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RESEARCH ARTICLE

THE TRANSFORM OF A LINE OF DESARGUES AFFINE PLANE IN AN ADDITIVE GROUP OF ITS POINTS

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ABSTRACT

In this paper we present a set transformation of points in a line of the Desargues affine plane in a additive group. For this, the first stop on the meaning of the Desargues affine plane, formulating first axiom of his that show proposition D1. Afterwards we show that little Pappus theorem, which we use in the construction of group proofs in additions of points on a line on desargues plane, also applies in the Desargues affine plane.

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INTRODUCTION

Desargues affine plane

Definition 1.1. (Francis Borceux, 2014; Orgest ZAKA, Kristaq FILIPI 2016) *Affine plane called the incidence structure* $\mathcal{A} = (\mathcal{P}, \mathcal{L}, \mathrm{I})$ *that satisfies the following axioms:*

A1: *For every two different points* P *and* $Q \in \mathcal{P}$*, there exists exactly one line* $\lambda \in \mathcal{L}$ *incident with that points.*
The line ℓ, determined from the point P and Q will denoted PQ.

A2: *For a point* $P \in \mathcal{P}$*, and an line* $\lambda \in \mathcal{L}$ *such such* $(P, \ell) \in \mathcal{I}$*, there exists one and only one line line* $r \in \mathcal{L}$*, incident with point* P *that such that* $\ell \cap r = \emptyset$.

A3: *In* $\mathcal{A}$ *there are three non-incident points with a line..*

The fact $(P, \ell) \in \mathrm{I}$ (equivalent to $P \,\mathrm{I}\, \ell$) we mark $P \in \ell$ and read *point* P *is incident with a line* ℓ *or a line* ℓ *passes by points* P (*contains point* P). Whereas a straight line of the affine plane we consider as sets of points of affine plane with her incidents. From axioms **A1** implicates that tow different lines of $\mathcal{L}$ many have an common point, in other words *tow different lines of* $\mathcal{L}$ *or no have common point or have only one common point.*

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Definition 1.2. *Two lines $\ell, m \in \mathcal{L}$ that matching or do not have common point of called parallel and in this case write $\ell || m$, and when they have only one common point say that they expected.*
For single line $r \in \mathcal{L}$, which passes by a point $P \in \mathcal{P}$ and it is parallel with line AB, that does not pass the point P, we will use the notation ℓ^P_{AB}.

PROPOSITION 1.1. (SADIKI, 2015) ***Parallelism relation*** $\| = \{(r, s) \in \mathcal{L}^2 | r \| s\}$ ***on*** $\mathcal{L}$ ***is an equivalence relation in*** $\mathcal{L}$.

Definition 1.3. *Three different points $P, Q, R \in \mathcal{P}$ wi called collineary, if there are incidents with the same line.*

Definition 1.4. *The set of three different non-collineary points **A, B, C** together with the line **AB, BC, CA** called three-vertex and marked **ABC**. Points **A, B, C** called vertices, while the line **AB, BC, CA** called side of three-vertex **ABC**.*
In affine Euclidian plane is true this

PROPOSITION *D*1. (Axiom I of Desargues) ***If*** AA_1, BB_1, CC_1 ***are the three different parallel line*** **(Fig. 1)*, then***

$$\left.\begin{array}{l} AB \parallel A_1B_1, \\ BC \parallel B_1C_1 \end{array}\right] \Rightarrow AC \parallel A_1C_1.$$

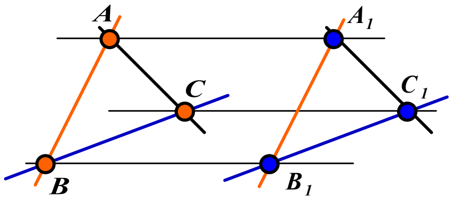

Fig. 1.Desargues configuration

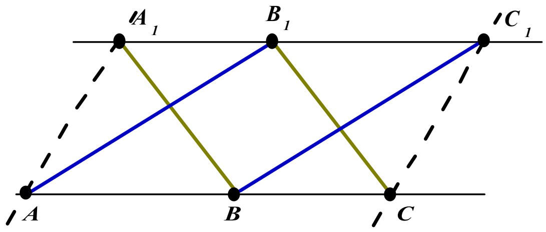

Fig. 2.Pappus configuration

There are affine plans where Propositioni D1 not valid. Such a is the Moulton plane (9).

