Lab 1: Entanglement and Bell's Inequalities

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Abstract

The purpose of this laboratory is to demonstrate polarization entanglement of a pair of photons created by spontaneous parametric down-conversion in two Type-1 beta barium borate crystals. We showed violation of the CHSH inequality by the number of coincident detections of the signal and idler photons on our two avalanche photo diode detectors using rotating linear polarizers to select 16 different incident polarizations for the purpose of evaluating the CHSH inequality. Achieving a value of $|S| = 2.64$, our results confirm that no local hidden variable theory accounts for the correlation between the photon polarizations, i.e. we successfully demonstrated photon polarization-entanglement.

1 Introduction

Entanglement is a property of multiparty quantum states that are not factorable into a product of states each describing just one party of the multiparty system: $|\psi_{12}\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$. Entangled states, or non-separable states, have the property that a complete description of any individual component of the system involves a description of the entangled partners as well. One may not completely describe an individual party of an entangled pair, for instance, without reference to the other party. Furthermore, measurement of an entangled party yields information about all parties in the entanglement. Thus, one may gain information about the state of an entangled party nonlocal to a measurement on its entangled partner. Entanglement, and its implied nonlocality, was unsettling to Einstein, Podolsky, and Rosen, who, in their famous EPR paper in 1935 set out to demonstrate how the prediction of entanglement in quantum mechanics was an indicator that the theory was incomplete [1]. However, in 1965, John Bell derived a rule to which any classical hidden variable theory must adhere [2] (see also [3]). He showed that nonlocal theories would violate an inequality that classical theories must obey, thus giving a testable way to determine if a system behaves classically, as EPR insisted it must, or demonstrates a type of quantum nonlocality that we now call entanglement.

In this laboratory we test the violation of the Clauser-Horne-Shimony-Holt version of Bell's inequality using polarization-entangled photons created by spontaneous parametric down-conversion in two orthogonal Type-1 BBO crystals [4]. We follow the experimental procedure developed by Kwiat *et. al.* [5] (see also [6], [7]).

1.1 Theory

We use the Clauser-Horne-Shimony-Holt (CHSH) version of Bell's inequality in this experiment [6]. The SPDC state that we will be measuring has the entangled form: $|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i)$, where V/H denote vertical/horizontal polarizations, and s/i indicate the signal/idler photons that result from the SDPC in the Type-I BBOs. We note that our entangled state is invariant under change of polarization basis, and show this by rewriting our state in an arbitrary polarization basis at an angle α to our original horizontal and vertical states. By simplifying, we see that our original state is intact.

$$\begin{aligned}
\frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i) &= \frac{1}{\sqrt{2}}[(\cos(\alpha)|V\rangle_s + \sin(\alpha)|H\rangle_s)(\cos(\alpha)|V\rangle_i + \sin(\alpha)|H\rangle_i) \\
&+ (-\sin(\alpha)|V\rangle_s + \cos(\alpha)|H\rangle_s)(-\sin(\alpha)|V\rangle_i + \cos(\alpha)|H\rangle_i)] \\
&= \frac{1}{\sqrt{2}}[\cos^2(\alpha)|V\rangle_s|V\rangle_i + \sin^2(\alpha)|H\rangle_s|H\rangle_i \\
&+ \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i + \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i \\
&+ \sin^2(\alpha)|V\rangle_s|V\rangle_i + \cos^2(\alpha)|H\rangle_s|H\rangle_i \\
&- \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i - \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i] \\
&= \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i) \\
&= |\Psi_{Bell}\rangle
\end{aligned}$$

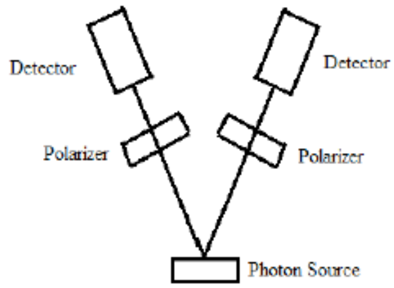


Figure 1: Depiction of polarization-entangled photon pairs passed through polarizers and incident on APDs.

