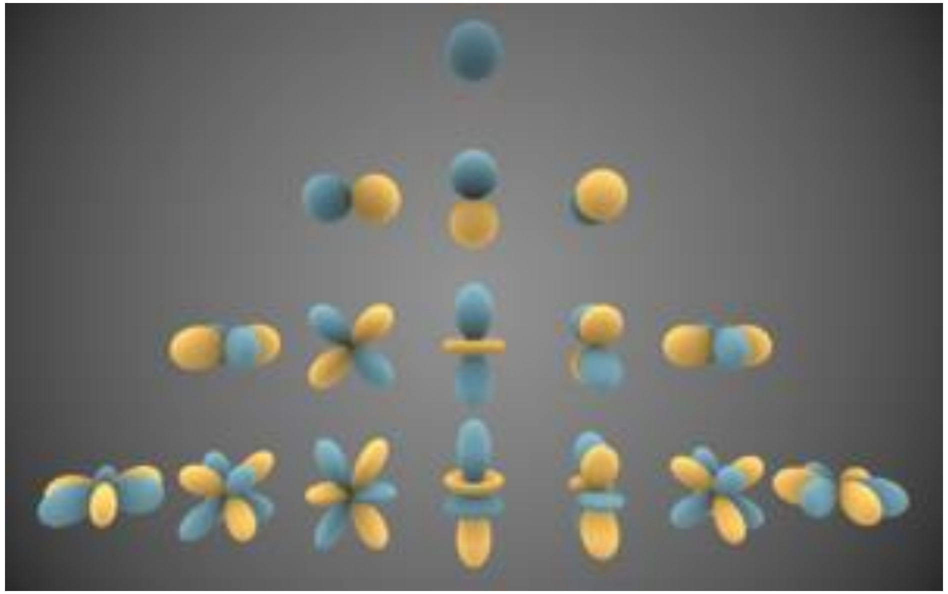
Speed of sound: $C_s = 10\text{mm/s}$
$L = 10^{-1}$-10^{-3}mm