Definition 1.5. (Francis Borceux, 2014; SADIKI, 2015; COXETER, 1969) *An affine plane complete with desargues axiom D1, we shall call Desargues affine plane.*

Let's be now ***A***, ***B***, ***C*** three different points of a line and A_1, B_1, C_1 three different points of one another straight-parallel to the first (Fig.2). If $AB_1 \| BC_1$ and $A_1B \| B_1C$ can contend that also $AA_1 \| CC_1$? Otherwise, we add the problem if it is true that
PROPOSITION 1.2 (FRANZ ROTHE, 2010; ROBİN HARTSHORNE, 2000) **(**Little Pappus Theorem**)**. ***Let us be A,B,C and*** A_1, B_1, C_1 ***two triple point located in two parallel lines*** **(Fig. 2). If** $AB_1 \| BC_1$ **and** $BA_1 \| CB_1$, ***then we have to*** $AA_1 \| CC_1$**.**

The answer is that

THEOREM 1.1 (FRANZ ROTHE, 2010) **(***the little Hessenberg Theorem***). In the Desargues plane is tru *Propositions*1.2, *to wit is worth the Little Pappus theorem.***

Proof. Let us have two triplets of points ***A, B, C*** and A_1, B_1, C_1 in two parallel lines such that $AB_1 \| BC_1$ and $BA_1 \| CB_1$ (Fig. 3).
We build a line $\ell^C_{AB_1}$ (a line that passes through points ***C*** and it is parallel to the line AB_1), and line $\ell^A_{BA_1}$ (a line that passes through points ***A*** and it is parallel to the line BA_1). We mark $D = \ell^C_{AB_1} \cap \ell^A_{BA_1}$. Also construct the line ***DB*** (Fig.3).

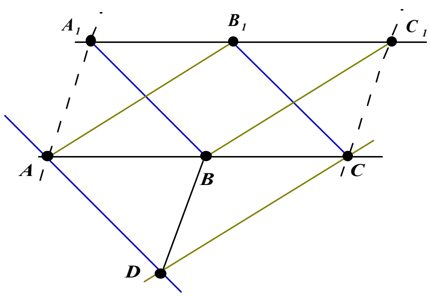

Fig. 3. The proof configuration.

By condition of Propositions 1.2 we have $AB_1 \| BC_1$ and $BA_1 \| CB_1$. This imply parallelism of straight lines AB_1, BC_1, $\ell^{C}_{AB_1}$, and parallelism of straight lines BA_1, CB_1, $\ell^{A}_{BA_1}$. In these conditions, three-vertex CC_1B_1 and DBA they have vertices in different parallel line AB_1, BC_1, $\ell^{C}_{AB_1}$ and their sides satisfy the condition $AB \| B_1C_1$ and $AD \| B_1C$. Hence, according to axiom D1, we have also $CC_1 \| DB$.

Also, three-vertex AA_1B_1 and DBC they have vertices in different parallel line BA_1, CB_1, $\ell^{A}_{BA_1}$ and their sides satisfy the condition $BC \| A_1B_1$ and $DC \| AB_1$. Hence, according to axiom D1, we have also $DB \| AA_1$. By comparing the two conclusions of the implementation of axiom D1, according to Propositions 1.1, we conclude $AA_1 \| CC_1$.

