Physics Internal Assessment

Investigating the Mpemba Effect: Why does hot water freeze faster than cool water?

Aim

The objective to this investigation is to determine the validity of the Mpemba Effect, a phenomenon in which hot water freezes faster than cool water, and to attempt to provide an explanation. For the purpose of this investigation, I will define the term "freeze" as when all liquid states of pure water transitions to solid state. The research question for this investigation is: How can the Mpemba Effect be justified?

Introduction

My interest in the Mpemba Effect began when I observed the phenomenon while preparing ice for a chemistry investigation. After searching online for an explanation of the effect, I was disappointed to find merely theories and assumptions. It is surprising that a phenomenon which seems so simple and so common has not been answered. Therefore, I am pleased to be able to use this opportunity to investigate the Mpemba Effect for my physics internal assessment and to attempt to produce my own explanation of the phenomenon from experimentation, analysis, and research. The conducting of this investigation has been both challenging and enlightening.

Background Information

The Mpemba Effect is defined as when there “exists a set of initial parameters (water mass, gas content of water, container shape and type, and refrigeration method), and a pair of temperatures, such that given two bodies of water identical in these parameters, and differing only in their temperatures, the hot one will freeze sooner.” This phenomenon was discovered in 1963 by Erasto Mpemba, and has sparked continuous discussion ever since the publication of his paper in 1969.

Through years of experimentation and analysis of the Mpemba Effect, many theories have been constructed, abolished, and debated, yet there has been no formal unanimous agreement upon a recognized explanation of this phenomenon. Among the many explanations proposed, I have concluded with three which I see to be the most reasonable, widely accepted, and relevant to the data collected in my experiment. The three explanations will be introduced and elaborated in the analysis.

Design and Methodology

Variables

The **independent variable** of the experiment is the initial temperature of water. The initial temperatures for this investigation are $20^{\circ c}$, $40^{\circ c}$, and $60^{\circ c}$.

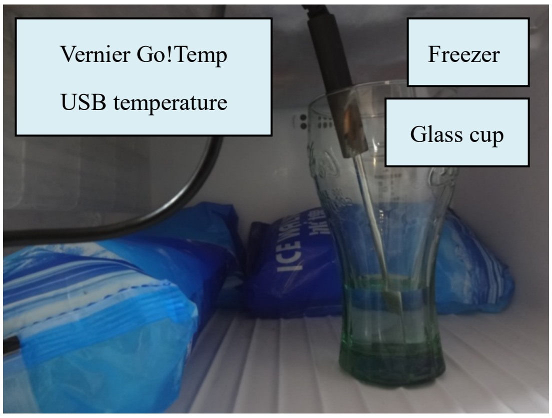
The **dependant variable** of the experiment is the time taken for the water to freeze. The freezing point may not be $0^{\circ c}$ exact due to the presence of supercooling.

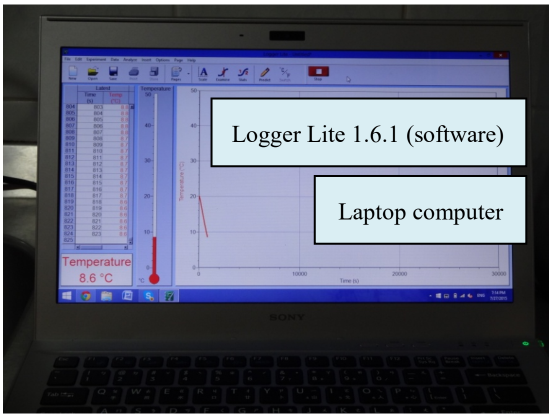
The **controlled variables** of the experiment are the freezing environment, the water contents, the water mass, and the surface area of water exposure. The controlled variables will be achieved by using the same freezer, using pure water, using 50 ml, and using the same glass cup, for all trials, accordingly.

