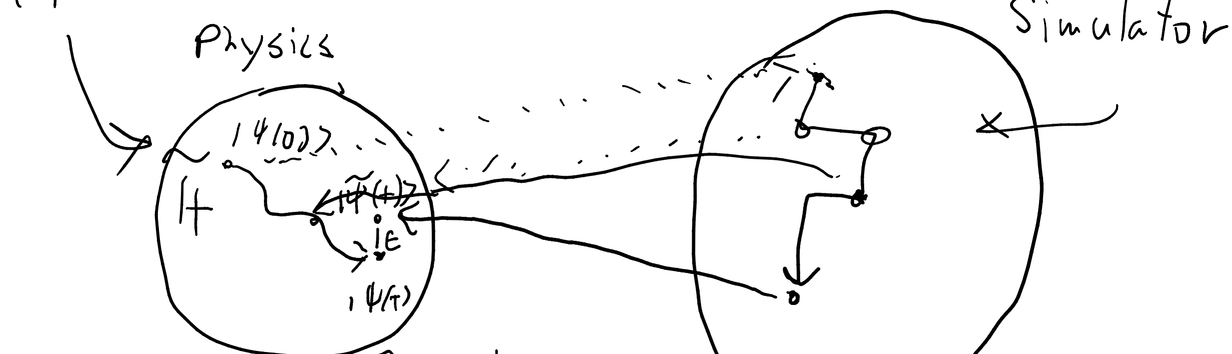
$$i\frac{\partial}{\partial t}\psi(t) = H\psi(t)$$

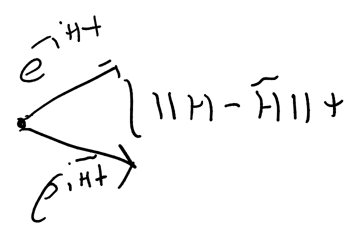
$$\hookrightarrow \quad |\psi(0)\rangle \longrightarrow |\psi(t)\rangle$$

$$e^{-iHt}|\psi(0)\rangle = |\psi(t)\rangle$$



$$\| |\psi(t)\rangle - |\tilde{\psi}(t)\rangle \| \le \epsilon$$

Thm: $\|e^{-iHt} - e^{-i\tilde{H}t}\| \le \|H - \tilde{H}\|t$



$$H'(s) = H(1-s) + \tilde{H}(s)$$

$$H'(0) = H \qquad H'(1) = \tilde{H}$$

$$\|e^{-iH'(1)t} - e^{-iH'(0)t}\| = \left\|\int_0^1 \frac{\partial}{\partial s} e^{-iH'(s)t}\, ds\right\|$$

$$\|e^{-iHt} - e^{-i\tilde{H}t}\| = \left\|\int_0^1 \frac{\partial}{\partial s} e^{-iH'(s)t}\, ds\right\|$$
$$\leq \int_0^1 \left\|\frac{\partial}{\partial s} e^{-iH'(s)}\right\| ds$$
$$\leq \int_0^1 \|H - \tilde{H}\|\, ds \leq \|H - \hat{H}\| t$$

=> Ham robustness

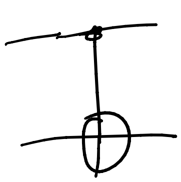
Box 4.1 Nielsen and Chuang *

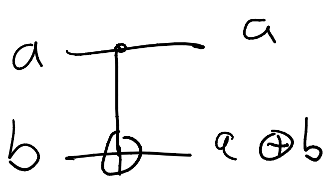
Thm: let $\forall j$ and unitary U_j V_j
$\|U_j - V_j\| \leq \epsilon$ then $\left\|\prod_{j=1}^{N} U_j - \prod_{j=1}^{N} V_j\right\| \leq N\epsilon$.

Gates:

$-\boxed{R_Z(\theta)}- = e^{-iZ\theta/2}$

$-\boxed{H}- = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} / \sqrt{2}$

 = CNOT.



↳ a+b mod 2 = parity(a,b)

Ex 1:

$H = \beta Z$

β

fuck off Bloch factor.

