3.35 WHEATSTONE BRIDGE

62. *What is a Wheatstone bridge ? When is the bridge said to be balanced ? Apply Kirchhoff's laws to derive the balance condition of the Wheatstone bridge.*

Wheatstone bridge. *It is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances.* This method was first suggested by a British physicist *Sir Charles F. Wheatstone* in 1843.

A Wheatstone bridge consists of four resistances P, Q, R and S ; connected to form the arms of a quadrilateral $ABCD$. A battery of emf ε is connected between points A and C and a sensitive galvanometer between B and D, as shown in Fig. 3.210.

Let S be the resistance to be measured. The resistance R is so adjusted that there is no deflection in the galvanometer. The *bridge is said to balanced* when the potential difference across the galvanometer is zero so that there is no current through the galvanometer. In the balanced condition of the bridge,

$$\frac{P}{Q}=\frac{R}{S}$$

Unknown resistance, $S=\frac{Q}{P}\cdot R$

Knowing the ratio of resistances P and Q, and the resistance R, we can determine the unknown resistance S. That is why the arms containing the resistances P and Q are called *ratio arms,* the arm AD containing R *standard arm* and the arm CD containing S the *unknown arm.*

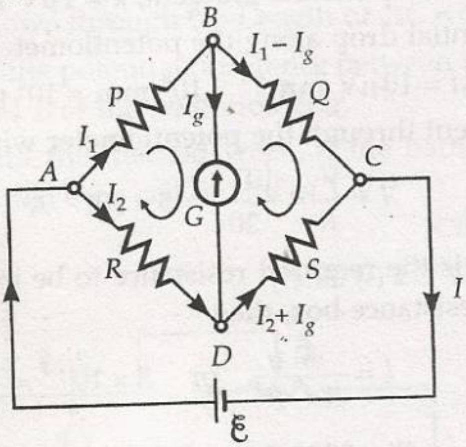


Fig. 3.210 Wheatstone bridge.

Derivation of balance condition from Kirchhoff's laws. In accordance with Kirchhoff's first law, the currents through various branches are as shown in Fig. 3.210.

Applying Kirchhoff's second law to the loop $ABDA$, we get

$$I_1 P + I_g G - I_2 R = 0$$

where G is the resistance of the galvanometer. Again applying Kirchhoff's second law to the loop $BCDB$, we get

$$(I_1 - I_g)Q - (I_2 + I_g)S - GI_g = 0$$

In the balanced condition of the bridge, $I_g = 0$. The above equations become

$$I_1 P - I_2 R = 0 \quad \text{or} \quad I_1 P = I_2 R \qquad ...(i)$$

and $$I_1 Q - I_2 S = 0 \quad \text{or} \quad I_1 Q = I_2 S \qquad ...(ii)$$

On dividing equation (*i*) by (*ii*), we get

$$\frac{P}{Q} = \frac{R}{S}$$

This proves the condition for the balanced Wheatstone bridge.

3.36 METRE BRIDGE OR SLIDE WIRE BRIDGE

65. *What is a metre bridge ? With the help of a circuit diagram, explain how it can be used to find an unknown resistance. Explain the principle of the experiment and give the formula used.*

Metre bridge or slide wire bridge. *It is the simplest practical application of the Wheatstone bridge that is used to measure an unknown resistance.*

Principle. Its working is based on the principle of Wheatstone bridge.

When the bridge is balanced,

$$\frac{P}{Q} = \frac{R}{S}$$

Construction. It consists of usually one metre long magnanin wire of uniform cross-section, stretched along a metre scale fixed over a wooden board and with its two ends soldered to two *L*-shaped thick copper strips *A* and *C*. Between these two copper strips, another copper strip is fixed so as to provide two gaps *ab* and a_1b_1. A resistance box R.B. is connected in the gap *ab* and the unknown resistance *S* is connected in the gap a_1b_1. A source of emf $\mathcal{E}$ is connected across AC. A movable jockey and a galvanometer are connected across BD, as shown in Fig. 3.211.

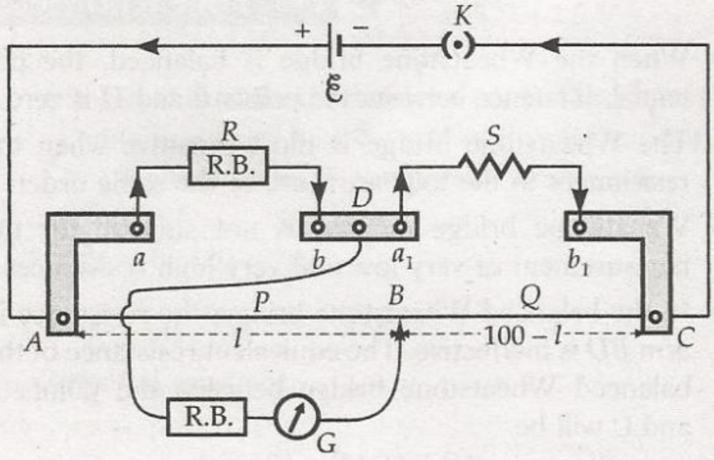


Fig. 3.211 Measurement of unknown resistance by a metre bridge.

Working. After taking out a suitable resistance R from the resistance box, the jockey is moved along the wire AC till there is no deflection in the galvanometer. This is the balanced condition of the Wheatstone bridge. If P and Q are the resistances of the parts AB and BC of the wire, then for the balanced condition of the bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

Let total length of wire $AC = 100$ cm and $AB = l$ cm, then $BC = (100 - l)$ cm. Since the bridge wire is of uniform cross-section, therefore,

resistance of wire ∝ length of wire

or $$\frac{P}{Q} = \frac{\text{resistance of } AB}{\text{resistance of } BC}$$

$$= \frac{\sigma\, l}{\sigma\,(100 - l)} = \frac{l}{100 - l}$$

where σ is the resistance per unit length of the wire.

Hence

$$\frac{R}{S} = \frac{l}{100 - l}$$

or $$S = \frac{R\,(100 - l)}{l}$$

Knowing l and R, unknown resistance S can be determined.

Determination of resistivity. If r is the radius of the wire and l' its length, then resistivity of its material will be

$$\rho = \frac{SA}{l'} = \frac{S \times \pi r^2}{l'}.$$

3.32 POTENTIOMETER

57. *What is a potentiometer ? Give its construction and principle.*

Potentiometer. An ideal voltmeter which does not change the original potential difference, needs to have infinite resistance. But a voltmeter cannot be designed to have an infinite resistance. Potentiometer is one such device which does not draw any current from the circuit and still measures the potential difference. So it acts as an ideal voltmeter.

A potentiometer is a device used to measure an unknown emf or potential difference accurately.

