4.3 BIOT-SAVART LAW

3. *State and explain Biot-Savart law for the magnetic field produced by a current element. Define the SI unit of magnetic field from this law.*

Biot-Savart law. Oersted experiment showed that a current carrying conductor produces a magnetic field around it. It is convenient to assume that this field is made of contributions from different segments of the conductor, called *current elements*. A current element is denoted by $\vec{dl}$, which has the same direction as that of current I. From a series of experiments on current carrying conductors of simple shapes, two French physicists *Jean-Baptiste Biot* and *Felix Savart*, in 1820, deduced an expression for the magnetic field of a current element which is known as Biot-Savart law.

Statement. As shown in Fig. 4.3, consider a current element $\vec{dl}$ of a conductor XY carrying current I. Let P be the point where the magnetic field $\vec{dB}$ due to the current element $\vec{dl}$ is to be calculated. Let the position vector of point P relative to element $\vec{dl}$ be $\vec{r}$. Let θ be the angle between $\vec{dl}$ and $\vec{r}$.

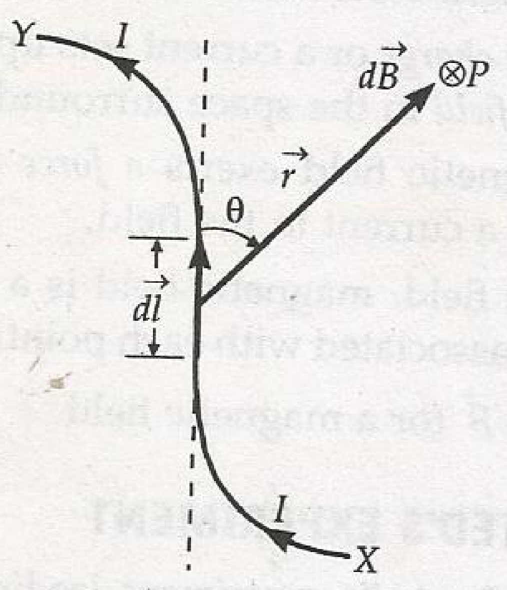

Fig. 4.3 Biot-Savart law.

According to *Biot-Savart law*, the magnitude of the field $\vec{dB}$ is

1. *directly proportional to the current I through the conductor,*

$$dB \propto I$$

2. *directly proportional to the length dl of the current element,*

$$dB \propto dl$$

3. *directly proportional to* $\sin\theta$,

$$dB \propto \sin\theta$$

4. *inversely proportional to the square of the distance r of the point P from the current element,*

$$dB \propto \frac{1}{r^2}$$

Combining all these four factors, we get

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

or
$$dB = K.\frac{I\,dl\sin\theta}{r^2}$$

The proportionality constant K depends on the medium between the observation point P and the current element and the system of units chosen. For free space and in SI units,

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{T\,mA^{-1}} \quad (\text{or } \mathrm{Wbm^{-1}A^{-1}})$$

Here μ_0 is a constant called *permeability* of free space. So the Biot-Savart law in SI units may be expressed as

$$dB = \frac{\mu_0}{4\pi}.\frac{I\,dl\sin\theta}{r^2}$$

We can write the above equation as

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3}$$

As the direction of $\vec{dB}$ is perpendicular to the plane of $\vec{dl}$ and $\vec{r}$, so from the above equation, we get the *vector form of the Biot-Savart law* as

$$\vec{dB} = \frac{\mu_0 I}{4\pi}\frac{\vec{dl}\times\vec{r}}{r^3}$$

4.5 MAGNETIC FIELD DUE TO A LONG STRAIGHT CURRENT CARRYING CONDUCTOR

6. *Apply Biot-Savart law to derive an expression for the magnetic field produced at a point due to the current flowing through a straight wire of infinite length. Also draw the sketch of the magnetic field. State the rules used for finding the direction of this magnetic field.*

Magnetic field due to a long straight current carrying conductor. As shown in Fig. 4.7, consider a straight conductor XY carrying current I. We wish to find its magnetic field at the point P whose perpendicular distance from the wire is a *i.e.*, $PQ = a$.

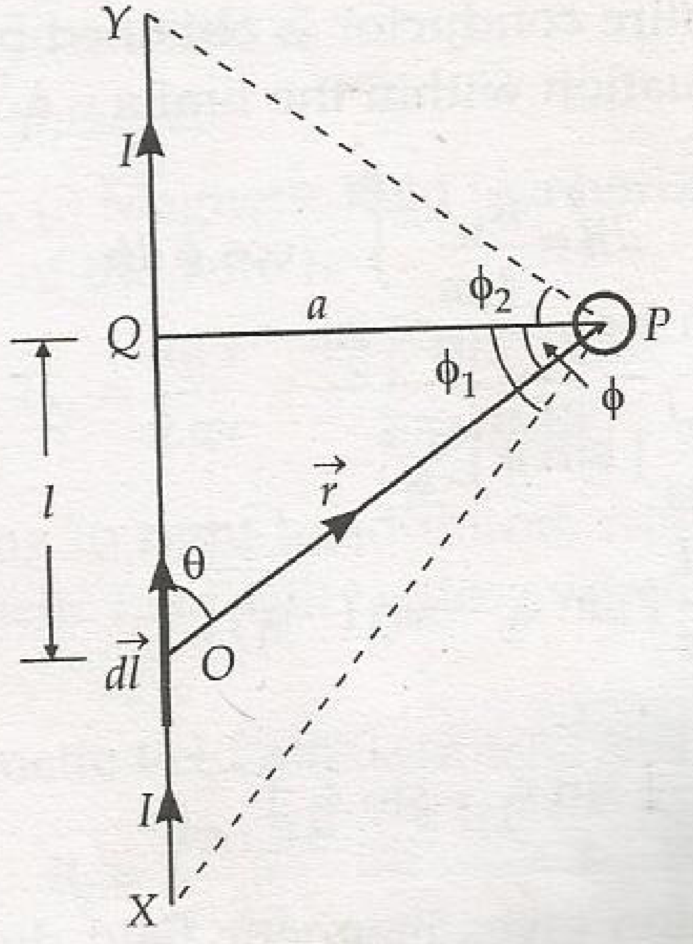

Fig. 4.7 Magnetic field due to a straight current carrying conductor.

Consider a small current element $\vec{dl}$ of the conductor at O. Its distance from Q is l *i.e.*, $OQ = l$. Let $\vec{r}$ be the position vector of point P relative to the current element and θ be the angle between $\vec{dl}$ and $\vec{r}$. According to Biot-Savart law, the magnitude of the field $\vec{dB}$ due to the current element $\vec{dl}$ will be

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2}$$

From right $\Delta\, OQP$,

$$\theta + \phi = 90°$$

or $$\theta = 90° - \phi$$

$$\therefore \quad \sin\theta = \sin(90° - \phi) = \cos\phi$$

Also $$\cos\phi = \frac{a}{r}$$

or $$r = \frac{a}{\cos\phi} = a \sec\phi$$

As $$\tan\phi = \frac{l}{a}$$

$$\therefore \quad l = a\tan\phi$$

On differentiating, we get

$$dl = a\sec^2\phi\, d\phi$$

Hence $$dB = \frac{\mu_0}{4\pi} \frac{I(a\sec^2\phi\, d\phi)\cos\phi}{a^2\sec^2\phi}$$

or $$dB = \frac{\mu_0 I}{4\pi a}\cos\phi\, d\phi.$$

According to right hand rule, the direction of the magnetic field at the P due to all such current elements will be in the same direction, namely, normally into the plane of paper. Hence the total field $\vec{B}$ at the point P due to the entire conductor is obtained by integrating the above equation within the limits $-\phi_1$ and ϕ_2.

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0 I}{4\pi a}\int_{-\phi_1}^{\phi_2} \cos\phi\, d\phi$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi]_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi_2 - \sin(-\phi_1)]$$

or $$B = \frac{\mu_0 I}{4\pi a}[\sin\phi_1 + \sin\phi_2]$$

This equation gives magnetic field due to a finite wire in terms of the angles subtended at the observation point by the ends of the wire.

