

SCALE NETWORKS AND DEBUSSY

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It is frequently observed that over the course of the nineteenth century the chromatic scale gradually supplanted the diatonic.¹ In earlier periods, non-diatonic tones were typically understood to derive from diatonic tones: for example, in C major, the pitch class F \sharp might be conceptualized variously as the fifth of B, the leading tone of G, or as an inflection of the more fundamental diatonic pitch class F \natural . By the start of the twentieth century, however, the diatonic scale was increasingly viewed as a selection of seven notes from the more fundamental chromatic collection. No longer dependent on diatonic scale for their function and justification, the chromatic notes had become entities in their own right.

Broadly speaking, composers approached this new chromatic context in one of two ways.² The first, associated with composers like Wagner, Strauss, and the early Schoenberg, de-emphasized scales other than the chromatic.³ Chord progressions were no longer constrained to lie within diatonic or other scalar regions. Instead, they occurred directly in chromatic space—often by way of semitonal or stepwise voice leading. Melodic activity also became increasingly chromatic, and conformed less frequently to recognizable scales. Chromaticism thus transformed not only the allowable chord progressions, but also the melodies they accompanied.

The second approach, associated with composers like Rimsky-Korsakov, Debussy, and Ravel, preserved a more conventional understanding of the relation between chord and scale, but within a significantly expanded musical vocabulary. New scales provided access to new chords, while new chords, in turn, suggested new scales. The *scalar tradition*, as I will call it, thus proposed a less radical, but also more hierarchically structured, conception of musical space: scales continued to exert their traditional influence in determining chord succession and melodic content, mediating between the musical surface and the underlying world of the total chromatic. By contrast, the chromatic tradition tended toward a flattening of musical space, as the chromatic scale exerted an increasingly direct influence on harmonic and melodic activity.

This paper proposes a general theoretical framework for understanding the scalar tradition in post-common-practice music. The argument has three parts. Section I discusses a number of intuitive constraints on scale formation, showing that the sets satisfying these constraints possess a number of interesting theoretical properties. This constitutes a “statics” of scalar collections. Section II turns to “dynamics”—techniques of moving between scales based on shared subsets or efficient voice leading. It presents graphs depicting common-tone and voice-leading relations among familiar scales, one of which is a three-dimensional *Tonnetz* for an important class of seven-note scales. Section III applies this theoretical framework to the analysis of four of Debussy’s piano pieces. The conclusion suggests that the ideas developed here have wider analytical applications as well.

Though my chief analytical concern will be with Debussy, my broader theoretical aim is to develop a set of concepts and structures useful for analyzing a wide range of post-common-practice scalar music. This paper draws examples from Debussy, Chopin, Liszt, Shostakovich, and Rzewski. In previous work I have used similar ideas to analyze the music of Ravel, Stravinsky, and several jazz improvisers.⁴ Other theorists, such as Elliott Antokoletz, Carlo Caballero, Clifton Callender, Ian Quinn, and Daniel Zimmerman, have used similar methods to analyze Bartók, Albrecht, Faure, Scriabin, Reich, and Prokofiev.⁵ Jose Antonio Martins uses closely-related structures to study Bartók, although his approach is importantly different from mine.⁶ The fact that similar methods can be used to describe such a broad range of music suggests that we can indeed speak of a scalar tradition—and even a “common practice”—in twentieth-century music: a set of concepts and techniques common to a variety of musical styles, including impressionism, jazz, and minimalism.

I. Scalar “Statics”: Three Scalar Collections

(a) Scales, Sets, and Modes: Some Terminological Preliminaries

A scale is a series of pitches ordered by register. This ordering underwrites a measure of musical distance distinct from the more general metrics provided by chromatic semitones and frequency ratios.⁷ The interaction between these contrasting metrics gives scalar music much of its complexity. Example 1 illustrates, presenting a series of three-note chords from Liszt. Understood in terms of the chromatic scale, the chords belong to three different set classes. Individual voices move by one of two intervals as they pass from chord to chord. Understood in terms of Example 1’s seven-note octave-repeating scale, however, each chord is an instance of the *same* set class: the triadic set class [024].⁸ From this perspective, individual voices always move by the same interval—a single scale step. Analytically, then, these chords belong either to three different types or to one single type, depending on whether we measure distance along the chromatic scale, or along the seven-note scale of Example 1. I will use the terms “chromatic distance” and “scalar distance” to refer to these two ways of measuring musical distance.⁹

Any pitch-class set can be associated with an infinite number of non-octave-repeating scales. There is, however, a unique *octave-repeating* scale associated with it: the ascending registral ordering of *all* the pitches belonging to *all* the pitch classes in the set. This infinite pitch series in turn defines a unique circular ordering of pitch classes, through which the pitch series repeatedly cycles.¹⁰ (As Maria puts it in *The Sound of Music*, the note “Ti” brings us *back* to “Do.”) In this article, the term “scale” refers to such circular orderings of pitch classes. Consequently, we can often dispense with talk of pitches and register, and speak instead about relations between pitch classes themselves.¹¹ The *scalar interval* from pitch class *a* to pitch class *b* is *one ascending scale step* if *a* immediately

(a)

(b)

Example 1. Parallel triads
(from Liszt’s “Angelus! Prière aux anges gardiens”)
a) mm. 177–182
b) the underlying scale

precedes b in the circular ordering; it is *two ascending scale steps* if a immediately precedes a note that immediately precedes b , and so on. Similarly, two notes form a “scalar second” or are “separated by one scale step,” if they are adjacent in the ordering; they form a “scalar third,” or are separated by two scale steps, if they are adjacent but for one note, and so on. Note that “scales” in this sense do not have tonic notes, pitch priority, or first or last notes. They are distinguished from sets chiefly in that their circular orderings give rise to the non-chromatic distance metric described above.

Since there is a unique octave-repeating scale associated with every pitch-class set, and vice versa, I will use the same names (or variables) to refer to both: thus “the set S ” (or “the collection S ”) refers to an unordered collection of pitch classes, while “the scale S ” is the unique circular ordering of those pitch classes described in the preceding paragraph. Octave-repeating scales and their associated sets share a great number of properties: every subset of set S can be uniquely associated with a “subscale” of scale S , and every voice-leading between sets S and T can be uniquely represented as a mapping between the elements of scales S and T . For this reason, it will sometimes be convenient to mix the terminology of scales and sets, as in: “the supersets of set S are scales with property P .” I trust this bit of linguistic shorthand will not confuse readers about the underlying distinction between sets and scales.¹² The terms “scale class” and “scale type” refer to classes of scales whose associated sets are related by transposition.¹³ Thus the C diatonic and D diatonic scales—considered as circular orderings of pitch classes—belong to the same scale class. Where the context is clear I will follow tradition, using the term “scale” to mean “scale class,” as in “*the diatonic scale*” and “*the octatonic scale*.”

Scales will be named according to their most familiar orderings.¹⁴ This is a mere terminological convenience, and is not meant to privilege any pitch or mode. The term “mode” will be used to describe a scale in which a pitch class has been singled out as having priority over the others. Though modes will not be a central concern of the theoretical portions of the paper, they play a role in the analyses presented in Section III. Diatonic modes are labeled using their familiar names: G \sharp natural minor, D dorian, and so forth. Non-diatonic modes are named with respect to the orderings in footnote 14. Thus the “first mode” of the C acoustic scale (C–D–E–F \sharp –G–A–B \flat) has C as its tonic note; the “second mode” has D, and so forth.

(b) Three Scalar Constraints

Imagine that you were an early-twentieth-century composer trying to devise scales and chords that were somehow similar to the scales and

Example 2. Three scalar passages (from the opening of Debussy's "Fetes")

chords of the tonal tradition. What kinds of "similarity" would you be interested in? How would you go about expanding the vocabulary of traditional classical practice?

This section suggests an answer to these questions, identifying three constraints on scale-formation that—I believe—may have influenced the post-common-practice exploration of non-diatonic materials.¹⁵ I begin by describing these constraints; later sections show that the scales satisfying them are exceedingly well-known. My hope is that the intrinsic plausibility of the constraints, the ubiquity of the objects they generate, and their analytical utility will jointly support the claim that the constraints played a role in the development of twentieth-century music.¹⁶

1. The "diatonic seconds" constraint. Example 2 presents a single series of scalar intervals in the context of three different scales. The three scales contain steps that are just one or two semitones large. This suggests the following constraint:

DS ("Diatonic Seconds"): If the scalar interval between two notes is one ascending step, then the chromatic interval between them is either one or two ascending semitones.

We can think of the DS constraint as generalizing a purely conventional feature of the diatonic scale; or we can think of it as expressing a deeper perceptual fact, perhaps related to the critical bandwidth of the auditory system.¹⁷ The difference between these perspectives is not crucial to the following discussion. What is important is that the DS constraint captures a clear and salient way in which a non-diatonic scale can resemble the traditional diatonic scale. This resemblance ensures an important degree of consistency between the scalar and chromatic distance metrics described in Section I(a).¹⁸ Composers can therefore transport a scalar

interval pattern from one scale to another, or within a single scale by means of scalar transposition, without radically transforming the size of its steps. This is why the three melodies in Example 2 above are recognizably similar.

2. *The “no consecutive semitones” constraint.* None of the scales shown in Example 2 contain successive one-semitone steps. This suggests a second constraint on scale formation:

NCS (“No Consecutive Semitones”): If the chromatic interval between any two notes is two ascending semitones, then the scalar interval between them is one ascending step; thus, a scale cannot contain successive one-semitone steps.

Where the DS constraint sets an upper limit on the size of a scale’s steps, NCS sets a lower limit on the size of its “leaps”: if a scale does not contain successive one-semitone steps, then any scalar interval that is larger than a step (a “leap”) must span at least three chromatic semitones. The two together ensure that there is a clear distinction between the size of a scale’s steps and leaps.¹⁹ This is illustrated in Example 2, where the brackets identify the scalar leaps.

It is also possible to understand the NCS constraint as operating directly on chords, or unordered sets. A set satisfies the NCS constraint only if it does not contain an [012] trichord as a subset. (Thus, an octave-repeating scale satisfies the NCS constraint if and only if its associated set satisfies the NCS constraint.) The motivation for applying the NCS constraint directly to sets is that the [012] trichord is often considered to be a particularly dissonant object.²⁰ Perhaps for this reason, there are a number of harmonically-rich, post-common-practice styles—including impressionism, jazz, and minimalism—in which sonorities containing [012] trichords are rare.²¹ One can think of the NCS constraint, when applied to sets, as a way of modeling these styles: the sets satisfying NCS constitute the harmonic space available to a composer who is willing to countenance virtually *any* chord as a potential sonority, except for the [012] trichord and its supersets.

Interpreting the NCS constraint as a constraint on chord formation provides another reason to think that it might have influenced the choice of scales themselves. For if an octave-repeating scale satisfies the NCS constraint, then all of the unordered chords that can be constructed from its elements also satisfy that constraint. This means that composers can treat such scales “pandiatonically,” freely choosing elements from the scale without generating collections that run afoul of NCS. Thus, insofar as avoidance of [012] and its supersets was indeed an aspect of early twentieth-century harmonic practice, and insofar as some of these composers were interested in treating every scalar subset as a potential har-

mony, it is natural that NCS might come to have an influence on the choice of scales themselves.²²

3. *The “diatonic thirds” constraint.* The scales shown in Example 2 possess a third interesting property: notes with one note between them (*scalar thirds*) are separated by either three or four semitones. This suggests a final constraint on scale formation:

DT (“Diatonic Thirds”): If the scalar interval between any two notes is two ascending steps, then the chromatic interval between them is three or four ascending semitones.

The DT constraint again ensures an important degree of consistency between scalar and chromatic distances. It permits a composer to transport a single pattern of scalar intervals from one DT scale to another, or within a single DT scale by scalar transposition, without radically changing the size of its two-step intervals. Consequently, all of the bracketed intervals shown in Example 2 span three or four chromatic semitones.

This fact is particularly important in light of the Western classical tradition’s emphasis on tertian harmonies. The DT constraint ensures that stacks of scalar thirds sound “tertian”—that is, they will be stacks of three- or four-chromatic-semitone intervals. This property allows composers to apply familiar routines of diatonic-scale composition in the context of a wider range of scales, obtaining results that are different from—but *not too different from*—from the music of the classical tradition. We can also imagine early twentieth-century composers happening onto the DT scales by the reverse process: beginning with a stack of three- and four-semitone intervals, a composer might explore the various ways it can be transformed into a scale by interleaving it with another such sonority. Such investigations could well lead to the DT scales themselves.

The three scalar constraints just described are often evoked—albeit tacitly—in accounts of the origin of the ascending form of the melodic minor scale. Commentators frequently refer to the “melodic awkwardness” of the harmonic minor scale’s augmented second.²³ This “awkwardness” presumably derives from the harmonic minor scale’s violation of the DS constraint. However, as Example 3 shows, there are a number of alternatives to the harmonic minor scale that do not violate the DS constraint. Standard accounts simply assume that the ascending melodic minor is the obvious choice from among these alternatives. Pressed to explain this, historians would no doubt attribute “melodic awkwardness”—or perhaps “harmonic awkwardness”—to those scales that add an additional note to the harmonic minor scale. Examples 3(b) and 3(c) illustrate. This awkwardness presumably reflects the fact that these scales violate the NCS and DT constraints. The ascending form of the melodic minor scale, by contrast, satisfies all three constraints. Indeed, it is the *only* scale sat-

(a) violates DS

(b) violates NCS & DT

(c) violates NCS & DT

(d) possesses DS, NCS, and DT

Example 3. Scalar constraints in the genesis of the melodic minor scale

- a) A harmonic minor
- b) a hypothetical alternative
- c) another alternative
- d) A melodic minor, ascending

isfying them that contains both a minor triad and its leading tone. In this sense it is the unique “non-awkward” alternative to the harmonic minor.

(c) The Locally Diatonic Scales

There are twenty-five types of scale satisfying the Diatonic Seconds constraint. To understand them, it is helpful to observe that DS is *inherited by supersets*. That is, if a scale S has this property, then so do all scales containing S . (This is because you cannot make a scale’s steps larger by adding notes to it!) The DS scales can therefore be characterized as supersets of the *minimal* DS scales. And the minimal DS scales are precisely those without consecutive semitones.²⁴

These scales belong to just four types: diatonic, octatonic, whole-tone, and acoustic.²⁵ In addition to satisfying the DS and NCS constraints, these scales are “locally diatonic” within a three-note span: any three consecutive notes of one of these scales will be enharmonically equivalent to

some three-note span of some diatonic scale. (They therefore satisfy the Diatonic Thirds constraint as well.) For this reason I refer to them as the *locally diatonic* scales. We can expect significant and audible similarities between traditional diatonic music and music that is based on locally diatonic scales: stepwise locally diatonic melodies will use one- and two-semitone intervals, while stacks of locally-diatonic scalar thirds will be stacks of three- and four-semitone intervals. The four locally diatonic scales represent natural objects of exploration for those early-twentieth-century composers who wanted to expand the resources of traditional tonal music without discarding such concepts as “triad” and “scale step.”

One point of clarification is in order. Although I have noted that all of the DS scales can be represented as supersets of locally diatonic scales, I do not think that it is always analytically profitable to do so. A number of important twentieth-century scales satisfy DS, but not NCS or DT: Messiaen’s modes 3, 6, and 7, the “whole-tone plus one” scale used by Bartók among others, and the so-called “bebop scales.”²⁶ Whether to treat these as supersets of the locally diatonic scales is a matter that very much depends on analytical context. In Messiaen’s music, for example, the “third mode” 9–12 [01245689T] is typically used as fundamental scale in its own right, rather than as a superset of the whole-tone and acoustic scales.

Nevertheless, I do think that there is a substantial body of music in which the locally diatonic scales have a privileged status. In such styles, supersets of the locally diatonic scales typically appear as embellishments of the locally diatonic scales. This is certainly true of the music of Debussy’s early and middle periods. A similar point applies to much jazz and minimalist music. The preceding discussion suggests numerous reasons why this might be so. The conjunction of the DS, DT, and NCS properties allow composers to obtain a wealth of nontraditional sounds while continuing to compose in fairly traditional ways. They therefore constitute an attractive set of tools for composers seeking to expand, rather than replace, the vocabulary of traditional tonality.

(d) the “Pressing Scales”

The NCS constraint is *inherited by subsets*: if a set S does not contain an 012 trichord, then neither do any of S ’s subsets. We can therefore characterize all the sets satisfying the NCS constraint as subsets of the *maximal* NCS sets. The situation is precisely the inverse of that which we encountered with the Diatonic Seconds property: the fact that DS is inherited by supersets led us to look for minimal DS sets. By contrast, because NCS is inherited by subsets, we look for maximal NCS sets.

It can be shown that a set S is a maximal NCS set if and only if S , when considered as a scale, possesses DT.²⁷ There are seven types of DT scale: the four locally diatonic scales, the familiar harmonic minor scale, its inversion (sometimes called the “harmonic major scale” because it can

locally diatonic

C acoustic (7–34)

C diatonic (7–35)

Octatonic Collection III (8–28)

Whole-Tone Collection II (6–35)

0145

C harmonic minor (7–32A)

0145

C harmonic major (7–32B)

0145 0145 0145

Hexatonic Collection IV (6–20)

Example 4. The seven Pressing scales

be obtained by raising the third degree of the harmonic minor), and the 6–20 [014589] “hexatonic scale” consisting of alternating one- and three-semitone steps.²⁸ Each of these last three scale-types contains a consecutive [0145] tetrachord, as illustrated in Example 4. None of the four locally diatonic scales can contain this tetrachord, since its three-semitone interval cannot be filled in without creating consecutive semitones. I will call these scales the “Pressing scales” since the late Jeff Pressing was the first person to write about them.²⁹

The Pressing scales represent a further extension of the locally diatonic scales. Each permits the construction of tertian harmonies, and—if we count the three semitone interval—more-or-less stepwise melodies.³⁰ Furthermore, these sets contain as subsets all and only those pitch-class sets that do not themselves contain an [012] trichord. Consequently, composers who are committed to NCS as a *harmonic* constraint will invariably find themselves using subsets of these scales. A particularly important corollary of this fact is that the seven scale types contain all the set classes that can be formed by superimposing two triads of any quality—major, minor, diminished, or augmented. They are therefore the natural

scalar concomitants to the kinds of polytriadic sonorities much favored by twentieth-century composers.

Thus a number of independent lines of investigation jointly converge on the Pressing scales. One could arrive at them through purely melodic processes, for instance by searching for the largest octave-repeating scales not containing consecutive one-semitone steps. Or one could arrive at these scales through purely harmonic processes, for instance by superimposing triadic sonorities. The DT constraint represents yet a third route to these scales, one that emphasizes the correspondence between scalar thirds and chromatic thirds. With so many independent considerations pointing in the same direction, we should not be shocked to find the Pressing scales reappearing in a variety of musical and theoretical contexts.

(e) The Maximal Anhemitonic Scales

We conclude by discussing the complements of the DS sets. These are the “anhemitonic” sets, since they cannot contain any semitones. (A set contains a semitone if and only if its complement’s octave-repeating scale contains a step of at least three semitones.) Since the locally diatonic scales are the minimal DS sets, they have as their complements the *maximal* anhemitonic set classes. These four set-classes are the only ones whose octave-repeating scales satisfy the following, enlarged version of the DS constraint:

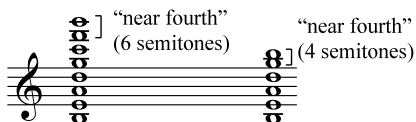
DS+: If the scalar interval between any two notes is one ascending step, then the chromatic interval between them is two or three ascending semitones.³¹

DS+ is similar to DS, except that chromatic distances have been increased by one semitone each. The four scale types possessing this property, and containing as subsets all the anhemitonic set classes, are the diminished seventh chord, the 5–35 [02479] pentatonic scale, the 5–34 [02469] “dominant ninth” pentachord, and the whole-tone scale.³²

The pentatonic scale is, in addition, one of just three scale types satisfying an enlarged version of the DT property.