2. Equipment of sets of points to a straight lines of the desargues affine plane with binary additive operations

In an Desargues affine plane $\mathcal{D} = (\mathcal{P}, \ , \mathrm{I})$ we fix two different points $O, I \in \mathcal{P}$, which, according to axiom A1, determine a line $OI \in$. Let us be A and B two whatever points of a line OI. Choosing in plane $\mathcal{D}$ a point B_1 non-incidents with OI: $B_1 \notin OI$. Construct line $\ell^{B_1}_{OI}$, which is only by axiom A2. Then construct line $\ell^{A}_{OB_1}$, which also is the only according to axiom A2. Marking their intersection $P_1 = \ell^{B_1}_{OI} \cap \ell^{A}_{OB_1}$. Finally construct line $\ell^{P_1}_{BB_1}$. For as much as BB_1 expects OI in point B, then this straight line, parallel with BB_1, expects line OI in a single point C (Fig. 4).

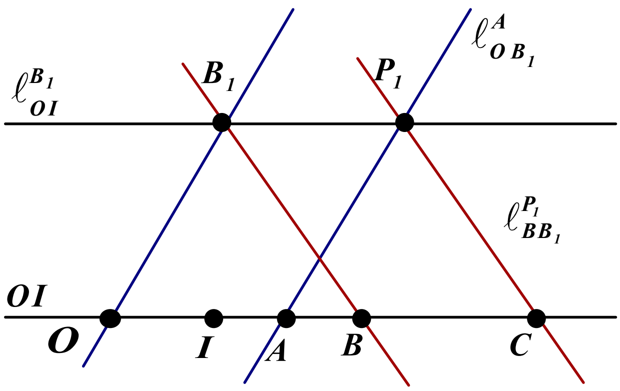

Fig. 4 The additions configuration.

The process of construct the points C, starting from two whatsoever points A, B of the line OI, is presented in the algorithm form

$$\left.\begin{array}{l} 1.\ B_1 \notin OI, \\ 2.\ \ell^{B_1}_{OI} \cap \ell^{A}_{OB_1} = P_1, \\ 3.\ \ell^{P_1}_{BB_1} \cap OI = C. \end{array}\right] \quad (3)$$

In the process of construct the points C, besides pairs (A, B) of points $A, B \in OI$, is required and the selection of point $B_1 \notin OI$, which we call the *auxiliary point* to point C. This choice affects the position of point C on the line OI?

THEOREM 2.1. ***For every two points*** $A, B \in OI$, ***algorithm*** (3) ***determines the a single point*** $C \in OI$, ***which does not depend on the choice of hers auxiliary point*** B_1.

Proof. Let it be (A, B) a pair points of the line OI. According to (3), by selecting point B_1, construct the point C.

Now choose another point B_2. Then but according to (3), construct the analog point C', in these conditions it takes view

$$\left.\begin{array}{l} 1.\ B_2 \notin OI, \\ 2.\ \ell_{OI}^{B_2} \cap \ell_{OB_2}^{A} = P_2, \\ 3.\ \ell_{BB_2}^{P_2} \cap OI = C'. \end{array}\right] \quad (3')$$

We distinguish these four cases the position of points ***A, B*** in relation to fixed point ***O*** the fitting line ***OI.***

Case I. ***A=B=O.*** In this case, by the choice of point B_1, according to (3) we have

$$P_1 = \ell_{OI}^{B_1} \cap \ell_{OB_1}^{O} = \ell_{OI}^{B_1} \cap OB_1 = B_1 \Rightarrow C = \ell_{OB_1}^{B_1} \cap OI = O;$$

whereas, from the choice of point B_2, according to (3') we have

$$P_2 = \ell_{OI}^{B_2} \cap \ell_{OB_2}^{O} = \ell_{OI}^{B_2} \cap OB_2 = B_2 \Rightarrow C' = \ell_{OB_2}^{B_2} \cap OI = O.$$