Special Cases

1. *If the conductor XY is infinitely long and the point P lies near the middle of the conductor,* then $\phi_1 = \phi_2 = \pi/2$.

$$\therefore \quad B = \frac{\mu_0 I}{4\pi a}[\sin 90° + \sin 90°]$$

or

$$B = \frac{\mu_0 I}{2\pi a}$$

2. *If the conductor XY is infinitely long but the point P lies near the end Y (or X),* then $\phi_1 = 90°$ and $\phi_2 = 0°$.

$$\therefore \quad B = \frac{\mu_0 I}{4\pi a}[\sin 90° + \sin 0°] = \frac{\mu_0 I}{4\pi a}.$$

Clearly, the magnetic field due to an infinitely long straight current carrying conductor at its one end is just half of that at any point near its middle, provided the two points are at the same perpendicular distance from the conductor.

3. *If the conductor is of finite length L and the point P lies on its perpendicular bisector,* then $\phi_1 = \phi_2 = \phi$ and

$$\sin\phi = \frac{L/2}{\sqrt{a^2 + (L/2)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

$$\therefore \quad B = \frac{\mu_0 I}{4\pi a}[\sin\phi + \sin\phi]$$

$$= \frac{\mu_0 I}{4\pi a} \cdot \frac{2L}{\sqrt{4a^2 + L^2}}$$

or

$$B = \frac{\mu_0 IL}{2\pi a\sqrt{4a^2 + L^2}}.$$

4.6 MAGNETIC FIELD AT THE CENTRE OF CIRCULAR CURRENT LOOP

7. *Apply Biot-Savart law to derive an expression for the magnetic field at the centre of a current carrying circular loop.*

Magnetic field at the centre of a circular current loop. As shown in Fig. 4.23, consider a circular loop of wire of radius r carrying current I. We wish to calculate its magnetic field at the centre O. The entire loop can be divided into a large number of small current elements.

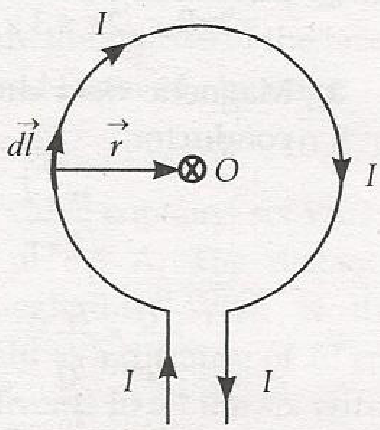

Fig. 4.23 Magnetic field at the centre of a circular current loop.

Consider a current element $\vec{dl}$ of the loop. According to Biot-Savart law, the magnetic field at the centre O due to this element is

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3}$$

The field at point O points normally into the plane of paper, as shown by encircled cross ⊗. The direction of $\vec{dl}$ is along the tangent, so $\vec{dl} \perp \vec{r}$. Consequently, the magnetic field at the centre O due to this current element is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin 90°}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2}$$

The magnetic field due to all such current elements will point into the plane of paper at centre O. Hence the total magnetic field at the centre O is

$$B = \int dB = \int \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2} = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi r^2} \cdot l = \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r$$

or $$B = \frac{\mu_0 I}{2r}.$$

If instead of a single loop, there is a coil of N turns, all wound over one another, then

$$B = \frac{\mu_0 N I}{2a}.$$

4.7 MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

8. *Apply Biot-Savart law to find the magnetic field due to a circular current carrying loop at a point on the axis of the loop. State the rules used to find the direction of this magnetic field.*

Magnetic field along the axis of a circular current loop. Consider a circular loop of wire of radius a and carrying current I, as shown in Fig. 4.24. Let the plane of the loop be perpendicular to the plane of paper. We wish to find field $\vec{B}$ at an axial point P at a distance r from the centre C.

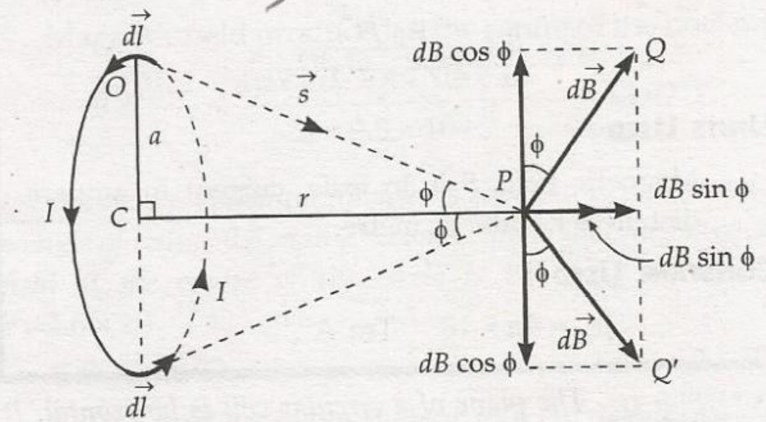

Fig. 4.24 Magnetic field on the axis of a circular current loop.

Consider a current element $\vec{dl}$ at the top of the loop. It has an outward coming current.

If $\vec{s}$ be the position vector of point P relative to the element $\vec{dl}$, then from Biot-Savart law, the field at point P due to the current element is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{s^2}$$

Since $\vec{dl} \perp \vec{s}$, *i.e.*, $\theta = 90°$, therefore

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{s^2}$$

The field $d\vec{B}$ lies in the plane of paper and is perpendicular to $\vec{s}$, as shown by $\vec{PQ}$. Let ϕ be the angle between OP and CP. Then dB can be resolved into *two* rectangular components.

1. $dB \sin\phi$ along the axis,
2. $dB \cos\phi$ perpendicular to the axis.

For any two diametrically opposite elements of the loop, the components perpendicular to the axis of the loop will be equal and opposite and will cancel out. Their axial components will be in the same direction, *i.e.*, along CP and get added up.

∴ Total magnetic field at the point P in the direction CP is

$$B = \int dB \sin\phi$$

But $\sin\phi = \frac{a}{s}$ and $dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{s^2}$

$$\therefore \quad B = \int \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{s^2} \cdot \frac{a}{s}$$

Since μ_0 and I are constant, and s and a are same for all points on the circular loop, we have

$$B = \frac{\mu_0 Ia}{4\pi s^3} \int dl = \frac{\mu_0 Ia}{4\pi s^3} . 2\pi a = \frac{\mu_0 Ia^2}{2s^3}$$

$$[\because \int dl = \text{circumference} = 2\pi a]$$

or $$B = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \qquad [\because s = (r^2 + a^2)^{1/2}]$$

As the direction of the field is along +ve X-direction, so we can write

$$\vec{B} = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}} \hat{i}$$

If the coil consists of N turns, then

$$B = \frac{\mu_0 N Ia^2}{2(r^2 + a^2)^{3/2}}$$

Special Cases

1. *At the centre of the current loop*, $r = 0$, therefore

$$B = \frac{\mu_0 N Ia^2}{2a^3} = \frac{\mu_0 NI}{2a}$$

or $$B = \frac{\mu_0 \, N \, IA}{2\pi a^3}$$

where $A = \pi a^2$ = area of the circular current loop. The field is directed perpendicular to the plane of the current loop.

2. *At the axial points lying far away from the coil*, $r >> a$, so that

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 \, N IA}{2\pi r^3}$$

This field is directed along the axis of the loop and falls off as the cube of the distance from the current loop.

3. *At an axial point at a distance equal to the radius of the coil i.e.*, $r = a$, we have

$$B = \frac{\mu_0 \, N I a^2}{2\,(a^2 + a^2)^{3/2}} = \frac{\mu_0 \, NI}{2^{5/2} \, a}.$$

Proof for a straight current carrying conductor. Consider an infinitely long straight conductor carrying a current I. From Biot-Savart law, the magnitude of the magnetic field $\vec{B}$ due to the current carrying conductor at a point, distant r from it is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