Measurement of the polarization of these photons in any basis thus preserves their entangled state. Indicated in Figure 1, our experiment places APDs so that two detectors sit on diametrically opposed sides of the SPDC cone of entangled light that is emitted from the BBOs. Thus, each APD will receive one of the entangled photons. Placing a polarizer in front of each APD, we can control the measurement basis of each APD, setting one polarizer to an angle α, and the other to an angle β. To evaluate the CHSH inequality we will want the probability that both photons are vertically polarized (both horizontally polarized, and one in each polarization) in their respective polarizer basis. Below we compute this first probability and state the resultant $\cos^2(\alpha-\beta)$ or $\sin^2(\alpha-\beta)$–dependence for the remaining combinations.

$$\begin{aligned}
P_{VV}(\alpha,\beta) &= |\langle V_\alpha|_i\langle V_\beta|_s|\Psi_{Bell}\rangle|^2 \\
&= \frac{1}{2}|[\langle V|_i\cos(\alpha)+\langle H|_i\sin(\alpha)][\langle V|_s\cos(\beta)+\langle H|_s\sin(\beta)][|V\rangle_i|V\rangle_s+|H\rangle_i|H\rangle_s]|^2 \\
&= \frac{1}{2}|\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)|^2 \\
&= \frac{1}{2}\cos^2(\alpha-\beta).
\end{aligned}$$

The remaining probabilities of polarization combinations follow the same way and are:

$$\begin{aligned}
P_{HH}(\alpha,\beta) &= \frac{1}{2}\cos^2(\alpha-\beta) \\
P_{VH}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta) \\
P_{HV}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta).
\end{aligned}$$

Let $N(\alpha,\beta)$ be the experimentally measured number of coincident signal and idler photon counts for polarization alignments at angles α and β respectively. Then, we may analogously define the above probabilities in terms of these data.

$$\begin{aligned}
P_{VV}(\alpha,\beta) &= \frac{N(\alpha,\beta)}{N_{Total}} \\
P_{HH}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta_\perp)}{N_{Total}} \\
P_{VH}(\alpha,\beta) &= \frac{N(\alpha,\beta_\perp)}{N_{Total}} \\
P_{HV}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta)}{N_{Total}}
\end{aligned}$$

We have introduced the notation $\alpha_\perp = \alpha + 90°$, and $\beta_\perp = \beta + 90°$, and

$N_{Total} = N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)$, which is the total number of coincident counts of signal/idler detections. Hence, we can measure the probabilities $P_{HH}, P_{HV}, P_{VH}, P_{VV}$ in our experiment by recording the number of coincident signal/idler photon pairs detected at the APDs when the polarizers are aligned to the different combinations of angles. Using this, we must choose angles for which it is possible to violate Bell's inequality. The CHSH inequality uses a correlation of the probabilities defined as follows:

$$\begin{aligned} E(\alpha,\beta) &= P_{VV} + P_{HH} - P_{VH} - P_{HV} \\ &= \frac{1}{2}\cos^2(\alpha-\beta) + \frac{1}{2}\cos^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) \\ &= \cos^2(\alpha-\beta) - \sin^2(\alpha-\beta) \\ &= \cos(2(\alpha-\beta)). \end{aligned}$$

We may also write the probability correlation above in terms of the photon counts using the experimental definition of the coincidence probabilities just defined. Then we have:

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)}.$$

The Clauser-Horne-Shimony-Holt inequality considers a convex combination of the correlation function above, using the four angle combinations, as displayed here: $S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')|$, where S is the quantity that is bounded for a classical system operating according to a local hidden variable theory; we must have $|S| \le 2$. Note, however, that for choices of angle $a = -\pi/4$, $b = -\pi/8$, $a' = 0, b' = \pi/8$, the predicted value of S from the above derivation starting with the rotationally invariant entangled Bell state that describes our setup of signal/idler entangled photon pair is $S = |-1/\sqrt{2} - 1/\sqrt{2}| + |1/\sqrt{2} + 1/\sqrt{2}| = 2\sqrt{2}$. So, the quantum mechanical description of our system violates the inequality.

The previous discussion assumes 100% visibility. Visibility is defined as follows:

$Visibility = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \xrightarrow{\text{for discrete measurements}} \frac{N_{\max} - N_{\min}}{N_{\max} + N_{\min}}$, where I is intensity, and N is the number of detections made. We compute visibility as a check on our data to ensure that we meet a minimum condition on observation of an entangled state: visibility larger than $1/\sqrt{2}$.

Our goal then is this: we seek to observe a value of S > 2 to suggest entanglement is present in our system. We note also that to violate Bell's Inequality in order to observe entanglement in our system, we should achieve a visibility larger than 0.71, in addition to a value of S > 2 and the $\cos^2(\alpha-\beta)$ dependence of coincident photons [8].