DT+: If the scalar interval between any two notes is two ascending steps, then the chromatic interval between them is four or five ascending semitones.

(The whole-tone and hexatonic scales also satisfy DT+, but in a less interesting way, since their two-step intervals are always four semitones large.) Example 5 shows that we can understand the pentatonic scale, like the diatonic scale, as a stack of five-semitone intervals that has been “perturbed” so that it returns to its starting point without cycling through all twelve pitch classes. In each case the perturbation is a minimal one. The diatonic scale can be arranged as a stack of six “perfect fourths” (five-



Example 5. The diatonic and pentatonic scales as “near” fourth chords

semitone intervals) plus one “near fourth” (or “augmented fourth”) of six semitones. The pentatonic scale can be presented as a stack of four “perfect fourths” plus one “near fourth” (here a “diminished fourth”) of four semitones. But whereas the five-semitone intervals in the diatonic scale are all *scalar fourths* (lying three scale steps away from one another), the five-semitone intervals in the pentatonic scale are *scalar thirds*. Thus the second chord shown in Example 5 is not a “fourth chord” at all. Instead, it is a stack of pentatonic *thirds*.

Example 6 presents a series of musical examples where what look like “fourths” are more properly described as pentatonic thirds. Example 6(a), from the end of Debussy’s “La fille aux cheveux de lin,” shows a passage in which diatonic fourths (left hand, m. 34) give way to pentatonic thirds (right hand, m. 35). (Related Debussian passages can be found in Examples 23 and 28 below.) Example 6(b), from the second movement of Ligeti’s Piano Concerto, features diatonic fifths in the right hand and pentatonic fourths (the registral inversions of pentatonic thirds) in the left. Example 6(c) shows the beginning of Miles Davis’s “So What.” The voicing that Bill Evans plays, which has become known as the “So What” chord, is a pentatonic ninth chord, or a stack of five pentatonic thirds.³³ Example 6(d) shows an excerpt from a Herbie Hancock solo that linearizes a series of parallel pentatonic triads.³⁴ Finally, Example 6(e) shows the chord formed by the open strings of a guitar, reminding us that many “fourth-based” instruments are in fact tuned as pentatonic thirds. All of this music has as much right to be called *tertian* as *quartal*, for the chords in question, though they contain five-semitone intervals, are built out of scalar thirds.

Example 7 summarizes the main points of Section I in graphic form. The upper black box contains those set classes whose associated scales possess Diatonic Seconds. At the bottom of this box are the minimal DS set classes, the locally diatonic scales. (Their supersets are shown, in condensed Forte notation, in the columns above.) The lower black box contains those set classes that satisfy the “No Consecutive Semitones” constraint. At the top of this box are the Pressing scales, which are both the maximal NCS sets and the only scales possessing the Diatonic Thirds property. (Note that Example 7 collapses the harmonic major and harmonic minor scales into a single “harmonic” scale type.) Finally, one can

(a)

(b) *both hands 15^{ma}*

(c) *piano (Bill Evans)*
bass (Paul Chambers)

(d) *(reduction)*

(e)

- Example 6. Pentatonic thirds and diatonic fourths
- Debussy's "La fille aux cheveux de lin"
 - Ligeti, *Piano Concerto*, II, mm. 60–61
 - Miles Davis's "So What"
 - Herbie Hancock's "Eye of the Hurricane" (beginning of the fifth chorus)
 - the open strings of the guitar (cf. Example 6[c], m. 2)

Diatonic Seconds (complements of the anhemitonic sets)

11-1	11-1	11-1	11-1
10-2, 4, 6	10-2, 3, 4, 5	10-2, 3, 4, 5, 6	10-3, 6
9-6, 8, 12	9-6, 7, 9, 11	9-6, 7, 8, 9, 10, 11	9-10
8-21, 24, 25	8-22, 23, 26	8-21, 22, 27	
7-33			
Whole-tone (6-35)	Diatonic (7-35)	Acoustic (7-34)	Octatonic (8-28)
			7-31
	6-Z25, Z26, 32, 33	6-Z23, Z24, 33, 34	6-Z13, Z23, 27, 30, 49, 50
5-33	5-Z12, 20, 23, 24, 25, 27, 29, 34, 35	5-19, Z17, 23, 24, 25, 26, 27, 28, 30, 33, 34, 35	5-10, 16, 19, 25, 28, 31, 32
4-21, 24, 25	4-8, 10, 11, 13, 14, 16, 20, 21, 22, 23, 26, 27, Z29	4-3, 10, 11, 12, 13, 14, Z15, 16, 19, 21, 22, 23, 24, 25, 26, 27, Z29	4-3, 9, 10, 12, 13, Z15, 17, 18, 25, 26, 27, 28, Z29
3-6, 8, 12	3-2, 4, 5, 6, 7, 8, 9, 10, 11	3-2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	3-2, 3, 5, 7, 8, 10, 11

Diatonic Thirds (Pressing Scales)

Harmonic (7-32)	Hexatonic (6-20)
6-Z19, Z24, Z25, 27, Z28, Z29, 31	
5-10, Z12, 16, Z17, Z18, 20, 21, 22, 23, 25, 26, 27, 29, 30, 31, 32	5-21
4-3, 7, 8, 10, 11, 12, 13, 14, Z15, 16, 17, 18, 19, 20, 22, 23, 24, 26, 27, 28, Z29	4-7, 17, 19, 20
3-2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	3-3, 4, 11, 12

No Consecutive Semitones

set inclusion

Example 7. A graphic portrayal of the relations between DS, DT, NCS, and the anhemitonic sets

take the complement of the DS sets (reading 11–1 as 1–1, 10–2 as 2–2, and so forth) to obtain the anhemitonic set classes, with the maximal anhemitonic set classes contained at the bottom of the box surrounded by the heavy black line.

The forgoing discussion will remind some readers of Richard Parks’s theory of pitch-class-set “genera.” Very much in the spirit of the present study, Parks identifies a series of scalar collections that he considers to be central to Debussy’s harmonic vocabulary—including the diatonic, octatonic, and whole-tone scales. Appendix I compares the two theories in some detail.

II. Scalar “Dynamics”: Common-Tone and Voice-Leading Relations Between Scales

One notable feature of traditional tonal practice is that modulation proceeds by way of efficient voice leading between scales sharing a large number of common tones. For example, the C diatonic scale shares six common tones with the G diatonic scale; moreover, these two scales can be linked by what Richard Cohn calls “maximally smooth voice leading”—voice leading in which only a single pitch class moves, and it moves by only a single semitone.³⁵ In this section we investigate common-tone and voice leading relationships among the scales discussed in Section I. Section II(a) considers common-tone relationships. Sections II(b)–(d) consider voice-leading in somewhat greater detail.

Note that in talking about common tones and voice leading we will be talking about properties that are shared by sets and the scales associated with them. Since the vocabulary of set theory is admirably well-suited for this, much of the following discussion will be couched in terms of “collections” and “sets.” In many musical styles, however, these relationships are manifested by objects that are clearly *scalar*—circular orderings of pitch classes that define their own distance metrics, as discussed in Section I(a). It is the conjunction of these two distinct sorts of attributes—the common-tone and voice-leading properties discussed here, and the scalar properties identified in Section I—that endows the following investigation with much of its interest.

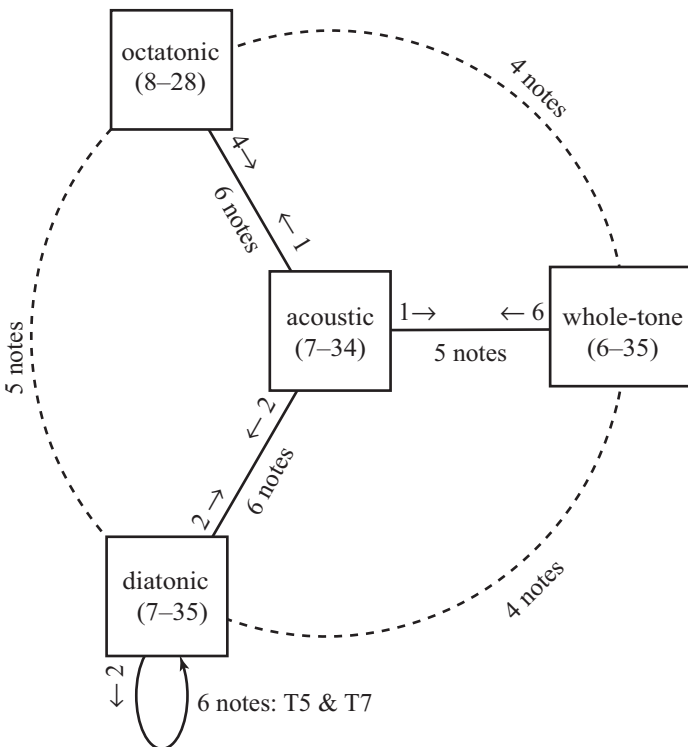
Finally, although a detailed discussion of this matter is beyond the scope of this paper, it is worth saying a word about *why* the scales discussed in Section I are linked by the voice-leading and common-tone relationships discussed here. The reason is that the DS, NCS, and DT constraints identify scales that resemble the *diatonic* collection—which divides the twelve-note chromatic scale into seven nearly equal parts.³⁶ Since the diatonic scale is “maximally even,” its scalar intervals come in just two consecutive-integer sizes: its “seconds” span one or two semitones; its “thirds” span three or four semitones; its “fourths” span five or

six semitones, and so forth.³⁷ In Section I we saw that this feature of the diatonic scale ensures an important degree of consistency between the scalar and chromatic measures of musical distance, and that this consistency is to some extent inherited by locally diatonic and Pressing scales. Elsewhere, I have shown that this same feature ensures that the various transpositions of the diatonic scale will be linked by unusually small voice leadings.³⁸ The scales described in Section I, by virtue of their resemblance to the diatonic scale, inherit these special voice-leading properties.³⁹ Thus the evenness of the diatonic collection plays two essential but independent roles in our inquiry: it ensures the consistency between the scalar and chromatic distance metrics, and it accounts for the unusually complex network of voice-leading we investigate below. Appendix II(a) explores this matter further.

(a) Common-Tone Relationships Between Scales

If S and T are sets, neither of which entirely contains the other as a subset, then they can share at most one note less than the cardinality of the smaller set.⁴⁰ Two sets, not related by inclusion, *maximally intersect* when they share this maximum number of notes. Thus if a whole-tone collection shares five notes with some other set, then the two maximally intersect, no matter how many notes that other set has. Two *set classes* maximally intersect if there exist maximally-intersecting members of those two set classes. Finally, a set class *maximally intersects itself* when two distinct forms of that set share all but one of their notes. Note that “maximal intersection” is a term of art that simply records the fact that two sets, not related by inclusion, share a maximum number of common tones. The term itself carries no implications about voice leading or registral realization.

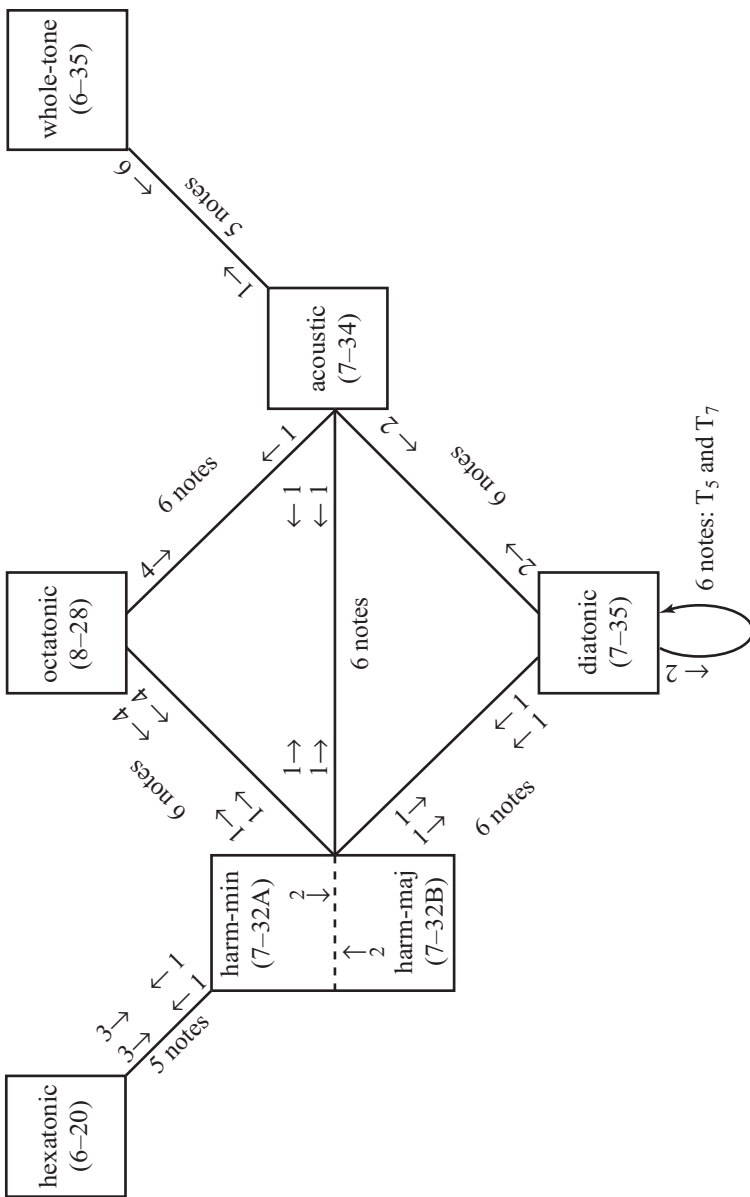
Example 8 shows which of the four locally diatonic set classes maximally intersect. Dark lines connect maximally intersecting set-classes, while the numbers next to the arrows identify how many distinct transpositions a given set maximally intersects. Thus the number “4” on the line connecting the octatonic to the acoustic set class indicates that every octatonic collection maximally intersects four distinct acoustic collections. The number “1” on the same line indicates that every acoustic collection maximally intersects just one octatonic collection. We see that the acoustic set class is central to the graph, maximally intersecting the other three set classes. Every acoustic collection can share six of its seven notes with some diatonic collection, six of its seven notes with some octatonic, and five of some whole-tone collection’s six notes.⁴¹ None of the other locally diatonic set classes maximally intersect. Instead, they can share at most one less than the maximum number of notes with the other two: every whole-tone collection shares four notes with some diatonic and some octatonic collection, and every diatonic collection shares five notes



Example 8. Maximal connections among the four locally diatonic scales

with some octatonic collection. Finally, notice that the diatonic set class maximally intersects itself: every diatonic collection shares six notes with two of its transpositions. This is not true of any of the other locally diatonic set classes.⁴² An acoustic collection shares five notes with its nearest transposition (by major second); an octatonic collection shares four notes with both of its transpositions; and the two whole-tone collections share no notes.

One can draw a similar graph for the seven Pressing set classes, as Example 9 demonstrates. The right side of Example 9 resembles Example 8, without the circular arcs. It shows that the acoustic set class maximally intersects the octatonic, whole-tone, and diatonic set classes. The left side of the graph shows that the octatonic, acoustic, and diatonic set classes all maximally intersect the harmonic set class. (Note that there are two arrows leading to and from the harmonic set class, indicating distinct



Example 9. Maximal intersections among the seven Pressing scales

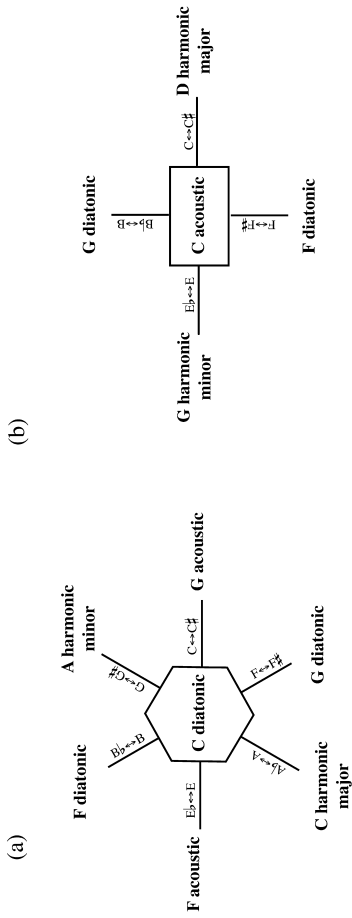
maximal intersections with its two inversional forms.) The harmonic set class in turn maximally intersects itself, as well as the hexatonic set class. All of the set classes not directly connected by lines can share one fewer than the maximal number of notes, with the exception of the whole-tone/hexatonic pair, which can share only three of six notes.

Finally, note that the transpositionally-symmetrical (or *T-symmetrical*)⁴³ Pressing set classes are at the top of Example 9, while the fifth-based diatonic set class is at the bottom. In between are the harmonic and acoustic set classes. These two seven-note set classes each maximally intersect the diatonic set class as well as two T-symmetrical set classes. For this reason, acoustic and harmonic scales can serve to mediate between diatonic and T-symmetrical scales, providing a smooth way of modulating between them. In much the same way, acoustic and harmonic scales can mediate between distinct T-symmetrical scales.⁴⁴ Thus the acoustic scale might be described as equally diatonic, octatonic, and whole-tone; just as the harmonic scales are equally diatonic, octatonic, and hexatonic. As we will see, these descriptions provide an important key to the scales' function in twentieth-century music.

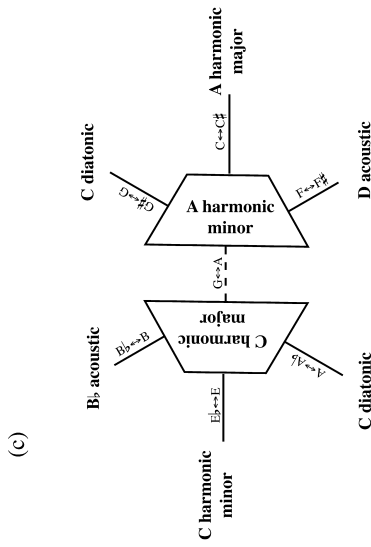
(b) Voice Leading Between Scales

Examples 8 and 9 show the maximal intersections among different set classes. We will now look in some detail at voice-leading and common-tone relationships between individual collections. We begin by considering the three seven-note Pressing set classes: the diatonic, acoustic, and harmonic. The six- and eight-note Pressing set classes, all of which are transpositionally symmetrical, will be considered in Section II(d) below.