Speed of light: C= $2.99\text{x}10^{11}$mm/s
L =2.99Km-2990Km

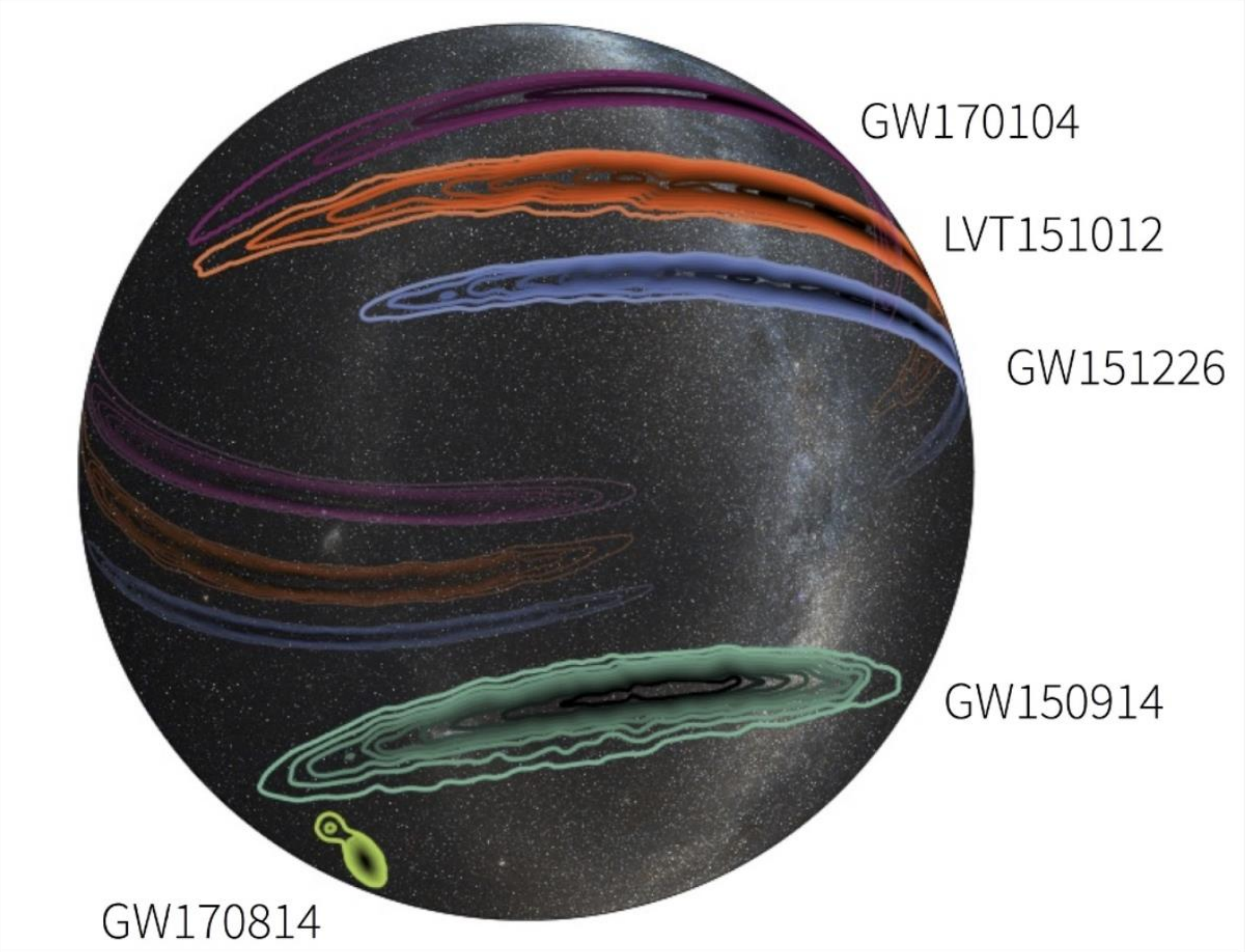
Broadband: 3D

Spherical BEC





We are studying different geometries

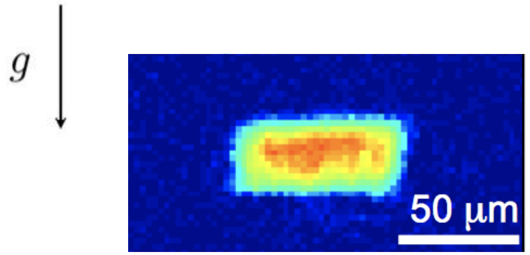


This three-dimensional projection of the Milky Way galaxy onto a transparent globe shows the probable locations of the three confirmed black-hole merger events observed by the two LIGO detectors—GW150914 (dark green), GW151226 (blue), GW170104 (magenta)—and a fourth confirmed detection (GW170814, light green, lower-left) that was observed by Virgo and the LIGO detectors. Also shown (in orange) is the lower significance event, LVT151012. Image credit: LIGO/Virgo/Caltech/MIT/Leo Singer (Milky Way image: Axel Mellinger).

Commercial applications

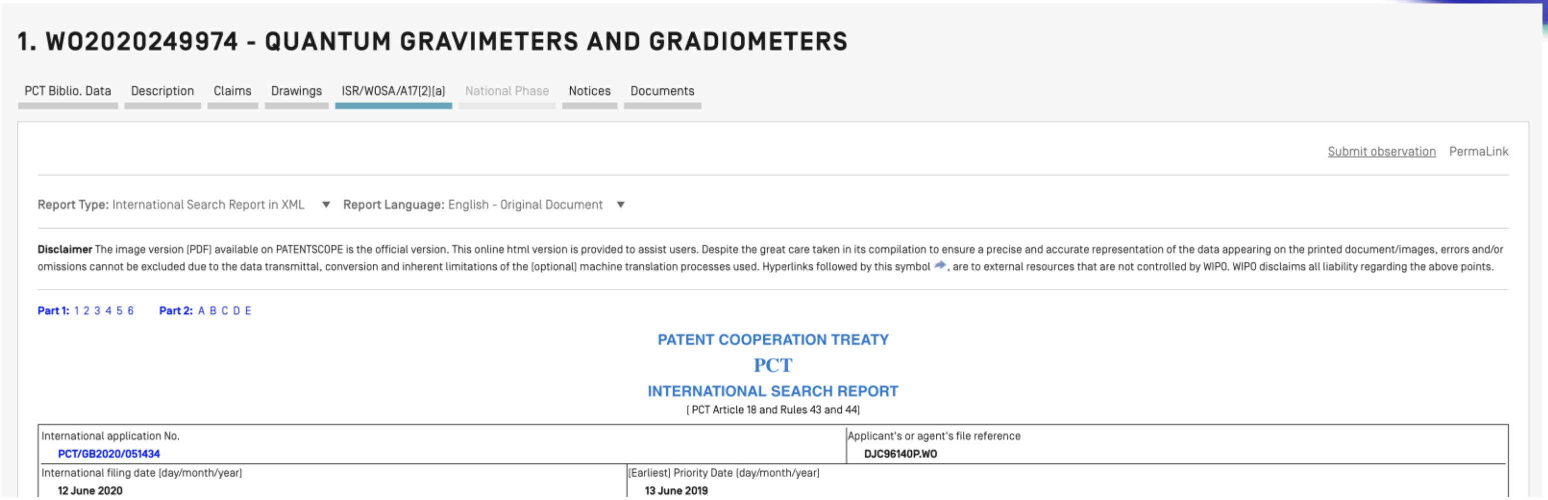
$$\boldsymbol{g} = \mathrm{diag}\left(-f(r), \frac{1}{f(r)}, r^2, r^2 \sin^2 \vartheta\right)$$

$$f(r) = 1 - r_S/r$$



Phononic gravimeter:
Same sensitivity but much smaller system.

