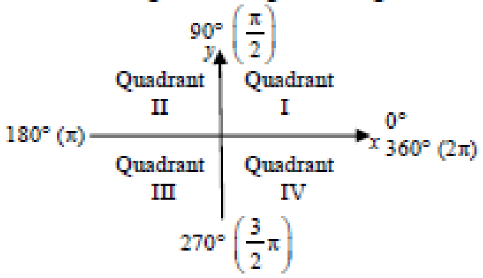
TRIGONOMETRIC FUNCTIONS

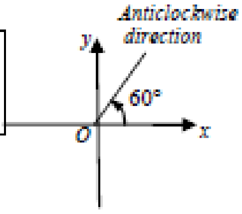
5.1 Positive Angle and Negative Angle



2π radian = 360°
π radian = 180°

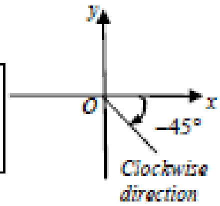
Positive Angle

A positive angle is measured in an *anticlockwise* direction from the positive x-axis.

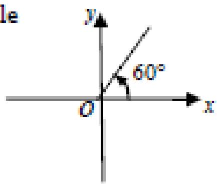
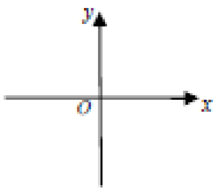
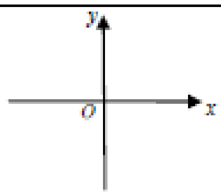
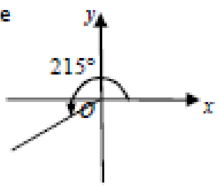
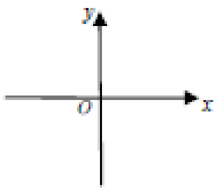
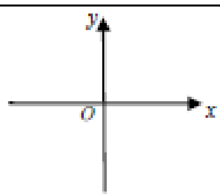
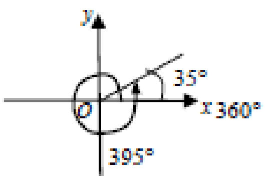
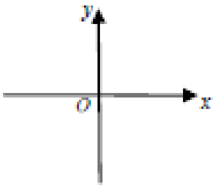
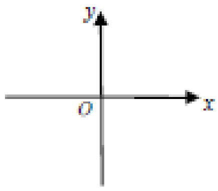
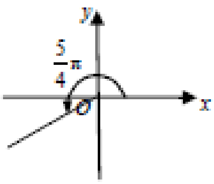
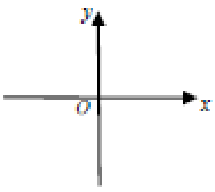
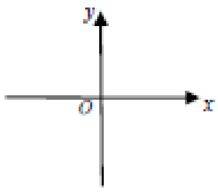
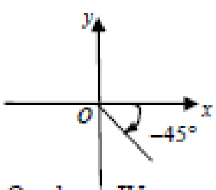
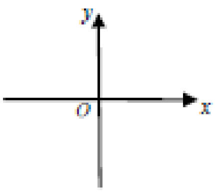
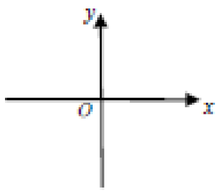


Negative Angle

A negative angle is measured in a *clockwise* direction from the positive x-axis.

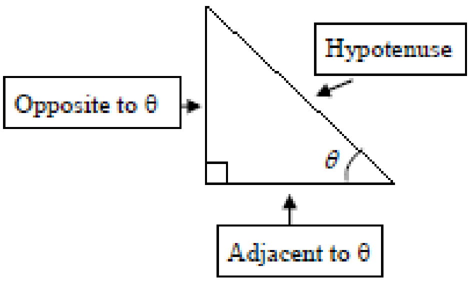
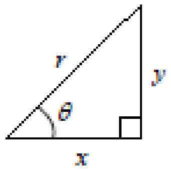
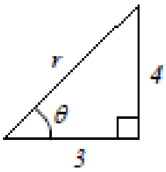
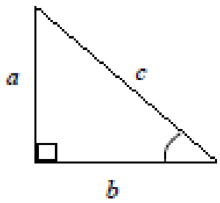


Represent each of the following angles in a Cartesian plane and state the quadrant of the angle.

