Questions

- 1. Let $\mathbf{p} = (1, 3, 4)$ and $\mathbf{q} = (5, 2, 7)$ be two vectors in \Re^3 . Calculate $\mathbf{p} \times \mathbf{q}$. Calculate $\mathbf{p} \cdot \mathbf{q}$. Verify for yourself that $\mathbf{p} \times \mathbf{q}$ is perpendicular to both \mathbf{p} and \mathbf{q} .
- 2. Give a vector that is perpendicular to the line passing between two points (x_0, y_0) and (x_1, y_1) .
- 3. Give a general expression for the perpendicular distance from a point (x, y) to a line that passes through (x_0, y_0) and (x_1, y_1) .
- 4. Given a vector (a, b, c), give at least three examples of vectors that are perpendicular to this vector.

Answers

1.

$$\mathbf{p} \times \mathbf{q} = (13, 13, -13) \qquad \mathbf{p} \cdot \mathbf{q} = 39$$

2. We want a vector that is perpendicular to $(x_1 - x_0, y_1 - y_0)$. Recall from linear algebra that $(-b, a) \cdot (a, b) = 0$ for any (a, b). Thus for our example

$$(-(y_1 - y_0), x_1 - x_0) \cdot (x_1 - x_0, y_1 - y_0) = 0$$

Thus $(-(y_1 - y_0), x_1 - x_0)$ is perpendicular to $(x_1 - x_0, y_1 - y_0)$.

3. Let

$$\mathbf{n} = \frac{1}{\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}} \ (-(y_1 - y_0), x_1 - x_0)$$

be a unit vector perpendicular to the line between the two given points.

One way to express the perpendicular distance from an arbitrary point (x, y) to the line is to consider the vector from one of the endpoints, say $(x - x_0, y - y_0)$, and take the dot product of that vector with **n**. The reason this works is that this vector $(x - x_0, y - y_0)$ can be written as the sum of two components: one component parallel to the line joining the two given points, and a second component that is parallel to the unit vector **n**.

Thus, the perpendicular distance from (x, y) to the line segment joining the two points is:

$$|(x - x_0, y - y_0) \cdot (n_x, n_y)|.$$

Note we have taken the absolute value because the direction of the surface normal is ambiguous; the sign could be flipped.

What is the normal? Since the normal is by definition perpendicular to the line, it must satisfy

$$(n_x, n_y) \cdot (x_1 - x_0, y_1 - y_0) = 0.$$

So we need a vector (n_x, n_y) that satisfies that equation.

4. Here are three: (-b, a, 0), (-c, 0, a), (0, c, -b). Note that these are generally not perpendicular to each other. Also note that any linear combination of these vectors will be perpendicular to (a, b, c).