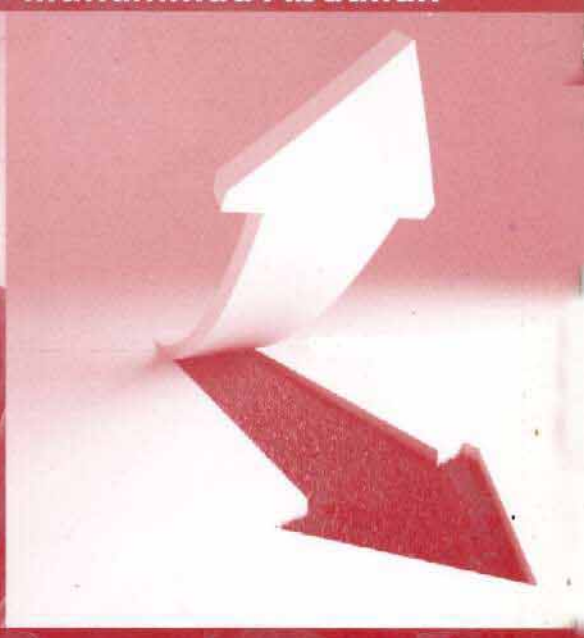
Muhammad Abdullah



KEY TO BUSINESS Statistics

I.Com. Part - II

Subjective + Objective

KEY

To

BUSINESS STATISTICS

For

I. Com Part II Students

By

Muhammad Abdullah

M. Sc. Statistics

Assistant Professor

Government College of Commerce

Faisalabad

CONTENTS

SUBJECTIVE SOLUTION

CONTENTS

OBJECTIVE SOLUTION

EXERCISE NO. 1

-:1.11:-

(1) Long term (2) Useful (3) Three ways (4) Quantitation (5) Average

-:1.12:-

(i) False	(ii) False	(iii) True	(iv) False	(v) True
(vi) False	(vii) True	(viii) True	(ix) True	(x) False
(xi) True	(xii) False			

-:1.13:-

Here Maximum value = 176
Minimum value = 119

Frequency Distribution

Classes	Tally	Frequency
118 – 123	I	1
124 – 129	II	2
130 – 135	III	3
136 – 141	III	3
142 – 147	𝍸	5
148 – 153	III	3
154 – 159	I	1
160 – 165	I	1
166 – 171	–	–
172 – 177	I	1
Total	-----	**20**

-:1.14:-

Class interval or groups are given, we make the frequency table as follows:

Marks	Tally	F
0 - 9	II	2
10 - 19	𝍸 𝍸 II	12
20 - 29	𝍸 I	6
30 - 39	𝍸 𝍸	10
40 - 49	𝍸 𝍸 II	12
50 - 59	𝍸 II	7
60 - 69	I	1
Total	----	**50**

-:1.15:-

Here Maximum value = 72
Minimum value = 62

Frequency Distribution

Classes	Tally	Frequency
60 – 62	II	2
62 – 64	II	2
64 – 66	~~IIII~~ III	8
66 – 68	IIII	4
68 – 70	IIII	4
70 – 72	IIII	4
72 – 74	I	1
Total	----	**25**

-:1.16:-

The lowest value $X_0 = 28$ and greatest value $X_m = 69$
Range $= X_m - X_0 = 69 - 28 = 41$
If we take class interval = h = 5, then

$$\text{The number of classes} = \frac{41}{5} = 8.2 = 9$$

The frequency distribution

Age	Tally	F
25 - 30	IIII	4
30 -35	~~IIII~~ ~~IIII~~	10
35 - 40	~~IIII~~ ~~IIII~~	10
40 - 45	~~IIII~~	5
45 - 50	~~IIII~~ ~~IIII~~ III	13
50 - 55	IIII	4
55 - 60	----	0
60 - 65	IIII	4
65 - 70	II	2
Total	----	**52**

-:1.17:-

Here $X_m = 4.7$, and $X_0 = 1.6$
Range = 4.7 - 1.6 = 3.1

Let the numbr of classes = 7, $h = \frac{3.1}{7} = 0.443$. We choose h = 0.5.

If we begin the lowest value at 1.5, the second class would begin at 2.0 and so fourth.

The Frequency Distribution

Class Interval	Tally	F
1.5 - 1.9	III	3
2.0 - 2.4	II	2
2.5 - 2.9	𝍸 II	7
3.0 - 3.4	𝍸 𝍸 𝍸 II	17
3.5 - 3.9	𝍸 𝍸 II	12
4.0 - 4.4	𝍸	5
4.5 - 4.9	IIII	4
Total	-----	**50**

-:1.18:-

The Frequency Distribution.

Weights	Tally	F
118 - 126	𝍸 I	6
127 - 135	𝍸 𝍸	10
136 - 144	𝍸 𝍸 IIII	14
145 - 153	𝍸 𝍸 𝍸 I	16
154 - 162	𝍸 I	6
163 - 171	𝍸 I	6
172 - 180	II	2
Total	-----	**60**

-:1.19:-

Here Maximum value = 95
Minimum value = 43

Frequency Distribution

Classes	Tally	Frequency
41 – 50	𝍸	5
51 – 60	III	3
61 – 70	IIII	4
71 – 80	𝍸	5
81 – 90	I	1
91 – 100	I	1
Total	----	**20**

-:1.20:-

Here Maximum value = 85, Minimum value = 50

Frequency Distribution

Classes	Tally	Frequency
50 – 54	III	3
55 – 59	II	2
60 – 64	𝍸	5
65 – 69	III	3
70 – 74	𝍸 II	7
75 – 79	II	2
80 – 84	II	2
85 – 89	I	1
Total	----	**25**

-:1.21:-

Here Maximum value = 54, Minimum value = 11, Interval size = 10

Frequency Distribution

Classes	Tally	Frequency
11 – 20	𝍸 I	6
21 – 30	𝍸 I	6
31 – 40	𝍸 𝍸	10
41 – 50	II	2
51 – 60	II	2
Total	----	**26**

-:1.22:-

Here Maximum value = 11

Minimum value = 1

Frequency Distribution

x	Tally	Frequency
1	I	1
2	I	1
3	I	1
4	I	1
5	III	3
6	𝍸	5
7	III	3
8	II	2
9	III	3
10	I	1
11	I	1
Total	----	**22**

-:1.23:-

Here Maximum value = 5, Minimum value = 0

Frequency Distribution

No of Heads	Tally	Frequency
0	II	2
1	IIII	4
2	~~IIII~~ I	6
3	~~IIII~~ II	7
4	~~IIII~~	5
5	I	1
Total	----	**25**

-:1.24:-

Here Maximum value = 9, Minimum value = 1

Frequency Distribution

Sales (Rs.) Thousand	Tally	Frequency
0.5 – 1.4	II	2
1.5 – 2.4	II	2
2.5 – 3.4	~~IIII~~	5
3.5 – 4.4	IIII	4
4.5 – 5.4	IIII	4
5.5 – 6.4	III	3
6.5 – 7.4	II	2
7.5 – 8.4	I	1
8.5 – 9.4	I	1
9.5 – 10.4	I	1
Total	----	**25**

-:1.25:-

$X_0 = 20$, $X_m = 97$

Range = 97 - 20 = 77

No. of classes = 8, $h = \frac{77}{8} = 9.625 = 10$

The Frequency Distribution

Marks	Tally	F
20 - 30	𝍸	5
30 - 40	𝍸 𝍸	10
40 - 50	𝍸 I	6
50 - 60	IIII	4
60 - 70	III	3
70 - 80	𝍸	5
80 - 90	𝍸 I	6
90 - 100	I	I
Total	----	**40**

-:1.26:-

This is problem of discrete variable.

There will be no class limits. We will deal the whole numbers.

No. of Members	Tally	F
1	II	2
2	IIII	4
3	𝍸	5
4	𝍸 I	6
5	𝍸 𝍸 I	11
6	𝍸 𝍸 III	13
7	𝍸 III	8
8	IIII	4
9	IIII	4
10	IIII	4
11	II	2
Total	----	**63**

-:1.27:-

It is the problem of discrete variable

No. of Child	Tally	F
0	I	1
1											9
2												10
3							5
4																14
5												10
6	||||| ||||| ||||| I	16
7																14
8												10
9	||||| I	6
10	||||| I	6
11	||||| I	6
12							5
12						4
Total	----	**116**

-:1.28:-

Let X represents the marks in statistics and y represents marks in Mathematics.