Apparatus

- Freezer (1)
- Pure water (450ml)
- Glass cup (1)
- Measuring cup (1) (1000ml ±5)
- Gas stove (1)
- Metallic pot with lid (1)
- Oven gloves (1)
- Laptop computer (1)
- Vernier Go!Temp USB temperature probe (1)
- Logger Lite 1.6.1 (software)

Diagram 1: Apparatus setup





Method

1. Measure 50ml of pure water via the measuring cup.
2. Pour the water into the metallic pot and close the lid (for 40^{oc}, 60^{oc}).
3. Heat the water to the desired initial temperature via the gas stove (for 40^{oc}, 60^{oc}).
4. Pour the (heated) water into the glass cup.
5. Place the glass cup into a fixed position within the freezer.
6. Allow for the water to freeze, recording the temperature for 10800 seconds, equivalent to 3 hours.
7. Clean the apparatus.
8. Repeat Step 1 to Step 7 three more times.
9. Repeat Step 1 to Step 8 three more times, setting the desired initial temperature to 20^{oc}, 40^{oc}, 60^{oc}, by this order, for each cycle.

Data Collection

The recording of the temperature of water in the freezer is conducted by the Vernier Go!Temp USB temperature probe connected to the laptop computer with Logger Lite 1.6.1 in operation. Data is collected with an interval of 1 second, a response time of 4 seconds, an accuracy of $\pm 0.5^{oc}$, stated by Vernier, the manufacturer.

Safety Precaution

The experimenter is advised to wear insulating oven gloves while transferring heated water for safety. This investigation does not have significant ethical or environmental issues.

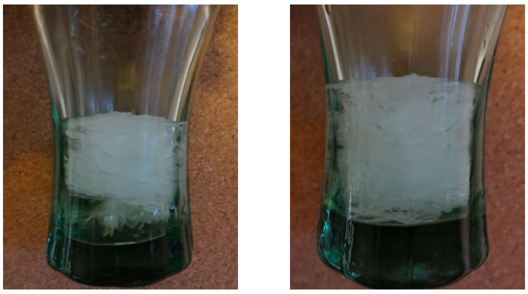
Results

Data were collected over a time span of 3 days. The total data collection time is 97200 seconds, equivalent to 27 hours.

Qualitative Data

Qualitative data were collected by the camera. The pictures were taken in approximately 5000 seconds after the start of the experiment. The experiments in which the pictures were taken were not accounted for quantitative data as the recording of qualitative data influenced the experimental procedure.

Diagram 2: Qualitative data



The left shows the qualitative results of the experiment conducted with an initial temperature of 20^{oc} at approximately 5000 seconds after the start of the experiment. The right shows the qualitative results of the experiment conducted with an initial temperature of 60^{oc} at approximately 5000 seconds after the start of the experiment.

Quantitative Data

Quantitative data were collected by the Vernier Go!Temp USB temperature probe. The following shows the quantitative data collected for each of the three trials of each of the three initial temperatures. It is worth noting that only 10 data points among the 10800 collected are shown for each trial for simplicity. The 10800 data points will be plotted to graph further on.

Table 1: Quantitative data of the first ten seconds with an initial temperature of 20°c

Time (sec ±1)	Temperature (°c ±0.5)			
	Trial 1	Trial 2	Trial 3	Average
0	20.0	20.0	20.0	20.0
1	20.0	20.0	20.0	20.0
2	20.0	20.0	20.0	20.0
3	19.9	19.9	19.9	19.9
4	19.9	19.9	19.9	19.9
5	19.9	19.9	19.9	19.9
6	19.9	19.9	19.9	19.9
7	19.9	19.9	19.9	19.9
8	19.9	19.9	19.9	19.9
9	19.9	19.9	19.8	19.9
10	19.8	19.9	19.8	19.8

Table 2: Quantitative data of the first ten seconds with an initial temperature of 40°c