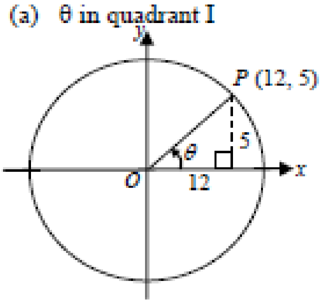
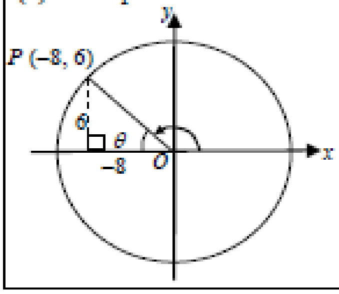
Example 60°  Quadrant 1	1(a) 70° 	(b) 150° 
Example 215°  Quadrant III	2(a) 195° 	(b) 345° 
Example 395°  Quadrant I	3(a) 415° 	(b) 480° 
Example $\frac{5}{4}\pi$  Quadrant III	4(a) $\frac{3}{4}\pi$ 	(b) $\frac{5}{3}\pi$ 
Example –45°  Quadrant IV	5(a) –130° 	(b) $-\frac{1}{3}\pi$ 

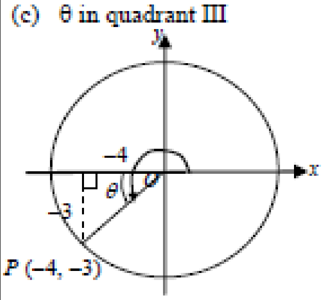
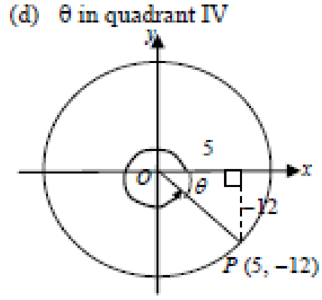
5.2 Six Trigonometric Functions of any Angle (1)

5.2.1 Define sine, cosine and tangent of any angle in a Cartesian plane

1  	 $\sin\theta = \frac{(\quad)}{(\quad)}$ $\cos\theta = \frac{(\quad)}{(\quad)}$ $\tan\theta = \frac{(\quad)}{(\quad)}$ Conclusion : $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$
2  $r^2 = 3^2 + 4^2$ $r = \sqrt{3^2 + 4^2}$ $r = 5$	Conclusion :  Pythagoras' Theorem : $c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2}$ $a^2 = c^2 - b^2 \Rightarrow a = \sqrt{c^2 - b^2}$ $b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2}$

3. Find the length of OA and the values of sine, cosine and tangent of θ.

(a) θ in quadrant I  	OP = =	$\sin\theta = \frac{5}{(\quad)}$	$\cos\theta = \frac{12}{(\quad)}$	$\tan\theta = \frac{5}{(\quad)}$
(b) θ in quadrant II  	OP = =	$\sin\theta = \frac{6}{(\quad)}$ =	$\cos\theta = \frac{-8}{(\quad)}$ =	$\tan\theta = \frac{6}{(\quad)}$ =

(c) θ in quadrant III 	OP = =	sin θ = $\frac{-3}{(\quad)}$	cos θ = $\frac{-4}{(\quad)}$	tan θ = $\frac{-3}{(\quad)}$ =
(d) θ in quadrant IV 	OP = =	sin θ = $\frac{-12}{(\quad)}$	cos θ = $\frac{5}{(\quad)}$	tan θ = $\frac{-12}{(\quad)}$

(e) Conclusion:

Sin θ is positive for θ in quadrant ……. and …….

Cos θ is positive for θ in quadrant ……. and …….

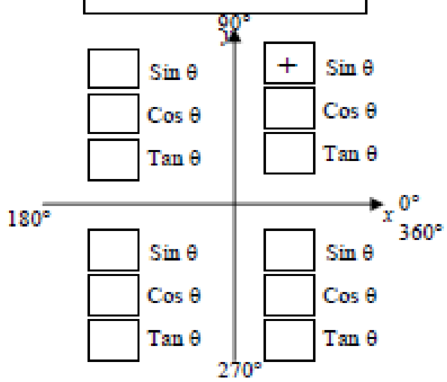
Tan θ is positive for θ in quadrant ……. and …….

Sin θ is negative for θ in quadrant ……. and …….