Phononic gradiometer:
Improves the state of the art by at least two orders of magnitude


1. WO2020249974 - QUANTUM GRAVIMETERS AND GRADIOMETERS

PCT Biblio. Data | Description | Claims | Drawings | ISR/WOSA/A17(2)(a) | National Phase | Notices | Documents

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Part 1: 1 2 3 4 5 6 Part 2: A B C D E

PATENT COOPERATION TREATY
PCT
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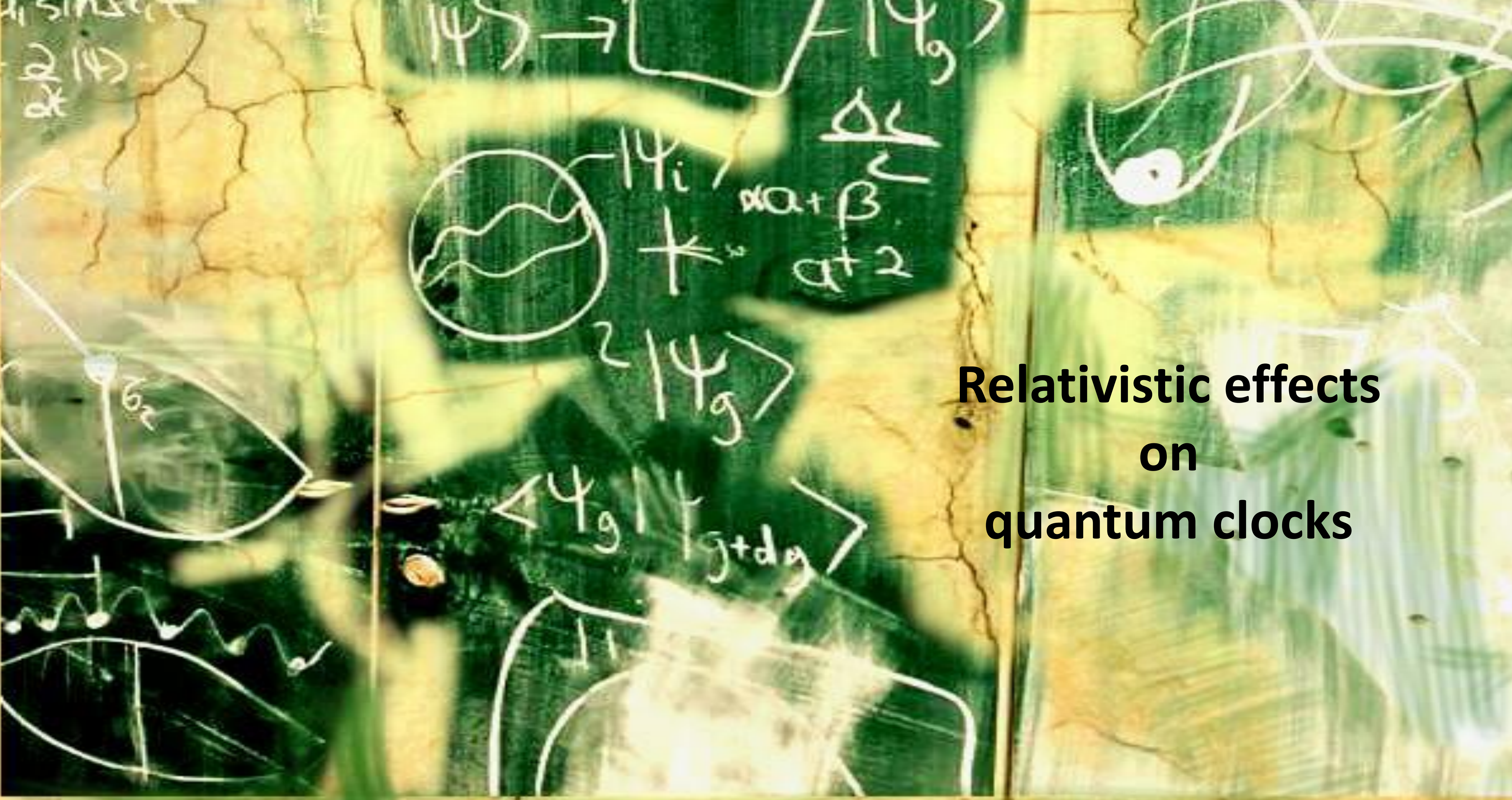
(PCT Article 18 and Rules 43 and 44)

International application No.	Applicant's or agent's file reference
PCT/GB2020/051434	DJC96140P.WO
International filing date (day/month/year) 12 June 2020	(Earliest) Priority Date (day/month/year) 13 June 2019

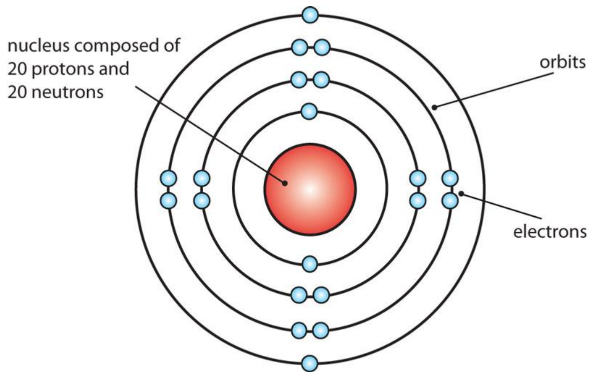
Setup	Running time	Length	$\Delta r_S/r_S$
[14, 15, 17] Atom. Int.	$100\,\mathrm{s} - 8\,\mathrm{h}$	$0.2 - 2.5\,\mathrm{m}$	10^{-9}
[16] BEC-chip	$100\,\mathrm{s}$	$10^{-2}\,\mathrm{m}$	10^{-10}
Phononic MZI	$6\,\mathrm{s}$	$10^{-4}\,\mathrm{m}$	10^{-8}