2 Experimental Setup and Procedure

2.1 Experimental Setup

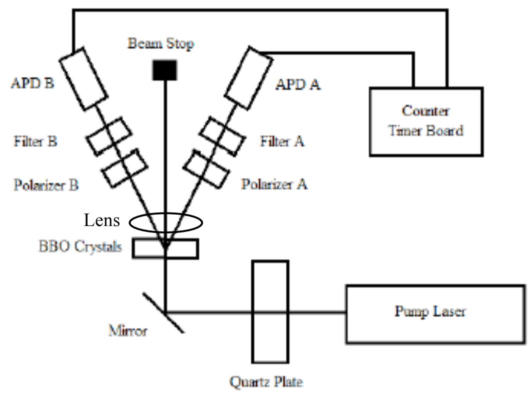


Fig. 2: The experimental setup: a pump laser of 363.8nm at 100mW power drives SPDC in the two type-I BBO crystals. Down-converted photons are detected by single-photon counting avalanche photodiodes (APDs). Polarizers are used to select the incident photon polarization on the APDs.

In this lab, we create polarization-entangled photons by pumping two type-I Beta Barium Borate (BBO) crystals with 363.8nm laser light from a 100mW argon-ion laser to induce spontaneous parametric down-conversion, a nonlinear $\chi^{(2)}$ process of very low probability – on the order of 10^{-10} (see, for example, Ref. [8]). When vertically/horizontally polarized photons of wavelength λ are incident on a type-I BBO crystal, two momentum and energy conserving horizontally/ vertically polarized photons of wavelength 2λ are emitted from the BBO. Thus, when two adjacent type-I BBO crystals are aligned with their optic axes orthogonal to each other and a pump beam, polarized at $45°$ to the BBOs optical axes, is incident on the BBOs, two SPDC polarization-entangled photons are produced with low probability from a single pump photon. For pump beam polarization at an angle θ to the optic axis of the vertical BBO, the signal/idler entangled state is written as we have shown in the Theory section,

$$|\psi_{SPDC}\rangle = \cos\theta|H\rangle_s|H\rangle_i + e^{i\phi}\sin\theta|V\rangle_s|V\rangle_i,$$

where here we have introduced an overall phase difference between the two out-coming entangled polarization states, which occurs because incident horizontally polarized photons travel a larger distance in the BBO crystals than incident vertically polarized photons do before they are down-converted. To correct for this phase shift ϕ, we use a quartz plate that be rotated along the horizontal and vertical axes. Our setup attempts to align the quartz plate such that $\phi = 0$. Then, aligning the pump beam so it is polarized at $\theta = 45°$ to the BBOs optical axes, the signal/idler coincidence counts at our two single-photon counting APD detectors is shown in the theory to have an expected dependence proportional to $\cos^2(\alpha - \beta)$, where α and β again are the polarizer angles in front of APD1 and APD2 respectively.

Figure 2 displays the main components of our experimental setup. We pass the argon ion laser light through a quartz plate and direct it toward the type-I BBO crystal pair, which is on a 3D rotating mounted with the crystals aligned so that their optic axes are orthogonal. The emitted SPDC light from the BBOs is then passed through a converging lens so that we may image the SPDC cones on our optical table. Two avalanche photodiodes (APDs) are mounted on alignment rails so that they are located on the diametrically opposite points of the down-converted light cone emitted by the BBOs. The single photon counting APDs detect the down-converted light, and are used to collect coincidence data. We place interference filters in front of each APD to allow only a narrow band of 10nm around the $2\lambda = 727.6nm$ (used 730nm filter) SPDC light to enter the detectors. Specifically, the BBO partially scatters the pump beam, and so we must block it at the APDs. A beam stop is used stop the pump beam, as well, at the end of our setup. Finally, the experimental part of our setup is two rotating polarizers, one placed in front of each APD. The polarizers are used to select the polarization of the incident SPDC photons on our APD detectors, where we indicate the angle of the signal polarizer A by α and the angle of the idler polarizer B by β. An incident SPDC photon on an APD triggers a TTL pulse from the APD to a counter/timer board in our computer running LabVIEW software. The computer records the results of the counter/timer board to display the number of incident photons on each APD per time. LabVIEW records the coincidence count, the number of times the signal and the idler photons are detected at the same time. We note that the counter/timer board has a timing resolution limit of 26ns, so our coincidence data is over counted by false readings of photons that are detected within this limit. Thus, we must correct for accidental coincidence detections, which we do by computing the net coincidences (denoted "net" in the data tables of section 4) [7]:

$$Net\ Coincidence = \frac{(Single\ Count\ A)(Single\ Count\ B)(Coincidence\ Window : \text{Time Resolution of Counter board} = 26ns)}{Aquistion\ Time = 5s}$$

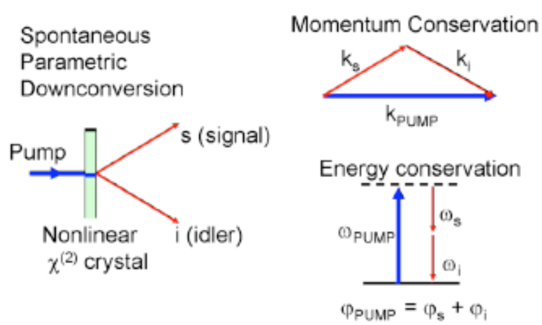


Figure 3: [from Dr. Lukishova lab-lecture] Spontaneous parametric down-conversion (SPDC): emission of a signal and idler photon in a nonlinear optical process of very low probability. The conservation of momentum and energy determine frequency and wave number of the SPDC photons given the corresponding parameters of the incident pump photon.