As a consequence (Fig. 5.*a*) we get

$$C=C'=O \quad (4)$$

Case II. ***A=O≠B.*** In this case, by the choice of point B_1, according to (3) we have

$$P_1 = \ell_{OI}^{B_1} \cap \ell_{OB_1}^{O} = \ell_{OI}^{B_1} \cap OB_1 = B_1 \Rightarrow C = \ell_{BB_1}^{B_1} \cap OI = BB_1 \cap OI = B;$$

whereas, from the choice of point B_2, according to (3') we have

$$P_2 = \ell_{OI}^{B_2} \cap \ell_{OB_2}^{O} = \ell_{OI}^{B_2} \cap OB_2 = B_2 \Rightarrow C' = \ell_{BB_2}^{B_2} \cap OI = BB_2 \cap OI = B.$$

As a consequence (Fig. 5.*b*) we get

$$C=C'=B \quad (5)$$

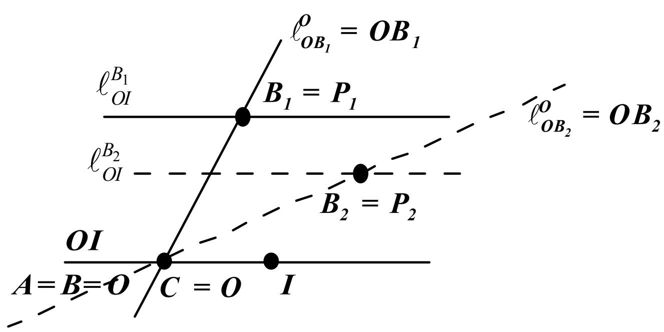

Fig. 5.*a* independence of addition

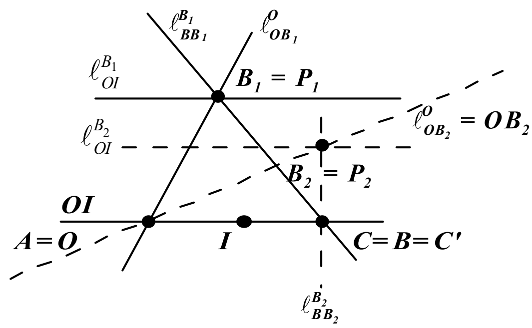

Fig. 5.*b* independence of addition

Case III. ***A≠O=B.*** In this case, by the choice of point B_1, according to (3) we have

$$P_1 = \ell_{OI}^{B_1} \cap \ell_{OB_1}^{A} \Rightarrow C = \ell_{OB_1}^{P_1} \cap OI = AP_1 \cap OI = A;$$

whereas, from the choice of point B_2, according to (3') we have

$$P_2 = \ell_{OI}^{B_2} \cap \ell_{OB_2}^{A} \Rightarrow C' = \ell_{OB_2}^{P_2} \cap OI = AP_2 \cap OI = A.$$