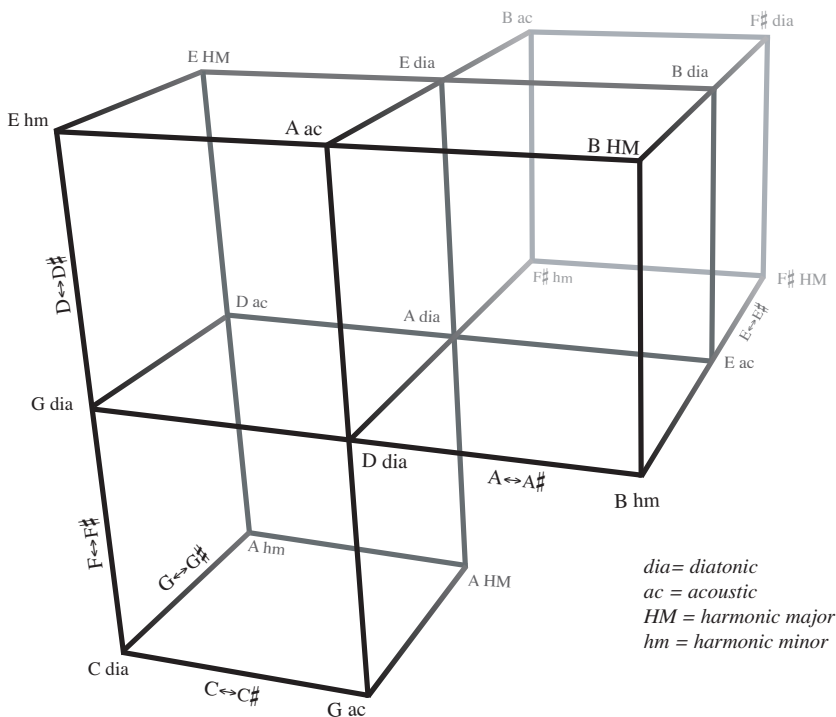
Example 10(a) identifies all the seven-note Pressing collections maximally intersecting the C diatonic collection. Example 10(b) does the same for C acoustic, while Example 10(c) shows the seven-note Pressing collections maximally intersecting A harmonic minor and C harmonic major. Two features of these graphs immediately attract attention. First, almost all of the maximally intersecting collections can be linked by maximally-smooth voice leading. (The one exception is the dotted line in Example 10(c), linking A harmonic minor to C harmonic major. It is discussed in Appendix II(b).) Equally striking is the fact that these maximally-smooth voice leadings come in inversionally-related pairs. For example, the C diatonic collection can be transformed into the G diatonic by shifting F to F \sharp ; similarly, C diatonic can be transformed into the F diatonic by shifting B to B \flat . The two voice-leading motions F \rightarrow F \sharp and B \rightarrow B \flat are symmetrical around the C diatonic collection's D/A \flat axis of inversional symmetry: I $_4$ takes F into B, F \sharp into B \flat , and the entire C diatonic collection into itself. Globally, the entire graph of Example 10(a) is inversionally symmetrical: I $_4$ simply rotates the graph 180 $^\circ$, exchanging I-related collections. (A glance at Examples 10(b) and 10(c) shows that



each graph is inversionally symmetrical around D/A_b . (\mathbf{I}_4 rotates each graph 180° .)



Example 10. Maximal intersections among the seven-note Pressing scales
 a) diatonic; b) acoustic; c) harmonic



Example 11. A portion of the scale lattice for the seven-note Pressing scales

they are similarly symmetrical around D/A b .) The inversive symmetry of the voice leadings is a byproduct of the inversive symmetry of the sets themselves.⁴⁵

The question now arises: is it possible to subsume the graphs of Examples 10(a)–(c) within one graph? Example 11 shows that it is.⁴⁶ The three-dimensional “scale lattice” of Example 11 is a *Tonnetz* for the seven-note Pressing collections. Like the familiar Oettingen/Riemann *Tonnetz*, it is a graphic representation that elegantly captures the maximally-smooth voice-leading possibilities between the relevant sets.⁴⁷ Unlike the triadic *Tonnetz*, however, it has a complex geometrical structure that cannot be perspicuously represented in two dimensions. Furthermore, Example 11 depicts objects belonging to three different set classes, whereas the Oettingen/Riemann *Tonnetz* contains triads belonging to just a single set class. The differences between these three set classes produces asymmetries that give Example 11 its distinctive geo-

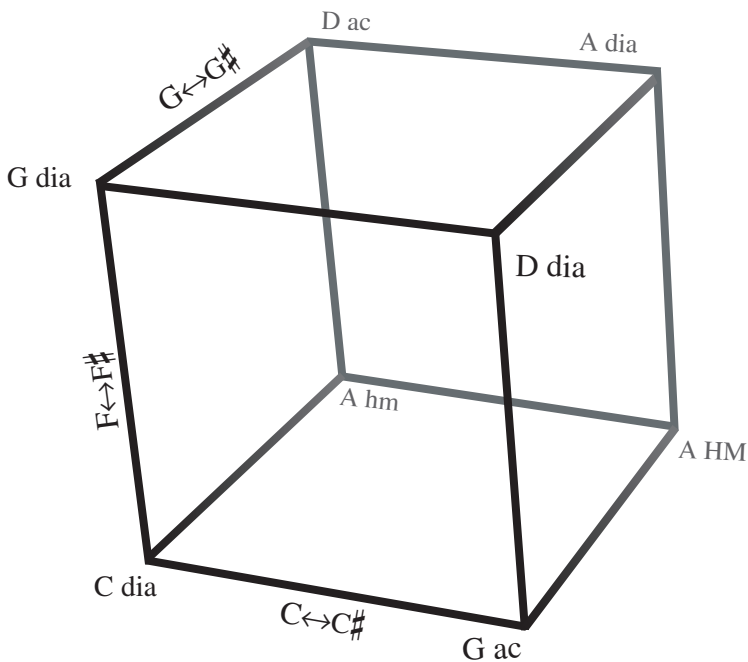
metrical structure: every diatonic collection is connected by maximally-smooth voice leading to six different seven-note Pressing collections; every acoustic collection is connected by maximally-smooth voice leading to four different seven-note Pressing collections; and every harmonic collection is connected by maximally-smooth voice leading to only three Pressing collections. For this reason, harmonic collections lie on the “corners” of the lattice, acoustic collections lie on “edges,” while diatonic collections lie within it.⁴⁸ The three set classes thus have different “degrees of connectedness,” giving rise to the complex structure shown in Example 11.

There are two ways to understand this lattice: as a series of stacked cubes, and as a series of intertwining strands. We will consider each in turn.

(c) The “Stacked Cubes” Interpretation

Example 12 shows one of Example 11’s cubic “units.” The graph contains eight collections: four diatonic, two acoustic, and one each of the harmonic major and minor. (This graph was originally devised by Richard Cohn, who brought it to my attention in a personal communication.) All of the collections in Example 12 share the four pitch classes D–A–E–B. The graph’s Cartesian coordinates represent pairs of semitone-related pitch classes: the x axis corresponds to the pair $\{0, 1\}$, here labeled $\{C, C\#\}$; the y axis corresponds to the pair $\{5, 6\}$ (labeled $\{F, F\#\}$); while the z axis corresponds to $\{7, 8\}$ (G and $G\#\$).⁴⁹ A collection’s coordinates determine which element of each pair it contains: thus, all collections with an x coordinate of 0 contain pitch class C^b , while all with an x coordinate of 1 contain pitch class $C\#$, and so forth. Since Example 12 is a cube, it contains a collection for every choice of one element from each pair. Finally, notice that collections on a given face jointly share five common tones. On the two faces with three diatonic collections, the common five-note collection is the familiar pentatonic scale. The four collections on the corners of these faces are the only Pressing scales containing that scale.

The lattice in Example 11 is composed of twelve cubes, each related by transposition to the one shown in Example 12. The first cube, in the lower left of the graph, is identical to that in Example 12. The second cube is stacked on top of it and shares a face with it. Every collection on the second cube is T_7 of some collection in the first.⁵⁰ Note, however, that the first and second cubes are oriented differently in three-dimensional space: the second is a (transposed) 120° rotation of the first around the C diatonic/A diatonic diagonal.⁵¹ Similarly, the third cube lies to the *right* of the second; it again shares a single face with it, and is its (transposed) 120° rotation around the same diagonal. The fourth cube lies *behind* the



Example 12. Richard Cohn's scale cube

third, and is oriented in the same way as the first cube: every collection on the fourth cube is T_3 of the collection in the corresponding position on the first cube. This pattern repeats to produce twelve transpositionally-related cubes, after which it returns to its starting point. Since it is difficult to depict the entire structure in two dimensions, Example 11 shows only an excerpt of the whole.⁵²

Example 11 also records shared “stack of fifths” subsets. Every diatonic hexachord (set class 6–32 [024579], a stack of five chromatic fifths) is contained by exactly two Pressing collections: both are diatonic and are connected by lines in Example 11. Every pentatonic collection (a stack of four chromatic fifths, set class 5–35 [02479]) is contained by exactly four Pressing scales, three diatonic and one acoustic. These share a cubic face on Example 11. Every stack of three chromatic fifths (set class 4–23 [0257]) is contained by eight collections, which—as discussed above—form one of Example 11’s cubes. Every stack of two fifths (set class 3–9 [027]) is contained by twelve collections, which appear on two cubes that share a face. Finally, there are twenty Pressing collections sharing a sin-

gle fifth. These are the sixteen collections contained by three consecutive cubes on Example 11, plus four others: one octatonic, one hexatonic, one harmonic major and one harmonic minor.⁵³

Careful investigation of Example 11 reveals many other interesting relationships which we cannot discuss here. Appendix II considers some of these in more detail.

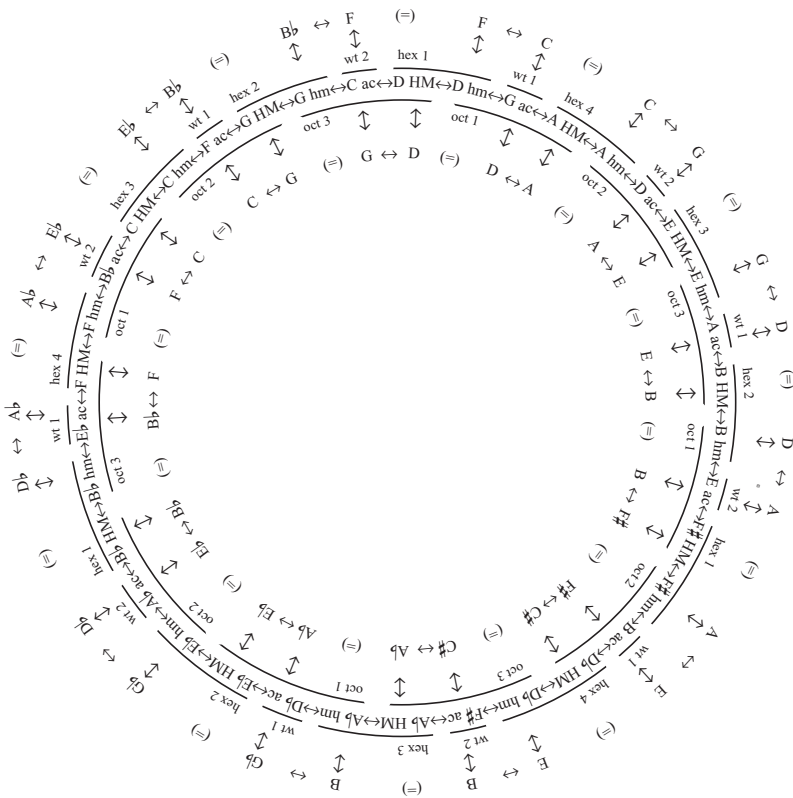
(d) The “Intertwining Strands” Interpretation

So far, we have been considering Example 11 as a series of stacked cubes. But it is equally instructive to view the graph as the product of two intertwined strands, one diatonic and one non-diatonic. The diatonic strand consists of the familiar circle of fifths: it takes a jagged path through Example 11, beginning in the lower left and moving up, right, and into the paper successively. The non-diatonic strand is a less familiar cyclic structure that also involves fifth transposition. Beginning with G acoustic, we find the following sequence of non-diatonic collections:

G acoustic ↔ A harmonic major ↔ A harmonic minor ↔
D acoustic ↔ E harmonic major ↔ E harmonic minor ↔ . . .

This is again a “circle of fifths,” but the unit of sequential repetition is three collections long. The two strands—one diatonic, one non-diatonic—are intertwined somewhat as the strands of a double helix. Their interactions are complicated by the fact that they do not have the same shape: the diatonic strand takes a right-angled turn after every step, whereas the non-diatonic strand moves in a straight line through each acoustic collection.

Example 13 unfolds the lattice of Example 11 along its non-diatonic strand. The central core of this graph features the full cycle of thirty-six non-diatonic seven-note Pressing collections. On the outside of this central circle runs the diatonic circle of fifths. Another diatonic circle of fifths can be found within the central circle. As can be seen from this graph, the circle of non-diatonic collections touches the diatonic circle of fifths at two different points. The A harmonic major collection, for example, is connected by maximally-smooth voice leading to the A diatonic collection. The next collection in the non-diatonic circle, A harmonic minor, is related by maximally-smooth voice leading to the C diatonic collection. Thus moving one step forward on the non-diatonic circle (from A harmonic major to A harmonic minor) brings us to a collection related by maximally-smooth voice leading to a diatonic collection three steps *backward* on the circle of fifths (C diatonic, which has three fewer sharps than A diatonic). Moving another step forward on the non-diatonic circle brings us to D acoustic, which is related by maximally-smooth voice leading to both G diatonic (a fifth above C) and A diatonic. In this way the non-diatonic circle of fifths continues to touch upon the



Example 13. A circle of fifth-related, nondiatonic scales

diatonic circle of fifths at two distinct points. This is yet another reason why the lattice is difficult to portray in two dimensions.

As Example 9 demonstrated, only the collections on the non-diatonic strand maximally intersect the T-symmetrical Pressing collections. Example 13 provides more information on these relationships. As can be seen from this graph, the non-diatonic cycle divides into triplets, each maximally intersecting the same octatonic collection. For example, A harmonic minor, D acoustic, and E harmonic major all maximally intersect Octatonic Collection II. (These triplets are collinear on Example 11.) Adjacent harmonic-major and harmonic-minor collections maximally intersect the same hexatonic collection. Acoustic collections maximally intersect alternate whole-tone collections. The most efficient voice leadings between these collections involve what Callender (1998) calls

	WT I	WT II	HEX I	HEX II	HEX III	HEX IV
OCT I	D \flat ac G ac	E ac B \flat ac	D hm F \sharp HM	B hm E \flat HM	A \flat hm C HM	F hm A HM
OCT II	F ac B ac	D ac A \flat ac	F \sharp hm B \flat HM	E \flat hm G HM	C hm E HM	A hm C \sharp HM
OCT III	E \flat ac A ac	C ac F \sharp ac	B \flat hm D HM	G hm B HM	E hm A \flat HM	D \flat hm F HM

Example 14. Maximal intersections involving T-symmetrical scales

“split” and “merge” transformations. In moving from the whole tone collection to the acoustic, or from the acoustic to the octatonic, a single pitch class “splits” into its two chromatic neighbors. Conversely, in moving from the octatonic to the acoustic, or from the acoustic to the whole tone, the two notes of an [02] dyad “merge” into the pitch class between them.⁵⁴ Voice leadings between T-symmetrical and harmonic collections involve a similar, though more complex, process of “splitting” and “merging”: here, either an [013] trichord becomes the semitone spanned by its outer notes, or the reverse process occurs.⁵⁵

One problem with Example 13 is that a single T-symmetrical collection appears in multiple places on the graph. The example does not, therefore, provide a particularly useful picture of how one might use seven-note scales to modulate between T-symmetrical scales. Example 14 rectifies this by displaying the non-diatonic Pressing collections in matrix form. The rows display all the collections maximally intersecting a given octatonic collection. The columns display all the collections maximally intersecting a given hexatonic or whole-tone collection. The table provides two modulatory routes between any octatonic/whole-tone pair, or between any octatonic/hexatonic pair. G acoustic and D \flat acoustic, for example, maximally intersect both the Octatonic I and Whole-Tone I collections, while E \flat harmonic minor and G harmonic major maximally intersect both the Octatonic II and Hexatonic II collections. There are no Pressing collections maximally intersecting both a whole-tone and a hexatonic collection. Instead, one must travel between them by way of two-collection path involving both a harmonic and an acoustic scale.

III. Scales in Debussy

We have now developed a rather formidable technical apparatus for understanding scales and their relationships. What remains to be shown is that this apparatus is analytically useful. This section considers four of Debussy’s piano compositions, all composed within a relatively short

time: two preludes, “Les Collines d’Anacapri” and “Le vent dans la plaine,” both written in 1910; *L’Isle Joyeuse*, written in 1904; and “Cloches à travers les feuilles,” the first in the second series of piano *Images*, written in 1907. These pieces show progressively more complex methods of scalar organization. “Les Collines d’Anacapri” is relatively simple, featuring a small network of seven-note scales surrounding a central diatonic collection. “Le vent dans la plaine” resembles “Collines,” but uses an expanded network that exploits the whole-tone scale’s transpositional symmetry. *L’Isle Joyeuse* is a much more sophisticated piece, involving a central *acoustic* scale and a greater variety of methods of scalar organization. Finally, “Cloches à travers les feuilles”—the most difficult of the works we will examine—uses a much wider range of scale-to-scale transformations, and is rather more elusive in its large-scale organization.

The following analyses will not aspire to completeness. Instead, they will attempt to demonstrate that Debussy makes systematic use of the voice-leading and common-tone relations between Pressing scales. My goals are twofold: first, to demonstrate that the scales discussed in Section I of this paper do indeed serve, to a good first approximation, as the basic scalar vocabulary of at least some of Debussy’s pieces; and second, that Debussy utilized the rich network of common-tone and voice-leading relationships between these scales. I will also be concerned, on occasion, to point out that the objects under discussion are indeed *scales* rather than unordered collections. (See, in particular, the discussion of Example 23, below.) But this will not be a major focus of the discussion. For the most part, the scalar nature of Debussy’s music is so obvious as to require little explicit commentary: his melodies tend to proceed in stepwise fashion, his chords tend to be tertian, and his harmonies frequently move in parallel within a scale. I trust that this is apparent even to readers only modestly familiar with Debussy’s music.

Before proceeding, I should say a few words to allay the worry that scales are, in general, very difficult or impossible to perceive. Although it can sometimes be hard to identify scales—as for instance when they are played as chords whose notes all lie within the span of a single octave—my own experience is that Debussy’s scalar vocabulary is quite perceptible. There are a number of reasons why this is so. First, his vocabulary is relatively limited. He makes most frequent use of just four scales—diatonic, pentatonic, *acoustic*, and whole-tone. Other scales, such as the octatonic, harmonic major, and harmonic minor, appear much less frequently. (Debussy almost never used the hexatonic scale.) Second, many of these scales are familiar, or otherwise distinctive, musical objects. Obviously, we are well-acquainted with the diatonic scale and its modes. The whole-tone scale is intrinsically a highly distinctive musical object, due to its limited interval content and its extreme transpositional

The musical score is presented in two systems. The first system contains three staves of music. The first staff is labeled "Theme α " with a measure number "1" below it. The second staff is labeled "Theme β ". The third staff is labeled "Theme γ " with a measure number "14" below it. The key signature is B major, indicated by two sharps (F# and C#) at the beginning of the first staff. The notation includes treble clefs, stems, beams, and various note values. The second system continues the musical ideas from the first system, with a brace under the first two staves and a "etc." label at the end of the second staff.

Example 15. Thematic ideas in the opening of "Les Collines d'Anacapri"

symmetry. And the pentatonic scale is not only aurally distinctive but also familiar from Western and non-Western folk sources. Third, Debussy tends to use some of his scales in just a few characteristic modes. In particular, the acoustic scale typically appears in the mode equivalent to the mixolydian mode with raised fourth degree.⁵⁶ (Other modes of the scale are occasionally found, however.) Fourth, Debussy tends to change scales reasonably slowly. One often hears several measures of music conforming to a single scalar collection, in which all of the scale's pitches appear. In some cases—such as “Voiles”—a single scale provides the pitch material for an entire musical section. Finally, Debussy often uses scales as *melodic* entities. Indeed, almost all of the pieces discussed below involve fairly extensive passages in which scales are explicitly stated melodically.