$X_m = 19$ $Y_m = 19$ Range $= Y_m - Y_0$

$X_0 = 0$ $Y_0 = 1$ $= 19 - 1 = 18$

Rane $= X_m - X_0 = 19 - 0 = 19$

Let the No. of classes = 4, then

Let the No. of classes = 5

$h = \frac{19}{4} = 4.75 = 5$ $h = \frac{18}{5} = 3.6 = 4$

TALLY SHEET

Stat / Math	0-4	5-9	10-14	15-19	Total
0-3	II	III			5
4-7	I	I	I		3
8-11	II	III	III	I	9
12-15		I	IIII	III	8
16-19	I		III	III	7
Total	**6**	**8**	**11**	**7**	**32**

FREQUENCY TABLE

Stat / Math	0-4	5-9	10-14	15-19	Total
0-3	2	3	---	---	5
4-7	1	1	1	---	3
8-11	2	3	3	1	9
12-15	---	1	4	3	8
16-19	1	---	3	3	7
Total	**6**	**8**	**11**	**7**	**32**

-:1.29:-

TALLAY SHEET

H / W	20-25	25-30	20-35	35-40	40-45	45-50	Total
20-25	III	II					5
25-30		III		II			5
30-35			I	III			4
35-40			I	I	I		3
40-45					I	I	2
45-50						I	1
Total	**3**	**5**	**2**	**6**	**2**	**2**	**20**

FREQUENCY TABLE

H / W	20-25	25-30	30-35	35-40	40-45	45-50	Total
20-25	3	2					5
25-30		3		2			5
30-35			1	3			4
35-40			1	1	1		3
40-45					1	1	2
45-50						1	1
Total	**3**	**5**	**2**	**6**	**2**	**2**	**20**

-:1.30:-

TALLY SHEET

Y \ X	60-62	63-65	66-68	69-71	Total
102-105	III		II		5
106-109	II	IIII		II	9
110-113		II	I	I	4
114-117			III	I	4
118-121		I	I	I	3
Total	**5**	**8**	**7**	**5**	**25**

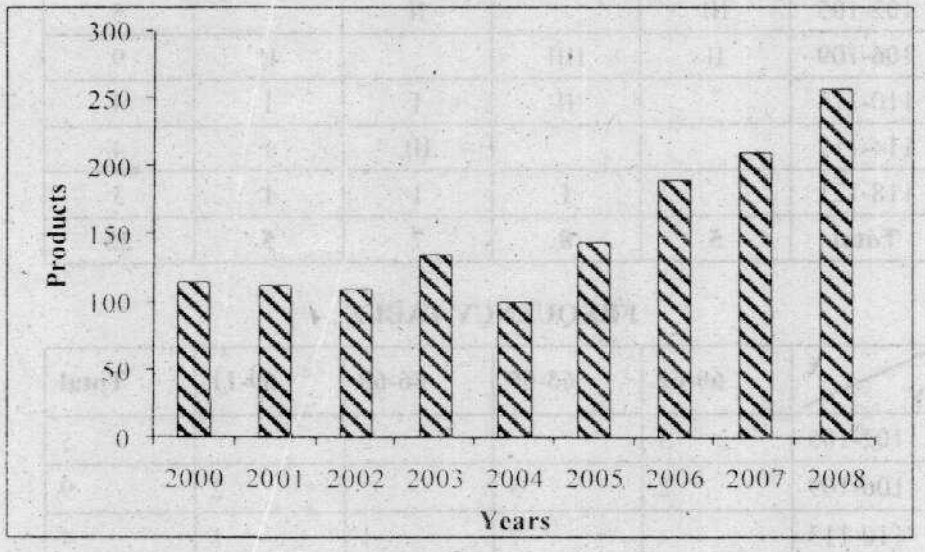
FREQUENCY TABLE

Y \ X	60-62	63-65	66-68	69-11	Total
102-105	3		2		5
106-109	2	5		2	9
110-113		2	1	1	4
114-117			3	1	4
118-121		1	1	1	3
Total	**5**	**8**	**7**	**5**	**25**

