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### MATH ANXIETY

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**H**ere is a math problem, a puzzle. You remove the plastic wrap, and open the lid. Pieces swim before you: amorphous blobs that overlap and stick to each other. As you begin to fit uncooperative pieces together, contours form, edges rigidify into straight lines or complex curves, knots appear and twist the goo of a piece into an unmanageable tangle, or a perfect sphere, or a Julia set. Pieces break apart, generate spontaneously, cover and eat each other. As pieces are brought into proximity, they take on not only contours or shapes, but also colors and patterns on their surfaces. Sometimes pieces “match,” sometimes they clash, sometimes you can’t tell. This makes the puzzle significantly more complex (?), and the pieces even harder to place; an exact fit is out of the question. When one solution loses its promise, you shuffle pieces and try others. In every solution, some pieces fit better than others, and some don’t fit at all. It needn’t be emphasized that this puzzle has no correct solution; it determines itself only as it is being fit together.

Such a puzzle might sound rather boring; no right answers, no logic, no *sense*; just pieces doing as they will. But look closer. Wherever you look, the pieces tangle, meld, and shift in relation to each other. Not just shape, but everything about a piece is in flux, from color to weight, strength to poverty, what it ate for breakfast to how it deals with gravity. This flux is not a chaos, far from it, for in chaos there can be no issue of color or weight, strength or poverty. Chaos does not admit of any sense, whereas this puzzle makes lots of sense, it generates sense as its pieces act and react in relation to each other. Color and weight, affect and defect are sensible, and the way that these sensible notions fit together is how sense is made.

How would you work on this puzzle? You already are working on it. I mean, you have problems. Doesn’t everyone? And problems are just

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this kind of puzzle; they demand to be solved, but one cannot say in advance what pieces may be brought to bear in response to this demand or how the problem will develop as one responds to it. Confronted with a problem, one draws on bits of history, geography, biology, physics, psychology, rhetoric, aesthetics, etc. Not just anything will make sense in relation to a problem, but one cannot foretell what might end up making sense. When your boyfriend threatens to call your wife and tell her “everything,” you’ve got a problem, and who knows what constellations this situation will lead to? When you have an hour to kill and aren’t sure what to do, this is a problem; problems may even be welcome, but pleasant or unpleasant, whatever the outcome, the problem calls for something, demands something not-yet-determined of you and of the world. I mean to emphasize here the complex and open nature of problems, the way they do not break down

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cooperatively, dissolve into clear and distinct elements. They drag otherwise unrelated people, places, and objects into the mix. Use your shoe for a pillow. Freeze your compact discs. Hire a philosopher to reorganize your company. Maybe dogs are good at sexing chicks. Your problem happens as solutions are attempted and discarded, feeding into the problem to alter its parameters; new obstacles and new paths may crop up at any time, any place. Think laterally, act locally.

When rain drips onto a leaf, the paths it takes across that uneven surface are solutions to a complex problem at the juncture of differential topology, fluid dynamics, evolutionary biology, and metaphysics. If this problem is a puzzle, then who is the puzzler?; the pieces – water, leaf, time, gravity, inertia, happenstance, etc. – are the only agents. As such, the problem becomes clear only in the course of its solution; one recognizes the problem and its boundaries after the fact, or as it is working itself out. The problem makes sense of its solutions, and these solutions make sense only when one grasps their unity in the problem.

I am still discussing puzzles, but these puzzle pieces do not come shrink-wrapped in a box. Instead, you find them all around you, wherever there is a tension, wherever something must be done, wherever things are created, destroyed, altered, augmented, combined, and juxtaposed. Anywhere that sense is made, it is made by a problem working itself through, a puzzle that demands to be worked and re-worked, and whose pieces cannot but respond to that demand. This puzzle is *ontogenesis*, the way things become what they become. At the beginning of Chapter Four of *Difference and Repetition*,<sup>1</sup> Gilles Deleuze offers an account of ontogenesis as a problematic puzzle, pieces that pose and respond to problems, acting and interacting to determine the relations that make sense of them or make them sensible, give them character. The problem, in working itself out, determines both the elements that figure in its solutions and the (sensible) qualities of those elements. To become is to become sensible. The dominant history of Western metaphysics holds that things first of all come to be, only later to assume the relationships

that give them meaning in the world. Sense is thus produced after the fact, and ontogenesis describes the generation of objects but not their relations. This standard metaphysics often explains genesis as a spontaneous modal flip (from possible to actual), and sense as an extraneous imposition of a human perspective on a fully formed pre-existent object. By contrast, Deleuze's account holds that genesis is not a sudden change of modes, but a process of determination, beginning not from a previous determined state (the possible), but from the undetermined. This process is a puzzle being fit together, an insistent demand or pressure. To understand why things are the way they are, we look at what problems they respond to, what forces create just these tensions, conflicts, and congruencies. Things make sense when connected to their problematic origins.

