6. *Prove that the nuclear density is same for all nuclei. Give an estimate of nuclear density.*

Nuclear density. The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A, *so the density of nuclear matter is independent of the size of the nucleus.* Thus the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of nucleus $= mA$

Volume of nucleus

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$\therefore$ Nuclear density,

$$\rho_{nu} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

or
$$\rho_{nu} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Clearly, *nuclear density is independent of mass number A or the size of the nucleus.*

Taking $m = 1.67 \times 10^{-27}$ kg

and $R_0 = 1.2 \times 10^{-15}$ m, we get

$$\rho_{nu} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.142 \times (1.2 \times 10^{-15})^3}$$

$$= 2.30 \times 10^{17} \text{ kg m}^{-3}$$

Thus the nuclear mass density is of the ord 10^{17} kg m^{-3}. This density is very large as comparec the density of ordinary matter, say water, for wl $\rho = 1.0 \times 10^3$ kg m^{-3}.

one another. This work gives a measure of the binding energy of the nucleus. Thus,

*The **binding energy** of a nucleus may be defined as the energy required to break up a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.*

The concept of binding energy may also be understood in terms of *Einstein's mass energy equivalence.* It is seen that the mass of a stable nucleus is always less than the sum of the masses of the constituent protons and neutrons in their free state. This mass difference is called *mass defect which accounts for the* ΔE_b energy released when a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass. Thus

$$\Delta E_b = \Delta m \times c^2$$

So *the binding energy may also be defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.*

The energy equivalent to the mass defect is radiated in the form of electromagnetic radiation when the nucleons combine to form a nucleus.

Expression for binding energy. The nucleus ${}^A_Z\text{X}$ contains Z protons and $(A-Z)$ neutrons. Its mass defect is

$$\Delta m = Zm_p + (A-Z)m_n - m_N \quad \text{...(1)}$$

where m_N is the nuclear mass of ${}^A_Z\text{X}$. From Einstein's mass-energy equivalence, the binding energy of the nucleus is

$$\Delta E_b = \Delta m \times c^2 = [Zm_p + (A-Z)m_n - m_N]\,c^2 \quad \text{...(2)}$$

Now, in an atom the electrons are bound to the nucleus by electrostatic forces. So they have a binding energy of their own, which from the mass-energy equivalence is given by

$$(\Delta E_b)_e = [(m_N + Zm_e) - m({}^A_Z\text{X})]\,c^2 \quad \text{...(3)}$$

where $m({}^A_Z\text{X})$ is the atomic mass. The binding energy of electrons ($\approx$ eV to keV) is negligible compared to the binding energy of nucleons ($\approx 10^3$ MeV). It will be a safe approximation to take,

$$(\Delta E_b)_e = 0$$

$$\therefore \quad m_N + Zm_e - m({}^A_Z\text{X}) = 0$$

or

$$m_N = m({}^A_Z\text{X}) - Zm_e$$

Thus, in terms of atomic mass the equation (2) becomes

$$\Delta E_b = [Zm_p + (A-Z)m_n - m({}^A_Z\text{X}) + Zm_e]\,c^2$$

$$= [Z(m_p + m_e) + (A-Z)m_n - m({}^A_Z\text{X})]\,c^2 \quad \text{...(4)}$$

But $m_p + m_e = m_H$ = mass of a hydrogen atom.

$\therefore$ The equation (4) can be written in terms of m_H as

$$\Delta E_b = [Zm_H + (A-Z)m_n - m({}^A_Z\text{X})]\,c^2 \quad \text{...(5)}$$

Binding energy per nucleon. *The binding energy per nucleon is the average energy required to extract one nucleon from the nucleus. It is obtained by dividing the binding energy of a nucleus by its mass number. The expression for binding energy per nucleon can be written as*

$$\Delta E_{bn} = \frac{\Delta E_b}{A} = \frac{[Zm_H + (A-Z)m_n - m({}^A_Z\text{X})]\,c^2}{A} \quad \text{...(6)}$$

The binding energy per nucleon gives a measure of the force which binds the nucleons together inside a nucleus.

13.19 RADIOACTIVE DECAY LAW

21. *State and deduce radioactive decay law. Hence define disintegration constant.*

Radioactive decay law. According to *Rutherford-Soddy theory* (*i*) The radioactive atoms are unstable and they decay spontaneously to emit α- or β-particles alongwith γ-rays. (*ii*) The disintegration is random. It is purely a matter of chance for any atom to disintegrate first. (*iii*) The disintegration is independent of all physical and chemical conditions and so it can neither be accelerated nor retarded.

