ETH zürich

Parametric reduced order modeling for cracked solids

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Outline

Activities of the chair of structural mechanics and monitoring

MOR for cracked solids

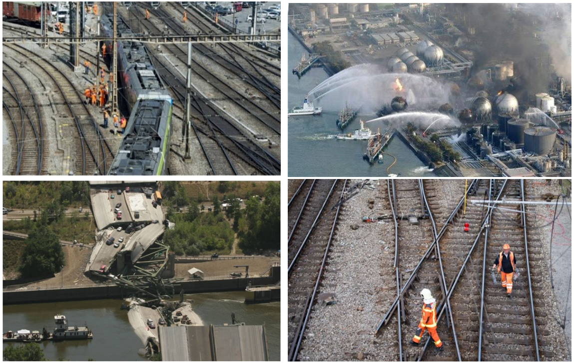
Numerical examples

Conclusions

Infrastructure

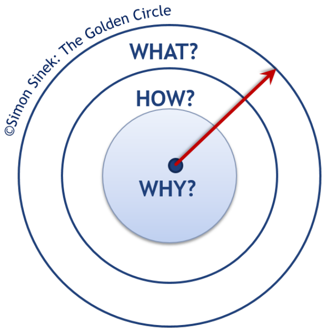
- General purpose $\Rightarrow$ Monitoring and assessment of infrastructure
- But what is infrastructure?

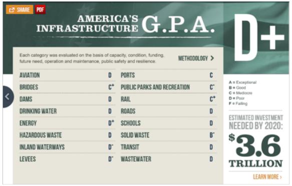
Reality check: structures...



- Experience motion
- Are exposed to hazards
- Age and deteriorate
- Undergo damage

Why?





UK's State of Nation, ICE

Required investment

USA $3.6 Trillion

from 2016 to 2025, each household will lose $ 3,400 per year in due to infrastructure deficiencies (ASCE 2013)

Germany $69 Billion

Germany needs to invest $69 billion in its roads to meet expected demand (McKinsey & Co. 2013)

Globally $57 Trillion

global investment needs to increase by 60 % to $ 57 trillion until 2030 (McKinsey & Co. 2013)

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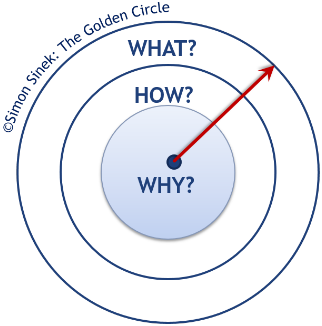
How?

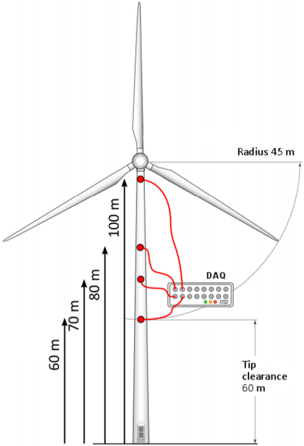


Low-cost and easily deployable sensors for:

- Recording & interpreting response
- Diagnosing the system's "health"
- Optimize operation & maintenance

Deployments

- Buildings
- Wind turbines
- Bridges



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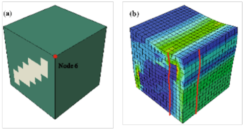
Data is not enough

Models are needed to:

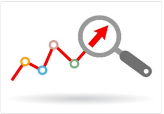
- investigate complex system behavior
- predict and learn performance
- identify vulnerabilities
- increase security, robustness, resilience, capacity

Simulation Paths

Physics-based



Data Driven



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MOR for cracked solids

Requirements:

- Parametrization with respect to crack location, size, orientation
- Widest possible range of parameters
- High accuracy
- Fast online evaluation for different crack configurations

Governing equations - Discretization

A simple static linear elastic problem is considered:

$$\int_{\Omega} \varepsilon(\boldsymbol{u}) : \boldsymbol{D} : \varepsilon(\boldsymbol{v}) \, d\Omega = \int_{\Omega} \boldsymbol{b} \cdot \boldsymbol{v} \, d\Omega + \int_{\Gamma_t} \bar{\boldsymbol{t}} \cdot \boldsymbol{v} \, d\Gamma$$

XFEM is used for discretization:

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{I \in \mathcal{N}} N_I(\boldsymbol{x}) \, \boldsymbol{u}_I + \sum_{J \in \mathcal{N}^j} N_J(\boldsymbol{x}) \left(H(\boldsymbol{x}) - H(\boldsymbol{x}_J)\right) \boldsymbol{b}_J + \sum_{T \in \mathcal{N}^t} \sum_{j} N_T(\boldsymbol{x}) \left(F_j(\boldsymbol{x}) - F_j(\boldsymbol{x}_T)\right) \boldsymbol{c}_{Tj}$$

