Goethe University Frankfurt
Macroeconomics 1 (BMAK)
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Macroeconomics 1 (BMAK)

IV. The Macroeconomy in the Medium Run

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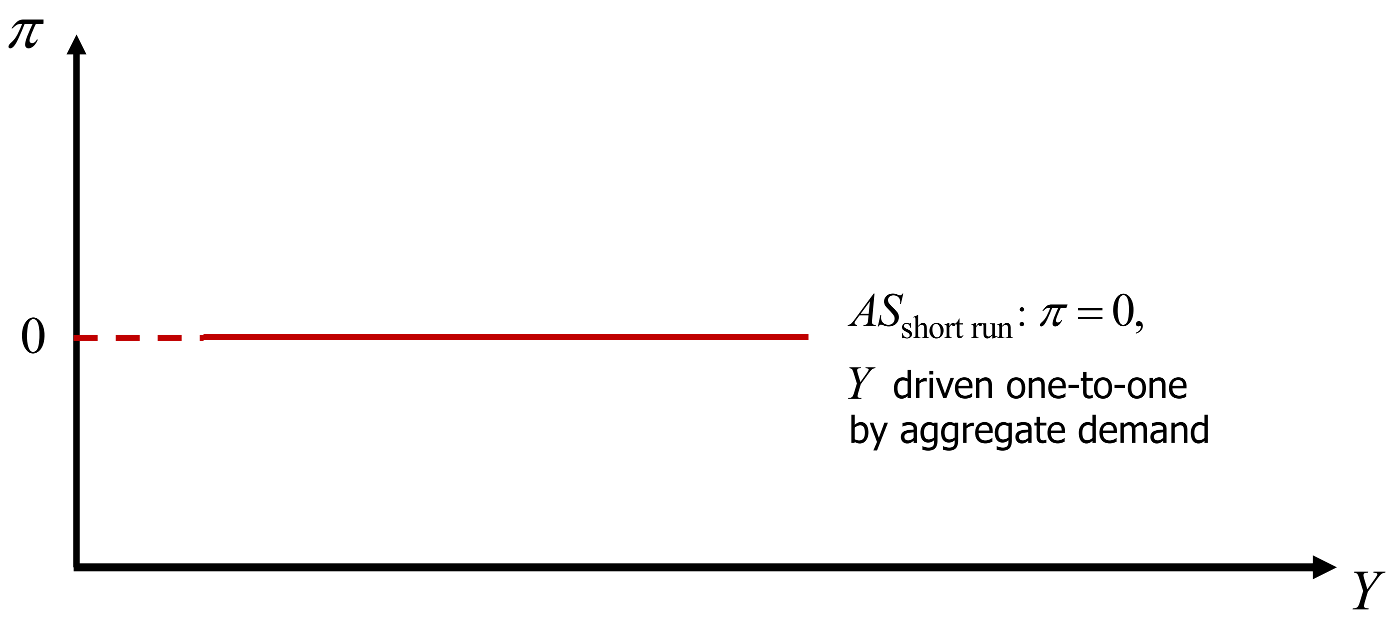
1. Prices and Wages: The *AS*-Curve

Readings:
- Burda and Wyplosz (2017), Chapter 13
- Challe (2019), Chapter 3
- Jones (2018), Chapter 12 and 13

- The model of business cycles we have developed in the previous chapter of the course was a short-run model.
- As we emphasized, in the short-run model, aggregate output supply responds one-to-one to changes in aggregate output demand, with prices of output being fixed.
- We now turn to studying the macroeconomy in the **medium run**. In the medium-run model, firms will be allowed to respond to changes in aggregate output demand by adjusting both output supply and prices.
- The medium run is implicitly defined as that time period during which macroeconomic outcomes adjust from their short-run to their long-run values. The long-run time paths themselves are not part of business cycles and will be studied when we turn to the analysis of economic growth.

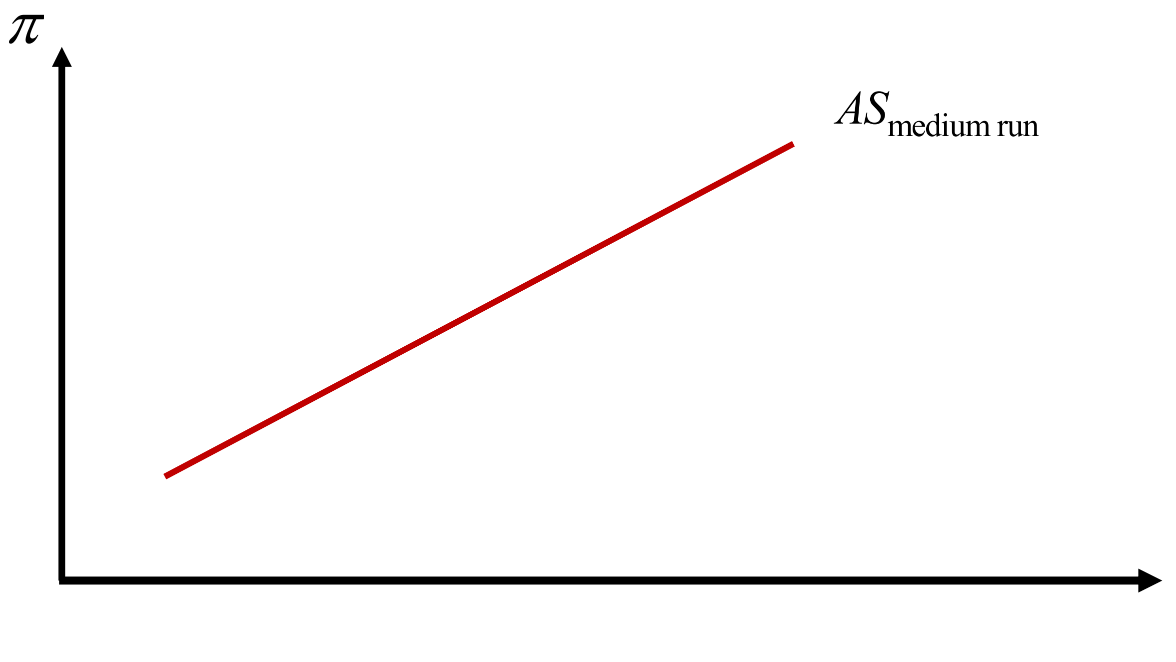
- To obtain the medium-run macroeconomic outcomes, we will need to understand how the prices of the goods and services change with the level of output produced.
- This **"aggregate supply" relationship (*AS*-curve)** is typically plotted in diagrams with the growth rate of prices (that is, the rate of inflation) on the vertical axis, and the level of output on the horizontal axis.
- In the short-run model, this relationship would look as follows:



- For the medium run, the aggregate supply relationship will, in contrast, turn out to be:

$$\pi = \underbrace{\pi^e}_{\text{expected rate of inflation}} + \underbrace{\omega}_{\omega > 0} \cdot \left(\frac{Y - \bar{Y}}{\bar{Y}} \right) + \underbrace{s}_{\text{shock to the cost of production ("supply" shock)}} , \qquad (1)$$

yielding the following graph of the medium-run AS-curve:



To derive this medium-run AS-curve, we will need to model the pricing decisions of firms, and how these entail responses to changes in their cost of production.

The Battle of Mark-Ups

- In the medium run, aggregate price dynamics are in large parts driven by
  -- firms with market power (such market power often achieved through product differentiation) attempting to set prices as a mark-up over costs of production (including wages), and
  -- employees or their wage negotiators attempting to increase real wages by pushing up nominal wages relative to expected prices and productivity.

Firms' Pricing

- We model firms as making their production decisions so as to maximize revenue relative to the costs of production:

$$\max_{Y} \underbrace{P}_{\substack{\text{s.t. } P=P(Y),\\ \text{the inverse}\\ \text{demand function}}} \cdot Y - \underbrace{W}_{\substack{\text{nominal wages, taken as}\\ \text{given by firms when}\\ \text{deciding about medium-run}\\ \text{output supply and its pricing}}} \cdot \underbrace{L}_{\text{labor hours}} - \underbrace{r^k \cdot K}_{\substack{\text{capital stock and rental rate}\\ \text{thereon, both taken as given}\\ \text{by firms when deciding about}\\ \text{medium-run output supply}\\ \text{and its pricing}}}, \qquad (2)$$

which implies the first-order condition

$$\frac{\partial P}{\partial Y} \cdot Y + P - W \cdot \frac{\partial L}{\partial Y} = 0$$

$$\Leftrightarrow \quad P \cdot \left(1 + \frac{\partial P / \partial Y}{P / Y}\right) = \frac{W}{\partial Y / \partial L}$$

$$\Leftrightarrow \quad \underbrace{P \cdot \left(1 + \frac{1}{\eta}\right)}_{= MR \text{ (marginal revenue)}} = \underbrace{\frac{W}{\lambda}}_{= MC \text{ (marginal cost)}}, \qquad (3)$$

where $\eta = (\partial Y / \partial P) / (Y / P) < 0$ denotes the price elasticity of output demand and λ denotes the marginal product of labor (labor productivity).

- Note from (3) that the firm will choose a level of output such that $MR > 0$ and thus $\eta < -1$:

$$1+\frac{1}{\eta}>0 \;\Rightarrow\; \frac{1}{\eta}>-1 \;\overset{(\text{as } \eta<0)}{\Rightarrow}\; -\eta>1 \;\Rightarrow\; \eta<-1.$$

- From Equation (3) we have

$$P = \frac{MC}{1+1/\eta}$$

$$\Leftrightarrow P = (1+\theta)\cdot MC, \qquad (4)$$

with the mark-up parameter $\theta = -1/(1+\eta)$.

- The degree of pricing power of the firms, that is, the extent to which they can charge mark-ups on their goods and services, is positively related to η; under perfect competition, we have that $\eta \to -\infty$ and thus $\theta \to 0$. Also note that under $\eta < -1$ we have that $\theta \to \infty$ as $\eta \to -1$.

- Moving from price levels to the rate of inflation, $\pi = \Delta P / P$, note from (3) and (4) that

$$\Delta P = \Delta\left[(1+\theta)\cdot\frac{W}{\lambda}\right] = \frac{\partial P}{\partial \theta}\cdot\Delta\theta + \frac{\partial P}{\partial W}\cdot\Delta W + \frac{\partial P}{\partial \lambda}\cdot\Delta\lambda$$

$$\Leftrightarrow \ \Delta P = \frac{W}{\lambda}\cdot\Delta\theta + \frac{1+\theta}{\lambda}\cdot\Delta W + \left(-\frac{1}{\lambda^2}\right)\cdot(1+\theta)\cdot W\cdot\Delta\lambda$$

and thus

$$\pi = \frac{\Delta P}{P} = \frac{W/\lambda}{(1+\theta)\cdot W/\lambda}\cdot\Delta\theta + \frac{(1+\theta)/\lambda}{(1+\theta)\cdot W/\lambda}\cdot\Delta W - \frac{(1+\theta)\cdot W/\lambda^2}{(1+\theta)\cdot W/\lambda}\cdot\Delta\lambda$$

$$\Leftrightarrow \ \pi = \underbrace{\frac{\Delta\theta}{1+\theta}}_{\text{inflation increasing as growth of mark-ups is rising}} + \underbrace{\frac{\Delta W}{W}}_{\text{inflation increasing as wage growth is rising}} - \underbrace{\frac{\Delta\lambda}{\lambda}}_{\text{inflation decreasing as productivity is rising}} . \qquad (5)$$

To obtain a complete picture about the rate of inflation, π, we turn to discussing the determinants of nominal wage growth, $\Delta W / W$.

Wage Bargaining

Nominal wages agreed upon in wage bargaining arguably should grow faster

- the higher the expected rate of inflation,
- the higher the growth rate of labor productivity, and
- the lower the level of unemployment, as worker turnover (that *ceteris paribus* always is costly for employers) then is more of a threat for employers, it under lower levels of unemployment being
  -- easier for workers to find a new job, and
  -- more difficult for employers to recruit.

- We formalize these considerations about wage bargaining using the following simple linear specification:

$$\frac{\Delta W}{W} = \pi^e + \frac{\Delta\lambda}{\lambda} + \underbrace{\underbrace{\omega}_{\omega>0} \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right)}_{\text{wage mark-up}} . \qquad (6)$$

- Note that the specification of the wage mark-up in Equation (6) reflects that unemployment from our Burns-Mitchell diagrams is countercyclical, and thus falls (rises) when actual output, Y, rises (falls) relative to long-run output, $\bar{Y}$.
- Substituting from the wage growth equation (6) back into the firm-pricing implied equation (5) for the rate of inflation, we obtain:

$$\pi = \frac{\Delta\theta}{1+\theta} + \pi^e + \underbrace{\omega}_{\omega>0} \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right) . \qquad (7)$$

- We have not yet accounted for changes in non-labor costs of production, including intermediate inputs and physical capital. When the non-labor costs of production vary significantly over the business cycle, then it is often due to exogenous shocks: oil or other primary commodity price shocks, or foreign exchange market shocks raising the prices of imported components.

- We refer to such shocks as supply shocks, and append them to Equation (7):

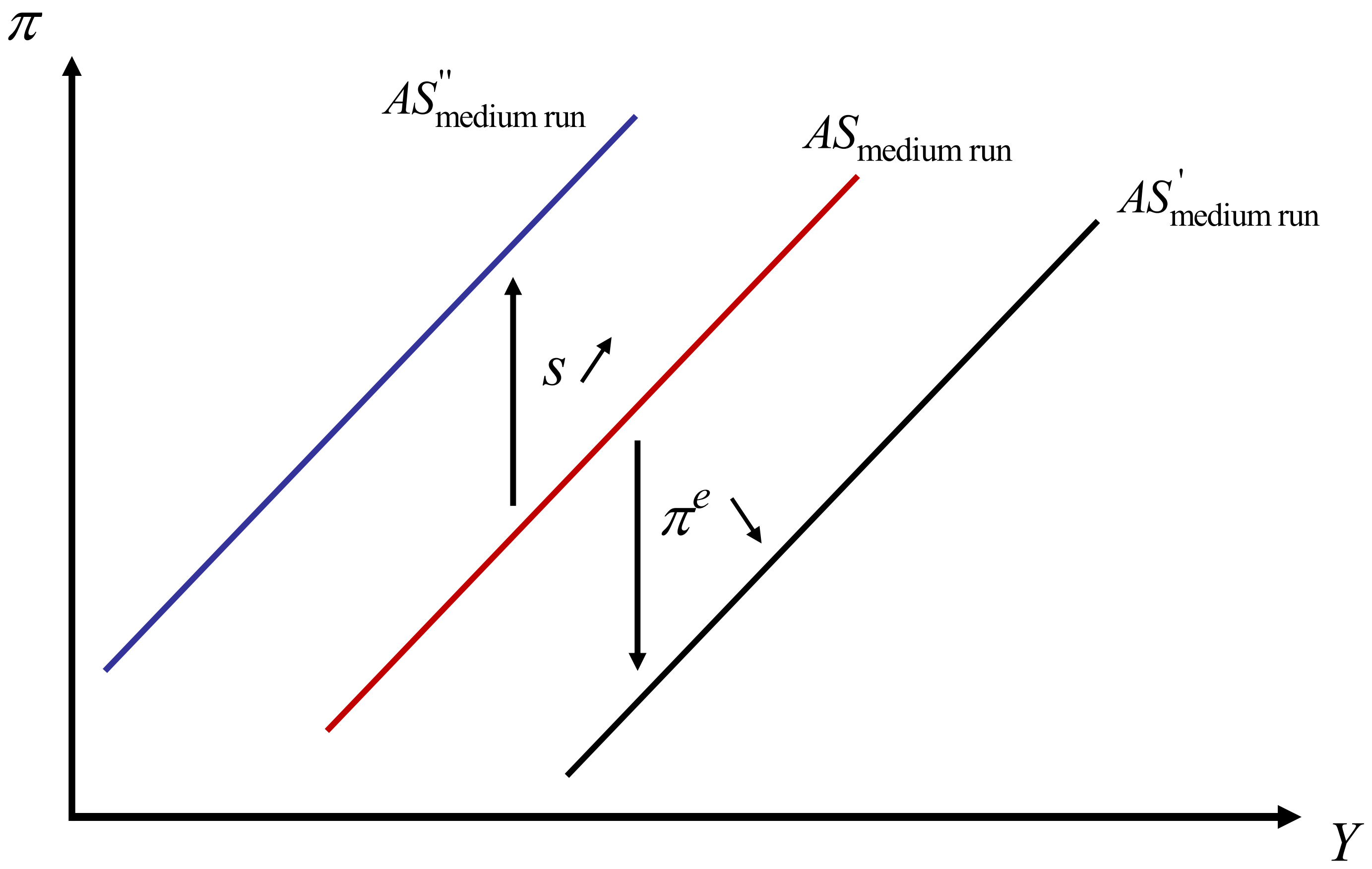
$$\pi = \frac{\Delta\theta}{1+\theta} + \pi^e + \omega \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right) + s. \qquad (8)$$

- The cyclical change of θ, the price mark-up, is ambiguous: In a boom, when actual output, Y, exceeds long-run output, $\bar{Y}$, then aggregate output demand is high, creating an opportunity to increase θ, but as competition among firms then tends to be higher as well, a reduction of θ could also be called for.

- For simplicity, our medium-run AS-curve in Equation (1) thus does not include a price mark-up:

$$\pi = \pi^e + \omega \cdot \left(\frac{Y - \bar{Y}}{\bar{Y}} \right) + s.$$

- Note that the medium-run AS-curve shifts, for example, with changes in expected inflation, π^e, or with supply shocks, s .



Workbook for the Medium-Run *AS*-Curve

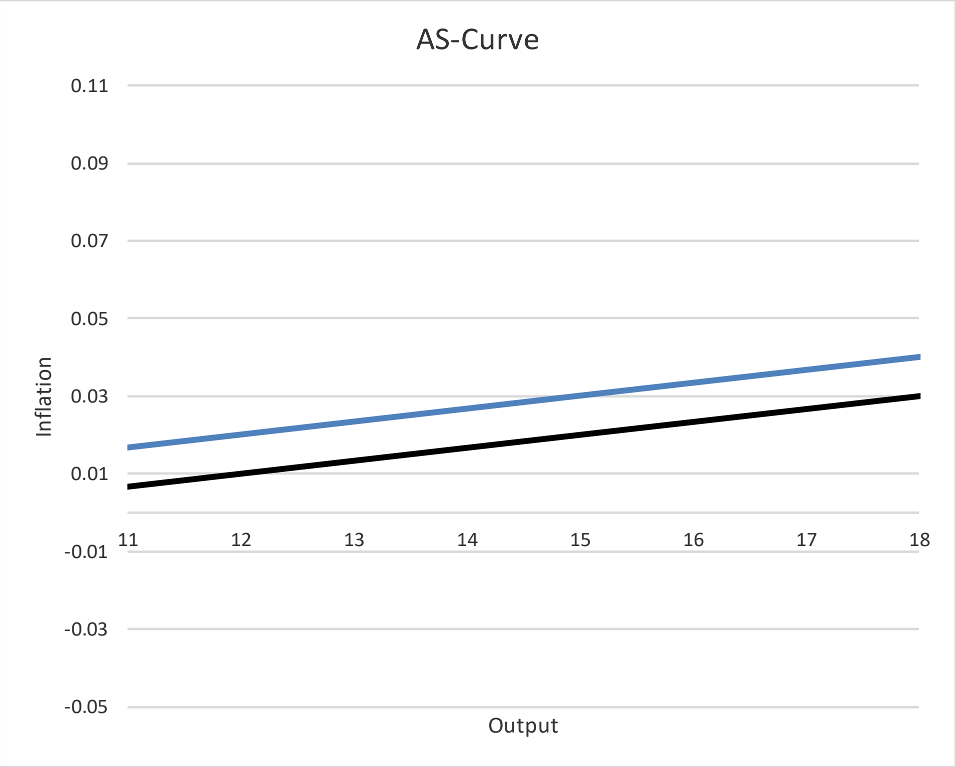
The workbook *AS-Curve.xlsm* allows to study all the determinants of the position and slope of the medium-run *AS*-curve.

AS-Curve: Specification

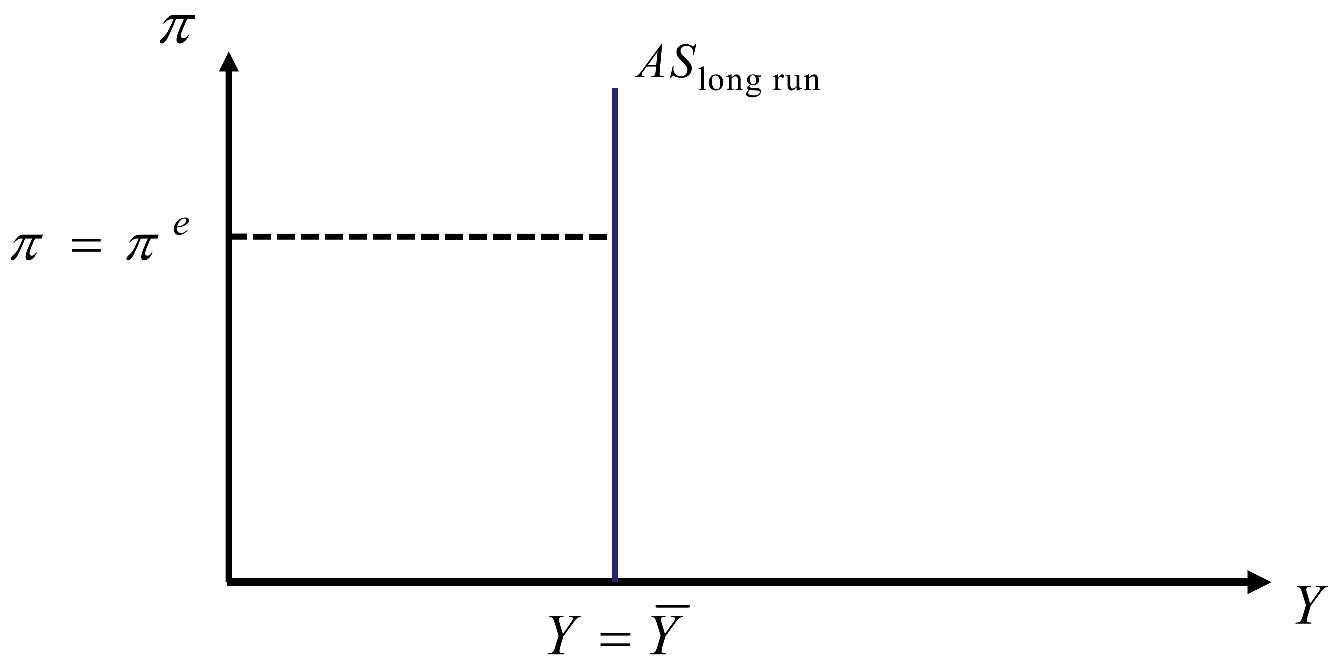
Parameters

	Initial	New
π^{ε}	0.020	0.020
$\bar{Y}$	15.000	15.000
ω	0.050	0.050
s	0.000	0.010

AS-Curve: Graph



- Also note that Equation (1) implies that the AS-curve in the long-run must be vertical: In the long run, $Y = \overline{Y}$, and the economy has adapted to supply shocks (so that we might as well set $s = 0$ in the long run), and from Equation (1) we then have that $\pi = \pi^e$.



- An aside: Recalling once more that unemployment is countercyclical, we may also write Equation (1) as:

$$\pi = \pi^e - \omega_u \cdot \underbrace{\left(\frac{u - \bar{u}}{\bar{u}}\right)}_{\text{unemployment gap}} + s\,. \qquad (9)$$

- Equation (9) is also known as the **Phillips curve**. This curve suggests that when actual unemployment, u, falls relative to the long-run level of unemployment, $\bar{u}$, then the rate of inflation rises (and vice versa). (Equation (9) does not suggest, though, that a higher rate of inflation, π, would lead to a lower rate of unemployment, u.)

2. Medium-Run Macroeconomic Outcomes: The *AS-AD* Model

Readings:
- Burda and Wyplosz (2017), Chapter 14
- Challe (2019), Chapter 4, 6 and 7
- Jones (2018), Chapter 13

To pin down medium-run macroeconomic outcomes, we will need to complement our analysis as to how the rate of inflation is determined as a function of output (as reflected in the medium-run *AS*-curve) by a relationship describing how output depends on the rate of inflation (wich will be called the ***AD*-curve**).

We will obtain the *AD*-curve by substituting the real interest rate determined in the financial sector (summarized by the *TR*-curve, as applicable in the medium run) into the equation determining the level of output (summarized by the *IS*-curve).

The AD-curve will thus incorporate the short-run model's capturing of the "**real sector**" of the economy (the interaction between the aggregated demand for goods and services with the production of the goods and services), as well as its capturing of the "**financial sector**" of the economy (how interest rates are set through the interaction between savers, borrowers, the central bank and commercial banks).

Initially, our medium-run model (that we will call the AS-AD model), will abstract from "deep crisis" issues, specifically the zero lower-bound for the monetary policy rate and the endogenous component of the risk premium, and thus be a model of "normal business cycle" outcomes.

We will later consider "deep crisis" issues also, in an extended version of the AS-AD model, the extended AS-AD model.