Bloch factor.

$$H = BZ$$

$$\hookrightarrow e^{-iHt} = e^{-iBZt} = \cancel{e}\; R_Z(2Bt)$$

Ex 2: $\uparrow$ spin $\longrightarrow \hat{x}$

$R_Z(\theta) \overset{\pm \epsilon}{\to} \{H, T\}^*$ using $4\log(1/\epsilon) + c$ T gates.

$$e^{-iBXt}$$

Thm: $U e^{A} U^{\dagger} = e^{UAU^{\dagger}}$

so $H e^{-iZt} H = e^{-iHZHt} = e^{-iXt}$

$$e^{-iXBt} = -\boxed{H}-\boxed{R_Z(2Bt)}-\boxed{H}-$$

Ex 3 $\quad H = a\, Z \otimes Z$

$$e^{-iaZ\otimes Zt} \neq e^{-iaZt} \otimes e^{-iaZt}$$

$$Z\otimes Z\,|00\rangle \to Z|0\rangle \otimes Z|0\rangle$$

$$= |0\rangle \otimes |0\rangle = |00\rangle$$

$$Z\otimes Z\,|10\rangle = -|1\rangle \otimes |0\rangle = -|10\rangle$$

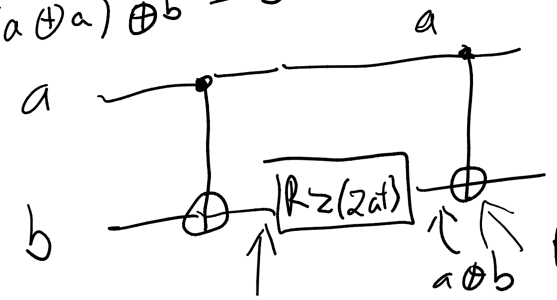
$$\begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array}$$

$$\begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{bmatrix}$$

$$e^{-iZ\otimes Z at} = \begin{bmatrix} e^{-iat} & & & 0 \\ & e^{iat} & & \\ & & e^{iat} & \\ 0 & & & e^{-iat} \end{bmatrix}$$

$a \oplus b \oplus a$

$= (a\oplus a)\oplus b = b$



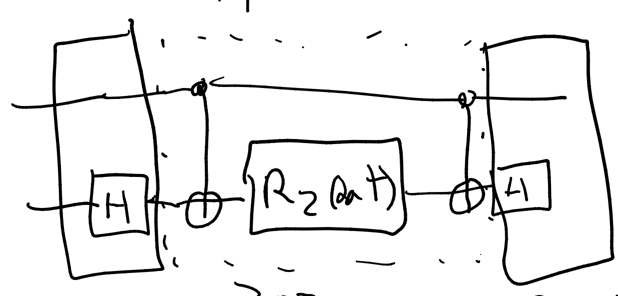
$$e^{-i(-1)^{\text{Parity}(a,b)} at}\,|ab\rangle$$