Construction. As shown in Fig. 3.195, a potentiometer consists of a long wire *AB* of uniform cross-section, usually 4 to 10 m long, of material having high resistivity and low temperature coefficient such as constantan or manganin. Usually, 1 m long separate pieces of wire are fixed on a wooden board parallel to each other. The wires are joined in series by thick copper strips. A metre scale is fixed parallel to the wires. The ends *A* and *B* are connected to a strong battery, a plug key *K* and a rheostat *Rh*. This circuit, called *driving* or *auxiliary circuit*, sends a constant current *I* through the wire *AB*. Thus, the potential gradually falls from *A* to *B*. A jockey can slide along the length of the wire.

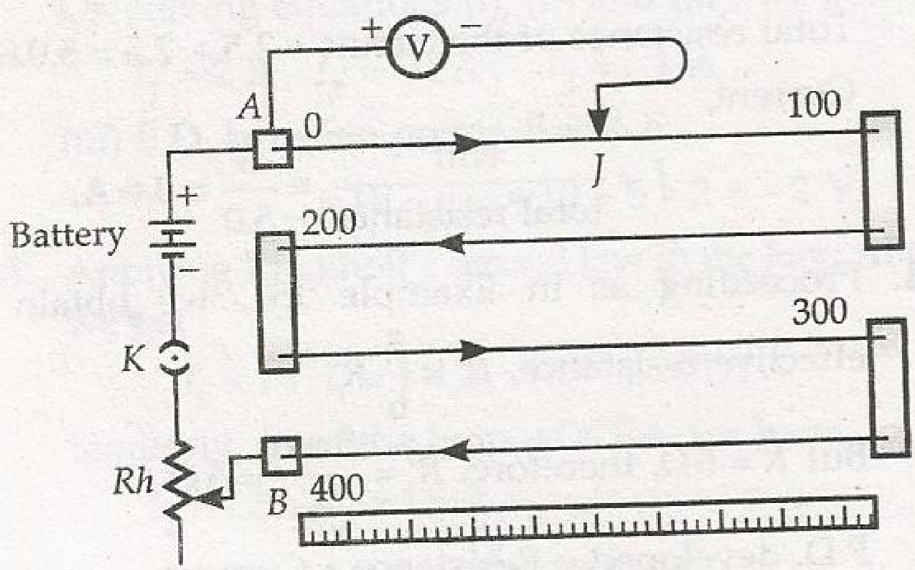


Fig. 3.195 Principle of a potentiometer.

Principle. *The basic principle of a potentiometer is that when a constant current flows through a wire of uniform cross-sectional area and composition, the potential drop across any length of the wire is directly proportional to that length.*

In Fig. 3.195, if we connect a voltmeter between the end A and the jockey J, it reads the potential difference V across the length l of the wire AJ. By Ohm's law,

$$V = IR = I \cdot \frac{\rho l}{A} \qquad \left[\because R = \rho \frac{l}{A}\right]$$

For a wire of uniform cross-section and uniform composition, resistivity ρ and area of cross-section A are constants. Therefore, when a steady current I flows through the wire,

$$\frac{I\rho}{A} = \text{a constant}, k$$

Hence $\quad V = kl \quad$ or $\quad V \propto l$

This is the principle of a potentiometer. A graph drawn between V and l will be a straight line passing through the origin O, as shown in Fig. 3.196.

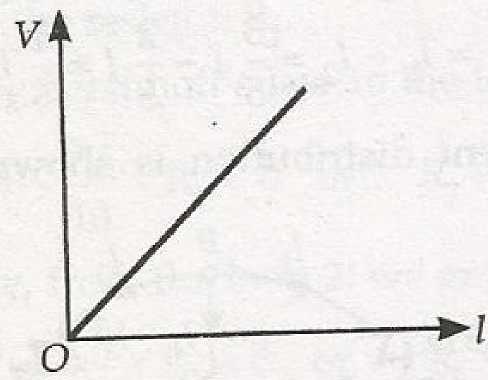


Fig. 3.196 Potential drop $V \propto$ length l

Potential gradient. *The potential drop per unit length of the potentiometer wire is known as potential gradient.* It is given by

$$k = \frac{V}{l}$$

SI unit of potential gradient = Vm^{-1}

Practical unit of potential gradient = V cm^{-1}.

3.33 APPLICATIONS OF A POTENTIOMETER

58. *With the help of a circuit diagram, explain how can a potentiometer be used to compare the emfs of two primary cells.*

Comparison of emfs of two primary cells. Fig. 3.197 shows the circuit diagram for comparing the emfs of two cells. A constant current is maintained in the potentiometer wire *AB* by means of a battery of emf ε through a key *K* and rheostat *Rh*. Let ε_1 and ε_2 be the emfs of the two primary cells which are to be compared. The positive terminals of these cells are connected to the end *A* of the potentiometer wire and their negative terminals are connected to a high resistance box R.B., a galvanometer G and a jockey J through a two way key. A high resistance R is inserted in the circuit from resistance box R.B. to prevent excessive currents flowing through the galvanometer.

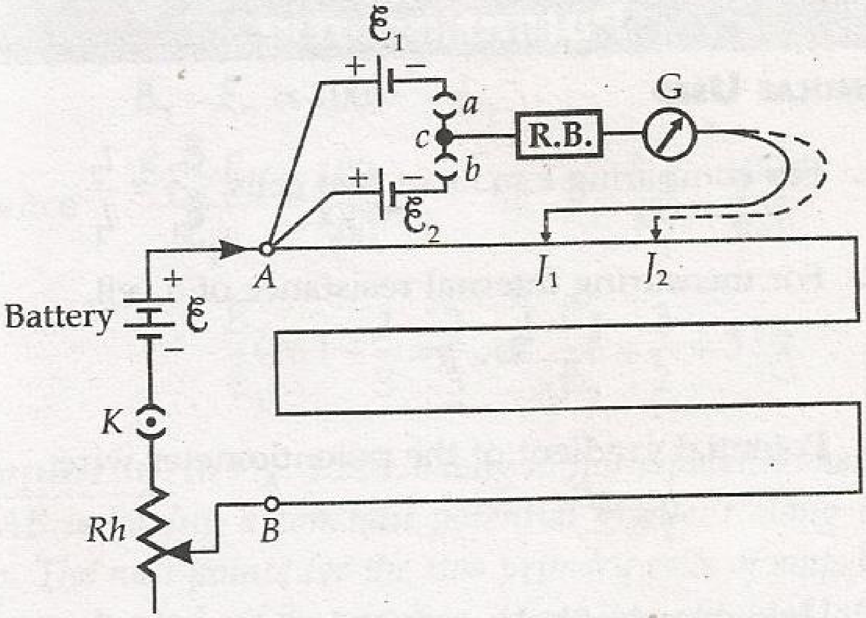


Fig. 3.197 Comparing emfs of two cells by a potentiometer.