Let us derive the AD-curve:

Medium-Run *TR*-Curve:

As in the medium run we allow for non-zero rates of inflation, the Taylor rule as formulated in Equation (60) of the chapter on "The Macroeconomy in the Short Run" is applicable:

$$R^{MP} = \overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right),$$

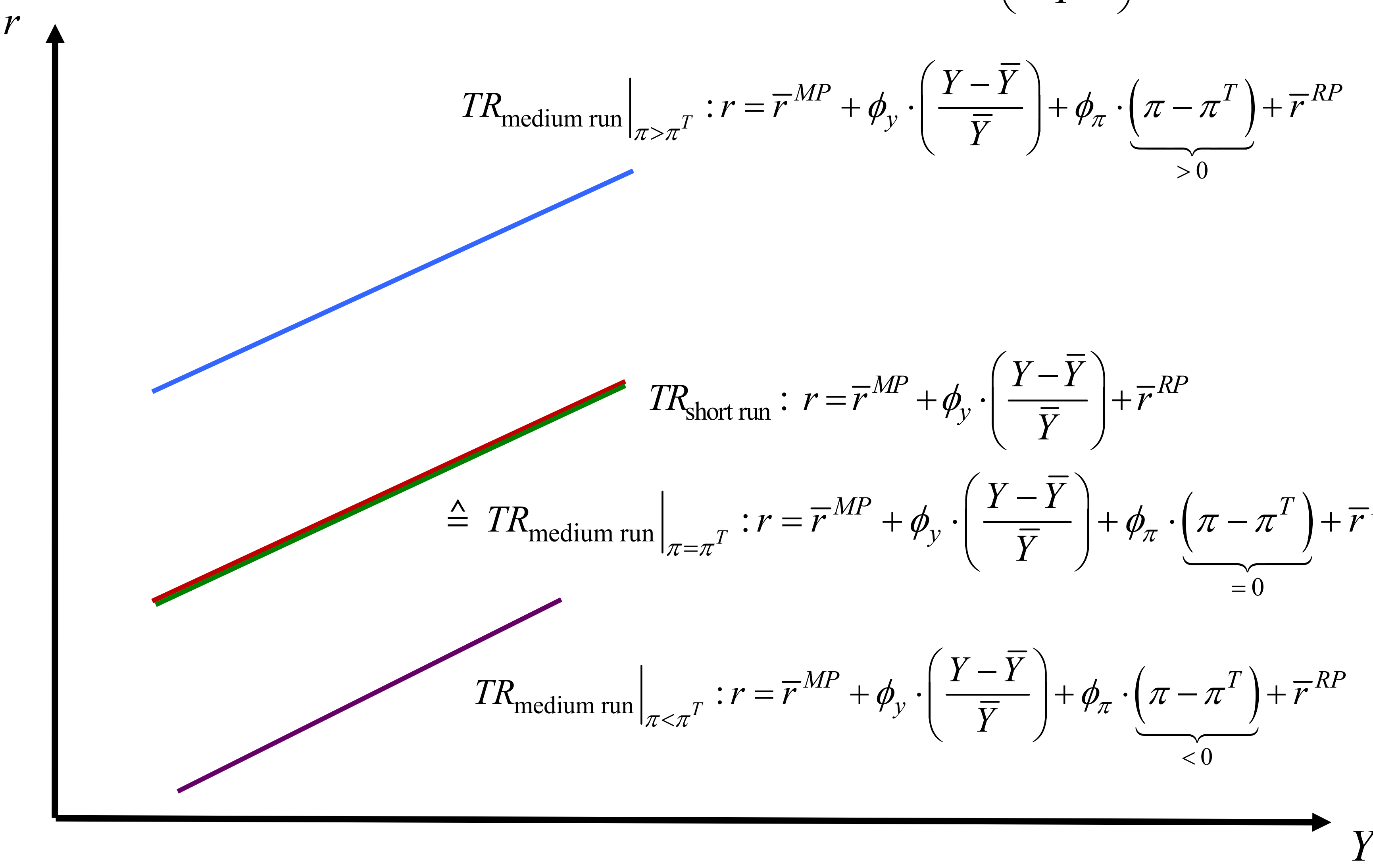
or, in terms of the real monetary policy rate,

$$r^{MP} \approx R^{MP} - \pi^e = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right). \qquad (10)$$

The medium-run *TR*-curve is thus given by:

$$r = r^{MP} + \overline{r}^{RP} = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right) + \overline{r}^{RP}. \qquad (11)$$

Plotting the medium-run *TR*-curve: $TR_{\text{medium run}}: r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi_t - \pi^T \right) + \overline{r}^{RP}$



$$TR_{\text{medium run}}\Big|_{\pi > \pi^T}: r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \underbrace{\left(\pi - \pi^T \right)}_{>0} + \overline{r}^{RP}$$

$$TR_{\text{short run}}: r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \overline{r}^{RP}$$

$$\triangleq TR_{\text{medium run}}\Big|_{\pi = \pi^T}: r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \underbrace{\left(\pi - \pi^T \right)}_{=0} + \overline{r}^{RP}$$

$$TR_{\text{medium run}}\Big|_{\pi < \pi^T}: r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \underbrace{\left(\pi - \pi^T \right)}_{<0} + \overline{r}^{RP}$$

IS-Curve:

We can use the same specification of the *IS*-curve as in the short-run model (as the short-run model's *IS*-curve involved only real quantities, and thus does not need to be adapted to the presence of non-zero rates of inflation). Equation (51) of the chapter on "The Macroeconomy in the Short Run" provided the *IS*-curve:

$$Y = \underbrace{\frac{A}{1-\left(C_y - TB_{im} - G_y\right)}}_{= IS_0} - \underbrace{\frac{C_r + I_r + TB_\varepsilon \cdot \varepsilon_r}{1-\left(C_y - TB_{im} - G_y\right)}}_{= IS_1} \cdot r. \qquad (12)$$

AD-Curve:

Combining the medium-run *TR*-curve (11) and the *IS*-curve (12), we obtain:

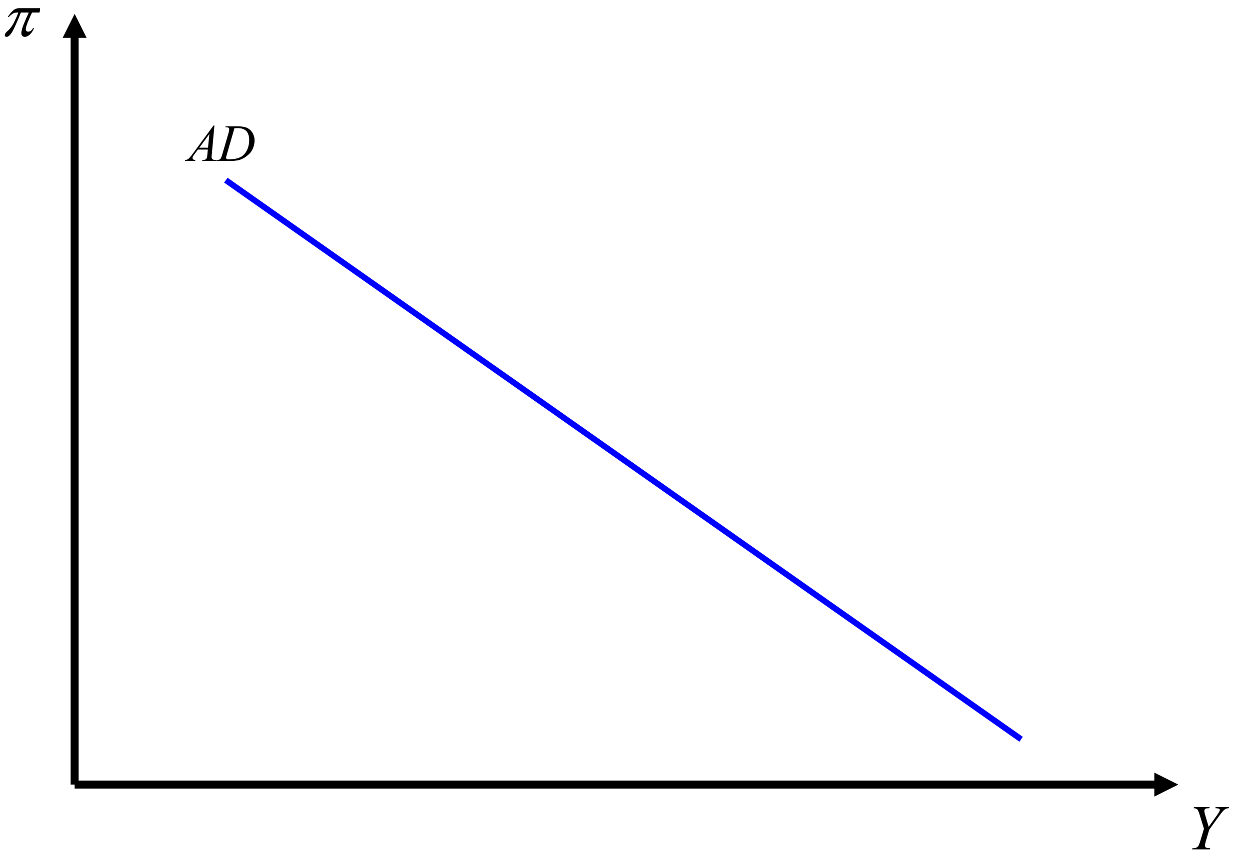
$$Y = IS_0 - IS_1 \cdot \left[\overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right) + \overline{r}^{RP} \right]. \qquad (13)$$

Equation (13) can be solved for the rate of inflation:

$$\pi = \frac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP} \right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \frac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}} \cdot Y. \qquad (14)$$

Equation (14) is the *AD*-curve, involving a negative relationship between output and inflation.

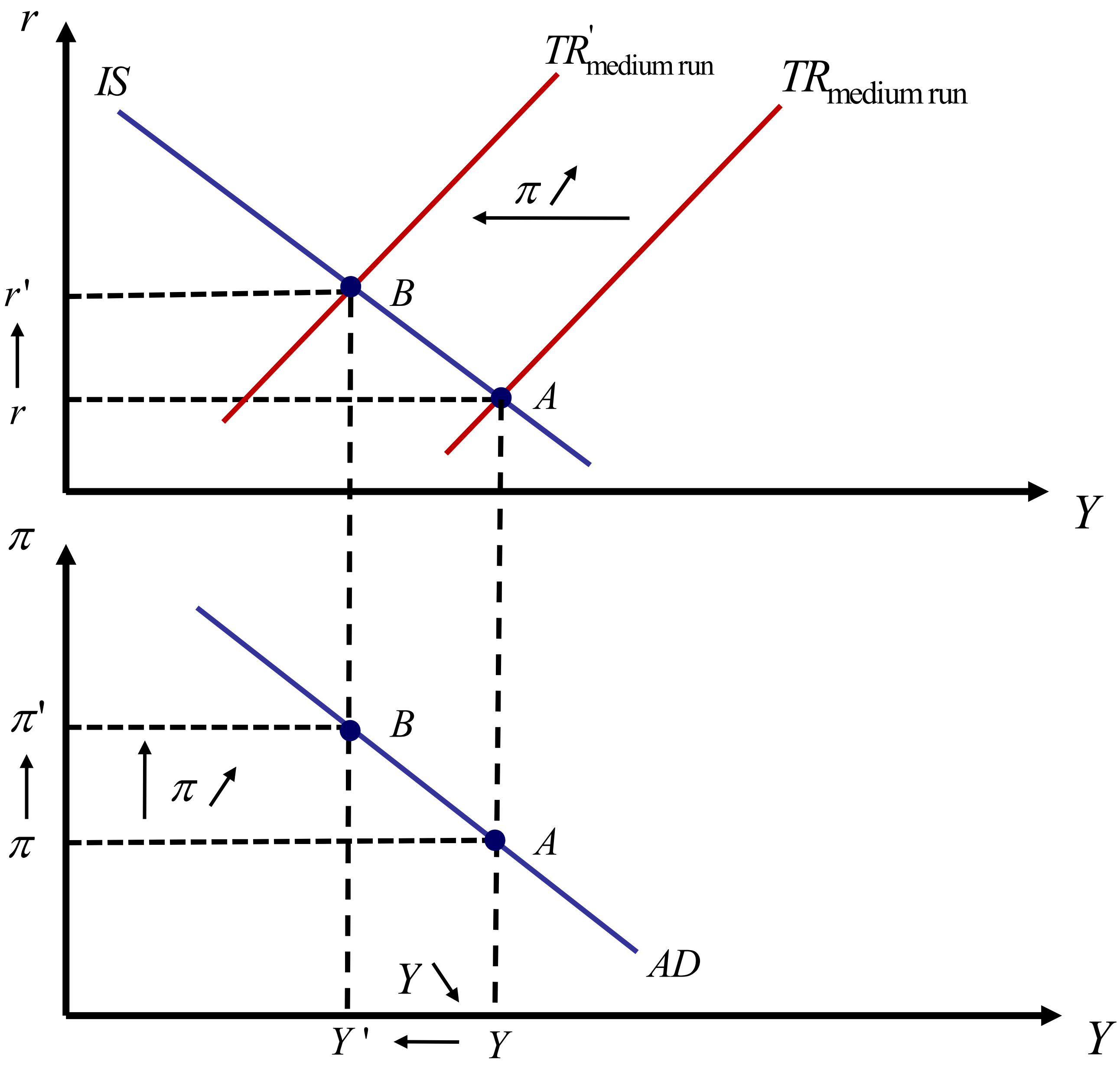
Plotting the AD-curve:



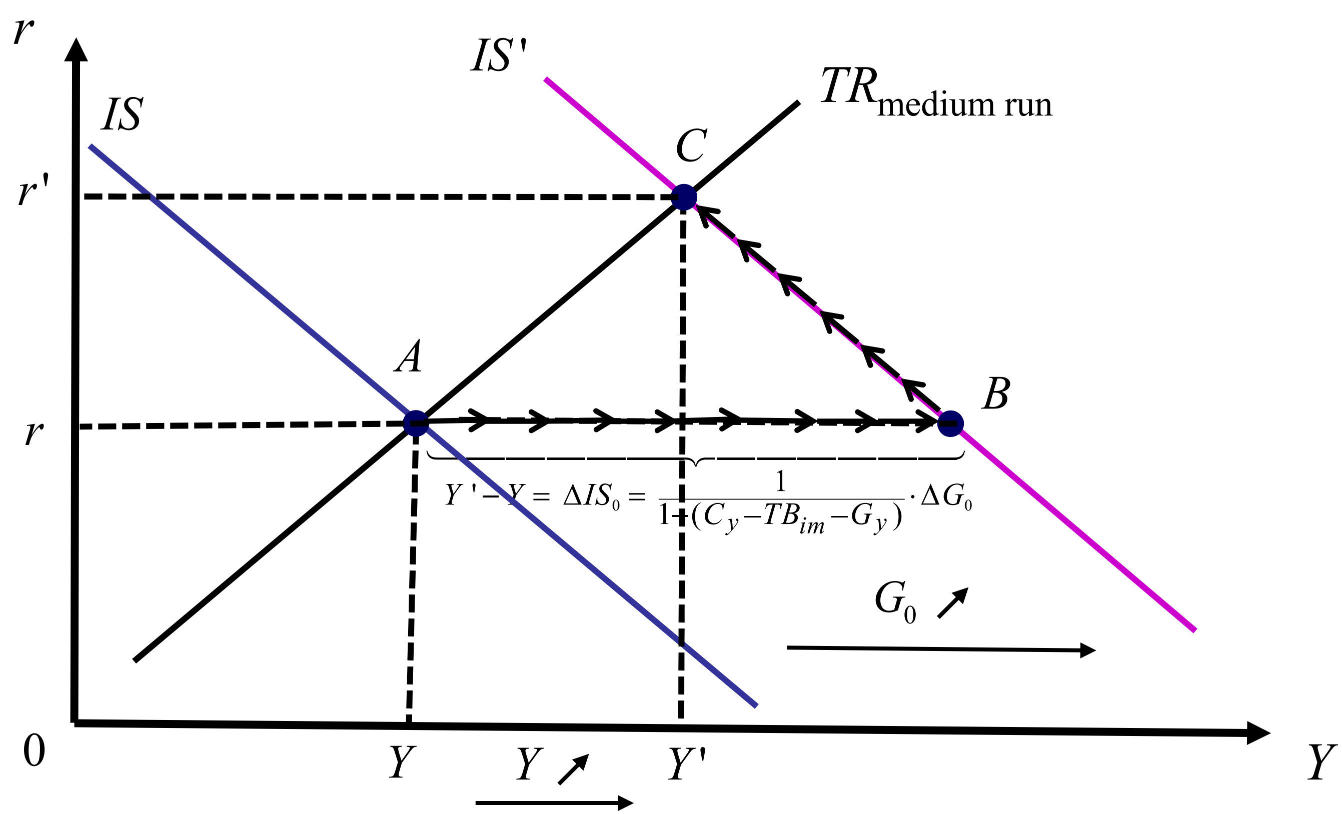
What is the economic rationale for the AD-curve being downward sloping?

- As the rate of inflation increases, the central bank, following the Taylor rule, raises the monetary policy rate.
- The increase in the monetary policy rate causes (for a given risk premium) the real interest rate at which households and firms can borrow to increase.
- Through the consumption, investment and exchange rate channels, this lowers output.

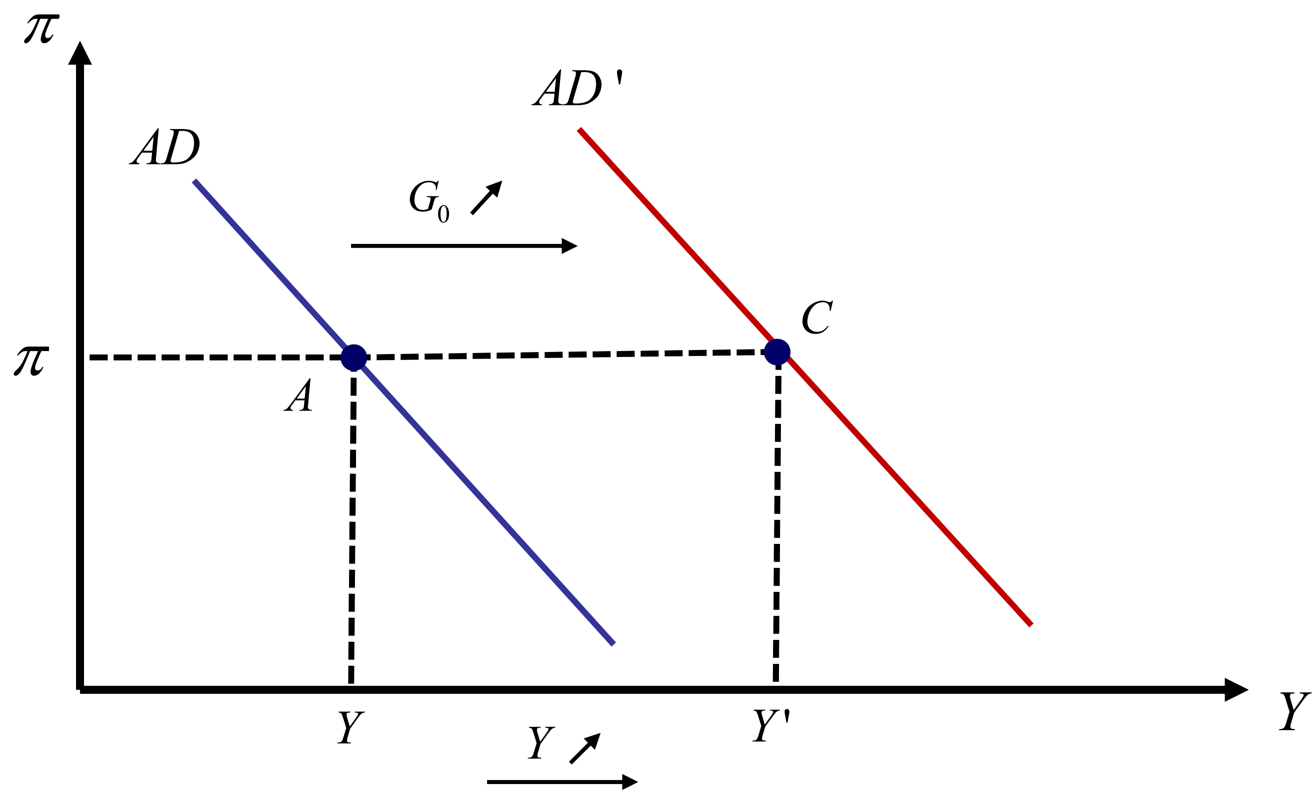
Let us illustrate this graphically:


r
IS
TR'medium run
TRmedium run
π ↗
r'
B
r
A
Y
π
π'
B
π ↗
π
A
AD
Y ↘
Y'
Y
Y

- Note that the AD-curve shifts when any of the parameters or exogenous variables entering the IS-curve and/or the medium-run TR-curve change.
- Consider, for example, an increase in G_0: This leads to a shift of the IS-curve. Graphing this in the medium-run IS-TR model diagram:



- Bringing this analysis to the AD-curve diagram:



- The move from Point A to Point C involves both the Keynesian multiplier effect (from Point A to Point B in the medium-run IS-TR diagram) and the crowding out effects due to the consumption, investment and exchange rate channels (from Point B to Point C in the medium-run IS-TR diagram).

Workbook for the *AD*-Curve

The workbook *AD-Curve.xlsm* allows to study all the determinants of the position and slope of the *AD*-curve.

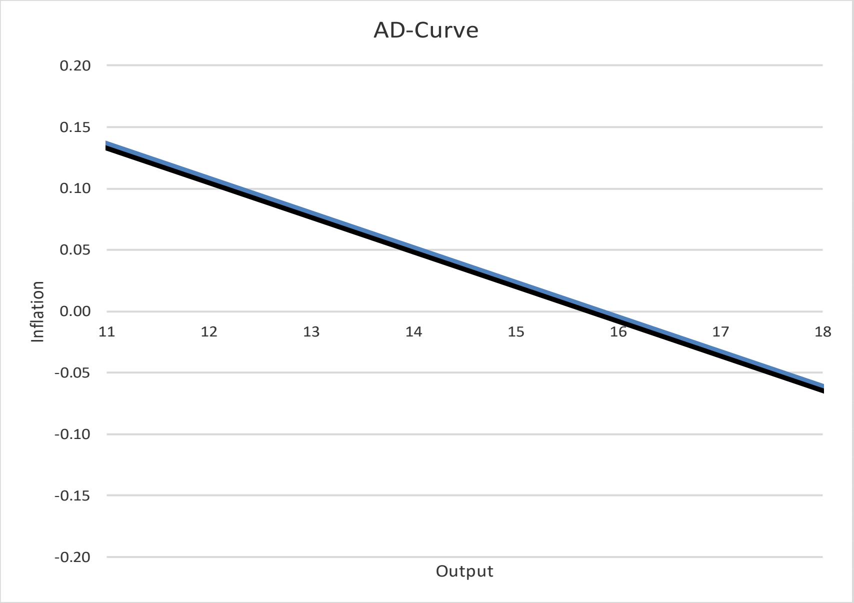
AD-Curve: Specification

Parameters

TR-Curve Parameters	Initial	New
$\bar{r}^{MP}$	0.020	0.020
$\bar{r}^{RP}$	0.010	0.010
ϕ_y	0.500	0.500
ϕ_π	2.000	2.000
$\bar{Y}$	15.000	15.000
π^T	0.020	0.020

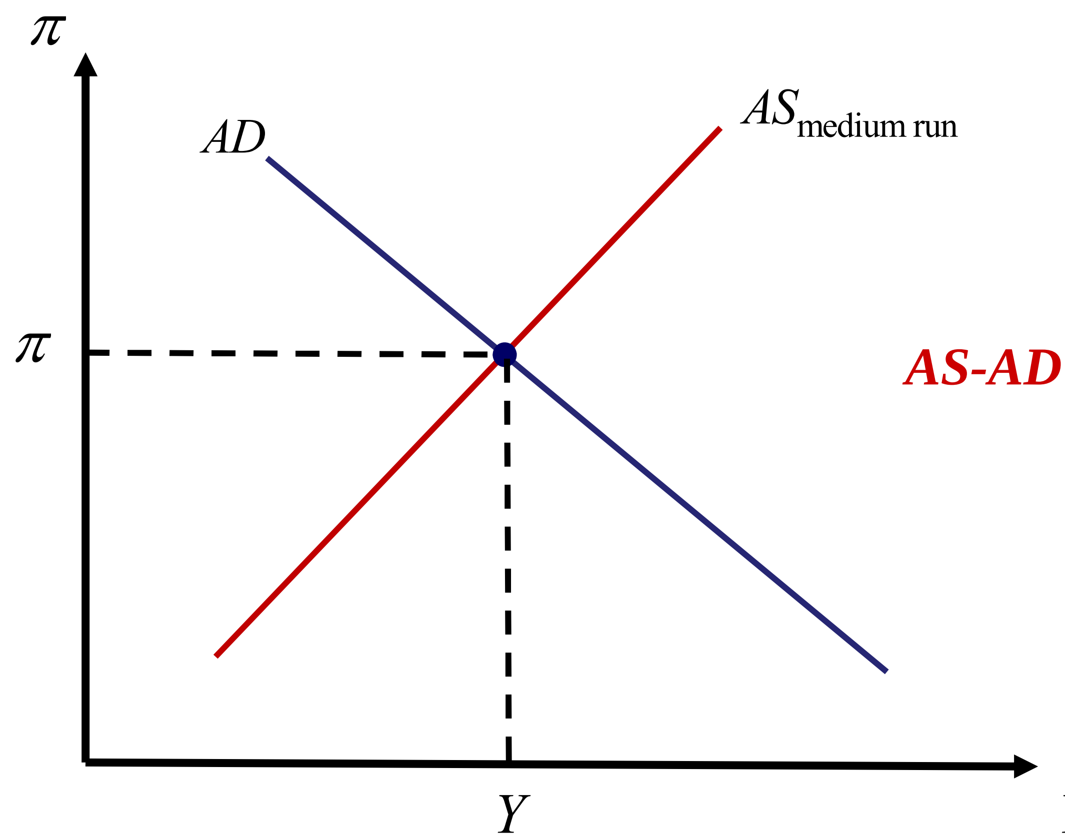
IS-Curve Parameters	Initial	New
C_r	4.000	4.000
I_r	16.000	16.000
TB_ε	1.000	1.000
ε_r	6.000	6.000
C_y	0.600	0.600
TB_{im}	0.050	0.050
G_y	0.150	0.150
A	9.780	9.980
C_0	1.500	1.500
I_0	3.000	3.000
G_0	2.800	3.000
TB_0	0.000	0.000
ε_0	1.000	1.000
T	3.000	3.000
TB_{ex}	0.010	0.010
Y^*	100.000	100.000
T^*	30.000	30.000
r^*	0.030	0.030

AD-Curve: Graph



The *AS-AD* Model

The medium-run macroeconomic outcome occurs at the intersection of the medium-run *AS*-curve with the *AD*-curve: it is the one combination of output and inflation that is compatible with the medium-run outcomes in both the real and the financial sector.



***AS-AD* model**

To appreciate better how the *AS-AD* model explains the functioning of the macroeconomy in the medium run, let us consider what happens following an increase in foreign income, Y^*.

Note at the outset, though, that as in the medium run the rate of inflation is allowed to vary, we need to pin down how inflation expectations, that enter the medium-run *AS*-curve, which from Equation (1) is given by

$$\pi_t = \pi_t^e + \omega \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + s_t ,$$

respond to changes in the actual rate of inflation.

Two often-used specifications of how inflation expectations are formed are:

(i) **inflation expectations** are **static**:

$$\pi_t^e = \pi^e ; \tag{15}$$

inflation expectations then do not respond to changes in the actual rate of inflation.