EXERCISE NO. 2

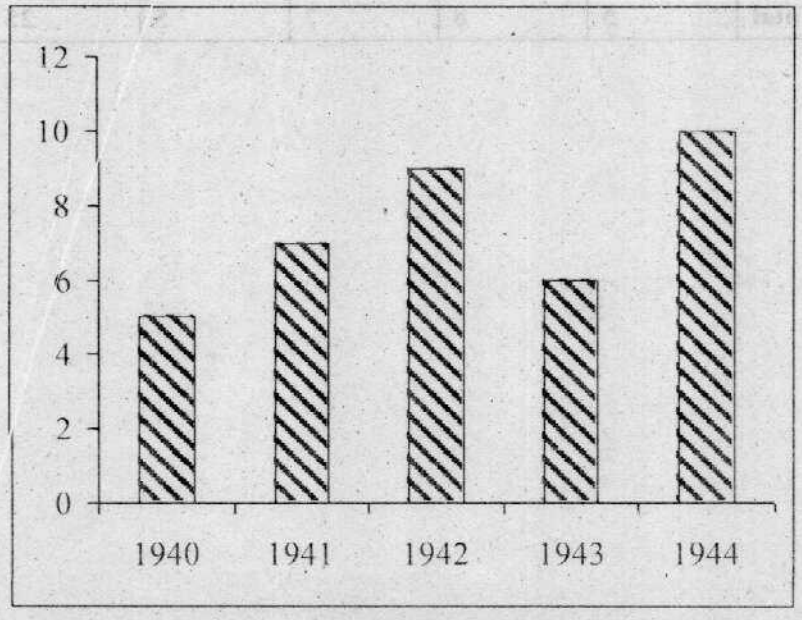
-:2.6:-

SIMPLE BAR DIAGRAM

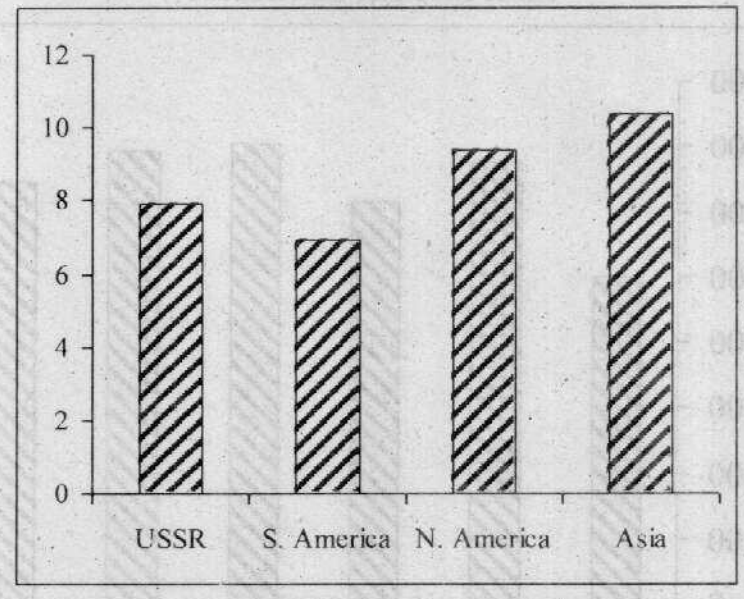


-:2.7:-

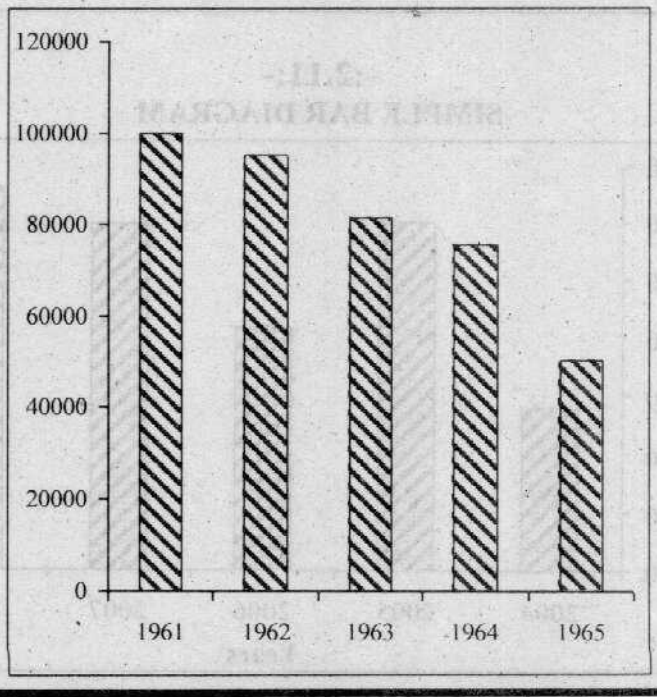
SIMPLE BAR DIAGRAM



-:2.8:-
SIMPLE BAR DIAGRAM

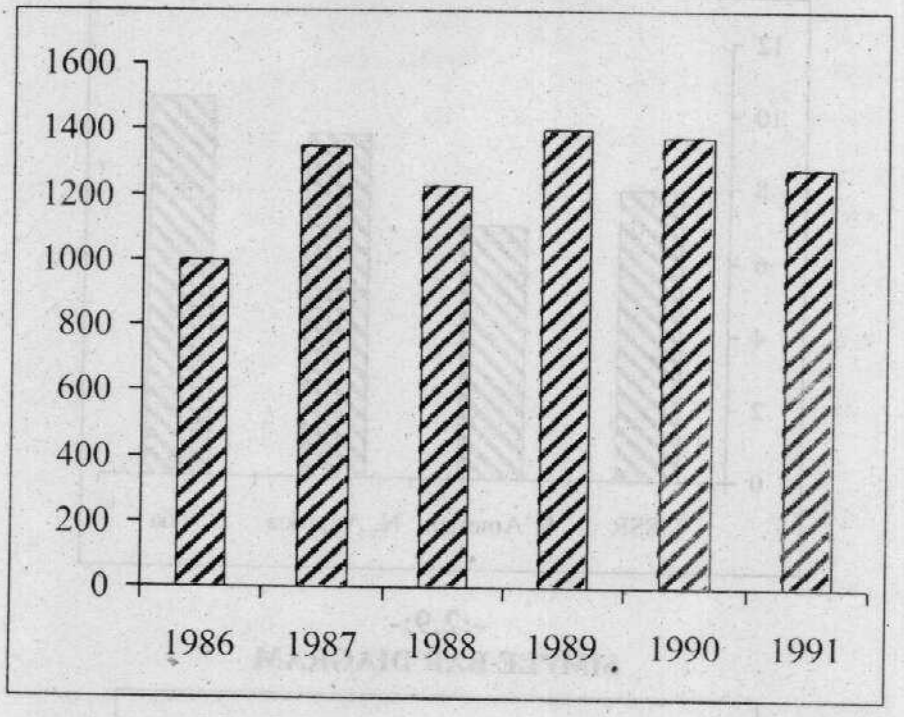


-:2.9:-
SIMPLE BAR DIAGRAM



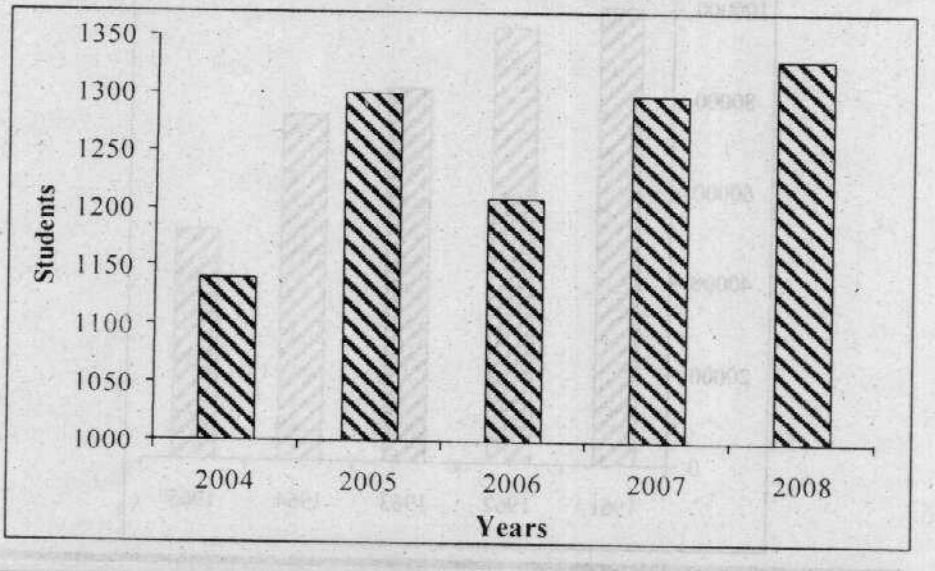
-:2.10:-

SIMPLE BAR DIAGRAM



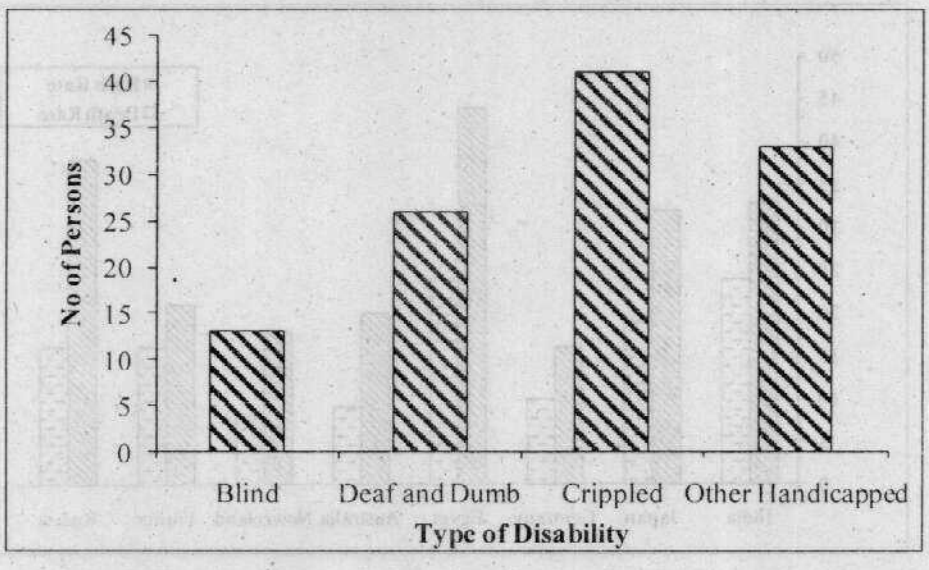
-:2.11:-

SIMPLE BAR DIAGRAM



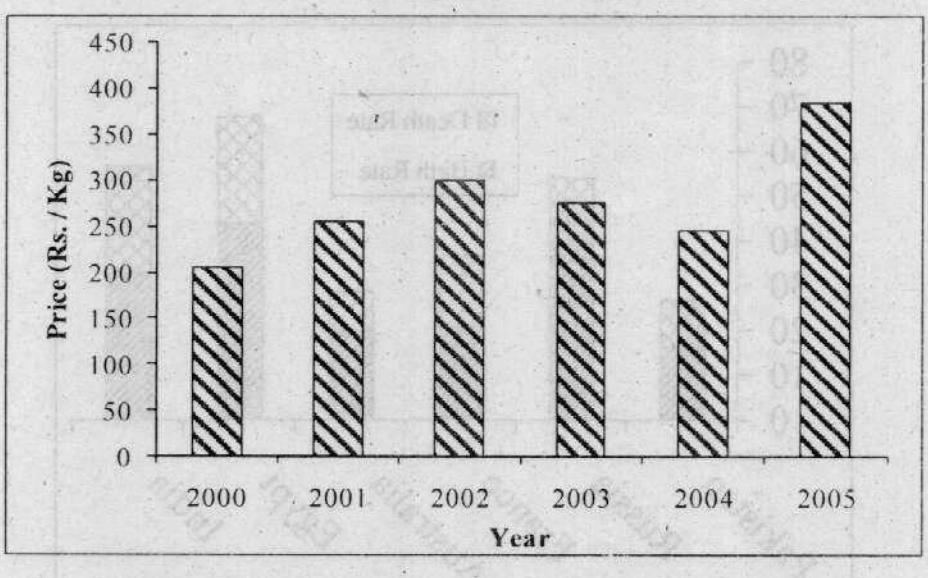
-:2.12:-

SIMPLE BAR DIAGRAM



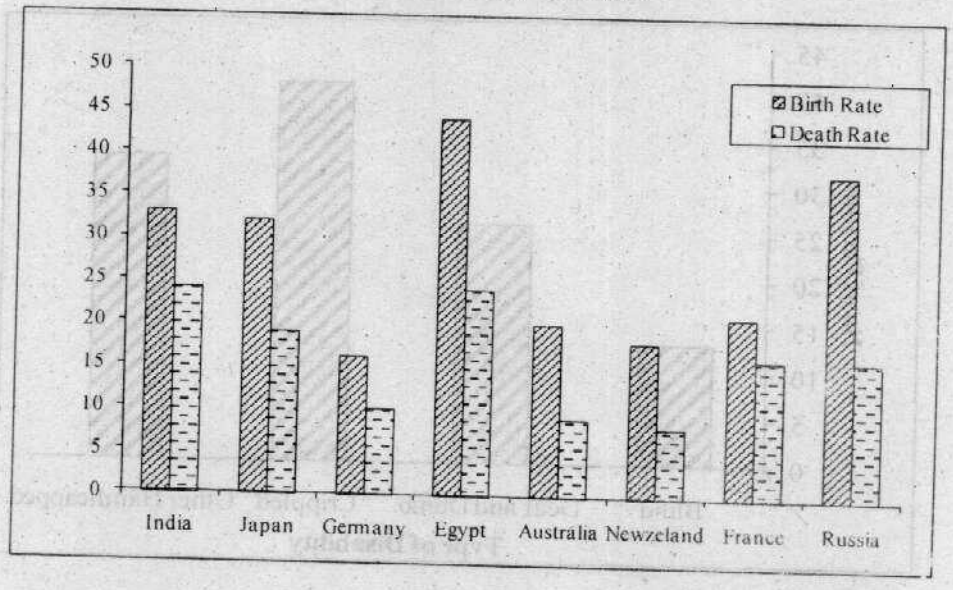
-:2.13:-

SIMPLE BAR DIAGRAM



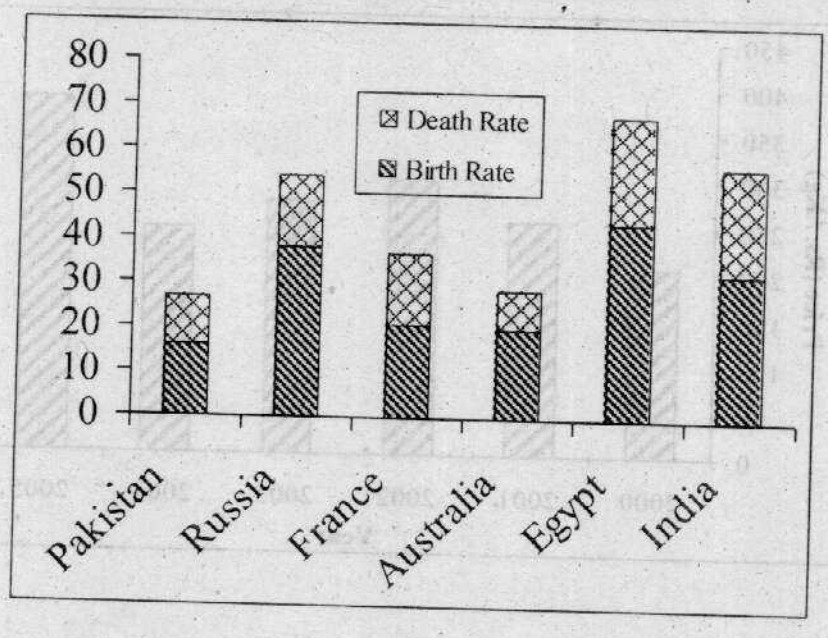
-:2.14:-

MULTIPLE DIAGRAM