It should also be emphasized that perceptibility is not the only criterion for evaluating musical analyses. Analysis does not simply describe the way we *hear* music; it can also show us *how the music was made*. It is possible to interpret the graphs presented in Section II as depicting an abstract, precompositional space within which Debussy's music moves.⁵⁷ The music may on occasion move quickly within this space, jumping between scales without clearly demarcating them. It may sometimes be hard to follow these transformations by ear. But this does not show that scales are, in these circumstances, analytically useless. A rapid series of closely-related scales can produce striking aural effects, even though the listener cannot explicitly identify the scales used in the progression. Scale-based analysis can, in turn, show us *how to produce* such effects. (Analysis in this sense shows composers how to steal from each other.) One might well compare Debussy's procedures here to the Impressionist painter's subtle use of closely related hues. In both cases, we perceive a single blended whole. Yet as theorists, analysts, or artists we may legitimately want to look beneath the dazzling, multicolored surface to the technical procedures animating it.⁵⁸

(a) “Les Collines d’Anacapri”

“Les Collines d’Anacapri” begins, as Example 15 shows, by introducing three thematic ideas: a rising pentatonic figure; a descending scale-fragment touching on six notes of the diatonic scale; and a melody, *joyeux* and *léger*, that starts with a six-note diatonic scale-fragment and ends by emphasizing the “black-note” pentatonic scale. (These three ideas are identified in the example as Themes α , β , and γ , respectively.) The first portion of the piece is purely and placidly diatonic. Its sixty-one measures involve just a few departures from the tonic B major⁵⁹: m. 9 introduces two accidentals, suggesting familiar chromatic passing motion between ii^7 and I^6 ; mm. 24–29 present the six pitch classes A–B \flat –C–D \flat –E \flat –F \sharp , common to both B \flat harmonic minor and Octatonic

CYCLE 2 Theme α mm. 66-72

8th Theme γ

G \sharp natural minor E \rightarrow E \sharp G \sharp dorian {E \sharp \rightarrow E \sharp , D \sharp \rightarrow D \sharp }

CYCLE 3 Theme α mm. 73-80

Theme γ m. 77

E acoustic chromatic G \sharp \rightarrow G \sharp \rightarrow G \sharp \rightarrow G \sharp
 B major B harmonic etc.
 major

CONCLUSION mm. 81-96

Theme α Theme β

B major A \sharp \rightarrow A \natural B mixolydian

The musical score is written in treble and bass clefs with a key signature of three sharps (F#, C#, G#). It consists of several sections:

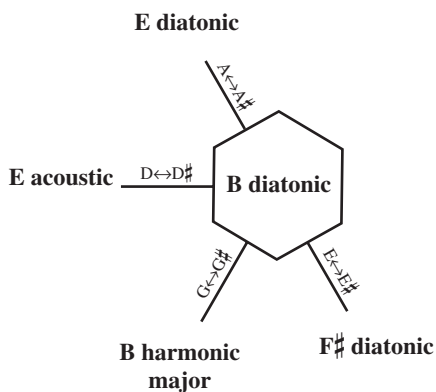
- CYCLE 2 (mm. 66-72):** Features Theme α in the bass clef and Theme γ in the treble clef. A dashed line connects the two themes. Harmonic annotations include G \sharp natural minor, E \rightarrow E \sharp , and G \sharp dorian. A bracketed note indicates {E \sharp \rightarrow E \sharp , D \sharp \rightarrow D \sharp }.
- CYCLE 3 (mm. 73-80):** Features Theme α in the bass clef and Theme γ in the treble clef. A bracketed note indicates m. 77. Harmonic annotations include E acoustic and chromatic G \sharp \rightarrow G \sharp \rightarrow G \sharp \rightarrow G \sharp , B major, B harmonic etc., and major.
- CONCLUSION (mm. 81-96):** Features Theme α in the bass clef and Theme β in the treble clef. Harmonic annotations include B major, A \sharp \rightarrow A \natural , and B mixolydian.

Example 16. Reharmonization in the concluding portion of “Les Collines d’Anacapri”

Collection III (with measure 28 interposing a D^7 chord that belongs neither to these collections nor to B major); while measures 46 and 51 involve straightforward chromatic passing notes.

The thematic ideas of Example 15 are largely absent from mm. 24–61, which has the character of an extended episode. When the themes return in the last third of the piece, they are detached from their original B major environment and set in the context of new scales. Example 16 shows how the end of the piece cycles through, and reharmonizes, the themes of Example 15. The second thematic cycle begins in m. 66 in G^\sharp natural minor. Halfway into Theme γ , however, the music replaces E with E^\sharp , producing G^\sharp dorian. (Theme β does not appear in this cycle.) The third cycle begins in m. 73. The B major scale has again been perturbed by the shift of a single pitch by a single semitone: D has replaced D^\sharp , yielding E acoustic. Theme α again leads directly to Theme γ , which, touching on D^\sharp , neutralizes the E acoustic scale. (There is an ascending chromatic line in the left hand of m. 77.) Measures 78–80 present an oscillation involving yet another single-semitone displacement: G^\sharp alternates with G^\natural , perhaps implying B major and B harmonic major respectively. (F^\sharp acoustic and Octatonic Collection III are also possible interpretations here.⁶⁰) Finally, m. 86 presents Themes α and β in the context of B mixolydian, shifting B major's A^\sharp to A^\natural .

Example 17 graphs the scales that I have identified in the latter portion of the piece. The graph is a subgraph of Example 10(a): it contains four of the six scales maximally intersecting the tonic B major. Several features of this example deserve comment. First, it is *centered*, in the sense that all of the scales on the graph maximally intersect a single collection. Second, this central collection also serves as a stable tonic scale,



Example 17. Scales used in “Les Collines d’Anacapri”

to which the music repeatedly returns. These are common, though not universal, features of Debussy's use of scales.⁶¹ ("Cloches à travers les feuilles," discussed below, provides an example of a non-centered scale network.) Third, the scales in Example 17 are presented with different degrees of explicitness. The diatonic and acoustic scales are all complete, and are given relatively extensive musical treatment. The harmonic major (like the octatonic) is incomplete, fleeting, and ambiguous. This is again typical of Debussy's style, which makes frequent use of the diatonic, whole-tone, and acoustic scales, but touches only occasionally—and often ambiguously—on octatonic and harmonic scales.

There is one further feature of "Collines" that distinguishes it from Debussy's other music. This is the way the themes themselves articulate the subsets shared by its maximally intersecting scales: Theme γ , which contains the six pitch classes common to the B diatonic and F# diatonic collections, appears as part of both scales. Likewise, Theme β contains the six pitch classes common to B diatonic and E diatonic, and again appears within both scales. Finally, Theme α contains the five pitch classes common to the three scales in which it appears: B diatonic, E diatonic, and E acoustic.⁶² We might therefore say that "Collines" *thematizes* the maximal intersections between its collections: not only does it present each of its themes, with a fair degree of systematic rigor, in each of its possible scalar contexts, but it also constructs its themes out of exactly the notes common to the relevant scales.⁶³

(b) "Le vent dans la plaine"

The opening of "Le vent dans la plaine," outlined in Example 18, is similar in structure to the end of "Collines," though its scales are less clearly articulated. The piece begins with an ostinato pattern involving the pitches B \flat –C \flat . Inside this ostinato there is a simple four-note melody that may suggest a pentatonic scale. If we take B \flat as the primary pitch of this section, and if we trust the key signature, then the mode is B \flat phrygian. (It is also possible—in light of what eventually happens—to hear E \flat as primary, in which case the mode is E \flat natural minor.) Measure 5 shifts E \flat to E $\flat\flat$, which I take to imply the third mode of G \flat harmonic major. As in mm. 78–80 of "Collines," the two forms of the note alternate before settling, in measure 7, on the ostinato and its implied return to B \flat phrygian.

Measure 9 introduces a new theme: a series of descending seventh chords over parallel fifths in the bass. C \sharp has replaced C \flat , yielding E \flat dorian. (Retrospectively, it may be possible to conceive the preceding music as an extended dominant chord, with its B \flat -centered music leading to Measure 9's tonic E \flat .) Four measures later, the ostinato reinstates the C \flat . In measure 15, B \flat becomes B $\flat\flat$; if we hear this new B $\flat\flat$ as central, then the mode is the seventh mode of the C \flat acoustic scale. The pen-

1 3 5 etc.

B \flat phrygian E \flat → E $\flat\flat$ → E \flat → E $\flat\flat$
G \flat HM B \flat phryg. G \flat HM

7 9

C \flat → C \sharp E \flat dorian

13 15 etc.

C \sharp → C \flat B \flat → B $\flat\flat$
B \flat phrygian C \flat acoustic
(seventh mode)

18 etc.

B $\flat\flat$ → B \flat → B \flat
B \flat phrygian C \flat acoustic

Example 18. Thematic and harmonic summary of the opening of “Le vent dans la plaine”

tatonic tune returns in the new acoustic context. Measure 18 briefly restores the original ostinato. In measure 19, the acoustic scale reappears; the pentatonic tune has now been transposed up by scale step.

Measures 20–22, shown in Example 19, present a more complex set of scalar interactions than we have thus far considered. The acoustic scale in mm. 15–20 maximally intersects the whole-tone collection in measure 22. This suggests a continuation of the compositional logic that has thus far animated the piece. However, this progression of maximally intersecting scales is interrupted by the surprising and more distantly-related

mm. 19-20

C diatonic

Whole Tone Collection I

G acoustic

21

{Gb, Ab → G} (from C_b acoustic)

{D, E → E_b} (from G acoustic)

Whole-Tone Collection I

D dorian

C_b acoustic, seventh mode

24

Whole-Tone Collection II

C → C_#

G acoustic

E_b dorian

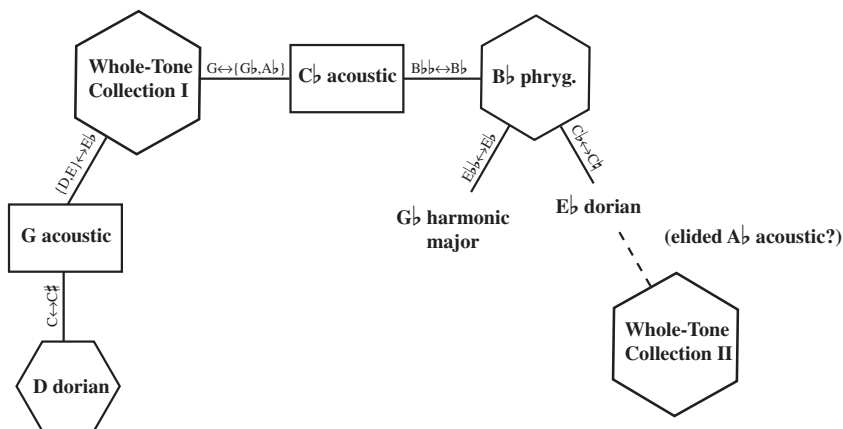
Example 19. "Le vent dans la plaine," mm. 19-26

white-note scale in measure 21. What is the reason for this dramatic shift, so different in character from the extremely smooth harmonic motion that has characterized the piece up to now?

Example 19 shows that this measure involves a complex set of scalar affiliations. The first three beats of m. 21 involve only white notes, suggesting D dorian. However, the fourth beat of the measure adds a $D\flat$ which, taken together with the earlier music, implies G acoustic.⁶⁴ This fleeting acoustic sound gives way, in the next measure, to the maximally intersecting Whole-Tone I collection.⁶⁵ Measures 21 and 22 thus suggest a sequence of maximally intersecting scales: D dorian→G acoustic→Whole-Tone I. This sequence is the inversion of the large-scale progression we have already discussed, $B\flat$ phrygian→ $C\flat$ acoustic→Whole-Tone I.⁶⁶ The two distinct progressions meet in the whole-tone scale of measure 22. Thus, what *sounds* like an interruption, the sudden appearance of the white-note diatonic collection in measure 21, can be analyzed as a continuation of the music's underlying logic.

The subsequent music reinforces this reading. The whole-tone scale of measure 22 gives way to D dorian “white note” music in m. 23. (The parallel fifths in the left hand recall the $E\flat$ dorian of mm. 9–12.) In m. 24, G acoustic returns, suggesting two “dominant ninth chords” (one with lowered fifth) a whole step apart.⁶⁷ In m. 25, the music slides chromatically, transposing mm. 22–23 up by half step. This produces the Whole-Tone Collection II, and brings back the $E\flat$ dorian collection heard in mm. 9–12. (Again, this connection is reinforced by the parallel fifths in the bass.) The transposition associates the $E\flat$ dorian mode with Whole-Tone Collection II, just as the D dorian mode of mm. 21 and 23 was associated with Whole-Tone Collection I. If the progression from $E\flat$ dorian to Whole-Tone Collection II were to proceed by way of maximally-smooth voice leading, it would involve an $A\flat$ acoustic collection not heard in the piece.⁶⁸

Example 20, which graphs all but one of the scales in the piece, attempts to capture this analysis.⁶⁹ The upper-right portion of the graph resembles Examples 10(a) and 17: it features a central diatonic scale (here, $B\flat$ phrygian) connected by maximally-smooth voice leading to three seven-note scales. Here, however, the graph is augmented by a series of additional scales: Example 20's $C\flat$ acoustic collection maximally intersects a scale (Whole-Tone Collection I) that does *not* maximally intersect the original diatonic collection. Furthermore, this whole-tone scale participates in a second progression involving additional diatonic and acoustic scales, neither of which maximally intersects the original $B\flat$ phrygian. Finally, $E\flat$ dorian is itself associated with the other whole-tone scale. While these scales do not maximally intersect, they can be interpreted an elided transposition of the D dorian→G acoustic→Whole-Tone I progression heard earlier in the piece.



Example 20. Scales in “Le vent dans la plaine”

Viewed in this light, “Le vent dans la plaine” can be seen to possess a fairly traditional scalar organization. The opening of the piece shows how the central $B\flat$ phrygian (or if one prefers, $E\flat$ natural minor) maximally intersects a series of closely related scales—much as the opening of a classical major-key sonata emphasizes the maximally-intersecting tonic and dominant scales. As we move into the middle part of the piece, these scales themselves give rise to additional (maximally intersecting) scales. Harmonic motion becomes somewhat freer, and mm. 20–22 feature more dramatic scalar shifts than those of the piece’s opening. What follows, in mm. 28–43, is a passage of largely nonscalar music (not discussed above) that briefly touches on, and rejects, the distantly-related $G\sharp$ phrygian scale (mm. 34–37). The final portion of the piece (mm. 44ff.) returns to the thematic and harmonic material of the opening. In short, the piece demonstrates a traditional musical logic, though in the context of an expanded scalar vocabulary.

(c) *L’Isle Joyeuse*

The previous two analyses featured networks largely centered on a single diatonic scale. *L’Isle Joyeuse* also features a central collection, but it is different from the previous pieces in three important ways. First, *L’Isle Joyeuse* uses only the four locally diatonic scales, rather than the seven Pressing scales: the hexatonic and harmonic scales do not appear in the piece. Second, the central collection in *L’Isle Joyeuse* is an acoustic rather than a diatonic scale. Third, whereas the previous two pieces featured maximally-smooth voice leading, *L’Isle Joyeuse* makes greater use of cardinality-changing shifts of scale. As we will see, these last two dif-

ferences permit a greater structural integration of the whole-tone and octatonic collections.

I hear the piece as composed of three large sections, each of which begins by exploring the same network of maximally intersecting scales.⁷⁰ Example 21 lists the main thematic ideas of the opening. The music of measures 1–6 is based on the whole-tone scale and serves to articulate the structural divisions of the piece: it appears at the beginning and end of the work, as well as at the beginning of the second section (mm. 52–63). Measures 7–9 are acoustic, and contain the main theme of the work. Measure 10 is diatonic; if we consider A to be its main pitch, it is in the lydian mode. Measures 11–12 are more complex. Here I have bracketed three spans of music, each suggesting different scales: the first contains the six notes common to both the acoustic and octatonic scales; the second adds an A \sharp which belongs only to the octatonic scale; while the third is in A major, with a passing chromatic F \sharp . Example 21 shows that measure 66 presents the relation between A acoustic and Octatonic Collection III more explicitly.

Example 22 graphs the scales used in the first section of the piece. Square boxes identify scales, which are linked by straight lines representing maximal intersection. The ovals contain measure numbers; they are linked with curved lines in places where the music progresses so as to reveal the maximal intersections between their underlying scales. Example 22, which we have derived purely analytically, is essentially identical to Example 8, which we derived from theoretical considerations. In the center is the A acoustic scale, from which the main theme is drawn. It maximally intersects one whole-tone, one octatonic, and one diatonic scale; this last diatonic scale in turn maximally intersects two further diatonic scales. The beginning of the piece emphasizes the maximal intersections between the scales, tracing out a path on Example 22 that relates the collections to the central A acoustic scale. I find the correspondence between Examples 8 and 22 to be remarkable. Theory and analysis are unusually close here, indicating that the ideas we have been exploring do indeed capture something important about this music.

The scales in Example 22 constitute the main harmonic region of *L'Isle Joyeuse*. They appear, in various combinations, at the beginning of each of the three main sections, as well as at the end of the piece. Overall they account for about three fifths of the music (roughly 150 of 255 measures).⁷¹ However, this harmonic region engenders further developments in each of the work's three sections. An in-depth exploration of these developments is beyond the scope of this paper, but it will be worth briefly summarizing each section in turn.

Section 1: mm. 1–51. Measures 1–35 have already been discussed in connection with Example 21. Measures 36–51, outlined in Example 23(a), contain a series of parallel triads in B major, twice interrupted by

1-6

Whole-Tone Collection I

7-9

F → {E, F#} A acoustic G → G# A lydian

G# → G A acoustic B → Bb Octatonic Collection III A major

cf. measure 66:

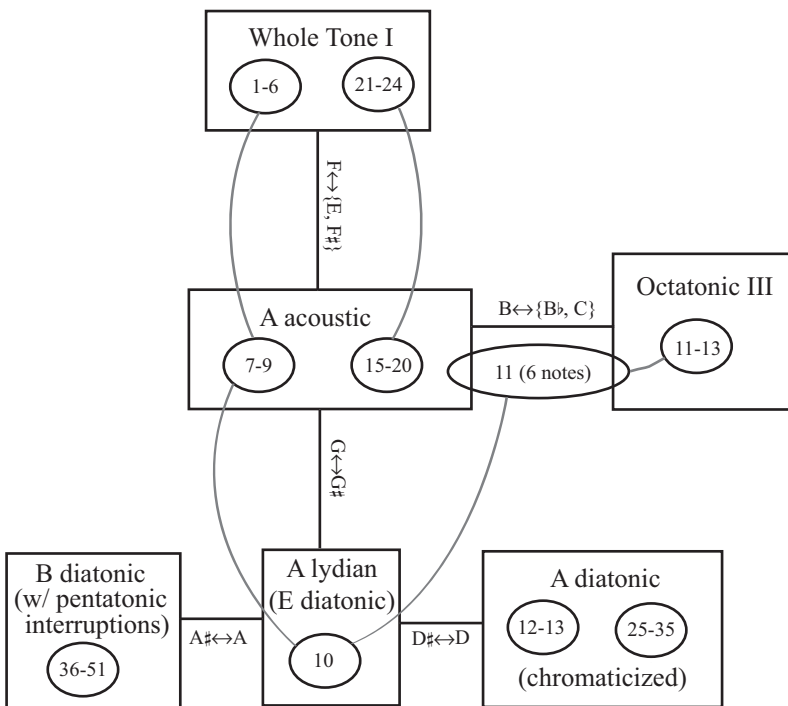
B → Bb A acoustic B Octatonic Collection III Bb A acoustic Octatonic Collection III

Example 21. The “tonic region” of *L’Isle Joyeuse*

mysterious and distantly related chords. Example 23(b) shows that that these “mysterious chords” are in fact parallel *first inversion* pentatonic triads. (To see why, note that D–G–B a “root position pentatonic triad” since it is a stack of two pentatonic thirds. Transposing the bass of the chord up by octave therefore generates a “first inversion pentatonic triad,” G–B–D.⁷²) The shift between the two scales occurs between the G# minor triad (a root position triad belonging to the B major scale) and the appar-

ent “root position G major triad,” which is in fact a first inversion triad belonging to the G pentatonic scale. Example 23(c) rewrites the passage on two staves, so that the two “G triads” are notationally equivalent.

