

$$I = \int \sin^4 x \cos^6 x \, dx =$$

$$= \int (\sin x \cos x)^2 (\sin x \cos x)^2 \cos^2 x \, dx =$$

$$= \int \left(\frac{\sin 2x}{2} \right)^4 \frac{1 + \cos 2x}{2} \, dx =$$

$$= \frac{1}{32} \int \sin^4 2x (1 + \cos 2x) \, dx =$$

$$= \frac{1}{32} \int \sin^4 2x \, dx + \frac{1}{32} \int \sin^4 2x \cos 2x \, dx$$

$$= \frac{1}{32} \int \left(\frac{1 - \cos 4x}{2} \right)^2 \, dx + \frac{1}{32} \int \sin^4 2x \cos 2x \, dx$$

$$= \frac{1}{128} \int (1 - 2\cos 4x + \cos^2 4x) \, dx + \frac{1}{32} \int \sin^4 2x \cos 2x \, dx$$

Set: $\sin 2x = u$

$$2 \cos 2x \, dx = du$$

$$\cos 2x \, dx = \frac{1}{2} du$$

$$\frac{1}{128} \int \left(1 - 2\cos 4x + \frac{1 + \cos 8x}{2} \right) dx + \frac{1}{64} \int u^4 du =$$

$$= \frac{x}{128} - \frac{\sin 4x}{256} + \frac{x}{256} + \frac{\sin 8x}{2048} + \frac{1}{64} \frac{u^5}{5} + C =$$

$$= \left[\frac{x}{128} - \frac{\sin 4x}{256} + \frac{x}{256} + \frac{\sin 8x}{2048} + \frac{\sin^5 2x}{320} + C \right]$$