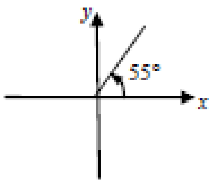
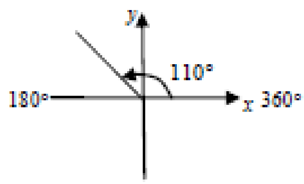
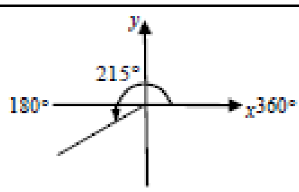
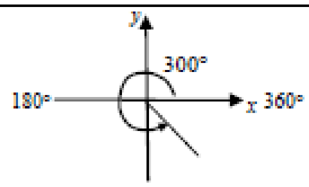
Cos θ is negative for θ in quadrant ……. and …….

Tan θ is negative for θ in quadrant ……. and …….

Fill in with – or + sign.

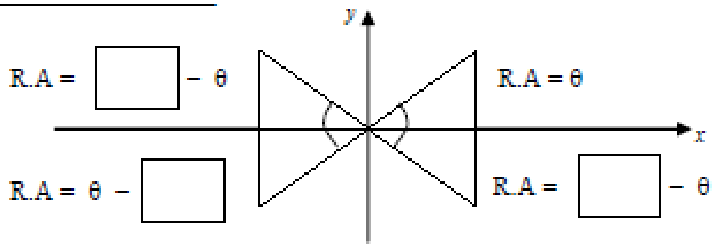


4. Find the corresponding reference angle of θ.

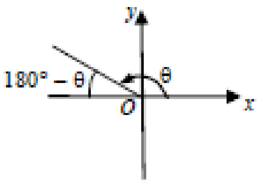
(a)  Reference angle = 55°	(b)  Reference angle = [ ] – 110° = 70°
(c)  Reference angle = 215° – [ ] = 35°	(d)  Reference angle = [ ] – 300° = 60°

(e) Conclusion:
Reference angle (RA) is the *acute angle* formed between the rotating ray of the angle and the

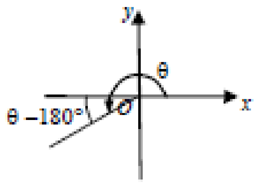
____



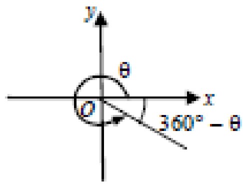
In Quadrant II:
sin θ = sin (180° − θ)
cos θ = −cos (180° − θ)
tan θ = −tan (180° − θ)



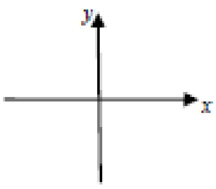
In Quadrant III
sin θ = −sin (θ − 180°)
cos θ = −cos (θ − 180°)
tan θ = tan (θ − 180°)



In Quadrant IV
sin θ = −sin (360° − θ)
cos θ = cos (360° − θ)
tan θ = −tan (360° − θ)



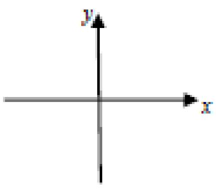
5. Given that cos 51° = 0.6293, find the trigonometric ratios of cos 231° without using a calculator or mathematical tables.



Reference angle of 231° = 231° − ☐
= ☐

cos 231° = ☐
= ☐

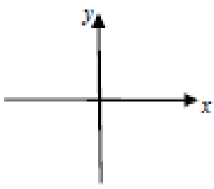
6. Given that sin 70° = 0.9397, find the trigonometric ratios of sin 610° without using a calculator or mathematical tables.



Reference angle of 610° = 610° − ☐ − ☐
= ☐

sin 610° = ☐
= ☐

7. Given that tan 25° = 0.4663, find the trigonometric ratios of tan 335° without using a calculator or mathematical tables.