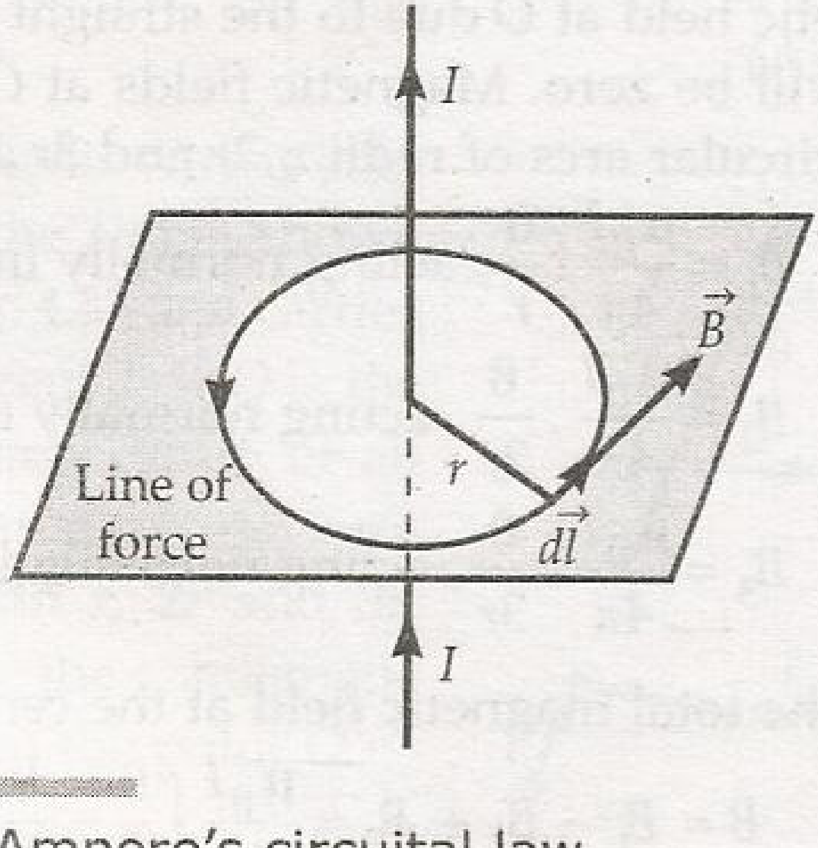

Fig. 4.45 Ampere's circuital law.

As shown in Fig. 4.45, the field $\vec{B}$ is directed along the circumference of the circle of radius r with the wire as centre. The magnitude of the field $\vec{B}$ is same for all points on the circle. To evaluate the line integral of the magnetic field $\vec{B}$ along the circle, we consider a small current element $\vec{dl}$ along the circle. At every point on the circle, both $\vec{B}$ and $\vec{dl}$ are tangential to the circle so that the angle between them is zero.

$$\therefore \qquad \vec{B} \cdot \vec{dl} = B\,dl \cos 0° = B\,dl$$

Hence the line integral of the magnetic field along the circular path is

$$\oint \vec{B} \cdot \vec{dl} = \oint B\,dl = B \oint dl = \frac{\mu_0 I}{2\pi r} \cdot l$$

$$= \frac{\mu_0 I}{2\pi r} \cdot 2\pi r$$

$$\therefore \oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

This proves Ampere's law. This law is valid for any assembly of current and for any arbitrary closed loop.

Application of Ampere's law to a straight conductor. Fig. 4.46 shows a circular loop of radius r around an infinitely long straight wire carrying current I. As the field lines are circular, the field $\vec{B}$ at any point of the circular loop is directed along the tangent to the

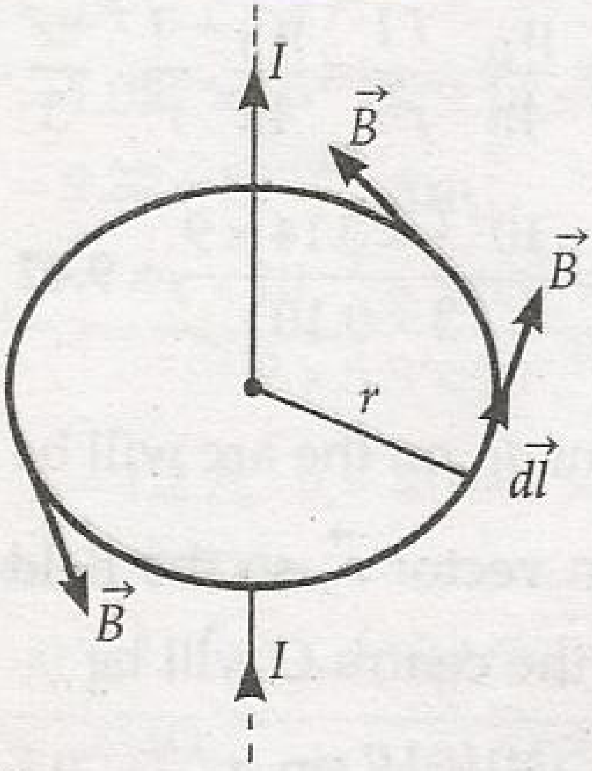

Fig. 4.46

circle at that point. By symmetry, the magnitude of field $\vec{B}$ is same at every point of the circular loop. Therefore,

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0° = B \oint dl = B \cdot 2\pi r$$

From Ampere's circuital law,

$$B \cdot 2\pi r = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

Calculation of magnetic field inside a long straight solenoid. The magnetic field inside a closely wound long solenoid is uniform everywhere and zero

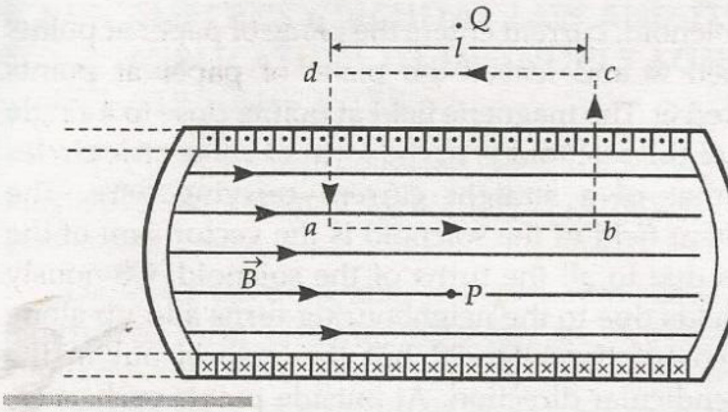

Fig. 4.50 The magnetic field of a very long solenoid.

outside it. Fig. 4.50 shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked $\odot$ and enters the plane of paper at points marked $\otimes$. To determine the magnetic field $\vec{B}$ at any inside point, consider a rectangular closed path *abcd* as the Amperean loop. According to Ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl}$$

$$= \mu_0 \times \text{Total current through the loop } abcd$$

$$\text{Now} \oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a \vec{B} \cdot \vec{dl}$$

$$\text{But} \int_b^c \vec{B} \cdot \vec{dl} = \int_b^c B\, dl \cos 90° = 0$$

$$\int_d^a \vec{B} \cdot \vec{dl} = \int_d^a B\, dl \cos 90° = 0$$

$$\int_c^d \vec{B} \cdot \vec{dl} = 0$$

as $B = 0$ for points outside the solenoid.

$$\therefore \quad \oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl}$$

$$= \int_a^b B\, dl \cos 0° = B \int_a^b dl = Bl$$

where,

l = length of the side *ab* of the rectangular loop *abcd*.

Let number of turns per unit length of the solenoid $= n$

Then number of turns in length l of the solenoid

$$= nl$$

Thus the current I of the solenoid threads the loop *abcd*, nl times.

$\therefore$ Total current threading the loop $abcd = nlI$

Hence $\quad Bl = \mu_0 nlI \quad$ or $\quad B = \mu_0 nI$

Magnetic field due to a toroidal solenoid. *A solenoid bent into the form of a closed ring is called a toroidal solenoid.* Alternatively, it is an anchor ring (torous) around which a large number of turns of a metallic wire are wound, as shown in Fig. 4.52. We shall see that the magnetic field $\vec{B}$ has a constant magnitude everywhere inside the toroid while it is zero in the open space interior (point P) and exterior (point Q) to the toroid.

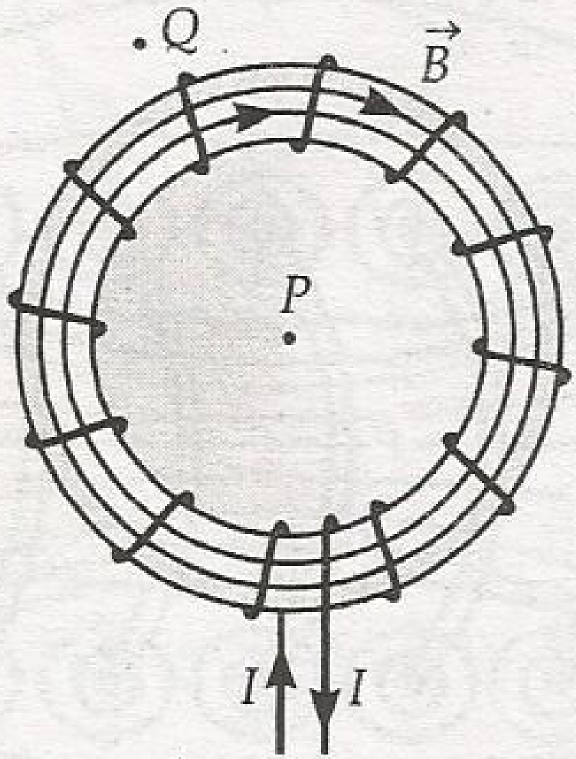

Fig. 4.52 A toroidal solenoid.