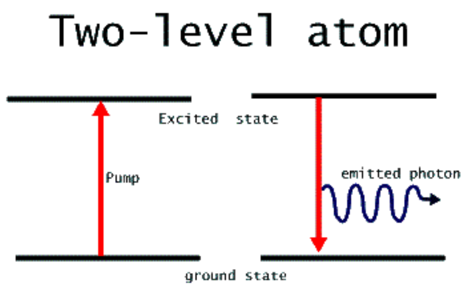


Relativistic effects on quantum clocks

Ivette Fuentes- University of Southampton

Quantum clocks (quantum 1.0)





Optical clocks routinely achieving 10^{-17} - 10^{-18} systematic uncertainty

Time

Quantum mechanics

Time is absolute (Galilean trans.).

Space and time are different.

  Time is a parameter.

  Space is an operator.

Particles can be in a superposition of positions at once.

Relativity

Space and time are not different.

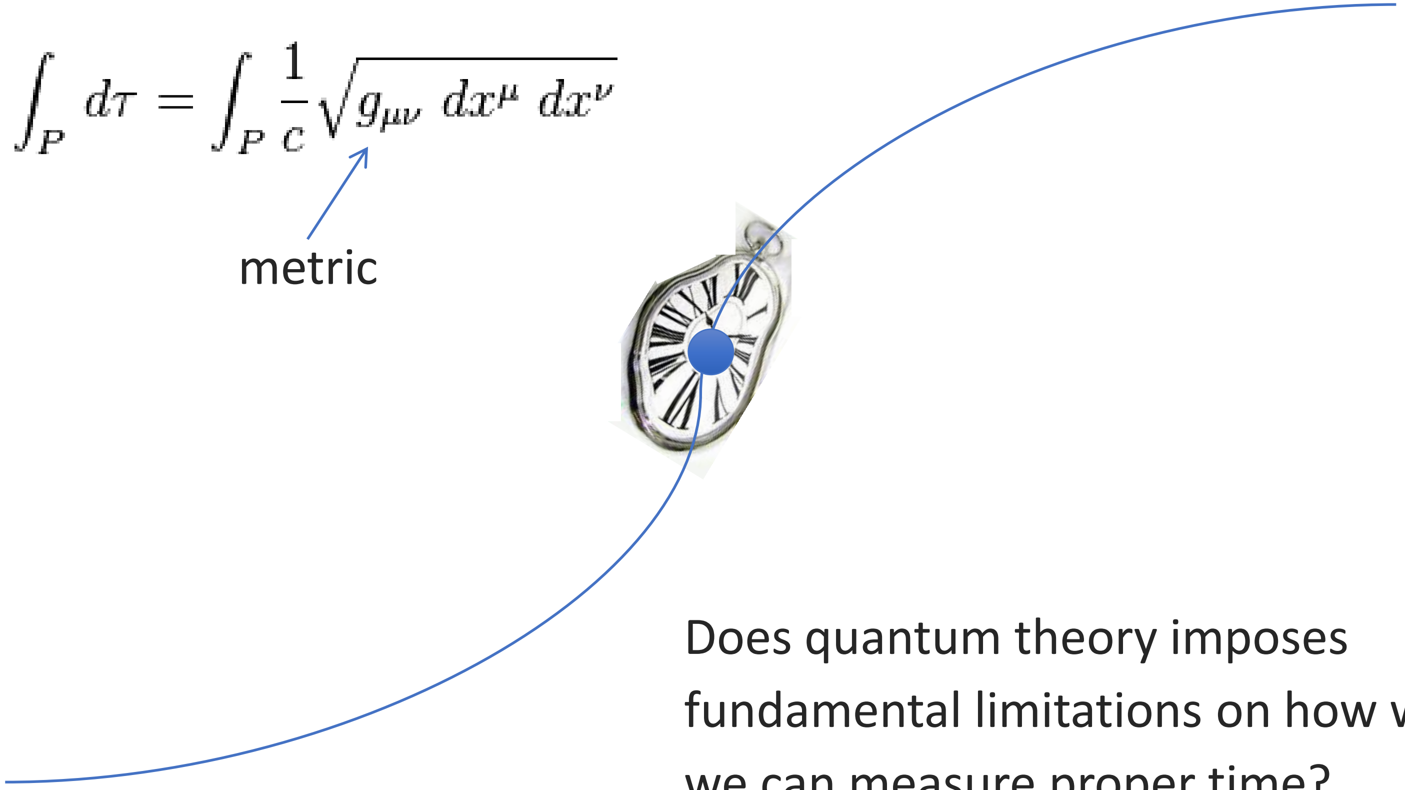
Time is observer dependent

Time flows at different rates in different points in space (Lorentz trans.)