The problem unifies its cases of solution, not because each element of the solution is interchangeable, not because each part of the solution responds to the problem in the same way, but because the entire solution responds to the problem together. The parts of a machine do not each do the same thing, but they are unified by virtue of the machine they constitute. The problem is a machine, a function or field, and this field orders and skews the objects to which it gives rise. The problem thus creates a kind of continuity, a continuity that connects its cases of solution in sense and sensibility, but the various parts of the solution are heterogeneous, each playing its own role in relation to the problem and to the other elements of the solution. Even to talk of solutions here is misleading, for the most problematic problems do not disappear in their solutions. Rather, the so-called solution is only that moment when the problem reaches its greatest degree of clarity, when the forces that constitute the problem ally themselves clearly on one side or another. If we consider only this ultimatum, this static snapshot of the problem, the problem disappears. If instead we focus on the process by which the problem comes to be, then we survey not only the elements of the problem, but the determination of their relations, the generation of their sense in what Deleuze refers to as the problematic Idea.

Why does he call this puzzle – that ties together its elements into the continuity of a sensible solution – an “Idea”? Platonic Ideas gather a number of objects under a common term, such as “beautiful” or “virtuous.” The participants in an Idea share that Idea in common, though there is nothing else in particular that they need share. That is, participation in the Idea is irreducible. Still, participation in an Idea is not haphazard, for it makes sense: the set of participants in an Idea forms a sensible continuum. For example, the Idea of human phenotype is irreducible: there are no necessary and sufficient elements that make a body human. But there are plenty of characteristics of human bodies, and these characteristics vary continuously—height, density, skin color, eye color, shape of head, etc. The Idea of the body yields a continuity, a mass of individual bodies that range continuously in a variety of ways.

Kant, as well, uses Ideas to make sense of things, to determine continuities, but unlike Plato, he does not begin with ready-made things which must be parceled out, sorted into the correct Ideas. Kant makes a great leap when he focuses on the *process* by which continuities form. The understanding makes sense of the world by fitting it into concepts and their relations. Concepts alone are inadequate to make sense of anything; they form isolated cases that offer no reason for their existence. Only by relating concepts in the problematic Idea do they bear relations that make sense of them, for they become not isolated instances but cases of solution, responses to a problem that determines their histories and gives them a sufficient reason. Whereas concepts are determinate things, comprising specific qualities, the manifold of intuition that we must make sense of is wholly indeterminate, a vague jumble with no particular characteristics, nothing accessible to the understanding. The process by which the world is divided into concepts is thus a process of determination, from the undetermined to the determinate or from manifold to concept.

In Kant, the Idea still gives rise to continuity, but it is no longer, as in Plato, the continuity of a common quality. The complex continuity generated by the problematic Idea brings things

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together meaningfully as a means of working out a problem. One cannot say in advance what will take shape, which forces will be significant, which events will stand out, who will play a pivotal role. Guests arrive at a cocktail party with all sorts of history, tendencies, characters, but with no set plan. The situation itself is a problem. What is going to happen? Whom will Ed talk to? Is it going to get political again? Are there enough hors d'oeuvres? And if the host poses these problems to herself, she may be mostly ineffectual as the problem of the party will undoubtedly find its own solution, agreeable or otherwise. Over the course of the evening, the guests permute their arrangements according to the problematic dynamic determined by their singular personalities, their prior relationships, the availability of alcohol, the style of music and locations of the loudspeakers, and who-knows-what else. The result is a complex continuity of movement that constitutes the only sense of the cocktail party and is unified only by the problem that inheres in the color of each tie, the strength of each drink, and the wit in every quip.

The problematic Idea, says Deleuze, provides a “systematic unity” (169) to its cases of solution; but the unity that the puzzle gives to its pieces is never final nor is the fit ever exact. Instead of a single big picture on its surface, the puzzle is always broken by unaligned edges and sudden changes of color from piece to piece. That is why the usual methods of investigating continuity are flawed. One attempts to start with the completed puzzle and work backwards, taking it apart into its pieces to determine what it’s made of and how it’s put together. But this method is doomed to fail, for breaking the puzzle into pieces yields only smaller puzzles but hides the process of determination by which those pieces take shape. The philosophical investigation of sense generally starts with one determined state of affairs and attempts to trace it back to another determined state of affairs whence it came, searching for the reasons for things among those very things. Ontogenesis is thought to proceed from determination to determination, thus ignoring the process by which things become determined. If we start with the continuous number line made of points, we can break that line down into

Fig. 1. A sequence of points,  $s_0, s_1$ , etc., that approaches the point  $P$ . Note that no matter how small an interval we consider around  $P$ , at some point in the sequence of  $s_n$ , all further points in the sequence will be contained in that interval.