Fig. 4.53 shows a sectional view of the toroidal solenoid. The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperean loops are shown by

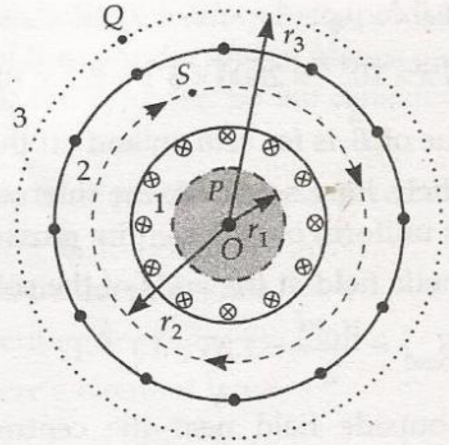

Fig. 4.53 A sectional view of the toroidal solenoid.

dashed lines. By symmetry, the magnetic field should be tangential to them and constant in magnitude for each of the loops.

1. *For points in the open space interior to the toroid.* Let B_1 be the magnitude of the magnetic field along the Amperean loop 1 of radius r_1.

Length of the loop 1, $L_1 = 2\pi r_1$

As the loop encloses no current, so $I = 0$

Applying Ampere's circuital law,

$$B_1 L_1 = \mu_0 I$$

or $$B_1 \times 2\pi r_1 = \mu_0 \times 0$$

or $$B_1 = 0$$

Thus the magnetic field at any point P in the open space interior to the toroid is zero.

2. *For points inside the toroid.* Let B be the magnitude of the magnetic field along the Amperean loop 2 of radius r.

Length of loop 2, $L_2 = 2\pi r$

If N is the total number of turns in the toroid and I the current in the toroid, then total current enclosed by the loop 2 = NI

Applying Ampere's circuital law,

$$B \times 2\pi r = \mu_0 \times NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

If r be the average radius of the toroid and n the number of turns per unit length, then

$$N = 2\pi r n$$

$\therefore$ $$B = \mu_0 nI$$

3. *For points in the open space exterior to the toroid.* Each turn of the toroid passes twice through the area enclosed by the Amperean loop 3. But for each turn, the current coming out of the plane of paper is cancelled by the current going into the plane of paper. Thus, $I = 0$ and hence $B_3 = 0$.

4.20 MOVING COIL GALVANOMETER

23. *Describe the principle, construction and working of a pivoted-type moving coil galvanometer. Define its figure of merit.*

Moving coil galvanometer. *A galvanometer is a device to detect current in a circuit.* The commonly used moving coil galvanometer is named so because it uses a current-carrying coil that rotates (or moves) in a magnetic field due to the torque acting on it.

In a *D'Arsonval galvanometer*, the coil is suspended on a phosphor-bronze wire. It is highly sensitive and requires careful handling. In *Weston galvanometer*, the coil is pivoted between two jewellel bearings. It is rugged and portable though less sensitive, and is generally used in laboratories. The basic principle of both types of galvanometers is same.

Principle. The operating principle of a moving coil galvanometer is that *a current-carrying coil placed in a magnetic field experiences a torque, the magnitude of which depends on the strength of current.*

Construction. As shown in Fig. 4.93, a Weston (pivoted-type) galvanometer consists of a rectangular coil of fine insulated copper wire wound on a light non-magnetic metallic (aluminium) frame. The two ends of the axle of this frame are pivoted between two jewelled bearings. The motion of the coil is controlled by a pair of hair springs of phosphor-bronze. The inner ends of the springs are soldered to the two ends of the coil and the outer ends are connected to the binding screws. The springs provide the restoring torque and serve as current leads. A light aluminium pointer attached to the coil measures its deflection on a suitable scale.

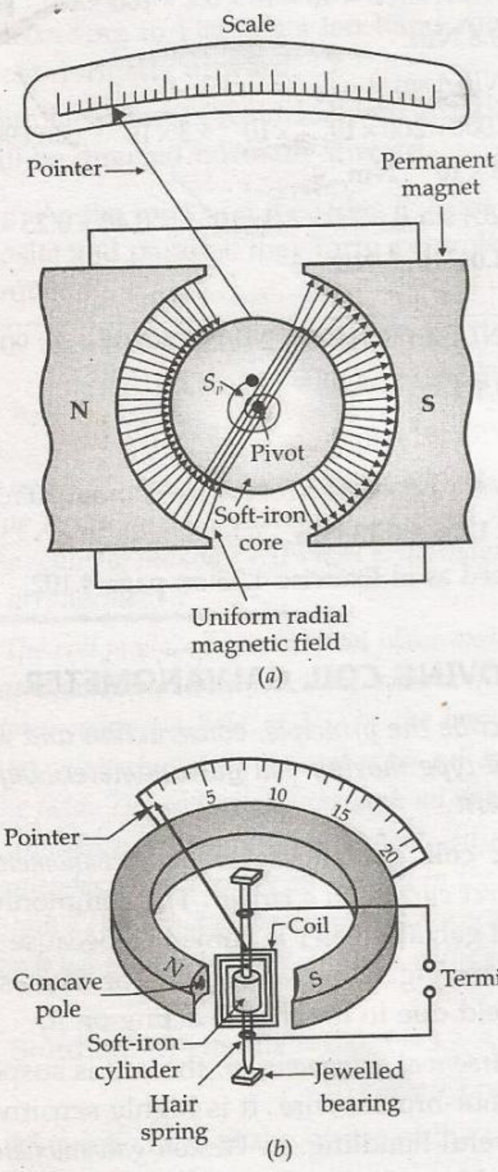

Fig. 4.93 (*a*) Top view (*b*) Front view of a pivoted-type galvanometer.

The coil is symmetrically placed between the cylindrical pole pieces of a strong permanent horse-shoe magnet.

A cylindrical soft iron core is mounted symmetrically between the concave poles of the horse-shoe magnet. This makes the lines of force pointing along the radii of a circle. Such a field is called a ***radial field***. The plane of a coil rotating in such a field remains parallel to the field in all positions, as shown in Fig. 4.93(*a*). Also, the soft iron cylinder, due to its high permeability, intensifies the magnetic field and hence increases the sensitivity of the galvanometer.

Theory and Working. In Fig. 4.94(*a*), we have

I = current flowing through the coil $PQRS$

a, b = sides of the rectangular coil $PQRS$

$A = ab$ = area of the coil

N = number of turns in the coil.

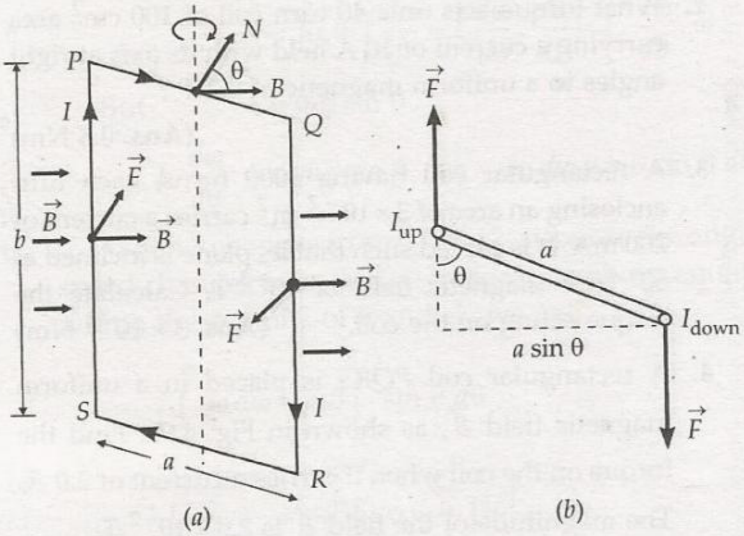

Fig. 4.94 (*a*) Rectangular loop *PQRS* in a uniform magnetic field. (*b*) Top view of the loop.