Proper time

$$\Delta\tau = \int_P d\tau = \int_P \frac{1}{c}\sqrt{g_{\mu\nu}\, dx^\mu\, dx^\nu}$$

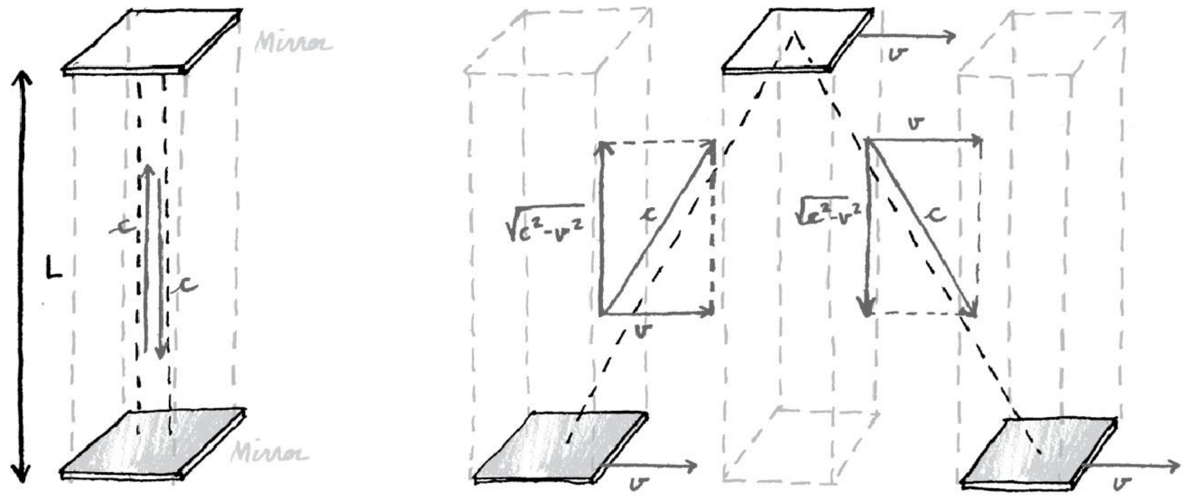
metric



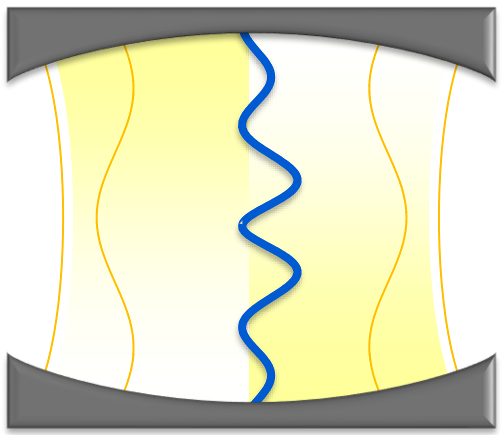
Does quantum theory imposes fundamental limitations on how well we can measure proper time?

Einstein's light clock

Classical light



Quantum: photons



Lindkvist, Sabin, Fuentes, Dragan, Svensson, Delsing, Johansson PRA (2014)