$$R_z(2at)|0\rangle = e^{-iat}|0\rangle$$

$a\oplus b = 1$ iff a and b have odd parity

$$R_z(2at)|1\rangle = e^{iat}|1\rangle$$

Ex: 4 $\quad H = aZ\otimes X$



Ex: 5 $\quad H = a\underbrace{Z\otimes Z}\otimes Z$

$+ZZZ|000\rangle$

$|000\rangle$

Ex: 3

$H = a\,\underbrace{\ldots}_{+/-1}$ $\quad |000\rangle$

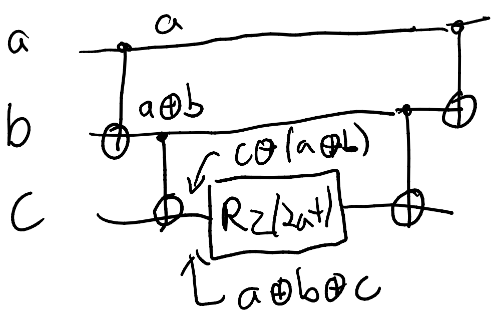
$-ZZZ \quad |001\rangle$

$(-1)|001\rangle$

$$e^{-iHt} \rightarrow e^{\pm iat} + ZZZ\,|101\rangle$$

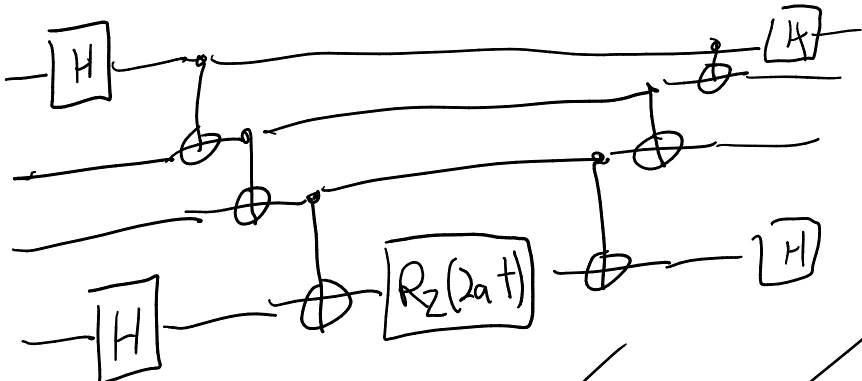
$= |101\rangle$

$$R_z(2at) \rightarrow e^{\pm iat}$$



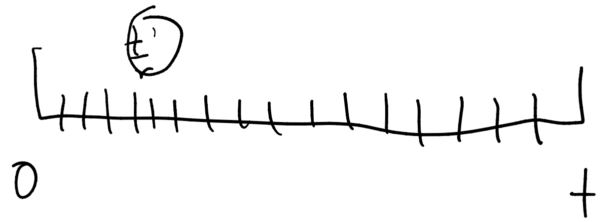
$a \oplus b \oplus c$

$$H = aX \otimes Z \otimes Z \otimes X$$



$$H = Z \otimes \mathbb{1} + \mathbb{1} \otimes Z + X \otimes X$$

$$e^{At+Bt} = e^{At}e^{Bt} + O(\|[A,B]\|t^2)$$



$$\|e^{-i(A+B)t} - (e^{-iAt/r}e^{-iBt/r})^r\|$$

$$= \|(e^{-i(A+B)t/r})^r - (e^{-iAt/r}e^{-iBt/r})^r\|$$

$$= \|\prod_{j=1}^{r} e^{-i(A+B)t/r} - \prod_{j=1}^{r} e^{-iAt/r}e^{-iBt/r}\|$$

$$\leq r\|e^{-i(A+B)t/r} - e^{-iAt/r}e^{-iBt/r}\|$$

$$\in O\left(\|[A,B]\|\frac{t^2}{r^2}\cdot r\right)$$

$$= O\left(\frac{\|[A,B]\|t^2}{r}\right) = \epsilon$$

$1/\epsilon$

$$r \in O\left(\frac{\|[A,B]\|t^2}{\epsilon}\right)$$

$$H = \sum_{j=1}^{m} H_j$$

$$\ldots\left(\prod^{m} e^{-iH_j t/r}\right)^r\|$$

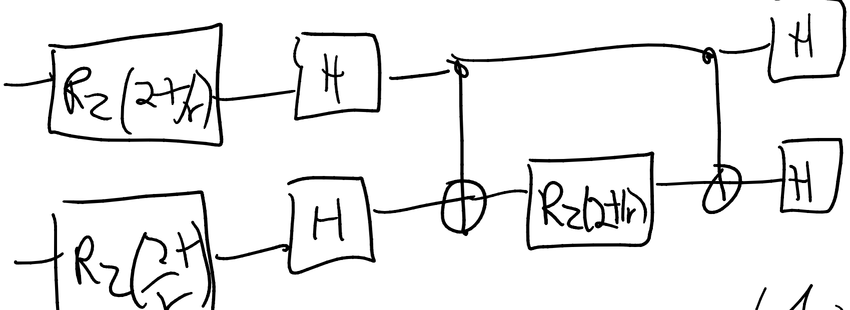
$$\left\| e^{-iHt} - \left(\prod_{j=1}^{m} e^{-iH_j t/r}\right)^{r} \right\|$$

$$\le O\left(\frac{m^2 \max_j \|H_j\| t^2}{r} \right)$$

$$H = \underbrace{Z\otimes \mathbb{1} + \mathbb{1}\otimes Z}_{A} + \underbrace{X\otimes X}_{B}$$

$$e^{-iHt/r} \approx e^{-i(Z\otimes\mathbb{1} + \mathbb{1}\otimes Z)t/r}\, e^{-iX\otimes X \frac{t}{r}}$$

$$= \underline{e^{-i(Z\otimes\mathbb{1})\frac{t}{r}}}\, e^{-i(\underline{\mathbb{1}\otimes Z})\frac{t}{r}} \cdot e^{-iX\otimes X t/r}$$