Since the field is radial, the plane of the coil always remains parallel to the field $\vec{B}$. The magnetic forces on sides PQ and SR are equal, opposite and collinear, so their resultant is zero. According to Fleming's left rule, the side PS experiences a normal inward force equal to $NIbB$ while the side QR experiences an equal normal outward force. The two forces on sides PS and QR are equal and opposite. They form a couple and exert a torque given by

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$= NIbB \times a \sin 90° = NIB(ab) = NIBA$$

Here $\theta = 90°$, because the normal to the plane of coil remains perpendicular to the field $\vec{B}$ in all positions.

The torque τ deflects the coil through an angle α. A restoring torque is set up in the coil due to the elasticity of the springs such that

$$\tau_{\text{restoring}} \propto \alpha \quad \text{or} \quad \tau_{\text{restoring}} = k\alpha$$

where k is the ***torsion constant*** of the springs *i.e.*, torque required to produce unit angular twist. In equilibrium position,

Restoring torque = Deflecting torque

$$k\alpha = NIBA$$

or $$\alpha = \frac{NBA}{k} \cdot I$$

or $$\alpha \propto I$$

Thus the deflection produced in the galvanometer coil is proportional to the current flowing through it. Consequently, the instrument can be provided with a scale with equal divisions along a circular scale to indicate equal steps in current. Such a scale is called ***linear scale.***

Also, $$I = \frac{k}{NBA} \cdot \alpha = G\alpha$$

The factor $G = k / NBA$ is constant for a galvanometer and is called ***galvanometer constant*** or ***current reduction factor*** of the galvanometer.

Figure of merit of a galvanometer. *It is defined as the current which produces a deflection of one scale division in the galvanometer and is given by*

$$G = \frac{I}{\alpha} = \frac{k}{NBA}$$

4.21 SENSITIVITY OF A GALVANOMETER

24. *When is a galvanometer said to be sensitive ? Define current sensitivity and voltage sensitivity of a galvanometer. State the factors on which the sensitivity of a moving coil galvanometer depends. How can we increase the sensitivity of a galvanometer ?*

Sensitivity of a galvanometer. *A galvanometer is said to be sensitive if it shows large scale deflection even when a small current is passed through it or a small voltage is applied across it.*

Current sensitivity. *It is defined as the deflection produced in the galvanometer when a unit current flows through it.*

Current sensitivity, $$I_S = \frac{\alpha}{I} = \frac{NBA}{k}$$

Voltage sensitivity. *It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.*

Voltage sensitivity, $$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{k\,R}$$

Clearly, $$\text{voltage sensitivity} = \frac{\text{Current sensitivity}}{R}$$

4.23 CONVERSION OF A GALVANOMETER INTO AN AMMETER

26. *Explain how can we convert a galvanometer into an ammeter of given range.*

Conversion of a galvanometer into an ammeter. *An ammeter is a device used to measure current through a circuit element.* To measure current through a circuit element, an ammeter is connected in *series* with that element so that the current which is to be measured actually passes through it. In order to ensure that its insertion in the circuit does not change the current, an ammeter should have zero resistance. So ammeter is designed to have very small effective resistance. In fact, *an ideal ammeter should have zero resistance.*

An ordinary galvanometer is a sensitive instrument. It gives full scale deflection with a small current of few microamperes. To measure large currents with it, a small resistance is connected in parallel with the galvanometer coil. The resistance connected in this way is called a ***shunt***. Only a small part of the total current passes through the galvanometer and remaining current passes through the shunt. The value of shunt resistance depends on the range of the current required to be measured.

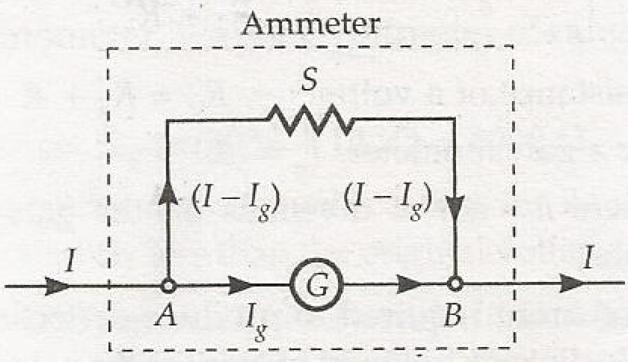

Fig. 4.95

Let G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - I$ = the required current range of the ammeter

S = shunt resistance

$I - I_g$ = current through the shunt.

As galvanometer and shunt are connected in parallel, so

P.D. across the galvanometer = P.D. across the shunt

$$I_g G = (I - I_g) S$$

or $$S = \frac{I_g}{I - I_g} \times G$$

So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Moreover,

$$I_g = \frac{S}{G + S} \times I$$

The deflection in the galvanometer is proportional to I_g and hence to I. So the scale can be graduated to read the value of current I directly.

Hence *an ammeter is a shunted or low resistance galvanometer*. Its effective resistance is

$$R_A = \frac{GS}{G + S} < S$$

Conversion of a galvanometer into a voltmeter. *A voltmeter is a device for measuring potential difference across any two points in a circuit.* It is connected in parallel with the circuit element across which the potential difference is intended to be measured. As a result, a small part of the total current passes through the voltmeter and so the current through the circuit element decreases. This decreases the potential difference required to be measured. To avoid this, the voltmeter should be designed to have very high resistance. In fact, *an ideal voltmeter should have infinite resistance.*

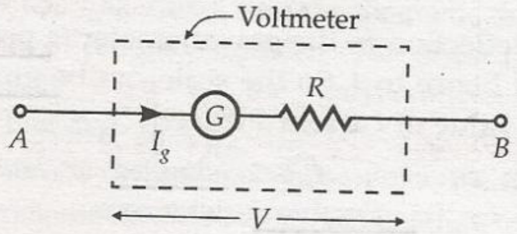

Fig. 4.96

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The value of this resistance is so adjusted that only current I_g which produces full scale deflection in the galvanometer, passes through the galvanometer.

Let

G = resistance of the galvanometer.

I_g = the current with which galvanometer gives full scale deflection

$0 - V$ = required range of the voltmeter, and

R = the high series resistance which restricts the current to safe limit I_g.

$\therefore$ Total resistance in the circuit = $R + G$

By Ohm's law,

$$I_g = \frac{\text{Potential difference}}{\text{Total resistance}} = \frac{V}{R+G}$$

or $$R + G = \frac{V}{I_g} \quad \text{or} \quad R = \frac{V}{I_g} - G$$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Moreover, the deflection in the galvanometer is proportional to current I_g and hence to V. The scale can be graduated to read the value of potential difference directly.

Hence *a voltmeter is a high resistance galvanometer*. Its effective resistance is

$$R_V = R + G >> G.$$

4.19 TORQUE EXPERIENCED BY A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

22. *Derive an expression for the torque acting on a current carrying loop suspended in a uniform magnetic field.*

Torque on a current loop in a uniform magnetic field. As shown in Fig. 4.88(*a*), consider a rectangular coil *PQRS* suspended in a uniform magnetic field $\vec{B}$, with its axis perpendicular to the field.