The above facts show that it is not possible to predict whether a particular nucleus will decay in a given time interval. By using the concept of probability, the decay behaviour of a collection of a large number of nuclei can be predicted accurately in terms of the ***radioactive decay law*** which states :

The number of nuclei disintegrating per second of a radioactive sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that instant.

Let

N_0 = the number of radioactive nuclei present initially at time $t = 0$ in a sample of radioactive substance.

N = the number of radioactive nuclei present in the sample at any instant t, and

dN = the number of radioactive nuclei which disintegrate in the small time interval dt.

According to radioactive law, the rate of decay at any instant is proportional to the number of undecayed nuclei, *i.e.*,

$$-\frac{dN}{dt} \propto N$$

or

$$-\frac{dN}{dt} = \lambda N \qquad ...(1)$$

where λ is a proportionality constant called the *decay* or *disintegration constant*. Here the negative sign shows that the number of undecayed nuclei, N decreases with time. The equation (1) can be written as

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating,

$$\int \frac{dN}{N} = -\lambda \int dt$$

or

$$\log_e N = -\lambda t + C \qquad ...(2)$$

where C is a constant of integration.

At $t = 0, N = N_0$, therefore from equation (2), we get

$$\log_e N_0 = C$$

Then the equation (2) becomes

$$\log_e N = -\lambda t + \log_e N_0$$

or

$$\log_e \frac{N}{N_0} = -\lambda t$$

or

$$\frac{N}{N_0} = e^{-\lambda t}$$

or

$$N = N_0 e^{-\lambda t} \qquad ...(3)$$

This equation represents the *radioactive decay law*. It gives the number of active nuclei left after time t.

Fig. 13.9 shows a graph between the number N of undecayed nuclei and time t. It reveals the following features :

1. The number of active nuclei in a radioactive sample decreases exponentially with time. The disintegration is fast in the beginning but becomes slower and slower with the passage of time.
2. The larger the value of decay constant λ, the higher is the rate of disintegration.
3. Irrespective of its nature, a radioactive sample will take infinitely long time to disintegrate completely.

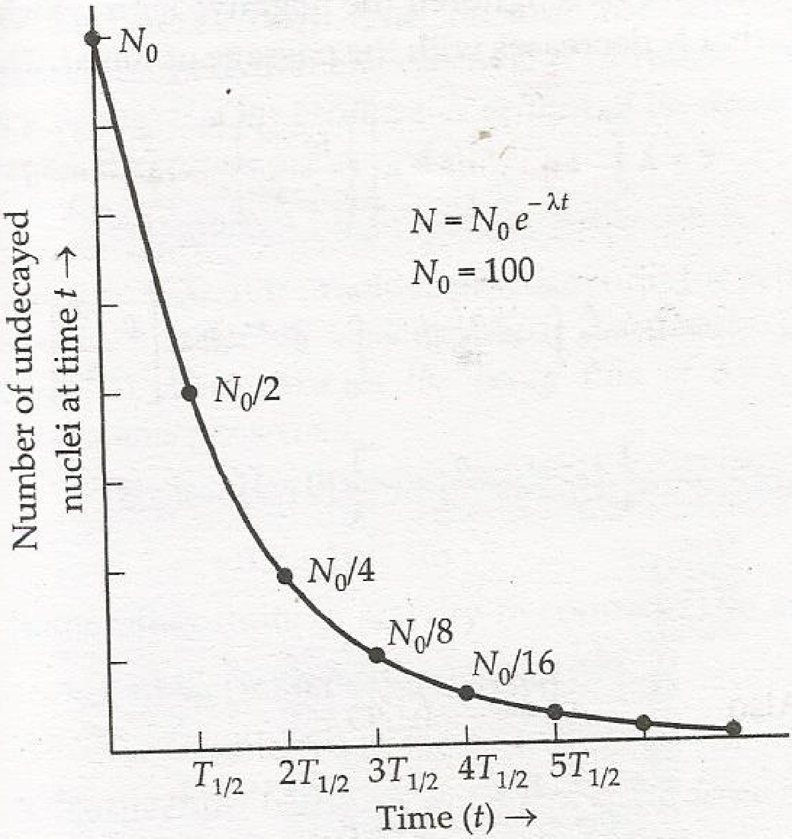

Fig. 13.9 Decay curve for a radioactive element.

Decay or disintegration constant :

If in equation (3), $t = \frac{1}{\lambda}$, then

$$N = N_0 e^{-1} = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368\ N_0$$

or

$$N = \frac{N_0}{e} = 36.8\% \text{ of } N_0 \qquad ...(4)$$

*The **radioactive decay constant** may be defined as the reciprocal of the time interval during which the number of active nuclei in a given radioactive sample reduces to 36.8% (or 1/e times) of its initial value.*

Decay constant may be defined in another way also, as follows :

As $\frac{dN}{dt} = -\lambda N$

$$\therefore \qquad \lambda = \frac{-\frac{dN}{dt}}{N} \qquad ...(5)$$

Thus *the radioactive decay constant may be defined as the ratio of the instantaneous rate of disintegration to the number of active nuclei present in the radioactive sample at the given instant.* It gives the *probability per unit time* for a nucleus of a radioactive substance to decay. The value of λ depends on the nature of the radioactive substance.

by $T_{1/2}$.