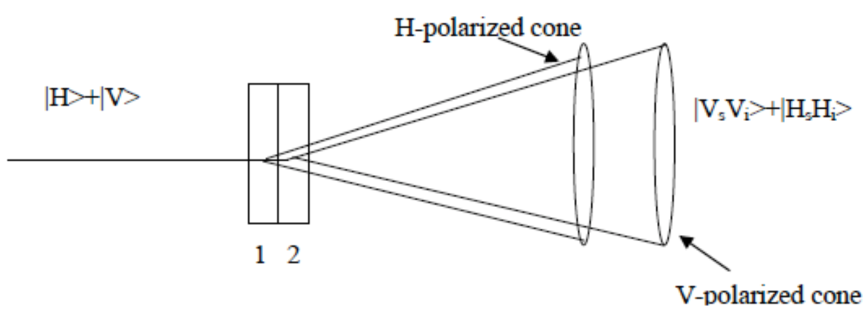


Figure 4: Type-I BBO crystals absorb H (V) polarized photons and emit polarization-entangled photons with Vertical (Horizontal) polarization in cones due to momentum conservation. The BBO crystal spacing causes down-converted photons emitted from one crystal to travel a longer distance than photons emitted from the other; this creates a phase difference between the H and V states of the down-converted signal/idler photons. (Incident: H → signal/idler: VV; incident: V → signal/idler: HH).

2.2 Experimental Procedure

The following is the procedure we used to align our setup and collect coincidence data of the SPDC light for the purpose of observing the violation of the CHSH inequality in order to demonstrate entanglement.

(1) *Imaging the SPDC light cone with an electron-multiplying (EM)-CCD camera:* An EM-CCD camera is used to detect the photons emerging from the BBO crystal and take images of the down-converted light cone to observe overlapping of the two cones from the two BBOs.

(2) *Polarizer and Detector Alignment:* We rotate the BBO crystal is along the horizontal and vertical axes to find the position of maximum SPDC photon counts in the two APDs. Polarizers A and B are placed at the same angle so that by adjusting their heights we find the maximum coincidence counts. We next check that the two SPDC cones coming from the BBOs are completely overlapping by checking the polarization of the light incident on our detectors; if the cones are overlapped properly the total SPDC light cone is unpolarized. This check is accomplished by rotating each of the polarizers, and observing that the single-photon counts in each APD do not have substantial fluctuation.

(3) *Quartz Plate Alignment:* To achieve consistent coincidence counts for as many polarization angles as possible of polarizers A and B, the quartz plate is rotated along its horizontal and its vertical axis, and then the coincidence counts are recorded for multiple polarizer positions. We used: $\alpha = \beta = 0$, $\alpha = \beta = 45$, $\alpha = \beta = 90$ and $\alpha = \beta = 135$ degrees. We plot the results of these measurements against the angular positions of the quartz plate, and choose the orientation of the quartz plate at the angles where the set of four curves are nearest to equal. We then fix this orientation for the experiment.

(4) *CHSH Test of Entanglement:* We measure the sixteen combinations of polarization angles for polarizers A and B. We do this by fixing a value of the angle α and varying the angle β from 0 to 360 degrees. We do this for four fixed α values. We record the corresponding coincidence counts and plot them as function of (α - β).

(5) *Evidence of Entanglement, CHSH Inequality violation:* After correcting for accidental coincidences, the measured coincidence counts for the 16 measurements of α and β are used to compute our S value, as described in the theory section.

3 Results and Analysis

We report here the results of the procedure described above, in following the work of Kwiat *et. al.* [10].

(1) *Imaging the SPDC light cone with an electron-multiplying (EM)-CCD camera:* The image of the down-converted light cone in Fig. 5 is taken by the EM-CCD camera. We see a spot inside the cone, which is a fluorescence artifact caused by a resonance inside the argon-ion laser cavity.

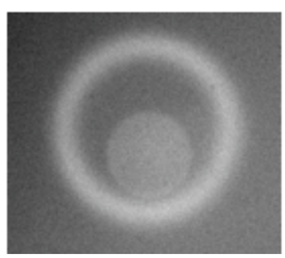


Fig. 5: Image of down-converted light cone taken by EM-CCD camera on 9/28/2010.

