Problem Set 1, Unofficial Solution Key

Chapters 1 & 2

Friday, 1st of January 2016

Problem 1. What is the principal difference, if any, between mathematical economics and econometrics?

Mathematical economics is deductive, aiming to establish logical conclusions from axioms, whereas econometrics is inductive, as it takes empirical data for its starting point.

For questions 2-4, use the given sets: $S_1 = \{1, 2, 3, 4\}$, $S_2 = \{2, 3, 5\}$, $S_3 = \{7, 8, 9\}$, $S_4 = \{1, 2, 4\}$.

Problem 2(a). What, if any, of the given sets are equal?

None are equal. This can be seen by taking any two sets and noticing that in every pair at least one set contains an element the other set does not contain (e.g. $S_1 \neq S_2$ because $1 \in S_1$ but $1 \notin S_2$).

Problem 2(b). What, if any, of the given sets are a subset of another?

$S_4 \subset S_1$

Problem 3. Use S_2, S_3, and S_4 to verify the distributive law.

$S_2 \cup (S_3 \cap S_4) = \{2, 3, 5\} \cup (\{7, 8, 9\} \cap \{1, 2, 4\}) = \{2, 3, 5\} \cup \emptyset = \{2, 3, 5\}$

$(S_2 \cup S_3) \cap (S_2 \cup S_3) = (\{2, 3, 5\} \cup \{7, 8, 9\}) \cap (\{2, 3, 5\} \cup \{1, 2, 4\}) = \{2, 3, 5, 7, 8, 9\} \cap \{1, 2, 3, 4, 5\} = \{2, 3, 5\}$

$S_2 \cap (S_3 \cup S_4) = 2, 3, 5 \cap (\{7, 8, 9\} \cup \{1, 2, 4\}) = \{2, 3, 5\} \cap \{1, 2, 4, 7, 8, 9\} = \{2\}$

$(S_2 \cap S_3) \cup (S_2 \cap S_3) = (\{2, 3, 5\} \cap \{7, 8, 9\}) \cup (\{2, 3, 5\} \cap \{1, 2, 4\}) = \emptyset \cup \{2\} = \{2\}$

Problem 4. How many subsets are within S_1?

Since S_1 has 4 elements, there are $2^4 = 16$ subsets of S_1.

Problem 5. Find the Cartesian product of $A = \{a, b\}$ and $B = \{c, d\}$.

$A \times B = \{(a, c), (a, d), (b, c), (b, d)\}$

Problem 6. Find the range of $y = 2x + 3$ on the domain $\{x : 0 < x < 3\}$. Write your answer in set notation.

The range is $\{y : y \in (3, 9)\}$.

Problem 7. In the theory of the firm, we say that cost (C) is a function of output (Q). Consider the function $C = 75 + 10Q$. If C represents TOTAL cost, what do 75 and $10Q$ represent, respectively? Explain your reasoning.

75 is the fixed cost, as it is a cost independent of the amount of output produced. $10Q$ is the variable cost, incurred as a proportion of the amount of output.

Problem 8. Graph the function given in question 6. Find the x and y intercepts. Are the intercepts in the given domain and its range?

Attached as Figure 1.

To find the y-intercept we set $x = 0$ and solve for y. Hence, the y-intercept is $(0, 3)$. Similarly, to find the x-intercept we set $y = 0$ and solve for x. The only x that solves the resulting equation is -1.5. Thus, the x-intercept is $(-1.5, 0)$.

Neither intercept is in the given domain or range, as neither -1.5 nor 0 are in the interval $(0, 3)$, nor are 0 or 3 in the interval $(3, 9)$.

Problem 9. Consider a quadratic function of the form $f(x) = a_0 + a_1x + a_2x^2$. Consider the sign of the leading coefficient, a_2. How might the shape of $f(x)$ change if the sign was negative rather than positive? Justify your answer.

If $a_2 < 0$, $f(x)$ would have the shape of a hill, increasing until it hit its "summit", after which it would decrease. If $a_2 > 0$, $f(x)$ would have its shape flipped vertically; decreasing first and then increasing.

Problem 10. Use the rules of exponents to condense the following expressions:

(a) $\dfrac{x^2}{x^{-2}} = x^{2-(-2)} = x^4$

(b) $\left(x^{1/3}y^{1/3}z^{1/3}\right)^5 = x^{5/3}y^{5/3}z^{5/3} = (xyz)^{5/3}$

(c) $\sqrt[n]{x^m} = x^{m/n}$