$-(1)$

$$-\boxed{R_2\left(\frac{2t}{r}\right)}-\boxed{\cdots}\ -\ \boxed{\ } = U_2$$

$$e^{(A+B)t} = e^{\frac{At}{2}} e^{Bt} e^{At/2} + O(t^3)$$

$$\left\| e^{(A+B)t} - \left(e^{At/2r} e^{Bt/r} e^{At/2r}\right)^r \right\| \le O\left(\frac{\max\|\{A,B\}\|^3 t^3}{r^2}\right)$$

$$r \in O\left(\frac{5^K (\max \|A\|, \|B\|)^{1+\frac{1}{2K}} t^{1+\frac{1}{2K}}}{\epsilon^{1/2K}}\right)$$

for any $K \geq 1$

0 $U_2(st)$ $U_2(st)$

$U_2((1-4s)t)$

$+$

$U_2(st)$ $U_2(st)$

$$\sim 4(st)^3 + ((1-4s)t)^3$$

$$= U_4 \quad s \text{ root of this}$$

$$\Rightarrow O(t^4) \times$$

$$\rightarrow O(t^5)$$

$$\left(e^{At/2r} e^{Bt/r} e^{At/2r}\right)^r$$

$$= e^{At/2r} e^{Bt/r} e^{At/r} \cdots e^{Bt/r} e^{At/2r})$$

<u>Progression</u>

$$\langle P_j, P_k \rangle = \frac{1}{2^n} \mathrm{Tr}\left(P_j P_k\right)$$

$$= \delta_{P_j = P_k}$$

$$P = \left\{ \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \right.$$

$$\begin{matrix} 2 & 0 & 0 & 0 \end{matrix}$$

only $\mathbb{1}$ has non-zero trace

$$\langle P_i, P_j \rangle = \frac{1}{2^n} \mathrm{Tr}(P_i P_j)$$

$$= 0 \quad \text{if } P_i P_j \neq \mathbb{1}$$

$$P_i P_j = \mathbb{1} \Rightarrow P_i = P_j$$

$$= \frac{1}{2^n} \mathrm{Tr}(\mathbb{1}) \qquad \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \leftarrow 2^n$$

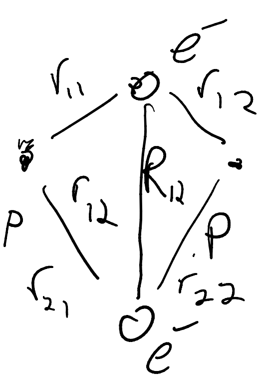
$$= \frac{2^n}{2^n} = 1$$

Thm Let $M \in \mathbb{C}^{2^n \times 2^n}$

$$M = \cancel{\phi} \sum_j a_j P_j \quad \text{where} \quad a_j = \langle M, P_j \rangle$$

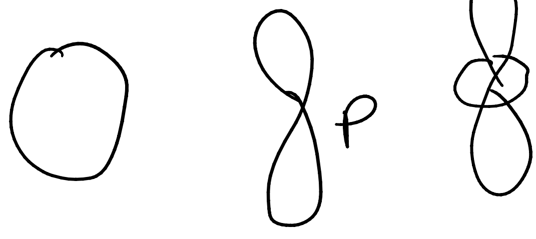
Chemistry

coulomb Ham



$$H = \underbrace{\frac{p_1^2}{2m_e} + \frac{p_2^2}{2me}}_{KE}$$

s

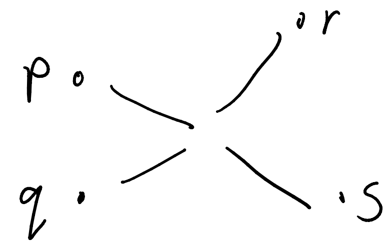


$$+ \frac{1}{R_{12}}$$

1 body "hopping" $- \frac{1}{r_{11}} - \frac{1}{r_{12}} - \frac{1}{r_{21}} - \frac{1}{r_{22}}$