Relativistic quantum clock model

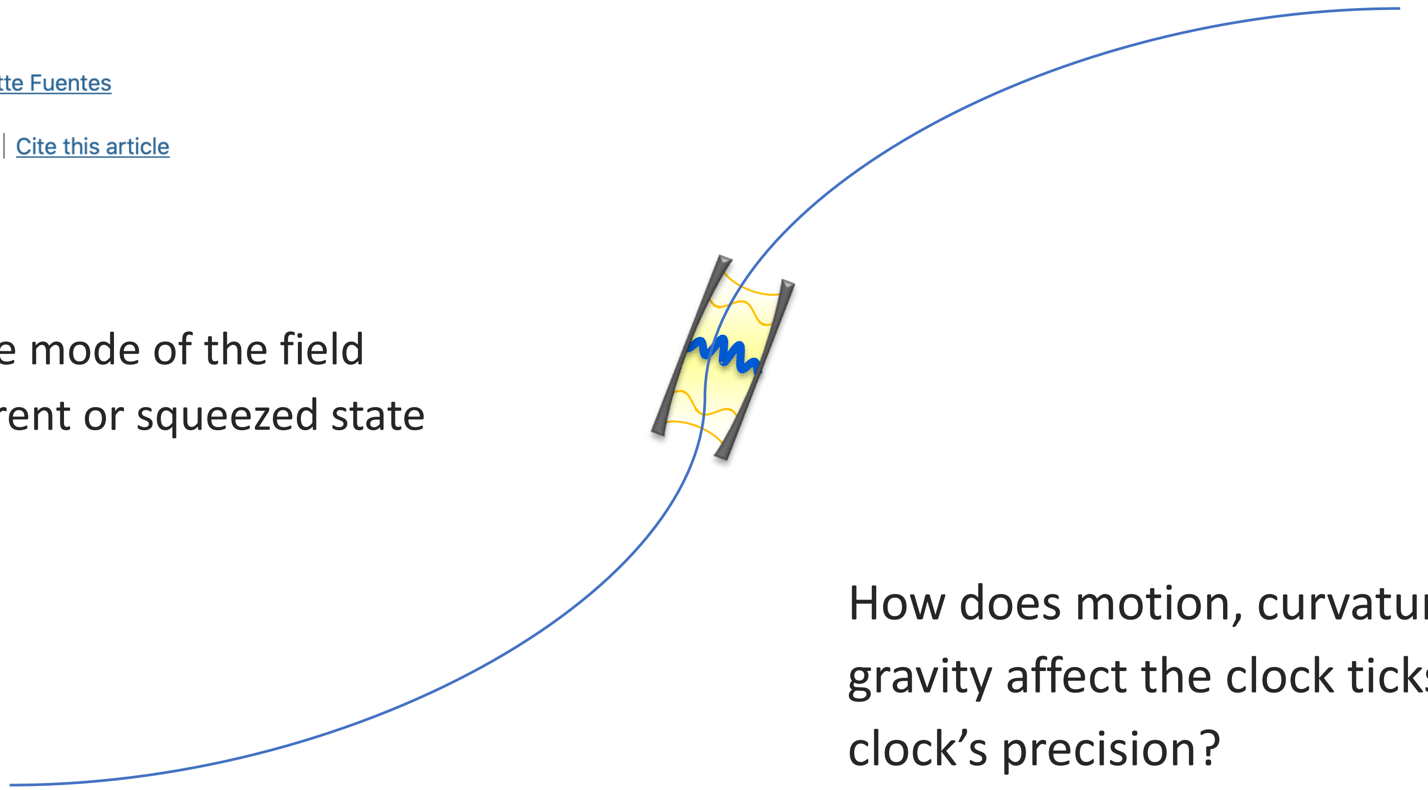
Open Access | Published: 19 May 2015

Motion and gravity effects in the precision of quantum clocks

Joel Lindkvist, Carlos Sabín, Göran Johansson & Ivette Fuentes

Scientific Reports **5**, Article number: 10070 (2015) | Cite this article

Clock: one mode of the field
in a coherent or squeezed state



How does motion, curvature and gravity affect the clock ticks and the clock's precision?

Precision: Quantum Fisher information

Lindkvist, Sabin, Johansson and Fuentes, Sci. Rep. (2015)

$$H_\theta = 4\alpha^2 P\left[\cosh(2r) + \sinh(2r)\cos\phi\right] + \frac{4\sinh^2(2r)}{1+P^2} \qquad P = \frac{1}{4\sqrt{\det\sigma}}$$

Squeezing

Coherent state amplitude

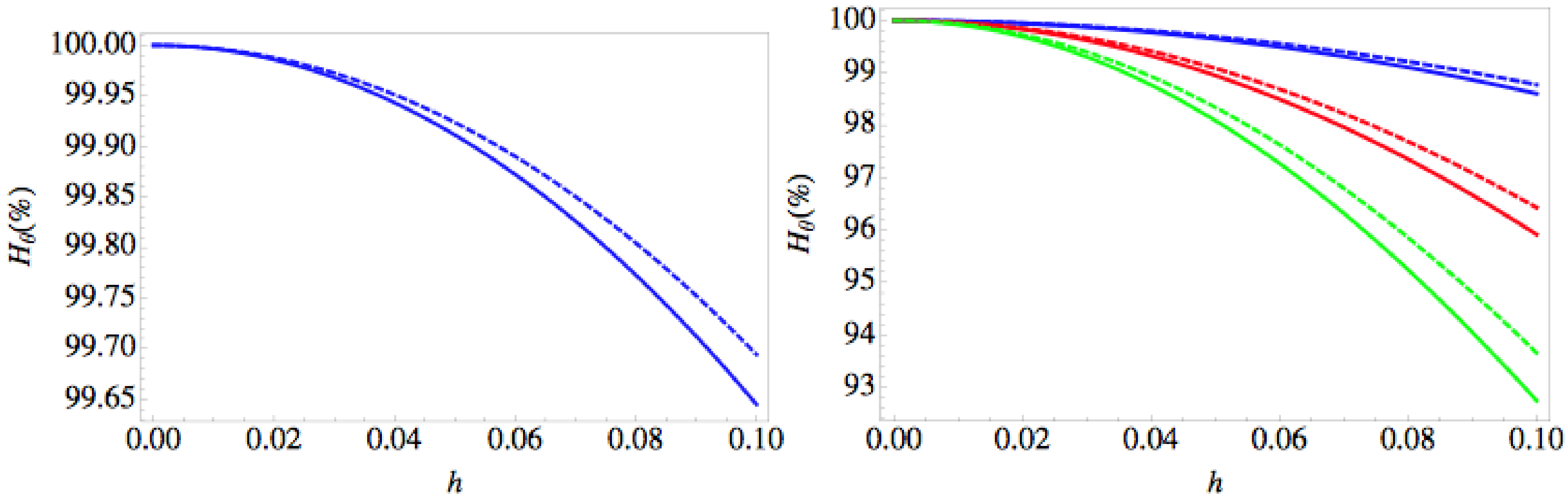
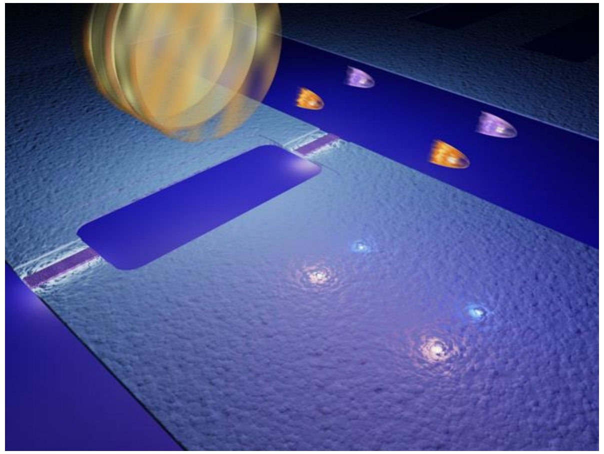
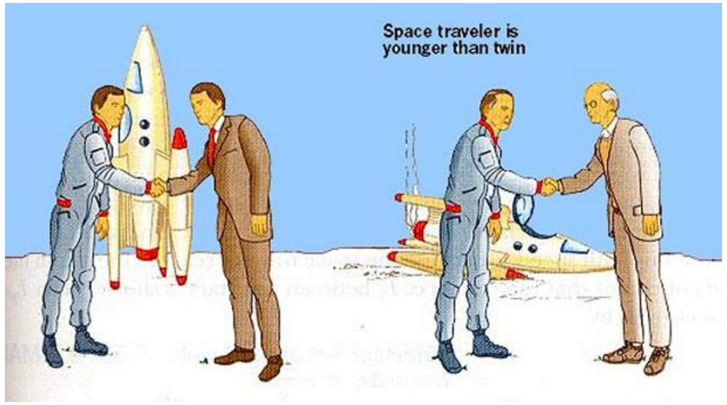