Reference angle of 335° = ☐ − 335°
= ☐

tan 335° = ☐
= ☐

TRIGONOMETRIC FUNCTION

POSITIVE ANGLES AND NEGATIVE ANGLES

THE SIX TRIGONOMETRIC FUNCTION

$\tan \theta = \dfrac{\sin \theta}{\square}$

$\sec \theta = \dfrac{1}{\cos \theta}$

$\text{cosec } \theta = \dfrac{1}{\sin \theta}$

$\cot \theta = \dfrac{\text{cosec } \theta}{\square}$

Special angles 30°, ___ and 60°

Complementary Angle

- $\sin\theta = \cos(90^\circ - \theta)$
- $\cos\theta = \sin(90^\circ - \theta)$
- $\tan\theta = \cot(90^\circ - \theta)$
- $\sec\theta = \cos ec(90^\circ - \theta)$
- $\cos ec\theta = \sec(90^\circ - \theta)$

Basic Identities

- $\sin^2 \theta + \cos^2 \theta \equiv 1$
- $\_\_\_\_ + \tan^2 \theta \equiv \sec^2 \theta$
- $1 + \cot^2 \theta \equiv \_\_\_\_\_\_$

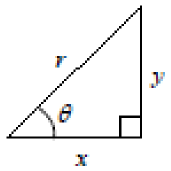
5.2.2 Define cotangent, secant and cosecant of any angle in a Cartesian plane.

1

$\sin\theta = \dfrac{(\quad)}{(\quad)}$

$\cos\theta = \dfrac{(\quad)}{(\quad)}$

$\tan\theta = \dfrac{(\quad)}{(\quad)}$

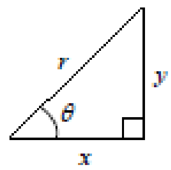


2

$\dfrac{1}{\sin\theta} = \dfrac{1}{\dfrac{(\quad)}{(\quad)}} = \dfrac{r}{y}$

$\dfrac{1}{\cos\theta} = \dfrac{1}{\dfrac{(\quad)}{(\quad)}} = \dfrac{(\quad)}{(\quad)}$

$\dfrac{1}{\tan\theta} = \dfrac{1}{\dfrac{(\quad)}{(\quad)}} = \dfrac{(\quad)}{(\quad)}$



3. Definition of cotangent θ, secant θ and cosecant θ.

$\dfrac{1}{\sin\theta} = \operatorname{cosec}\theta$

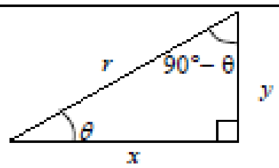
$\dfrac{1}{\cos\theta} = \sec\theta$

$\dfrac{1}{\tan\theta} = \cot\theta$

4. Since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, then

$\cot\theta = \dfrac{\boxed{}}{\boxed{}}$

5.



$\sin\theta = \dfrac{y}{r}$ $\qquad \sin(90-\theta) = \dfrac{x}{r}$

$\cos\theta = \dfrac{x}{r}$ $\qquad \cos(90-\theta) = \dfrac{y}{r}$

$\tan\theta = \dfrac{y}{x}$ $\qquad \tan(90-\theta) = \dfrac{x}{y}$

6.

Complementary angles:

$\sin\theta = \cos(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$

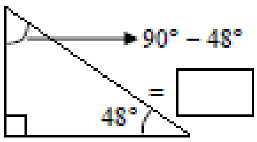
$\tan\theta = \cot(90° - \theta)$

$\operatorname{cosec}\theta = \sec(90° - \theta)$

$\sec\theta = \operatorname{cosec}(90° - \theta)$

$\cot\theta = \tan(90° - \theta)$

7. Given that sin 48° = 0.7431, cos 48° = 0.6991 and tan 48° = 1.1106, evaluate the value of cos 42°.

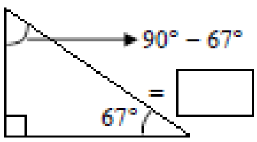


= ☐

cos 42° = ☐

= ☐

8. Given that sin 67° = 0.9205, cos 67° = 0.3907 and tan 67° = 2.3559, evaluate the value of cot 23°.

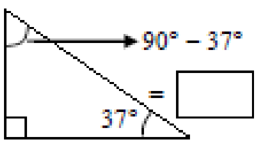


= ☐

cot 23° = ☐

= ☐

9. Given that sin 37° = 0.6018, cos 37° = 0.7986 and tan 37° = 0.7536, evaluate the value of sec 53°.



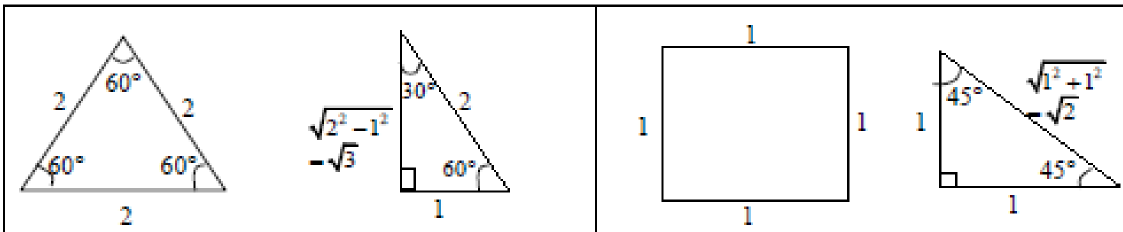
= ☐

sec 53° = ☐

= ☐

5.2.3 Find values of six trigonometric functions of any angle

1. Complete the table below.

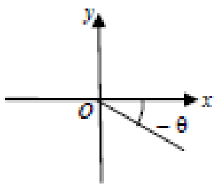


	30°	45°	60°
sin θ	$\frac{1}{2}$		
cos θ		$\frac{1}{\sqrt{2}}$	
tan θ		1	

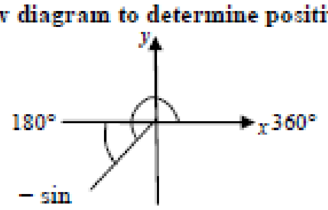
cos (− θ) = cos θ

sin (− θ) = − sin θ

tan (− θ) = − tan θ



2. Use the values of trigonometric ratio for the special angles, 30°, 45° and 60°, to find the value of the trigonometric functions below

Example: Evaluate sin 210°	a. Evaluate tan 300°
Draw diagram to determine positive or negative 	**Draw diagram to determine positive or negative**
Find reference angle Reference angle of 210° = 210° − 180° = 30°	**Find reference angle**
Solve sin 210° = − sin 30° $= -\frac{1}{2}$	**Solve**

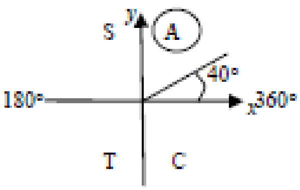
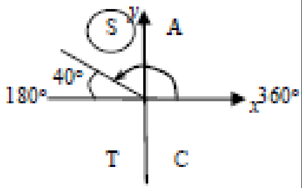
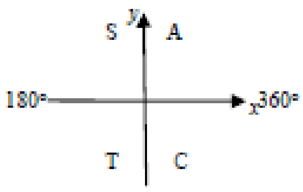
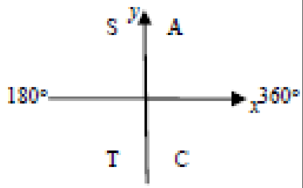
b. Evaluate cos 150°	c. Evaluate sec 135°
Draw diagram to determine positive or negative	**Draw diagram to determine positive or negative**
Find reference angle	**Find reference angle**
Solve	**Solve**

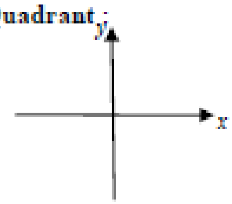
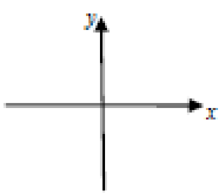
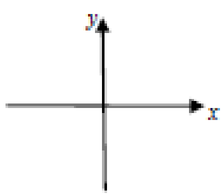
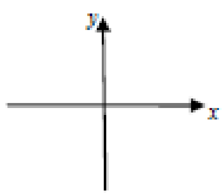
5.2.4 Solve trigonometric equations

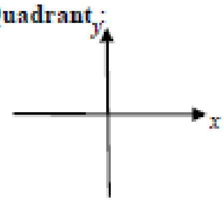
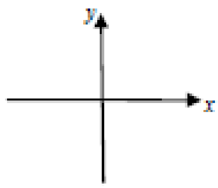
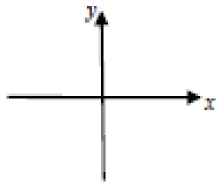
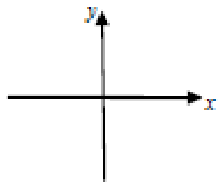
A. Steps to solve trigonometric equation

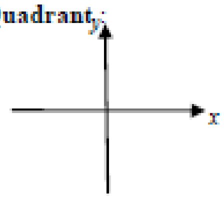
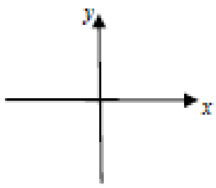
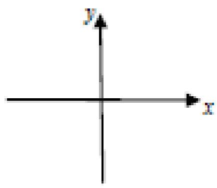
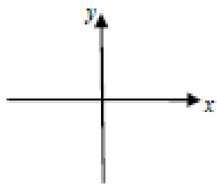
1. Determine the range of the angle.
2. Find the reference angle using tables or calculator.
3. Determine the quadrant where the angle of the trigonometric function is placed.
4. Determine the values of angles in the respective quadrants.