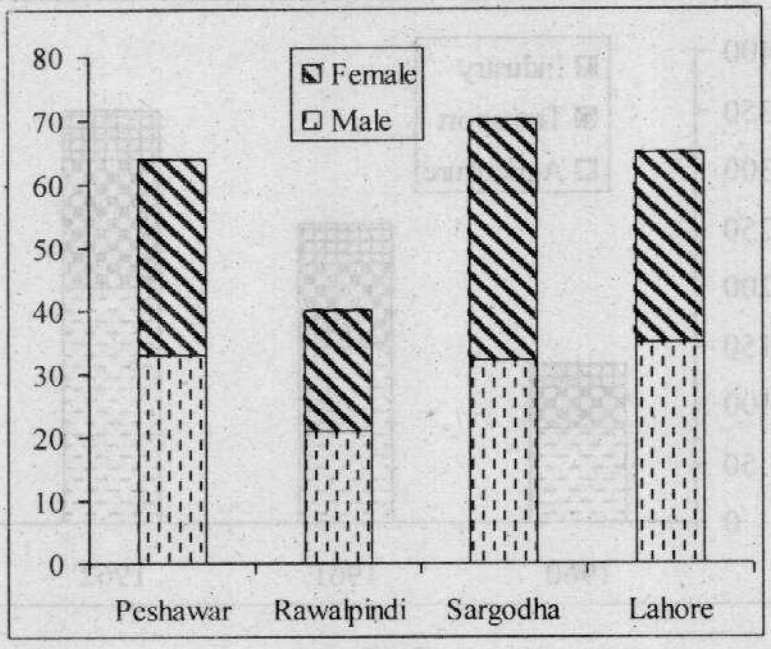


-:2.15:-

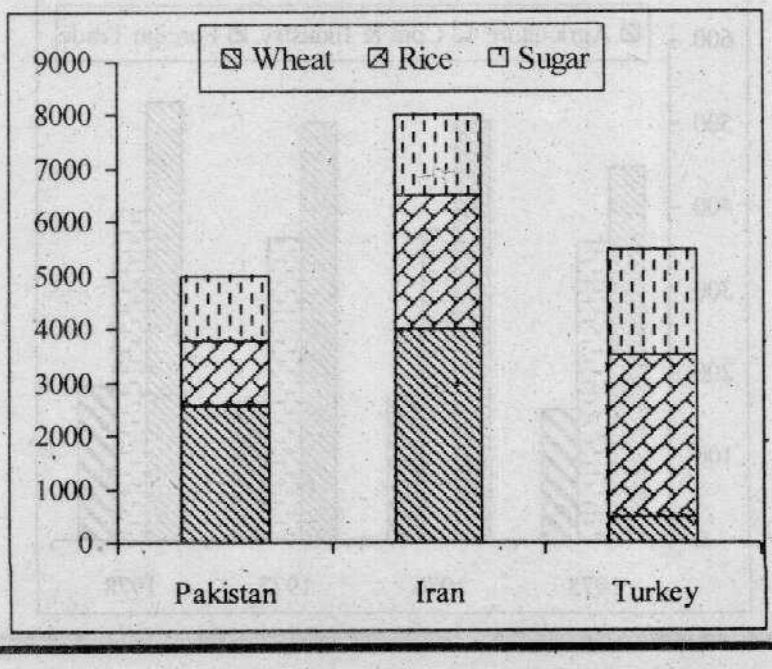
SIMPLE DIVIDED BAR DIAGRAM



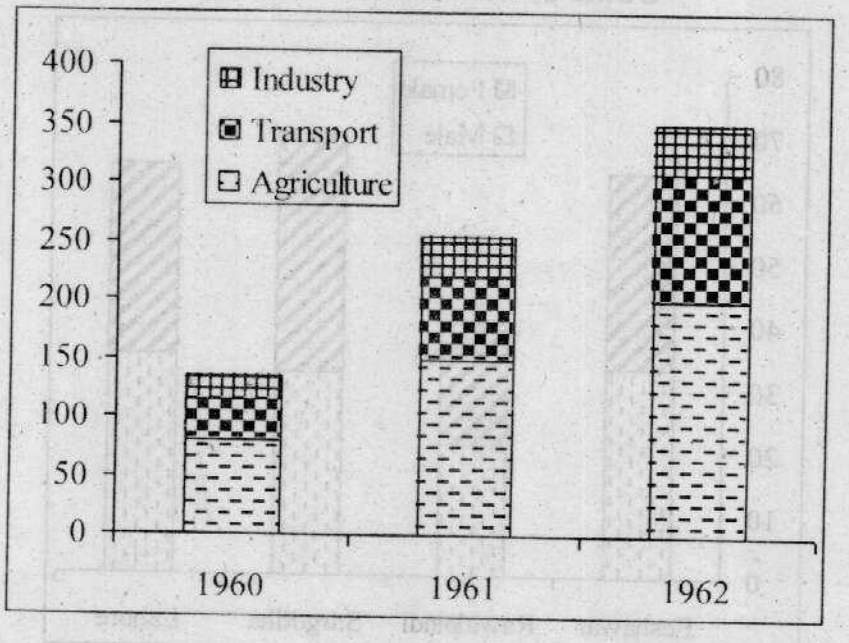
-:2.16:-
COMPONENT BAR DIAGRAM



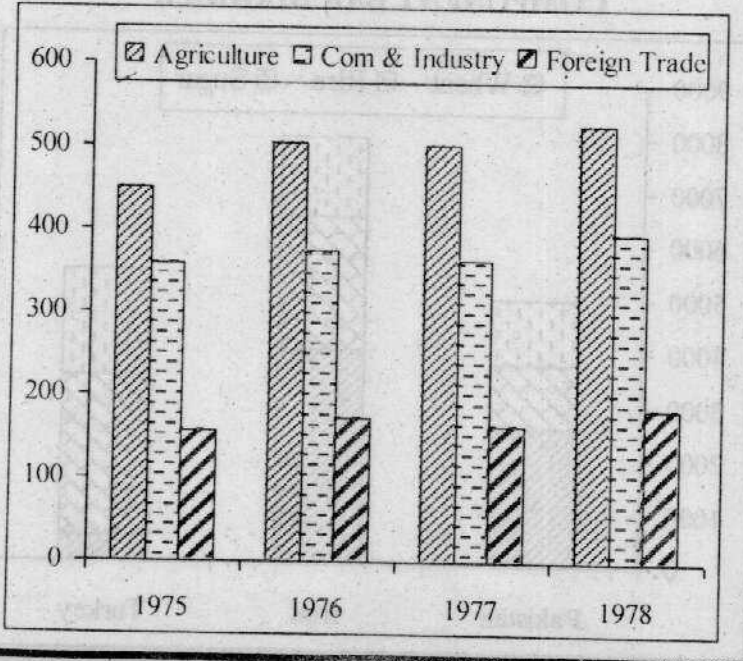
-:2.17:-
COMPONENT BAR DIAGRAM



-:2.18:-
SUB DIVIDED BAR DIAGRAM

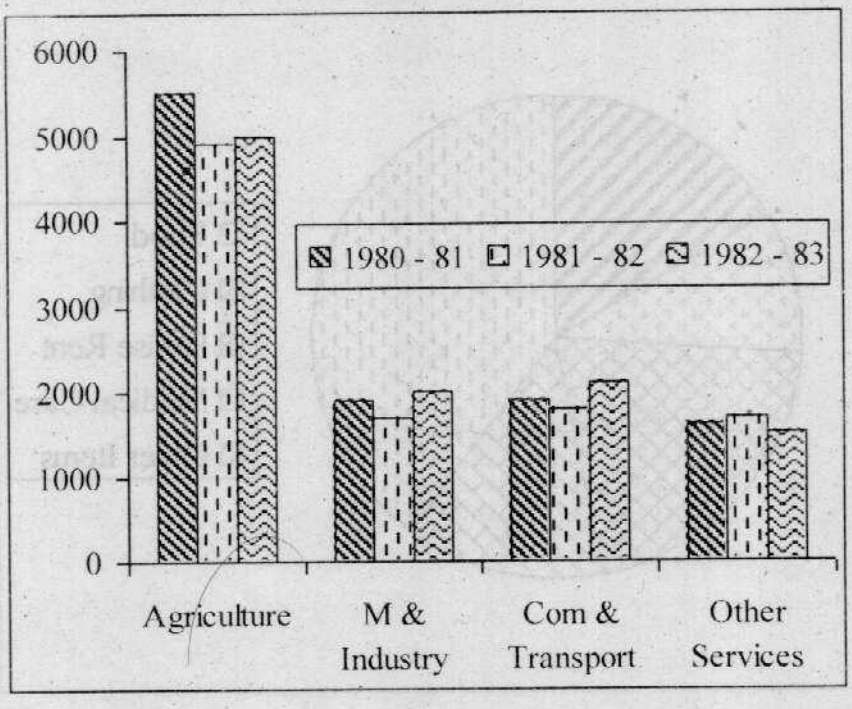


-:2.19:-
MULTIPLE BAR DIAGRAM



-:2.20:-

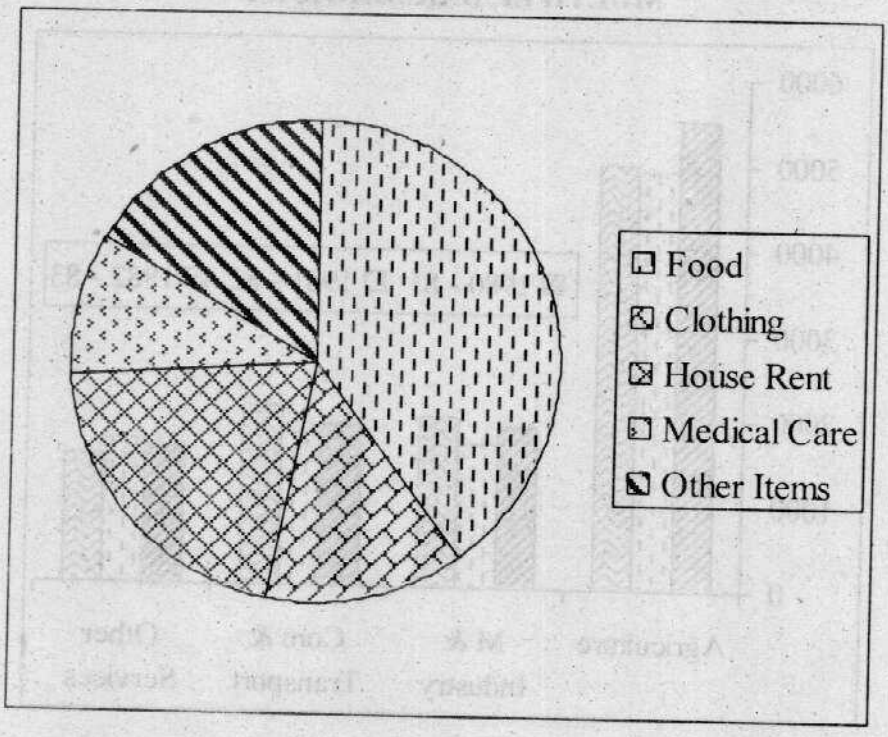
MULTIPLE BAR DIAGRAM



-:2.21:-

Items	Expenditure (Rs.)	Angles of Sector (Degrees)	Cum. Angles
Food	95	142.5	142.5
Clothing	32	48.0	190.5
House Rent	50	75.0	265.5
Medical Care	23	34.5	300.0
Other Items	40	60.0	360.0
Total	**240**	**260**	**----**