(2) *Polarizer and Detector Alignment:* In Fig. 6 the plot of the change in single photon counts at both APD A and APD B with the angle β is shown for several values of the α polarizer angle. From this data we conclude that the alignment of the two SPDC light cones is accurately overlapped enough to provide us with unpolarized light, though the sinusoidal oscillations indicate that we do not have completely unpolarized SPDC light.

(3) *Quartz Plate Alignment:* In Fig. 7 we see the results of aligning the quartz plate. Fig. 7(a) plots the coincidence counts from APD1 and APD2 against the orientation of the horizontal

axis of the quartz plate, taking four different polarizer angles for A and B: $\alpha = \beta = 0$, $\alpha = \beta = 45$, $\alpha = \beta = 90$ and $\alpha = \beta = 135$ degrees. The curves approach closest to each other at an angle of 359.5 degrees (on the scale). Fig. 7(b) likewise displays the coincidence counts plotted against the orientation of the quartz plate's vertical axis, taking the same four positions of the polarizers A and B. For the vertical angle, the curves approach most closely for 36.5 degrees.

(4) *CHSH Test of Entanglement:* Figure 8 plots the coincidence counts as a function of angle β. We plot against just the angle β, since α is fixed, so the dependence is the same as the predicted dependence on the angle difference: $\cos^2(\alpha - \beta)$, just shifted by α. We see a $\cos^2(\alpha - \beta)$-dependence is apparent, in agreement with the theory purporting entanglement. We thus suspect that the signal/idler photons are polarization-entangled. Lastly, we note that the average visibility of our measurements in Fig. 7 is 90%, which is better than the visibility requirements noted in the theory section.

(5) *Evidence of Entanglement, CHSH Inequality violation:* We report the data for calculation of the test of the Clauser-Horne-Shimony-Holt inequality in Table 1. From this data table we compute our measurement's associated S value to find that $|S| = 2.64 \pm 0.08$. So we have successfully shown a violation of Bell's inequality, $|S| < 2$.

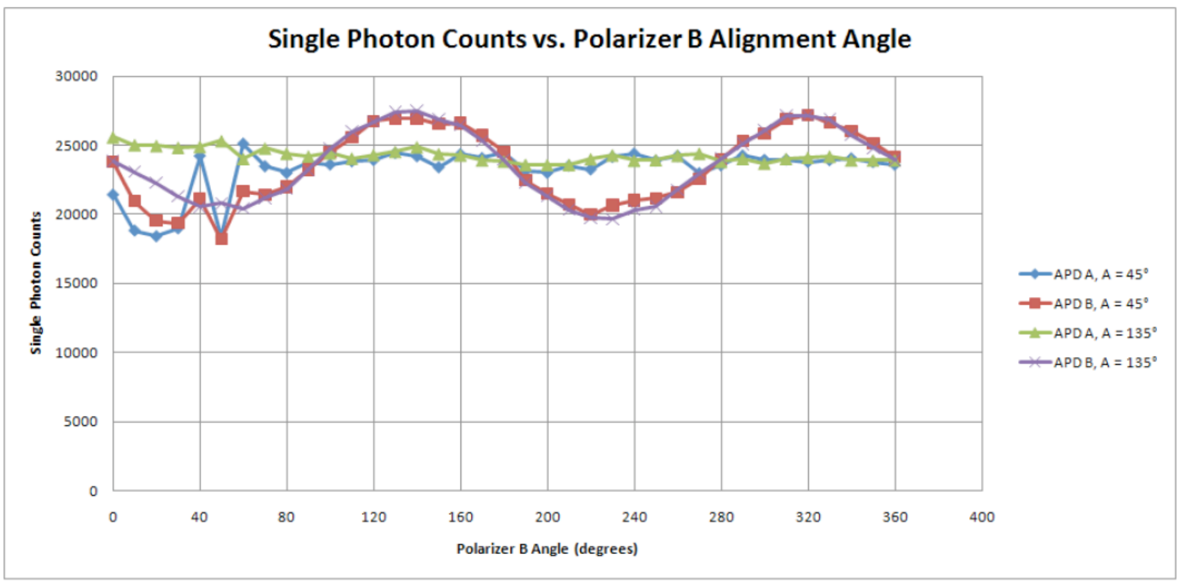
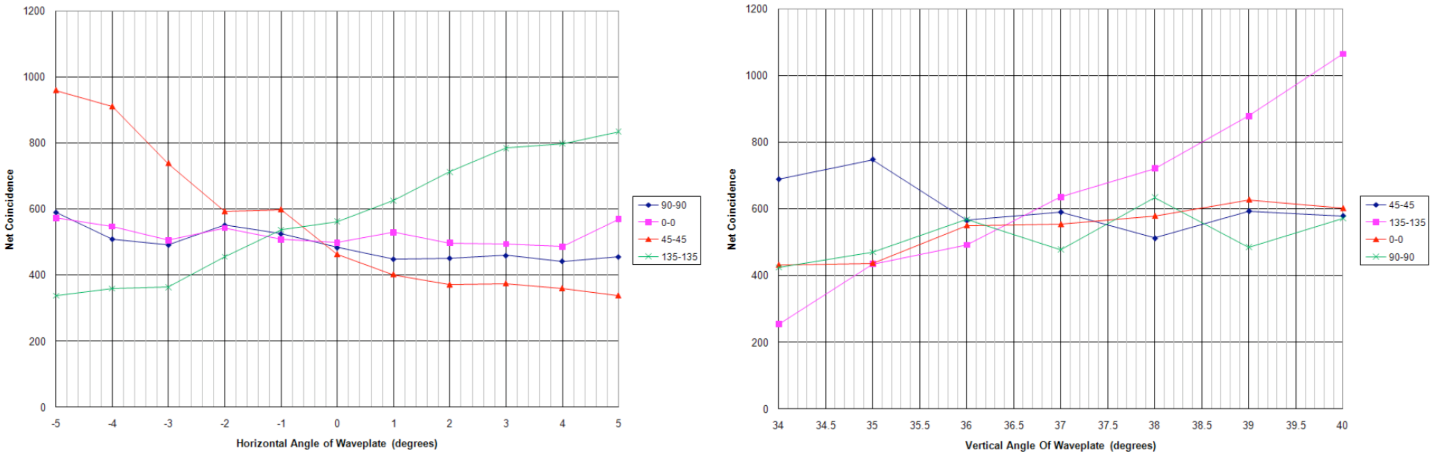