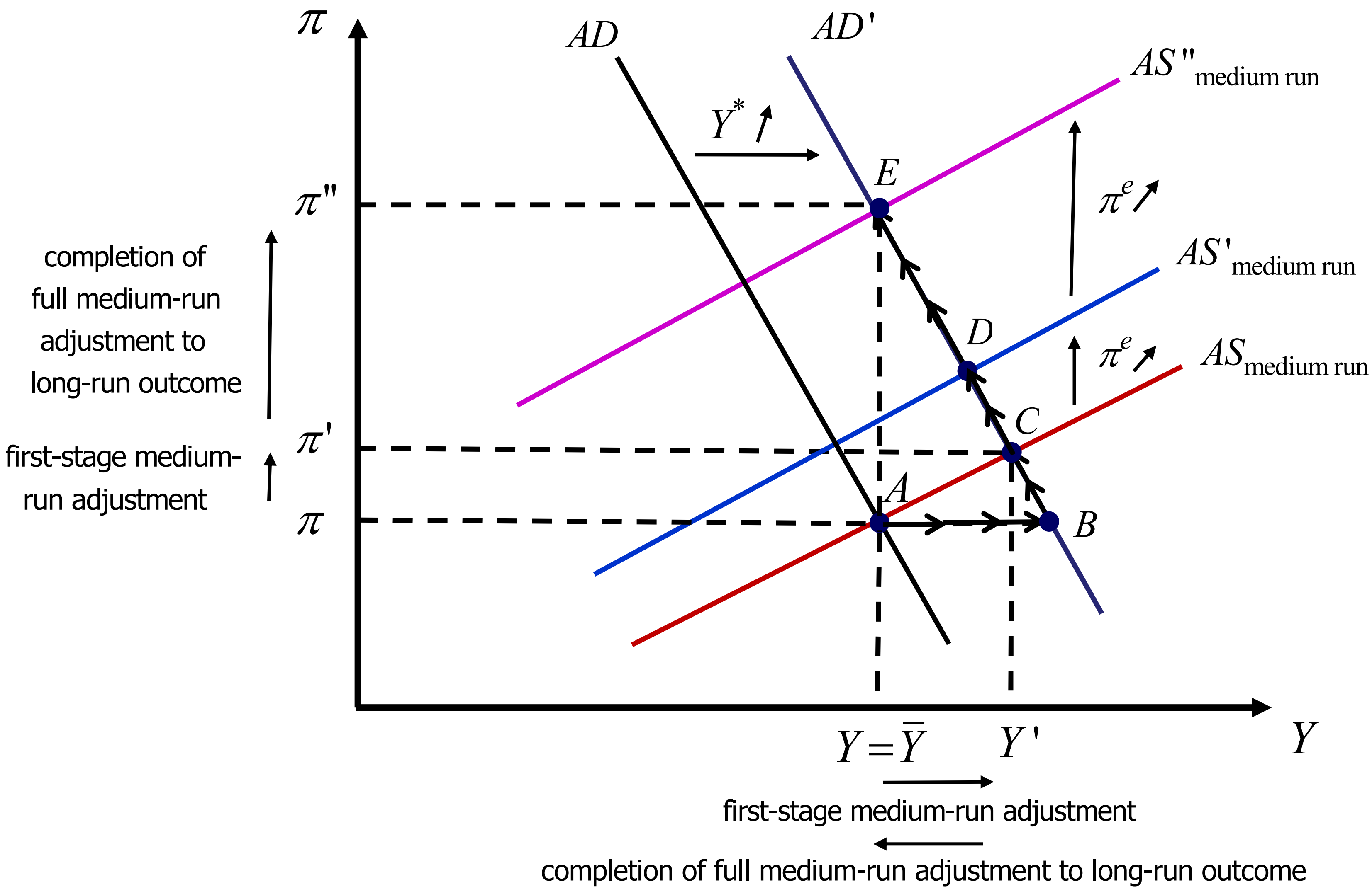
(ii) **inflation expectations** are **adaptive** and adjust one-to-one to the previous period's actual rate of inflation:

$$\pi_t^e = \pi_{t-1} . \tag{16}$$

For now, let us choose the less restrictive specification, that of adaptive expectations. Substituting from (16) into the medium-run AS-curve, Equation (1), we obtain:

$$\pi_t = \pi_{t-1} + \omega \cdot \left(\frac{Y_t - \bar{Y}}{\bar{Y}} \right) + s_t . \tag{17}$$

Set of adjustments following an increase in foreign income, Y^* (suppposing that initially output in the domestic economy, Y, is equal to long-run output, $\bar{Y}$):



- Starting from the original long-run outcome at Point A, after the increase in $Y*$ the AD-curve shifts to the right.

- The move from Point A to Point B reflects the short-run adjustments of the macroeconomy:
  -- the initial increase in desired aggregate demand,
  -- the resultant Keynesian multiplier effects, and
  -- the crowding out of consumption, investment and the trade balance in response to the central bank in line with the Taylor rule raising the monetary policy rate.

- At Point B, we are off the medium-run AS-curve, however: As output is expanding, wage mark-ups rise, leading to accelerated growth of production costs and thus also accelerated growth of the prices of goods and services, that is, rising inflation.

- The rise in inflation causes the central bank, again in line with the Taylor rule, to further raise the monetary policy rate, causing further crowding out. For given inflation expectations, this crowding out occurs as we move along the AD-curve from Point B to Point C: Point C is a medium-run outcome in both the real and the financial sector.

- Under adaptive inflation expectations overall medium-run adjustment does not stop at Point C, though (it would stop there if inflation expectations were static).

- As actual inflation is increasing, under adaptive inflation expectations the inflation expectations start to rise as well, and the medium-run AS-curve in the next period of medium-run adjustment shifts upwards, implying higher rates of inflation for given levels of output.

- The central bank responds to this further rise of inflation, again raising the monetary policy rate in an attempt to curb the inflationary process. The implied further crowding out of consumption, investment and the trade balance is reflected in the move along the AD-curve from Point C to Point D. Point D is the medium-run outcome in both the real and the financial sector in the second period of adjustment.

- At Point D, relative to the starting point at A, we are still in an expansionary phase for output. As actual inflation has increased again and inflation expectations are adaptive, the inflation expectations continue to rise.

- The medium-run AS-curve thus continues to shift upwards, implying yet higher rates of inflation for given levels of output (leading the central bank, in its continued attempt to curb the inflationary process, to keep raising the monetary policy rate, in turn driving continued crowding out of consumption, investment and the trade balance).

- Inflation expectations only stop rising when the medium-run AS-curve has shifted so far upwards that it intersects the prevailing AD-curve at the initial, long-run level of output. At Point E, we have that $Y_t = \bar{Y}$, and thus from the medium-run AS-curve, Equation (17), it then holds that $\pi_t = \pi_{t-1} = \pi_t^e$ (as $s_t = 0$).
- At Point E, when medium-run adjustment has completed, the foreign income-driven aggregate demand stimulus has caused a higher rate of inflation; all short- and medium-run increases of the level of output in the domestic economy have been crowded out.

Quantitative Analysis:

The medium-run macroeconomic outcome implied by the *AS-AD* model is obained by combining the *AD*-curve, Equation (14), amended to include time subscripts,

$$\pi_t = \frac{\left(\phi_y + \phi_\pi \cdot \pi^T - \bar{r}^{MP} - \bar{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \frac{\bar{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \bar{Y}} \cdot Y_t \,, \qquad (18)$$

with the medium-run *AS*-curve, Equation (17),

$$\pi_t = \pi_{t-1} + \omega \cdot \left(\frac{Y_t - \bar{Y}}{\bar{Y}} \right) + s_t \,.$$

These are two equations in two unknowns, the rate of inflation, π_t , and the output level, Y_t .

The level of output in the medium-run macroeconomic outcome is given by:

$$\pi_t = \underbrace{\frac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \frac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}} \cdot Y_t}_{\text{from } AD\text{-curve (18)}} = \underbrace{\pi_{t-1} + \omega \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}}\right) + s_t}_{\text{from medium-run } AS\text{-curve (17)}} = \pi_t . \qquad (19)$$

Note that (19) is one equation in one unknown, namely Y_t. We can thus use Equation (19) to solve for Y_t:

$$\frac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \pi_{t-1} + \omega - s_t = \left(\frac{\omega}{\overline{Y}} + \frac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}}\right) \cdot Y_t .$$

$$\Leftrightarrow \quad Y_t = \frac{\dfrac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \pi_{t-1} + \omega - s_t}{\dfrac{\omega}{\overline{Y}} + \dfrac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}}} . \qquad (20)$$

Equation (20) gives us the *AS-AD* model-implied level of **output** in the medium-run macroeconomic outcome.

Having obtained the level of output, we can calculate the AS-AD model-implied rate of **inflation** in the medium-run macroeconomic outcome from the medium-run AS-curve, Equation (17),

$$\pi_t = \pi_{t-1} + \omega \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + s_t \ ,$$

$$\Longleftrightarrow \quad \pi_t = \pi_{t-1} - \omega + s_t + \frac{\omega}{\overline{Y}} \cdot Y_t \ , \qquad (21)$$

substituting for Y_t from Equation (20). Note that Equation (21) describes a dynamic process for the rate of inflation.

The real interest rate can now be calculated as well: Given the level of output and the rate of inflation, the medium-run TR-curve in Equation (11), amended to include time subscripts, yields the AS-AD model-implied **real interest rate** in the medium-run macroeconomic outcome:

$$r_t = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi_t - \pi^T \right) + \overline{r}^{RP}, \quad (22)$$

with Y_t given by Equation (20), and π_t given by Equation (21).

Given output, inflation and the real interest rate, the medium-run macroeconomic outcomes of the other key macroeconomic aggregates, including consumption, investment, government expenditure, the trade balance and the exchange rate, may be retrieved from the IS-portion of the model.

Numerical Example:

While the algebra is tedious, a numerical example will be illuminating. Note from Equations (20) and (21) that we need to initialize the value of inflation to calculate a first medium-run macroeconomic outcome for, say, period $t = 1$.

Suppose therefore that we start out in period $t = 0$ in the long-run macroeconomic outcome, so that $Y_0 = \overline{Y}$, $r_0 = \overline{r}^{MP} + \overline{r}^{RP}$, and $\pi_0 = \pi^T$.

Also suppose $A = 9.78$, $C_y = 0.6$, $G_y = 0.15$, $TB_{im} = 0.05$, $TB_{ex} = 0.01$, $C_r = 4$, $I_r = 16$, $TB_\varepsilon = 1$, $\varepsilon_r = 6$, $\overline{Y} = 15$, $\overline{r}^{MP} = 0.02$, $\overline{r}^{RP} = 0.01$, $\phi_y = 0.5$, $\phi_\pi = 2$, $\pi^T = 0.02$, $\omega = 0.05$ and $s = 0$.

Then for period $t = 1$ we obtain from Equation (20), under $\pi_0 = \pi^T$, as the medium-run level of output $Y_1 = 15$. Using this value of Y_1 in Equation (21), we find that the medium-run rate of inflation in period $t = 1$ is given by $\pi_1 = 0.02$. Furthermore, from Equation (22), the medium-run real interest rate in period $t = 1$ for these values of Y_1 and π_1 is given by $r_1 = 0.03$.

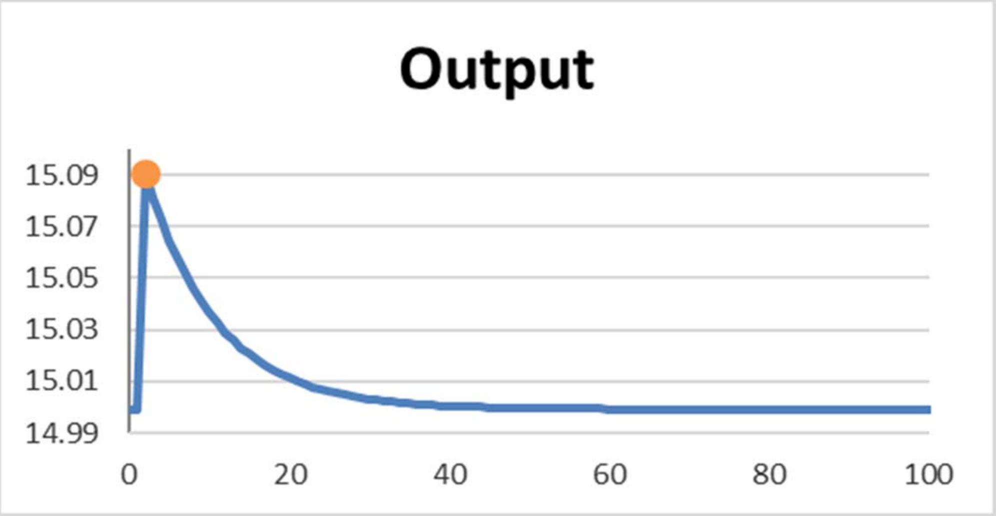
Now suppose that in period $t=2$ the foreign income, Y^*, increases by 15, that is, $\Delta Y^* = 15$. As in the short-run analysis, this increase of foreign income implies a change in the autonomous component of aggregate demand by

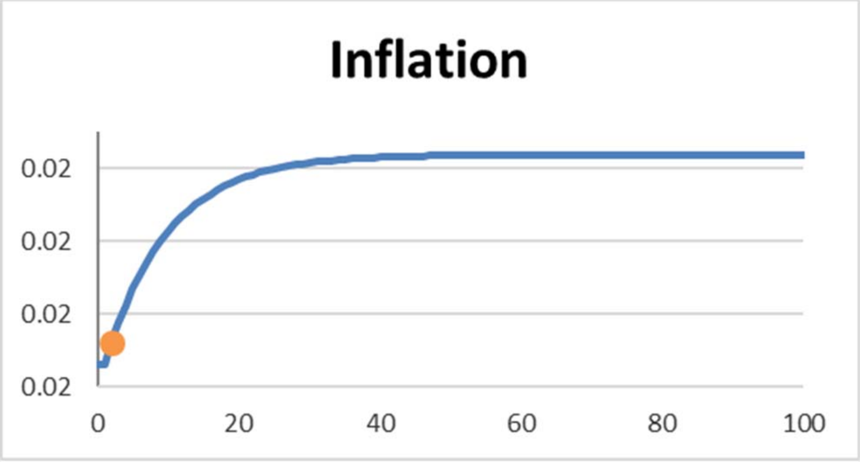
$$\Delta A = \frac{\partial A}{\partial Y^*} \cdot \Delta Y^* = TB_{ex} \cdot \Delta Y^* = 0.15.$$

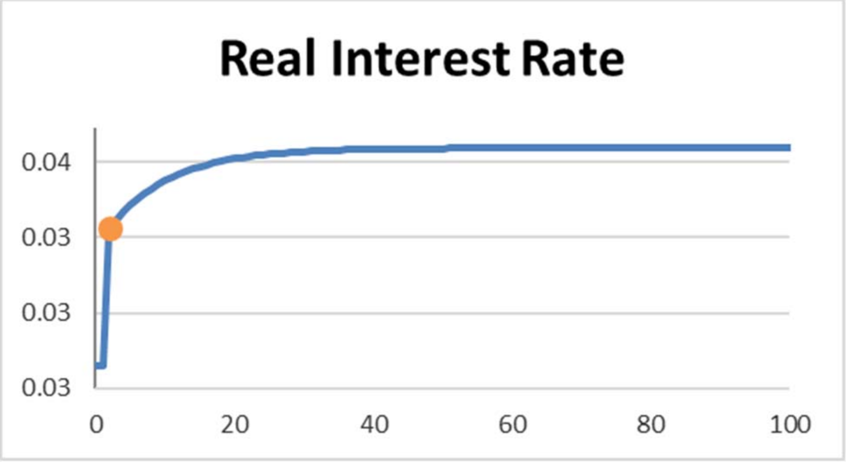
Using the implied new value of IS_0, $IS_0 = \left(A + \Delta A\right) / \left[1 - \left(C_y - TB_{im} - G_y\right)\right] = 16.55$, as well as $\pi_1 = 0.02$ in Equation (20), we obtain as the period $t=2$ medium-run level of output $Y_2 \approx 15.094$. Using the (exact) level of Y_2 in Equation (21), we find the medium-run rate of inflation in period $t=2$: $\pi_2 \approx 0.020$. Furthermore, from Equation (22), the medium-run real interest rate in period $t=2$ for the (exact) values of Y_2 and π_2 can be calculated to be $r_2 \approx 0.034$ (following the Taylor rule, the central bank has increased the monetary policy rate, and so the real interest rate has risen).

Given that the (exact) value of the rate of inflation has increased from period $t=1$ to period $t=2$ and inflation expectations are adaptive, the adjustment process does not end in period $t=2$. Using the same steps as for period $t=2$, we may calculate the values of medium-run output, inflation and the real interest rate for all subsequent periods, $t = 3, 4, \ldots$, until medium-run adjustment is complete and we have reached the long-run levels.

Clearly, such calculations can be quite time-consuming, and we confine ourselves here to reporting the results, based on automated calculations within the workbook *AS-AD-Model.xlsm*. The following graphs show the full medium-run adjustment paths implied by Equations (20), (21) and (22):


Output
15.09
15.07
15.05
15.03
15.01
14.99
0
20
40
60
80
100


Inflation
0.02
0.02
0.02
0.02
0
20
40
60
80
100


Real Interest Rate
0.04
0.03
0.03
0.03
0
20
40
60
80
100

Workbook for the *AS-AD* Model

The workbook *AS-AD-Model.xlsm* allows to study how changes in the various parameters and exogenous variables of the *AS-AD*-model …

AS-AD Model: Parameter Specification

	TR-Curve Parameters						IS-Curve Parameters																		AS-Curve Parameters	
Period	$\bar{r}^{MP}$	$\bar{r}^{RP}$	ϕ_y	ϕ_π	$\bar{Y}$	π^T	C_r	I_r	TB_ε	ε_r	C_y	TB_{im}	G_y	A	C_0	I_0	G_0	TB_0	ε_0	T	TB_{ex}	Y^*	T^*	r^*	ω	s
0	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
1	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
2	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
3	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
4	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
5	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
6	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
7	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
8	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
9	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
10	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
11	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
12	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
13	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
14	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
15	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
16	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
17	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
18	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
19	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
20	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
21	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
22	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
23	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
24	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
25	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
26	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
27	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
28	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
29	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
30	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
31	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
32	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
33	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
34	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
35	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
36	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
37	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
38	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
39	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000
40	0.020	0.010	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.930	1.500	3.000	2.800	0.000	1.000	3.000	0.010	115.000	30.000	0.030	0.050	0.000

Notes: It is advised not to change the parameter values for Period 0. These are set such that the model initialization in Period 0 occurs in the long-run macroeconomic outcome. From Period 41 onwards, all parameter values are constrained to be the same as for Period 40.

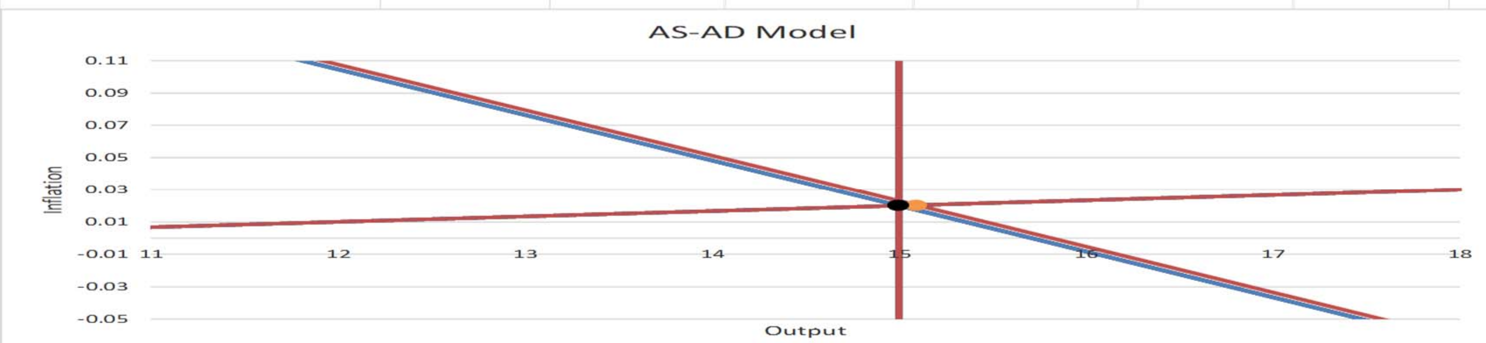
… affect output, inflation and the other endogenous variables in the medium-run macroeconomic outcome, …

AS-AD Model: Quantitative Results and AS-AD Diagram

Medium-Run Macroeconomic Outcomes

	Period			
	0	1	2	1000
Inflation	0.020	0.020	0.020	0.023
Output	15.000	15.000	15.091	15.000
Real Interest Rate	0.030	0.030	0.034	0.036
Consumption	8.820	8.820	8.890	8.843
Investment	2.520	2.520	2.461	2.428
Government Expenditure	2.800	2.800	2.786	2.800
Trade Balance	1.100	1.100	1.223	1.215
Real Exchange Rate	1.000	1.000	0.978	0.965
Nominal Monetary Policy Rate	0.040	0.040	0.044	0.046
Nominal Interest Rate	0.050	0.050	0.054	0.056

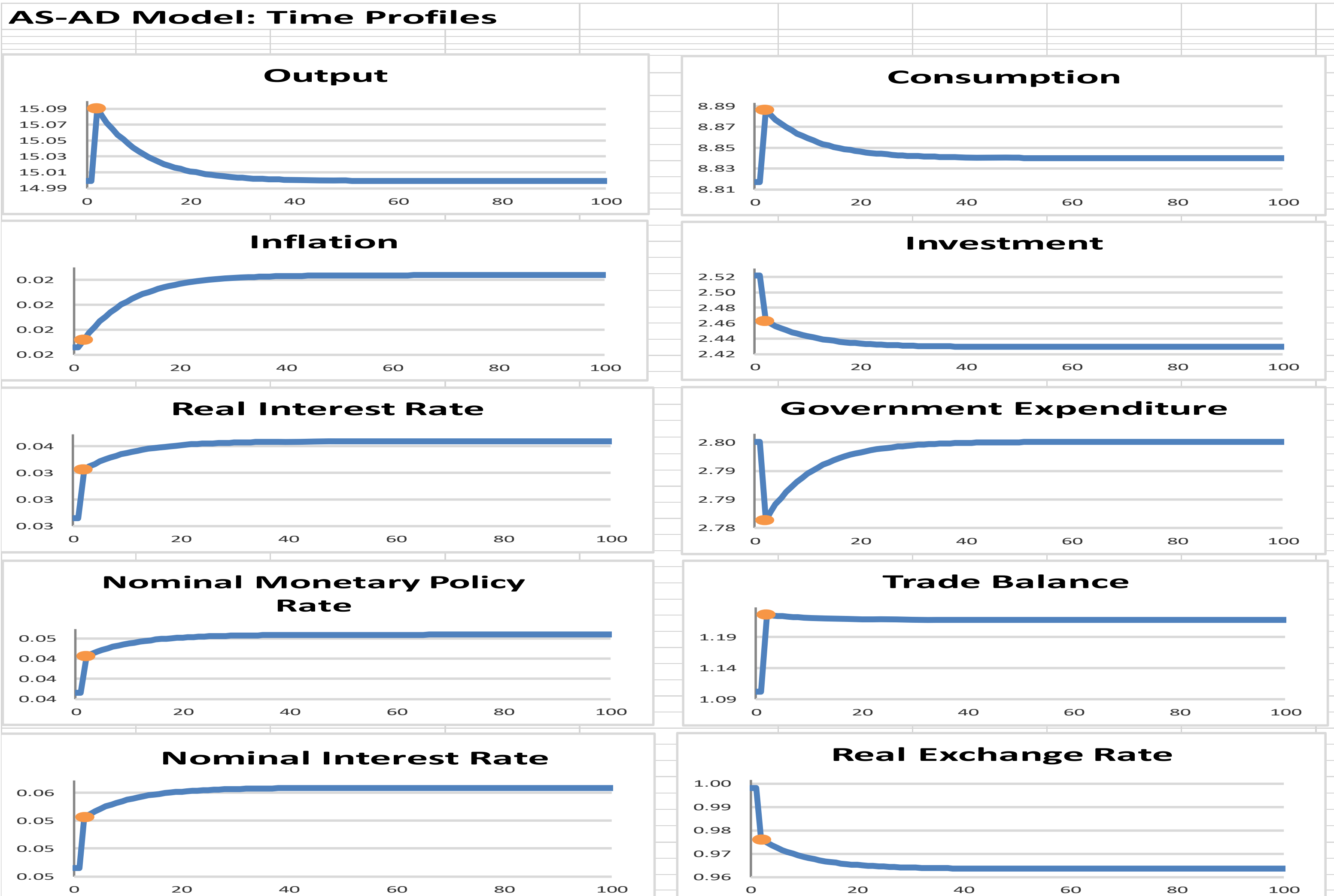
	Current	
Period	2	Plus / Minus



Rescale All Graphs

Horizons	
tmin	tmax
0	100

... and also graphs the transition dynamics (from the initialization onwards, and depicting the full medium-run adjustment):



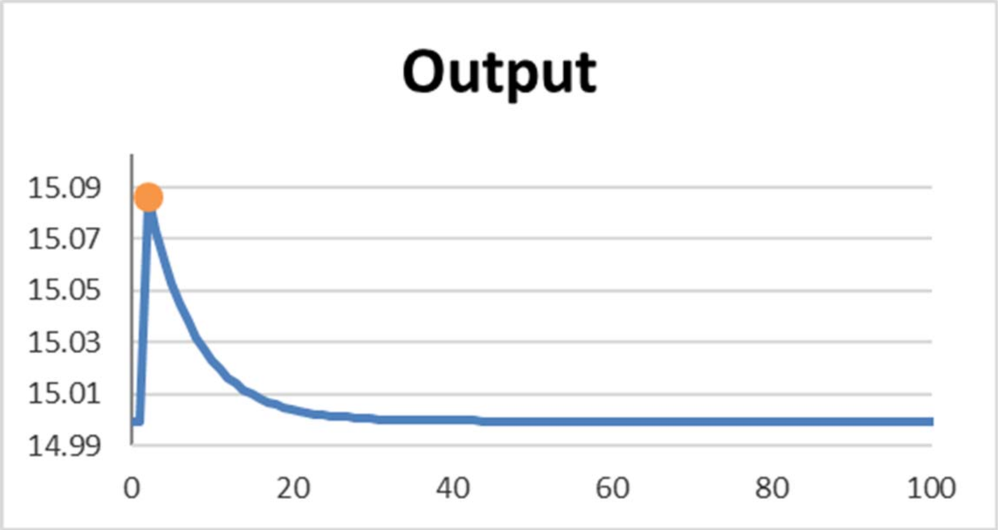
- Under the scenario considered in the numerical example we have considered (a permanent positive stimulus to aggregate demand, following an initial medium-run macroeconomic outcome that is equal to the long-run macroeconomic outcome),

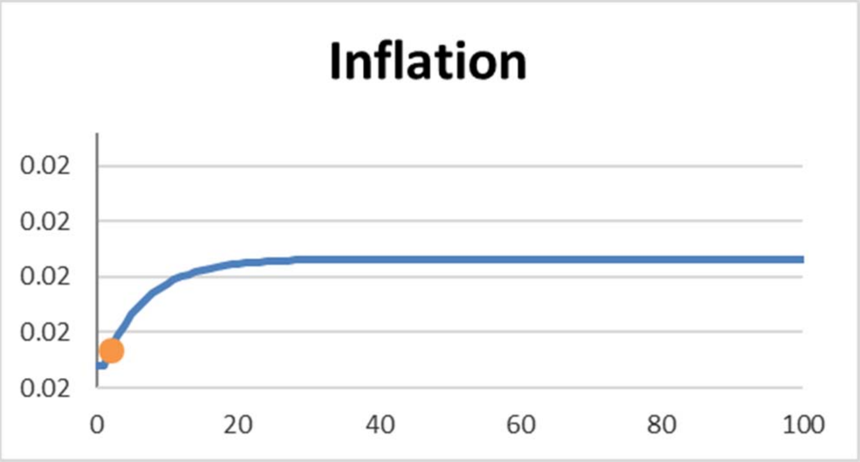
  -- the level of output eventually returns to its long-run level, but

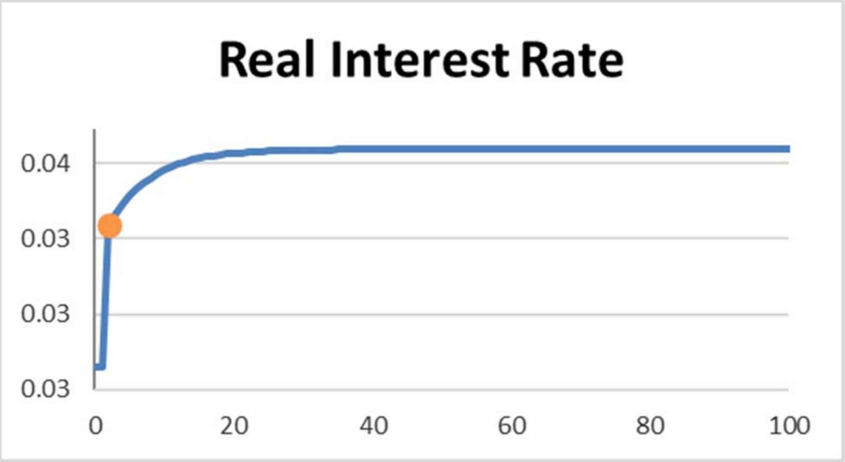
  -- the rate of inflation and the real interest rate rise permanently.