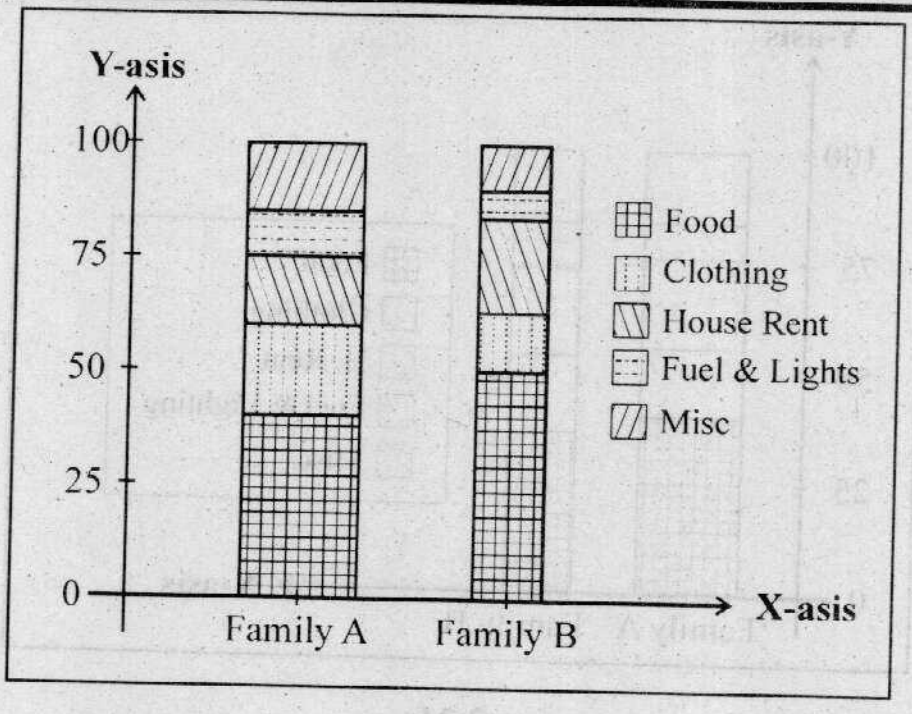
PIE DIAGRAM



-:2.22:-

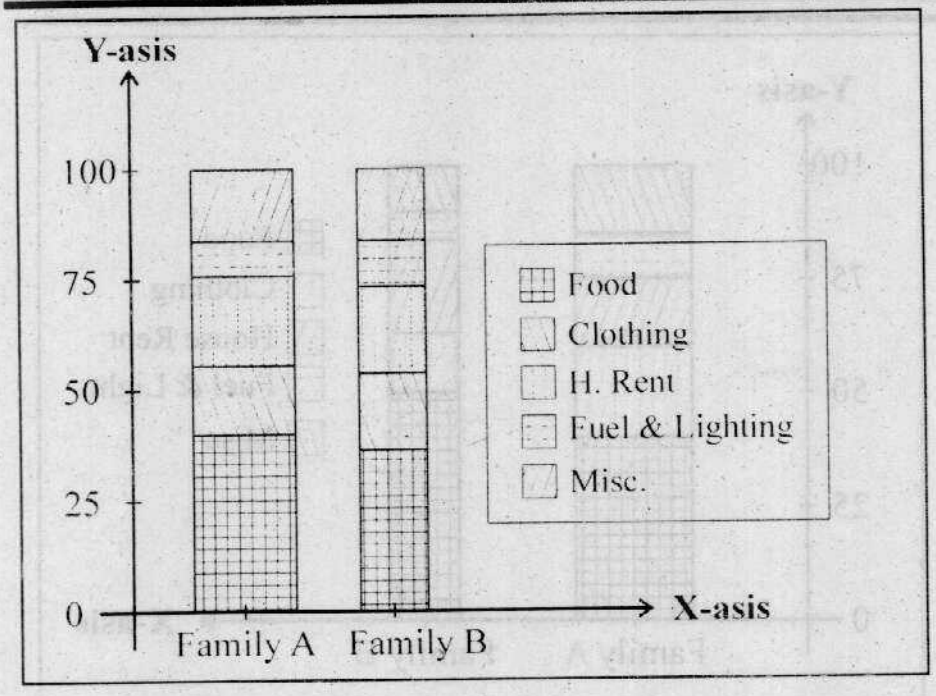
Items of Expenditure	Family A	%	Cumu-lative	Family B	%	Cumu-lative
Food	160	40	40	120	50	50
Clothing	80	20	60	32	13.33	63.33
House Rent	60	15	75	48	20	83.33
F. & Lighting	20	5	80	16	6.67	90
Miscellaneous	80	20	100	24	10	100
Total	**400**	**100**	----	**240**	**100**	----

400 : 240 → 40 : 24 → 10 : 6



-:2.23:-

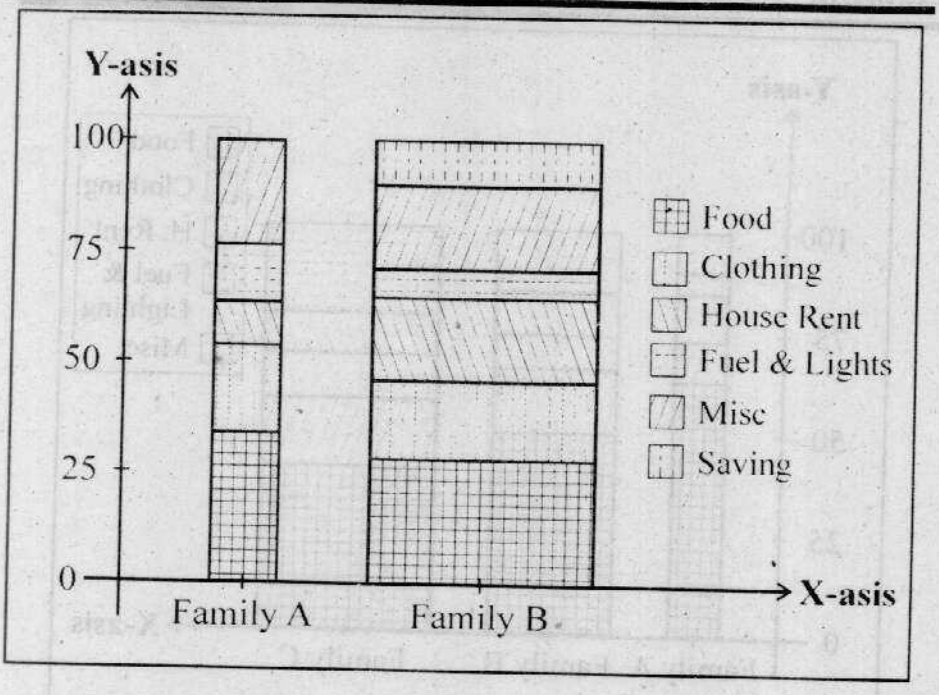
Items of Expenditure	Family A	%	Cumu-lative	Family B	%	Cumu-lative
Food	180	40	40	110	36.67	36.67
Clothing	70	15.55	55.56	50	16.67	53.34
House Rent	90	20	75.56	60	20	73.34
F. & Lighting	35	7.78	83.34	30	100	83.34
Miscellaneous	75	16.66	100	50	16.66	100
Total	**450**	**100**	----	**300**	**100**	----



-:2.24:-

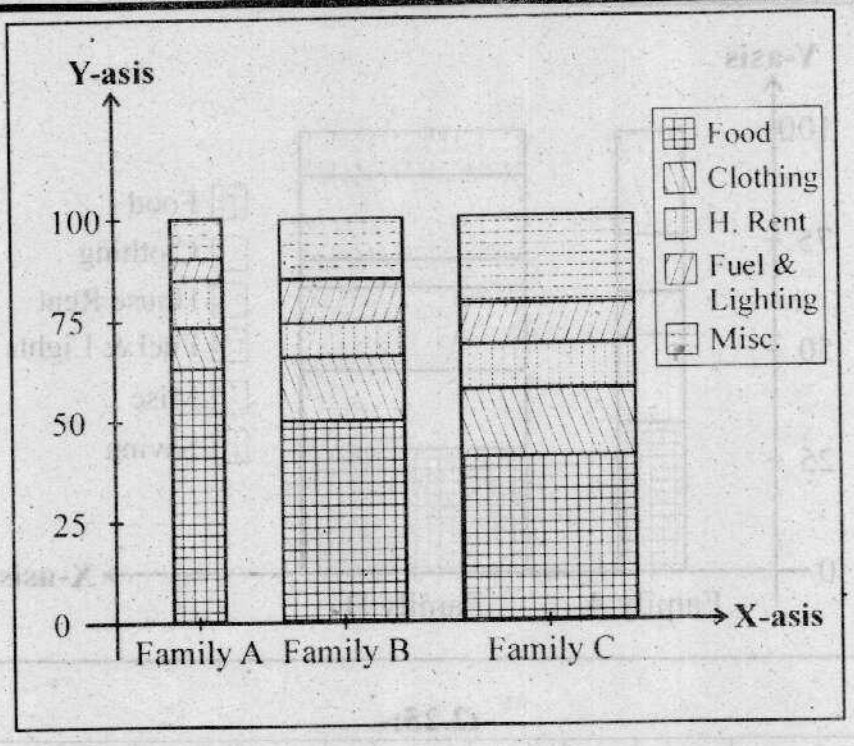
Items of Expenditure	Family A	%	Cumu-lative	Family B	%	Cumu-lative
Food	50	33.33	33.33	140	28	28
Clothing	30	20	53.33	90	18	46
House Rent	20	13.33	66.66	95	19	65
F. & Lighting	15	10	76.66	35	7	72
Miscellaneous	35	23.34	100	90	18	90
Saving	---	---	100	50	10	100
Total	**150**	**100**	----	**500**	**100**	----