As the plug is inserted between a and c, the cell ε_1 gets introduced in the circuit. The jockey J is moved along the wire AB till the galvanometer shows no deflection. Let the position of the jockey be J_1 and length of wire $AJ_1 = l_1$. If k is the potential gradient along the wire AB, then at null point,

$$\varepsilon_1 = kl_1$$

By inserting the plug between b and c, the null point is again obtained for cell ε_2. Let the balancing length be $AJ_2 = l_2$. Then

$$\varepsilon_2 = kl_2$$

Hence,
$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1}$$

If one of the two cells is a standard cell of known emf, then emf of the other cell can be determined.

$$\varepsilon_2 = \frac{l_2}{l_1} . \varepsilon_1$$

In order to get the null point on the potentiometer wire, it is necessary that the emf, ε of the auxiliary battery must be greater than both ε_1 and ε_2.

59. *With the help of a circuit diagram, explain how can a potentiometer be used to measure the internal resistance of a primary cell.*

Internal resistance of a primary cell by a potentiometer. As shown in the Fig. 3.198, the +ve terminal of the cell of emf $\mathcal{E}$ whose internal resistance r is to be measured is connected to the end A of the potentiometer wire and its negative terminal to a galvanometer G and jockey J. A resistance box R.B. is connected across the cell through a key K_2.

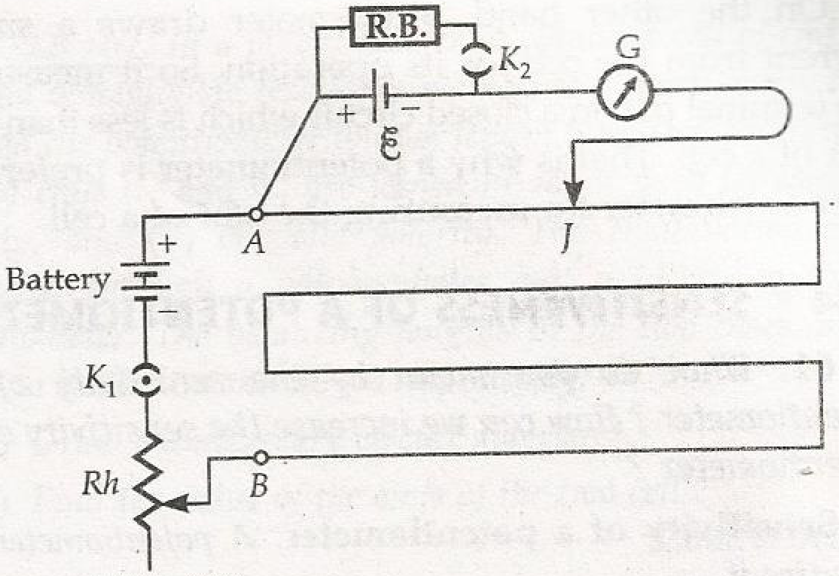


Fig. 3.198 To determine the internal resistance of a cell by a potentiometer.

Close the key K_1. A constant current flows through the potentiometer wire. With key K_2 kept open, move the jockey along AB till it balances the emf $\mathcal{E}$ of the cell. Let l_1 be the balancing length of the wire. If k is the potential gradient, then emf of the cell will be

$$\mathcal{E} = kl_1$$

With the help of resistance box R.B., introduce a resistance R and close key K_2. Find the balance point for the terminal potential difference V of the cell. If l_2 is the balancing length, then

$$V = kl_2$$

$$\therefore \quad \frac{\mathcal{E}}{V} = \frac{l_1}{l_2}$$

Let r be the internal resistance of the cell. If current I flows through cell when it is shunted with resistance R, then from Ohm's law we get

$$\mathcal{E} = I(R + r) \quad \text{and} \quad V = IR$$

$$\therefore \quad \frac{\mathcal{E}}{V} = \frac{R + r}{R} = \frac{l_1}{l_2}$$

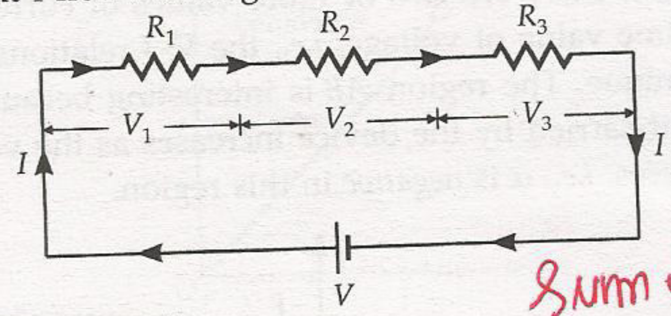
$$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

or
$$\frac{r}{R} = \frac{l_1 - l_2}{l_2}$$

$\therefore$ Internal resistance, $$r = R\left[\frac{l_1 - l_2}{l_2}\right].$$

32. *When are the resistances said to be connected in series ? Find an expression for the equivalent resistance of a number of resistances connected in series.*

Resistances in series. *If a number of resistances are connected end to end so that the same current flows through each one of them in succession, then they are said to be connected in series.* Fig. 3.30 shows three resistances R_1, R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.



Sum of V
I same

Fig. 3.30 Resistances in series.

By Ohm's law, the potential drops across the three resistances are

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

If R_s is the equivalent resistance of the series combination, then we must have

$$V = IR_s$$

But V = Sum of the potential drops across the individual resistances

or $$V = V_1 + V_2 + V_3$$

or $$IR_s = IR_1 + IR_2 + IR_3$$

or $$R_s = R_1 + R_2 + R_3$$

The equivalent resistance of n resistances connected in series will be

$$R_s = R_1 + R_2 + R_3 + \ldots + R_n$$

Thus when a number of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

Laws of resistances in series

(*i*) *Current through each resistance is same.*

(*ii*) *Total potential drop = Sum of the potential drops across the individual resistances.*

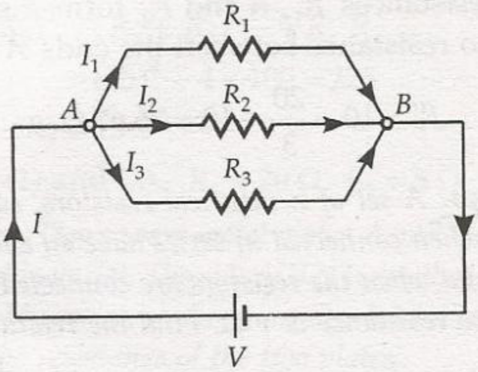
(*iii*) *Individual potential drops are directly proportional to individual resistances.*

(*iv*) *Equivalent resistance = Sum of the individual resistances.*

(*v*) *Equivalent resistance is larger than the largest individual resistance.*

33. *When are the resistances said to be connected in parallel ? Find the equivalent resistance of a number of resistances connected in parallel.*

Resistances in parallel. *If a number of resistances are connected in between two common points so that each of them provides a separate path for current, then they are said to be connected in parallel.* Fig. 3.31 shows three resistances R_1, R_2 and R_3 connected in parallel between points A and B. Let V be the potential difference applied across the combination.



sum of I
V same

Fig. 3.31 Resistance in parallel.