  Thus, the cost to the economy of benefitting from a temporary expansion of output includes a permanent loss in the degree of price stability prevailing in the economy.

- Everything else equal, if the central bank in its setting of the monetary policy rate was yet more responsive to deviations of actual inflation from target inflation, and we had, say, $\phi_{\pi} = 3$, then the time paths for output, inflation and the real interest rate would change to:


Output
15.09
15.07
15.05
15.03
15.01
14.99
0
20
40
60
80
100


Inflation
0.02
0.02
0.02
0.02
0.02
0
20
40
60
80
100


Real Interest Rate
0.04
0.03
0.03
0.03
0
20
40
60
80
100

- These graphs show the trade-off between

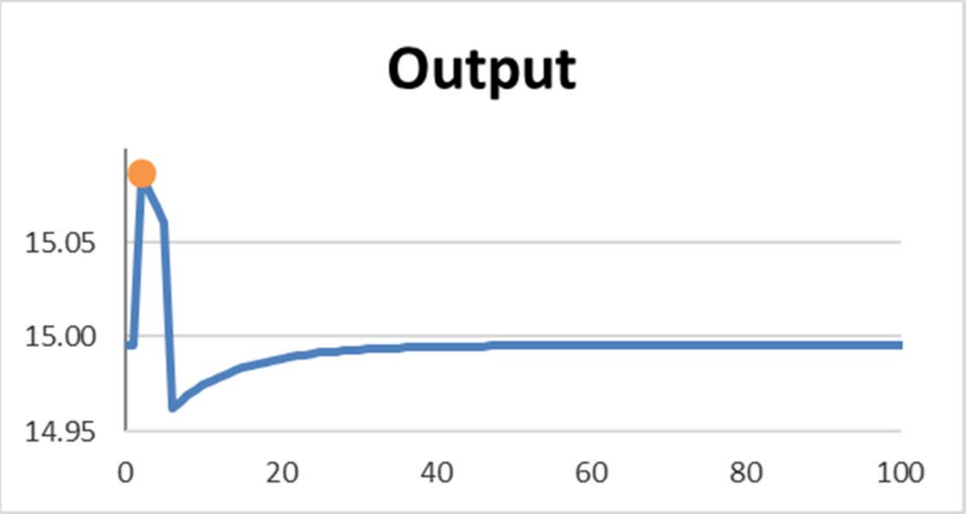
-- more sustained medium-run output gains facilitated by a "less inflation responsive" central bank, coming at the cost of a stronger increase of long-run inflation,

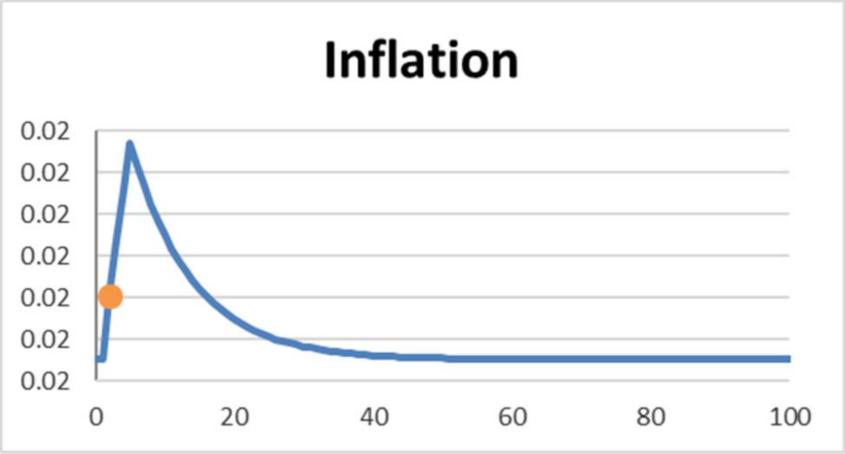
and

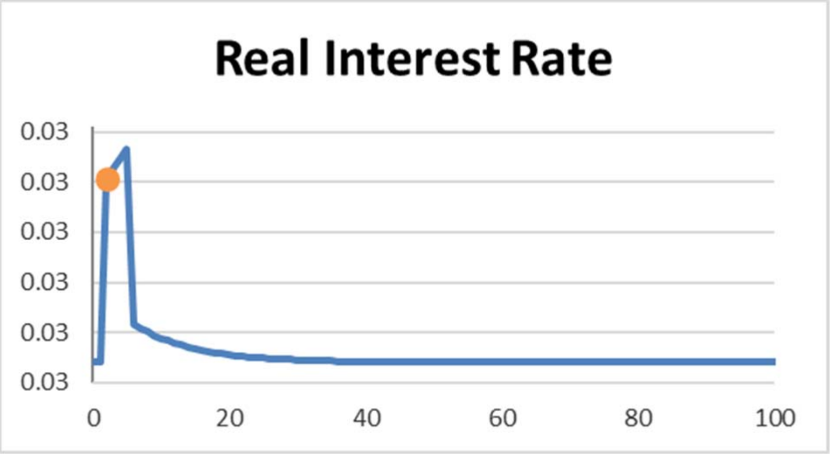
-- the success of a "more inflation responsive" central bank in reducing the rise of long-run inflation, coming at the cost of shorter-lived medium-run output gains.

All the scenarios considered so far involve the specification that the stimulus to aggregate demand persists forever (from $t=2$ onwards for all periods). This is rather unlikely, though, taking into account also that we started from the long-run macroeconomic outcome.

- Consider thus (setting $\phi_\pi = 2$ again) the case of a temporary aggregate demand stimulus, say an increase of foreign income, Y^*, by 15 that lasts for four periods only (from $t=2$ to $t=5$), before foreign income in period period $t=6$ is coming back down to its original level of $Y^* = 100$.
- As the following graphs illustrate, it is then not just output that eventually converges back to its long-run level, but also the real interest rate; furthermore, inflation then converges back to the target rate of inflation, π^T:


Output
15.05
15.00
14.95
0
20
40
60
80
100


Inflation
0.02
0.02
0.02
0.02
0.02
0.02
0.02
0
20
40
60
80
100


Real Interest Rate
0.03
0.03
0.03
0.03
0.03
0.03
0
20
40
60
80
100

- The adjustment path for output is particularly intriguing: following the peak output gain that occurs in period $t=2$ (the first period of the temporary aggregate demand stimulus), the adjustment path involves a full business cycle (a contraction between $t=2$ and $t=6$, and then an expansion after $t=6$ in the return to the long-run level of output).
- The contraction occurs as in periods $t=2$, 3, 4 and 5 we observe crowding out due to the central bank raising the monetary policy rate in its effort to curb rising inflation, and in period $t=6$ due to aggregate demand falling back down to its original level (and this far outweighing the crowding-in effects caused by the central bank lowering the monetary policy rate in period $t=6$ responding to the decline in aggregate demand).
- The expansion occurs as from period $t=7$ onwards aggregate demand is stable and the central back keeps lowering the monetary policy rate while output is below its long-run level.
- Using the workbook *AS-AD-Model.xlsm*, these dynamics can also be traced within the *AS-AD* model diagram.

- Given what we have learned so far, it should not come as a surprise that

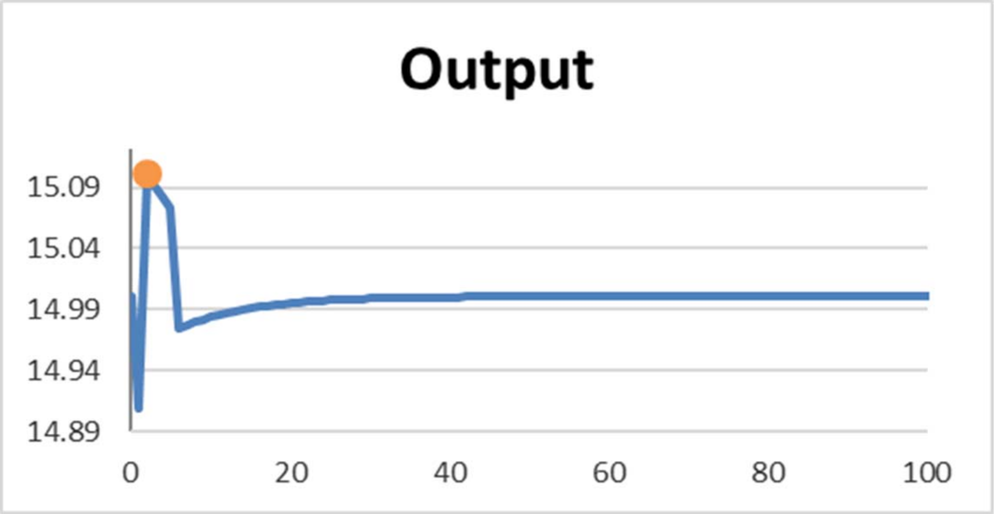
  -- if the period $t=1$ medium-run macroeconomic outcome was not equal to the long-run macroeconomic outcome,

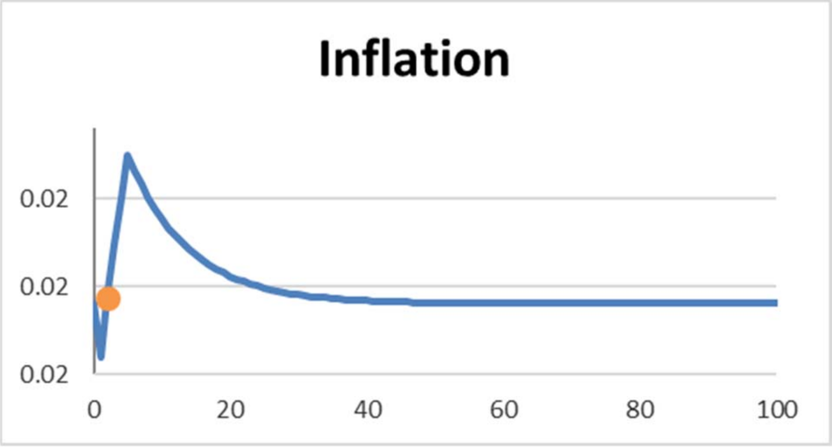
  then following a temporary aggregate demand stimulus,

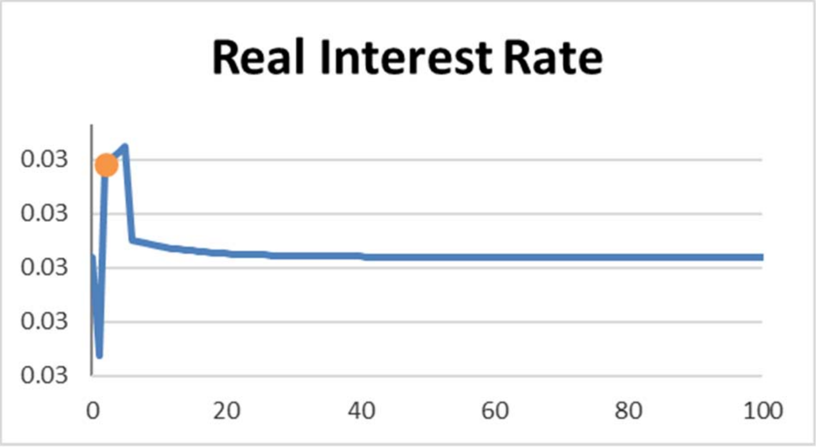
  -- output, inflation and the real interest rate would eventually converge back to their long-run/target levels (rather than their period $t=1$ values).

- To illustrate this point explicitly, consider a scenario in which in period $t=1$ we have that $Y^*=85$, so that the level of output in the period of the first medium-run macroeconomic outcome is below the long-run level of output. As in the previous scenario, foreign income then rises temporarily for four periods, from $t=2$ to $t=5$, to $Y^*=115$, before it falls back down in period $t=6$ to $Y^*=100$.

- The time paths of output, inflation and the real interest rate then are as follows, showing that these indeed converge back to their long-run/target levels:


Output
15.09
15.04
14.99
14.94
14.89
0
20
40
60
80
100


Inflation
0.02
0.02
0.02
0
20
40
60
80
100


Real Interest Rate
0.03
0.03
0.03
0.03
0.03
0
20
40
60
80
100

3. Financial Crises and the Macroeconomy in the Medium Run: The Extended *AS*-*AD* Model

Readings:
- Challe (2019), Chapter 8
- Jones (2018), Chapter 14

- We return to studying macroeconomic outcomes following the onset of the global financial crisis, combining the medium-run "aggregate supply" relationship (medium-run *AS*-curve) with the insights from the extended *IS*-*TR* model, to obtain the **extended *AS*-*AD* model**.
- In analogy to our modelling strategy for the macroeconomy in the short run, we will augment the financial sector within the *AS*-*AD* model (that is, the medium-run *TR*-curve) by two features:

- First feature: a zero lower bound on the nominal monetary poliy rate, R^{MP}, which in the medium-run context is captured through the following **extended monetary policy rule**:

$$R^{MP} = \max\left\{0, \underbrace{\overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y-\overline{Y}}{\overline{Y}}\right) + \phi_\pi \cdot \left(\pi - \pi^T\right)}_{\text{Taylor Rule}}\right\}. \tag{23}$$

-- Under Equation (23), if according to the Taylor rule the central bank would choose to set a negative R^{MP}, it is constrained by the zero lower bound and instead sets $R^{MP} = 0$.

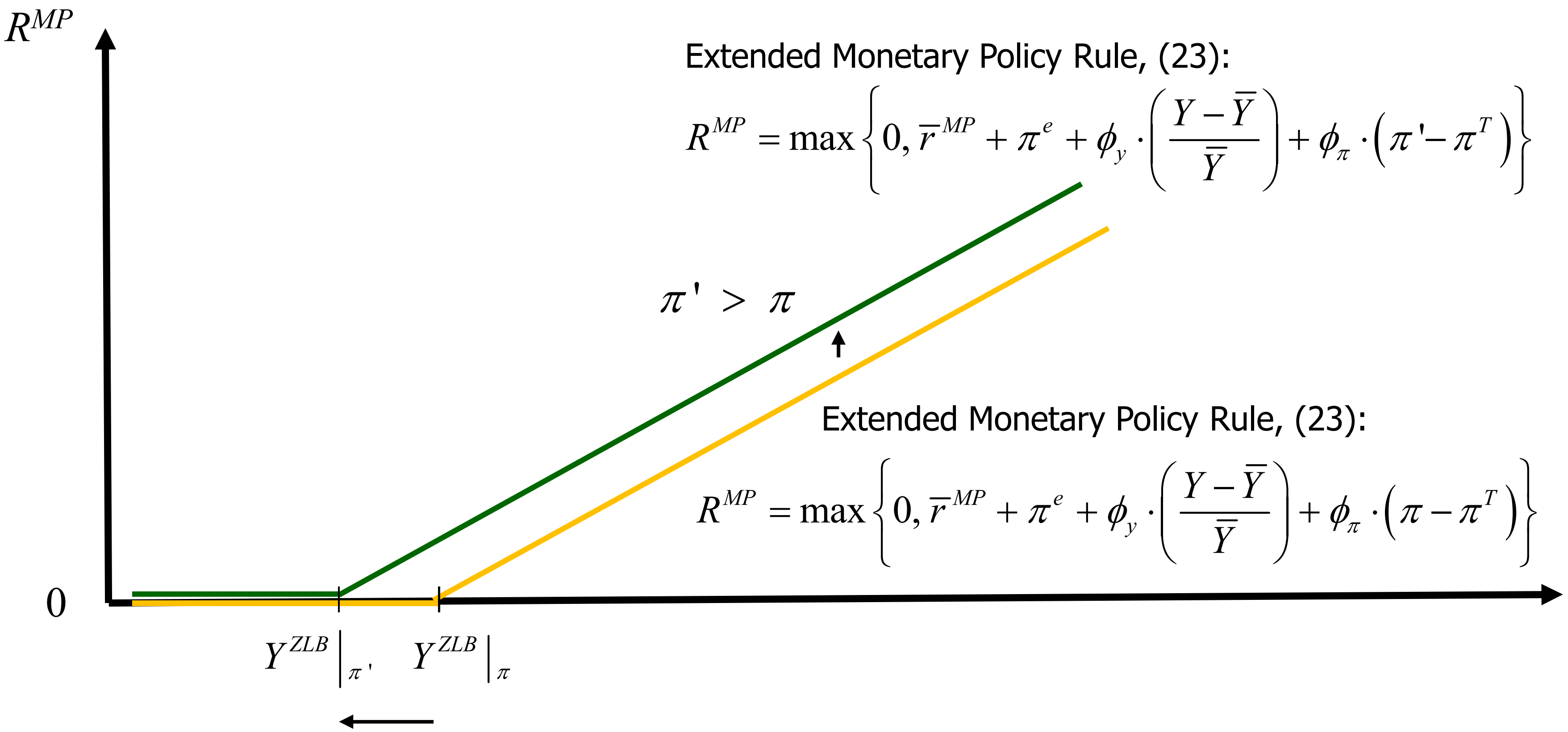
-- When the zero lower bound on R^{MP} is binding, we label the macroeconomy as being in a **"deep crisis"**.

-- Note from Equation (23) that the highest level of output at which the zero lower bound on R^{MP} is binding in the medium run depends on the rate of inflation, and is thus endogenous. To indicate this dependence, we denote the zero lower bound level of output as $Y^{ZLB}\big|_{\pi}$:

$$\bar{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y^{ZLB}\big|_{\pi} - \bar{Y}}{\bar{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right) = 0 \qquad (24)$$

$$\Leftrightarrow \quad Y^{ZLB}\big|_{\pi} = \frac{\bar{Y}}{\phi_y} \cdot \left[\phi_y - \bar{r}^{MP} - \pi^e - \phi_\pi \cdot \left(\pi - \pi^T \right) \right]. \qquad (25)$$

-- (25) implies that the zero lower bound level of output depends negatively on the rate of inflation: as the rate of inflation, say, increases, then the central bank sets the monetary policy rate to be higher (as long as the new zero lower bound constraint is not binding), and thus reaches the zero lower bound for R^{MP} at a lower level of output:


R^{MP}
Extended Monetary Policy Rule, (23):
$R^{MP} = \max\left\{0, \overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y-\overline{Y}}{\overline{Y}}\right) + \phi_\pi \cdot \left(\pi' - \pi^T\right)\right\}$
$\pi' > \pi$
Extended Monetary Policy Rule, (23):
$R^{MP} = \max\left\{0, \overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y-\overline{Y}}{\overline{Y}}\right) + \phi_\pi \cdot \left(\pi - \pi^T\right)\right\}$
0
$Y^{ZLB}\big|_{\pi'}$
$Y^{ZLB}\big|_{\pi}$
Y

- Second feature: an endogenous component to the risk premium, that in "deep crisis" times drives the mark-up between

  -- the interest rate, r, at which households and firms can borrow from the commercial banks to fund (part of) their consumption and investment expenditure, and

  -- the real value of the monetary policy rate, $r^{MP} \approx R^{MP} - \pi^e$, set by the central bank:

$$r^{RP} = r - r^{MP} = \overline{r}^{RP} \underbrace{- r_y \cdot \left(Y - Y^{ZLB}\big|_{\pi}\right) \cdot I\left(Y \leq Y^{ZLB}\big|_{\pi}\right)}_{\text{endogenous component of risk premium}} . \qquad (26)$$

- Combining Equations (23) and (26), we obtain the **extended *TR*-curve**, as applicable in the **medium run**:

$$
\begin{aligned}
r &= r^{MP} + r^{RP} \approx R^{MP} - \pi^{e} + r^{RP} \\
&= \max\left\{ -\pi^{e},\ \overline{r}^{MP} + \phi_{y} \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_{\pi} \cdot \left(\pi - \pi^{T} \right) \right\} \\
&\quad + \overline{r}^{RP} - r_{y} \cdot \left(Y - Y^{ZLB}\big|_{\pi} \right) \cdot I\left(Y \le Y^{ZLB}\big|_{\pi} \right). \qquad (27)
\end{aligned}
$$

- The medium-run extended *TR*-curve features a discontinuity at that level of output for which the Taylor rule suggests switching from a positive to a zero monetary policy rate, $Y^{ZLB}\big|_{\pi}$:

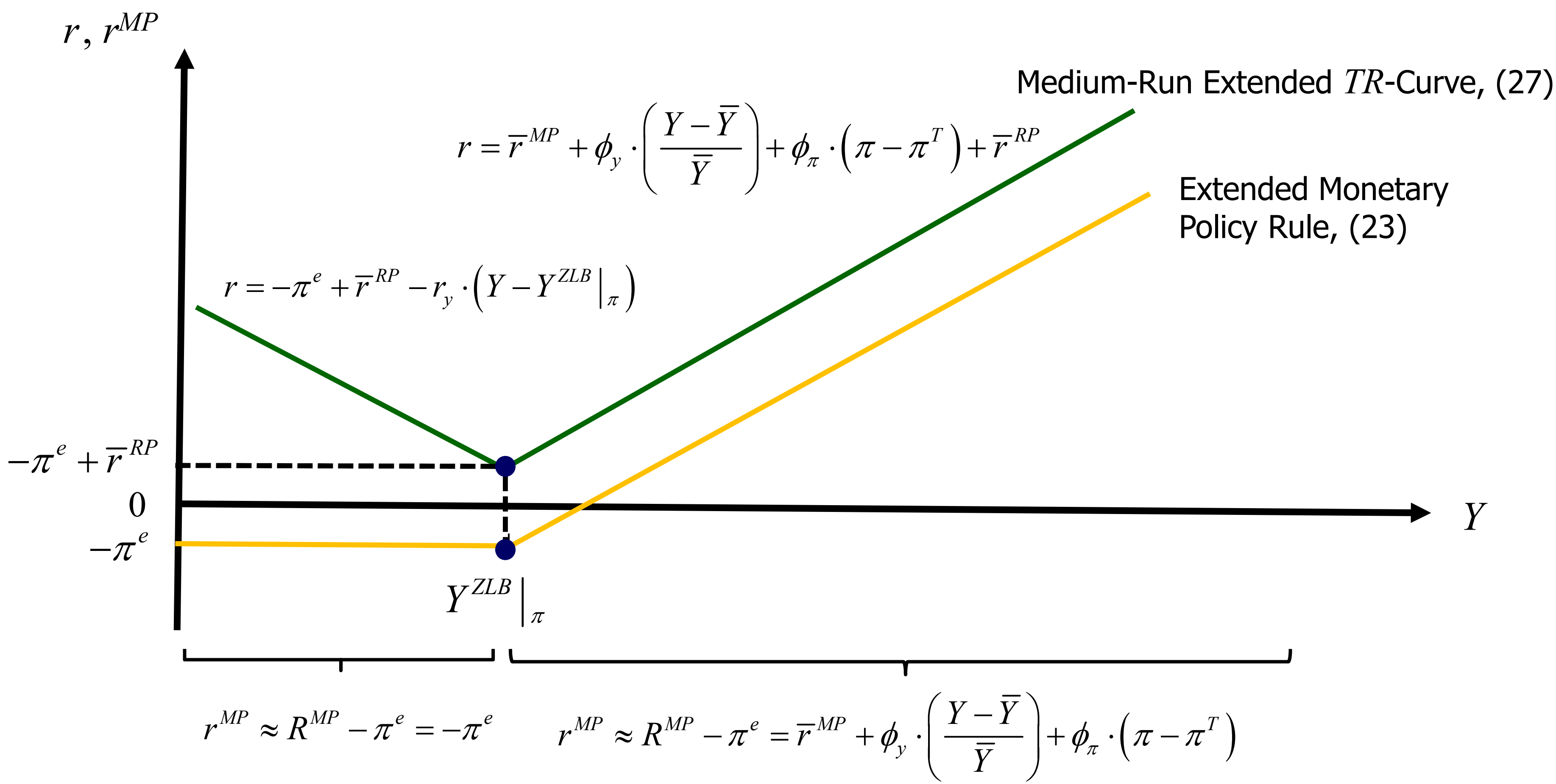
-- For "normal business cycle" outcomes, the zero lower bound on R^{MP} is not binding ($R^{MP} > 0$ and $Y > Y^{ZLB}\big|_{\pi}$), and (27) reduces to the medium-run TR-curve given by (11):

$$r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi - \pi^T \right) + \overline{r}^{RP}.$$

-- For "deep crisis" outcomes, the zero lower bound on R^{MP} is binding ($R^{MP} = 0$ and $Y \leq Y^{ZLB}\big|_{\pi}$), and (27) reduces to:

$$r = -\pi^e + \overline{r}^{RP} - r_y \cdot \left(Y - Y^{ZLB}\big|_{\pi} \right). \qquad (28)$$

Depicting the medium-run extended TR-curve graphically:



Workbook for the Medium-Run Extended *TR*-Curve

The workbook *Medium-Run-Extended-TR-Curve.xlsm* allows to study all the determinants of the position and slope of the medium-run extended *TR*-curve.