smaller and smaller continuous pieces, but we will never discover what makes the line or those pieces continuous, no matter how small the parts get. Working backwards from continuity to its cause will not succeed. Instead, we must discover an ideal cause of continuity, and examine how that cause can generate the continuity alongside the continuous things. We must acknowledge that things and their senses come to be simultaneously and interdependently. As that ideal cause of continuity, the problematic Idea that draws things together into sense, Deleuze proposes the mathematical differential,  $dx$ :

I did mention at the outset that this was a math problem. A math problem, even a calculus problem, seems at first to be a rather poor example of a problem. The math problems one finds at the end of the chapter in math textbooks just sit there, seemingly indifferent to whether or not they are solved. (No doubt, because they have already been solved.) Indeed, most of us feel little compulsion to attempt a solution. They are not very problematic problems. Fermat's Last Theorem, on the other hand, certainly was a problem: it begged to be solved.<sup>2</sup> Deleuze discovers in archaic interpretations of the differential a problematic power, the power to problematize.

How does the differential,  $dx$ , cause problems? In the modern calculus, the differential is ill-suited to be the ideal cause of continuity, because it is understood on a finitist model, or, at best, as an infinitesimal. In other words, the modern interpretation of the differential makes it out to be an arbitrarily small quantity. In modern calculus, when we use the differential (say, to prove something or to take a derivative), what we do is to demonstrate that something-or-other is true for an arbitrarily small quantity. Usually, it is a question of intervals: a line "approaches" a point when, no matter how tiny an interval around the point you take, it still encloses the end of the line. That line, we reason, must get arbitrarily

close to the center of the interval, since no matter how small we make the interval, the tail of the line is still contained in it. The fundamental concept of calculus, the concept of the limit, is defined in just such finitist terms. A sequence of terms is said to approach a limit when it can be shown that no matter how small an interval you choose around that limit, there is some point in the sequence after which every term is contained within that interval (Figure 1). So, in proving something about limit points, we only ever consider intervals of finitely small size, for one can of course always find a smaller interval. Likewise, in proofs using the differential, we never actually involve the infinite; rather, we use variables, which are manipulated as arbitrarily small (but still finitely small) numbers. The important point here is that as long as our symbolism reduces the limit, and by reference the differential as well, to finite quantities and variables that represent such quantities, the limit and the differential become just more quantities, a finite value that is special only in that it is unspecified or unknown. Without the power of problem, the differential loses its unique dynamic, and we lose sight of its unique powers.

The limit is Lesson One in a modern calculus class. This lesson teaches the mathematical formalisms that correspond to the notion of "arbitrarily small." So, one learns which symbols to use and what rules govern the manipulation of those symbols, or what constitutes legitimate proof in the domain of the arbitrarily small. Lesson Two makes use of Lesson One to show how to take a derivative. Though we are introduced to it by means of the limit or differential at its core, the derivative is no sooner learned than the differential is discarded, and even its symbol,  $dx$ , is often omitted. Calculating derivatives becomes a matter of the rote manipulation of symbols, and the intimate relationship between the derivative and the differential is

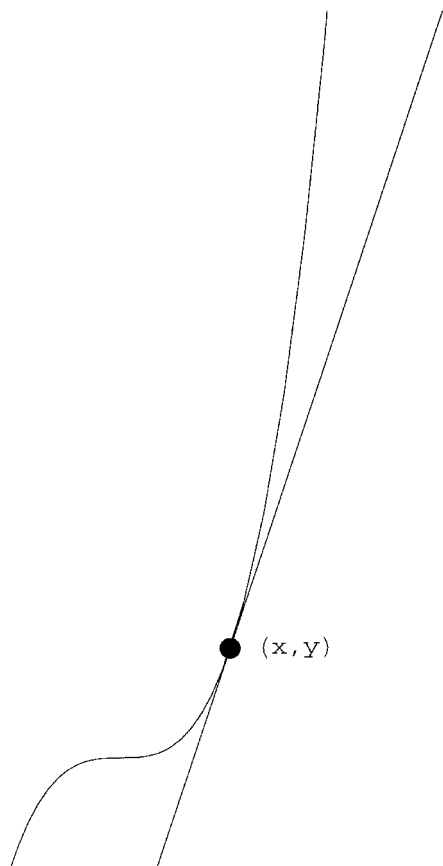


Fig. 2. A curve and the line tangent to it at the point  $(x, y)$ .