To understand Example 23(a) it is necessary to think in terms of scales and scalar distances. The first part of the phrase features scalar transposition, here generating parallel diatonic triads in the key of B major. The end of the passage has exactly the same structure, although the chords now move along the pentatonic rather than diatonic scale. One can therefore depict Example 23(a) as deriving from a single, continuous process—parallel motion within a scale—that is expressed, at a more surface level, using two distinct scales. A key aspect of this reading is that the progression from the diatonic G# minor triad to the pentatonic G major triad results not simply from the movement of chord voices, but from the shift of the underlying scale. This is clear from the spatial layout of Example 23(c), which portrays the shift between the chords as motion within a scale, but as a “jump” from one staff to another.



Example 22. Scales in *L'Isle Joyeuse*, mm. 1–51

(a) Musical notation showing a reduction of measures 36–51. It features a treble clef with a key signature of three sharps (F#, C#, G#). The notation includes two endings: "1." and "2.", both marked with "???" below them. Brackets above the notes indicate intervals: "third", "third", and "fourth".

(b) Diagrams of triads and pentatonic triads. The first diagram is labeled "(= root position triad)" and shows intervals of "third", "third", and "fourth". The second diagram is labeled "(= first inversion pentatonic triad)" and shows intervals of "third", "second", and "third". The third diagram is also labeled "(= first inversion pentatonic triad)" and shows intervals of "third", "second", and "third".

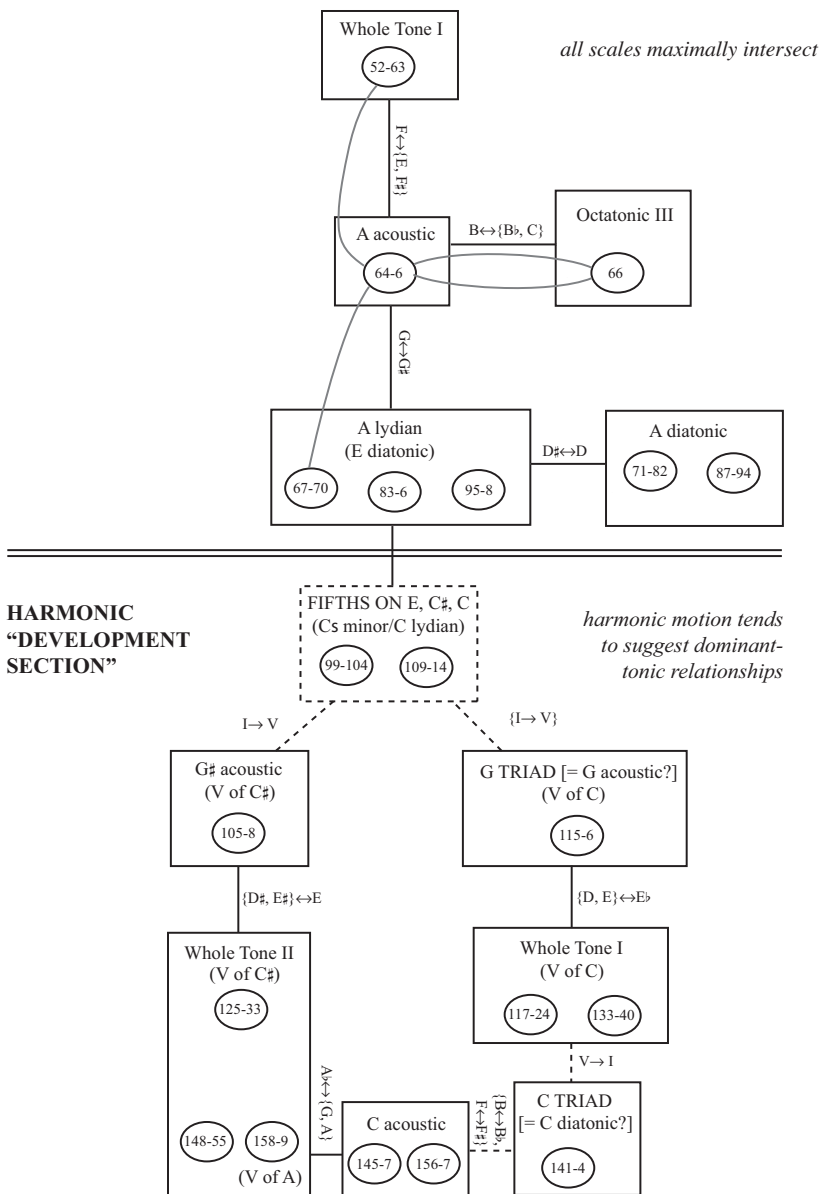
(c) Comparison of two scales. The top staff is labeled "first inversion triads in G pentatonic" and shows four triads. The bottom staff is labeled "root position triads in B major" and shows four triads. A vertical box highlights the final triad in both scales, showing they are identical.

Example 23. Parallel diatonic and pentatonic triads in Debussy’s *L’Isle Joyeuse*, mm. 36–51

- a) a reduction of measures 36–51
- b) diatonic and pentatonic triads
- c) mm. 36–43 as composed of parallel chords belonging to two scales

Section 2: mm. 52–159. Example 24 graphs the collections used in this portion of the piece. The section begins with a return to the whole-tone music of the opening, this time colored by the appearance of C# major triads. Measures 64–66 briefly touch on the music of Example 21 before launching, in m. 67, into a new theme. Some analysts, such as Roy Howat, hear this new theme as articulating a major structural division in the piece; I do not, since it continues to inhabit the same scalar region graphed in Example 22.

Measures 99–159 comprise what I have called the harmonic “development section.” The music begins with parallel fifths articulating a pair of alternating thirds (E–C#, E–Cb). The music then falls into a long whole-tone passage. Example 25 portrays these whole-tone scales as possessing attenuated dominant-chord functionality.⁷³ The G# acoustic scale in m. 105 can be understood as an altered dominant of the immediately-preceding fifths on C#. When the music repeats, the G#



Example 24. Scales in *L'Isle Joyeuse*, mm. 52–159

105 115 121 133 141 145 152

1. Theme α 3
2. Theme β 3
Theme α 3
Theme β

V V V V I I

A major: V

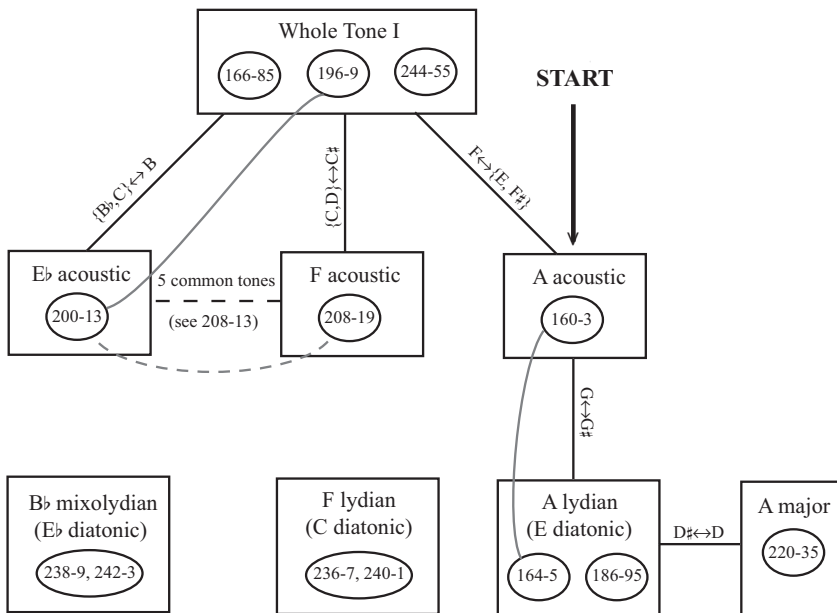
Scale: C \sharp dorian/Clydian G \sharp acoustic [G ac?] G w.t. A \flat w.t. G w.t. C acoustic C w.t.

Example 25. Dominant-tonic relations in mm. 99–159 of *L'Isle Joyeuse*

acoustic scale is replaced by a G^{\sharp} triad (perhaps suggesting G acoustic) that immediately gives way to the Whole-Tone I collection. Example 25 associates this music with the fifths on C^{\sharp} . The reduction suggests that the whole passage can be interpreted as an indecisive oscillation between I and (altered) V, in both C^{\sharp} and C^{\flat} . Of course, as this whole-tone music continues (in mm. 117–140), it begins to lose its “dominant” quality, sounding more and more like a stable collection in its own right. But in measure 141, the whole-tone music “resolves” triumphantly to a C major triad, again suggesting—if only retroactively—that it had dominant function. (Example 25 shows that this “resolution” articulates a I–V–I progression spanning almost fifty measures.) This C major triad eventually gives way to the C acoustic scale (m. 145), which in turn dissolves into Whole-Tone Collection II (m. 148).⁷⁴ A descending whole-tone scale in mm. 158–159 prepares for the return of the tonic A.

Section 3: mm. 160–255. Example 26 graphs the final section of the piece. The first 26 measures return again to the tonic network of scales described in Example 22. Measure 186 marks a break in the musical texture, and the subsequent music demonstrates Debussy’s sophisticated handling of scalar relationships. Example 27 summarizes. Measures 186–195 (not shown on Example 27) are in C^{\sharp} natural minor. Measure 196 turns to the whole-tone scale, and initiates a long melodic ascent by whole-tones lasting until m. 220. The harmony of mm. 200–207 implies the E^{\flat} acoustic scale; measures 208–215 repeat 200–207 a major second higher, now in F acoustic. Three features of mm. 196–220 are noteworthy. First, these acoustic scales share five of their seven notes, which is the most two acoustic scales can share. Second, the two acoustic scales maximally intersect Whole-Tone Collection I, which appears in mm. 196–199 and controls the melodic ascent in the passage. Third, the five pitch classes common to both acoustic collections are precisely the pitches that appear in the fanfare-like theme in mm. 208–211. These measures therefore belong to *both* the E^{\flat} and F acoustic collections, as indicated by the overlapping brackets on Example 27. Measure 220 brings a return to the second theme and A lydian. The rest of the piece explores the tonic scalar network depicted in Example 22, with brief digressions to B^{\flat} mixolydian and F lydian.

The intricate scalar relationships in *L’Isle Joyeuse* pose a critical and interpretive challenge: how do we reconcile the improvisatory surface of the piece with what seems to be its sophisticated scalar structure? More generally, how is it that a joyous and exuberant work celebrating the flowering of Debussy’s relationship with Emma Bardac, later to become his second wife, could be among the most systematic of his musical constructions? Even more: how is it that the composer who despised rules and declared “pleasure is the law” could have written a piece that so consistently hews to its own compositional laws?⁷⁵ There seems to be almost



Example 26. Scales in *L'Isle Joyeuse*, mm. 160–255

too much structure in *L'Isle Joyeuse*—too much to be consistent with what we think we know about Debussy and his attitude toward composition.

One way to approach these important and difficult questions is suggested by what Robert Morris calls a “compositional space.” Compositional spaces, as Morris defines them, are “out-of-time networks of pitch classes that can underlie compositional or improvisational action.”⁷⁶ Thinking of scale networks in this way may ameliorate the cognitive dissonance produced by the preceding analysis. For if Debussy had managed to internalize some of the scalar relationships we have been discussing, if he could move fluidly and easily between maximally-related scalar collections, then it is clear how his music could be at once highly spontaneous and highly structured. A good part of the “rigorous structure” would reside in the precompositional network of related scales; the music itself would remain free to move spontaneously within this network, gamboling lightheartedly in a field whose boundaries are drawn according to strict principles. I find this image to be appealing, as it helps to reconcile the results of the current investigation with our sense—both historical and aural—that Debussy’s music is locally very free. Debussy’s music *is* in-

Musical score for Example 27, *L'Isle Joyeuse*, mm. 196-223. The score is written for guitar and includes performance instructions such as "fanfare", "A w.t.", "Eb acoustic", "F acoustic", and "A major". Measure numbers 196, 200, 204, 208, 212, and 220 are marked. The score features a treble and bass clef, a key signature of two sharps (D major), and a 3/4 time signature. The piece is divided into sections: "A w.t." (measures 196-200), "Eb acoustic" (measures 200-208), "F acoustic" (measures 208-212), and "A major" (measures 212-223). The "fanfare" section is marked with a bracket and the word "fanfare" in quotes. The "A w.t." section is marked with a bracket and "A w.t.". The "Eb acoustic" section is marked with a bracket and "Eb acoustic". The "F acoustic" section is marked with a bracket and "F acoustic". The "A major" section is marked with a bracket and "A major".

Example 27. *L'Isle Joyeuse*, mm. 196-223

deed spontaneous, but it operates within a compositional space that is intricate and sophisticated.

(d) “Cloches à travers les feuilles”

The three pieces we have analyzed so far are quite similar: each presents a central scale surrounded by a number of maximally related collections. “Cloches à travers les feuilles,” the first in the second series of piano *Images*, is a very different sort of piece. Not only are its scales less fully articulated, but the relations between them are somewhat more opaque. “Cloches” therefore provides an opportunity to test the limits of the theoretical perspective I have presented. Do these concepts help us to interpret even Debussy’s more mysterious and elusive compositions?

Example 28 presents a summary of the piece. We can see that maximally intersecting scales play a role at four places, two of which are local, and two of which connect distant passages of music. The first instance is found in the opening measures. The piece begins, as Example 28 shows, with a series of whole-tone fragments moving in different directions at different speeds. (Arthur Wenk suggests that this music was inspired by the gamelan.) The first non-whole-tone music, in measure 6, uses a six-note subset of the E \flat acoustic scale.⁷⁷ We then hear a return of the opening whole-tone material. When the piece next leaves the whole-tone realm it reaches something much closer to E \flat mixolydian: the first three beats of mm. 9 and 11 contain six of the seven notes of that mode, while the next beat adds the missing A \flat , albeit clouded by nonharmonic passing tones. Thus the shift between whole-tone and diatonic is accomplished in stages, with the acoustic scale serving as a midpoint between the two extremes. This modulatory schema here is, as we have seen, extremely common throughout Debussy’s music.⁷⁸

Measures 13–19 comprise the second thematic region of “Cloches” (labeled “Theme β ” on the graph); they also contain the second instance of maximally intersecting scales. Measures 13–16 are octatonic, while measures 17–19 feature a series of extended sonorities that imply fifth-related diatonic collections: B \flat mixolydian–E \flat mixolydian–E \flat dorian. By themselves, of course, these maximally-intersecting diatonic scales are unremarkable. However, their appearance here underscores the similarity between these traditional fifth-based progressions and the more unusual scale-to-scale transformations we have been investigating.

Measures 20–23 present the first return to the whole-tone music of the opening. As can be seen from Example 28, the music has been transformed so that it now conforms to the D \flat acoustic scale, maximally intersecting the original Whole-Tone Collection I. (The reduction omits the dominant ninth chords, on G and B, which interrupt the music.) Notice, however, that the D \flat acoustic scale in mm. 20–23’s is “colored” by the presence of a non-scalar D \sharp that accompanies the melodic B \flat throughout

the passage. To my ear, these nonscalar tones complicate, but do not dislodge, the acoustic-scale quality of the passage as a whole; this is largely because the left hand very clearly states an acoustic-scale melody.⁷⁹ (I hear the right hand in mm. 20–23 as suggesting flamenco-style parallel triads on B \flat , C \flat , and D \flat .) Although m. 1 and mm. 20–23 are separated by a significant quantity of music, their thematic similarity makes the close harmonic connection palpable.

Measures 44–46 involve a second long-range relationship. This passage, a reprise of Theme β , suggests the B harmonic minor scale, maximally intersecting Theme β 's original Octatonic Collection I. (Again, the scalar qualities of the reprise are somewhat obscured, this time by a descending chromatic line in the left hand of mm. 45–46.) As before, the thematic similarities dramatize the harmonic relationship, helping our ears to associate two passages separated by a considerable quantity of intervening music.

Unfortunately, the rest of the piece proves rather more difficult to unravel. Looking again at Example 28, we can see that the opening presents a series of closely-related, but not always maximally-intersecting scales. Measures 24–39 constitute a contrasting middle section, featuring distantly-related scales (E lydian, B \flat pentatonic, and C \sharp major), and contrasting figuration in pentatonic thirds with figuration in diatonic fourths. The conclusion (mm. 40–49) brings back earlier material, but features dramatic harmonic shifts whose underlying logic is not immediately clear. Overall, one gets the impression of a meandering tour through a series of basically unrelated harmonic regions.

Let us see what happens when we plot these scales on the scale lattice of Example 11. Example 29 graphs almost all of the scales used in the piece, laid out three-dimensionally as in Example 11. (The three scales in brackets are “placeholders,” not appearing in the piece but necessary to complete the graph.) For reasons that will become clear shortly, I have omitted the T-symmetric scales, as well as B harmonic minor. What remains are a series of adjacent faces from the cubic lattice of Example 11. Each contains four scales, three diatonic and one acoustic. Scales on a single face have a common pentatonic subset. The pentatonic scale associated with the topmost face is B \flat pentatonic, which appears in measures 31, 32, and 47 of the piece.

Example 29 is a portion of an interesting structure: the lattice displaying maximal intersections among the four locally diatonic scales. Example 30 flattens Example 29 into two dimensions. The diatonic “circle of fifths” runs in a zigzag fashion through the center, like the stripe on Charlie Brown’s sweater. Above and below the diatonic zigzag of fifths are a sequence of T₂-related acoustic scales, each sharing five notes with its neighbors, and maximally intersecting the same whole-tone collection. Neighboring acoustic scales maximally intersect distinct octatonic col-

Theme α
mm. 1-5

6 9

return to
w.t. collection I

Whole-Tone Collection I B \rightarrow {B \flat , C} Eb acoustic A \rightarrow A \flat Eb mixolydian C triad

11 13 17 18 19

Theme β

Whole-Tone Collection I {G \rightarrow G \flat , D \flat \rightarrow D} Ab acoustic Octatonic Collection I B \flat mixolydian D \rightarrow D \flat Eb mixolydian G \rightarrow G \flat Eb dorian {G \flat \rightarrow G, C \rightarrow C \flat }

Theme α
20-23

Db acoustic
maximally intersecting Whole Tone Collection I

Example 28. Thematic and harmonic summary of “Cloches à travers les feuilles”

Theme γ
24

31

pentatonic thirds

33

pentatonic thirds

E Lydian

$B\flat$ pentatonic

Theme α
40

37

diatonic fourths

42

Whole-Tone Collection I
common tones: A, G, F

F major

Theme β
44-46

47-49

pentatonic fourths

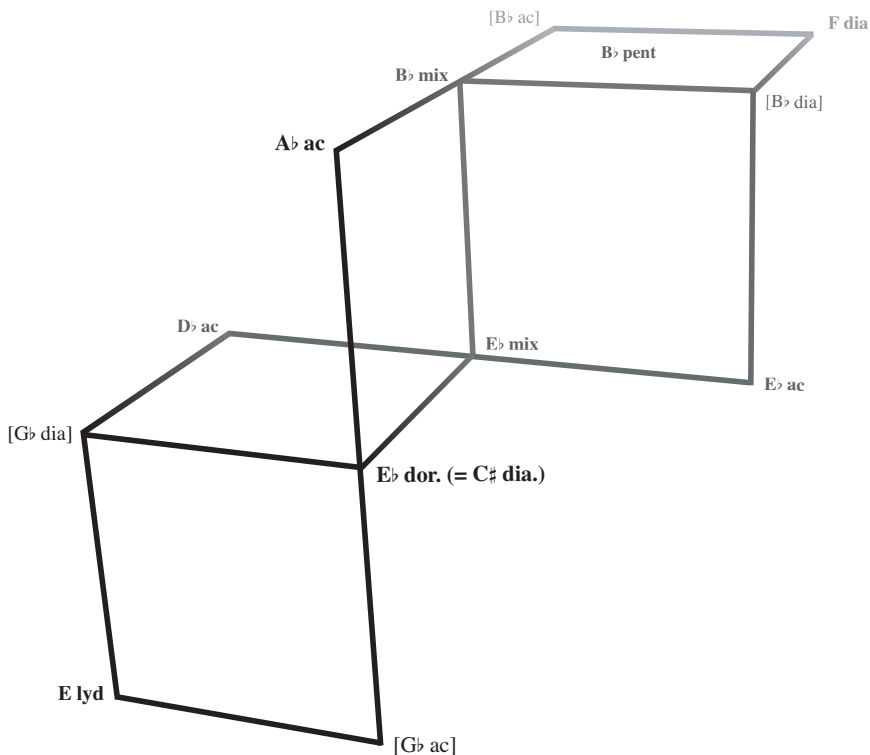
chromatic slide

B harmonic minor
maximally intersecting octatonic collection I

$B\flat$ pentatonic

B harm. minor?
Oct. Col. I?