Medium-Run Extended TR-Curve: Specification

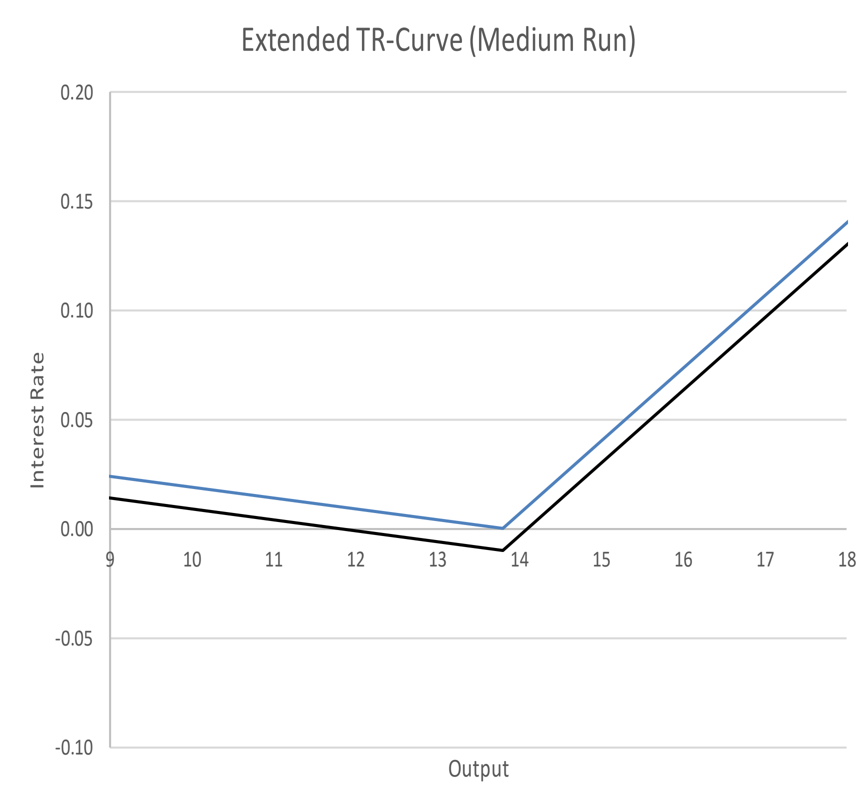
Parameters

	Initial	New
$\bar{r}^{MP}$	0.020	0.020
$\bar{r}^{RP}$	0.010	0.020
ϕ_y	0.500	0.500
$\bar{Y}$	15.000	15.000
r_y	0.005	0.005
ϕ_π	2.000	2.000
π^T	0.020	0.020
π	0.020	0.020
π^e	0.020	0.020

Quantitative Results

	Initial	New
$Y^{ZLB}\|_\pi$	13.800	13.800

Medium-Run Extended TR-Curve: Graph



The Extended *AD*-Curve

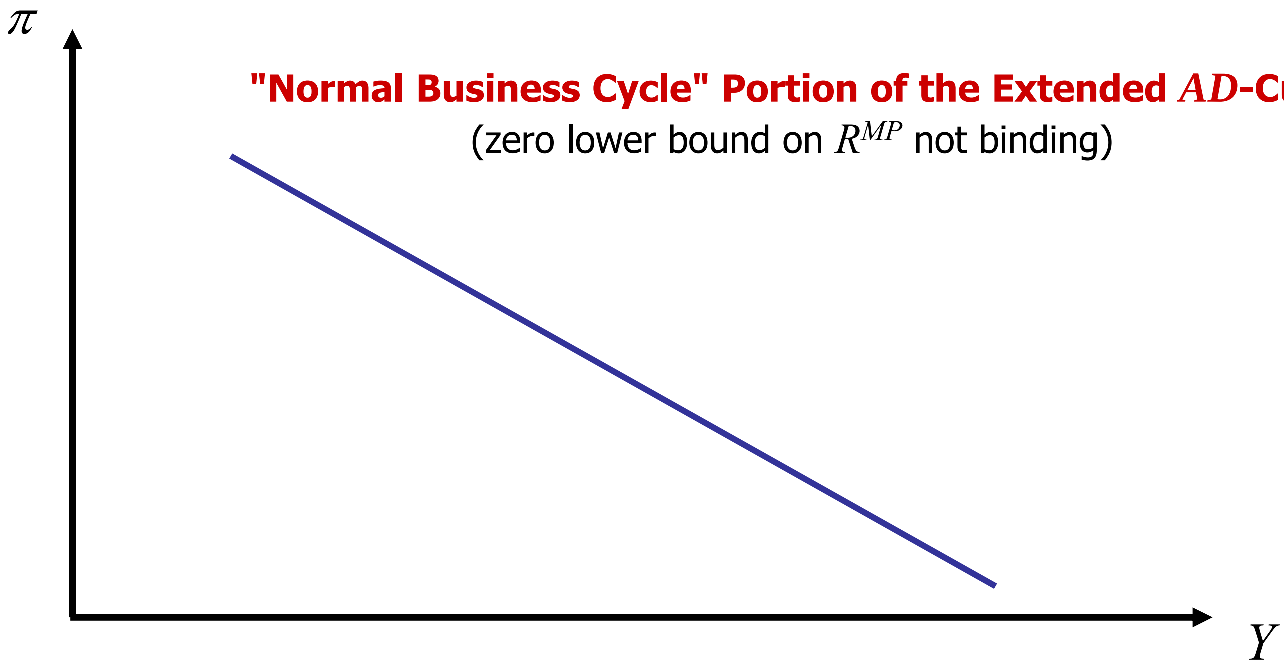
The extended *AS*-*AD* model combines the medium-run extended *TR*-curve with the standard *IS*-curve to yield an extended *AD*-curve.

To construct the extended *AD*-curve, let us first consider **"normal business cycle" outcomes**:

- When the zero lower bound on R^{MP} is not binding, then the extended *AD*-curve is the same as the *AD*-curve in the *AS*-*AD* model.
- To see this, note that in the *AS*-*AD* model the *AD*-curve (14) was obtained from combining the medium-run *TR*-curve (11) that holds when $R^{MP} > 0$ with the *IS*-curve (12), yielding

$$\pi = \frac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \frac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}} \cdot Y.$$

- Thus, when the zero lower bound on R^{MP} is not binding, then the extended AD-curve features the familiar negative relationship between output and inflation.
- This still reflects that as the rate of inflation, say, decreases, then the central bank, following the Taylor rule, lowers its monetary policy rate, R^{MP}. For a given exogenous risk premium, this increases output through the consumption, investment and exchange rate channels.



Turn second to **"deep crisis" outcomes**:

- When the zero lower bound on R^{MP} is binding, we can obtain the "deep crisis" portion of the extended AD-curve that is applicable then by substituting the real interest rate that holds when $R^{MP} = 0$ (see the medium-run extended TR-curve, Equations (27) and (28)),

$$r = -\pi^e + \overline{r}^{RP} - r_y \cdot \left(Y - Y^{ZLB}\big|_{\pi} \right),$$

into the IS-curve (12), obtaining

$$Y = IS_0 - IS_1 \cdot \left[-\pi^e + \overline{r}^{RP} - r_y \cdot \left(Y - Y^{ZLB}\big|_{\pi} \right) \right], \tag{29}$$

where, from Equation (25),

$$Y^{ZLB}\big|_{\pi} = \frac{\overline{Y}}{\phi_y} \cdot \left[\phi_y - \overline{r}^{MP} - \pi^e - \phi_\pi \cdot \left(\pi - \pi^T \right) \right].$$

- To complete derivation of the "deep crisis" portion of the extended AD-curve, we need to solve (29) for the rate of inflation. Substituting from (25) back into (29), we obtain

$$Y = IS_0 - IS_1 \cdot \left[-\pi^e + \overline{r}^{RP} - r_y \cdot \left(Y - \frac{\overline{Y}}{\phi_y} \cdot \left[\phi_y - \overline{r}^{MP} - \pi^e - \phi_\pi \cdot \left(\pi - \pi^T \right) \right] \right) \right]$$

$$<=> \left(1 - IS_1 \cdot r_y\right) \cdot Y = IS_0 - IS_1 \cdot \left[-\pi^e + \overline{r}^{RP} + \frac{r_y \cdot \overline{Y}}{\phi_y} \cdot \left[\phi_y - \overline{r}^{MP} - \pi^e - \phi_\pi \cdot \left(\pi - \pi^T \right) \right] \right]$$

$$<=> \left(1 - IS_1 \cdot r_y\right) \cdot Y = \frac{IS_1 \cdot r_y \cdot \overline{Y} \cdot \phi_\pi}{\phi_y} \cdot \pi$$

$$+ IS_0 - IS_1 \cdot \left[-\pi^e + \overline{r}^{RP} + \frac{r_y \cdot \overline{Y}}{\phi_y} \cdot \left(\phi_y - \overline{r}^{MP} - \pi^e + \phi_\pi \cdot \pi^T \right) \right]$$

$$<=> \pi = \left(\frac{\phi_y \cdot \left(1 - IS_1 \cdot r_y\right)}{IS_1 \cdot r_y \cdot \overline{Y} \cdot \phi_\pi} \right) \cdot Y$$

$$- \left(\frac{\phi_y}{IS_1 \cdot r_y \cdot \overline{Y} \cdot \phi_\pi} \right) \cdot \left[IS_0 - IS_1 \cdot \left(-\pi^e + \overline{r}^{RP} + \frac{r_y \cdot \overline{Y}}{\phi_y} \cdot \left(\phi_y - \overline{r}^{MP} - \pi^e + \phi_\pi \cdot \pi^T \right) \right) \right].$$

(30)

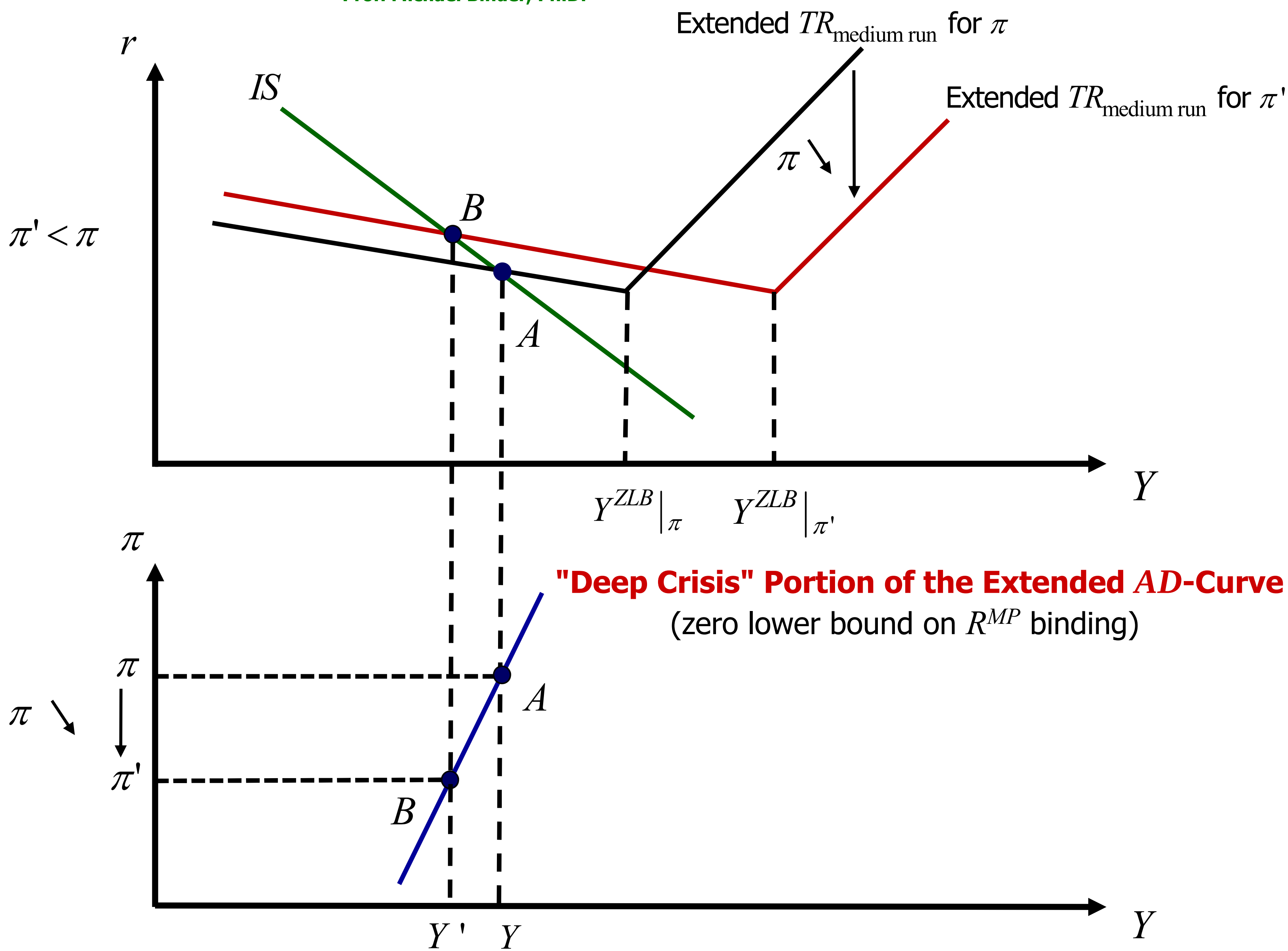
- Note that the **"deep crisis" portion of the extended *AD*-curve** in Equation (30) exhibits (for empirically sensible parameter values, including $0 < IS_1 \cdot r_y < 1$) a **positive relationship between output**, Y, **and inflation**, π, and is thus **upward sloping**.

- This reflects that as the rate of inflation decreases, $Y^{ZLB}\big|_{\pi}$ from Equation (25) increases (as the central bank for "normal business cycle" outcomes then levies lower monetary policy rates), implying that

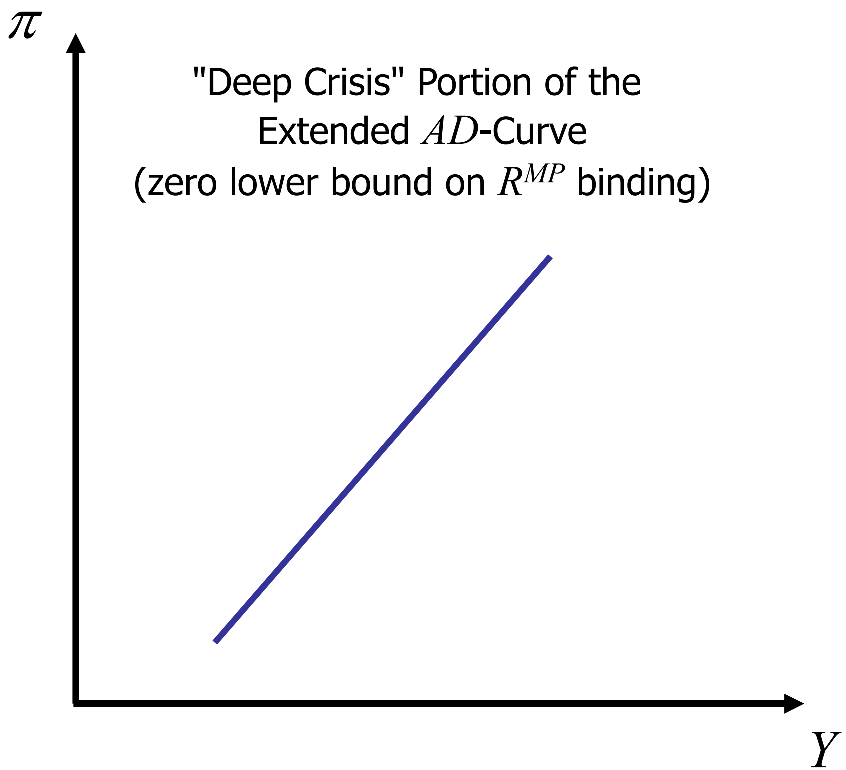
  -- the endogenous component of the risk premium becomes positive at higher levels of output than before, and

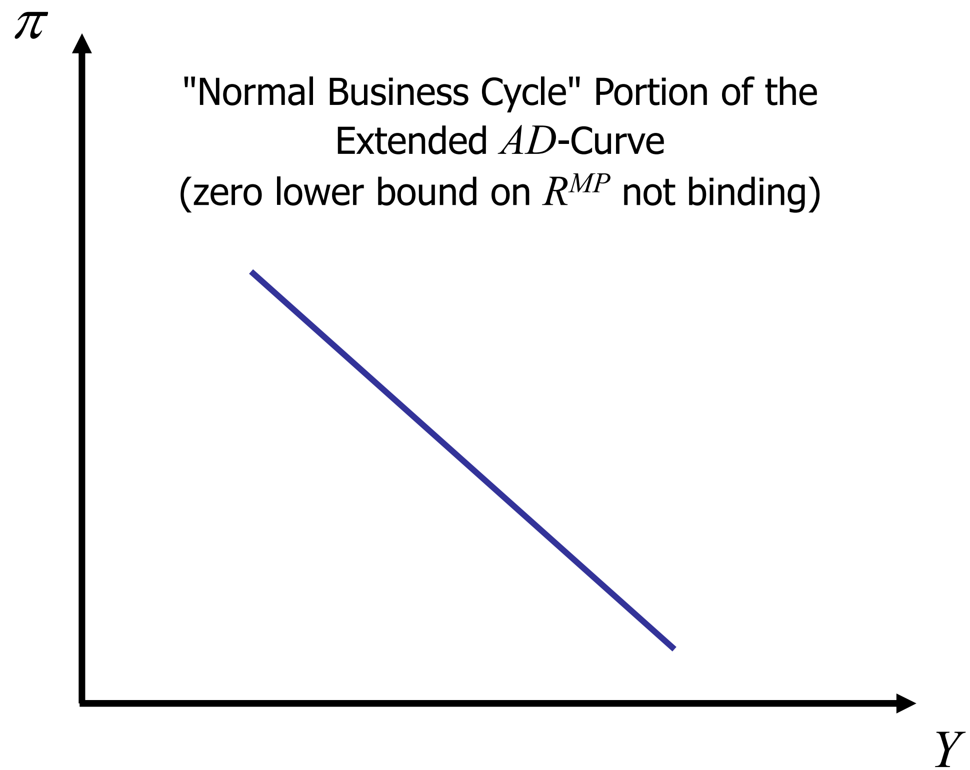
  -- for any level of output that is below the level of $Y^{ZLB}\big|_{\pi}$ that prevailed prior to the decrease of the rate of inflation, the endogenous component of the risk premium is now higher than before, in turn crowding out more of aggregate demand.

Let us illustrate graphically also that the "deep crisis" portion of the extended AD-curve is upward sloping:


Extended $TR_{\text{medium run}}$ for π
Extended $TR_{\text{medium run}}$ for π'
r
IS
$\pi \searrow$
$\pi' < \pi$
B
A
Y
$Y^{ZLB}|_{\pi}$
$Y^{ZLB}|_{\pi'}$
π
"Deep Crisis" Portion of the Extended AD-Curve
(zero lower bound on R^{MP} binding)
π
π'
$\pi \searrow$
A
B
Y'
Y
Y

How to combine the "deep crisis" and the "normal business cycle" portions of the extended AD-curve graphs into a single extended AD-curve graph?



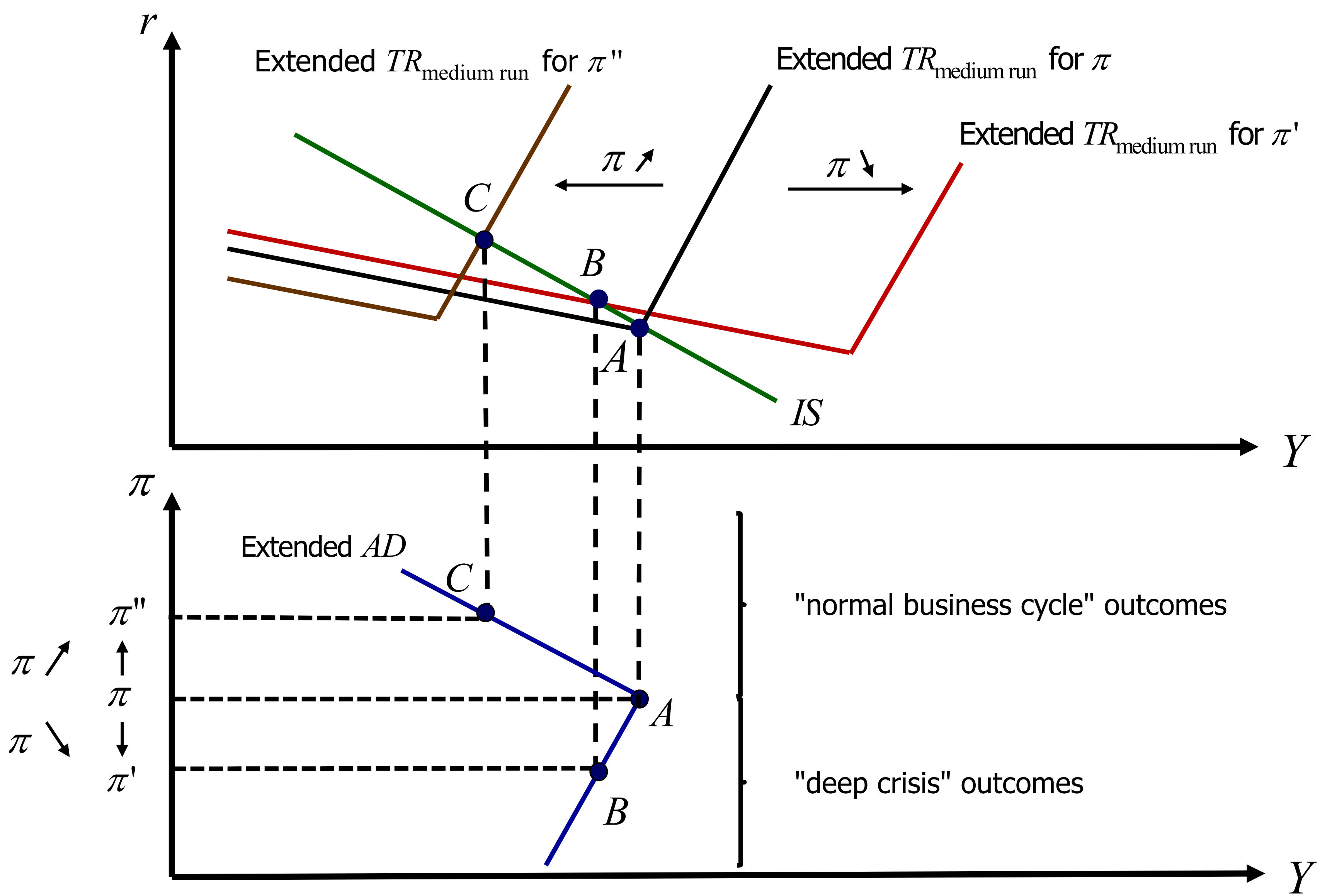


- We have already seen that the zero lower bound level of output depends on the rate of inflation. It is thus plausible to expect that in the extended AD-curve diagram, whether the AD-curve reflects a "normal business cycle" or a "deep crisis" outcome depends on the rate of inflation.

- When the rate of inflation is relatively low, from the Taylor Rule, the nominal monetary policy rate ceteris paribus is relatively low, rendering it more likely that we may reach the zero lower bound, $R^{MP} = 0$.

- When the rate of inflation is relatively high, from the Taylor Rule, the nominal monetary policy rate ceteris paribus is relatively high, rendering it less likely that we may reach the zero lower bound, $R^{MP} = 0$.

The following graph shows that this line of reasoning is correct:


r
Extended $TR_{\text{medium run}}$ for π''
Extended $TR_{\text{medium run}}$ for π
Extended $TR_{\text{medium run}}$ for π'
π ↗
π ↘
C
B
A
IS
Y
π
Extended AD
C
π''
π ↗
π
π ↘
π'
A
B
"normal business cycle" outcomes
"deep crisis" outcomes
Y

Let us inspect the **extended AD-curve** in more detail:

- At which rate of inflation does the extended AD-curve switch from being downward sloping ("normal business cycle" outcomes, $R^{MP} > 0$) to being upward sloping ("deep crisis" outcomes, $R^{MP} = 0$)?

- The discontinuity in the extended AD-curve occurs at the highest rate of inflation at which the zero lower bound on R^{MP} is binding. From the extended monetary policy rule, Equation (23), this rate of inflation depends on the level of output. To indicate this dependence, we denote the zero lower bound rate of inflation as $\pi^{ZLB}\big|_Y$:

$$\overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi^{ZLB}\big|_Y - \pi^T \right) = 0$$

$$\Leftrightarrow \quad \pi^{ZLB}\big|_Y = \pi^T - \frac{1}{\phi_\pi} \cdot \left[\overline{r}^{MP} + \pi^e + \phi_y \cdot \left(\frac{Y - \overline{Y}}{\overline{Y}} \right) \right]. \qquad (31)$$

- To obtain the unconditional π^{ZLB}, we need to eliminate Y in Equation (31).

- Since at the point of discontinuity in the extended AD-curve it not only holds that $R^{MP} = 0$, but it also holds that the endogenous component of the risk premium is equal to zero, from the medium-run extended TR-curve, Equations (27) and (28), we have that at the point of discontinuity in the extended AD-curve it holds that

$$r = -\pi^{e} + \overline{r}^{RP}. \tag{32}$$

- From the IS-curve relationship in Equation (12) we therefore have that at the point of discontinuity in the extended AD-curve it holds that

$$Y = Y^{ZLB} = IS_0 - IS_1 \cdot \left(-\pi^{e} + \overline{r}^{RP}\right). \tag{33}$$

- Substituting from Equation (33) back into the right-hand side of Equation (31), we have that at the point of discontinuity in the extended AD-curve it also holds that:

$$\pi^{ZLB} = \pi^{T} - \frac{1}{\phi_\pi} \cdot \left[\overline{r}^{MP} + \pi^{e} + \frac{\phi_y}{\overline{Y}} \cdot \left[IS_0 - IS_1 \cdot \left(-\pi^{e} + \overline{r}^{RP}\right)\right] - \phi_y \right]. \tag{34}$$

- Note from Equations (33) and (34) that both Y^{ZLB} and π^{ZLB} are functions of the expected rate of inflation, π^e. Note furthermore from Equation (30) that the intercept of the "deep crisis" portion of the extended AD-curve also is a function of π^e. Therefore, as under adaptive inflation expectations we know from the AS-AD model that the medium-run adjustment paths following, say, stimuli to aggregate demand can involve continued changes in π^e, such changes would lead to continued shifts of the extended AD-curve. This will be important to keep in mind later when analyzing medium-run adjustment paths in the extended AS-AD model.

To understand what outcomes may constitute a "deep crisis" outcome, it will be useful to close our discussion of the extended AD-curve with a **numerical example**:

- Suppose $A = 9.78$, $C_y = 0.6$, $G_y = 0.15$, $TB_{im} = 0.05$, $TB_{ex} = 0.01$, $C_r = 4$, $I_r = 16$, $TB_\varepsilon = 1$, $\varepsilon_r = 6$, $\overline{Y} = 15$, $\overline{r}^{MP} = 0.02$, $\overline{r}^{RP} = 0.01$, $\phi_y = 0.5$, $\phi_\pi = 2$ and $\pi^T = 0.02$.
- Substituting these parameter values into Equations (33) and (34), using the definitions of IS_0 and IS_1 given in Equation (12), and under the further specification $\pi^e = \pi^T$, we obtain $Y^{ZLB} \approx 16.733$ and $\pi^{ZLB} \approx -0.029$.
- This numerical example thus illustrates that in the extended AS-AD model (unlike in the extended IS-TR model for the macroeconomy in the short run), a "deep crisis" outcome need not be a crisis outcome in terms of the level of output. Rather, the central bank may set $R^{MP} = 0$ even though actual output is above the long-run level of output (!), and does so because actual inflation is well below the target level of inflation.

- A "deep crisis" outcome could also be a crisis outcome in terms of the level of output, though: Under the same parameter values as above except that we had that $\pi^e = \pi^T - 0.06$, we would obtain $Y^{ZLB} \approx 14.133$ and $\pi^{ZLB} \approx 0.044$.

- Note that the central bank may thus set $R^{MP} = 0$ even though actual inflation is above the target rate of inflation, and does so because actual output is well below the long-run level of output. This second numerical example illustrates that this type of "deep crisis" outcome may occur particularly if there is expectation of (sizable) deflation.

Workbook for the Extended *AD*-Curve

The workbook *Extended-AD-Curve.xlsm* allows to study all the determinants of the position and slope of the extended *AD*-curve.