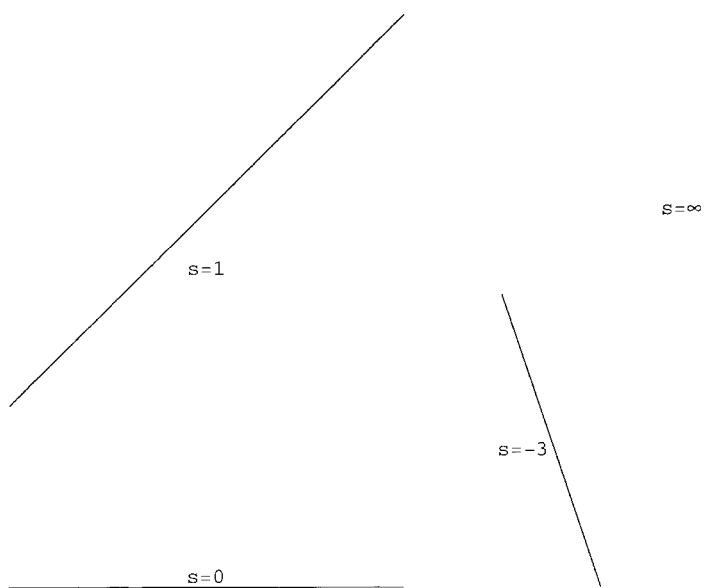


Fig. 3. Four line segments, with their slopes indicated.

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forgotten, a distant murky memory that won't be on the exam. If the differential still appears in certain formulas, it is only an annoying vestige of the higher mathematics that birthed it.

So, let me remind you of the relationship between limits and derivatives. Derivatives are first introduced by posing this problem: given a curve that represents some function, called the *primitive function*, and a point on that curve, what is the slope of the tangent to the curve at that point? (See Figure 2.) Well, why do we care? First, we must define some terms (and these are paraphrases with attendant inaccuracies). The *tangent* at a given point is the line that touches the curve at that point without crossing the curve. The *slope* of the tangent is how steep it is, mathematically a matter of the relation between the change in the vertical direction and the change in the horizontal, or the increase in  $y$  divided by the increase in  $x$ , or "the rise over the run" (see Figure 3).

What is the significance of the slope of the tangent to the primitive function at a given point? It tells us the slope of the curve at that point. And the slope of a curve at a point is how fast it is changing there, how quickly the curve is "moving" up or down. A steep tangent means a steep curve at that point, one that is rising or

falling quickly. A flat tangent means a curve that is not moving up or down at all at that point. If the curve represents distance over time, then the slope of the tangent at a point tells us the velocity at that time. If the curve represents population over time, then the slope tells us the growth rate. And if the curve represents the elevation of land not over time but over some area, then the slope of the tangent represents the slope of the land at any given point. In all, it is quite a useful calculation to make.

But finding the slope of the line tangent to a curve turns out not to be such a simple matter. Since the tangent touches the curve without crossing it (nearly), it only touches it in one point (locally). But it takes two points to define a line and determine its characteristics such as slope. Given two points, it is easy to find the slope of the line that runs through them: it is just the difference between the  $y$  values divided by the difference between the  $x$  values. If the two points are  $(x_1, y_1)$  and  $(x_2, y_2)$ , then the slope of the line they define is  $(y_2 - y_1) / (x_2 - x_1)$  (see Figure 4).

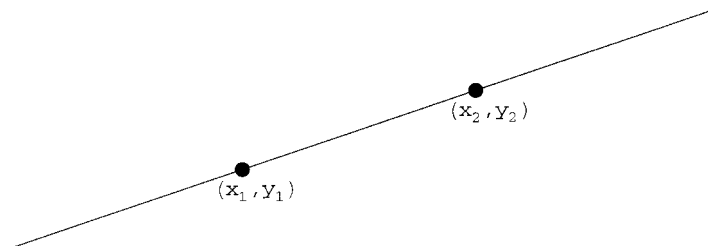


Fig. 4. The slope of this line is  $(y_2 - y_1) / (x_2 - x_1)$ .

Since we know the function, and can calculate it at any point, we know at least one point on the line in question, the point of tangency; but how do we calculate the slope with only one point specified? The method is to choose a second point on the curve some distance away, and calculate the slope of the line that runs through both of those points. This yields an approximation of the slope of the tangent. The closer the second point is to the point of tangency, the closer will be the approximation of the slope of the tangent.<sup>3</sup> To find the slope of the tangent at a point, we must take the limit – as the “other” point approaches the point of tangency – of the slope of the line between the two points. Instead of  $x_1$  and  $x_2$ , we can represent the two  $x$  coordinates as  $x$  (the  $x$ -value of the point of tangency) and  $x + \Delta x$  (the  $x$ -value of a point  $\Delta x$  away from  $x$ ), and allow  $\Delta x$  to approach 0. The two points are  $(x, f(x))$  and  $(x + \Delta x, f(x + \Delta x))$ . The slope is the difference between the  $y$  values ( $f(x)$  and  $f(x + \Delta x)$ ) divided by the difference between the  $x$  values ( $x$  and  $x + \Delta x$ ). Taking the limit,

$$\lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x + \Delta x)}{x - (x + \Delta x)}$$

which simplifies to

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provides the general formula for the derivative of  $f(x)$  (see Figure 5).