1 body "hopping" $-\frac{1}{r_{11}} - \frac{1}{r_{12}} - \frac{}{r_{21}} \; \frac{}{r_{22}}$

$$H = \sum_{pq} h_{pq} \, a_p^{\dagger} a_q + \sum_{pqrs} h_{pqrs} \, a_p^{\dagger} a_q^{\dagger} a_r a_s$$



$$a_p^{\dagger} |0\rangle_p = |1\rangle_p \leftarrow \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$a_p^{\dagger} |1\rangle_p = 0 \qquad \uparrow \; \frac{X - iY}{2}$$

$$\{a_p^{\dagger}, a_q^{\dagger}\} = 0 \qquad \{Z, X\} = 0$$

$$\{Z, Y\} = 0$$

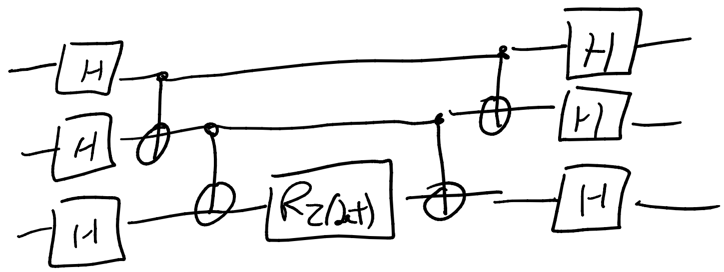
$$a_1^{\dagger} = \frac{X - iY}{2} \otimes \mathbb{1} \otimes \mathbb{1}$$

$$a_2^{\dagger} = Z \otimes \frac{X - iY}{2} \otimes \mathbb{1}$$

$$a_3^\dagger = Z \otimes Z \otimes \frac{X - iY}{2}$$

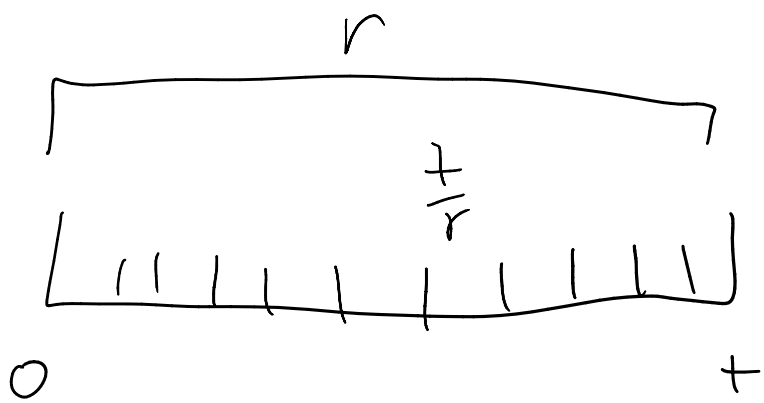
Qsim Alg Day 2

$$H = a\, X \otimes X \otimes X$$



$$e^{-i(Z \otimes \mathbb{1} + X \otimes X)t} \approx e^{-i Z \otimes \mathbb{1} t}\, e^{-i X \otimes X t}$$

$$+\, O(t^2)$$



$$O\left(\frac{t^2}{r}\right)$$

$$e^{At/2}\, e^{Bt}\, e^{At/2} \leftarrow O(t^3)$$

$$O\left(\frac{t^{1+1/2k}}{\epsilon^{1/2k}}\right)$$

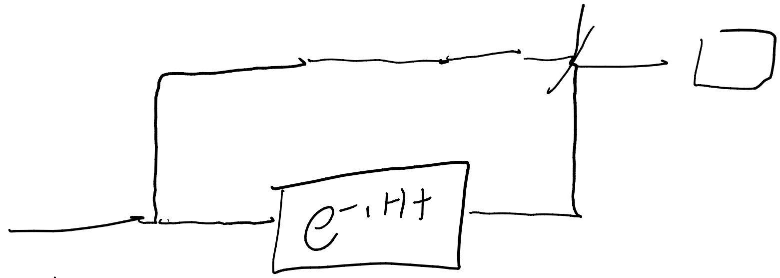
Chemistry

$$H = \sum_{pq} h_{pq}\, a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} h_{pqrs}\, a_p^\dagger a_q^\dagger a_r a_s$$