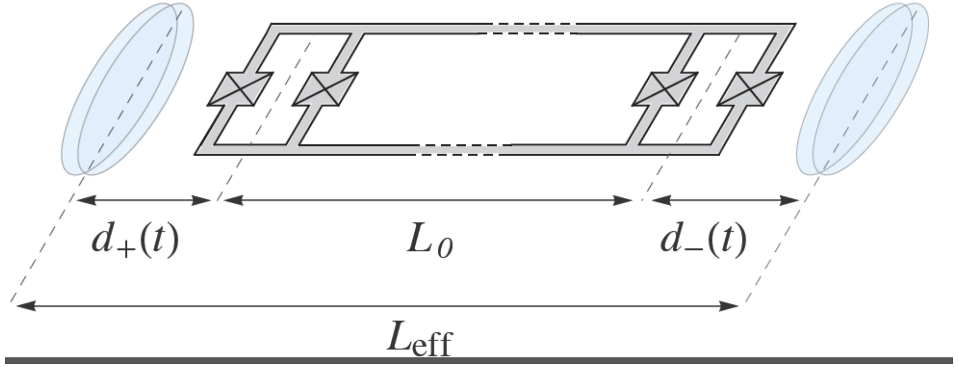
FIG. 2: a) Ratio of the transformed and original QFI for an initial coherent state as a function of h, for $\theta_a = \pi$ and initial photon numbers $N > 1$. The solid(dashed) curve is for $\theta_0 = 0(\pi/2)$. b) Ratio of the transformed and original QFI for an initially squeezed vacuum as a function of h, for $\theta_a = \pi$ and initial photon numbers $N = 1$ (blue), $N = 5$ (red) and $N = 10$ (green). The solid(dashed) curves are for $\theta_0 = 0(\pi/2)$.

Implementing the twin-paradox

Lindkvist, Sabin, Fuentes, Dragan, Svensson, Delsing, Johansson PRA (2014)







What new did we learn:

Quantum particle creation makes clock tick slower

[Submitted on 16 Apr 2022]

Gravitational time dilation in extended quantum systems: the case of light clocks in Schwarzschild spacetime

Tupac Bravo, Dennis Rätzel, Ivette Fuentes

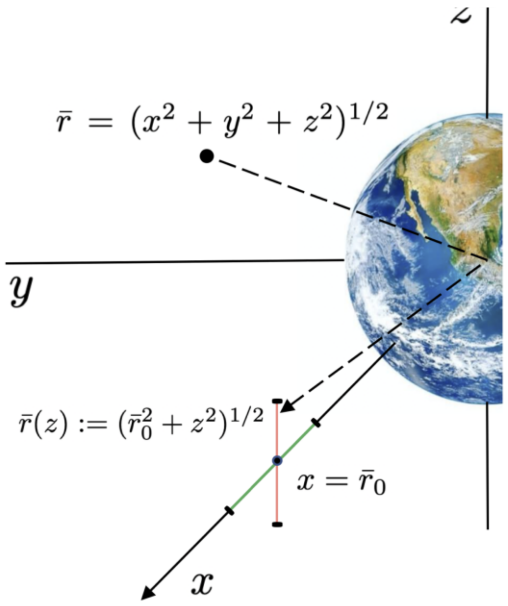
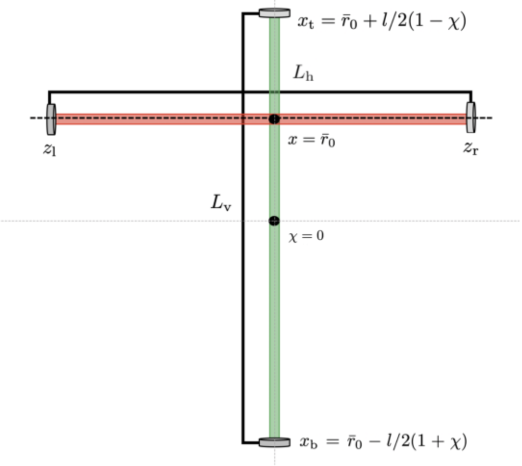
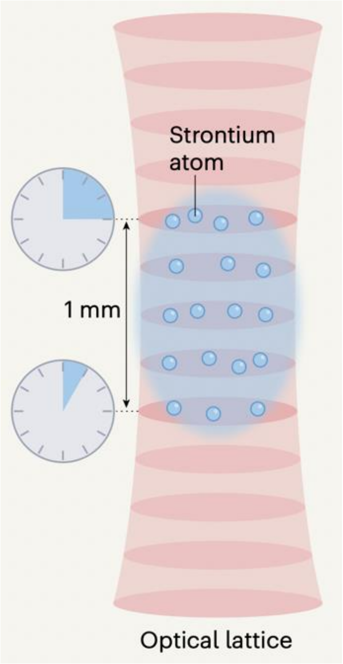


