

Starter

- Match the following sequences to their names:

3, 9, 15, 21, 27, 33...	Cube
1, 1, 2, 3, 5, 8...	Fibonacci
1, 3, 6, 10, 15, 21...	Geometric
1, 4, 9, 16, 25, 36...	Linear
4, 12, 36, 108, 324...	Quadratic
1, 8, 27, 64, 125, 216...	Square
3, 10, 21, 36, 55, 78...	Triangular

Recurrence Relations

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- Know the names of the different types of sequence
- Understand the subscript notation of a recurrence relation
- Able to generate terms of a sequence given the n th term

Recurrence Relations

- ▶ Recurrence Relations describe the relationship between consecutive terms in a sequence

- ▶ Example: A sequence is defined by the n th term:

$$U_n = 2U_{n-1}$$

Given that $U_1 = 2$ write down the first four terms of the sequence

Recurrence Relations

U is used here to represent a term

n is the term number

- Example: A sequence is defined by the nth term:

$$U_n = 2U_{n-1}$$

U₁ means the 1st term of the sequence

Given that $U_1 = 2$ write down the first four terms of the sequence

► $U_1 = 2$	$U_2 = 2 \times U_{2-1}$	$U_3 = 2 \times U_{3-1}$	$U_4 = 2 \times U_{4-1}$
	$= 2 \times U_1$	$= 2 \times U_2$	$= 2 \times U_3$
	$= 2 \times 2$	$= 2 \times 4$	$= 2 \times 8$
	$= 4$	$= 8$	$= 16$

$\therefore 2, 4, 8, 16 \dots$

Recurrence Relations

- Example: A sequence is defined by the nth term:

$$U_{n+1} = U_n^2 - 8U_n + 17$$

Current term

Previous term

Given that $U_1 = 4$ find U_2 and U_3

► $U_1 = 4$	$U_2 = U_1^2 - 8U_1 + 17$	$U_3 = U_2^2 - 8U_2 + 17$
	$= 4^2 - 8(4) + 17$	$= 1^2 - 8(1) + 17$
	$= 16 - 32 + 17$	$= 1 - 8 + 17$
	$= 1$	$= 10$

Extension: Given, instead, that $U_1 = 2$, find U_2 , U_3 and U_{100}

Recurrence Relations

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► For each of the following, find U_2, U_3 , and U_4

a) $U_n = 10U_{n-1}$ $U_1 = 4$ $U_2 = 40$ $U_3 = 400$ $U_4 = 4000$

b) $U_n = 5U_{n-1} + 3$ $U_1 = 2$ $U_2 = 13$ $U_3 = 68$ $U_4 = 343$

c) $U_n = \frac{1}{2}U_{n-1} - 2$ $U_1 = 6$ $U_2 = 1$ $U_3 = -1.5$ $U_4 = -2.75$

d) $U_{n+1} = 4U_n + 7$ $U_1 = 1$ $U_2 = 11$ $U_3 = 51$ $U_4 = 211$

e) $U_{n+1} = U_n^2 - 2U_n$ $U_1 = 1$ $U_2 = -1$ $U_3 = 3$ $U_4 = 3$

f) $U_{n+1} = 2 - \frac{4}{U_n}$ $U_1 = 4$ $U_2 = 1$ $U_3 = -2$ $U_4 = 4$

Extension: Given, instead, that $U_1 = 3$ in e) and f), find U_{100}