Example 28 (continued)



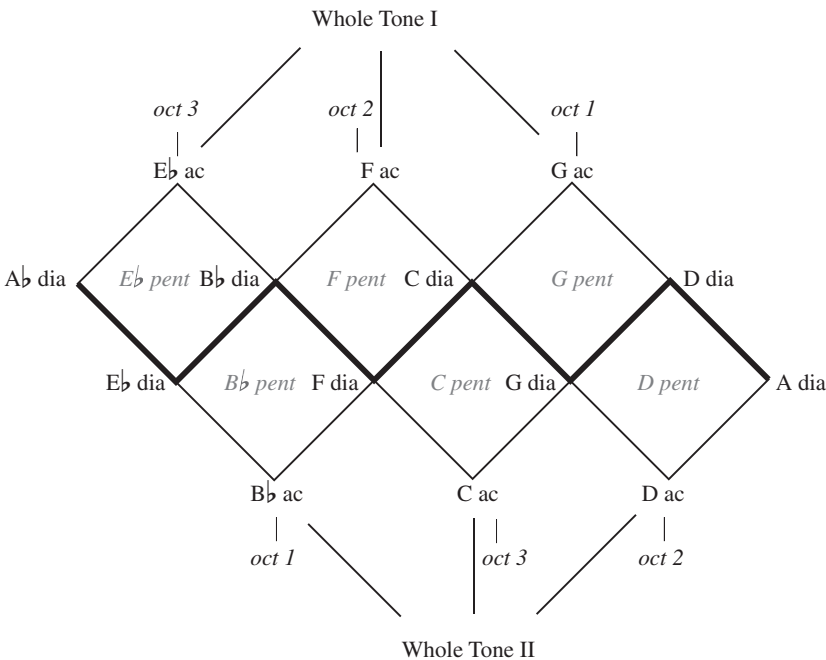
Example 29. Scales used in “Cloches à travers les feuilles”

lections. (These octatonic collections must therefore appear at multiple points on the graph.) Finally, each square of the graph contains all the Pressing scales sharing a single pentatonic collection.⁸⁰

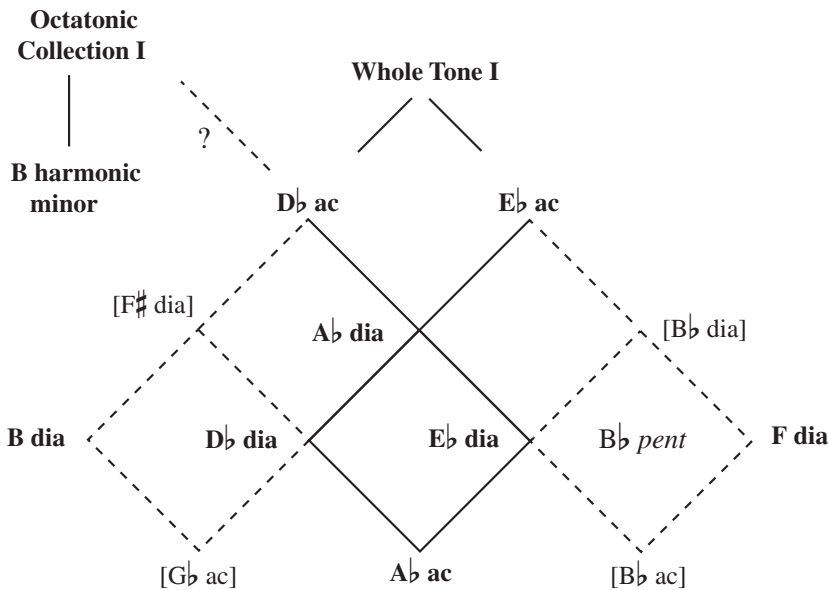
Example 31 displays all the scales used in “Cloches,” laid out as in Example 30. This graph somewhat clarifies the mysterious structure of the piece. It shows that the opening explores a small collection of relatively closely-related scales. We begin at the top with Whole-Tone Collection I. We then move to the maximally intersecting E^b acoustic scale, back to the Whole-Tone I collection, and then on to A^b diatonic (E^b mixolydian).⁸¹ A^b diatonic jumps briefly to the non-scalar and distantly related C major triad, but then returns to a point vertically below it, A^b acoustic. This acoustic scale again leaps to a distant region—here Octatonic Collection I—before returning to the nearby E^b diatonic (m. 17). (Octatonic Collection I maximally intersects D^b acoustic, as shown by the dotted line on Example 31. I am unsure how much weight to place on

this connection.⁸²) We now progress along the circle of fifths through $A\flat$ diatonic (m. 18) to $D\flat$ diatonic (m. 19), then vertically upward to $D\flat$ acoustic. Though it is perhaps an overstatement to describe these harmonic moves as systematic, it would be just as wrong to dismiss them as entirely capricious. For “Cloches,” no less than “Le vent dans la plaine” or *L’Isle Joyeuse*, begins by exploring a small group of closely-related scales.

Furthermore, as in those other pieces, “Cloches” explores more distant regions as it progresses. The middle section of the piece is chiefly in B diatonic (E lydian), lying on the far left side of Example 31. B diatonic/E lydian alternates with the very distantly-related $B\flat$ pentatonic (shared by all the scales on the rightmost square of Example 31) before moving toward the more centrally located $C\sharp$ ($D\flat$) diatonic, heard previously at m. 19. The conclusion, after returning briefly to the whole-tone ideas of the opening, shifts dramatically to F diatonic. (The music emphasizes the three tones common to both collections, A–G–F, thereby smoothing the transition somewhat.) This F diatonic scale might be understood to “balance” or compensate for the B diatonic of the central



Example 30. A portion of the locally diatonic scale lattice



Example 31. Scales used in “Cloches à travers les feuilles”

section: F diatonic and B diatonic are both a minor third away from the central A \flat diatonic (E \flat mixolydian) scale, the first diatonic collection in the piece.⁸³ Example 31 thus suggests some new ways of understanding the piece’s distantly-related scalar regions.

Finally, Example 31 suggests a way of thinking about the different roles of the various scales used in “Cloches.” The graph represents the whole-tone, acoustic, and diatonic scales as structurally central: these collections form a systematic network along which motion tends to occur in fairly comprehensible ways. By contrast, the octatonic and harmonic minor scales are represented as outliers, with the shift to octatonic Theme β being somewhat abrupt. Moreover, the octatonic and harmonic minor collections, although maximally intersecting each other, do not clearly relate to the other collections used in the piece. (Recall in this regard the music in mm. 24–29 of “Collines.”) These facts comport with my belief that Debussy made the most systematic use of the whole-tone, acoustic, and diatonic scales. Although octatonic and harmonic scales do appear in his music, they play a fleeting and more adventitious role.

So should we then conclude that “Cloches a travers les feuilles” is *not* improvisatory, free, or loosely-structured? No. For in comparison to the three pieces studied earlier, “Cloches” exhibits considerably greater permissiveness, a “wandering” quality that we as analysts should be pre-

pared to acknowledge. The point I want to insist upon, however, is that this freedom is not arbitrariness. The procedures exemplified by a more structured piece like *L'Isle Joyeuse* are also present here, though in more veiled form. We find striking instances of maximally intersecting scales, expressed in both local and long-range ways. We see the acoustic scale mediating between the fifth-based world of the diatonic collection and the T-symmetrical world of the whole-tone scale. The harmonic minor scale is clearly associated with the octatonic. And the scales in "Cloches" participate in a venerable formal schema: the piece begins by exploring a series of closely-related scales, broadens in its middle sections to encompass more distantly-related scales, and concludes by recapitulating important collections presented earlier in the piece. In short, there is structure to be found here, structure that has been clarified by the theoretical investigation undertaken in Sections I and II of the paper.

IV. Debussy in Context

The analytical challenge in approaching Debussy's music has always been to say something about the way it balances between common-practice tonality and radical atonality. What makes this particularly difficult is that Debussy's music is situated in the no-man's land between our two most popular analytic methodologies: Schenkerian analysis, which has its most natural application to music that is more squarely conventional than Debussy's; and set theory, which is most comfortably at home with music that embraces the total chromatic. I do not want to suggest that these methods cannot produce interesting insights into this music. But I do think that one often has to engage in analytical contortions to apply them, and I think there are cases where neither technique (nor any combination of the two) tells the full story about what the composer was up to. I hope to have begun to outline a more indigenous approach, developing concepts and techniques that have a natural home in this most alluring, and elusive, of musical styles.

More generally, the scales derived in Sections I and II themselves represent a natural way-station between the poles of traditional tonality and complete atonality. Structurally similar to the diatonic scale, they are capable of participating in the regime of tonally functional harmony. This is precisely their role in jazz.⁸⁴ Yet at the same time, they possess an extraordinarily rich variety of subsets, and are linked by a complex network of common-tone and voice-leading relationships. One might go so far as to say that the scales themselves are perched on the boundary between tonality and atonality. As such, they are potentially of interest to composers who seek rapprochement with tonal methods of musical organization. I would hope that the preceding discussion, in addition to enriching our understanding of existing music, may also interest com-

a) mm. 5-8

b) mm. 73-76

E → E#

Example 32. Maximally-intersecting scales in Chopin's Mazurka Op 41, no. 1

posers looking to explore new musical techniques. Certainly, my own compositional practice has been enriched by this perspective.

I will conclude with a few final musical examples that indicate our investigation has identified features of a larger musical practice. The first two examples come from the nineteenth century. Example 32 presents two excerpts from Chopin's C# minor Mazurka, Op. 41 no. 1. The piece opens in what seems to be C# phrygian. When this music returns, in mm. 73–80, the C# phrygian scale is transformed, by way of maximally-smooth voice leading E→E#, into a mode of the F# harmonic minor scale. The resulting combination of modality with maximally related scales anticipates Debussy's more systematic use of the same techniques.

Example 33 presents a strikingly Debussian passage from Liszt's third "Mephisto Waltz." The passage implies three scales: D dorian, G acoustic (D melodic minor ascending), and the Whole-Tone Collection I, with G acoustic (D melodic minor) maximally intersecting the other two scales. As occurs frequently in Debussy's music, the acoustic scale mediates between diatonic and whole-tone collections. (This is just one of a number of remarkable passages in Liszt's late works that anticipate Debussy's compositional procedures.⁸⁵) These examples from Chopin and Liszt suggest a more general question: what was the scope of pre-twentieth-century exploration of non-diatonic scales, and how does it relate to the more systematic scalar thinking we find in Debussy? Both questions deserve further investigation.⁸⁶

Example 34 is an excerpt from Stravinsky's *Petrouchka*, in which a Bb lydian scale progresses to a maximally intersecting Bb acoustic scale. This is one of a large number of similar passages to be found in the works

D dorian D melodic minor ascending (G acoustic)

C→C#

Whole-Tone Collection I

{D→D#, E→E#}

Example 33. Maximally intersecting scales in Liszt's third "Mephisto Waltz," mm. 19–26

of disparate early twentieth-century composers, including Scriabin, Prokofiev, Ravel, and others. (*Klokkeklang*, one of Grieg’s more radical works, could also be mentioned here.) Again, we might ask: How systematically did these twentieth-century composers explore non-diatonic scales? Do the similarities between them result from historical influence or convergent evolution? And what happened to this “scalar tradition” in early twentieth-century music? Did it continue, unnoticed by theorists, to play a significant role in later styles? Or was it displaced by more radical

Cls.
Str.

B \flat lydian A \rightarrow A \flat B \flat acoustic

Example 34. Maximally intersecting scales in in *Petrouchka*, R65:5–8

G harmonic major

{C \rightarrow C \sharp , F \sharp phrygian
E \flat \rightarrow E}

Example 35. Shostakovich’s Fs-minor fugue, mm. 21–29

B \flat acoustic

A \flat → A F major

Example 36. Rzewski's *Les Moutons de Panurge*

methods such as polytonality and atonality? Example 35 hints at an answer to these questions: it presents an excerpt of the F \sharp minor fugue from Shostakovich's *Preludes and Fugues*. This passage, which appears multiple times throughout the fugue, involves a two-semitone shift between G harmonic major and F \sharp phrygian. It gives us reason to suspect that the scalar tradition plays a role in Shostakovich's music, interacting in complex ways with post-Wagnerian chromaticism, polytonality, and atonality.

Finally, it is clear that the late twentieth century witnessed the gradual reemergence of diatonic thinking, largely under the influence of minimalism. But non-diatonic scales also play an important role in this music as well. Example 36 presents an early minimalist piece, Fredrick Rzewski's *Les Moutons de Panurge*.⁸⁷ The entire composition centers on a maximally-smooth progression from B \flat acoustic to F diatonic. (The scales involved are the same as those appearing in Example 34.) This should lead us to ask: how does the scalar music in early twentieth-century art music relate to the scalar music we find at the end of the century? What are the similarities uniting these superficially quite distinct musical styles?

I would not want, at this point, to make too much of the connections uniting Examples 32–36, nor of their relation to the Debussy pieces studied in this paper. But I do think it is likely that further investigation will reveal something like a “hidden common practice,” a way of thinking about scales that is the exclusive property of no one composer. This “common practice” is, to be sure, just one of many threads that run through the heterogeneous cloth that is twentieth-century music. But it is an important and colorful thread, one that—almost a hundred years after the fact—we are just beginning to understand.

Appendix I: The “Genera” of Richard Parks

In *The Music of Claude Debussy*, Richard Parks identifies five “genera” that he takes to be important in Debussy’s music: the “diatonic,” “whole-tone,” “octatonic,” “8–17/18/19” and “chromatic.”⁸⁸ The diatonic, whole-tone, and octatonic genera consist of all the subsets and supersets of the diatonic, whole-tone, and octatonic collections. These genera center on three of my four “locally diatonic” scales. Though Parks acknowledges that Debussy makes frequent use of the fourth locally diatonic scale, the acoustic, he does not consider it to engender a genus of its own. He prefers to interpret it, like the harmonic major and minor scales, as a “distortion” of the diatonic. Parks considers the prevalence of these scales in Debussy’s music to be “a manifestation of Debussy’s post-Romantic tendencies.”⁸⁹

Parks’s fourth genus, the “8–17/18/19” genus, consists of all the subsets and supersets of the octochords 8–17, 8–18, and 8–19. His fifth, the “chromatic,” is derived according to substantially different principles, which bear little resemblance to the theory developed here.⁹⁰ I will therefore restrict my attention to Parks’s treatment of the “diatonic,” “whole-tone,” and “octatonic” genera, since it is here that his “genus theory” comes closest to the concerns of this paper. The similarities between our two approaches should be clear, even from this brief description: in the grand scheme of things, Parks’s views are quite similar to my own. However, more detailed consideration shows that there are also interesting differences between us.

1. As mentioned above, Parks does not consider the acoustic or harmonic scales to be objects in their own right, treating them as mere modifications of the diatonic scale. By contrast, I consider them to be independent scales that interact with the diatonic and T-symmetrical scales in complex and interesting ways. I can see no theoretical reason, and certainly no reason in Debussy’s music, for assimilating these non-diatonic scales to the diatonic. Indeed, it is a central point of the present paper that the octatonic, whole-tone, and hexatonic scales relate to the diatonic scale in much the same way as the acoustic and harmonic scales do.

2. Unlike Parks, I attempt to provide an explanation of *why* Debussy and other composers used the scales they did. Parks’s genera are merely descriptive—they are simply the collections he finds most frequently in Debussy’s music. My own approach, though it is also useful for describing Debussy’s music, attempts to push deeper, into the realm of underlying principles. Thus I emphasize, not just that Debussy used the scales he did, but also that these scales are *related* in interesting and specifiable ways. This in turn leads me to suggest reasons why Debussy—as well as so many other composers—might have used these scales.

3. Parks’s approach is largely set-theoretic, typically treating the dia-

tonic, whole-tone, and octatonic collections as unordered sets. By contrast, I emphasize that Debussy typically uses these objects as *scales* whose constituents are ordered in pitch-class space (and typically, register). This ordering underwrites a notion of distance that permits analysts to talk about scalar intervals, transposition within a scale, parallel motion within a scale, and so on. Examples 2 and 6, among many others, demonstrate that these concepts are useful in analyzing Debussy's music.

4. Parks's genera consist in the subsets *and supersets* of a central "cynosural" set class. However, I am not at all convinced that it is necessary to appeal to large scalar supersets in analyzing Debussy's music. Not only is it hard to hear such sets but there is scant precedent for their use in any of the music that Debussy might have known. Furthermore, the seeming appearance of these scalar supersets can easily be explained as the result of modulation (change of scale) or nonharmonic tones.

5. Finally, I disagree with Parks on the importance of complementation. Unlike Allen Forte, Parks does not consider complementation to be essential to the process of genus formation. Nevertheless, the complement relation does play a major role in his analyses. I am suspicious of this, chiefly because scales satisfying the DS constraint *necessarily contain their own complements*. (As discussed in Section I, the complements of these sets are anhemitonic, sharing no common tones with their semitone transpositions.) Thus, any analysis that identifies an abundance of subsets and supersets of the locally diatonic scales will *ipso facto* identify an abundance of complement-related pairs. It is therefore incumbent on the analyst to show that he or she is revealing something interesting about the music's structure, rather than describing an inevitable byproduct of the artist's choice of materials. Parks, to my knowledge, never addresses this fundamental issue; instead, he takes the mere presence of *complements* as evidence of the analytical and musical relevance of *complementation*.

Appendix II: Advanced Scale Geometry

This appendix discusses further features of scale geometry. I begin by providing a general explanation of the geometrical representations contained in this paper. I then investigate voice leading between harmonic major and minor scales.