As a consequence (Fig. 5.*c*) we get

$$C=C'=A \quad (5')$$

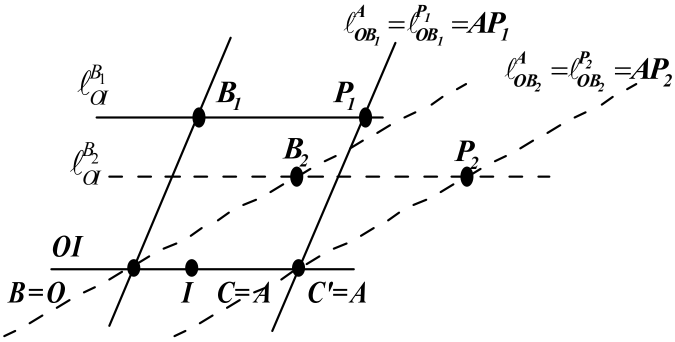

Fig. 5.*c* independence of addition

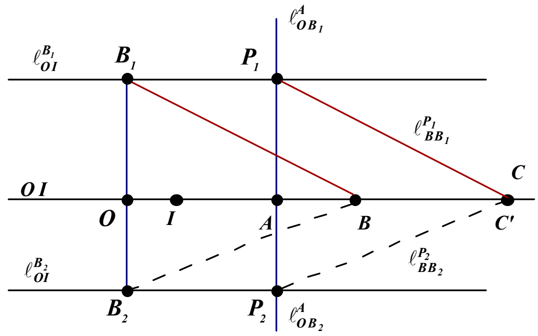

Fig. 5.*d* independence of addition

Case IV. ***A≠B≠O.*** Here we distinguish two sub-cases.

a*)** Points ***O, B₁, B₂ are collineary. In this case, by the choice of point B_1, according to (3) we have

$$P_1 = \ell_{OI}^{B_1} \cap \ell_{BB_1}^{A} \Rightarrow C = \ell_{BB_1}^{P_1} \cap OI;$$

whereas, from the choice of point B_2, according to (3') we have

$$P_2 = \ell_{OI}^{B_2} \cap \ell_{BB_2}^{A} \Rightarrow C' = \ell_{BB_2}^{P_2} \cap OI.$$

From (3) and (3') imply alsi, collinearity of the points O, B_1, B_2 imply collinarity of points A, P_1, P_2. Suppose now that $C \neq C'$ (Fig. 5.*d*).

We examine three-vertex BB_1B_2 and CP_1P_2. We note that $AP_1 = \ell_{OB_1}^{A} \parallel OB_1$, $P_2 \in AP_1$, $B_2 \in OB_1$, that imply $B_1B_2 \parallel P_1P_2$. But $C \in \ell_{BB_1}^{P_1} \parallel BB_1$, therefore $BB_1 \parallel CP_1$. From here, from axioms D1 of Desargues, results $B_2B \parallel P_2C$. On the other hand, $C' \in \ell_{BB_2}^{P_2}$, that imply $P_2C' \parallel B_2B$, which is parallel to P_2C. As a consequence $C' \in P_2C$. But P_2C and OI received in a single point, which imply $C = C'$, in contradiction with supposition that $C \neq C'$.
***b*)** Points O, B_1, B_2 are non-collineary. In this case, by the choice of point B_1, according to (3) we have

$$P_1 = \ell_{OI}^{B_1} \cap \ell_{BB_1}^{A} \Rightarrow C = \ell_{BB_1}^{P_1} \cap OI;$$

whereas, from the choice of point B_2, according to (3') we have

$$P_2 = \ell_{OI}^{B_2} \cap \ell_{BB_2}^{A} \Rightarrow C' = \ell_{BB_2}^{P_2} \cap OI.$$

Suppose now that $C \neq C'$ (Fig. 5.*e*). From (3) and (3') we have, non-colinearity of points O, B_1, B_2 imply non-colinearity of the points A, P_1, P_2 .

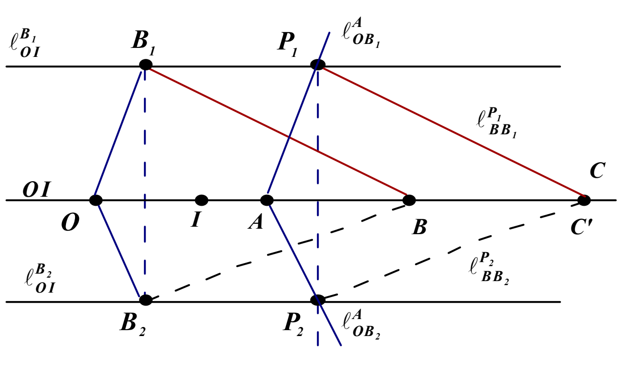

Fig. 5.*e*. Independence of addition.