Jordan wigner

$$a_p^\dagger = \frac{(X - iY)}{2} \otimes Z^{\otimes p-1} \qquad a_3^\dagger = Z \otimes Z \otimes \frac{X - iY}{2}$$

$$a_1^\dagger = \frac{X - iY}{2} \qquad a_2^\dagger = Z \otimes \frac{X - iY}{2}$$



$\leftarrow P(0) = \cos^2(Et)$

$|\psi\rangle$

$A \leftarrow$ matrix not necceserily unitary or Hermitian

U is unitary Block encodes A

if $\exists$ $\alpha > 0$ s.t

if $\exists$ $\alpha > 0$ s.t.

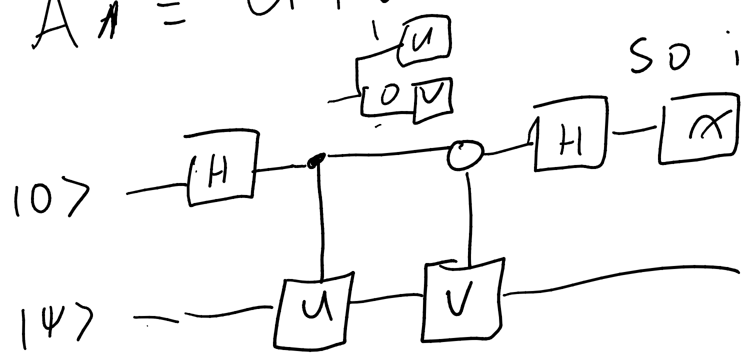
$$U = \begin{bmatrix} \boxed{A/\alpha} & ? \\ ? & ? \end{bmatrix}$$

$$\frac{A}{\alpha} = \langle 0| \otimes \mathbb{1} \;\; U \;\; |0\rangle \otimes \mathbb{1}$$

$U \begin{bmatrix} |\psi\rangle \\ 0 \end{bmatrix}$ and measured 0 in the end on anc then $|\psi\rangle \rightarrow \frac{A|\psi\rangle}{\alpha}$

LCU

$A = U + V$ U is unitary, so is V

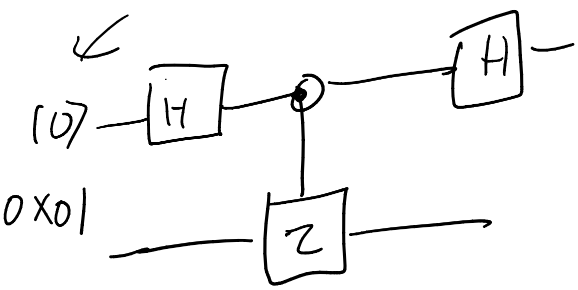


$$|0\rangle|\psi\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle\right)$$

$$\rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle V|\psi\rangle + |1\rangle U|\psi\rangle\right)$$

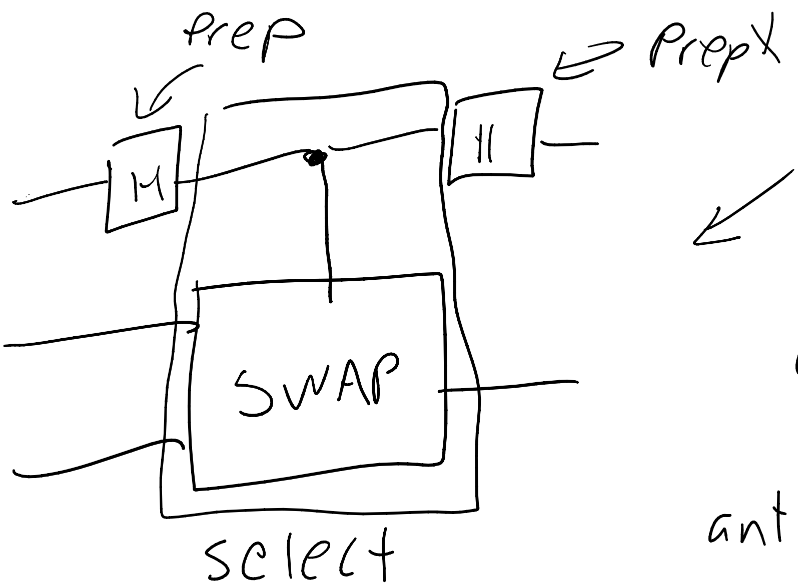
$$\to \frac{1}{2}\left(|0\rangle(U+V)|\psi\rangle + |1\rangle(V-U)|\psi\rangle\right)$$