FIG. 1. Coordinate system used to quantize the field in horizontal (red) and vertical (green) cavities.





$$\boldsymbol{g} = \boldsymbol{\eta} + \frac{r_S}{\bar{r}}\,\mathbb{1} + \frac{r_S^2}{2\bar{r}^2}\mathrm{diag}\left(-1, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}\right)$$

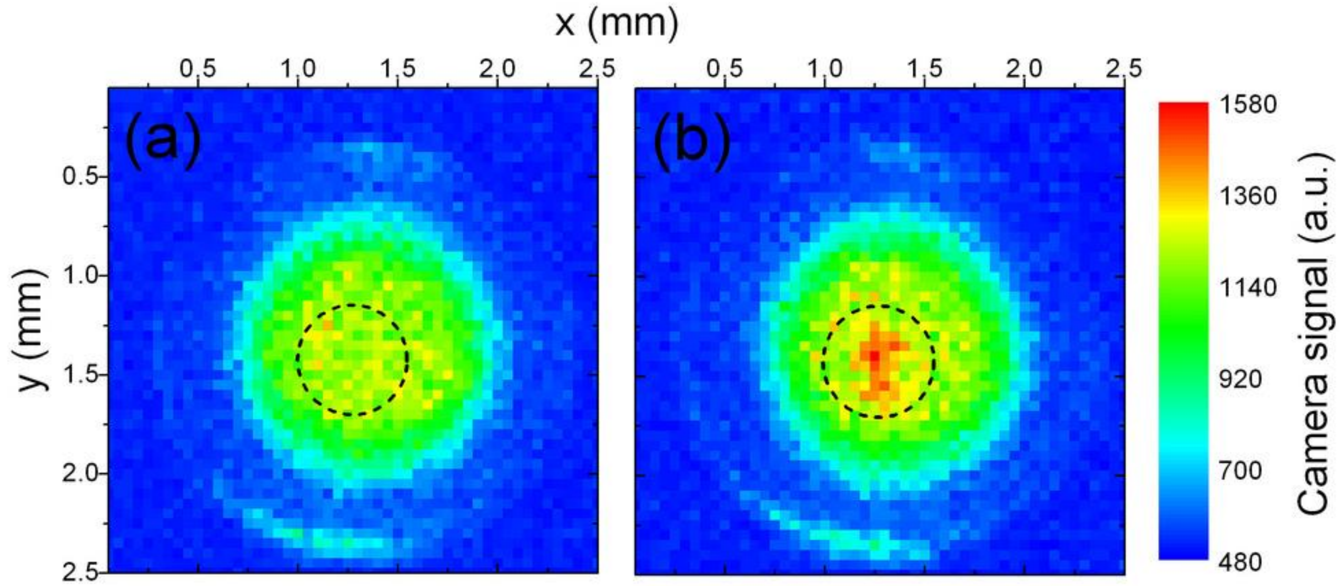
And for quantum clocks with squeezed vacuum input states

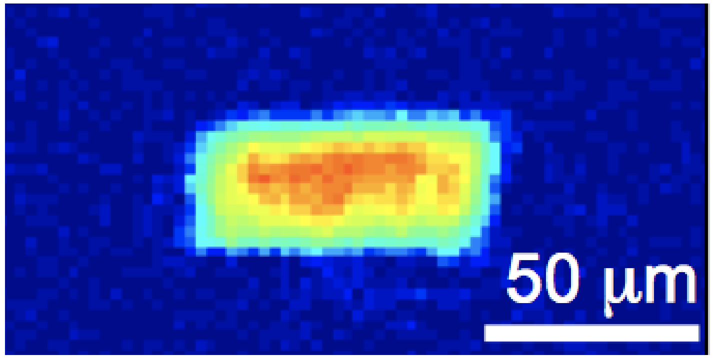
$$\Delta_{h,k}(\tau_0) = \frac{1}{2\sqrt{\mathcal{M}}}\frac{1}{\sqrt{N_p(N_p+1)}}\frac{L}{\pi\,c\,k}$$

$$\Delta_{v,k}(\tau_0) = \Delta_{h,k}(\tau_0)\left(1 + \frac{r_S L}{4\bar{r}_0^2}\chi\right)$$

Clock made of atoms in a trap

Notion of time?





Use collective modes of vibration

Using Quantum technologies we hope to understand better physics at the interplay of quantum and gravity.