Fig. 6: The results of single photon counting under rotation of the β angle of polarizer B. We observe essentially linear behavior for an angle $\alpha = 45°$, but slight sinusoidal behavior for $\alpha = 135°$. This indicates imperfect alignment of the SPDC cones due to the orientation of the BBOs and/or the quartz plate. Since the count rate fluctuates in a range of only ~7000 counts, we take this as satisfactorily unpolarized light since the variation expected in the cosine-squared dependence on the polarization angle difference should be significantly larger.



(a) Horizontal Alignment of the Quartz Plate

(b) Vertical Alignment of the Quartz Plate

Fig. 7: [plot format from J. Winkler Lab 1] Data from (a) horizontal, and (b) vertical quartz plate alignment showing the point of closest approach of the curves representing the coincident detection at the APDs for a series of polarizer angle values. Coincidence detection was taken over 5 seconds in four trials using polarizers A and B set at 0, 45, 90, and 135 degrees.

4 Conclusion

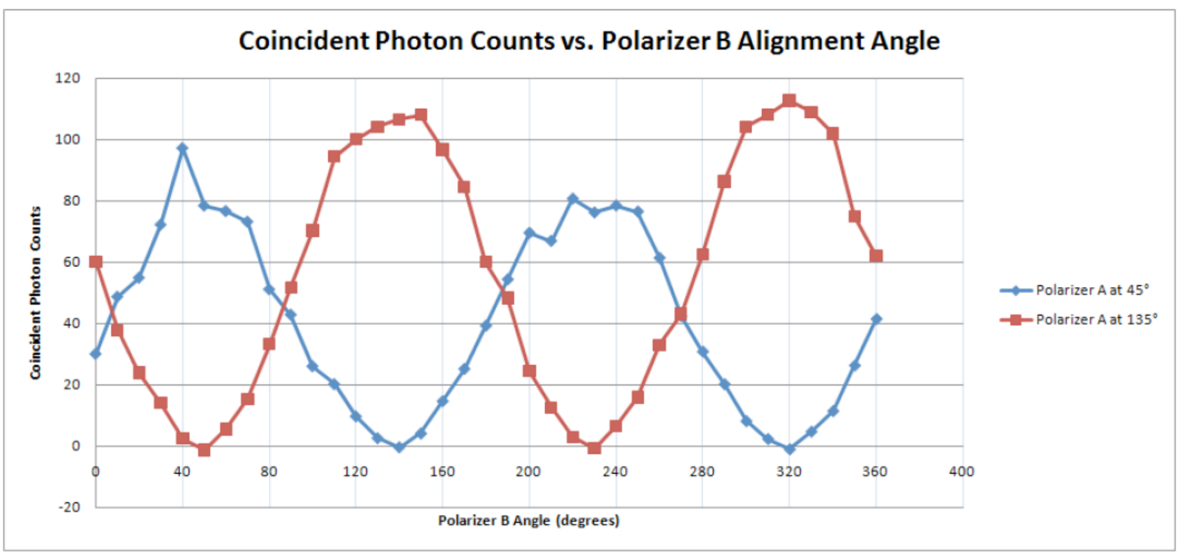


Fig. 8: Plot of coincident photon counts as function of the polarizer B angle β, for angles of 45 and 135 degrees of polarizer A. Our results have the entanglement-predicted $\cos^2(\alpha-\beta)$ dependence, and demonstrate a phase change of $90°$ for the $90°$ change in polarizer A angle α.

