

1.

6. What common relations on  $\mathbb{Z}$  are the transitive closures of the following relations?

(a)  $a S b$  if and only if  $a + 1 = b$ .

(b)  $a R b$  if and only if  $|a - b| = 2$ .

(c) What common relation on  $\mathbb{Z}$  is the transitive closure of the relation  $T$ , where  $a T b$  if and only if  $|a - b|$  equals either 4 or 6? Explain.

2.

C. Suppose that  $r$  and  $s$  are relations on the set  $A$ . Show that if  $r \subseteq s$  then  $r^+ \subseteq s^+$ .

3.

2. Let  $A$  be a set and let  $S$  be any subset of  $A$ . Let  $\chi_S : A \rightarrow \{0, 1\}$  be defined by

$$\chi_S(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$$

The function  $\chi_S$  is called the *characteristic function* of  $S$ ,

(a) If  $A = \{a, b, c\}$  and  $S = \{a, b\}$ , list the element of  $\chi_S$ .

(b) If  $A = \{a, b, c, d, e\}$  and  $S = \{a, c, e\}$ , list the element of  $\chi_S$ .

(c) If  $A = \{a, b, c\}$ , what are  $\chi_\emptyset$  and  $\chi_A$ .

4.

4. Find the ranges of the following functions on  $\mathbb{Z}$ :

(a)  $g = \{(x, 4x + 1) \mid x \in \mathbb{Z}\}$

(b)  $h(x) =$  least integer that is greater than or equal to  $\sqrt{|x|}$ .

(c)  $P(x) = x + 10$ .

5.

4. Which of the following are injections, surjections, or bijections on  $\mathbb{R}$ , the set of real numbers?

(a)  $f(x) = -2x$ .

(b)  $g(x) = x^2 - 1$ .

(c)  $h(x) = \begin{cases} x & x < 0 \\ x^2 & x \geq 0 \end{cases}$

(d)  $q(x) = 2^x$

(e)  $r(x) = x^3$

(f)  $s(x) = x^3 - x$ .