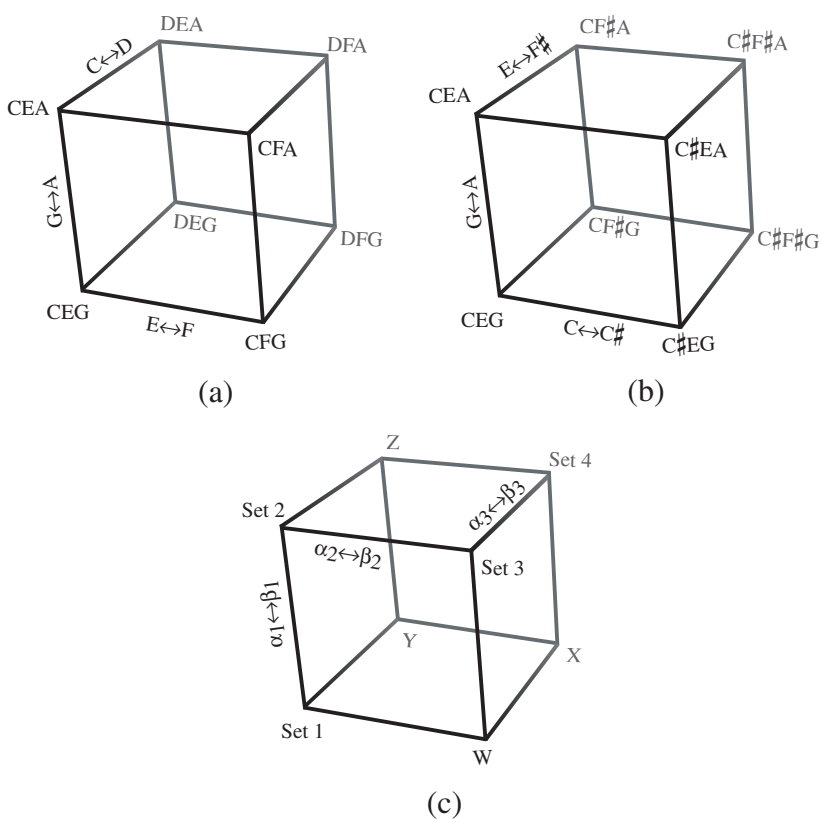
(a) Where Does the Scale Lattice Come From?

The scale lattice of Example 11 belongs to a family of closely-related structures. Indeed, virtually the same lattice depicts single-step voice leading relations among trichords belonging to four of the five diatonic trichord-types, as well single-step voice leading among trichords belonging to four of the eight *octatonic* trichord types. Examples 37(a) and (b) illustrate, providing analogues to Example 12's "scale cube." These cubes can be stacked in the manner described in Section II(c). The resulting lattices can be decomposed into "intertwining strands" as described in Section II(d). The only difference between the respective lattices is the number of cubes involved: there are twelve distinct transpositions of each seven-note Pressing scale, producing a lattice of twelve cubes; since there are only seven transpositions of each diatonic trichord, and only eight transpositions of octatonic trichord, these lattices involve seven and eight cubes, respectively.

How can this possibly be? What is the relation between Pressing scales, diatonic trichords, and octatonic trichords? The answer is that any maximally even set that is not transpositionally symmetrical can be used to build a "generalized circle of fifths"—a cycle of transpositionally related sets linked by single-step voice leading.⁹¹ It follows that we can always build a cubic lattice as shown in Example 37(c). Here, our "generalized circle of fifths" takes Set 1 to (transpositionally related) Set 2 by way of the single-step voice leading $\alpha_1 \rightarrow \beta_1$, Set 2 to Set 3 by way of the single-step voice leading $\alpha_2 \rightarrow \beta_2$, and Set 3 to Set 4 by way of the single-step voice leading $\alpha_3 \rightarrow \beta_3$. The cube in Example 37(c) associates each voice leading motion with a spatial dimension: here, the y -axis is associated with $\alpha_1 \rightarrow \beta_1$, the x -axis is associated with $\alpha_2 \rightarrow \beta_2$, and the z -axis is associated with $\alpha_3 \rightarrow \beta_3$. To build the rest of the cube, we simply apply these single-step voice leadings to the appropriate sets. For example, we get from Set 1 to W , the set in the lower front-right corner, by applying the voice leading $\alpha_2 \rightarrow \beta_2$. It follows that this set will be linked to Scale 3 by the single-step voice leading $\alpha_1 \rightarrow \beta_1$. It is fairly straightforward to describe the structural properties of the sets that result: for example, set W must be inversionally symmetrical and transpositionally related to Z , the set in the rear upper-left corner; meanwhile, sets X and Y , on the lower rear corners, are related by inversion.⁹²

The lattices described here are also related to another group of cubic

lattices. When an n -note set divides the octave perfectly evenly, then one can build a circular lattice of n -dimensional cubes, each sharing a single vertex with its neighbors. For example, single-semitone voice leading between major, minor, and augmented trichords can be represented using four cubes, each of which shares a vertex with its neighbors. This structure was first investigated by Douthett and Steinbach, who call it “Cube Dance.”⁹³ Similarly, single-semitone voice leading between diminished seventh chords, half-diminished seventh chords, minor seventh chords, dominant seventh chords, and [0268] “French Sixth” chords can



Example 37. Other scale lattices
 (a) Diatonic trichords 024, 014, 013, and 023
 (b) Octatonic trichords 025, 024, 014, 034
 (c) A recipe for forming a generalized “scale lattice”

be represented using a necklace of three four-dimensional *hypercubes*, each of which again shares a vertex with its neighbors.⁹⁴

A deep understanding of all of these lattices, as well as the relations between them, is well beyond the scope of this paper. I believe it can only be obtained when we adopt a *geometrical* perspective on set classes. In Tymoczko 2006 I show that there exist geometrical spaces in which points represent unordered pitch-class sets and line segments represent voice leadings between their endpoints, with the length of a line segment equal to the size of the voice leading it represents. These spaces are complete, representing *all* pitch class sets—in every imaginable tuning system—of a given cardinality. All of the lattices we have discussed appear naturally in these geometrical spaces. Their structure is largely determined by the “twists” of the space containing them. Readers who wish to explore the matter further will find additional resources on the internet: on my website, there is a draft of an extensive collaborative project with Ian Quinn and Clifton Callender, in which we explore the geometrical perspective in great depth.⁹⁵ I have also written a free software program that produces 3D models of these geometrical spaces.⁹⁶ This program can be used to display the complete trichord lattices generated by Examples 37(a) and (b).

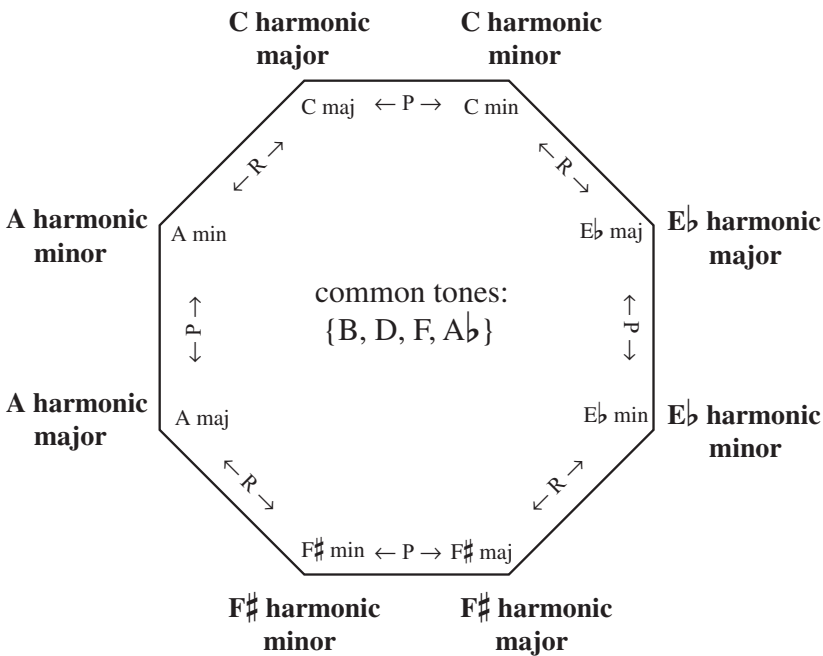
(b) The Circle of Harmonic Scales

Example 10(c) shows that the C harmonic major scale shares six of its seven notes with both the C and A harmonic minor scales. The voice leading between C harmonic major and C harmonic minor is maximally smooth, since it involves the single-semitone displacement $E \rightarrow E\flat$. But in moving from C harmonic major to A harmonic minor, the pitch class G is displaced by *two* semitones to A, leading to an enharmonic respelling of $A\flat$ as $G\sharp$. Alternatively, one can represent this voice leading as involving two single-semitone shifts: G moves to $G\sharp$, producing the multiset C-D-E-F- $G\sharp$ - $A\flat$ -B, which combines an augmented triad with a diminished seventh chord.⁹⁷ Then $A\flat$ moves to A. In Douthett and Steinbach’s terminology, the C harmonic major and A harmonic minor scales are $P_{0,1}$ -related. We now turn our attention to the maximal intersection—but not maximally-smooth voice leading—between them.

To understand these relationships, it is useful to note that every harmonic major or minor scale can be partitioned into a triad and a nonintersecting diminished seventh chord. The four harmonic scales in Example 10(c)—C harmonic minor, C harmonic major, A harmonic minor, and A harmonic major—all share the same diminished seventh chord (B-D-F- $A\flat/G\sharp$). The triads not intersecting this diminished seventh chord are related by the neo-Riemannian P and R voice leadings.⁹⁸ Example 38 demonstrates, presenting the eight harmonic scales containing the diminished seventh chord B-D-F- $A\flat/G\sharp$. The scales are identi-

fied on the outside of the octagon; inside it, one finds eight triads, linked to their neighbors by the parallel and relative voice leadings and together comprising an octatonic scale. Scales whose triads are linked by the P voice leading are adjacent on Example 11. (For example, Ahm↔AHM is an edge of the lowest cube.) Scales linked by the R operation are not adjacent on Example 11. Instead, they correspond to “short cuts” that leap ten places on the non-diatonic circle of fifths, cutting diagonally through empty space. On Example 11, for instance, the A harmonic major scale in the *first* cube is linked, via the R operation, to the F# harmonic minor scale in the *fourth* cube.⁹⁹

The effect of considering these additional voice leadings is to transform the circle of non-diatonic scales, shown in Example 13, into a more complex structure: a doubly periodic lattice lying on the surface of a 2-torus. One can think of this structure as a series of three octagons, each transpositionally equivalent to Example 38. One moves from octagon to octagon by way of maximally smooth voice leading involving an intervening acoustic scale. For example, one can move from C harmonic minor, which lies on the octagon shown in Example 38, to G harmonic major, which lies on a different octagon, by way of the F acoustic scale.



Example 38. The circle of harmonic scales

Scale Type	diatonic	acoustic	harmonic	octatonic	whole-tone	hexatonic	total #
whole-tone		6					6
hexatonic			6				6
octatonic		4	8				12
diatonic	2	2	2				6
acoustic	2		2	1	1		6
harmonic	1	1	2	1		1	6

Example 39. Maximal intersections by scale type

(Four acoustic scales thus lie between any two octagons, serving to connect a harmonic major scale on one octagon to a harmonic minor scale on the other.) The 36-scale “circle of non-diatonic fifths” winds its way along this structure, alternately moving along each octagon and moving between octagons by way of acoustic scales.

We are now in a position to appreciate an attractively simple fact: all of the Pressing scales, with the exception of the octatonic, maximally intersect precisely six other Pressing scales; the octatonic maximally intersects twelve other Pressing scales. Example 39 explains. Every whole-tone scale maximally intersects six acoustic scales. Every hexatonic scale maximally intersects six harmonic scales, three harmonic major, and three harmonic minor. Every octatonic scale maximally intersects four acoustic, four harmonic major, and four harmonic minor scales. All of these intersections involve scales with different cardinalities, and hence do not give rise to maximally-smooth voice leading. Every diatonic scale maximally intersects six seven-note scales (two diatonic, two acoustic, and two harmonic). It can be linked to all six of these by maximally smooth voice leading. The acoustic scale maximally intersects two T-symmetrical scales (one whole-tone, one octatonic) and four seven-note scales (two diatonic, and two harmonic). It can be linked by maximally-smooth voice leading to all four of the seven-note scales. Finally, every harmonic scale maximally intersects two T-symmetrical scales (one hexatonic, one octatonic), and four seven note scales (two harmonic, one diatonic, and one acoustic). It can be linked to three of the four seven-note scales by maximally-smooth voice-leading, and to the remaining one by way of the non-maximally-smooth R relation discussed above. There is therefore an inverse relationship between a scale’s transpositional symmetry and the number of different scale-types it maximally intersects. The highly T-symmetrical whole-tone scale maximally intersects only one type of scale; while the highly variegated, T-asymmetrical harmonic scales (which contain three different step sizes and a very diverse collection of subsets) maximally intersect five of the six scale types.

NOTES

1. See, for example, McCreless 1996, especially pp. 110–111.
2. Samson 1977 proposes a similar dichotomy.
3. Schoenberg 1911/1978, ch. 20, explicitly asserts the primacy of the chromatic scale.
4. Tymoczko 1997, 2002, 2003.
5. See Antokoletz 1993, Caballero 2004, Callender 1998, Quinn 2002, and Zimmerman 2002.
6. Martins 2005.
7. We typically interpret the term “one chromatic semitone” to mean “one scale step in the standard chromatic scale.” However, since the chromatic scale is equal-tempered, we can also take the term “semitone” to refer to a *scale-independent* unit of distance, namely 1/12 of an octave. Tymoczko 2005 develops this perspective at length.
8. Clough 1979 explores diatonic set theory in detail.
9. Clough and Douthett 1991 use the terms “chromatic length” and “diatonic length,” abbreviated clen and dlen, to refer to these two ways of measuring distance.
10. For example, corresponding to the pitch-class set [024579E] there is the infinite ascending ordering of pitches (. . . , 55, 57, 59, 60, 62, 64, 65, 67, . . .). Integers here represent MIDI note numbers, which provide a spelling-free alternative to the more familiar, spelling-specific system of designating pitches; see Tymoczko 2005. If one replaces each pitch by the pitch class containing it, the infinite, ordered series of pitches becomes a unique *circular* ordering of pitch classes. We can notate this as (0, 2, 4, 5, 7, 9, 11, [0]), with the bracket indicating a circular sequence that returns to its starting point.
11. The language of pitch classes provides an efficient way to make general claims about pitches. For instance, when one says “pitch class B is one semitone, and one scale step below C in the C diatonic scale,” one means: “for every pitch C, there is a pitch B exactly one semitone and one scale step below it in the C major scale.” See Tymoczko 2005.
12. Neologisms such as “subscale” and “superscale” are in my judgment unnecessarily confusing, as are such convoluted locutions as “the set associated with scale S has subsets that are themselves associated with scales that . . .” The mixed terminology is significantly more direct.
13. In general, I will not consider inversionally-related scales to belong to the same class: thus, the harmonic major and harmonic minor scales are different.
14. The C diatonic scale is the unique octave-repeating scale containing only the pitch classes 0, 2, 4, 5, 7, 9, and 11. The C acoustic scale is the unique octave-repeating scale containing only the pitch classes 0, 2, 4, 6, 7, 9, and 10—in letters C–D–E–F#–G–A–Bb. (The scale is familiar from the classical tradition as the ascending form of G melodic minor.) The C harmonic minor scale contains the pitch classes 0, 2, 3, 5, 7, 8, and 11, or C–D–Eb–F–G–Ab–B. The C harmonic major scale contains pitch classes 0, 2, 4, 5, 7, 8, 11, or C–D–E–F–G–Ab–B. The C pentatonic collection contains 0, 2, 4, 7, 9, or C–D–E–G–A. “Whole-Tone Collection I” is the whole-tone collection containing C# (=1, in integer notation). “Whole-Tone Collection II” is the collection containing D# (=2). “Octatonic Collection I” is the octatonic collection containing the dyad C#–D (= 1, 2). “Octatonic

Collection II” contains D–D \sharp (= 2, 3), and “Octatonic Collection III” contains D \sharp –E (=3, 4). “Hexatonic Collection I” contains the dyad C \sharp –D, “Hexatonic Collection II” contains D–D \sharp and so forth.

15. Lerdahl (2001, 268–274) likewise proposes a set of desirable scalar properties. Though these properties differ from those discussed in this paper, there is considerable overlap between his scales and mine.
16. Music theorists sometimes shy away from historical explanations of this sort. At the end of Section I(b), however, I argue that conventional accounts of the origin of the melodic minor scale presuppose the three scalar constraints described here. My account simply generalizes the widely-accepted explanation of the origin of the melodic minor scale, adapting it to a twentieth-century, post-common-practice context.
17. Lerdahl (2001, 316) has suggested that something like the DS property reflects a natural, psychoacoustically-based boundary between the “melodic” and “harmonic” realms. Pitches separated by one or two semitones are subject to forces of “melodic attraction” that interfere with their ability to coexist in stable harmonies; pitches separated by three or more semitones, since they are subject to much smaller “melodic attractions,” are more likely to coexist harmonically.
18. Jay Rahn (1991) investigates a related kind of consistency between scalar and chromatic distance metrics.
19. This is an instance of what Rahn (1991) calls the absence of “contradictions” in a scale’s interval spectrum.
20. Zimmerman 2002 attributes the dissonance of the [012] trichord to the fact that “all three of its intervals would be judged dissonant in any tonal context.” Huron 1994 presents a quantitative model of acoustical dissonance in which the 012 trichord is the most dissonant of the trichords. Friedman (1990), though he does not emphasize the term “dissonance,” asserts the distinctiveness of the [012] trichord, advising students to rely on its presence or absence when identifying larger set classes aurally. Of course, the notion of musical dissonance is both conventional and contextual, and is notoriously hard to quantify. (Even the more restrictive notion of acoustic or sensory dissonance is complicated, since it depends on timbre, register, and loudness.) The assertion in the text should therefore be taken as a reflection of what some composers and theorists have believed, rather than a simple fact of nature.
21. A number of musical analysts, working on a wide range of music, have arrived at the NCS constraint independently. Jeff Pressing (1978, 1982) initially identified the constraint as a key feature of jazz harmony. Philip Russum (1985) applied the constraint in analyzing Ravel. (It is unclear whether Russum, who has since left music theory, developed the NCS constraint on his own, or took it from Pressing.) The constraint also plays a role in Larson 1992. Daniel Zimmerman (2002) independently formulated the NCS constraint in considering Prokofiev. My own use of the constraint in analyzing jazz and impressionism (1997) was likewise carried out without knowledge of this other work. The simplest explanation for this theoretical convergence is that it results from the music itself—that there indeed are styles of post-common-practice music that employ a wide range of harmonies, but not those containing [012] subsets.
22. Explicit articulations of this “pandiatonic” approach to harmony can be found in

the works of jazz theorists who assert that chord voicings can be constructed out of any of the notes of the appropriate scale. See, for instance Holdsworth 1993 and Levine 1989.

23. See, for instance, *The New Harvard Dictionary of Music*.
24. The only way you can remove a pitch class from a DS scale, and still end up with another DS scale, is if you remove the middle note of an [012] trichord.
25. See Tymoczko 1997.
26. See Messiaen 1956, Martins 2005, and Baker 1985, respectively.
27. Suppose S is a maximal NCS set. Consider any consecutive trichord of the scale S . Since the trichord cannot be an instance of set class [012], it must span at least three semitones. But since S is maximal, the trichord can span no more than four semitones. (If the trichord spanned five semitones, then we could, contrary to hypothesis, insert another note within it without violating NCS.) Therefore the scale S possesses the DT property. Conversely, suppose a scale S possesses DT. Its consecutive trichords span either three or four semitones, which means they belong to one of three types: [013], [024], and [014]. (The argument that follows mentions only the prime forms of the [013] and [014] trichords. However, it also applies to their inversions [023] and [034].) Furthermore, if a DT scale contains a consecutive [014] trichord, that trichord must be embedded within an [0145] tetrachord. (This is because the middle note of the [014] trichord must lie two scale steps, and four semitones, below a scale tone.) Now let us see whether any note can be inserted within the span of these trichords. [013] and [024] are “internally NCS maximal,” since adding a note within the span of either trichord violates NCS. [014] is not itself internally NCS maximal, since a note can be inserted within the trichord to create an [0134] tetrachord. However, we have determined that the [014] trichord must be embedded within an [0145] tetrachord. [0145] is internally NCS maximal. We conclude that the set S must be a maximal NCS set. This shows that every DT scale is a maximal NCS set, and vice versa.
28. See Pressing (1978), Tymoczko (1997). The computer scientist David Rappaport (2006) has observed that the *multiset* C–C–D#–E–F#–G#–A also possesses diatonic thirds.
29. The theorists who have independently discovered the NCS constraint have all gone on to identify the Pressing scales: first Jeff Pressing (1978), then Larson (1992), Tymoczko (1997), and Zimmerman (2002). Allan Holdsworth (1993) explores related ideas. I recently learned that Sam Davis, who teaches guitar privately in Cambridge, Massachusetts, has developed a substantially similar scale theory. The fact that so many musicians have independently gravitated toward the same scalar collections helps support the idea that a similar convergence occurred among early twentieth-century composers.
30. Note that the two harmonic scales minimally violate DS, since they contain only a single three-semitone scale-step. The hexatonic scale involves a more radical violation of DS, since it contains *three* three-semitone steps. For this reason, perhaps, it is less frequently encountered as a melodic entity.
31. Since a DS+ scale contains no one-semitone steps, its complement contains no steps larger than two semitones. Since a DS+ scale contains no steps larger than three semitones, its complement contains no consecutive semitone steps.
32. Note that since an anhemitonic set class shares no notes with its T_1 and T_{11} forms,

it must be contained twice within its complement; in the case of the whole-tone scale, the T_1 and T_{11} forms are equivalent. This becomes relevant in Appendix I, which discusses Richard Parks's work.