Here,  $\Delta x$  represents a finitely small quantity, a finite and measurable distance, that is allowed to approach zero. So it comes as no surprise that the differential,  $dx$ , which is defined as  $\Delta x$  “on its way to 0,” should also be regarded in modern calculus in finite terms. In school, we begin with discrete whole numbers: 0, 1, 2, 3, and so on. Eventually, we fill in the gaps between these numbers using ratios of them to produce rational numbers (fractions). Then, we fill gaps between rationals by taking sequences of rational numbers such that the numbers in the sequence get closer and closer together – in other words,

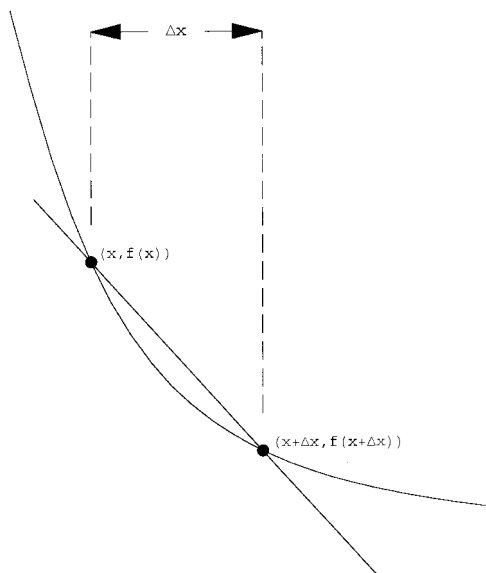


Fig. 5. Finding the slope of the tangent: a line drawn between two points on a curve. As  $\Delta x$  gets smaller, the lower point gets closer to the upper point, and the line gets closer to being tangent. The formula for the derivative is the slope of this line as  $\Delta x$  approaches 0:

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

we take a limit – and this produces irrational numbers, completely filling out the number line. Finally, this concept of an arbitrarily small interval, or limit, is applied not just to the numbers but to their relations, *functions*, where it becomes the differential. Instead of producing the differential from the number line in this manner by an operation on arbitrarily small intervals, Deleuze reverses the usual priority to start with the differential and to generate number from it. That is, he places the differential at the origin of number, as the power of difference that deviates from itself to generate the entire number line and eventually the points that populate it. The differential is not a finite distance, not even an ever smaller one. If the differential is an infinitesimal, it is a substantive and positive infinitesimal; not a finite approach toward zero as modern calculus would have it, but a movement of zero away from itself. The differential,  $dx$ , is a torsion that inflects the point,  $x$ . It is the instantaneous velocity of  $x$ ;  $x$  being only the apparent point of departure for a movement that precedes any departure.

Though we can make some sense of  $x$  by itself, as a variable that represents any of certain specifiable values, we can make almost no sense of  $dx$  by itself. Inasmuch as it is a movement, it precedes those coordinates ( $x$  and  $y$ ) that eventually measure it, an arrow set against the void. As Deleuze says, on its own it is “strictly nothing” (171). It obtains a value only in relation.  $dx$ , though undetermined, points toward its own determination in relation to another differential,  $dy$ . The above formula for the derivative necessarily involves a comparison, change in  $y$  versus change in  $x$ . Differential calculus is keenly interested in this relation,  $dy/dx$ ; the differential rise over the differential run. The terms of this relation are neither constants nor variables, for  $dx$  and  $dy$  mean nothing outside of their relation. They determine their relation reciprocally, determining each other in relation as they determine the primitive function that relates  $x$  to  $y$ .

Deleuze’s reversal of the priority of the differential and the finite numbers that normally define it thus carries over from Lesson One to Lesson Two. It is not just that the limit itself

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precedes numbers and intervals of finitely small size, but the differential relation,  $dy/dx$ , precedes the “primitive” function whose slope it is said to represent. In calculus class we are presented with a function and told to differentiate it, to take the derivative or produce the differential relation. In Deleuze’s rereading of the calculus, the primitive function does not precede the differential relation, but is only the ultimate result or byproduct of the progressive determination of that relation. The differential is a problem, and its solution leads to the primitive function.

We have thus far been considering just one point at a time, though it could have been any point. But inasmuch as it generates the primitive function, the differential relation characterizes not only one point but a whole range of points, an entire function. It is not a relation of variables, where one is considered independent ( $x$ ), and the other dependent ( $y$ ). The differentials,  $dx$  and  $dy$ , do not take on single values nor do they vary over a range of values; they are neither particular nor general, but universal, inasmuch as their relation establishes an irreducible character that is not constant, but incorporates multiplicity.