150 : 500 → 15 : 50 → 3 : 10



-:2.25:-

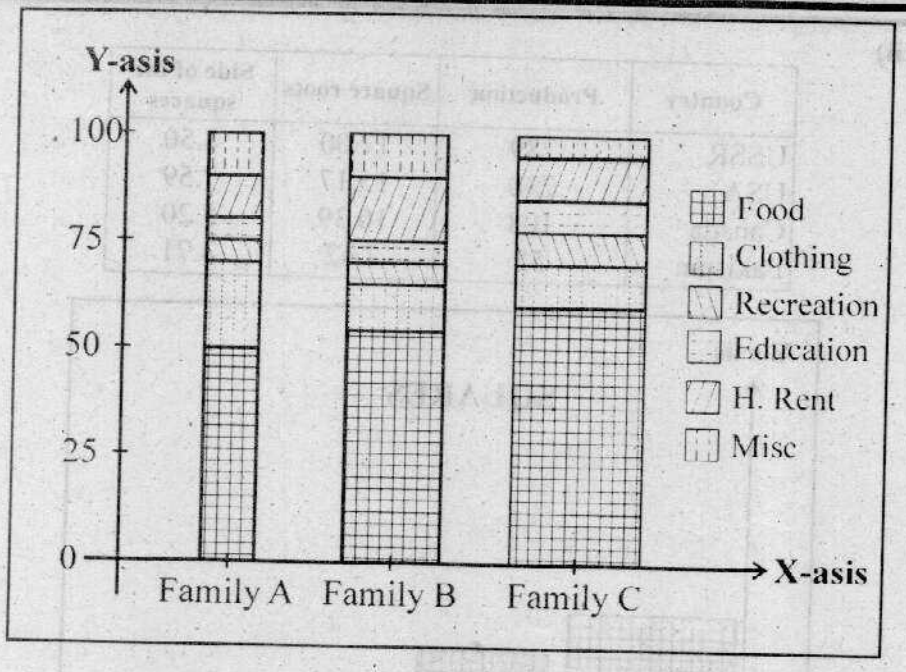
Items of Expenditure	Family A	%	Cumu-lative	Family B	%	Cumu-lative	Family C	%	Cumu-lative
Food	120	63.15	63.15	250	50	50	300	42.86	42.86
Clothing	20	10.53	73.68	80	16	66	100	14.28	57.14
House Rent	20	10.53	84.21	40	8	71	80	11.43	68.57
Education	10	5.26	89.43	50	10	84	70	10.00	78.54
Miscellaneous	20	10.53	100	80	16	100	150	21.43	100
Total	**190**	**100**	----	500	100	----	700	100	----



-:2.26:-

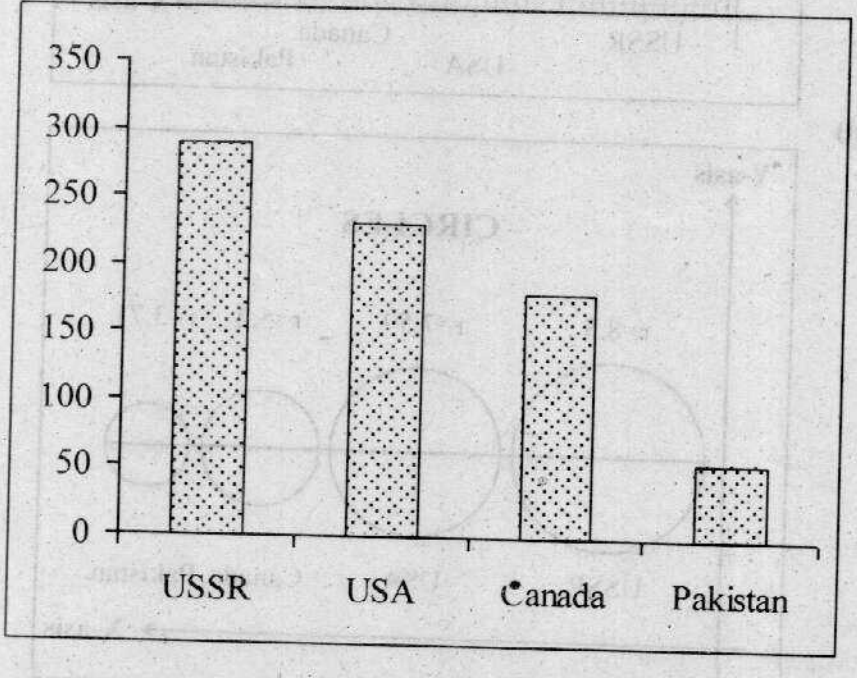
Items of Expenditure	Family A	%	Cumu-lative	Family B	%	Cumu-lative	Family C	%	Cumu-lative
Food	43	50.59	50.59	87	54.72	54.72	120	60	60
Clothing	18	21.17	71.76	17	10.69	65.41	25	12.5	72.5
Recreation	3	3.54	75.30	10	6.29	71.7	12	6	78.5
Education	5	5.88	81.18	9	5.66	77.36	15	7.5	86.0
House Rent	10	11.76	92.94	21	13.21	90.57	17	8.5	94.5
Misc	6	7.06	100	15	9.33	100	11	5.5	100
Total	**85**	**100**	----	159	100	----	200	100	----

85 : 159 : 200 → 5 : 9.35 : 12 → 5 : 9 : 12



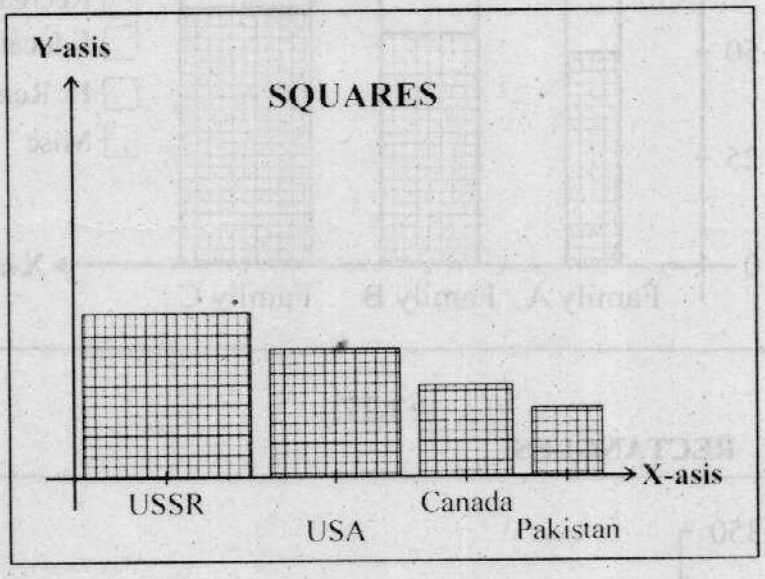
-:2.27:-

(i) **RECTANGLES**

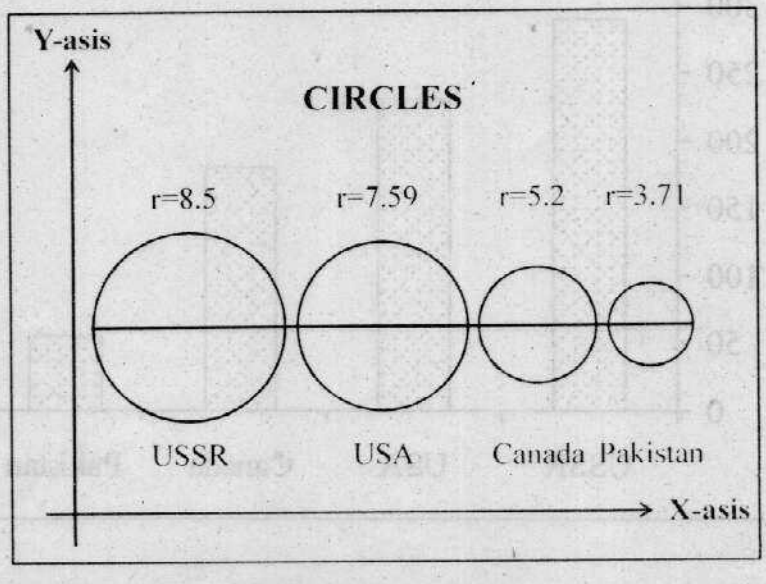


(ii)

Country	Production	Square roots	Side of the squares
USSR	289	17.00	8.50
USA	230	15.17	7.59
Canada	108	10.39	5.20
Pakistan	55	7.42	3.71