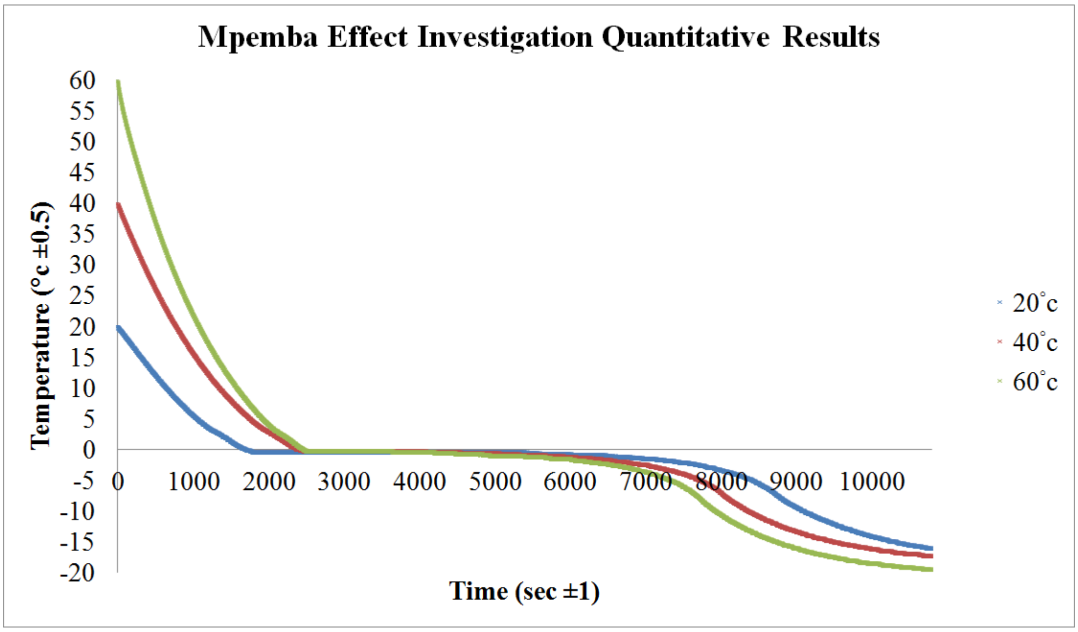
Time (sec ±1)	Temperature (°c ±0.5)			
	Trial 1	Trial 2	Trial 3	Average
0	40.0	40.0	40.0	40.0
1	40.0	40.0	39.9	40.0
2	39.9	30.9	39.9	36.9
3	39.9	39.9	39.9	39.9
4	39.9	39.8	39.8	39.8
5	39.9	39.8	39.8	39.8
6	39.8	39.7	39.8	39.8
7	39.8	39.7	39.7	39.7
8	39.7	39.7	39.7	39.7
9	39.7	39.6	39.6	39.6
10	39.7	39.6	39.6	39.6

Table 3: Quantitative data of the first ten seconds with an initial temperature of 60°c

Time (sec ±1)	Temperature (°c ±0.5)			
	Trial 1	Trial 2	Trial 3	Average
0	60.0	60.0	60.0	60.0
1	59.9	60.0	59.9	59.9
2	59.8	59.9	59.9	59.9
3	59.8	59.9	59.7	59.8
4	59.7	59.8	59.7	59.7
5	59.7	59.7	59.6	59.7
6	59.6	59.7	59.6	59.6
7	59.6	59.6	59.5	59.6
8	59.5	59.5	59.4	59.5
9	59.4	59.4	59.3	59.4
10	59.3	59.4	59.3	59.3

After calculating the average for the results of each of the three trials of distinct initial temperatures, the 32400 data points (10800 data points for each initial temperature) were plotted onto the graph below. The graph is plotted by temperature against time. Error bars are negligible in Graph 1 as they are insignificantly small (±0.5°c and ±1 sec). It is important to note that the curves are not lines of best fit nor created by joining data points. The curves are produced by the high frequency of data collection where each point of the curves display actual collected data points. Maximum and minimum slopes are not applicable to Graph 1. Data linearization is not applicable to Graph 1.

Graph 1: Mpemba Effect Investigation Quantitative Results



Raw data collection for this investigation was fairly time consuming (27 hours) while data processing was relatively straightforward, shown by Graph 1. Written explanations will be explored in further depth in the Analysis to make further use of the data collected and to bridge them to the objective of the research question.