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Full order problem

Discretized equilibrium equations:

$$\boldsymbol{K}(\boldsymbol{\mu})\,\hat{\boldsymbol{u}}(\boldsymbol{\mu}) = \boldsymbol{f}(\boldsymbol{\mu})$$

where:

$\boldsymbol{K} \in \mathbb{R}^{N \times N}$

$\bar{\boldsymbol{u}} \in \mathbb{R}^{N}$

$\boldsymbol{f} \in \mathbb{R}^{N}$

$\boldsymbol{\mu} \in \mathbb{R}^{N^p}$ is a vector of parameters

N the problem size

N^p the number of parameters

General case of parametrized MOR

A solution is sought in a lower dimensional space of size $k \ll N$:

$$\hat{\boldsymbol{u}} \approx \boldsymbol{V}\boldsymbol{y}$$

with:

$\boldsymbol{V} \in \mathbb{R}^{N\times k}$ a basis of the subspace $\qquad \boldsymbol{y} \in \mathbb{R}^{k}$

Then, a problem of reduced size can be derived:

$$\boldsymbol{V}^T\boldsymbol{K}(\mu)\,\boldsymbol{V}\boldsymbol{y} = \boldsymbol{V}^T\boldsymbol{f}(\mu) \Rightarrow \tilde{\boldsymbol{K}}(\mu)\,\boldsymbol{y} = \tilde{\boldsymbol{f}}(\mu)$$

General case of parametrized MOR

To form $\boldsymbol{V}$:

- Solutions, called “snapshots”, are obtained for different values of the parameters through a greedy sampling process
- Snapshots are collected in a matrix
- An SVD is performed on the snapshot matrix

Application to problems with discontinuities

For cracks/discontinuities:

- μ should geometrically describe the crack
- Snapshots of different cracks should be computed
- Snapshots of different cracks would be linearly combined
- The resulting basis would not be accurate

Proposed approach

Alternatively:

- A reference configuration can be defined
- Mesh morphing can be employed
- The reference crack can be moved to match different cracks
- All snapshots can be referred to the reference crack

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Mesh morphing

For mesh morphing

- A method based on inverse distance weighting is developed
- Modifications are made to increase efficiency
- Points on the boundaries of the considered domain are also morphed to match target cracks
- Optimization is employed to maximize morphed mesh quality

Projection into the lower dimensional basis

Once the lower dimensional basis is formed, the projections

$$\tilde{\boldsymbol{K}}(\boldsymbol{\mu}) = \boldsymbol{V}^T \boldsymbol{K}(\boldsymbol{\mu}) \boldsymbol{V}, \quad \tilde{\boldsymbol{f}}(\boldsymbol{\mu}) = \boldsymbol{V}^T \boldsymbol{f}(\boldsymbol{\mu})$$

should be computed.

Two alternatives are employed:

- Hyperreduction $\Rightarrow$ ECSW
- System matrix interpolation $\Rightarrow$ Spline interpolation

Edge crack in a tension/shear specimen

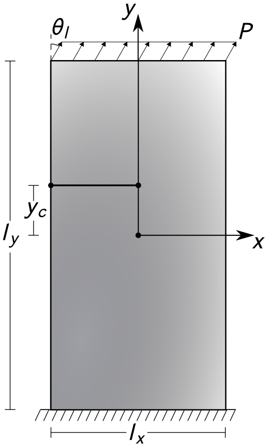
ly = 16 units

lx = 7 units

$\theta_l \in \{0, \pi/8, \pi/4, 3\pi/8\pi/2\}$

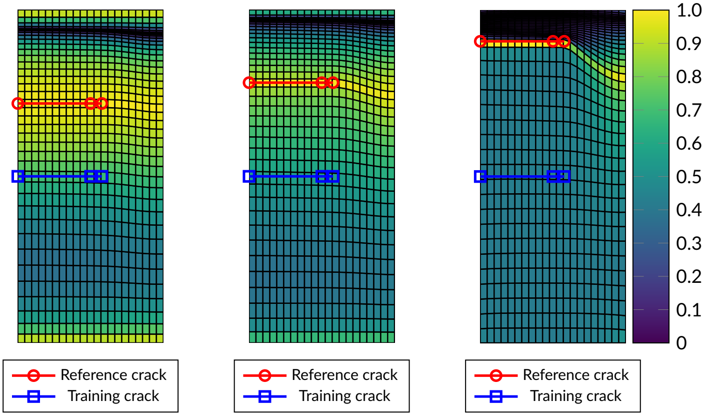
$y_c \in [-6.5, 6.5]$

Structured 41 $\times$ 81 quad mesh



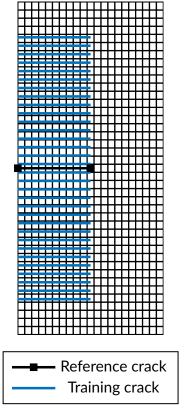
Edge crack in a tension/shear specimen

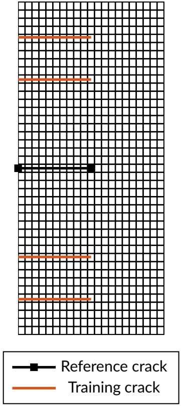
Morphed mesh and mesh quality index:

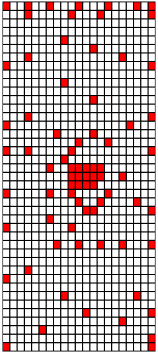


Edge crack in a tension/shear specimen

Training set and elements used:







Edge crack in a tension/shear specimen

Size, errors and timings of the models tested:

	FOM	ROM1	ROM2
size	1,920	10	10
elements	861	56	-
parameter space samples	-	-	10
maximum error %	0.0000	1.6971	2.1538
mean error %	0.0000	0.4885	0.9203
solution time (s)	0.1271	0.0172	0.0014
morphing time (s)	-	0.0029	-
speedup	1.0000	6.1401	90.7857

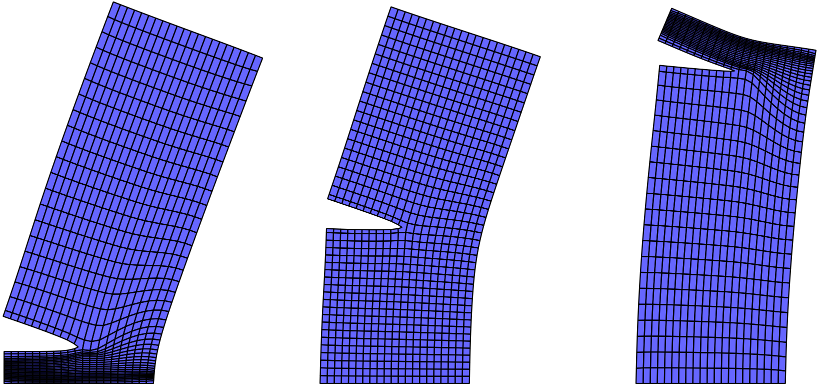
FOM, full order model

ROM1, reduced order model with hyper reduction

ROM2, reduced order model with matrix interpolation

Edge crack in a tension/shear specimen

Deformed mesh for three different crack locations:



Rib of an aircraft wing

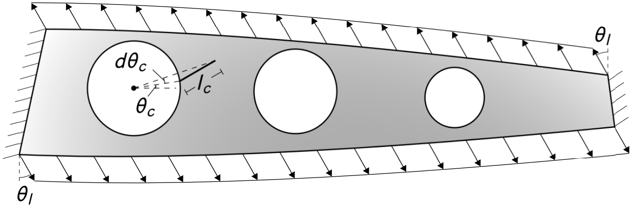
$\theta_l \in \{0, \pi/8, \pi/4, 3\pi/8\pi/2\}$

$\theta_c \in [0, 0.15\pi]$

$l_c \in [40, 60]$

$d\theta_c \in [-0.05\pi, 0.05\pi]$

Unstructured mesh



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Rib of an aircraft wing

Size, errors and timings for the models tested:

	FOM	ROM1	ROM2
size	13,912	24	24
elements	6,526	258	-
parameter space samples	-	-	250 $(10 \times 5 \times 5)$
maximum error %	0.0000	1.7017	1.6345
mean error %	0.0000	0.8676	0.9478
solution time (s)	0.9901	0.0440	0.0073
morphing time (s)	-	0.0274	-
speedup	1.0000	13.8670	135.6301

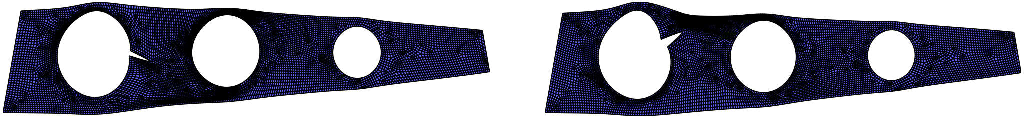
FOM, full order model

ROM1, reduced order model with hyper reduction

ROM2, reduced order model with matrix interpolation

Rib of an aircraft wing

Deformed mesh for two different crack locations:



Conclusions

Advantages of the proposed approach:

- Allows parametrization of crack geometry
- Provides high accuracy
- Is efficient

Disadvantages:

- Topology and number of cracks must be known
- Possible range can be limited for higher numbers of parameters

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Directions for future work

Possible extensions:

- Introduction of several reference configurations
- Dynamic problems
- Shell structures

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