1. Solve the following equation for $0° \le \theta \le 360°$.

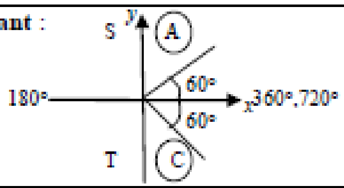
Example: $\sin\theta = 0.6428$	a. $\cos\theta = 0.3420$
Range : $0° \le \theta \le 360°$	$0° \le \theta \le 360°$
Reference angle : $\theta = \sin^{-1} 0.6428$ $\theta = 40°$	
Quadrant : Quadrant I Quadrant II	Quadrant ____ Quadrant ____
Actual angles $\theta = 40°$, $\theta = 180° - 40°$ $\theta = 40°, 140°$	

b. $\tan\theta = 1.192$	c. $\cos\theta = -0.7660$
Range :	
Reference angle :	
Quadrant : Quadrant ____ Quadrant ____	Quadrant ____ Quadrant ____
Actual angles	

d. $\sin \theta = -0.9397$	e. $\tan \theta = -0.3640$
Range :	
Reference angle :	
Quadrant :   Quadrant ___ Quadrant ___	  Quadrant ___ Quadrant ___
Actual angles	

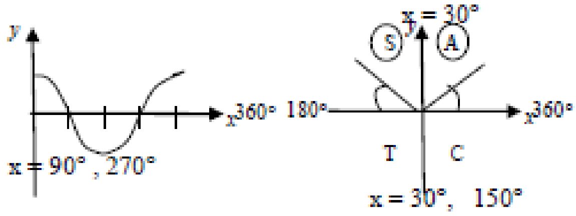
f. $\cot \theta = -1.4826$	g. $\text{cosec}\, \theta = -2.2027$
Range :	
Reference angle :	
Quadrant :   Quadrant ___ Quadrant ___	  Quadrant ___ Quadrant ___
Actual angles	

2. Solve the following equation for $0° \le \theta \le 360°$.

example : $\sec 2\theta = 2$	a. $2 \sin 2\theta = 1.6248$
Range : $0° \le \theta \le 360°$ $0° \le 2\theta \le 720°$	
Reference angle : $\frac{1}{\cos 2\theta} = 2$ $\cos 2\theta = \frac{1}{2}$ $2\theta = 60°$	
Quadrant : 	
Actual angles $2\theta = 60°, 360° - 60°, 60° + 360°, (360° - 60°) + 360°$ $\theta = 60°, 300°, 420°, 660°$	

b. $\cos 3\theta = -0.9781$	c. $\tan \frac{\theta}{2} = -2.05$
Range	
Reference angle :	
Quadrant :	
Actual angles	

d. $\sin(\theta + 10°) = 0.7660$	e. $\cos(\theta + 40°) = 0.7071$
f. $\tan(\theta + 15°) = 1$	g. $\cos(\theta - 20°) = 0.5$
h. $\tan(2\theta - 10°) = -2.082$	i. $\sin(2\theta - 30°) = 0.5$

j. $\sin \theta = \cos 20°$	k. $\cos \theta = -\sin 55°$
Example : $2 \sin x \cos x = \cos x$ $2 \sin x \cos x - \cos x = 0$ $\cos x (2 \sin x - 1) = 0$ $\cos x = 0$, $2 \sin x - 1 = 0$ $\sin x = \frac{1}{2}$ $x = 30°$  $\therefore x = 30°, 90°, 150°, 270°$	m. $2\sin x \cos x = \sin x$
n. $2 \cos^2 \theta + 3 \cos \theta = -1$	o. $2 \sin^2 \theta + 5 \sin \theta = 3$
p. $\tan^2 \theta = \tan \theta$	q. $3 \sin \theta = 2 + \text{cosec}\, \theta$