$$\begin{bmatrix} \frac{U+V}{2} & \cancel{\emptyset} & ? \\ \frac{U-V}{2} & \cancel{\emptyset} & ? \end{bmatrix} \qquad \begin{bmatrix} |0\rangle\langle 0| & ? \\ |1\rangle\langle 1| & ? \end{bmatrix}$$



$U = \mathbb{1}$ $V = Z$

$$\frac{\mathbb{1}+Z}{2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = |0\rangle\langle 0|$$



symmetric space

$$\frac{\mathbb{1}+\text{SWAP}}{2}$$

or $\dfrac{\mathbb{1}-\text{SWAP}}{2}$

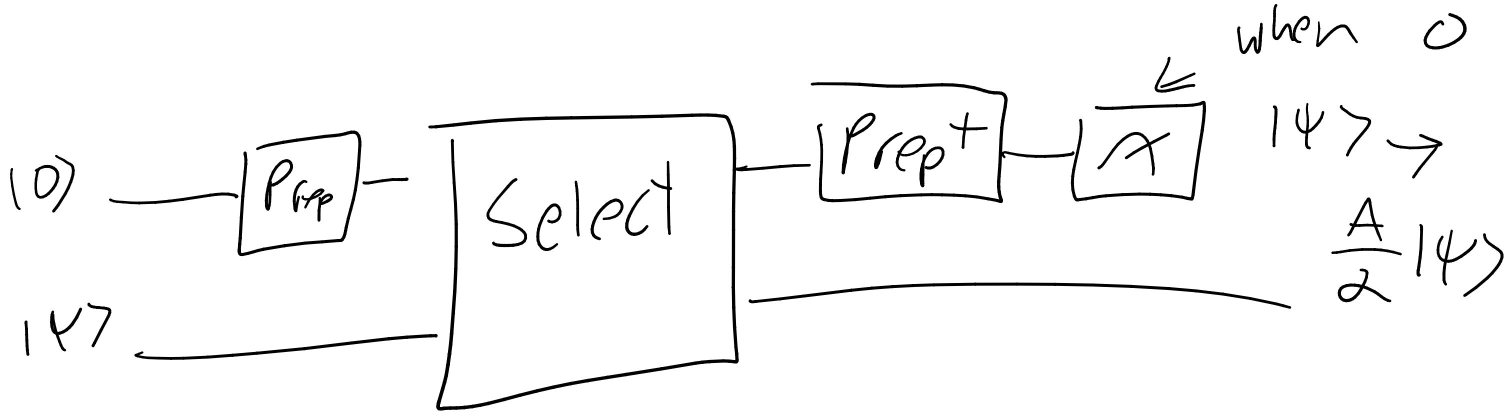
antisymmetric

Generalization

$A = \sum_j \alpha_j U_j$ $\qquad \alpha_j \geq 0$ $\qquad U_j$ unitary

<u>Prep</u> $: |0\rangle^{\otimes m} \longrightarrow \sum_j \sqrt{\alpha_j}\, |j\rangle$

$\underline{\text{select}}$ $|j\rangle|\psi\rangle \longrightarrow |j\rangle U_j|\psi\rangle$



$$e^{-iHt} \approx \sum_{k=0}^{K} \frac{(-iHt)^k}{k!}$$

$$H = \sum_j \beta_j U_j \qquad O\left(\sum_j |\beta_j| t \log(1/\epsilon)\right)$$

$$\text{vs} \quad O\left(\frac{\left(\sum_j |\beta_j| t\right)^{1+1/2K}}{\epsilon^{1/2K}}\right)$$