In other words, the differential relation is not a formula that relates  $x$  to  $y$  over some range of values for  $x$ , though this is how we are taught to interpret it: in school, the differential relation, or derivative, is just another formula, another function akin to the primitive function. Rather, the differential relation relates  $x$  to  $y$  not in breadth, over a range of values, but in depth; it operates in each point on the function, condensing the quality, the character of the entire function into every point. If you are given the value of some differentiable function at a point, then that’s almost all you know about the function. You have little idea how the function behaves at any other point; you know the function only at that one point. However, the existence of a derivative means that you can take a limit at that point, and in order to take a limit the function must be relatively smooth, not broken up or wildly chaotic. The function does not jump around *too* much in the vicinity of that point. Which means that near the point whose value you know, the function stays relatively close to that value. If you are told that the differentiable function passes through

the point  $(3,5)$ , then you know that as long as  $x$  is close to 3,  $y$  will be close to 5.<sup>4</sup> You can approximate the function near that one point: since it stays close to 5 near 3, it must look *something like* a horizontal line passing through  $(3,5)$ . However, if you also are given the slope of the tangent to the function at that point, then you know how quickly and in what direction the function is moving near that point. Let's say the slope at the point  $(3,5)$  is steep, maybe 100. You can now take a better guess at the values of nearby points. Specifically, you can assume that, at least near that one point, the function looks like a steep line passing through  $(3,5)$  with a slope of 100. If you also know how fast the slope is changing at that point – the slope of the slope or the value of the second derivative – then you can make an even better estimate of the behavior of the function near that point. The function may be steep, but is it getting steeper, or more shallow (at that point)? The second derivative answers this question, and gives you an even better idea of the shape of the function near the one point you know. Third derivative: the function is steep and getting steeper, but how quickly is it getting steeper? Etc.

Note that your increasingly refined estimates of the behavior of the function do not rely on the formula for the derivative nor on any formula, but only on the actual values of the derivatives. You are given one point on a function and a sequence of numbers representing the values of the derivatives of the function at that point, and from these numbers, you can reconstruct an approximation of the whole function not just at that one point, but also in an area around that point. Each successive (value of the) derivative at that point provides a more accurate sense of the shape of the whole function. This process of successive approximation is not merely heuristic; in fact, if you know the values of all the derivatives of a function at a given point, you can construct a polynomial, another function, that is equivalent to the primitive function near that point. This is the sense in which the differential is a universal: the differential packs into each point the nature of the entire function, for the differential relation generates not the *value* of the function, but its *behavior*, its character, what the

function is doing at each point. It's not that the differential relation represents the slope of the function at each point; it's that by representing the slope of the function at each point, the differential relation presents or characterizes the whole function in each of its points.

The differential relation captures the universal character of the whole function, and this overall character or shape of the function is a matter of how many times it changes direction, how many bumps it has and how regularly they occur, how often it becomes infinite, and how often it crosses the  $x$ -axis. These criteria divide the function into pieces, whose endpoints are defined by the critical points that mark the changes in direction, the local maxima and minima, the zero-crossings, the approach to infinity (Figure 6). Deleuze refers to these critical or distinctive points as singular points, for their number and relations uniquely determine the singular character of the function itself. They are points of articulation where the function alters its behavior: the function gets steeper to the left of a singular point and shallower to the right; or it's positive on one side and negative on the other; or it's curvy and then suddenly flat. These singular points break the function into parts and so determine the type or species of function in question. We can judge the kind of function according to the number and arrangement of its distinctive points. Trigonometric functions tend to have an infinite number of distinctive points with a certain regular recurrence. (A sine wave has an infinite number of peaks and valleys, an infinity of maxima and minima.) Polynomial functions have a finite number of distinctive points, one fewer than the degree of the polynomial. (Think of the parabola, a binomial, with its single minimum or maximum.) Capturing the character of the primitive function, the differential also holds the key to the singular points, for where the primitive function has a peak, its tangent will be horizontal, so its slope will be zero; and where it approaches infinity, its slope too will become infinite (see Figure 6). By examining the number and distribution of zeros and infinities in the differential relation, we can determine the species of primitive function.