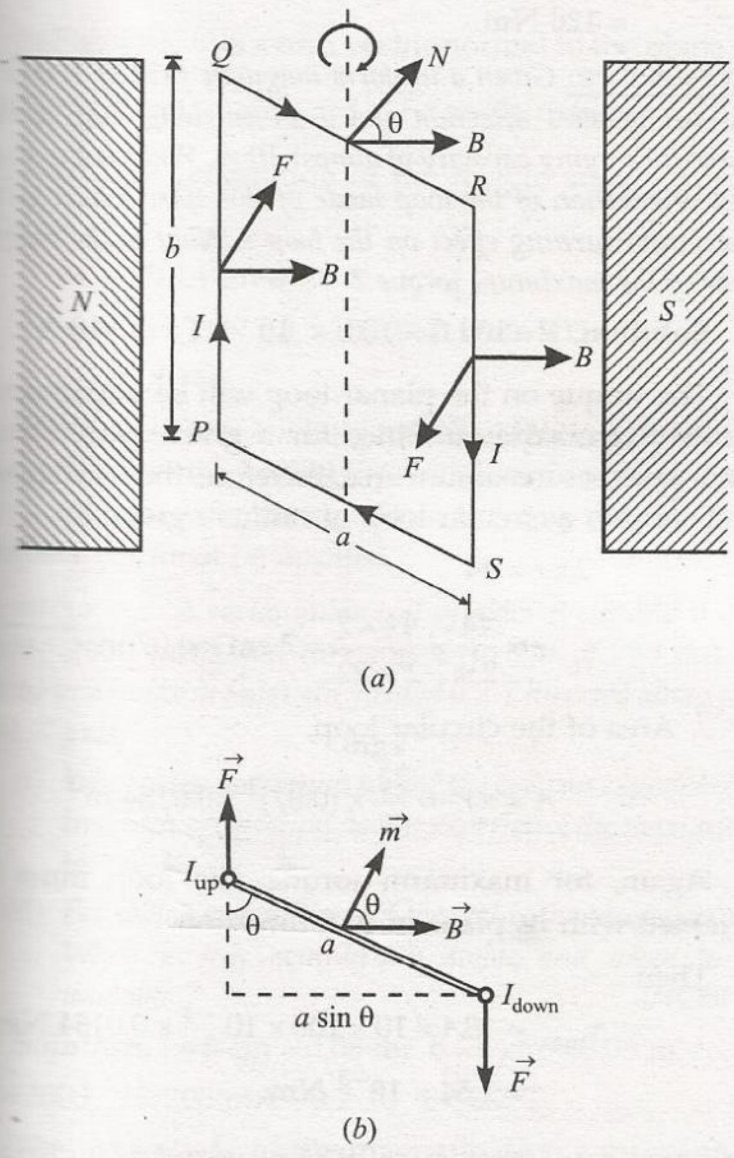

Fig. 4.88 (*a*) A rectangular loop *PQRS* in a uniform magnetic field $\vec{B}$. (*b*) Top view of the loop, magnetic dipole moment $\vec{m}$ is shown.

Let I = current flowing through the coil *PQRS*

a, b = sides of the coil *PQRS*

$A = ab$ = area of the coil

θ = angle between the direction of $\vec{B}$ and normal to the plane of the coil.

According to Fleming's left hand rule, the magnetic forces on sides *PS* and *QR* are equal, opposite and collinear (along the axis of the loop), so their resultant is zero.

The side PQ experiences a normal inward force equal to IbB while the side RS experiences an equal normal outward force. These two forces form a couple which exerts a torque given by

$$\tau = \text{Force} \times \text{perpendicular distance}$$

$$= IbB \times a \sin\theta = IBA \sin\theta$$

If the rectangular loop has N turns, the torque increases N times *i.e.*,

$$\tau = NIBA \sin\theta$$

But $NIA = m$, the magnetic moment of the loop, so

$$\tau = mB \sin\theta$$

In vector notation, the torque $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{m} \times \vec{B}$$

The direction of the torque $\vec{\tau}$ is such that it rotates the loop clockwise about the axis of suspension.

21. *Derive an expression for the force per unit length between two infinitely long straight parallel current carrying wires. Hence define one ampere. Also define coulomb in terms of ampere.*

Expression for the force between two parallel current-carrying wires. As shown in Fig. 4.81(a), consider two long parallel wires AB and CD carrying currents I_1 and I_2. Let r be the separation between them.

The magnetic field produced by current I_1 at any point on wire CD is

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

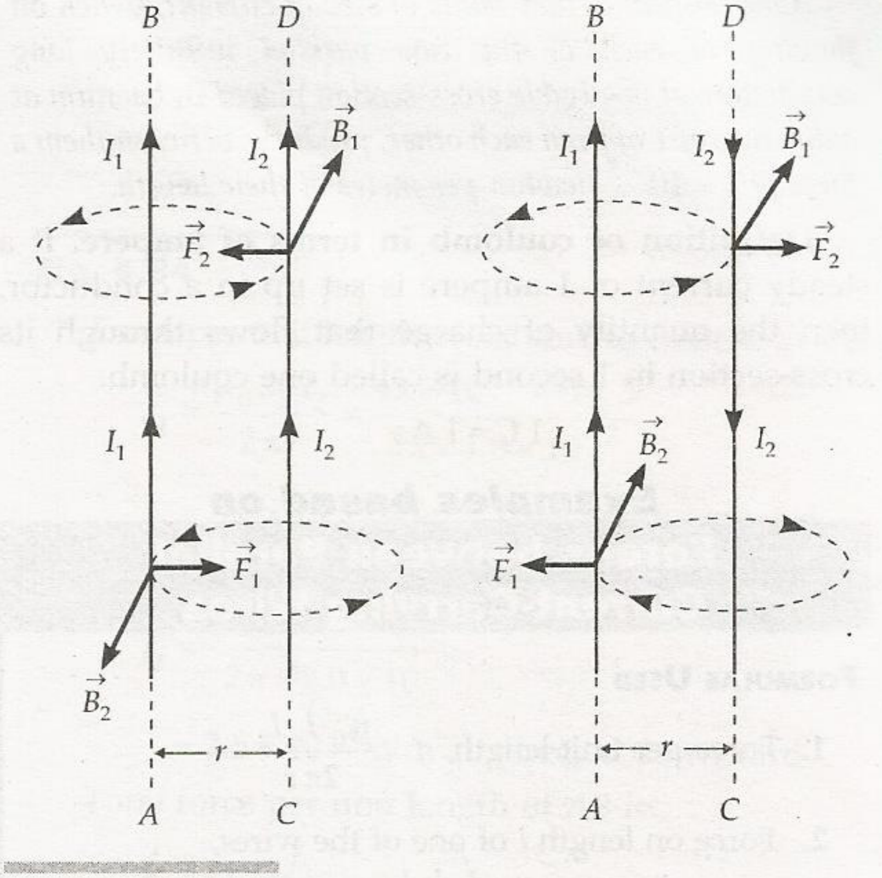

Fig. 4.81 (*a*) Parallel currents attract, (*b*) Antiparallel currents repel.

This field acts perpendicular to the wire *CD* and points into the plane of paper. It exerts a force on current carrying wire *CD*. The force acting on length *l* of the wire *CD* will be

$$F_2 = I_2 l B_1 \sin 90° = I_2 l \cdot \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 I_1 I_2}{2\pi r} \cdot l$$

Force per unit length,

$$f = \frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

According to Fleming's left hand rule, this force acts at right angles to *CD*, towards *AB* in the plane of the paper. Similarly, an equal force is exerted on the wire *AB* by the field of wire *CD*. Thus when the currents in the two wires are in the same direction, the forces between them are attractive. It can be easily seen that

$$\vec{F}_1 = -\vec{F}_2$$

As shown in Fig. 4.81(*b*), when the currents in the two parallel wires flow in opposite directions (anti-parallel), the forces between the two wires are repulsive. Thus,

Parallel currents attract and antiparallel currents repel.

force ?

Expression for the force on a current carrying conductor in a magnetic field. As shown in Fig. 4.70, consider a conductor PQ of length l, area of cross-section A, carrying current I along +*ve* Y-direction. The field $\vec{B}$ acts along +*ve* Z-direction. The electrons drift towards left with velocity $\vec{v}_d$. Each electron experiences a magnetic Lorentz force along +*ve* X-axis, which is given by

$$\vec{f} = -e(\vec{v}_d \times \vec{B})$$

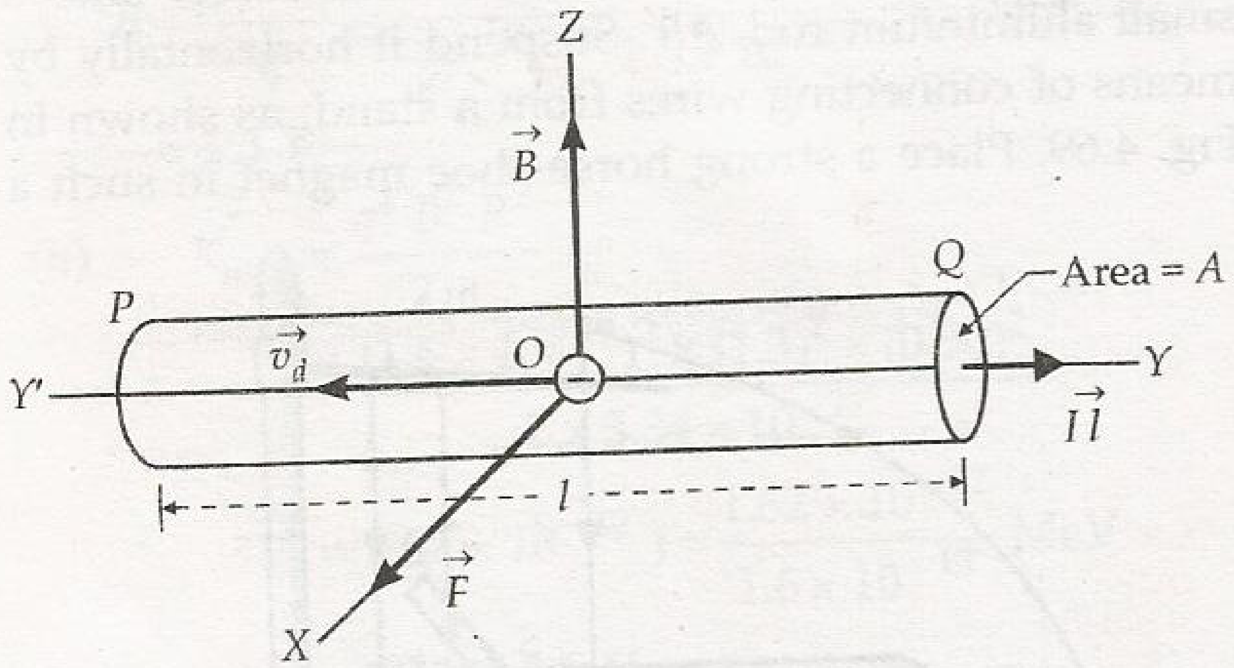

Fig. 4.70 Force on a current in a magnetic field.