Extended AD-Curve

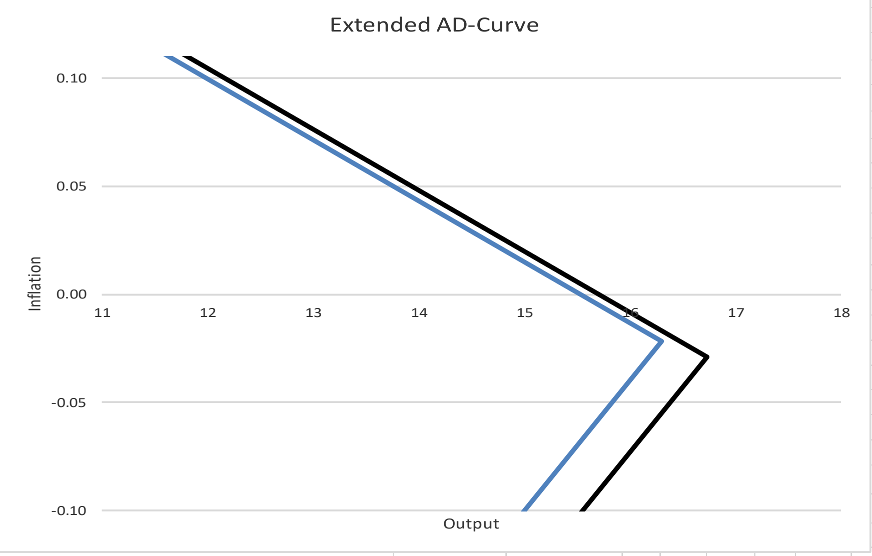
Specification

Extended AD-Curve Parameters		
Extended TR-Curve Parameters	Initial	New
$\bar{r}^{MP}$	0.020	0.020
$\bar{r}^{RP}$	0.010	0.020
r_y	0.005	0.005
ϕ_y	0.500	0.500
ϕ_π	2.000	2.000
$\bar{Y}$	15.000	15.000
π^T	0.020	0.020
π^e	0.020	0.020
IS-Curve Parameters	Initial	New
c_r	4.000	4.000
I_r	16.000	16.000
TB_ε	1.000	1.000
ε_r	6.000	6.000
c_y	0.600	0.600
TB_{im}	0.050	0.050
G_y	0.150	0.150
A	9.780	9.780
c_0	1.500	1.500
I_0	3.000	3.000
G_0	2.800	2.800
TB_0	0.000	0.000
ε_0	1.000	1.000
T	3.000	3.000
TB_{ex}	0.010	0.010
Y^*	100.000	100.000
T^*	30.000	30.000
r^*	0.030	0.030

Quantitative Results

	Initial	New
π^{ZLB}	-0.029	-0.022

Graph



The Extended AS-AD Model

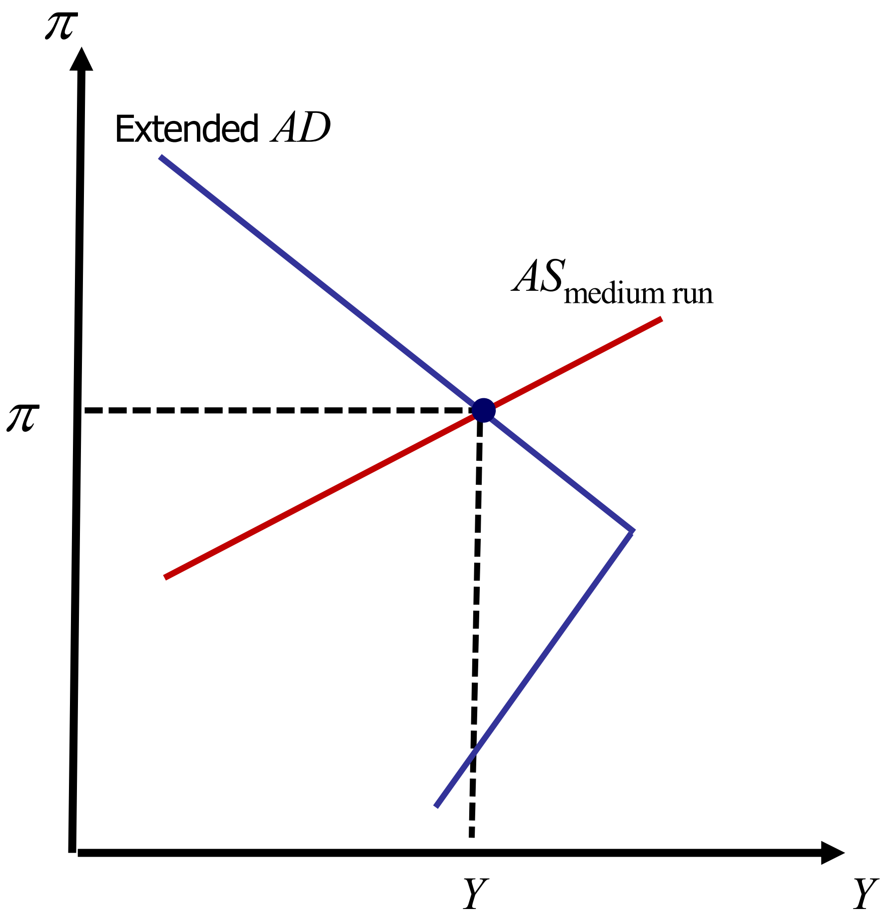
Medium-run macroeconomic outcomes in the **extended AS-AD model**, that graphically occur at the intersection of the extended AD-curve with the medium-run AS-curve, may materialize either at

-- a combination of inflation and output for which we are in a "normal business cycle" outcome,

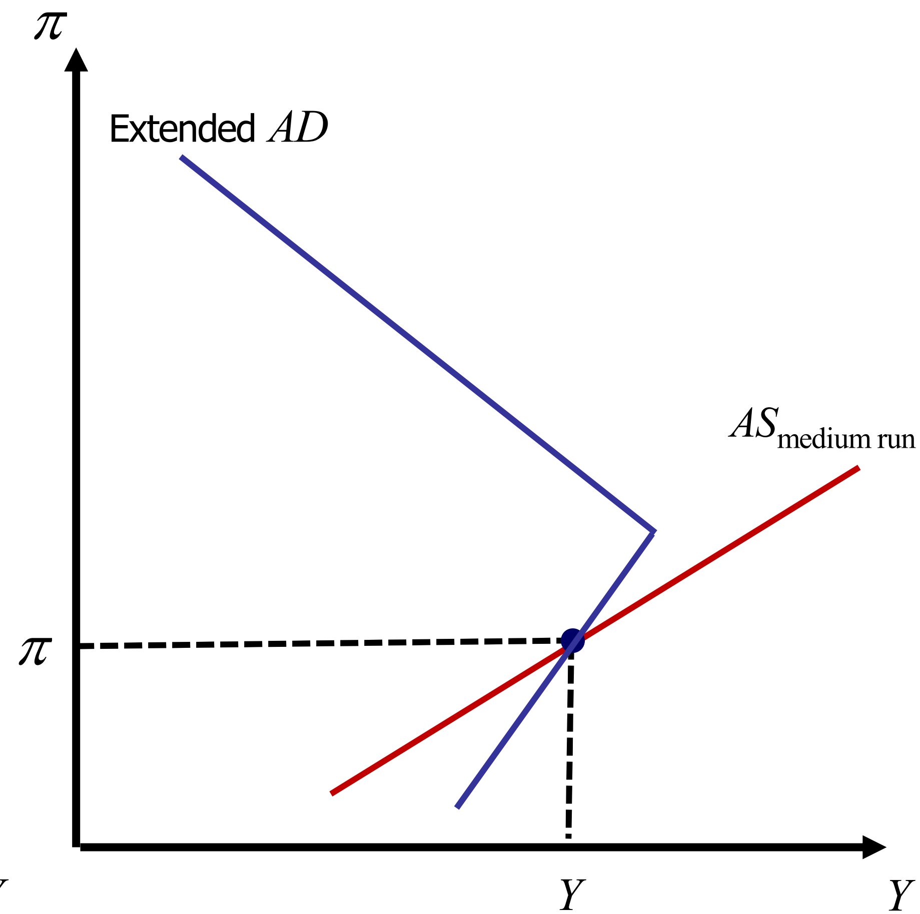
or

-- a combination of inflation and output for which we are in a "deep crisis" outcome.

The following graph illustrates this:



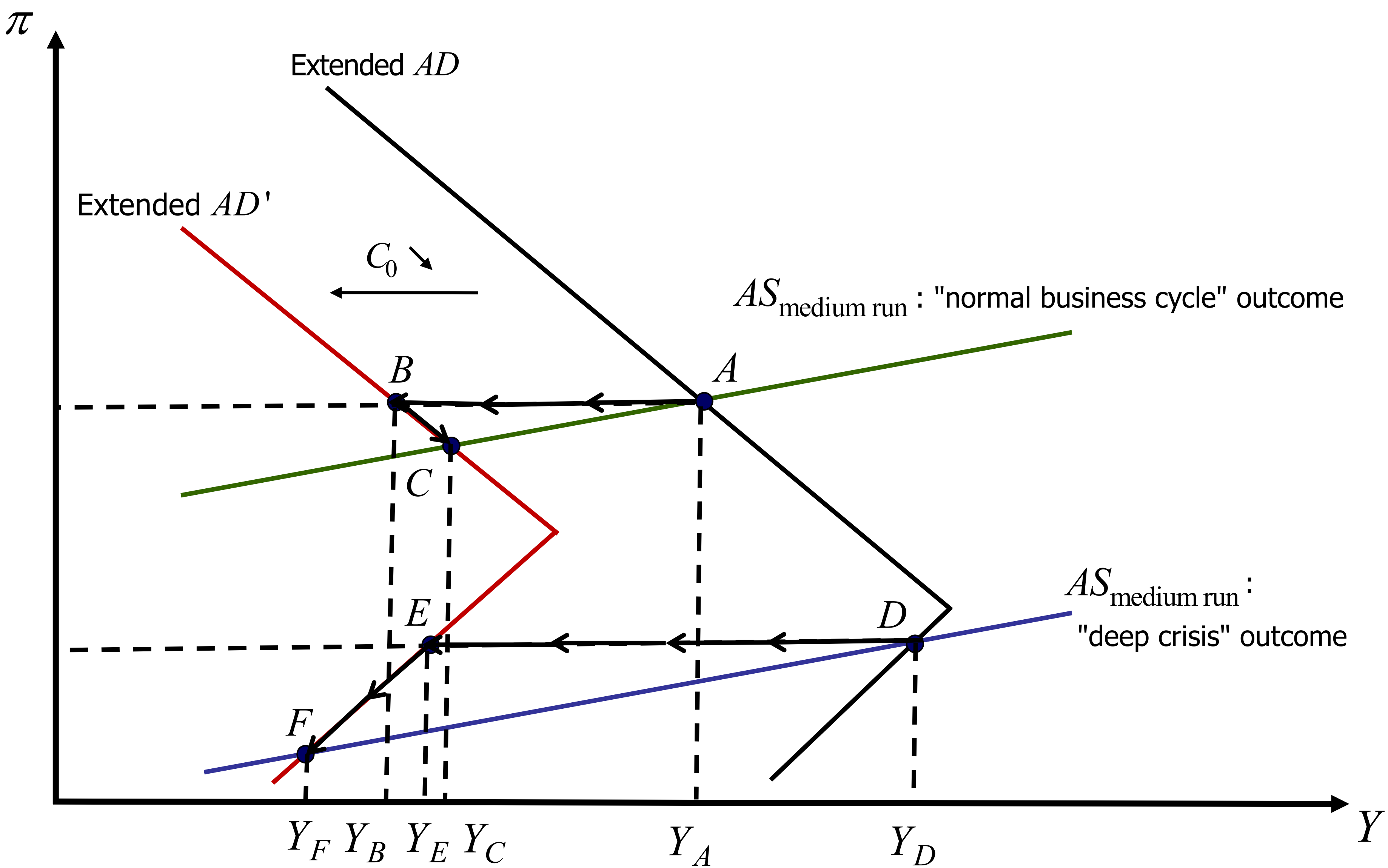
"normal business cycle" outcome
(along the downward-sloping
portion of the extended AD-curve)



"deep crisis" outcome
(along the upward-sloping
portion of the extended AD-curve)

Can the extended AS-AD model explain severe *and* prolonged recessions, such as the one that occurred in the wake of the global financial and European sovereign debt crises, and if so, what are the extended AS-AD model's novel propagation channels relative to those of the standard AS-AD model?

To address these questions, let us examine first how the magnitudes of the losses of output in the extended AS-AD model that occur after a drop in the autonomous component of aggregate demand (say, a collapse in housing prices, implying $C_0\searrow$) compare across the "normal business cycle" and "deep crisis" outcomes. To keep this analysis brief, let us suppose for purposes of this comparison that inflation expectations are fixed:


π
Extended AD
Extended AD'
C0 ↘
AS medium run : "normal business cycle" outcome
AS medium run :
"deep crisis" outcome
A
B
C
D
E
F
Y_F Y_B Y_E Y_C
Y_A
Y_D
Y

- In regards to the shift of the extended AD-curve, following the decrease in C_0 (that in turn leads to a decrease of IS_0), for a given rate of inflation there is a lower level of aggregate demand and thus output, and the extended AD-curve generally shifts to the left. As for the point of discontinuity in the extended AD-curve, from (33) we observe that Y^{ZLB} must decrease, and from (34) we observe that π^{ZLB} must increase. Also note from Equations (14) and (30) that the slopes of both portions of the extended AD-curve do not depend on C_0 and thus remain unchanged.

- Note that when we are not in the "deep crisis" case, the decrease in C_0 through the lower level of aggregate demand for a given rate of inflation leads to a decrease in output (including through Keynesian multiplier effects, that are only partially offset by the central bank lowering the monetary policy rate in response to the decrease in output) (movement from Point A to Point B).

- As output falls, the wage mark-ups and subsequently inflation fall as well. The central bank will thus lower the monetary policy rate in response also to the decrease in the rate of inflation, leading to rises of consumption, investment and the trade balance, that partially offset the initial decline in output (movement from Point B to Point C).
- These various monetary policy rate adjustments are absent in the "deep crisis" case (due to the zero lower bound constraint being binding), and this is the first reason why the loss of output following the decrease in C_0 is more severe in the "deep crisis" case than in the "normal business cycle" case.
- The initial fall of output in the "deep crisis" case (movement from Point D to Point E) reflects the decrease in C_0, augmented by Keynesian multiplier effects and a rise of the endogenous component of the risk premium, leading (despite monetary policy operating at the zero lower bound) to an increase of the interest rate at which households and firms can borrow, and thus crowding out of consumption, investment and the trade balance.

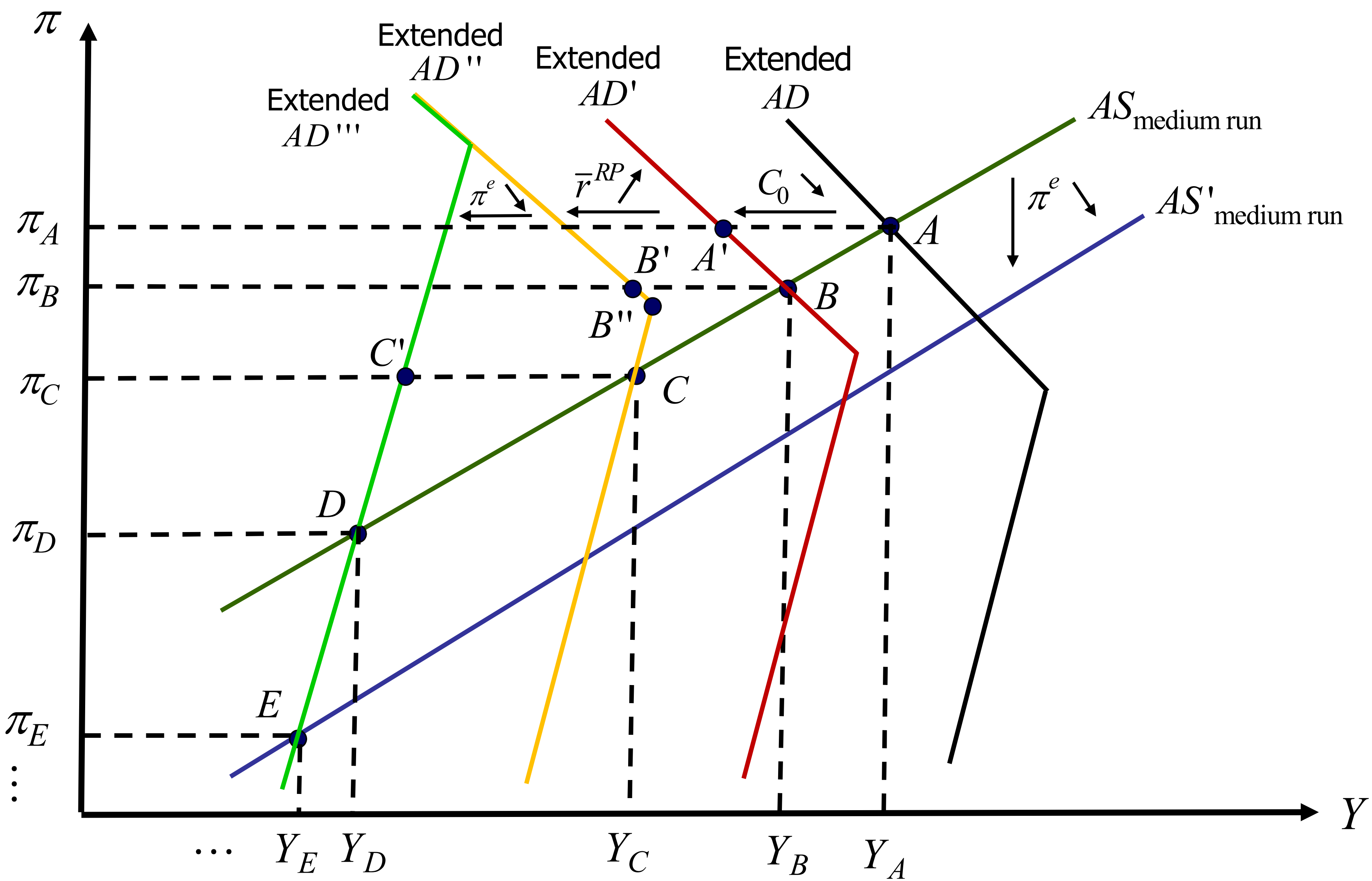
- As output falls, again the wage mark-ups and subsequently inflation fall as well. $Y^{ZLB}\big|_{\pi}$ increases, implying that the endogenous component of the risk premium rises yet further (it becomes positive at higher levels of output than before, and for any level of output that is below the level of $Y^{ZLB}\big|_{\pi}$ that prevailed prior to the decrease of the rate of inflation, the endogenous component of the risk premium is now higher than before), crowding out yet more of aggregate demand (movement from Point E to Point F).
- The various increases in the endogenous component of the risk premium are a propagation mechanism only present in the "deep crisis" case, and are the second reason why the loss of output following the decrease in C_0 is more severe in the "deep crisis" case than in the "normal business cycle" case.

To see that the extended *AS-AD* model can explain recessions that are not just deeper, but also more prolonged than those in the standard *AS-AD* model, let us examine within the extended *AS-AD* model the medium-run macroeconomic effects of the two shocks that seemed so important for the global financial crisis:

-- a collapse in housing prices, implying $C_0 \searrow$, coupled with

-- an increase in the exogenous component of the risk premium, $\bar{r}^{RP} \nearrow$.

To provide complete reasoning, we now take into account again that inflation expectations are adaptive.


π
Extended AD''
Extended AD'
Extended AD
Extended AD'''
AS medium run
AS' medium run
π^e
r̄^RP
C0
π^e
π_A
π_B
π_C
π_D
π_E
A
A'
B
B'
B''
C
C'
D
E
Y
Y_E
Y_D
Y_C
Y_B
Y_A

Movement from Point A to Point B:

- As noted before, following the decrease of C_0 (that in turn leads to a decrease of IS_0), the extended AD-curve shifts to the left and upwards, as the point of discontinuity of the extended AD-curve shifts to the left and upwards, and the slopes of both portions of the extended AD-curve remain unchanged.

- Following the shift of the extended AD-curve, for a given rate of inflation there is now a lower level of aggregate demand and thus output (initial drop in aggregate demand plus Keynesian multiplier effects), partially offset by a crowding in of consumption, investment and the trade balance, as the central bank in response to the decrease in output lowers the monetary policy rate (from Point A to Point A').

- As output falls, wage mark-ups and subsequently inflation fall as well. The central bank lowers the monetary policy rate further, inducing further crowding in (from Point A' to Point B).

Movement from Point B to Point C:

- Following the increase of $\overline{r}^{RP}$, the extended AD-curve shifts to the left and upwards: from (33) we observe that Y^{ZLB} decreases, and from (34) we observe that π^{ZLB} increases. The point of discontinuity of the extended AD-curve thus again shifts to the left and upwards.
- As $\overline{r}^{RP}$ increases, so does the real interest rate, causing, for a given rate of inflation, crowding out of consumption, investment and the trade balance (from Point B to Point B').
- As output falls further, wage mark-ups and subsequently inflation fall further as well. The central bank lowers the monetary policy rate further, causing partial crowding in of consumption, investment and the trade balance, but, as graphed, hits the zero lower bound in the process (from Point B' to Point B'').

- At Point B'', the economy has moved into a "deep crisis" outcome. As the rate of inflation keeps falling in the adjustment towards the medium-run AS-curve, the endogenous component of the risk premium comes into effect and then increases, reflecting continued increases in $Y^{ZLB}\big|_{\pi}$, as per Equation (25). (As the zero lower bound for the monetary policy rate becomes effective at increasingly higher levels of output, the endogenous component of the risk premium keeps increasing.)

- There is thus crowding out of consumption, investment and the trade balance in the movement from Point B'' to Point C. Point C is a medium-run outcome in both the real and the financial sector.

- Under adaptive inflation expectations overall medium-run adjustment does not stop at Point C, though (it would stop there if inflation expectations were static).

Movement from Point C to Point D:

- Under adaptive inflation expectations, given the decrease in the actual rate of inflation from Point A to Point C, inflation expectations will be revised downward in the second period of adjustment. While this does not affect the downward-sloping portion of the extended AD-curve (see (14)), the intercept of the upward-sloping portion of the extended AD-curve increases (see (30)). Also, from from (33) we observe that Y^{ZLB} decreases, and from (34) we observe that π^{ZLB} increases. The point of discontinuity of the extended AD-curve thus once more shifts to the left and upwards.
- As π^e decreases, the "deep crisis" real interest rate increases further, recalling from the medium-run extended TR-curve, Equation (27), that

$$r = -\pi^e + \overline{r}^{RP} - r_y \cdot \left(Y - Y^{ZLB} \Big|_{\pi} \right).$$

This further increase of the real interest rate for a given actual rate of inflation causes further crowding out of consumption, investment and the trade balance (from Point C to Point C').

- As the actual rate of inflation keeps falling in the adjustment towards the medium-run AS-curve, the endogenous component of the risk premium increases further, reflecting continued increases in $Y^{ZLB}\big|_{\pi}$, as per Equation (25). (As the zero lower bound for the monetary policy rate becomes effective at increasingly higher levels of output, the endogenous component of the risk premium increases further.)
- There is thus yet further crowding out of consumption, investment and the trade balance in the movement from Point C' to Point D.

Movement from Point D to Point E:

- Not to be neglected, the decrease of inflation expectations in the second period of adjustment also causes a downward shift of the medium-run AS-curve (for a given level of output, actual inflation decreases, as wage growth and thus the growth rate of production costs fall).
- As the rate of inflation keeps falling in the adjustment towards the new medium-run AS-curve, the endogenous component of the risk premium increases yet further, reflecting further continued increases in $Y^{ZLB}\big|_{\pi}$, as per Equation (25). (As the zero lower bound for the monetary policy rate becomes effective at yet increasingly higher levels of output, the endogenous component of the risk premium increases yet further.)
- There is thus yet further crowding out of consumption, investment and the trade balance in the movement from Point D to Point E. Point E is the medium-run outcome in both the real and the financial sector in the second period of adjustment.

Full medium-run adjustment does not end at Point E, as under adaptive inflation expectations the spiral of decreases in the actual rate of inflation causing in the following period further decreases in inflation expectations (implying yet higher real interest rates), that in turn imply a yet lower actual rate of inflation (with the latter leading to yet further rises of the endogenous component of the risk premium and thus again of the real interest rate), keeps operating.

In comparison to the standard AS-AD model, then, the extended AS-AD model can not only explain deeper recessions (the various increases in the endogenous component of the risk premium are a propagation mechanism that is only present for "deep crisis" outcomes), but can also explain more prolonged recessions (reflecting that for "deep crisis" outcomes the decreases of inflation expectations that occur following losses of output lead to further losses of output, and thus mutually re-inforcing spirals of decreases of output, actual inflation and expected inflation can arise).

Quantitative Analysis:

As was the case in the short-run analysis involving the extended *IS-TR* model, a complication in the quantitative analysis of the extended *AS-AD* model is that (in general) we do not know prior to our calculations whether the medium-run outcome is a "normal business cycle" outcome or a "deep crisis" outcome.

Suppose initially, though, that we knew that the outcome was a **"normal business cycle" outcome**, with $R_t^{MP} > 0$. We already derived the level of output holding in this case when considering the standard *AS-AD* model, see in particular Equation (20):

$$Y_t = \frac{\dfrac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \pi_{t-1} + \omega - s_t}{\dfrac{\omega}{\overline{Y}} + \dfrac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}}} .$$

The rate of inflation in the "normal business cycle" medium-run outcome can in turn be obtained from Equation (21),

$$\pi_t = \pi_{t-1} - \omega + s_t + \frac{\omega}{\overline{Y}} \cdot Y_t \ ,$$

with Y_t given by Equation (20). Using these levels of output and the rate of inflation, the real interest rate in the "normal business cycle" medium-run outcome from Equation (22) is equal to

$$r_t = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi_t - \pi^T \right) + \overline{r}^{RP} .$$

The nominal monetary policy rate corresponding to this real interest rate from the medium-run extended TR-curve in Equation (27), amended to include time subscripts and accounting for adaptive expectations, is given by

$$R_t^{MP} = \overline{r}^{MP} + \pi_{t-1} + \phi_y \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi_t - \pi^T \right). \qquad (35)$$

Given output, inflation and the real interest rate, the "normal business cycle" medium-run macroeconomic outcomes of the other key macroeconomic aggregates, including consumption, investment, government expenditure, the trade balance and the exchange rate, may be retrieved from the IS-portion of the model.

Suppose next that we knew that the outcome was a **"deep crisis" outcome**, with the zero lower bound constraint on the monetary policy rate binding, $R_t^{MP} = 0$.

To derive the level of output holding in this case, we set the rate of inflation as implied by the medium-run AS-curve, Equation (17), equal to the rate of inflation implied by the "deep crisis" portion of the extended AD-curve, Equation (30), amended to include time subscripts and to incorporate adaptive expectations,

$$\underbrace{\pi_t = \pi_{t-1} + \omega \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + s_t}_{\text{medium-run } AS\text{-curve (17)}} = \qquad (36)$$

$$= \underbrace{\left(\frac{\phi_y \cdot (1 - IS_1 \cdot r_y)}{IS_1 \cdot r_y \cdot \overline{Y} \cdot \phi_\pi} \right) \cdot Y_t - \left(\frac{\phi_y}{IS_1 \cdot r_y \cdot \overline{Y} \cdot \phi_\pi} \right) \cdot \left[IS_0 - IS_1 \cdot \left(-\pi_{t-1} + \overline{r}^{RP} + \frac{r_y \cdot \overline{Y}}{\phi_y} \cdot \left(\phi_y - \overline{r}^{MP} - \pi_{t-1} + \phi_\pi \cdot \pi^T \right) \right) \right]}_{\text{from "deep crisis" portion of extended } AD\text{-curve (30)}} = \pi_t .$$

Note that (36) is one equation in one unknown, namely Y_t. We can thus use Equation (36) to solve for Y_t:

$$\left(\frac{\omega}{\overline{\overline{Y}}}-\frac{\phi_y\cdot\left(1-IS_1\cdot r_y\right)}{IS_1\cdot r_y\cdot\overline{\overline{Y}}\cdot\phi_\pi}\right)\cdot Y_t =$$

$$-\left(\frac{\phi_y}{IS_1\cdot r_y\cdot\overline{\overline{Y}}\cdot\phi_\pi}\right)\cdot\left[IS_0-IS_1\cdot\left(-\pi_{t-1}+\overline{r}^{RP}+\frac{r_y\cdot\overline{\overline{Y}}}{\phi_y}\cdot\left(\phi_y-\overline{r}^{MP}-\pi_{t-1}+\phi_\pi\cdot\pi^T\right)\right)\right]-\pi_{t-1}+\omega-s_t$$

$$<=>\quad Y_t=\frac{\left(\frac{\phi_y}{IS_1\cdot r_y\cdot\overline{\overline{Y}}\cdot\phi_\pi}\right)\cdot\left[IS_0-IS_1\cdot\left(-\pi_{t-1}+\overline{r}^{RP}+\frac{r_y\cdot\overline{\overline{Y}}}{\phi_y}\cdot\left(\phi_y-\overline{r}^{MP}-\pi_{t-1}+\phi_\pi\cdot\pi^T\right)\right)\right]+\pi_{t-1}-\omega+s_t}{\frac{\phi_y\cdot\left(1-IS_1\cdot r_y\right)}{IS_1\cdot r_y\cdot\overline{\overline{Y}}\cdot\phi_\pi}-\frac{\omega}{\overline{\overline{Y}}}}\,. \tag{37}$$

Equation (37) gives us the level of **output** in the "deep crisis" medium-run macroeconomic outcome.