We examine three-vertexes OB_1B_2 and AP_1P_2. We note that $AP_1 = \ell^A_{OB_1} \parallel OB_1$ and $AP_2 = \ell^A_{OB_2} \parallel OB_2$, therefore the axioms D1 results $B_1B_2 \parallel P_1P_2$. We examine now three-vertexes BB_1B_2 and CP_1P_2. The fact that $C \in \ell^{P_1}_{BB_1} \parallel BB_1$, imply $BB_1 \parallel CP_1$. From the above, we also $B_1B_2 \parallel P_1P_2$. Therefore, again by axioms D1 results $B_2B \parallel P_2C$. On the other hand, $C' \in \ell^{P_2}_{BB_2}$, that imply $P_2C' \parallel B_2B$. As a consequence $C' \in P_2C$, that imply $C=C'$, in contradiction with supposition that $C \neq C'$.
The above theorem creates the possibility of introduction of a binary operation, that we call the additions, in set of points to line OI, as follows.

Let us be A and B two whatsoever points of the line OI. I associate pairs $(A,B) \in OI \times OI$ point $C \in OI$, that determines algorithm (3). According to the preceding Theorems, point C is determined in single mode by (3). Thus we obtain a application $OI \times OI \to OI$.

Definition 2.1. *In the above conditions, application*

$$+: OI \times OI \to OI,$$

defined by $(A, B) \quad C$ *for all* $(A, B) \in OI \times OI$ *we call the addition in* OI.
According to this Definitioni, can write

$$\forall A, B \in OI, \quad \left.\begin{array}{l} 1.\ B_1 \notin OI, \\ 2.\ \ell^{B_1}_{OI} \cap \ell^{A}_{OB_1} = P_1, \\ 3.\ \ell^{P_1}_{BB_1} \cap OI = C. \end{array}\right] \Leftrightarrow A + B = C. \tag{6}$$

3. GROUPOID (OI, +) IS COMMUTATIVE GROUP

With reference to cases I, II, III of Theorem 2.1, appears immediately true this
PROPOSITION 3.1. ***Additions in OI there areelement zero the point O***:

$$\forall A \in OI, \; O + A = A + O = A. \tag{7}$$

As well as worth and below propositions.

PROPOSITIONI 3.2. ***Additions is commutative in OI***:

$$\forall A, B \in OI, \; A+B = B+A \tag{8}$$

Proof. In the case where $A=B$ the statement is evident, whereas when $A=O$ or $B=O$, propositions is tru goes according to (7). Stopped in case when $A, B \neq O$ and $A \neq B$. We mark $A+B=C$ and $B+A=C'$. Auxiliary point B_1 the sum $A+B$ and auxiliary point A_1 the sum $B+A$ we get the same (Fig.7). In this case, according to (6), we have

$$\left.\begin{array}{l} 1.\ B_1 \notin OI, \\ 2.\ \ell^{B_1}_{OI} \cap \ell^{A}_{OB_1} = P_1, \\ 3.\ \ell^{P_1}_{BB_1} \cap OI = C. \end{array}\right] \Leftrightarrow A + B = C \text{ and } \left.\begin{array}{l} 1.\ A_1 \notin OI, \\ 2.\ \ell^{A_1}_{OI} \cap \ell^{B}_{OA_1} = P_2, \\ 3.\ \ell^{P_2}_{AA_1} \cap OI = C'. \end{array}\right] \Leftrightarrow B + A = C'. \tag{6'}$$

It is clear that $A+B=B+A$ means that the points C and C' are the same points. For this use Proposition 1.2. Suppose now that $C \neq C'$ (Fig. 6). We examine trio collinary points A, B, C and other trio of points collinary B_1, P_1, P_2, that are in parallel lines. According to (6'), $AP_1 \parallel BP_2$ and $BB_1 \parallel CP_1$. We are in conditions of little Pappus Theorems, thus resulting $CP_2 \parallel AB_1$, otherwise $CP_2 \parallel AA_1$. But from (6 ') have also $C'P_2 \parallel AA_1$, that imply $C=C'$, in contradiction with supposition that $C \neq C'$.
PROPOSITIONI 3.3. ***Addition is associative in OI:***