If n is the number of free electrons per unit volume, then total number of electrons in the conductor is

$$N = n \times \text{volume} = nAl$$

Total force on the conductor is

$$\vec{F} = N\vec{f} = nAl[-e(\vec{v}_d \times \vec{B})]$$

$$= enA[-l\,\vec{v}_d \times \vec{B}]$$

If $I\vec{l}$ represents a current element vector in the direction of current, then vectors $\vec{l}$ and $\vec{v}_d$ will have opposite directions and we can take

$$-l\,\vec{v}_d = v_d\,\vec{l}$$

$$\therefore \quad \vec{F} = enAv_d\,(\vec{l} \times \vec{B})$$

But $enAv_d$ = current, I

Hence $\vec{F} = I(\vec{l} \times \vec{B})$

Magnitude of force. The magnitude of the force on the current carrying conductor is given by

$$F = IlB \sin\theta$$

where θ is the angle between the direction of the magnetic field and the direction of flow of current.

4.16 CYCLOTRON

17. *What is a cyclotron ? Discuss the principle, construction, theory and working of a cyclotron. What is the maximum kinetic energy acquired by the accelerated charged particles ? Give the limitations and uses of a cyclotron.*

Cyclotron. *It is a device used to accelerate charged particles like protons, deutrons, α-particles, etc., to very high energies.* It was invented by *E.O. Lawrence* and *M.S. Livingston* in 1934 at Berkeley, California University.

Principle. *A charged particle can be accelerated to very high energies by making it pass through a moderate electric field a number of times. This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit.*

Construction. As shown in Fig. 4.68, a cyclotron consists of the following main parts :

1. It consists of two small, hollow, metallic half-cylinders D_1 and D_2, called *dees* as they are in the shape of D.
2. They are mounted inside a vacuum chamber between the poles of a powerful electromagnet.
3. The dees are connected to the source of high frequency alternating voltage of few hundred kilovolts.
4. The beam of charged particles to be accelerated is injected into the dees near their centre, in a plane perpendicular to the magnetic field.
5. The charged particles are pulled out of the dees by a deflecting plate (which is negatively charged) through a window *W*.
6. The whole device is in high vacuum (pressure ~ 10^{-6} mm of Hg) so that the air molecules may not collide with the charged particles.

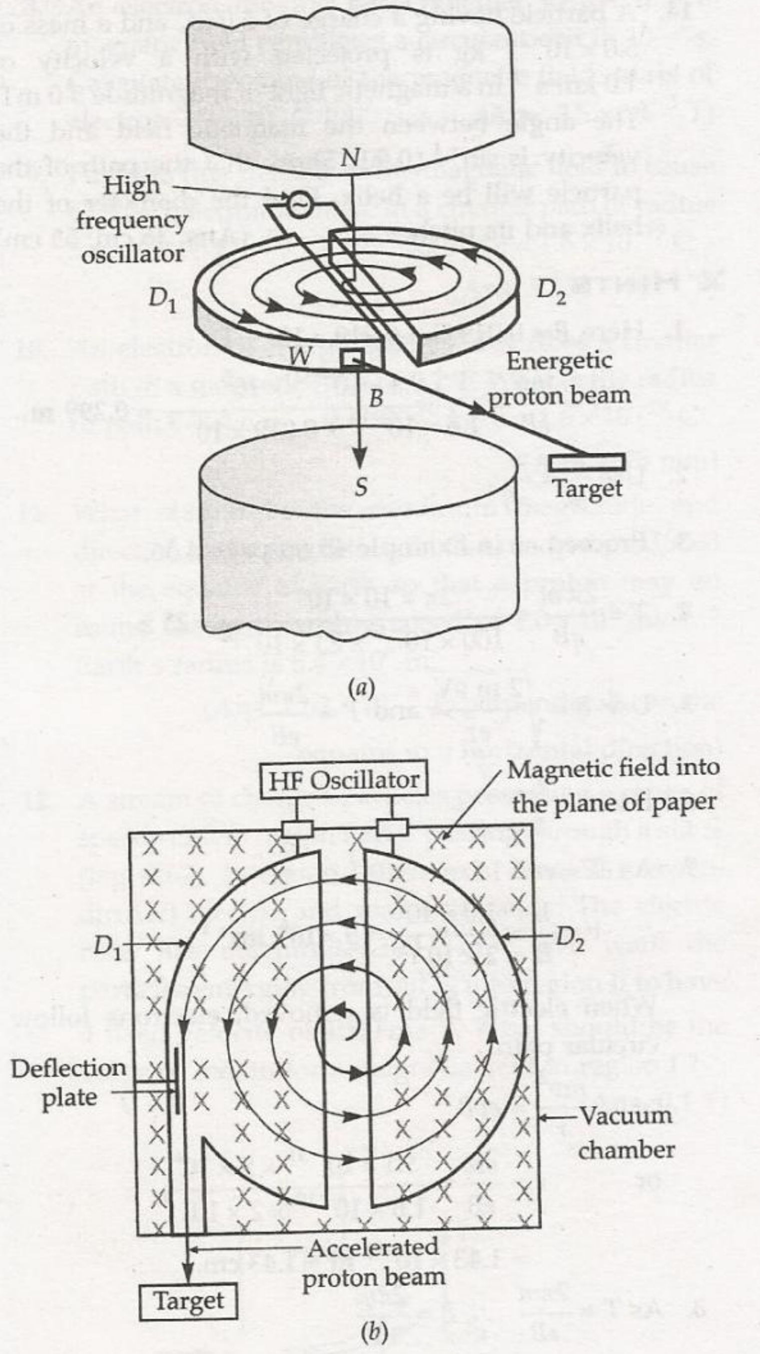

Fig. 4.68 Cyclotron (*a*) Front view (*b*) Section diagram.

Theory. Let a particle of charge *q* and mass *m* enter a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$, normal to the field $\vec{B}$. The particle follows a circular path, the necessary centripetal force being provided by the magnetic field. Therefore,

Magnetic force on charge *q*

= Centripetal force on charge *q*

or $$q v B \sin 90° = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

Period of revolution of the charged particle is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Hence frequency of revolution of the particle will be

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

Clearly, this frequency is *independent* of both the velocity of the particle and the radius of the orbit and is called ***cyclotron frequency*** or ***magnetic resonance frequency***. This is the key fact which is made use of in the operation of a cyclotron.

Working. Suppose a positive ion, say a proton, enters the gap between the two dees and finds dee D_1 to be negative. It gets accelerated towards dee D_1. As it enters the dee D_1, it does not experience any electric field due to shielding effect of the metallic dee. The perpendicular magnetic field throws it into a circular path. At the instant the proton comes out of dee D_1, it finds dee D_1 positive and dee D_2 negative. It now gets accelerated towards dee D_2. It moves faster through D_2 describing a larger semicircle than before. Thus if the frequency of the applied voltage is kept exactly the same as the frequency of revolution of the proton, then every time the proton reaches the gap between the two dees, the electric field is reversed and proton receives a push and finally it acquires very high energy. This is called the cyclotron's *resonance condition.* The proton follows a spiral path. The accelerated proton is ejected through a window by a deflecting voltage and hits the target.