Let I_1, I_2 and I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. Then the current in the main circuit must be $I = I_1 + I_2 + I_3$

Since all the resistances have been connected between the same two points A and B, therefore, potential drop V is same across each of them. By Ohm's law, the currents through the individual resistances will be

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}.$$

If R_p is the equivalent resistance of the parallel combination, then we must have

$$I = \frac{V}{R_p}$$

But $$I = I_1 + I_2 + I_3$$

or $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent resistance R_p of n resistances connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}.$$

Thus *when a number of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.*

Laws of resistances in parallel

(*i*) *Potential drop across each resistance is same.*

(*ii*) *Total current = Sum of the currents through individual resistances.*

(*iii*) *Individual currents are inversely proportional to the individual resistances.*

(*iv*) *Reciprocal of equivalent resistance = Sum of the reciprocals of the individual resistances.*

(*v*) *Equivalent resistance is less than the smallest individual resistance.*

3.20 RELATION BETWEEN INTERNAL RESISTANCE, EMF AND TERMINAL POTENTIAL DIFFERENCE OF A CELL

35. *Define terminal potential difference of a cell. Derive a relation between the internal resistance, emf and terminal potential difference of a cell.*

Terminal potential difference. *The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference (V).*

Relation between r, $\mathcal{E}$ and V. Consider a cell of emf $\mathcal{E}$ and internal resistance r connected to an external resistance R, as shown in Fig. 3.90. Suppose a constant current I flows through this circuit.

By definition of emf,

$\mathcal{E}$ = Work done by the cell in carrying a unit charge along the closed circuit

= Work done in carrying a unit charge from A to B against external resistance R

+ Work done in carrying a unit charge from B to A against internal resistance r

or $$\mathcal{E} = V + V'$$

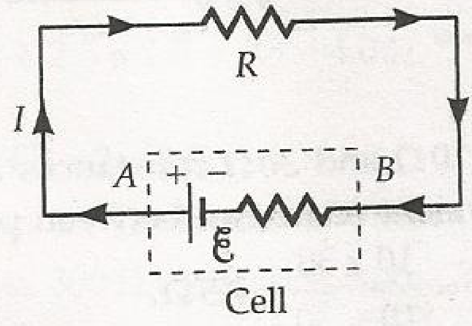


Fig. 3.90 Cell of emf ℰ and internal resistance r.

By Ohm's law,

$$V = IR$$

and $$V' = Ir$$

$$\therefore \quad \mathcal{E} = IR + Ir = I(R + r)$$

Hence the current in the circuit is

$$I = \frac{\mathcal{E}}{R + r}$$

Thus to determine the current in the circuit, the internal resistance r combines in series with external resistance R.

The terminal p.d. of the cell that sends current I through the external resistance R is given by

$$V = IR = \frac{\mathcal{E}R}{R + r}$$

Also $$V = \mathcal{E} - V' = \mathcal{E} - Ir$$

or terminal p.d. = emf – potential drop across the internal resistance

Again, from the above equation, we get

$$r = \frac{\mathcal{E} - V}{I} = \frac{\mathcal{E} - V}{V / R} = \left(\frac{\mathcal{E} - V}{V}\right) R.$$

37. ***What do you mean by a series combination of cells ? Two cells of different emfs and internal resistances are connected in series. Find expressions for the equivalent emf and equivalent internal resistance of the combination.***

Cells in series. *When the negative terminal of one cell is connected to the positive terminal of the other cell and so on, the cells are said to be connected in series.*

As shown in Fig. 3.102, suppose two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in series between points A and C. Let I be the current flowing through the series combination.

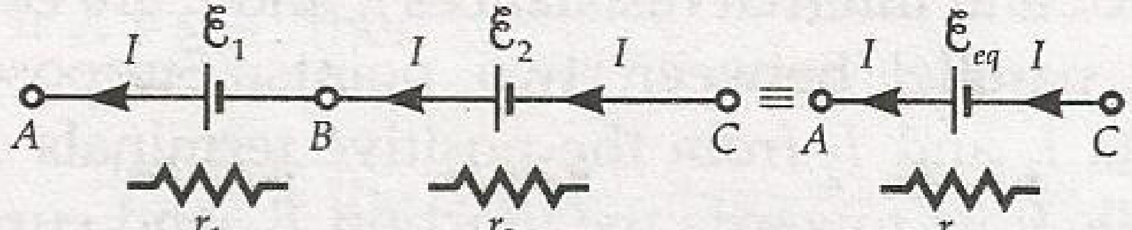


Fig. 3.102 A series combination of two cells is equivalent to a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}

Let V_A, V_B and V_C be the potentials at points A, B and C respectively. The potential differences across the terminals of the two cells will be

$$V_{AB} = V_A - V_B = \mathcal{E}_1 - Ir_1$$

and

$$V_{BC} = V_B - V_C = \mathcal{E}_2 - Ir_2$$

Thus the potential difference between the terminals A and C of the series combination is

$$V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

$$= (\mathcal{E}_1 - Ir_1) + (\mathcal{E}_2 - Ir_2)$$

or

$$V_{AC} = (\mathcal{E}_1 + \mathcal{E}_2) - I(r_1 + r_2)$$

If we wish to replace the series combination by a single cell of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}, then

$$V_{AC} = \mathcal{E}_{eq} - Ir_{eq}$$

Comparing the last two equations, we get

$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 \quad \text{and} \quad r_{eq} = r_1 + r_2$$

We can extend the above rule to a series combination of any number of cells :

1. The equivalent emf of a series combination of n cells is equal to the sum of their individual emfs.

$$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + ... + \mathcal{E}_n$$

2. The equivalent internal resistance of a series combination of n cells is equal to the sum of their individual internal resistances.

$$r_{eq} = r + r_2 + r_3 + ... + r_n$$

3. The above expression for $\mathcal{E}_{eq}$ is valid when the n cells assist each other *i.e.*, the current leaves each cell from the positive terminal. However, if one cell of emf $\mathcal{E}_2$, say, is turned around 'in opposition' to other cells, then

$$\mathcal{E}_{eq} = \mathcal{E}_1 - \mathcal{E}_2 + \mathcal{E}_3 + ... + \mathcal{E}_n.$$