$$\forall A, B, D \in OI, \; (A+B)+D = A+(B+D). \tag{9}$$

Proof. In the case where at least one of the point A, B, D is O proposition is tru according to (7), whereas when $A=D$, proposition is tru according to (8). Stopped in case the $A, B, D \neq O$ and $A \neq B \neq D$, (the reasoning is the same in other cases). Construct the first sum $(A+B)+D$. In this case (Fig. 7), according to (6), for $A+B$ have

$$\left.\begin{array}{l} 1.\ B_1 \notin OI, \\ 2.\ \ell_{OI}^{B_1} \cap \ell_{OB_1}^{A} = P_1, \\ 3.\ \ell_{BB_1}^{P_1} \cap OI = C. \end{array}\right] \Rightarrow \boldsymbol{A+B} = \ell_{BB_1}^{P_1} \cap OI \ .$$

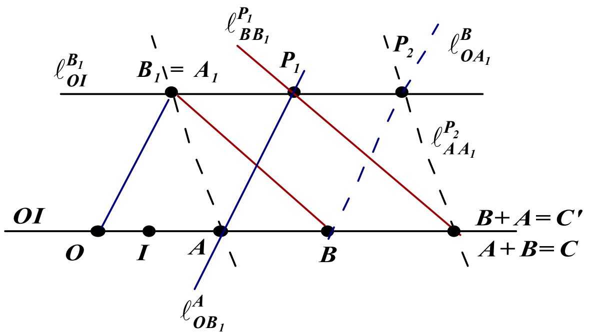

Fig. 6.Commutative property

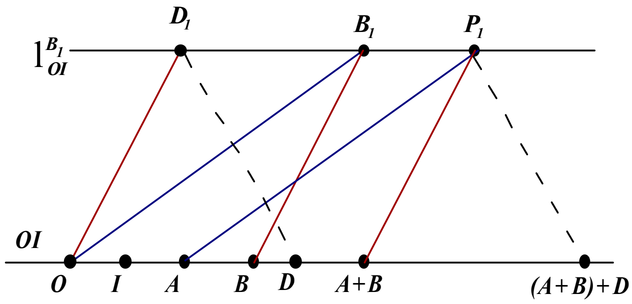

Fig. 7. Associative property

Construct line $\ell^{O}_{(A+B)P_1}$ and write down $\boldsymbol{D_1} = \ell^{O}_{(A+B)P_1} \cap \boldsymbol{OI}$. Select point $\boldsymbol{D_1}$ as auxiliary points for construction of the sum ***(A+B)+D.*** Then, according to (6) have

$$\left.\begin{array}{l} 1.\ D_1 \notin OI, \\ 2.\ \ell_{OI}^{D_1} \cap \ell_{OD_1}^{A+B} = P_1, \\ 3.\ \ell_{DD_1}^{P_1} \cap OI = C. \end{array}\right] \Rightarrow \boldsymbol{(A+B)+D} = \ell_{DD_1}^{P_1} \cap OI \ . \qquad (*)$$

On the order of same construct now sum ***A+(B+D).*** In this case, we choose as auxiliary points for ***B+D*** point $\boldsymbol{D_1}$ (Fig. 8). With this, the role of point $\boldsymbol{P_1}$ is the point $\boldsymbol{B_1}$. By constructed line $\ell^{B_1}_{DD_1}$, according to (6), we find $\boldsymbol{B+D} = \ell^{B_1}_{DD_1} \cap OI$. Whence imply that $\boldsymbol{(B+D)B_1} \,//\, \boldsymbol{DD_1}$. Select now as auxiliary points for sum ***A+(B+D)*** point $\boldsymbol{B_1}$.