Maximum K.E. of the accelerated ions. The ions will attain maximum velocity near the periphery of the dees. If v_0 is the maximum velocity acquired by the ions and r_0 is the radius of the dees, then

$$\frac{mv_0^2}{r_0} = qv_0B \quad \text{or} \quad v_0 = \frac{qBr_0}{m}$$

The maximum kinetic energy of the ions will be

$$K_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{qBr_0}{m}\right)^2$$

or $$K_0 = \frac{q^2B^2r_0^2}{2m}.$$

4.13 WORK DONE BY A MAGNETIC FORCE ON A CHARGED PARTICLE IS ZERO

14. *Show that the work done by a magnetic field on a moving charged particle is always zero.*

Work done by a magnetic force on a charged particle. The magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$ always acts perpendicular to the velocity $\vec{v}$ or the direction of motion of charge q. Therefore,

$$\vec{F} . \vec{v} = q(\vec{v} \times \vec{B}) . \vec{v} = 0$$

According to Newton's second law,

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\therefore \quad m\frac{d\vec{v}}{dt} . \vec{v} = 0$$

or $$\frac{m}{2}\left[\frac{d\vec{v}}{dt} . \vec{v} + \vec{v} . \frac{d\vec{v}}{dt}\right] = 0$$

or $$\frac{m}{2}\frac{d}{dt}(\vec{v} . \vec{v}) = 0$$

or $$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \qquad [\vec{v} . \vec{v} = v^2]$$

or $$\frac{dK}{dt} = 0$$

or $$K = \text{constant}$$

Thus *a magnetic force does not change the kinetic energy of the charged particle.* This indicates that the *speed of the particle does not change.* According to the work-energy theorem, the change in kinetic energy is equal to the work done on the particle by the net force. Hence the *work done* on the charged particle *by the magnetic force is zero.*

2. ***When the initial velocity is perpendicular to the magnetic field.*** Here $\theta = 90°$, so $F = qvB \sin 90° = qvB$ = a maximum force. As the magnetic force acts on a particle perpendicular to its velocity, it does not do any work on the particle. It does not change the kinetic energy or speed of the particle.

Fig. 4.62 shows a magnetic field $\vec{B}$ directed normally into the plane of paper, as shown by small crosses. A charge $+q$ is projected with a speed v in the plane of the paper. The velocity is perpendicular to the

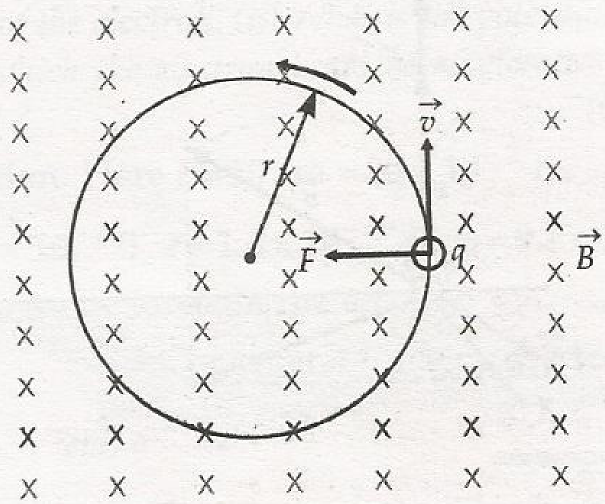

Fig. 4.62 A positively charged particle moving in a magnetic field directed into the plane of paper.

magnetic field. A force $F = qvB$ acts on the particle perpendicular to both $\vec{v}$ and $\vec{B}$. This force continuously deflects the particle sideways without changing its speed and the particle will move along a circle perpendicular to the field. Thus the magnetic force provides the centripetal force. Let r be the radius of the circular path. Now

$$\text{Centripetal force, } \frac{mv^2}{r} = \text{Magnetic force, } qvB$$

or
$$r = \frac{mv}{qB}$$

Thus the radius of the circular orbit is inversely proportional to the specific charge (charge to mass ratio q/m) and to the magnetic field.

$$\text{Period of revolution} = \frac{\text{Circumference}}{\text{Speed}}$$

or
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Clearly, the time period is independent of v and r. If the particle moves faster, the radius is larger, it has to move along a larger circle so that the time taken is the same.

The frequency of revolution is

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

This frequency is called ***cyclotron frequency***.

16. *Electric and magnetic fields are applied mutually perpendicular to each other. Show that a charged particle will follow a straight line path perpendicular to both of these fields, if its velocity is E / B in magnitude.*

Velocity selector. Suppose a beam of charged particles, say electrons, possessing a range of speeds passes through a slit S_1 and then enters a region in which crossed (perpendicular) electric and magnetic fields exist. As shown in Fig. 4.65, the electric field $\vec{E}$ acts in the downward direction and deflects the electrons in the upward direction. The magnetic field $\vec{B}$ acts normally into the plane of paper and deflects the electrons in the downward direction.

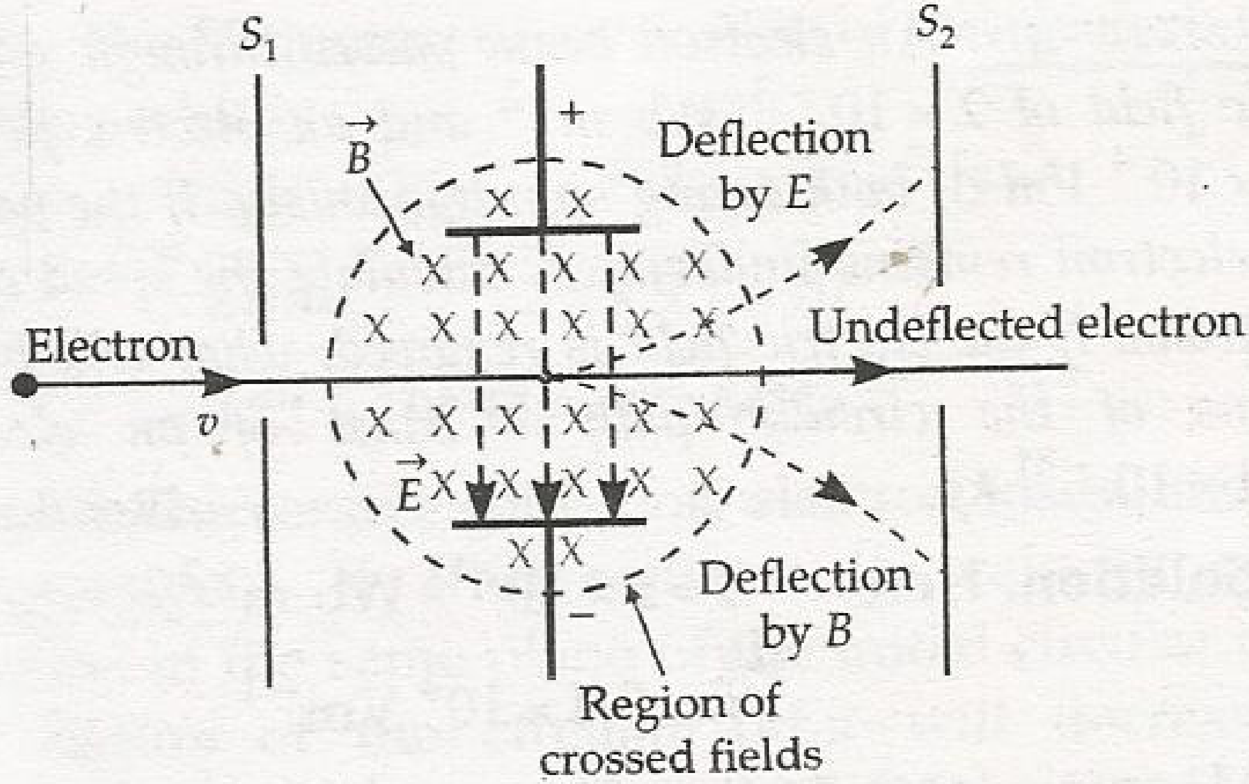

Fig. 4.65 Motion of an electron in a region of crossed magnetic and electric fields.

Only those electrons will pass undeflected through the slit S_2 on which the electric and magnetic forces are equal and opposite. The velocity v of the undeflected electrons is given by

$$eE = evB \quad \text{or} \quad v = \frac{E}{B}$$

Such an arrangement can be used to *select* charged particles of a particular velocity out of a beam in which the particles are moving with different speeds. This arrangement is called ***velocity selector*** or ***velocity filter***. This method was used by *J.J. Thomson* to determine the charge to mass ratio (e/m) of an electron.