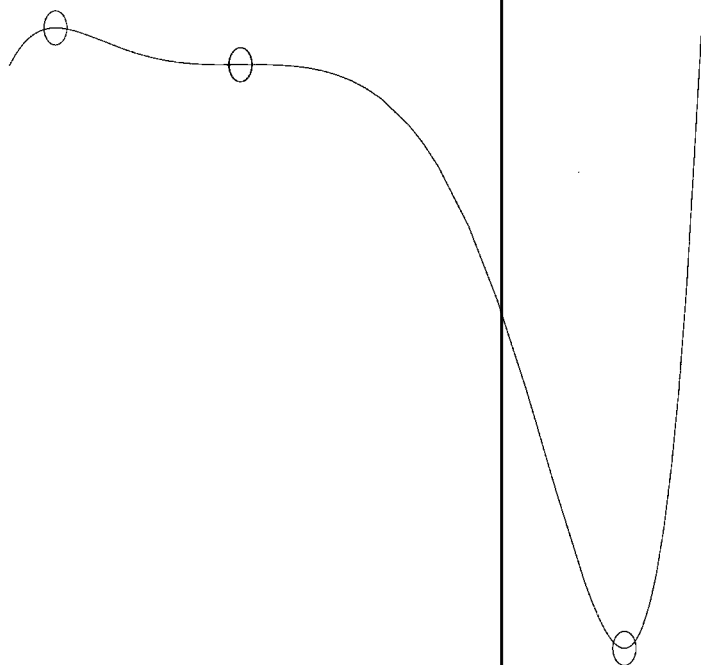


Fig. 6. Each of the circles surrounds a singular or distinctive point, where the curve has a local maximum or minimum. Note that the tangent at each of the circled points is flat, so its slope equals 0.

The differential relation,  $dy/dx$ , applies to each point and also to the universal character of the entire function. Thus, the reciprocal determination of  $dx$  and  $dy$  in each point is simultaneously a reciprocal determination of the character of the whole function. Reciprocal determination occurs at once point by point and part by part, determining the shape of the whole function as it determines the points that constitute it. The function thus takes shape gradually, progressively, as the singular points shift and glide relative to each other, tense and relax to alter their configuration. A problem forms like a soap bubble stretched across the wire outline of an abstract geometric figure. How to connect the vertices most efficiently, how to find the correct degree of curvature, how to distribute density so as to bend without breaking? And when a weakness is stretched beyond its breaking point, the bubble snaps into a new shape, determining new criteria, new boundary conditions, posing and solving a new problem in a flash, so that one would never suspect the whole network of differential calcula-

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tions that take place in this instant. Problems determine themselves incrementally and always in relation; the pieces of the problem don't fall into place until they are already solved, hesitating, negotiating, calculating in a game of strategy and diplomacy with goals to be determined as the game is played. Problems do not come out of nowhere, though it may sometimes seem so, as they collapse, in an instant, the gap between the vague dread of reciprocal determination and the abject terror of complete determination.

Consider the increasingly successful attempts to harness the differential power of the problem in quantum computers. Classical computers deal only with pseudo-problems, problems already solved, since the conditions of the problem and the number of possible solutions are strictly determined in advance, and always boil down to a choice among two alternatives. There is no problem in the problems tackled by classical computers; we can't even pose a real problem to such a computer, much less arrive at a solution. Quantum computers, on the other hand, survey not one value but a range at a time, allowing each "variable" not only to vary but to statistically inhabit multiple and infinitesimally distinct values at once. A physical quantum bit can be 0, or 1, or in between, or something else altogether, since the bit is in fact a wave, and the differences between one value and another are not immaculate, but shifting and subtle. Of course, the equations that describe the value of the quantum bit (qubit) are filled with differentials, and these do not disappear until the wave collapses into its final determination. Quantum bits do not collapse into specific values until they have solved the problem, until they find a mutually acceptable solution that deter-

mines not only the final outcome but also the nature of the problem itself. Qubits can be entangled, such that they only operate conjointly, posing a differential and statistical problem of reciprocal determination. Quantum computers are thus unlike their classical cousins, which begin and end with already determined possibilities for individual bits. They pose and solve problems more like artists, who must generate the right stroke or choose the right materials or invent the right sound that will advance the composition. (What artist would not be tortured? They deal exclusively with problems, problems of determination.)

I discussed earlier how the values of the derivatives for a function at a given point allow the construction of a polynomial that is equivalent to that function near that point. Specifically, these values are used to determine the coefficients of a polynomial called a power series or Taylor series.<sup>5</sup> Incidentally, we are probably now up to Lesson Fifteen or Twenty in first-year calculus. Given a differentiable function, by taking derivatives of that function at a particular point, much as I described earlier, we can produce a power series representation of the function. The point at which the derivatives are taken is called the *center* of the power series. For example, the power series for  $\sin x$  centered at 0 is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$$

Again, Deleuze reverses the usual priority. Whereas we normally begin with the primitive function and generate the power series, he regards the power series as a way of constructing the primitive function. Since the power series is itself derived from the differential relation, it is the differential which is powerful, and exercises its power to generate the primitive function, the sensible solution. Sine and cosine, logarithm and exponent become just a convenient shorthand for