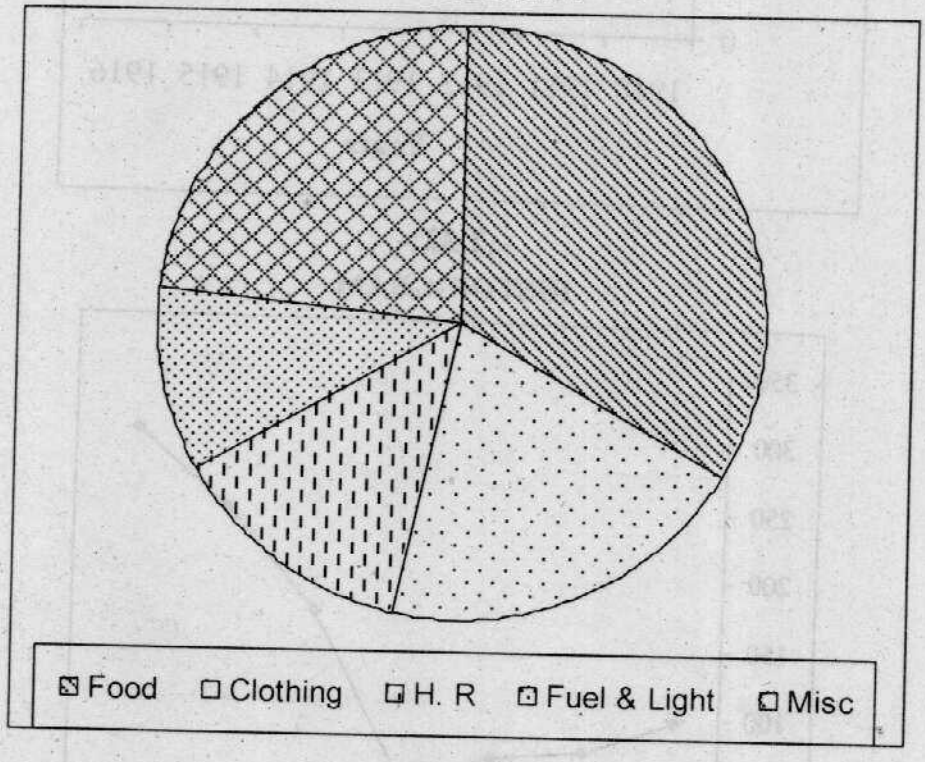
(iii)



-:2.28:-

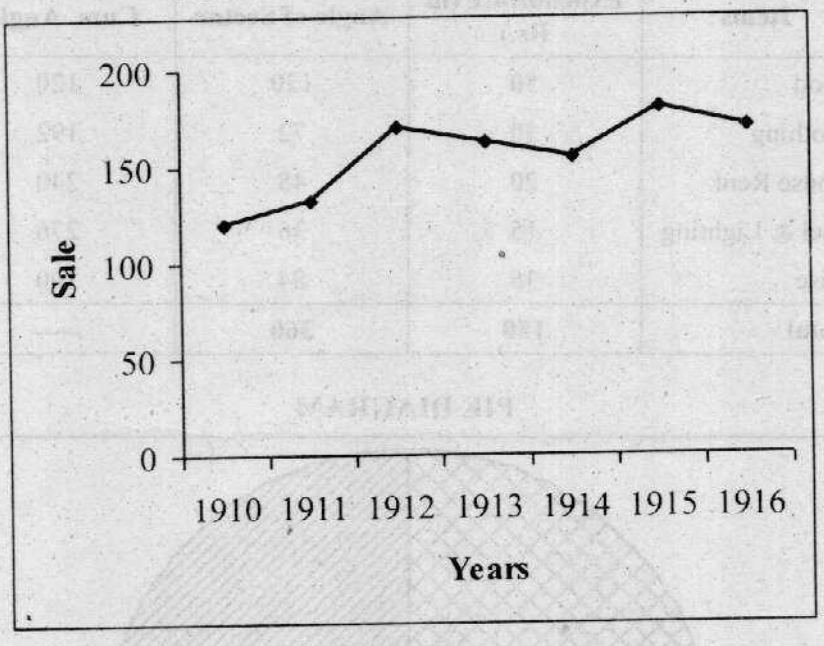
Items	Expenditure (in Rs.)	Angle of Sector	Cum. Angles
Food	50	120	120
Clothing	30	72	192
House Rent	20	48	240
Fuel & Lighting	15	36	276
Misc	35	84	360
Total	**150**	**360**	-----

PIE DIAGRAM



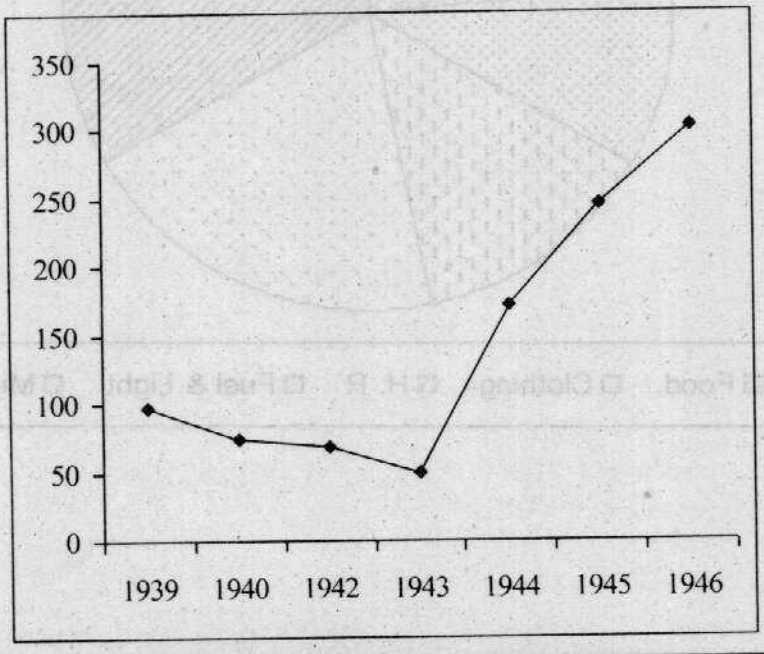
-:2.31:-

SALE OF FACTORY 1910 – 1916



-:2.32:-

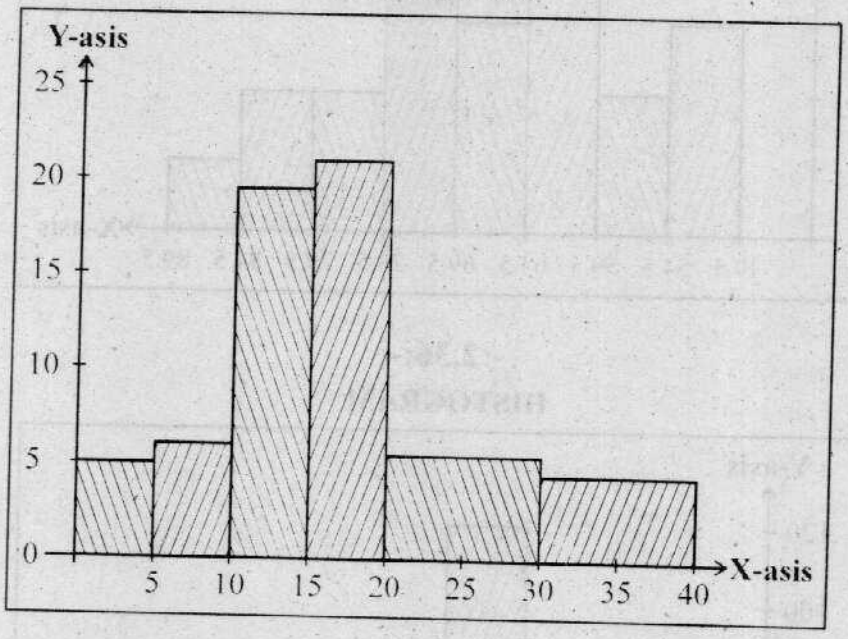
HISTORIGRAM



-:2.34:-

Group	Frequency	Length of Class Interval	Adjusted Height of Rectangle
0 – 5	5	5	$^5/_1 = 5$
5 – 10	7	5	$^7/_1 = 7$
10 – 15	19	5	$^{19}/_1 = 19$
15 – 20	22	5	$^{22}/_1 = 22$
20 – 30	12	10	$^{12}/_2 = 6$
30 – 40	8	10	$^8/_2 = 4$