Analysis

Considering the definition of the Mpemba Effect allows us to conclude that for the Mpemba Effect to occur, the following condition must be satisfied: As the hotter water reaches the initial temperature of the cooler water, it's properties must be distinct from those of the cooler water such that the rate of further cooling is increased or the temperature of the freezing point is significantly raised. If this condition is not met, the Mpemba Effect does not occur. This is because the rate of cooling is identical for two identical samples of water of identical temperature and that the hotter water must reach the initial temperature of the cooler water during the cooling process. I will be discussing the cause of the Mpemba Effect through the following three viewpoints:

1. Newton's Law of Cooling and the Boltzmann Distribution Curve
2. Convection and thermal conductivity
3. Supercooling and nucleation temperature

1. Newton's Law of Cooling and the Boltzmann Distribution Curve

Newton's Law of Cooling states that the rate of heat loss by a given mass is directly proportional to the temperature difference between the system and its surroundings, and can be expressed through the equation: $\frac{dT}{dt} = k(T - R)$, where T is the temperature of the given mass at time t, R is the temperature of the system's surroundings, and k is a constant of proportionality. By this law, we can deduce that the rate of cooling is faster for water of high initial temperature than water of a low initial temperature.

Diagram 3: Maxwell-Boltzmann Velocity Distribution

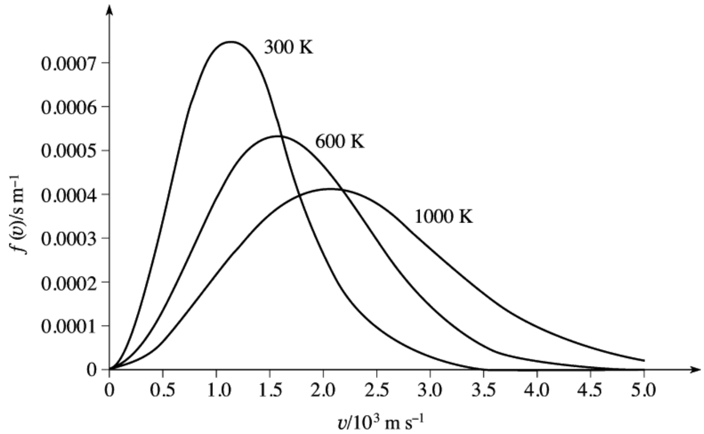


Diagram 3 shows the Maxwell-Boltzmann Velocity Distribution curves for temperatures of 300K, 600K, and 1000K. While the Maxwell-Boltzmann Distribution is usually applied to molecules of a gas, it can also be applied to the kinetic energy of the particles of a liquid, and in the context of this investigation, water. From the distribution, we can recognize as temperature increases, the average kinetic energy of the liquid increases, since $v \propto ke$. In addition, the number of particles with high kinetic energy also increases.

When heated water undergoes cooling process, the average kinetic energy of its particles decrease, but a large quantity of its high-energy particles remain. Conversely, the unheated water does not have the equivalent quantity of high-energy particles while the majority of its particles have the kinetic energy close to the system's low average. Therefore, the cooled heated water will have a higher radiation heat transfer rate in comparison to the unheated cool water of the same temperature. The Mpemba Effect is thus induced, where the heated water experiences a complete frozen state slower than the unheated water.

2. Convection and thermal conductivity

The Mpemba Effect is observed by the state change of water from liquid to solid, achieved by cooling its temperature. However, we can also view the process of cooling water as decreasing the kinetic energy of its molecules. As temperature decreases, the molecules vibrate closer and closer until motion is decreased sufficiently for water to change state. Thus, we can say that freezing water is not only about reducing its temperature to 0°c, but also the process of turning water into ice crystals – that the Mpemba Effect can be more easily explained by investigating how water crystallizes during the experiment as to investigating its thermodynamics.