Having obtained the level of output, we can calculate the rate of **inflation** in the "deep crisis" medium-run macroeconomic outcome from the medium-run AS-curve, Equation (17), to obtain, as in Equation (21),

$$\pi_t = \pi_{t-1} - \omega + s_t + \frac{\omega}{\overline{Y}} \cdot Y_t \, ,$$

with Y_t given by Equation (37).

The **real interest rate** in the "deep crisis" medium-run macroeconomic outcome may be obtained from the medium-run extended TR-curve, Equations (27) and (28), amended to include time subscripts and accounting for adaptive expectations, as

$$r_t = -\pi_{t-1} + \overline{r}^{RP} - r_y \cdot \left(Y_t - Y_t^{ZLB} \Big|_{\pi} \right), \qquad (38)$$

with Y_t given by Equation (37), and with $Y_t^{ZLB} \Big|_{\pi}$ given by Equation (25), also amended to include time subscripts and accounting for adaptive expectations,

$$Y_t^{ZLB} \Big|_{\pi} = \frac{\overline{Y}}{\phi_y} \cdot \left[\phi_y - \overline{r}^{MP} - \pi_{t-1} - \phi_{\pi} \cdot \left(\pi_t - \pi^T \right) \right], \qquad (39)$$

where π_t is given by Equation (21).

Given output, inflation and the real interest rate, the "deep crisis" medium-run macroeconomic outcomes of the other key macroeconomic aggregates, including consumption, investment, government expenditure, the trade balance and the exchange rate, may be retrieved from the IS-portion of the model.

Numerical Example:

Suppose that we start out in period $t = 0$ in the long-run macroeconomic outcome where $Y_0 = \overline{Y}$, $r_0 = \overline{r}^{MP} + \overline{r}^{RP}$, and $\pi_0 = \pi^T$.

Also suppose $A = 9.78$, $C_y = 0.6$, $G_y = 0.15$, $TB_{im} = 0.05$, $TB_{ex} = 0.01$, $C_r = 4$, $I_r = 16$, $TB_\varepsilon = 1$, $\varepsilon_r = 6$, $\overline{Y} = 15$, $\overline{r}^{MP} = 0.02$, $\overline{r}^{RP} = 0.01$, $\phi_y = 0.5$, $\phi_\pi = 2$, $r_y = 0.005$, $\pi^T = 0.02$, $\omega = 0.05$ and $s = 0$.

Note that (in general) we will not know prior to our calculations whether the medium-run macroeconomic outcomes are "normal business cycle" outcomes or "deep crisis" outcomes.

One approach to resolving this complication is as follows:

***Step 1*:**

Conjecture that in period t we are in a "normal business cycle" outcome. (We make this conjecture as in the medium-run analysis the algebra to solve the model is a bit less involved for the "normal business cycle" outcome than for the "deep crisis" outcome). Based on this conjecture, calculate the implied level of output from Equation (20):

$$Y_t^{Conjecture} = \frac{\dfrac{\left(\phi_y + \phi_\pi \cdot \pi^T - \overline{r}^{MP} - \overline{r}^{RP}\right) \cdot IS_1 + IS_0}{\phi_\pi \cdot IS_1} - \pi_{t-1} + \omega - s_t}{\dfrac{\omega}{\overline{Y}} + \dfrac{\overline{Y} + \phi_y \cdot IS_1}{\phi_\pi \cdot IS_1 \cdot \overline{Y}}} .$$

***Step 2*:**

From the medium-run extended TR-curve in Equation (27), amended to include time subscripts and accounting for adaptive expectations,

$$r_t \approx \max\left\{-\pi_{t-1},\ \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y_t - \overline{Y}}{\overline{Y}}\right) + \phi_\pi \cdot \left(\pi_t - \pi^T\right)\right\} + \overline{r}^{RP} - r_y \cdot \left(Y_t - Y_t^{ZLB}\Big|_{\pi_t}\right) \cdot I\left(Y_t \leq Y_t^{ZLB}\Big|_{\pi_t}\right),$$

it is clear that whether in period t we we are in a "normal business cycle" or a "deep crisis" outcome for a given level of output depends on the highest level of output at which the zero lower bound on the monetary policy rate is binding, that is, $Y_t^{ZLB}\Big|_{\pi_t}$.

Using $Y_t^{Conjecture}$ calculated in Step 1, we thus calculate $Y_t^{ZLB}\Big|_{\pi_t^{Conjecture}}$ from Equation (39) as

$$Y_t^{ZLB}\Big|_{\pi_t^{Conjecture}} = \frac{\overline{Y}}{\phi_y} \cdot \left[\phi_y - \overline{r}^{MP} - \pi_{t-1} - \phi_\pi \cdot \left(\pi_t^{Conjecture} - \pi^T\right)\right],$$

where $\pi_t^{Conjecture}$ from Equation (21) is given by

$$\pi_t^{Conjecture} = \pi_{t-1} - \omega + s_t + \frac{\omega}{\overline{Y}} \cdot Y_t^{Conjecture} .$$

Step 3:

If $Y_t^{Conjecture}$ (calculated in Step 1) was larger than $Y_t^{ZLB}\Big|_{\pi_t^{Conjecture}}$ (calculated in Step 2),

$$Y_t^{Conjecture} > Y_t^{ZLB}\Big|_{\pi_t^{Conjecture}},$$

we know that in period t we are indeed in a "normal business cycle" outcome, for which it holds that R_t^{MP} is not constrained at the zero lower bound, that is, from the Taylor rule in Equation (35) we have that

$$R_t^{MP} = \overline{r}^{MP} + \pi_{t-1} + \phi_y \cdot \left(\frac{Y_t^{Conjecture} - \overline{Y}}{\overline{Y}} \right) + \phi_\pi \cdot \left(\pi_t^{Conjecture} - \pi^T \right) > 0.$$

We can then calculate the outcomes for all variables based on the relations that hold for the "normal business cycle" outcome, including the real interest rate from Equation (22), using that $Y_t = Y_t^{Conjecture}$ and that $\pi_t = \pi_t^{Conjecture}$ (note that $\pi_t^{Conjecture}$ was calculated in Step 2).

Step 4:

If, however, the conjectured level of output calculated in Step 1 was less than or equal to Y_t^{ZLB},

$$Y_t^{Conjecture} \leq Y_t^{ZLB}\Big|_{\pi_t^{Conjecture}},$$

then a contradiction has arisen, and the period t outcome must be a "deep crisis" outcome, so that we need to calculate all outcomes based on the relations that hold when the zero lower bound on the monetary policy rate is binding:
In particular, we can need to calculate short-run output from Equation (37),

$$Y_t = \frac{\left(\dfrac{\phi_y}{IS_1 \cdot r_y \cdot \bar{Y} \cdot \phi_\pi}\right) \cdot \left[IS_0 - IS_1 \cdot \left(-\pi_{t-1} + \bar{r}^{RP} + \dfrac{r_y \cdot \bar{Y}}{\phi_y} \cdot \left(\phi_y - \bar{r}^{MP} - \pi_{t-1} + \phi_\pi \cdot \pi^T \right) \right) \right] + \pi_{t-1} - \omega + s_t}{\dfrac{\phi_y \cdot \left(1 - IS_1 \cdot r_y\right)}{IS_1 \cdot r_y \cdot \bar{Y} \cdot \phi_\pi} - \dfrac{\omega}{\bar{Y}}}.$$

Using this level of output, we can then calculate the rate of inflation from Equation (21) and the real interest rate from Equation (38).

Implementing this procedure for our numerical example:

- Presume at first that the period $t=1$ macroeconomic outcome is a "normal business cycle" outcome. From Equation (20) we obtain $Y_1^{Conjecture} = 15$. From Equation (21), we obtain that $\pi_1^{Conjecture} = 0.02$, and from Equation (39), we obtain that $Y_1^{ZLB}\Big|_{\pi_1^{Conjecture}} = 13.8$. We are thus indeed in a "normal business cycle" outcome.
- The fact that we are in a "normal business cycle" outcome may also be confirmed from the Taylor rule: From Equation (35) we obtain that $R_1^{MP} = 0.04 > 0$.

Now suppose that from period $t=2$ onwards we observe for four periods (that is, from $t=2$ to $t=5$) the two shocks that seemed so important for the global financial crisis: a decrease of the intercept of the consumption function, $\Delta C_0 = -0.7$, and an increase of the exogenous component of the risk premium, $\Delta \overline{r}^{RP} = 0.04$.

- Presume at first that the period $t=2$ medium-run macroeconomic outcome is a "normal business cycle" outcome. From Equation (20), noting that following the two shocks to the consumption function and the risk premium we have that $\bar{r}^{RP} = 0.05$ and $IS_0 = \left(A + \Delta C_0\right) / \left[1 - \left(C_y - TB_{im} - G_y\right)\right] \approx 15.133$, we obtain $Y_2^{Conjecture} \approx 13.939$. From Equation (21) (and using the exact level of $Y_2^{Conjecture}$), we obtain that $\pi_2^{Conjecture} \approx 0.016$. Using the (exact) levels of $Y_2^{Conjecture}$ and of $\pi_2^{Conjecture}$ in Equation (39) , we obtain that

$$13.939 \approx Y_2^{Conjecture} < Y_2^{ZLB}\Big|_{\pi_2^{Conjecture}} \approx 14.012\,.$$

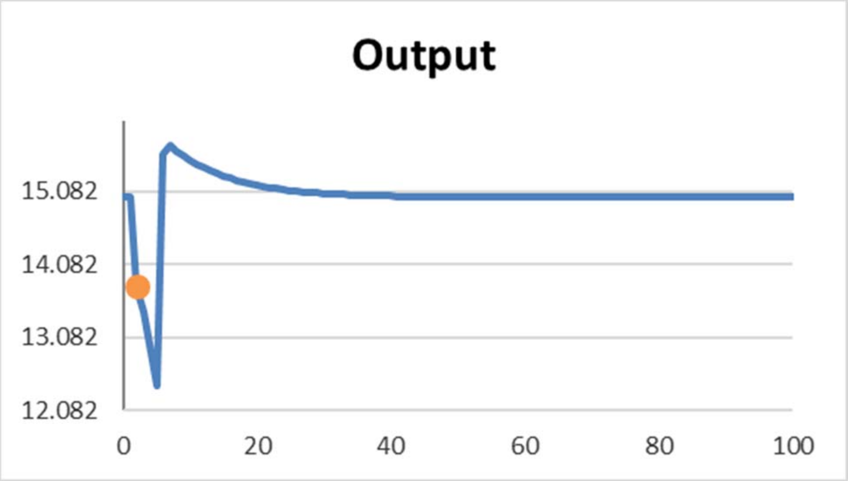
  We thus cannot be in a "normal business cycle" outcome in period $t=2$, and must rather be in a "deep crisis" outcome.

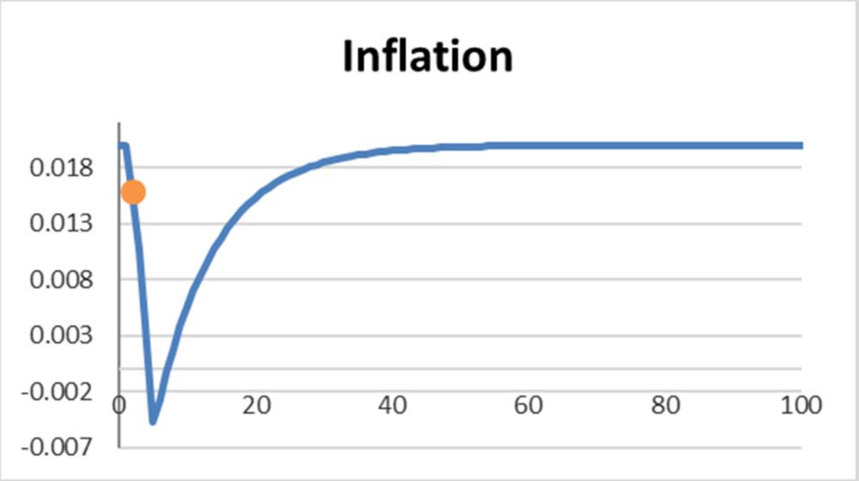
- The fact that the period $t=2$ outcome must be a "deep crisis" outcome may also be confirmed from the Taylor rule: Using the (exact) levels of $Y_2^{Conjecture}$ and of $\pi_2^{Conjecture}$ in Equation (35), we obtain that $R_2^{MP} \approx -0.002 < 0$.

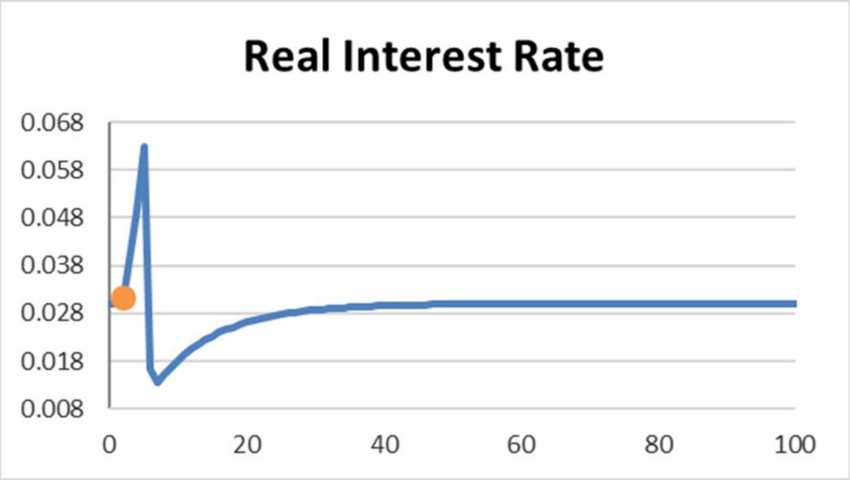
- Calculating the level of output in period $t=2$ from Equation (37), we obtain $Y_2 \approx 13.775$. From Equation (21) (and using the exact level of Y_2), it then follows that $\pi_2 \approx 0.016$. From Equation (35), the Taylor rule for these values of Y_2 and π_2 would imply that $R_2^{MP} \approx -0.009$, once again confirming that the zero lower bound restriction is binding in period $t=2$.

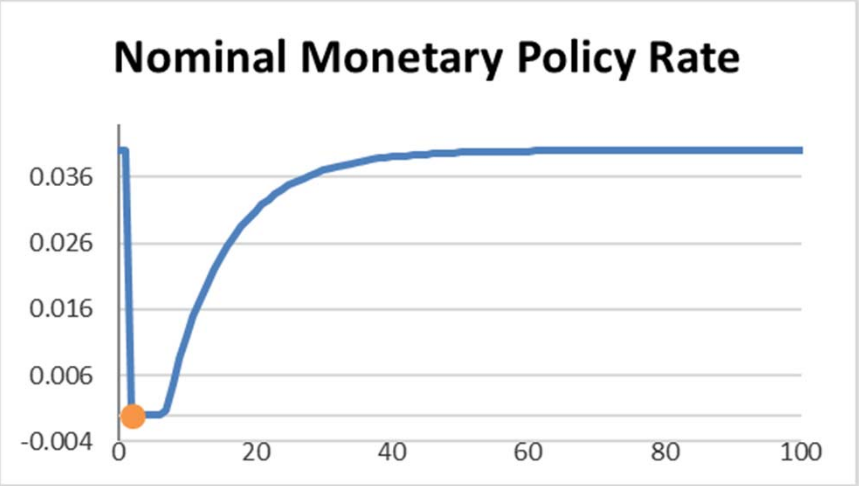
The following graphs show the full medium-run adjustment paths implied by the extended *AS-AD* model, based on automated calculations within the workbook *Extended-AS-AD-Model.xlsm*, and presuming that from period $t=6$ onwards, the decrease of the intercept of the consumption function and the increase of the exogenous component of the risk premium are reversed: relative to periods $t=2$ to $t=5$, we have that $\Delta C_0 = 0.7$ and $\Delta \overline{r}^{RP} = -0.04$, so that from period six onwards all parameter values are again as in period one.

Using the workbook *Extended-AS-AD-Model.xlsm*, these dynamics can also be traced within the extended *AS-AD* model diagram.


Output
15.082
14.082
13.082
12.082
0
20
40
60
80
100


Inflation
0.018
0.013
0.008
0.003
-0.002
-0.007
0
20
40
60
80
100


Real Interest Rate
0.068
0.058
0.048
0.038
0.028
0.018
0.008
0
20
40
60
80
100


Nominal Monetary Policy Rate
0.036
0.026
0.016
0.006
-0.004
0
20
40
60
80
100

Workbook for the Extended *AS-AD* Model

The workbook *Extended-AS-AD-Model.xlsm* allows to study how changes in the various parameters and exogenous variables of the *Extended-AS-AD*-model …

Extended AS-AD Model: Parameter Specification

	TR-Curve Parameters							IS-Curve Parameters																		AS-Curve Parameters	
Period	$\bar{r}^{MP}$	$\bar{r}^{RP}$	r_y	ϕ_y	ϕ_π	$\bar{Y}$	π^T	C_r	I_r	TB_ε	ε_r	C_y	TB_{im}	G_y	A	C_0	I_0	G_0	TB_0	ε_0	T	TB_{ex}	Y^*	T^*	r^*	ω	s
0	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
1	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
2	0.020	0.050	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.080	0.800	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
3	0.020	0.050	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.080	0.800	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
4	0.020	0.050	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.080	0.800	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
5	0.020	0.050	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.080	0.800	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
6	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
7	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
8	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
9	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
10	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
11	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
12	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
13	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
14	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
15	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
16	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
17	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
18	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
19	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
20	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
21	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
22	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
23	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
24	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
25	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
26	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
27	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
28	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
29	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
30	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
31	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
32	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
33	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
34	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
35	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
36	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
37	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
38	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
39	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000
40	0.020	0.010	0.005	0.500	2.000	15.000	0.020	4.000	16.000	1.000	6.000	0.600	0.050	0.150	9.780	1.500	3.000	2.800	0.000	1.000	3.000	0.010	100.000	30.000	0.030	0.050	0.000

Notes: It is advised not to change the parameter values for Period 0. These are set such that the model initialization in Period 0 occurs in the long-run macroeconomic outcome. From Period 41 onwards, all parameter values are constrained to be the same as for Period 40.

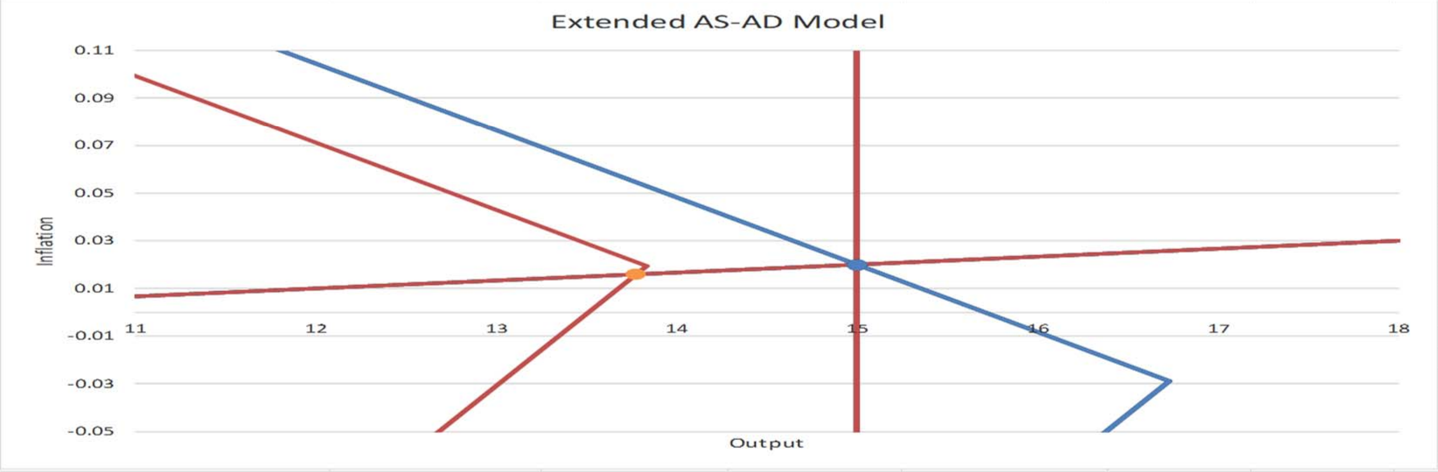
… affect output, inflation and the other endogenous variables in the medium-run macroeconomic outcome, …

Extended AS-AD Model: Quantitative Results and Extended AS-AD Diagram

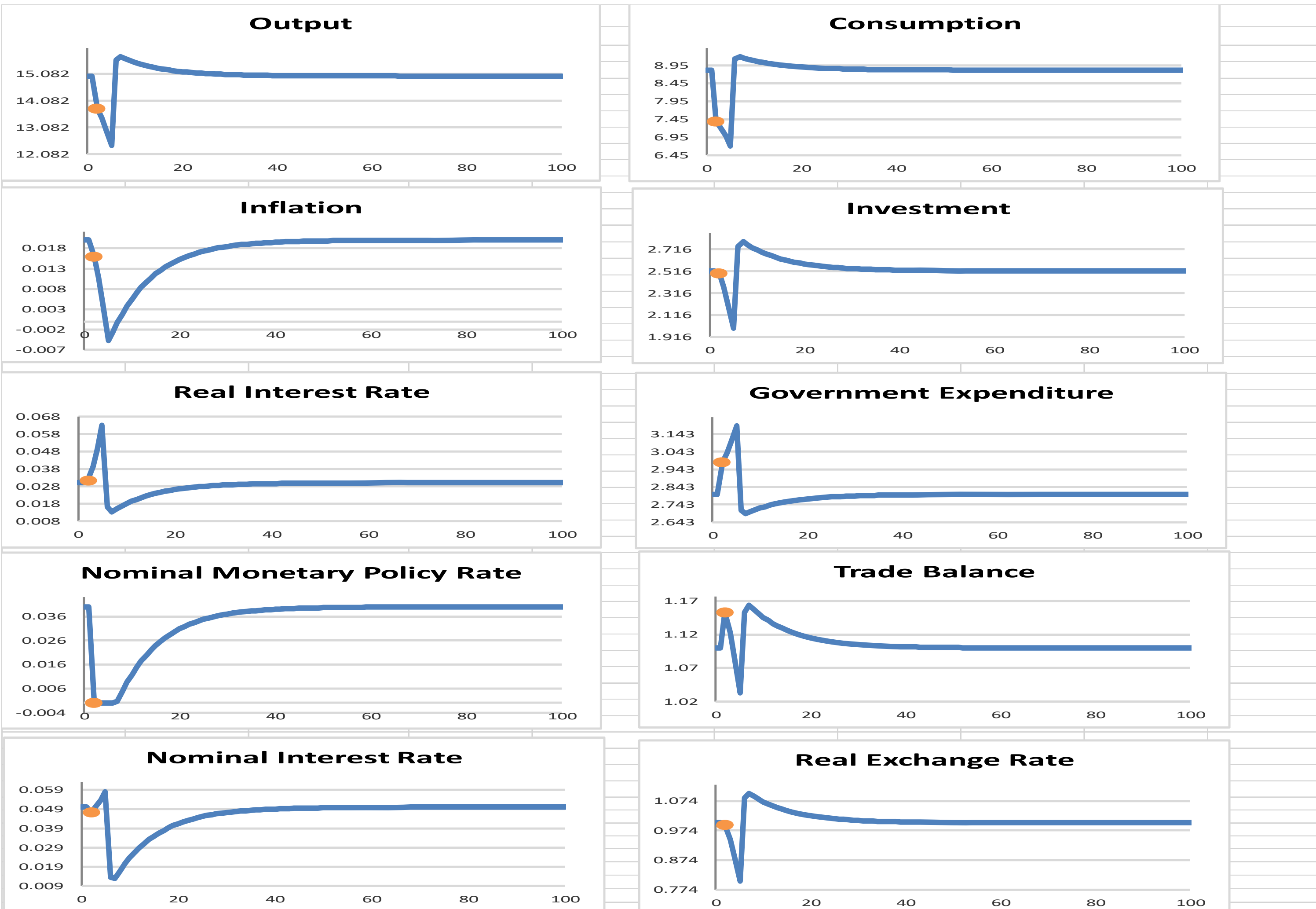
Medium-Run Macroeconomic Outcomes

	Period			
	0	1	2	1000
Inflation	0.020	0.020	0.016	0.020
Output	15.000	15.000	13.775	15.000
Real Interest Rate	0.030	0.030	0.031	0.030
Consumption	8.820	8.820	7.390	8.820
Investment	2.520	2.520	2.498	2.520
Government Expenditure	2.800	2.800	2.984	2.800
Trade Balance	1.100	1.100	1.153	1.100
Real Exchange Rate	1.000	1.000	0.992	1.000
Nominal Monetary Policy Rate	0.040	0.040	0.000	0.040
Nominal Interest Rate	0.050	0.050	0.047	0.050

	Current	
Period	2	Plus / Minus



… and also graphs the transition dynamics (from the initialization onwards, and depicting the full medium-run adjustment):



4. Fiscal and Monetary Policy Options in "Deep Crises"

Readings:
- Challe (2019), Chapters 9 and 10
- Jones (2018), Chapter 14

Using the extended AS-AD model, we can evaluate the effectiveness of fiscal and monetary stabilization measures across "normal business cycle" and "deep crisis" outcomes, and from a short- and medium term-perspective.

Let us first turn to **fiscal policy**. Recall that in our short-run analysis, we had argued that expansionary fiscal policy will be more effective in "deep crisis" situations than in "normal business cycle" situations, because when $R^{MP} = 0$, (i) there is no crowding out through the consumption, investment and exchange rate channels, and (ii) the endogenous component of the risk premium falls as expansionary fiscal policy increases output.

Using the extended AS-AD model, let us now examine the medium-run effects of expansionary fiscal policy, including in "deep crisis" situations.

For the subsequent analysis, suppose we start out (in period $t=0$) in the long-run macroeconomic outcome where $Y_0 = \overline{Y}$, $r_0 = \overline{r}^{MP} + \overline{r}^{RP}$, and $\pi_0 = \pi^T$.

Also suppose that for period $t=1$ we use our previous baseline parameterization, including that $I_0 = 3$, $G_0 = 2.8$, $A = 9.78$, $C_y = 0.6$, $G_y = 0.15$, $TB_{im} = 0.05$, $TB_{ex} = 0.01$, $TB_\varepsilon = 1$, $\varepsilon_r = 6$, $\overline{Y} = 15$, $\overline{r}^{MP} = 0.02$, $\overline{r}^{RP} = 0.01$, $\phi_y = 0.5$, $\phi_\pi = 2$, $C_r = 4$, $I_r = 16$, $r_y = 0.005$, $\pi^T = 0.02$, $\omega = 0.05$ and $s = 0$.

