## Questions

1. Let $\mathbf{p}=(1,3,4)$ and $\mathbf{q}=(5,2,7)$ be two vectors in $\Re^{3}$.

Calculate $\mathbf{p} \times \mathbf{q}$. Calculate $\mathbf{p} \cdot \mathbf{q}$. Verify for yourself that $\mathbf{p} \times \mathbf{q}$ is perpendicular to both $\mathbf{p}$ and $\mathbf{q}$.
2. Give a vector that is perpendicular to the line passing between two points $\left(x_{0}, y_{0}\right)$ and $\left(x_{1}, y_{1}\right)$.
3. Give a general expression for the perpendicular distance from a point $(x, y)$ to a line that passes through $\left(x_{0}, y_{0}\right)$ and $\left(x_{1}, y_{1}\right)$.
4. Given a vector $(a, b, c)$, give at least three examples of vectors that are perpendicular to this vector.

## Answers

1. 

$$
\mathbf{p} \times \mathbf{q}=(13,13,-13) \quad \mathbf{p} \cdot \mathbf{q}=39
$$

2. We want a vector that is perpendicular to $\left(x_{1}-x_{0}, y_{1}-y_{0}\right)$. Recall from linear algebra that $(-b, a) \cdot(a, b)=0$ for any $(a, b)$. Thus for our example

$$
\left(-\left(y_{1}-y_{0}\right), x_{1}-x_{0}\right) \cdot\left(x_{1}-x_{0}, y_{1}-y_{0}\right)=0
$$

Thus $\left(-\left(y_{1}-y_{0}\right), x_{1}-x_{0}\right)$ is perpendicular to $\left(x_{1}-x_{0}, y_{1}-y_{0}\right)$.
3. Let

$$
\mathbf{n}=\frac{1}{\sqrt{\left(x_{1}-x_{0}\right)^{2}+\left(y_{1}-y_{0}\right)^{2}}}\left(-\left(y_{1}-y_{0}\right), x_{1}-x_{0}\right)
$$

be a unit vector perpendicular to the line between the two given points.
One way to express the perpendicular distance from an arbitrary point $(x, y)$ to the line is to consider the vector from one of the endpoints, say $\left(x-x_{0}, y-y_{0}\right)$, and take the dot product of that vector with $\mathbf{n}$. The reason this works is that this vector $\left(x-x_{0}, y-y_{0}\right)$ can be written as the sum of two components: one component parallel to the line joining the two given points, and a second component that is parallel to the unit vector $\mathbf{n}$.
Thus, the perpendicular distance from $(x, y)$ to the line segment joining the two points is:

$$
\left|\left(x-x_{0}, y-y_{0}\right) \cdot\left(n_{x}, n_{y}\right)\right| .
$$

Note we have taken the absolute value because the direction of the surface normal is ambiguous; the sign could be flipped.
What is the normal? Since the normal is by definition perpendicular to the line, it must satisfy

$$
\left(n_{x}, n_{y}\right) \cdot\left(x_{1}-x_{0}, y_{1}-y_{0}\right)=0 .
$$

So we need a vector $\left(n_{x}, n_{y}\right)$ that satisfies that equation.
4. Here are three: $(-b, a, 0),(-c, 0, a),(0, c,-b)$. Note that these are generally not perpendicular to each other. Also note that any linear combination of these vectors will be perpendicular to $(a, b, c)$.