38. *What do you mean by a parallel combination of cells ? Two cells of different emfs and internal resistances are connected in parallel with one another. Find the expressions for the equivalent emf and equivalent internal resistance of the combination.*

Cells in parallel. *When the positive terminals of all cells are connected to one point and all their negative terminals to another point, the cells are said to be connected in parallel.*

As shown in Fig. 3.103, suppose two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in parallel between two points. Suppose the currents I_1 and I_2 from the positive terminals of the two cells flow towards the junction B_1, and current I flows out. Since as much charge flows in as flows out, we have

$$I = I_1 + I_2$$

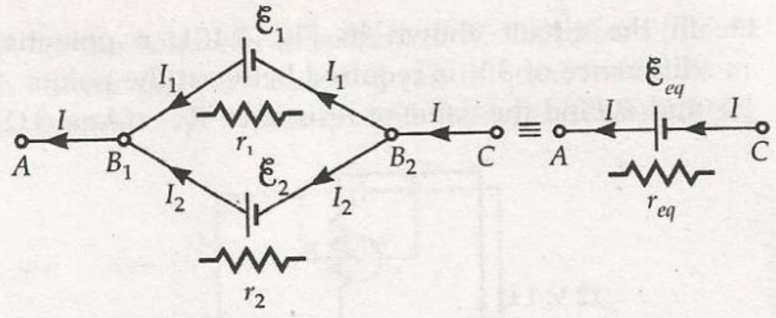


Fig. 3.103 A parallel combination of two cells is equivalent to a single cell of emf ε_{eq} and internal resistance r_{eq}.

As the two cells are connected in parallel between the same two points B_1 and B_2, the potential difference V across both cells must be same.

The potential difference between the terminals of first cell is

$$V = V_{B_1} - V_{B_2} = \varepsilon_1 - I_1 r_1$$

$$\therefore \quad I_1 = \frac{\varepsilon_1 - V}{r_1}$$

The potential difference between the terminals of ε_2 is

$$V = V_{B_1} - V_{B_2} = \varepsilon_2 - I_2 r_2$$

$$\therefore \quad I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Hence $$I = I_1 + I_2 = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or $$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$

or $$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\frac{r_1 r_2}{r_1 + r_2}$$

If we wish to replace the parallel combination by a single cell of emf ε_{eq} and internal resistance r_{eq}, then

$$V = \varepsilon_{eq} - I r_{eq}$$

Comparing the last two equations, we get

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad \text{and} \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

We can express the above results in a simpler way as follows :

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For a parallel combination of n cells, we can write

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \ldots\ldots + \frac{\varepsilon_n}{r_n}$$

and
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_n}.$$

39. *Derive the condition for obtaining maximum current through an external resistance connected across a series combination of cells.*

Condition for maximum current from a series combination of cells. As shown in Fig. 3.104, suppose n similar cells each of emf $\mathcal{E}$ and internal resistance r be connected in series. Let R be the external resistance.

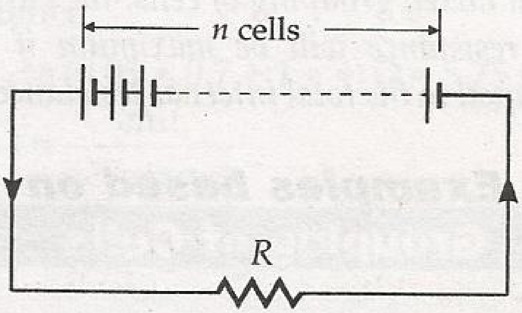


Fig. 3.104 A series combination of n cells.

Total emf of n cells in series

$$= \text{Sum of emfs of all cells} = n\mathcal{E}$$

Total internal resistance of n cells in series

$$= r + r + r + \ldots\ldots n \text{ terms} = nr$$

Total resistance in the circuit $= R + nr$

The current in the circuit is

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{n\mathcal{E}}{R + nr}$$

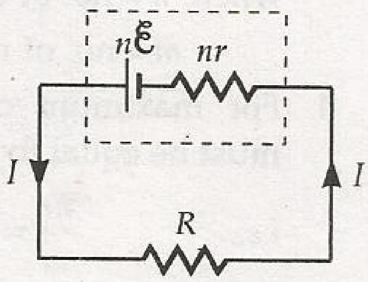


Fig. 3.105 Equivalent circuit.

Special Cases

(*i*) If $R >> nr$, then

$$I = \frac{n\mathcal{E}}{R}$$

$= n$ times the current $(\mathcal{E} / R)$ that can be drawn from one cell.

(*ii*) If $R << nr$, then

$$I = \frac{n\mathcal{E}}{nr} = \frac{\mathcal{E}}{r}$$

$=$ the current given by a single cell

Thus, *when external resistance is much higher than the total internal resistance, the cells should be connected in series to get maximum current.*

40. *Derive the condition for obtaining maximum current through an external resistance connected to a parallel combination of cells.*

Condition for maximum current from a parallel combination of cells. As shown in Fig. 3.106, suppose m cells each of emf ε and internal resistance r be connected in parallel between points A and B. Let R be the external resistance.

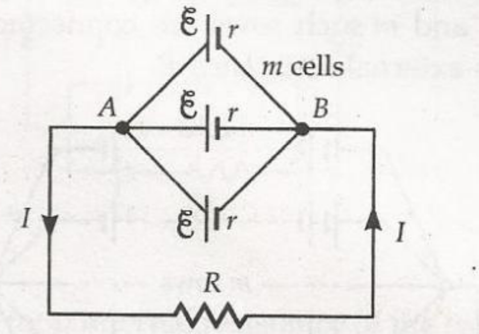


Fig. 3.106 A parallel combination of m cells.

Since all the m internal resistances are connected in parallel, their equivalent resistance R' is given by

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots\ldots m \text{ terms} = \frac{m}{r}$$

or $$R' = \frac{r}{m}$$

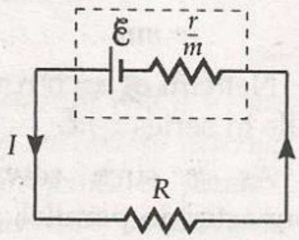


Fig. 3.107 Equivalent circuit.