33. For an earlier jazz example featuring pentatonic thirds, see Art Tatum's 1933 recording of "Tea for Two," chorus 3, bars 5–6. A transcription can be found in Edstrom and Schiff 1996.
34. This solo is transcribed in Waters 1996.
35. See Cohn 1996. The voice leading I have in mind is, of course, $F \rightarrow F\sharp$. For an account of scale-to-scale transformations as voice leadings between pitch-class sets, see Tymoczko 2005.
36. Clough and Douthett 1991.
37. See Clough and Meyerson 1985, Clough and Douthett 1991.
38. This point, which is developed in Tymoczko 2005 and 2006, generalizes an observation first made by Richard Cohn (1996, 1997). It can, for example, be shown that the minimal bijective voice leading between two transpositionally-related diatonic sets is at least as small as the minimal bijective voice leading between any other seven-note sets related by the same transposition operation.
39. For example, the smallest bijective voice leading between any DS set and its T_1 - or T_2 -form will be at least as small as the smallest bijective voice leading between any other T_1 - or T_2 -related sets of the same cardinality. Similar statements can be made for DT scales and their T_3 - or T_4 -forms. These facts follow from the structure of the relevant "scalar path matrices" as described in Tymoczko 2005.
40. This sounds more complicated than it is: if the larger set were to share all the notes of the smaller set, then it would contain that set!
41. This is one reason why the acoustic collection is so often mistaken for other scales (Tymoczko 2002, 2003). Among theorists who have discussed it, Perle (1984) and Callender (1998) have both pointed out that it maximally intersects the whole-tone and octatonic scales. Taruskin (1996) makes a similar observation with regard to the "mystic chord," a six-note subset of the acoustic scale. Howat (1983) notes the maximal intersection between the whole-tone and acoustic scales. And Antokoletz (1993) notes the maximal intersection with the diatonic collection.
42. An inversionally symmetrical scale must be an incomplete interval cycle to have this property, and the diatonic is the only such scale in the group.
43. I will use "T-symmetrical" and "I-symmetrical" as shorthand for "transpositionally symmetrical" and "inversionally symmetrical."
44. Two distinct T-symmetrical sets can never maximally intersect.
45. If one can transform set A into set B by raising a single pitch class by a single semitone, then one can transform the *inversion* of set A into some *inversion* of set B by *lowering* a single pitch class by a single semitone. In the special case where A and B are, like the diatonic and acoustic scales, inversionally symmetrical, inversion yields a second voice leading between set A and a set of B 's type.
46. Example 11 continues to omit the non-maximally-smooth voice leading between A harmonic minor and C harmonic major, discussed in Appendix II(b).
47. My graph most closely resembles that version of the triadic *Tonnetz* in which triads appear as nodes, rather than faces, of the graph. This is the "geometric dual" of the *Tonnetz* featuring pitch classes as its nodes. See Douthett and Steinbach 1998. Note that not all the connections on the triadic *Tonnetz* involve maximally-smooth voice leading whereas all the connections on my scale lattice do.

48. On Example 11, only the A diatonic collection is surrounded by six scales. This is because the example shows only a portion of the complete lattice. The subgraphs shown in Example 10 may provide a clearer perspective.
49. I use spelling-specific pitch-class labels for pitch-classes here because they are somewhat more convenient than numerical labels. However, I do not mean to privilege any one spelling over any other: “C” refers to the *same* pitch class as “B#,” just as “Bob Dylan” refers to the same individual as “Robert Zimmerman.”
50. Since each cube is itself inversionally symmetrical, the second cube is also the inversion (around E) of the first.
51. If this is difficult to visualize, produce a physical model of Example 12 by drawing the scale names on the corners of a cardboard box. Then rotate the box so that the line connecting C major and G major occupies the position held by the line connecting C major to G acoustic. Then transpose each scale by seven semitones.
52. The first cube in Example 11 should share a face with another cube that is in front of it, while the last cube should share a face with another cube on top of it, and so on.
53. I describe the relationship that gives rise to these additional harmonic scales in Appendix II(b).
54. For example, Octatonic Collection I can be transformed into G acoustic by replacing the dyad {G \flat , A \flat } with the pitch class A. G acoustic can be transformed into Whole-Tone Collection I by replacing the dyad {D, E} with the pitch class E \flat .
55. Octatonic Collection III can be transformed into C# harmonic minor by replacing the trichord {G, A, B \flat } with the dyad {G#, A}. It can be transformed into F harmonic major by replacing the trichord {D#, E, F#} with the dyad {E, F}. Similarly, C# harmonic minor can be transformed into Hexatonic Collection IV by replacing the trichord {D#, E, F#} with the dyad {E, F}. F harmonic major can be transformed into Hexatonic Collection IV by replacing the trichord {G, A, B \flat } with the dyad {G#, A}. Observe that the two harmonic scales define two pathways between Octatonic Collection III and Hexatonic Collection IV; the two pathways involve the same trichord-for-dyad substitutions, but in reverse order.
56. The term “acoustic scale” is often used to refer to this particular mode, though in this paper I have appropriated it for the scale itself.
57. On the notion of a compositional “space” see Morris 1995 and 1998. I will return to this topic at the end of Section III(c).
58. Analysis can also give us useful information about how a composer thought about his or her music, even if those thoughts do not correspond to any perceptual reality whatsoever. Analysis in this sense is part of the history of ideas, a kind of conceptual archaeology.
59. While I have no qualms about describing “Collines” as being in B major, it is worth mentioning that centrality in post-common-practice is a subtle matter, as it may involve a tonic *note*, a tonic *collection*, or both at once. These notions correspond only vaguely to the traditional conception of “key.” For instance, a piece may have the white-note scale as its tonic scale, and pitch class C as its tonic note, and yet it may follow few or none of the harmonic and voice-leading conventions of traditional tonality. It is slightly misleading to describe such a piece as being “in C major”; the more neutral description, “in C ionian,” seems more appropriate.
60. My analysis of mm. 78–80 is somewhat speculative. The total pitch content of the first halves of mm. 78–80 is C#–D#–E–F#–G#–A#, common to both B major and

F \sharp acoustic. The total pitch content of the second halves of mm. 78–80 is C \sharp –D \sharp –E–F \sharp –G–A \sharp , common to both B harmonic major and Octatonic Collection III. This music can therefore be analyzed using a variety of underlying scales. I favor B major for the first halves because Debussy uses diatonic scales more frequently than acoustic scales, and because I find nothing in the music that leads me to postulate a “missing B \sharp .” My hypothesis of “B harmonic major” for the second half may strike readers as more questionable, especially if they—like me—are inclined to analyze this same set class as octatonic when it is presented in mm. 24–29. I can give two reasons for my decision: first, the local context of mm. 78–80 is quite different from that of 24–29, clearly suggesting a series of scales connected to the tonic B major by maximally-smooth voice leading; second, the music of 24–29, with its distinctive half-step, whole-step melodic pattern, is more stereotypically “octatonic” than the music of 78–80. Other readers may prefer to conceive of mm. 78–80 in terms of a fleeting chromatic inflection rather than full-blown change of scale. My response is that Debussy had a distinct preference for those fleeting inflections that produce scales that obey NCS. Thus, to characterize Debussy’s practice of “fleeting inflection” it helps to think in terms of the Pressing scales. Finally, I should emphasize that my analysis of these measures represents a hypothesis about the techniques embodied in the music, rather than the way we perceive them, and that this hypothesis is supported not just by the piece in question, but by a broader investigation of Debussy’s compositional practices.

61. The prelude “Des pas sur la neige” features a similar network of scales. Here the tonic collection is D natural minor (mm. 1–4, 19), surrounded by the maximally-intersecting D dorian (mm. 5–7, 20–21), B \flat acoustic (mm. 16–18), and D harmonic minor (mm. 32–36). Other scales appear in the piece as well, including A \flat mixolydian, G \flat major, and Whole-Tone Collection II.
62. Each of the themes is, furthermore, a stack of perfect fifths: Themes β and γ are diatonic hexachords, or stacks of five perfect fifths, while Theme α is a pentatonic scale, or stack of four fifths. David Kopp’s 1997 article makes a related point, emphasizing the “pentatonic” quality of this piece’s themes.
63. Theme α could also appear in an A diatonic context. However, the A diatonic scale does not maximally intersect the tonic B major. So one should really say: “Collines” presents each of its themes in the context of every possible Pressing scale that maximally intersects the tonic collection.
64. The entrance of the D \flat on the fourth beat of the measure—which can be conceptualized as a C \sharp —suggests a retrospective reinterpretation of the earlier part of the measure in terms of the G acoustic scale. Since the notes in the first three beats of the measure belong to both scales, this reinterpretation complements, rather than supplanting, the D dorian reinterpretation.
65. Note that the fourth beat of m. 21, taken together with the first beat of m. 22, contains the five pitches common to both the G acoustic and the Whole-Tone I collection.
66. I $_{10}$ maps C diatonic to G \flat diatonic, B acoustic to G acoustic, and Whole-Tone Collection I to itself. As a result of this inversional equivalence, the progression D dorian→G acoustic raises one note by semitone, while the progression B \flat phrygian→B acoustic lowers one note by semitone. The voice leading between acoustic and whole-tone scales involves an inversionally symmetrical “merge.”

67. I use the term “dominant ninth chord” to refer to a collection rather than a tonally-functional object.
68. One can hear hints of this “missing” $A\flat$ acoustic collection on the downbeats of mm. 25 and 26.
69. Not appearing on the graph is the $G\sharp$ phrygian scale implied by mm. 35–37. This collection does not maximally intersect the other collections in the piece. This serves as a reminder that, although Debussy’s music often features networks of maximally-intersecting scales, he has no qualms about leaping between distantly-related scalar regions.
70. These are not the divisions proposed in Howat 1983, nor would I insist on their unqualified correctness. *L’Isle Joyeuse*, like *Prelude to the Afternoon of a Faun*, has an ambiguous and somewhat elusive formal structure. It is not surprising that different analysts parse it differently. Fortunately, my remarks do not depend on the precise location of section boundaries.
71. Specifically, they account for the music of mm. 1–98, 160–185, and most of 220–255. Note that this count includes the largely B major music of 36–51. Although the B major collection does not reappear in the piece, it is maximally connected to the tonic network shown in Example 22.
72. Note that a first-inversion *diatonic* triad (in close position) contains, from bottom to top, scalar intervals third-fourth-third. A close-position first-inversion *pentatonic* triad contains, from bottom to top, scalar intervals third-second-third.
73. In Tymoczko 1997, I describe how jazz and impressionism both use non-diatonic scales in tonally functional ways.
74. The C acoustic collection briefly resurfaces in 156–157, acting as a kind of “neighbor scale” to the Whole-Tone II collection. Note that if one continues to interpret the music functionally then the acoustic and whole-tone scales in mm. 145–157 have tonic function. By contrast, in jazz the acoustic and whole-tone scales almost invariably have dominant function.
75. Lockspeiser 1962: Volume 1, 206.
76. Morris 1998, 177.
77. The spelling here is $C\sharp$ – $D\sharp$ – F – G – A – $C\sharp$, rather than the more usual $D\flat$ – $E\flat$ – F – G – A – C . In discussing this passage, Richard Parks (1989) considers the C in measure 6 to be part of the whole-tone superset 7–33 [012468A], the complement of the set heard in m. 1. This strikes me as odd: the $C\sharp$ *displaces* the $B\sharp$ of mm. 1–5; it does not commingle or coexist with it. Our disagreement here testifies to a more fundamental difference in analytical approach: Parks tends to think in terms of large scalar supersets, whereas I am more inclined to postulate changes of scale. See Appendix I for further discussion.
78. We have already encountered this schema in both “Le vent dans la plaine” and *L’Isle Joyeuse*. “Mouvement,” the third in the first series of piano *Images*, provides another instance at mm. 97–114. The opening of “Fetes,” shown in Example 2, involves an interesting variant of the schema: the dorian music of mm. 1–8 moves to the $D\flat$ acoustic music of mm. 11–14, which moves to the whole-tone music of mm. 15–18. F dorian maximally intersects $D\flat$ acoustic, but the Whole-Tone Collection II of mm. 15–18 does not. Whole-Tone Collection I appears in m. 19, however.
79. Debussy and Ravel frequently harmonize a melodic scale in ways that imply other

- scales. For related passages, see Example 27, above, and Tymoczko 1997, Example 26. Satyendra 1992 uses the term “stacked space” to refer to this situation.
80. It may be helpful to contemplate how the lattice of Example 11 results from folding up the lattice of Example 29, connecting fifth-related acoustic collections by way of harmonic scales.
 81. From here on, I will revert to speaking of scales rather than modes: E \flat diatonic will stand equally for B \flat mixolydian, F dorian, and so forth.
 82. In my initial thinking about the piece, I was not at all tempted to associate the D \flat acoustic collection with the octatonic scale in m. 13. However, I now find myself intrigued by the thought that mm. 20–23 represent a hybrid of the whole-tone Theme α with the octatonic Theme β . On this interpretation, the D \flat acoustic scale is a whole-tone collection that has been “infected” with elements—specifically an [0134] tetrachord—acquired from the octatonic second theme.
 83. Note that the E lydian scale appears on the left of Example 31, suggesting motion in the flat direction from the “central collections” of the beginning. In this sense the sharp-based notation is misleading. Note also that Example 31 is globally I-symmetrical. Reflecting the graph through a vertical line containing A \flat major is equivalent to inverting each collection around E/B \flat . This operation preserves the graph’s acoustic and diatonic scales; in particular, it maps the middle section’s (distant) B major onto the closing section’s (distant) F major.
 84. See Tymoczko 1997.
 85. Liszt’s late works use a variety of “Debussian” devices: fourth chords, non-functional harmony, whole-tone scales, acoustic scales, and perhaps even the harmonic major collection. Semitonal voice leading, as Ramon Satyendra (1992, 1997) emphasizes, is a pervasive feature of the style.
 86. See Taruskin 1996, Satyendra 1992, 1997.
 87. The piece is meant to be played by multiple players in unison at rapid tempo. Performers begin by playing the first note, then the first and second, then the first, second, and third, and so forth, until the whole sequence is played completely. At that point musicians begin eliminating notes from the beginning of the sequence, playing the second through the last notes, then the third through the last, and so forth. It is expected that players will make mistakes and lose their place; Rzewski instructs them *not to try to return to the fold*. The “lost sheep” are to proceed at their own pace, creating a pandiatonic “halo” that surrounds the main line. (NB: the existence of this “halo” means that the transition from B \flat acoustic to F major will occur by way of a superimposition of notes belonging to both scales.)
 88. Allen Forte has also developed a theory of “genera” that is quite different from Parks’s—and much farther from the spirit of the current investigation. Unlike Parks, whose theory is principally motivated by analytical considerations relating to Debussy’s music, Forte aims to provide a fully general taxonomy of pitch-class sets, not bound to the procedures associated with any one composer. A fuller comparison of Parks’s and Forte’s theories can be found in Parks 1998.
 89. Parks 1989, 59.
 90. Parks (1998, 207) claims that the chromatic genus is a “simple genus,” in the sense that it consists of all the subsets and supersets of a single “cynosural” set. Elsewhere, Parks (1989, 74) explicitly contradicts this statement. I do not see how to resolve the discrepancy.
 91. See Lewin 1996. Moreover, any incomplete interval cycle will give rise to an anal-

- ogous scale lattice whose elements are connected by single-voice (but not necessarily single-step) voice leading.
92. Any maximally even set that is not transpositionally symmetrical can be written as an ordered series whose successive elements are separated by the interval c , and the last element is separated from the first by the interval $c + i$, with $i \neq \pm 1$. For example, the diatonic scale can be represented as the series (F, C, G, D, A, E, B), with $c = 7$ and $i = -1$. The voice leading motion $\alpha_1 \rightarrow \beta_1$ displaces the first element in the pitch class series by $-i$, producing a series whose successive elements are separated by the intervals $(c + i, c, c, \dots, c)$, with the first element now separated from the last by interval c . Similarly, $\alpha_2 \rightarrow \beta_2$ displaces the second element by $-i$, producing a set with interval series $(c, c + i, c, \dots, c)$, and $\alpha_3 \rightarrow \beta_3$ displaces the third element by $-i$, producing the interval series $(c, c, c + i, \dots, c)$. It follows that applying $\alpha_2 \rightarrow \beta_2$ to the initial element in the cycle produces the series $(c - i, c + i, c, \dots, c)$. Since the interval from the last element to the first is $c + i$, this element is inversionally symmetrical. It is the analogue of the acoustic scale on Exx. 11–12. Similar reasoning can be used to derive analogues of the harmonic minor and harmonic major scales, and to show that they will be inversionally related. One needs to make slight adjustments to the argument when the underlying chords have only three notes, but the essential idea is the same.
 93. See Douthett and Steinbach 1998. “Cube Dance” contains within it the familiar triadic *Tonnetz*. In some ways, however, “Cube Dance” is a far better model of voice-leading relationships than the *Tonnetz*: on “Cube Dance,” the F minor triad is closer to the C major triad than the F major triad, whereas this is not true on the *Tonnetz*. Thus “Cube Dance” can be used to represent the fact that the F minor triad can appear as a passing chord between F major and C major.
 94. This graph subsumes Douthett and Steinbach’s “Power Towers,” but is not equivalent to it. It can be obtained from “Power Towers” by incorporating the six “French Sixth” chords omitted from that graph. Jack Douthett informs me that he and Peter Steinbach omitted the “French Sixth” from “Power Towers” merely for orthographic reasons.
 95. See <http://music.princeton.edu/~dmitri>.
 96. See <http://music.princeton.edu/~dmitri/ChordGeometries.html>.
 97. As David Rappaport points out, this multiset possesses the DT property. If one includes it on the scale lattice of Example 11, the resulting structure is locally isomorphic to the graph of single-step voice-leading relationships between diatonic trichords.
 98. The neo-Riemannian P (parallel) voice leading holds constant the root and fifth of a (major or minor) triad, while changing the quality of its third. The R (relative) voice leading holds constant the major third of a triad, while moving the remaining note by two semitones to form another major or minor triad. The L (leading-tone exchange) voice leading is similar, though it holds fixed the minor third.
 99. Note that although these scales are far apart on the “circle of non-diatonic scales,” they are not far apart on Example 11 itself. For instance, A harmonic major can move to F# harmonic minor by way of an intervening A diatonic collection. This two-step path is shown on Example 11.

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