Diagram 4: Comparison of heat gradient induced convection currents

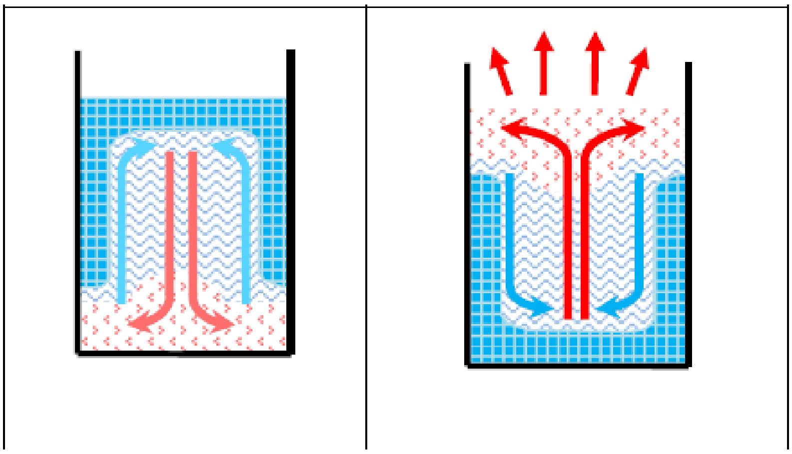


Diagram 4 shows the comparison of heat gradient induced convection currents between cool water (left) and hot water (right). In cool water, the heat gradient is relatively less than that of hot water. Thus, cool water undergoes slow freezing and an insulative ice cap is formed on the upper surface of the container (shown on left) as heat is lost from the surface. The remaining water, however, does not have sufficient heat to melt the ice cap before it reforms. The location of cooling moves to the bottom of the container, the secondary location of heat loss, until the process repeats and an insulative layer of ice is formed on the bottom of the container as well. Therefore, the water in the center of the container crystallizes the last. Conversely, the insulative ice cap on the surface of hot water is melted once formed or does not form due to the relatively large heat gradient between water and the freezing environment (shown on right). The surface of the water acts as the primary location of cooling throughout the entire cooling process as hot water continuously rises, leading to ice formation from the bottom of the container, without the presence of an insulative layer of frost on the surface. It is to be noted that the initial temperatures affect the systems' heat gradient. Thus, when hot water is cooled to the initial temperature of cool water, its heat gradient differs from that of cool water.

Therefore, it can be said that even though cool water begins freezing before hot water, hot water finishes the process before cold water, supporting my definition of "freeze" stated in the Introduction. This is evident from Diagram 2 of qualitative data, identifying a small volume of unfrozen water near the bottom of the container. Furthermore, this is also apparent from Graph 1, through the blue curve reaching $0^{\circ c}$ before the green curve, yet leaving $0^{\circ c}$ after the green curve, indicating that water of an initial temperature of $20^{\circ c}$ took longer to undergo full state change than water of an initial temperature of $60^{\circ c}$. The regions of zero gradients of the curves in Graph 1 indicate the state change of water from liquid to solid.

3. Supercooling and nucleation temperature

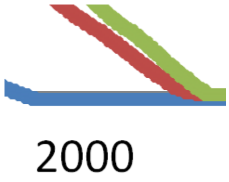
Supercooling, in the context of this investigation, is the process of lowering the temperature of water below its freezing point without the occurrence of crystallization. Water crystallizes under the presence of a seed crystal or nucleus, an impurity, which a crystal structure forms around, creating a solid, ice. This form of crystallization is known as heterogeneous nucleation. However, when any form of nucleus is absent, the liquid phase of water is maintained until crystal homogenous nucleation occurs. While it is known for water to freeze at $0^{\circ c}$, extremely pure water can be supercooled to nearly $-48.3^{\circ c}$ under standard pressure before homogenous crystallization.

From Graph 1, we can identify water to begin freezing at approximately $-0.3^{\circ c}$ and $-0.1^{\circ c}$ (for different initial temperatures), where the gradient of the curves approach zero. Therefore, even though the use of pure water was intended for the conducting of this investigation, the presence of the impurities in the experiment cannot be ignored.

When the freezing point is lowered, the time required for water to reach this temperature considerably increases, as the rate of cooling diminishes significantly during the final few degrees near freezing point. Therefore, a slight change in the impurity of water may influence the time taken for it to freeze greatly.

It is reasonable to assume that the impurity of hot water is higher than that of cool water. This is due to the experimental procedure of the investigation. Water for initial temperatures of $40^{\circ c}$ and $60^{\circ c}$ were heated by pouring it into a metal pot (closed with a lid) and heated by the fire of the gas stove, while water for an initial temperature of $20^{\circ c}$ had not gone through such process. Thus, we can suspect that heated water were contaminated in the process, leading to a slight upward shift in freezing point, decreasing the time required for the water to freeze. This can effect can be identified in an enlarged diagram of Graph 1, Diagram 5.