Total resistance in the circuit

$$= R + R' = R + \frac{r}{m}$$

As the only effect of joining m cells in parallel is to get a single cell of larger size with the same chemical materials, so

total emf of parallel combination

$$= \text{emf due to single cell} = \varepsilon$$

$\therefore$ The current in the circuit is

$$I = \frac{\varepsilon}{R + r/m} = \frac{m\varepsilon}{mR + r}$$

Special Cases

(*i*) If $R << \frac{r}{m}$, then

$$I = \frac{m\varepsilon}{r} = m \text{ times the current due to a single cell.}$$

(*ii*) If $R >> \frac{r}{m}$, then

$$I = \frac{\varepsilon}{R} = \text{the current given by a single cell.}$$

Thus, *when external resistance is much smaller than the net internal resistance, the cells should be connected in parallel to get maximum current.*

3.27 POWER CONSUMPTION IN A COMBINATION OF APPLIANCES

47. *Prove that the reciprocal of the total power consumed by a series combination of appliances is equal to the sum of the reciprocals of the individual powers of the appliances.*

Power consumed by a series combination of appliances. As shown in Fig. 3.125, consider a series combination of three bulbs of powers P_1, P_2 and P_3 ; which have been manufactured for working on the same voltage V.

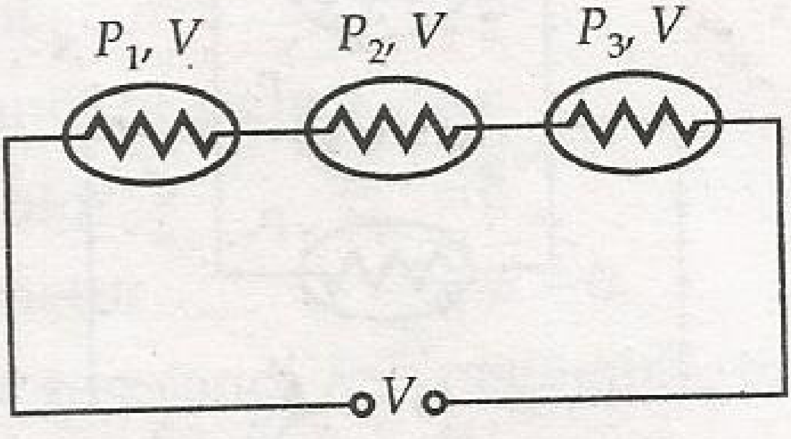


Fig. 3.125 Series combination of bulbs.

The resistances of the three bulbs will be

$$R_1 = \frac{V^2}{P_1}, \; R_2 = \frac{V^2}{P_2}, \; R_3 = \frac{V^2}{P_3}$$

As the bulbs are connected in series, so their equivalent resistance is

$$R = R_1 + R_2 + R_3$$

If P is the effective power of the combination, then

$$\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$$

or

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

Thus *for a series combination of appliances, the reciprocal of the effective power is equal to the sum of the reciprocals of the individual powers of the appliances.*

Clearly, when N bulbs of same power P are connected in series,

$$P_{eff} = \frac{P}{N}$$

As the bulbs are connected in series, the current I through each bulb will be same.

$$I = \frac{V}{R_1 + R_2 + R_3}$$

The brightness of the three bulbs will be

$$P_1' = I^2 R_1, \; P_2' = I^2 R_2, \; P_3' = I^2 R_3$$

As $R \propto \frac{1}{P}$, the bulb of lowest wattage (power) will have maximum resistance and it will glow with maximum brightness. When the current in the circuit exceeds the safety limit, the bulb of lowest wattage will be fused first.

fused first.

48. *Prove that when electrical appliances are connected in parallel, the total power consumed is equal to the sum of the powers of the individual appliances.*

Power consumed by a parallel combination of appliances. As shown in Fig. 3.126, consider a parallel combination of three bulbs of powers P_1, P_2 and P_3, which have been manufactured for working on the same voltage V.

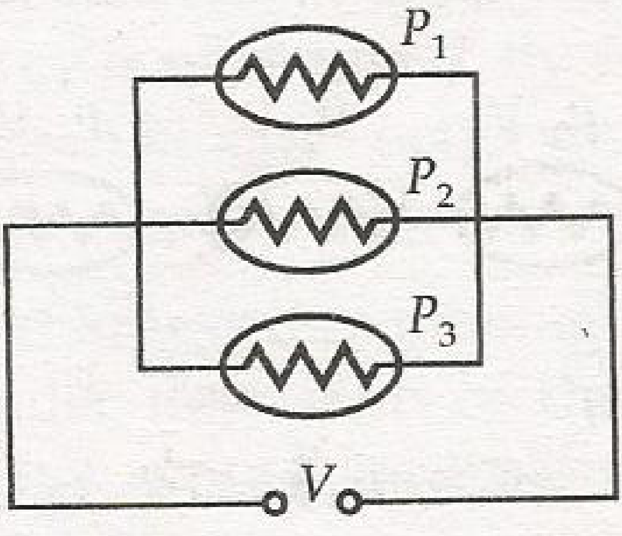


Fig. 3.126 Parallel combination of bulbs.

The resistances of the three bulbs will be

$$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}, \qquad R_3 = \frac{V^2}{P_3}$$

As the bulbs are connected in parallel, their effective resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Multiplying both sides by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or $$P = P_1 + P_2 + P_3$$

Thus *for a parallel combination of appliances, the effective power is equal to the sum of the powers of the individual appliances.*

If N bulbs, each of power P, are connected in parallel, then

$$P_{eff} = NP$$

The brightness of the three bulbs will be

$$P_1 = \frac{V^2}{R_1}, \; P_2 = \frac{V^2}{R_2}, \; P_3 = \frac{V^2}{R_3}.$$

As the resistance of the highest wattage (power) bulb is minimum, it will glow with maximum brightness. If the current in the circuit exceeds the safety limit, the bulb with maximum wattage will be fused first. For this reason, the appliances in houses are connected in parallel.

3.14 MOBILITY OF CHARGE CARRIERS

24. *Define mobility of charge carrier. Write relations between electric current and mobility for (i) a conductor and (ii) a semiconductor. Hence write an expression for the conductivity of a semiconductor.*

Mobility. The conductivity of any material is due to its mobile charge carrier. These may be electrons in metals, positive and negative ions in electrolytes ; and electrons and holes in semiconductors.

The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field. It is given by

$$\mu = \frac{v_d}{E}$$

As drift velocity, $v_d = \frac{qE\tau}{m}$

$$\therefore \quad \mu = \frac{v_d}{E} = q\frac{\tau}{m}$$

For an electron, $\mu_e = \frac{e\tau_e}{m_e}$

For a hole, $\mu_h = \frac{e\tau_h}{m_h}$

The mobilities of both electrons and holes are positive ; although their drift velocities are opposite to each other.