Table 1: Data collected for testing violation of CHSH inequality; from data file "BellInequality_Oct_7_2010.xls". This set of data yield a violation of the CHSH inequality with $|S| = 2.64 \pm 0.08$.

AngleOf Polari-zationA	AngleOf Polari-zationB	1st Trial Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	2nd Trial Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	3rd Trial Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Average Coinci-dence	Net	SD Coinci-dence	Net
0	22.5	41813	32753	384	7	377	41362	32378	307	7	300	42054	32658	363	7	356	351	344	40	40
0	67.5	41717	33007	100	7	93	41398	32442	105	7	98	41229	32519	102	7	95	102	95	3	3
0	112.5	41597	33288	55	7	48	41146	33160	79	7	72	41895	33462	87	7	80	74	66	17	17
0	157.5	41708	33101	312	7	305	41869	33566	310	7	303	41445	32906	306	7	299	309	302	3	3
45	22.5	46468	36537	403	9	394	46932	36412	373	9	364	46639	36135	369	9	360	382	373	19	19
45	67.5	41238	32781	309	7	302	40764	32016	348	7	341	40837	32194	327	7	320	328	321	20	20
45	112.5	41459	33505	71	7	64	41175	32917	86	7	79	41442	33039	83	7	76	80	73	8	8
45	157.5	41946	33293	42	7	35	41788	33252	44	7	37	41764	33142	51	7	44	46	38	5	5
90	22.5	46391	37309	99	9	90	45764	36720	87	9	78	46416	37333	82	9	73	89	80	9	9
90	67.5	46115	37632	390	9	381	47504	38383	391	9	382	46905	38308	373	9	364	385	375	10	10
90	112.5	47849	38984	423	10	413	47731	39452	408	10	398	47127	39266	391	10	381	407	398	16	16
90	157.5	46668	38168	134	9	125	47658	38210	124	9	115	47053	38215	122	9	113	127	117	6	7
135	22.5	45759	37576	111	9	102	47443	38686	103	10	93	47465	38623	103	10	93	106	96	5	5
135	67.5	48317	39571	51	10	41	48288	39508	50	10	40	47966	39372	56	10	46	52	42	3	3
135	112.5	47410	39216	372	10	362	47706	39494	417	10	407	47234	38993	383	10	373	391	381	23	23
135	157.5	46982	38442	453	9	444	47506	38012	476	9	467	46853	38204	465	9	456	465	455	12	12

From the net coincidences (coincidence counts corrected for accidental coincidences) in table 1, we evaluate the correlation functions, E(a,b), and compute S, with a total error found via propagation of error in the net coincidence count standard deviations.

$$E(135°,157.5°) = 0.81 \pm 0.09$$
$$E(135°,22.5°) = -0.63 \pm 0.02$$
$$E(0°,157.5°) = 0.52 \pm 0.01$$
$$E(0°,22.5°) = 0.67 \pm 0.04$$
$$\Rightarrow |S| = 2.64 \pm 0.08$$

Table 2: This second table contains a second set of data from trials yielding violation of Bell's inequality using random angles for Polarizer B. Here we obtain $|S| = 2.24 \pm 0.18$. This data was taken on 10/7/2010.