Diagram 5: Evidence of nucleation temperature variation



From Diagram 5, the blue curve indicates water of an initial temperature of $20^{\circ c}$ to begin freezing at approximately $-0.3^{\circ c}$, while the red curve and the green curve indicates water of an initial temperature of $40^{\circ c}$ and $60^{\circ c}$ to begin freezing at approximately $-0.1^{\circ c}$.

Yet although the explanation of supercooling and nucleation temperature is applicable to this investigation, it does not give a universal explanation to the Mpemba Effect. In fact, the heating of water up to temperatures of $105^{\circ c}$ may actually shift the freezing point downwards, as the water is now more pure. Thus, the explanation of supercooling and nucleation temperature suggests that the Mpemba Effect may or may not occur under different methods of experimentation, and is explanatory to the lack of reproducibility of the phenomenon and why it is still debated within the science community.

Conclusion

From the quantitative and qualitative data collected in this investigation, the validity of the Mpemba Effect is justified. Through theoretical induction and correlation with the experiment's results in the analysis, we can conclude the cause for the Mpemba Effect as the following:

1. When hot water is cooled to the initial temperature of cool water, its properties are different from those of the cool water, leading to distinct cooling rates. In comparison, the cooled hot water has a higher radiation heat transfer rate due to the presence relatively more high-energy particles.

2. In addition, the cooling time for water of high initial temperature is relatively shorter due to the system's high heat gradient and absence of an insulative layer of ice on the surface of the water and container during the cooling process.

3. Furthermore, the nucleation temperature of water may be varied through preheating, influenced by the nature of impurities of the sample and container, and when the nucleation temperature is shifted upwards, the Mpemba Effect is induced.

Evaluation

Preliminary Experiment

Before conducting the experiment via the procedure stated within this investigation, I conducted a preliminary experiment by a different method which was unsuccessful. Instead of using a freezer for cooling, I placed the glass cup filled with pure water into a bath of ice cubes. I initially presumed that this method would allow the water to freeze faster. However, this method failed because the ice cubes melted before water within the glass cup reached freezing point, causing water to not only not freeze, but to gradually warm once a sufficient amount of ice cubes have melted. I later modified my experimental procedure to the one used for this investigation.

Limitations and Errors

The data collected were both reasonably accurate and precise as they were collected through a data logger with relatively constant experimental environments. Nonetheless, the limitations of errors are present and may be categorized into random errors and systematic errors. The calculation of total percentage uncertainty is unnecessary for this investigation.

Random Error

The random error of the experiment may be identified as the human reaction time of transferring the heated water into the freezer. The delays of transferring heated water into the freezer may lead to slight downward shift in the initial temperatures of heated water.

Systematic Errors

A systematic error of the experiment may be identified as the limitations of the freezer. When inputting the heated water into the freezer, the freezer's environment experiences a slight shift upwards in temperature before shifting back to its controlled temperature. This causes slight errors in initial cooling rates between different initial temperatures of water.

Other systematic errors of the experiment may be identified as the apparatus uncertainties of the experiment, including the uncertainties of the measuring cup (±5ml) and the Vernier Go!Temp USB temperature probe ($\pm 0.5^{oc}$ and ±0.1 sec).

Improvement and Extension

To improve my experiment by minimizing its uncertainties and errors, I should improve the accuracies of my apparatus and lower the time period of transferring the heated water into the freezer via either a closer distance between the heater and the freezer or by transferring the heated water by an instantaneous automated machine. Furthermore, I should switch my apparatus from a household freezer to a laboratory or industrial freezer to decrease temperature fluctuation errors.

As an extension to my experiment, I would like to investigate on developing a methodology that guarantees the occurrence of the Mpemba Effect, which addresses explanation 3 in the Analysis. My current thoughts upon this extension are to determine the maximum or minimum temperatures in which the water should be heated to in reference to effects of water supercooling and nucleation temperature.

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