SI unit of mobility = $m^2V^{-1}s^{-1}$

Practical unit of mobility = $cm^2\ V^{-1}s^{-1}$.

$$1\ m^2V^{-1}s^{-1} = 10^4\ cm^2\ V^{-1}s^{-1}$$

Relation between electric current and mobility for a conductor

In a metallic conductor, the electric current is due to its free electrons and is given by

$$I = enAv_d$$

But $v_d = \mu_e E \quad \therefore \quad I = enA\mu_e E$

This is the relation between electric current and electron mobility.

Relation between electric current and mobility for a semiconductor

The conductivity of a semiconductor is both due to electrons and holes. So electric current in a semiconductor is given by

$$I = I_e + I_h = enAv_e + epAv_h$$
$$= enA\mu_e E + epA\mu_h E$$
$$= eAE\,(n\mu_e + p\mu_h) \qquad ...(i)$$

where n and p are the electron and hole densities of the semiconductor.

Conductivity of a semiconductor. According to Ohm's law,

$$I = \frac{V}{R} = \frac{El}{\rho l/A} = \frac{EA}{\rho} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$\frac{EA}{\rho} = eAE\,(n\mu_e + p\mu_h)$$

or $$\frac{1}{\rho} = e\,(n\mu_e + p\mu_h)$$

But $1/\rho$ is the electrical conductivity σ. Therefore,

$$\sigma = e\,(n\mu_e + p\mu_h)$$

them.

Mechanism of the flow of electric charges in a metallic conductor : Concepts of drift velocity and relaxation time. Metals have a large number of free electrons, nearly 10^{28} per cubic metre. In the absence of any electric field, these electrons are in a state of continuous random motion due to thermal energy. At room temperature, they move with velocities of the order of 10^5 ms^{-1}. However, these velocities are distributed randomly in all directions. There is no preferred direction of motion. On the average, the number of electrons travelling in any direction will be equal to number of electrons travelling in the opposite direction. If $\vec{u}_1, \vec{u}_2, ..., \vec{u}_N$ are the random velocities of N free electrons, then *average velocity* of electrons will be

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0$$

Thus, there is no net flow of charge in any direction.

In the presence of an external field $\vec{E}$, each electron experiences a force $-e\vec{E}$ in the opposite direction of $\vec{E}$ (since an electron has negative charge) and undergoes an acceleration $\vec{a}$ given by

$$\vec{a} = \frac{\text{Force}}{\text{Mass}} = -\frac{e\vec{E}}{m}$$

where m is the mass of an electron. As the electrons accelerate, they frequently collide with the positive metal ions or other electrons of the metal. Between two successive collisions, an electron gains a velocity component (in addition to its random velocity) in a direction opposite to $\vec{E}$. However, the gain in velocity lasts for a short time and is lost in the next collision. At each collision, the electron starts afresh with a random thermal velocity.

If an electron having random thermal velocity $\vec{u}_1$ accelerates for time τ_1 (before it suffers next collision), then it will attain a velocity,

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\,\tau_1$$

Similarly, the velocities of the other electrons will be

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\,\tau_2,$$

$$\vec{v}_3 = \vec{u}_3 + \vec{a}\,\tau_3, \ldots,$$

$$\vec{v}_N = \vec{u}_N + \vec{a}\,\tau_N$$

The average velocity $\vec{v}_d$ of all the N electrons will be

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots + \vec{v}_N}{N}$$

$$= \frac{(\vec{u}_1 + \vec{a}\,\tau_1) + (\vec{u}_2 + \vec{a}\,\tau_2) + \ldots + (\vec{u}_N + \vec{a}\,\tau_N)}{N}$$

$$= \frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_N}{N} + \vec{a}\,\frac{\tau_1 + \tau_2 + \ldots + \tau_N}{N}$$

$$= 0 + \vec{a}\,\tau$$

where $\tau = (\tau_1 + \tau_2 + \ldots + \tau_N)/N$ is the average time between two successive collisions. *The average time that elapses between two successive collisions of an electron is called **relaxation time**.* For most conductors, it is of the order of 10^{-14} s. The velocity gained by an electron during this time is

$$\vec{v}_d = \vec{a}\,\tau = -\frac{e\,\vec{E}\,\tau}{m}.$$

The parameter $\vec{v}_d$ is called ***drift velocity*** of electrons. *It may be defined as the average velocity gained by the free electrons of a conductor in the opposite direction of the externally applied electric field.*

It may be noted that although the electric field accelerates an electron between two collisions, yet it does not produce any net acceleration. This is because the electron keeps colliding with the positive metal ions. The velocity gained by it due to the electric field is lost in next collision. As a result, it acquires a constant average velocity $\vec{v}_d$ in the opposite direction of $\vec{E}$. The motion of the electron is similar to that of a small spherical metal ball rolling down a long flight of stairs. As the ball falls from one stair to the next, it acquires acceleration due to the force of gravity. The moment it collides with the stair, it gets decelerated. The net effect is that after falling through a number of steps, the ball begins to roll down the stairs with zero average acceleration *i.e.*, at constant average speed. Moreover, as the average time τ between two successive collisions is small, an electron slowly and steadily drifts in the opposite direction of $\vec{E}$, as shown in Fig. 3.18.

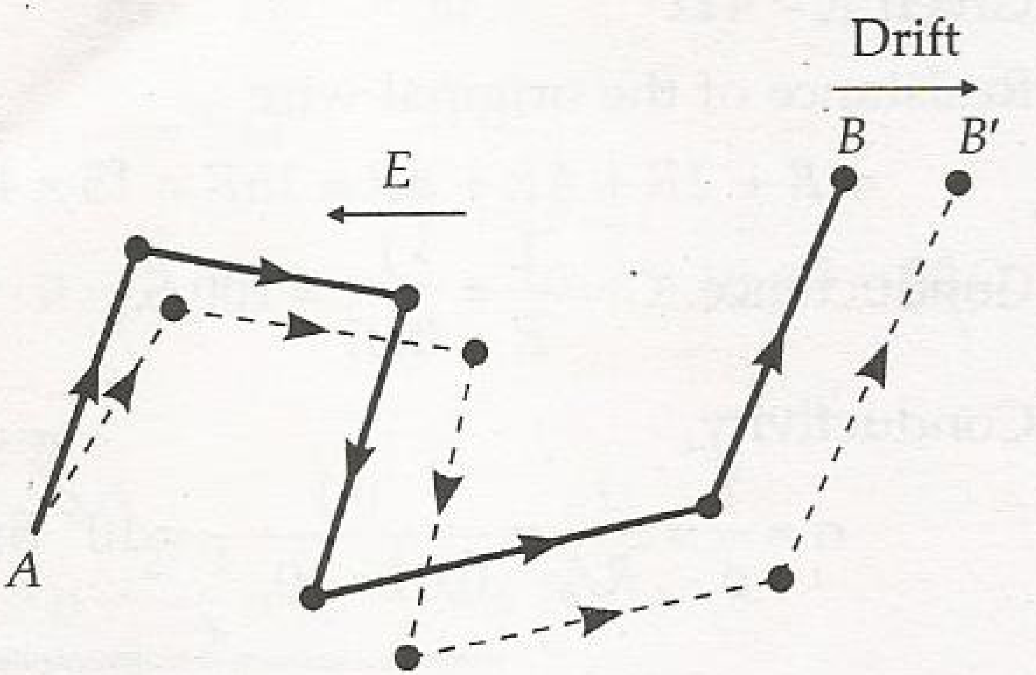


Fig. 3.18 Slow and steady drift of an electron in the opposite direction of $\vec{E}$. The solid lines represent the path in the absence of $\vec{E}$ and dashed lines in the presence of $\vec{E}$.