From Equation (20), under $\pi_0 = \pi^T = 0.02$, we then still obtain for period $t=1$ as the medium-run level of output $Y_1 = 15$. Using this value of Y_1 in Equation (21), the medium-run rate of inflation in period $t=1$ still is equal to $\pi_1 = 0.02$.

For period $t=2$ and subsequent periods, consider the following scenarios for shocks to the intercept of the investment function and to the exogenous component of the risk premium, as well as possible responses to the resultant recessions in the form of expansionary fiscal policy increasing the intercept of the government expenditure function, G_0:

- "Normal business cycle" recession:

  $t = 2, 3, ..., 6: I_0 = 2.4; \quad t = 7, 8, ... : I_0 = 3$

  (temporary decrease of intercept of investment function)

- "Deep crisis" recession:

  $t = 2, 3, ..., 6: I_0 = 2.4$ and $\overline{r}_t^{RP} = 0.04; \quad t = 7, 8, ... : I_0 = 3$ and $\overline{r}_t^{RP} = 0.01$

  (temporary decrease of intercept of investment function and temporary increase of exogenous component of risk premium)

combined with the following specifications of fiscal policy:

- Constant intercept of government expenditure function ("without fiscal stimulus"):

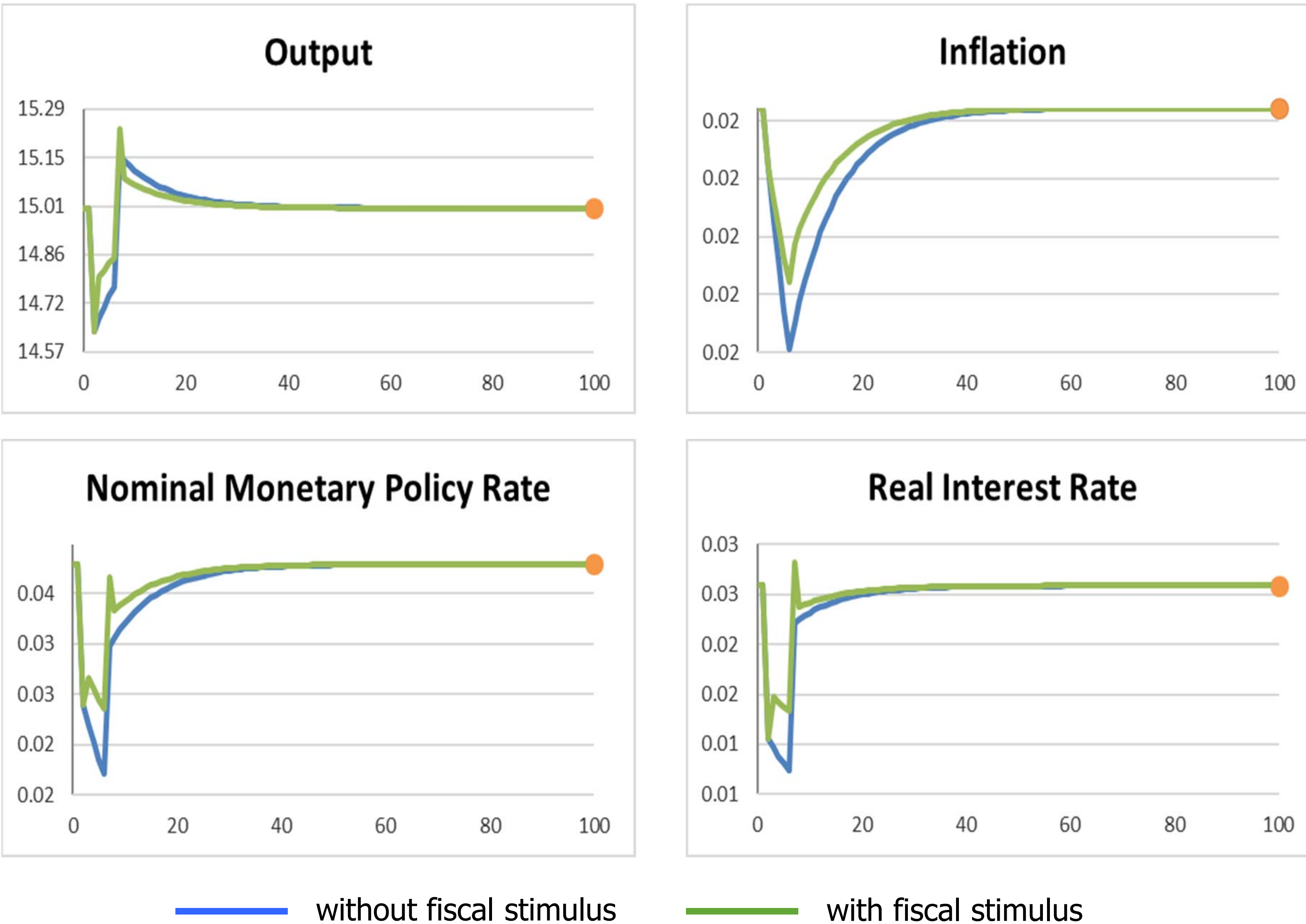
  $t = 2, 3, ... : G_0 = 2.8.$

- Temporary increase of intercept of government expenditure function ("with fiscal stimulus"):

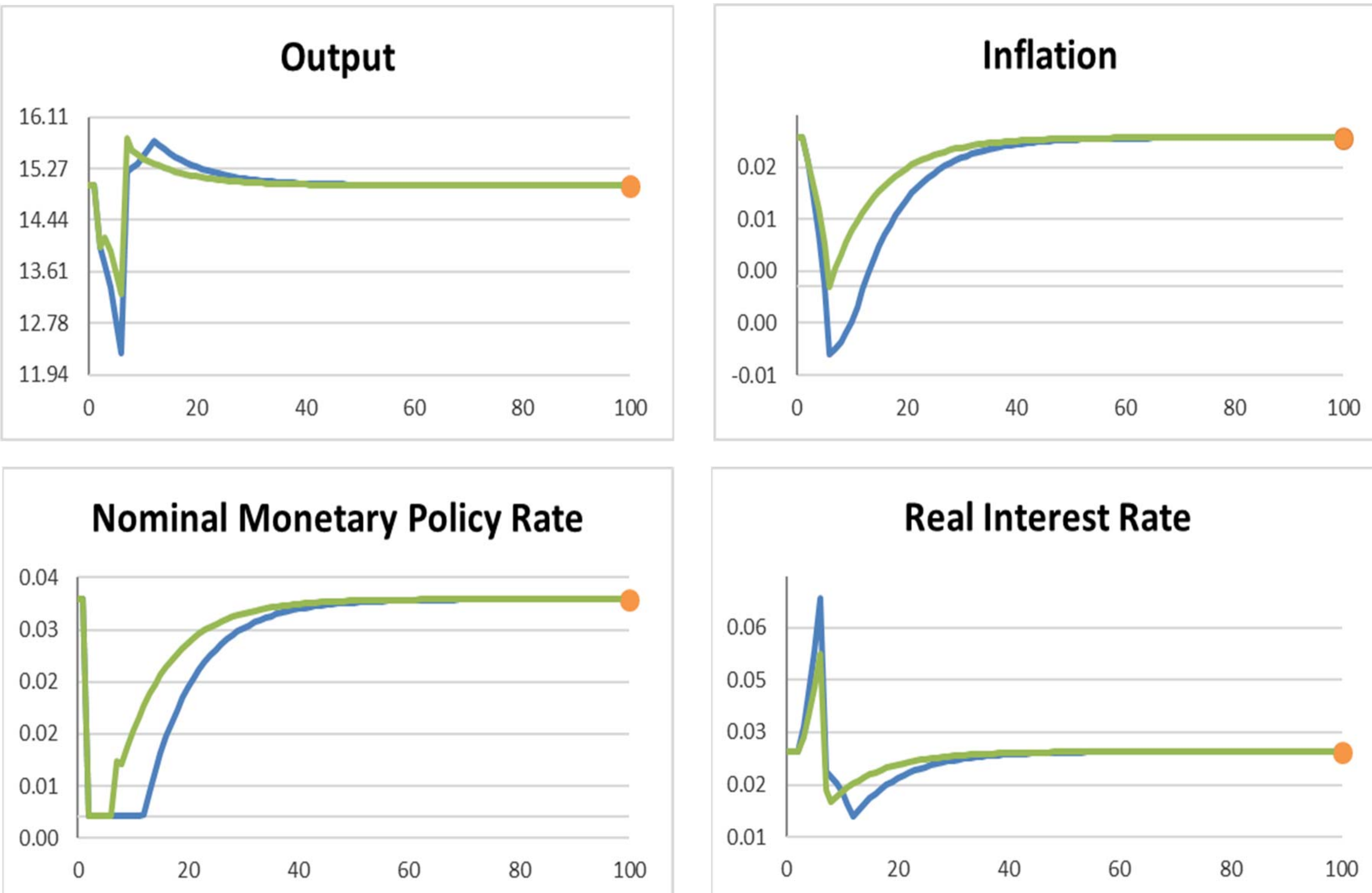
  $t = 2 : G_0 = 2.8; \quad t = 3, 4, ..., 7: G_0 = 3; \quad t = 8, 9 ... : G_0 = 2.8.$

The following graphs depict the resultant time paths for output, inflation, the real interest rate and the monetary policy rate for all four resultant scenarios for periods $t = 0, 1, ..., 100$ (based on automated calculations within the workbook *Extended-AS-AD-Model-PolicyOptions.xlsm*):

"Normal Business Cycle" Recession



"Deep Crisis" Recession



without fiscal stimulus   with fiscal stimulus

The stabilization of output through fiscal stimulus clearly is notably more pronounced in the "deep crisis" recession, in line with the economic rationales advanced in the short- and medium-run (extended) IS-TR and (extended) AS-AD models, specifically that only in the "deep crisis" recession (i) there is no crowding out through the consumption, investment and exchange rate channels, and (ii) the endogenous component of the risk premium falls as the fiscal stimulus increases output.

Notice also, though, that in the absence of fiscal stimulus, when investment demand rebounds and the exogenous component of the risk premium is back to its original level, output overshoots its long-run level for a longer period of time than in the presence of fiscal stimulus. As the graphs show, this is due to the different paths that monetary policy takes in the two scenarios: In the scenario without fiscal stimulus, the rate of inflation from period $t=3$ onwards is lower than in the scenario involving temporary fiscal stimulus. The central bank in the scenario without fiscal stimulus therefore sets a lower monetary policy rate, until output and inflation have converged back to their long-run/target levels.

We may assess the overall impact of fiscal stimulus on output through a single measure, the cumulated Keynesian multiplier:

$$\Delta KM = \frac{\sum_{t=2}^{100}\left(Y_t^{fiscal\ stimulus} - Y_t^{no\ fiscal\ stimulus}\right)}{\sum_{t=2}^{100}\left(G_t^{fiscal\ stimulus} - G_t^{no\ fiscal\ stimulus}\right)} . \tag{40}$$

Under the above scenarios of "normal business cycle" and "deep crisis" recessions, one obtains

$$\Delta KM = 0 . \tag{41}$$

Even putting aside questions as to how the fiscal stimulus is to be funded, to make the case for government authorities to pursue fiscal stimulus, on the basis of the extended AS-AD model one would thus need to argue along lines such as

-- near-future events at the time of a "normal business cycle" / "deep crisis" recession (when the output losses in the absence of fiscal stimulus occur) matter more than further-ahead future events (when the output gains in the absence of fiscal stimulus occur),

or

-- in the absence of fiscal stimulus, the macroeconomy may become unstable, that is, not return to the long-run outcome anymore.

Let us next turn to unconventional **monetary policy** as a second policy option in "deep crises".

Suppose the economy again is in the long-run outcome in periods $t=0$ and $t=1$. For period $t = 2$ and the subsequent periods, consider the following scenarios for shocks to the intercept of the investment function and to the exogenous component of the risk premium, as well as possible responses to the resultant recession in the form of unconventional monetary policy succeeding in lowering the exogenous component of the risk premium:

- "Deep crisis" recession, without an unconventional monetary stimulus being initiated:

  $t = 2, 3, ..., 6$: $I_0 = 2.4$ and $\overline{r}_t^{RP} = 0.04$; $t = 7, 8, ...$: $I_0 = 3$ and $\overline{r}_t^{RP} = 0.01$

  (temporary decrease of intercept of investment function and temporary increase of exogenous component of risk premium)

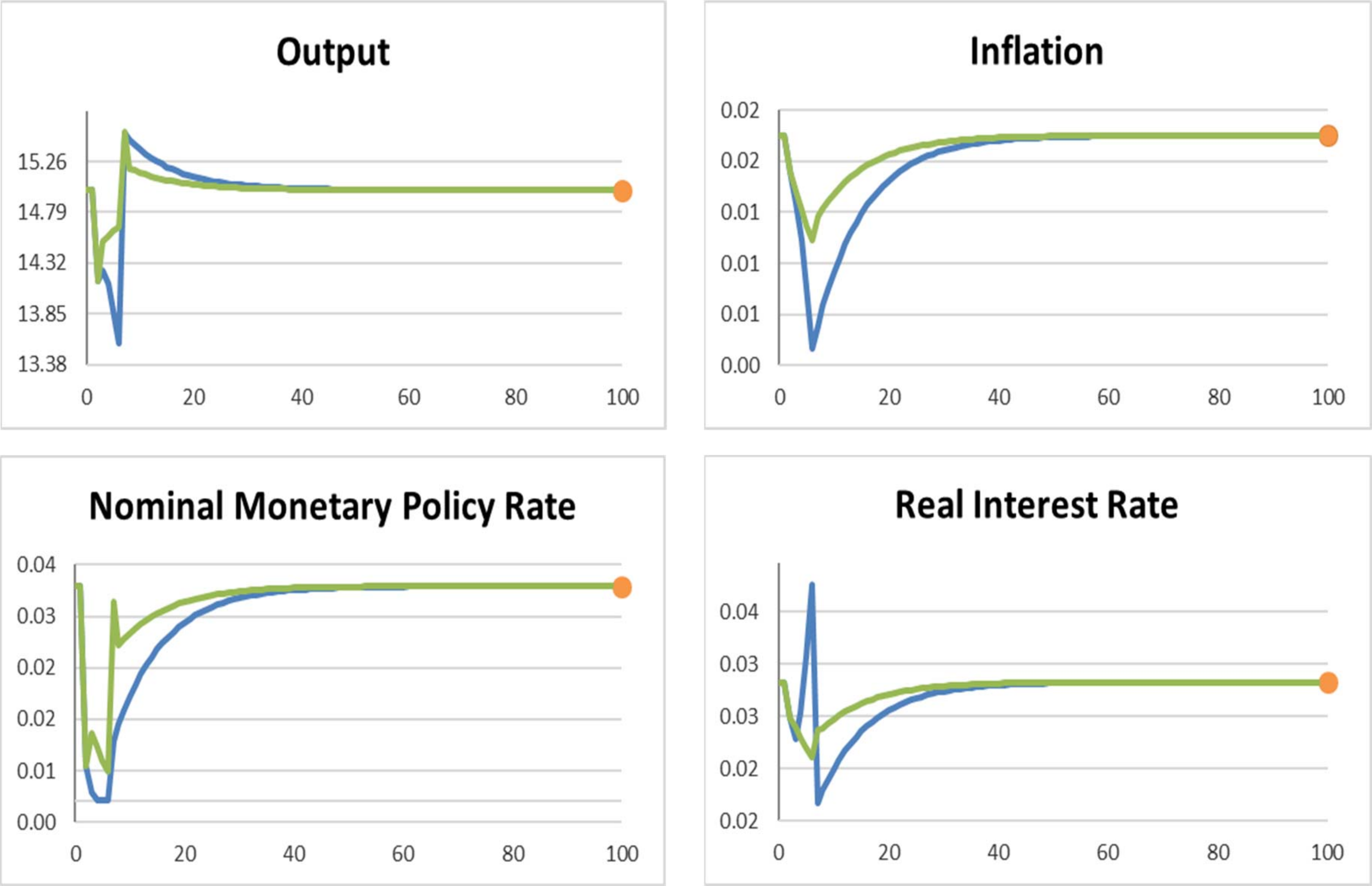
- "Deep crisis" recession, with an unconventional monetary stimulus being initiated to partially mitigate the increase of the exogenous component of the risk premium:

  $t = 2, 3, ..., 6$: $I_0 = 2.4$; $t = 7, 8, ...$: $I_0 = 3$

  $t = 2$: $\overline{r}_t^{RP} = 0.04$; $t = 3, 4, ..., 6$: $\overline{r}_t^{RP} = 0.03$; $t = 7$: $\overline{r}_t^{RP} = 0$; $t = 8, 9 ...$: $\overline{r}_t^{RP} = 0.01$.

The following graphs depict the resultant time paths for output, inflation, the real interest rate and the monetary policy rate for both resultant scenarios for periods $t = 0, 1, ..., 100$ (again based on automated calculations within the workbook *Extended-AS-AD-Model-PolicyOptions.xlsm*):

"Deep Crisis" Recession



without unconventional monetary stimulus
with unconventional monetary stimulus

In the "deep crisis" recession, there is a pronounced stabilization of output through the unconventional monetary stimulus, in line with the economic rationale advanced in the short- and medium-run (extended) IS-TR and (extended) AS-AD models.

Notice also, though, that in the absence of an unconventional monetary stimulus, when investment demand rebounds and the exogenous component of the risk premium is back to its original level, output overshoots its long-run level for a longer period of time than in the presence of an unconventional monetary stimulus. As the graphs show, this is due to the different paths that conventional monetary policy takes in the two scenarios: In the scenario without unconventional monetary stimulus, the rate of inflation from period $t=3$ onwards is lower than in the scenario involving temporary unconventional monetary stimulus. The central bank in the scenario without unconventional monetary stimulus therefore sets a lower monetary policy rate, until output and inflation have converged back to their long-run/target levels.

We may assess the overall impact of unconventional monetary policy on output through a single measure, the cumulated output change:

$$\left(\Delta Y\right)^{sum} = \sum_{t=2}^{100}\left(Y_t^{unconventional\ monetary\ stimulus} - Y_t^{no\ unconventional\ monetary\ stimulus}\right). \qquad (42)$$

Under the above scenario, one obtains

$$\left(\Delta Y\right)^{sum} = 0. \qquad (43)$$

Even putting aside questions as to how the unconventional monetary stimulus is to be funded, to make the case for the monetary authorities to pursue an unconventional monetary stimulus, on the basis of the extended *AS-AD* model one would thus need to argue along lines such as

-- near-future events at the time of "deep crisis" recession (when the output losses in the absence of unconventional monetary stimulus occur) matter more than further-ahead future events (when the output gains in the absence of unconventional monetary stimulus occur), or

-- in the absence of unconventional monetary stimulus, the macroeconomy may become unstable, that is, not return to the long-run outcomes anymore.

5. The Big Picture Concerning Business Cycles

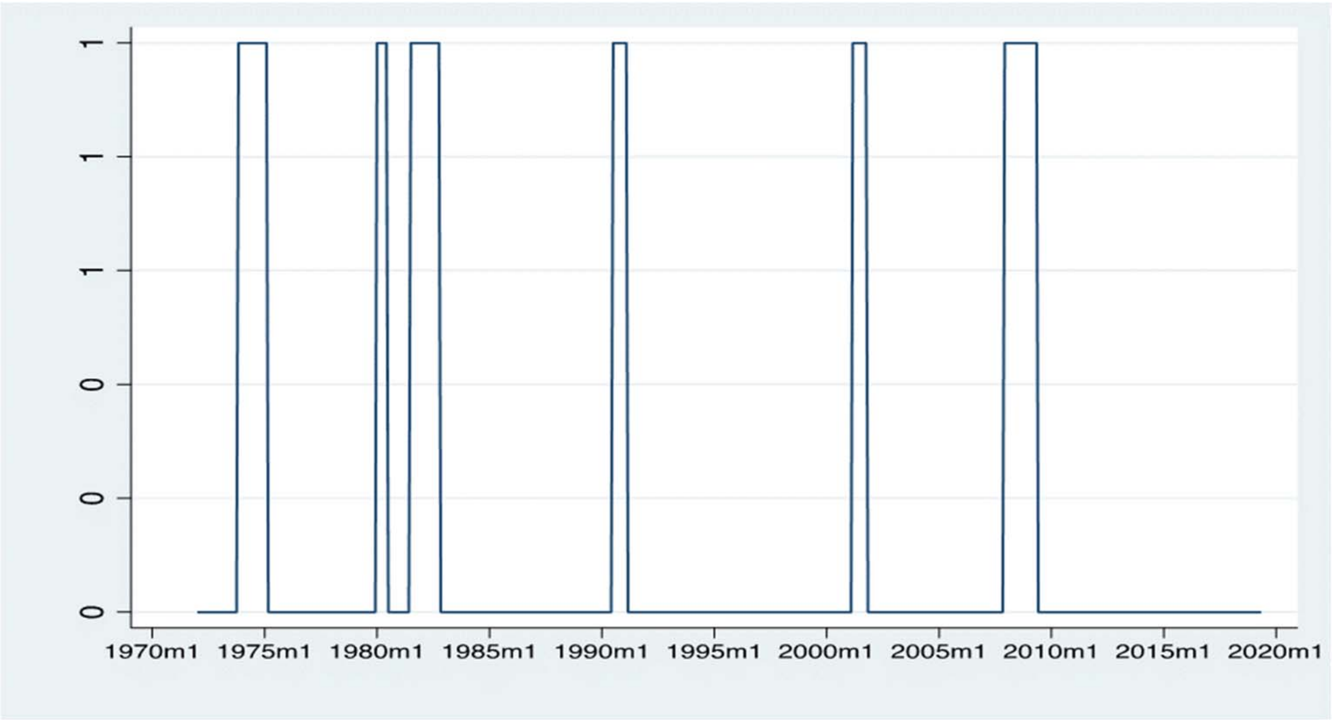
The (extended) *IS-TR* and (extended) *AS-AD* models we have developed to study macroeconomic outcomes in the short and medium runs suggest that business cycles, and the co-movements between core macroeconomic aggregates that they involve, are initially due to exogenous shocks to components of aggregate demand, to financial market risk premia, or to policy rules ("impulses") .

These shocks are then propagated through mechanisms such as the Keynesian multiplier, endogenous risk premia, as well as crowding out through the consumption, investment and exchange rate channels ("propagation mechanisms"). We have seen that these latter mechanisms can be quantitatively rather strong.

Inspecting business cycle dates gives us more insight into the likely types of shocks that in practice give rise to (deep) recessions. See, for example, the following diagrams, depicting business cycle dates in the United States and the Euro Area:

NBER Recession Indicator for the United States

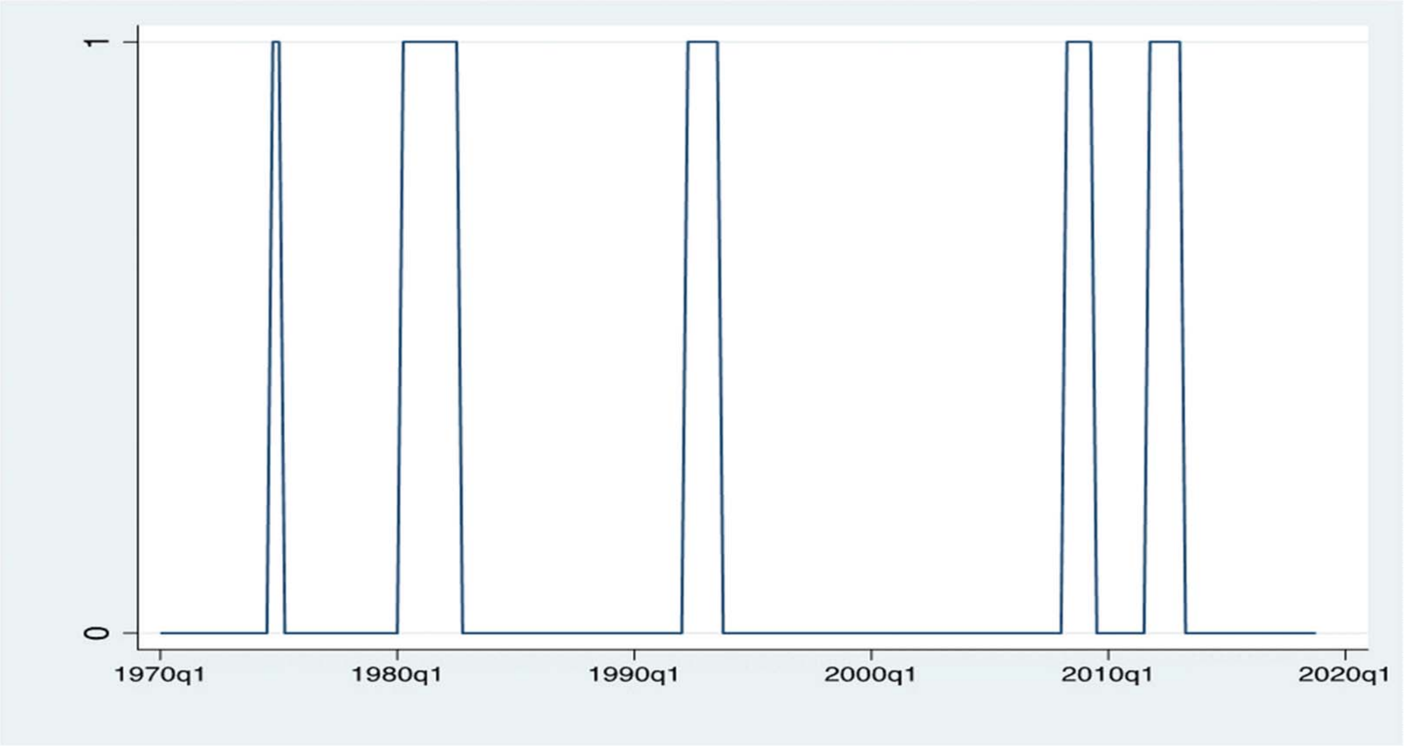
(a Value of 1 is a Recessionary Period; a Value of 0 is an Expansionary Period)



Source of Data: Federal Reserve Bank of St. Louis (2019)

CEPR Recession Indicator for the Euro Area

(a Value of 1 is a Recessionary Period; a Value of 0 is an Expansionary Period)



Source of Data: CEPR (2019)

In regards to assessing the quantitative performance of the business cycle models we have studied in this course, this would involve the use of econometric techniques that are well beyond the scope of this course.

Business-cycle models used by macroeconomists working at the research frontier share numerous of the mechanisms of the (extended) *IS*-*TR* and (extended) *AS*-*AD* models, but go well beyond these. From a methodological perspective, such models

-- are explicitly dynamic (modelling for example also capital accumulation),

-- capture uncertainty through being fully stochastic,

-- feature a range of non-linearities, for example through non-linear constraints such as occasionally binding borrowing constraints,

-- feature further institutional details, for example restrictions on exchange rate adjustment,

-- may feature yet more elaborate microfoundations (decision rules of households, firms and government institutions explicitly derived from intertemporal optimization problems, possibly involving elements of bounded rationality).

Such models typically cannot be solved analytically (that is, in closed form) anymore, and require the use of numerical analysis techniques. Moving beyond the baseline business cycle models studied in this course to explore such modelling choices effectively constitutes the agenda of all more advanced macroeconomics courses.

Even when yet more sophisticated business cycle models are considered, it should also be acknowledged, though, that uncertainty regarding the specific model structure to use is bound to remain, with the macroeconomic modeller having many choices to make, within national and global environments that are constantly evolving.