Pol. A	Pol. B	1st Trial S.C. A	S.C. B	Coinc.	Acc	Net	2nd Trial S.C. A	S.C. B	Coinc	Acc	Net	3rd Trial S.C. A	S.C. B	Coinc	Acc	Net	AC	AN	SDCoinc.	SDNet
0	32	40592	31603	277	7	270	40940	32336	302	7	295	42050	33085	303	7	296	294	287	17.67767	14.5018
0	90	41867	34348	50	7	43	42273	34678	39	8	31	42430	34831	46	8	38	45	37	149.3464	5.628011
0	122	43284	36843	108	8	100	43828	37070	126	8	118	44101	36927	129	8	121	121	113	41.96824	11.26158
0	180	44613	36422	394	8	386	44591	36570	440	8	432	44245	36850	424	8	416	419	411	167.8601	23.33616
60	32	45579	35765	400	8	392	45418	35780	371	8	363	45932	35938	395	9	386	389	380	26.53928	15.46591
60	90	46383	37212	369	9	360	46613	37234	375	9	366	46418	37626	355	9	346	366	357	13.61372	10.30114
60	122	46703	38279	139	9	130	46304	38325	141	9	132	46456	37856	116	9	107	132	123	124.1343	13.82748
60	180	45262	36148	125	9	116	45486	36935	138	9	129	45000	36629	140	9	131	134	126	11.06044	8.071652
90	32	44824	35269	143	8	135	44896	35550	141	8	133	44910	35009	144	8	136	143	134	1.527525	1.590135
90	90	45213	36748	425	9	416	45277	36524	419	9	410	45143	36705	441	9	432	428	420	160.5314	11.36895
90	122	45391	36800	362	9	353	45361	37016	348	9	339	45359	37836	335	9	326	348	340	50.14313	13.62134
90	180	45642	36531	56	9	47	45284	36524	44	9	35	45029	36918	56	9	47	52	43	164.6542	6.895457
150	32	45404	35660	139	8	131	46083	35523	153	9	144	46005	35433	155	8	147	149	141	52.43091	8.676811
150	90	45854	36589	123	9	114	45637	36745	124	9	115	45093	36769	129	9	120	125	117	18.19341	3.272178
150	122	45552	37742	383	9	374	45654	37030	352	9	343	45360	37050	352	9	343	362	354	138.5677	17.79687
150	180	45381	36491	417	9	408	45621	36316	359	9	350	45174	36341	412	9	403	396	387	35.67913	32.16121

Table 3: This third table contains the first set of data we collected (which we took before aligning the quartz plate); these trials did not conclusively yield a violation of Bell's inequality. Here we obtained $|S| = 1.98 \pm 0.17$. This data was taken on 10/7/2010.

AngleOf	AngleOf	1st Trial					2nd Trial					3rd Trial					Average		SD	
Polari-zationA	Polari-zationB	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Coinci-dence	Net	Coinci-dence	Net
0	22.5	88255	97272	253	11	242	88255	97272	256	11	245	88255	97272	222	11	211	244	233	19	19
0	67.5	88670	98447	91	11	80	88670	98447	103	11	92	88670	98447	94	11	83	96	85	6	6
0	112.5	93516	97795	165	12	153	93516	97795	151	12	139	93516	97795	146	48	98	154	130	10	28
0	157.5	84740	89381	296	10	286	84740	89381	242	10	232	84740	89381	267	39	228	268	249	27	33
45	22.5	81171	74592	279	8	271	81171	74592	286	8	278	81171	74592	259	31	228	275	259	14	27
45	67.5	77660	73401	234	7	227	77660	73401	221	7	214	77660	73401	238	30	208	231	216	9	9
45	112.5	88844	77887	51	9	42	88844	77887	39	9	30	88844	77887	83	36	47	58	40	23	9
45	157.5	88053	76084	71	9	62	88053	76084	66	9	57	88053	76084	35	35	0	57	40	20	35
90	22.5	90654	96811	140	11	129	90654	96811	122	11	111	90654	96811	138	46	92	133	111	10	18
90	67.5	90148	93100	267	11	256	90148	93100	271	11	260	90148	93100	239	44	195	259	237	17	36
90	112.5	90094	89758	271	11	260	90094	89758	243	11	232	90094	89758	251	42	209	255	234	14	26
90	157.5	92459	92044	117	11	106	92459	92044	142	11	131	92459	92044	150	44	106	136	114	17	14
135	22.5	96572	117723	48	15	33	96572	117723	54	15	39	96572	117723	56	59	-3	53	23	4	23
135	67.5	97563	118603	161	15	146	97563	118603	157	15	142	97563	118603	145	60	85	154	124	8	34
135	112.5	92178	108582	408	13	395	92178	108582	378	13	365	92178	108582	391	52	339	392	366	15	28
135	157.5	85934	100883	313	11	302	85934	100883	351	11	340	85934	100883	335	45	290	333	310	19	26

In this lab we tested the violation of the CHSH version of Bell's inequality to show the polarization-entanglement of SPDC photons from type-I BBO crystals. The experimental difficulties in accomplishing this laboratory center on alignment of the quartz plate and the BBO crystals in order to overlap the SPDC cones in the direction of our detectors. That our results did not exactly match the theoretically predicted value of $S = 2\sqrt{2}$ is an indication that we did not have a perfect setup, nor did we achieve 100% visibility; nevertheless, we did demonstrate a violation of the CHSH version of Bell's Inequality in two sets of trials indicating entanglement in our SPDC photons.

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