3.13 RELATION BETWEEN ELECTRIC CURRENT AND DRIFT VELOCITY : DERIVATION OF OHM'S LAW

19. *Derive relation between electric current and drift velocity. Hence deduce Ohm's law. Also write the expression for resistivity in terms of number density of free electrons and relaxation time.*

Relation between electric current and drift velocity. Suppose a potential difference V is applied across a conductor of length l and of uniform cross-section A. The electric field E set up inside the conductor is given by

$$E = \frac{V}{l}$$

Under the influence of field $\vec{E}$, the free electrons begin to drift in the opposite direction $\vec{E}$ with an average drift velocity v_d.

Let the number of electrons per unit volume or electron density $= n$

Charge on an electron $= e$

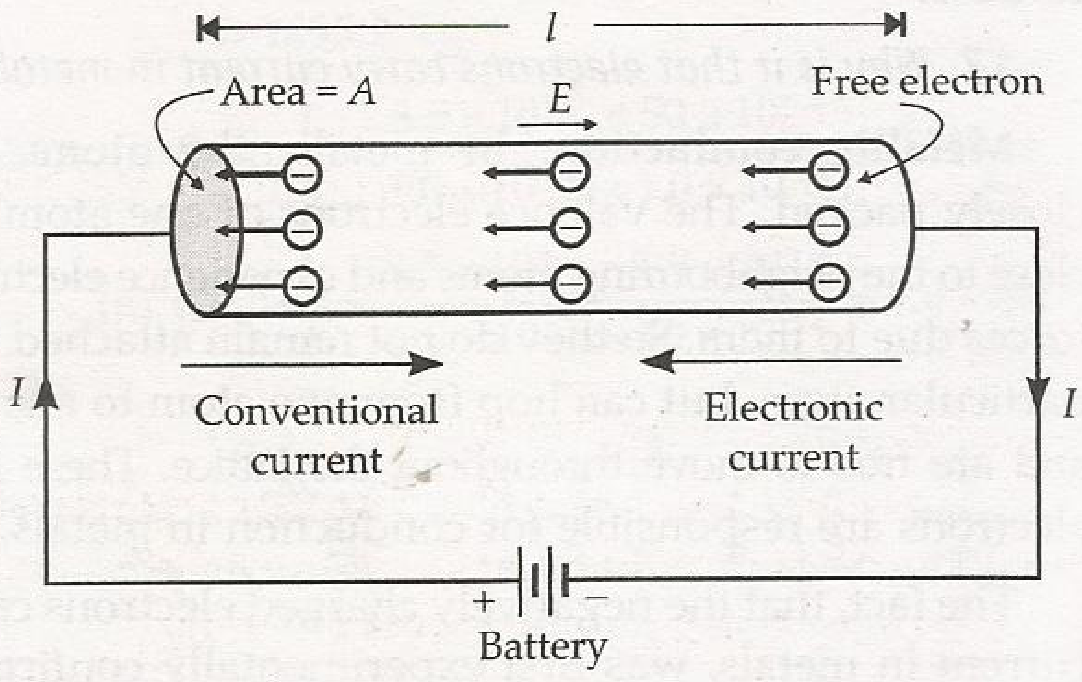


Fig. 3.19 Drift of electrons and electric field inside a conductor.

Number of electrons in length l of the conductor

$$= n \times \text{volume of the conductor} = n\,Al$$

Total charge contained in length l of the conductor is

$$q = en\,Al$$

All the electrons which enter the conductor at the right end will pass through the conductor at the left end in time,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{l}{v_d}$$

$$\therefore \text{Current,} \quad I = \frac{q}{t} = \frac{enAl}{l/v_d} \quad \text{or} \quad I = enAv_d$$

This equation relates the current I with the drift velocity v_d.

The current density 'j' is given by

$$j = \frac{I}{A} = env_d$$

In vector form $\quad \vec{j} = en\,\vec{v}_d$

The above equation is valid for both positive and negative values of q.

negative values of q.

Deduction of Ohm's law. When a potential difference V is applied across a conductor of length l, the drift velocity in terms of V is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

If the area of cross-section of the conductor is A and the number of electrons per unit volume or the electron density of the conductor is n, then the current through the conductor will be

$$I = enAv_d = enA \cdot \frac{eV\tau}{ml}$$

or $$\frac{V}{I} = \frac{ml}{ne^2 \tau A}.$$

At a fixed temperature, the quantities m, l, n, e, τ and A, all have constant values for a given conductor. Therefore,

$$\frac{V}{I} = \text{a constant, } R$$

This proves Ohm's law for a conductor and here

$$R = \frac{ml}{ne^2 \tau A}$$

is the resistance of the conductor.

Resistivity in terms of electron density and relaxation

Resistivity in terms of electron density and relaxation time. The resistance R of a conductor of length l, area of cross-section A and resistivity ρ is given by

$$R = \rho \frac{l}{A}$$

But $$R = \frac{ml}{ne^2 \tau A}$$

where τ is the relaxation time. Comparing the above two equations, we get

$$\rho = \frac{m}{ne^2 \tau}$$

Obviously, ρ is independent of the dimensions of the conductor but depends on its two parameters :

1. Number of free electrons per unit volume or electron density of the conductor.
2. The relaxation time τ, the average time between two successive collisions of an electron.

Relation between $\vec{j}$, σ and $\vec{E}$. For an electron,

$$q = -e$$

and $$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

$$\therefore \quad \vec{j} = nq\vec{v}_d = n(-e)\left(-\frac{e\vec{E}\tau}{m}\right) = \frac{ne^2\tau}{m}\vec{E}$$

But $\frac{ne^2\tau}{m} = \frac{1}{\rho} = \sigma$, conductivity of the conductor

$$\therefore \quad \vec{j} = \sigma\vec{E} \quad \text{or} \quad \vec{E} = \rho\vec{j}$$

This is Ohm's law in terms of vector quantities like current density $\vec{j}$ and electric field $\vec{E}$.