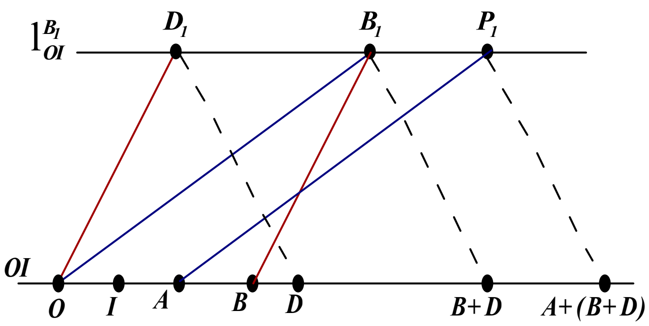

Fig. 8. Associative property

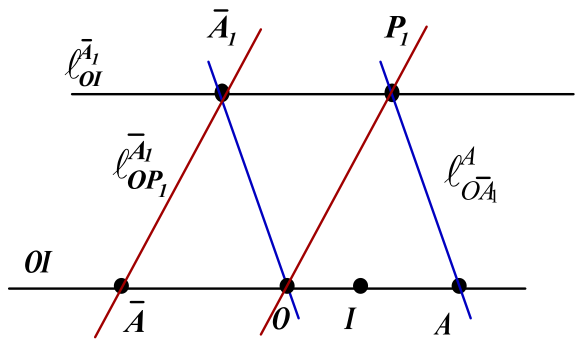

Fig. 9.The inverse

Then, according to (6) have

$$\left.\begin{array}{l} 1.\ B_1 \notin OI, \\ 2.\ \ell_{OI}^{B_1} \cap \ell_{OB_1}^{A} = P_1, \\ 3.\ \ell_{(B+D)B_1}^{P_1} \cap OI = C. \end{array}\right] \Rightarrow \boldsymbol{(A+B)+D} = \ell_{(B+D)B_1}^{P_1} \cap OI \ .$$

But $\boldsymbol{(B+D)B_1} \,//\, \boldsymbol{DD_1}$, that imply $\ell^{P_1}_{(B+D)B_1} = \ell^{P_1}_{DD_1}$. Eventually, according to (*), we have:

$$(A+B)+D=\ell_{DD_1}^{P_1}\cap OI = \ell_{(B+D)B_1}^{P_1}\cap OI = (A+B)+D.$$

PROPOSITION 3.4. ***For every point in OI exists her right symmetrical according to addition***:

$$\forall\ A\in OI,\ \exists\overline{A}\in OI,\ A+\overline{A}=O. \tag{10}$$

Proof. We distinguish two cases: $A = O$ and $A \neq O$.

If $A = O$, then $\overline{A} = O$, because, according to (7), $O + O = O$.

If $A \neq O$, requested points $\overline{A}$ such that

$$\left.\begin{array}{l} 1.\ \overline{A}_1 \notin OI, \\ 2.\ \ell_{OI}^{\overline{A}_1}\cap \ell_{O\overline{A}_1}^{A} = P_1, \\ 3.\ \ell_{\overline{A}\overline{A}_1}^{P_1}\cap OI = O. \end{array}\right]$$

Given this, we get initially a point $\overline{A}_1 \notin OI$ and construct line $\ell_{OI}^{\overline{A}_1}$ and then line $\ell_{O\overline{A}_1}^{A}$, which intersect at the point $\boldsymbol{P_1}$. Furthermore construct $\boldsymbol{OP_1}$ and parallel with her by the points $\overline{A}_1$ line $\ell_{OP_1}^{\overline{A}_1}$. This last is not parallel with line ***OI,*** therefore awaits him at one point. It is clear that this point is the point of demanding $\overline{A}$, therefore $\overline{A} = \ell_{OP_1}^{\overline{A}_1} \cap OI$ (Fig. 9).

Propositions 3.1, 3.2, 3.3, 3.4 proved that is true this

THEOREM 3.1. ***Groupoid (OI, +) is commutative(abeljan) Group***

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