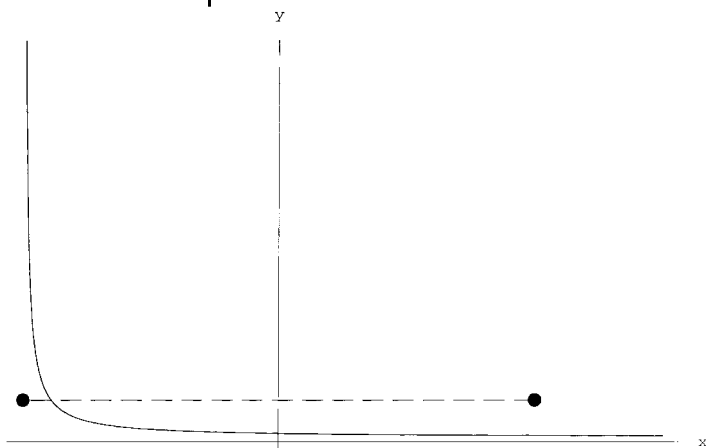


Fig. 7. The curve is a graph of part of the function,  $1/(1+x)$ . The dashed line shows the domain of convergence of a power series for this function centered at 0. Outside of this domain, the power series is undefined, so we would require more than one power series to approximate the entire function.

the series of infinite order polynomials they represent. But there is a problem with power series: they only represent the primitive function in some interval, possibly a very small one, around their center point. They are only guaranteed to represent part of a function. Far from its center, a power series may no longer be equal to the primitive function, and it may even become undefined. Singular points divide a function into parts, and each part can have a different power series to represent it. Deleuze thus raises this question: for any particular function, can it be constructed from just one power series over its entire domain, or must it be built in pieces from multiple power series (see Figure 7)?

Much rides on this question of the number of power series. If only one is required, then the entire function is built according to one overriding principle, and there is a smooth and unbroken path from any point on the function to any other point. A function can be regarded as a rule for transformation, and a single universal power series guarantees that this transformation will carry any point on the function to any other with no complications, no convolutions. But if any point can be smoothly transformed into any other, then we have eliminated radical difference. Deleuze opposes himself here to Leibniz, who effectively builds his differential calculus and his

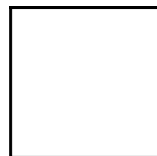
monadic philosophy one out of the other. Leibniz insists on smooth transformation, for only then do the differences among monads boil down to mere shifts of perspective, reconcilable by a continuous movement without a hitch. Every argument is resolvable, every point of view comprehensible in terms of every other, though vast differences might be crossed before debating parties see eye to eye. The entire function, the entire universe is summarized, captured in a single, overarching power series; and this one power series, that covers the whole of existence and encompasses every point of view, is God himself, God as the ultimate mathematical entity. For Leibniz, God and the universe include only reconcilable difference, only solvable problems – such is the best of all possible worlds.

But what is so great about a world where disagreements are only apparent, where difference is always superficial? On the contrary, Deleuze wishes to allow for disagreement that cuts to the bottomless bottom of the ontology itself, a difference that cannot heal and does not want to. Numerous power series are required, uncountably many, which can make for some ugly and contorted functions, bent and broken, but it also means that problems can form in spite of, or even because of, radical difference. The lesson never taught in calculus, for the teachers do not know it, is that sense is problematic, and the problems that give sense to things are not limited to the solvable or the reconcilable. If we look into the process of determination, if we look at more than the actual states of things, but follow the differential lines that connect solutions to the problems that make sense of them, then the world becomes a Pirandello play, or a garden of forking paths. Only a problem makes sense, and problematic sense does not tie things into a neat bundle, does not wrap up all the loose ends, but always points in new directions, demands to be worked on, shaped into another problem with another sense.

So your puzzle really was a math problem, for it was a differential puzzle, a puzzle of determination. The puzzle never fits together, and there would be no sense in its doing so, or only a dead sense, a totalizing sense that would homogenize the puzzle, flatten its surface, and make each piece uniform. Instead, let the world be a puzzle

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that must be constantly pieced together again, that is never the same thing twice or for two different puzzlers. And *this* makes sense, for sense is not a complacent and comfortable fit, but a problematic puzzle that never quits asking. The best puzzles, the ones that make the most sense, are also the hardest problems and the most compelling. To solve a problem is to follow the differential through its twisted calculus. And you thought you didn't like math.



notes

1 Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton (New York: Columbia UP, 1994) 168ff. All further page references are to this text.

2 We might conclude that mathematicians work on real problems, while math students deal only with neutralized problems. Really though, math students, from infants to grade schoolers to graduate students, are already working on real problems. Learning always proceeds by confronting a problem, so that the transition from math student to mathematician feels relatively smooth; one continues to work in the familiar manner.

3 This formulation is not strictly true. But you *can* find an interval around the point of tangency in which it is true.

4 “Close to” is a very relative term here. In fact, the function can vary by an arbitrarily large amount over an arbitrarily small interval near the point. Strictly speaking, the smoothness (or *continuity*) of the function at a given point means that no matter how small a variance you want, you can find some neighborhood around that point in which the variance *is* that small.

5 The general formula for a power series is  $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n + \dots$ . The  $a$  values are calculated using the values of the derivatives of the primitive function.

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