Relation between half-life and decay constant. Let

N_0 = Number of radioactive nuclei present in the radioactive sample initially (at $t = 0$).

N = Number of radioactive nuclei left at any instant t.

At $t = T_{1/2}$, $N = \frac{N_0}{2}$

Now $N = N_0 e^{-\lambda t}$, where λ is the radioactive decay constant.

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \text{or} \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

or $$e^{\lambda T_{1/2}} = 2$$

Taking natural logarithm, we get

$$\lambda T_{1/2} \log_e e = \log_e 2$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{2.303 \log 2}{\lambda}$$

$$= \frac{2.303 \times 0.3010}{\lambda} \quad [\because \log_e e = 1]$$

or $$T_{1/2} = \frac{0.693}{\lambda}.$$

Thus the half-life of a radioactive substance is inversely proportional to its decay constant and is independent of the number N_0, the number of radioactive nuclei present initially in the sample.

... The value of the half-life

13.21 MEAN LIFE

23. *Define mean life of a radioactive example. Deduce its relation with decay constant and half-life.*

Mean life. All the nuclei of a radioactive sample do not disintegrate at the same time. While one nucleus may disintegrate right at the beginning and some other may disintegrate at the end of the process. So the life time of the different nuclei may vary from zero to infinity.

*The average time for which the nuclei of a radioactive sample exist is called **mean life** or **average life** of that sample. It is equal to the ratio of the combined age of all the nuclei to the total number of nuclei present in the given sample.* It is denoted by τ.

$$\text{Mean life} = \frac{\text{Sum of the lives of all the nuclei}}{\text{Total number of nuclei}}$$

Relation between mean life and decay constant. Suppose a radioactive sample contains N_0 nuclei at time $t = 0$. After time t, this number reduces to N. Furthermore, suppose dN nuclei disintegrate in time t to $t + dt$. As dt is small, so the life of each of the dN nuclei can be approximately taken equal to t.

$\therefore$ Total life of dN nuclei $= t\,dN$

$$\text{Total life of all the } N_0 \text{ nuclei} = \int_0^{N_0} t\,dN$$

$$\text{Mean life} = \frac{\text{Total life of all the } N_0 \text{ nuclei}}{N_0}$$

or
$$\tau = \frac{1}{N_0}\int_0^{N_0} t\,dN$$

As
$$N = N_0\,e^{-\lambda t}$$

$\therefore$
$$dN = -\lambda\,N_0\,e^{-\lambda t}\,dt$$

When $N = N_0$, $t = 0$ and when $N = 0$, $t = \infty$.

Changing the limits of integration in terms of tir we get

$$\tau = \frac{1}{N_0}\int_0^{\infty} t\,\lambda\,N_0\,e^{-\lambda t}\,dt$$

Here we have ignored the negative sign which just tells that N decreases with the passage of time t. Thus

$$\tau = \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda \left[\left\{ \frac{te^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= 0 + \frac{\lambda}{\lambda} \int_0^\infty e^{-\lambda t} dt = \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty$$

$$= -\frac{1}{\lambda} [e^{-\infty} - e^0] = -\frac{1}{\lambda} [0 - 1]$$

or $$\tau = \frac{1}{\lambda}$$

Also $$T_{1/2} = \frac{0.693}{\lambda} = 0.693\,\tau$$

or $$\tau = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}.$$

Decay rate or activity of a radioactive sample. *The rate of decay or activity of a sample is defined as the number of radioactive disintegrations taking place per second in the sample.*

If a radioactive sample contains N radioactive nuclei at any time t, then its decay rate or activity R at the same time t will be

$$R = -\frac{dN}{dt}$$

The negative sign shows that the activity of the sample decreases with the passage of time.

According to the radioactive decay law,

$$-\frac{dN}{dt} = \lambda N$$

$$\therefore \qquad R = \lambda N$$

As $N = N_0 e^{-\lambda t}$, so we can write

$$R = \lambda N_0 e^{-\lambda t}$$

or

$$R = R_0 e^{-\lambda t}$$

This is *another form of the radioactive decay law*. Here $R_0 = \lambda N_0$, is the decay rate at time $t = 0$ and R is the lecay rate at any subsequent time t. Like N, obviously also decreases exponentially with time.