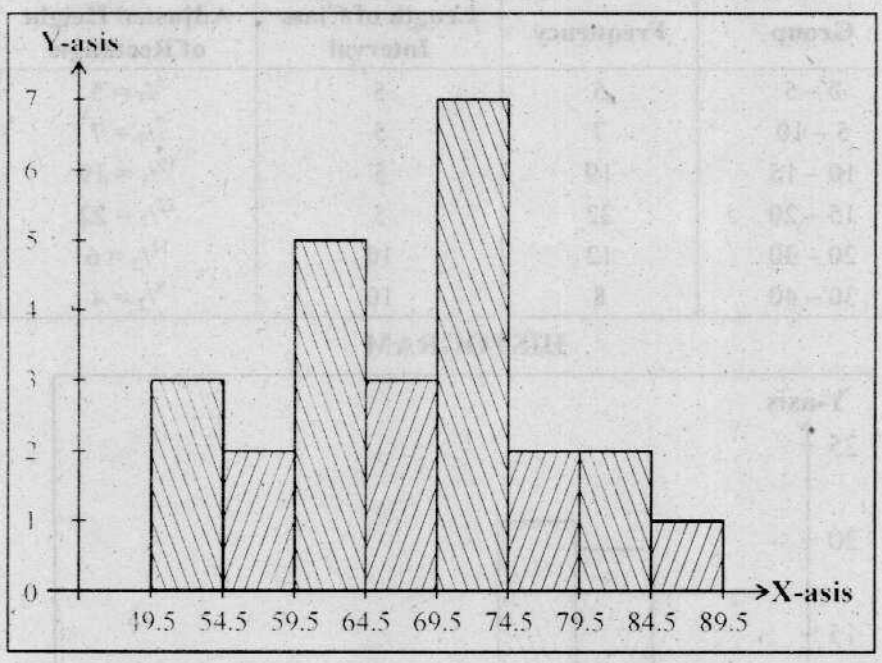
HISTOGRAM



-:2.35:-

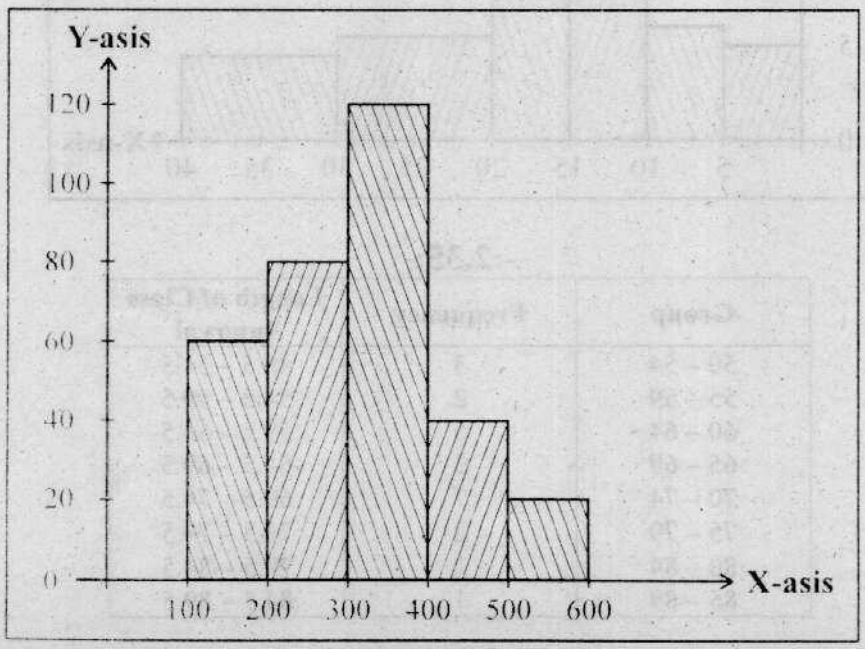
Group	Frequency	Length of Class Interval
50 – 54	3	49.5 – 54.5
55 – 59	2	54.5 – 59.5
60 – 64	5	59.5 – 64.5
65 – 69	3	64.5 – 69.5
70 – 74	7	69.5 – 74.5
75 – 79	2	74.5 – 79.5
80 – 84	2	79.5 – 84.5
85 – 89	1	84.5 – 89.5

HISTOGRAM



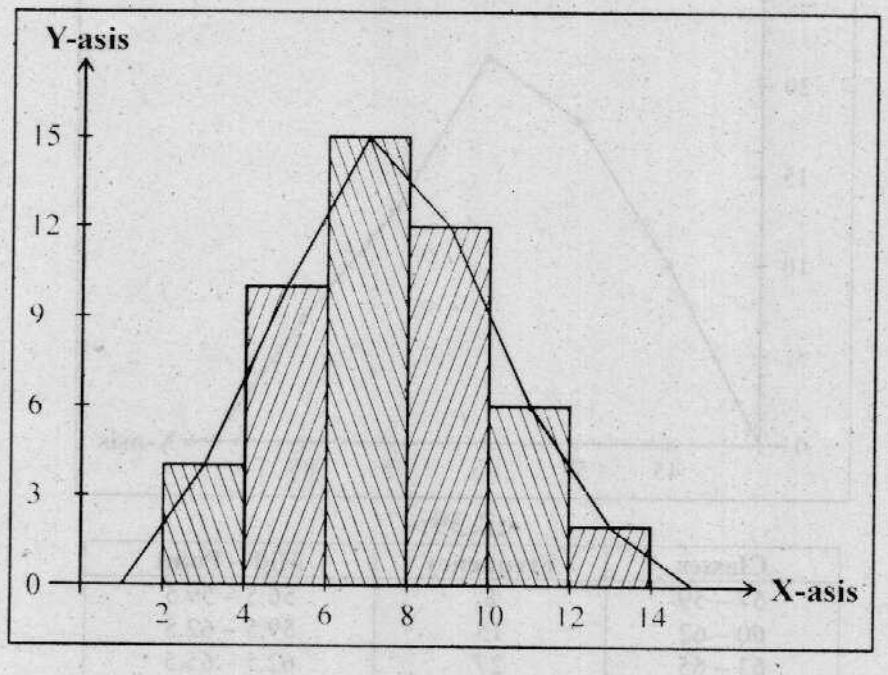
-:2.36:-

HISTOGRAM



-:2.37:-

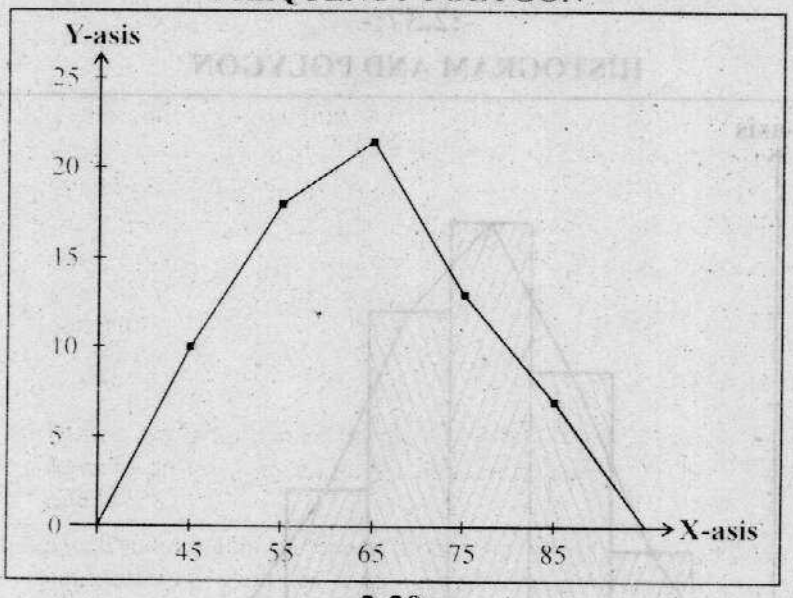
HISTOGRAM AND POLYGON



-:2.38:-

Classes	Frequency	Mid – Point
40 – 50	10	45
50 – 60	18	55
60 – 70	22	65
70 – 80	13	75
80 – 90	7	85

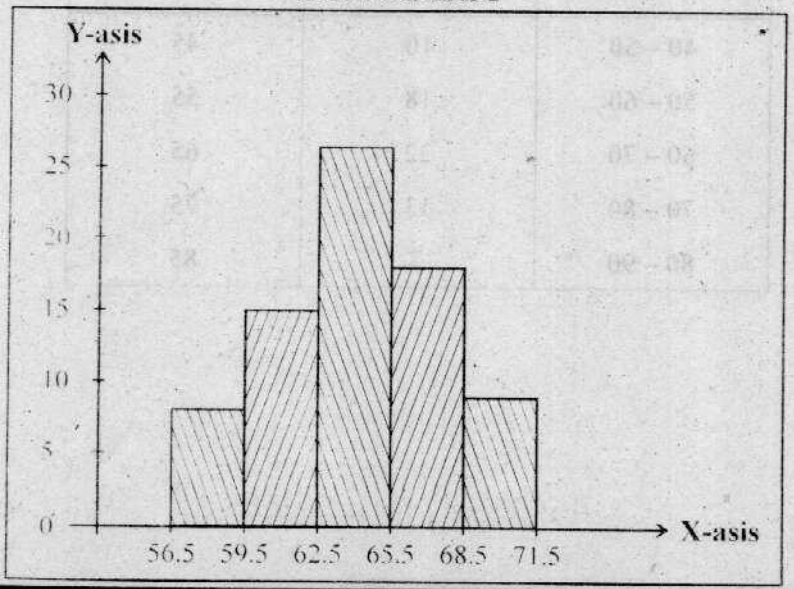
FREQUENCY POLYGON



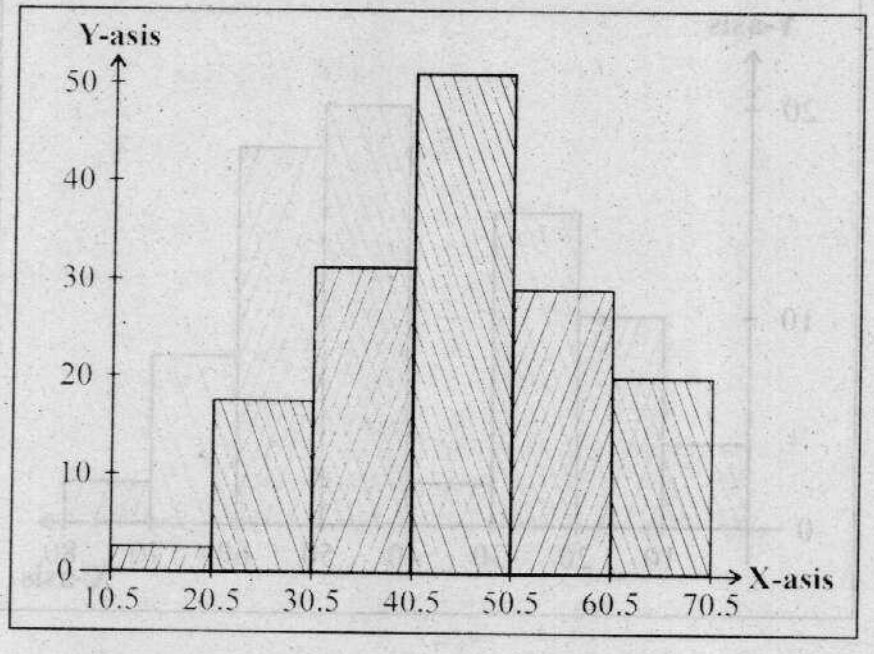
-:2.39:-

Classes	Frequency	Mid – Point
57 – 59	8	56.5 – 59.5
60 – 62	15	59.5 – 62.5
63 – 65	27	62.5 – 65.5
66 – 68	18	65.5 – 68.5
69 – 71	9	68.5 – 71.5

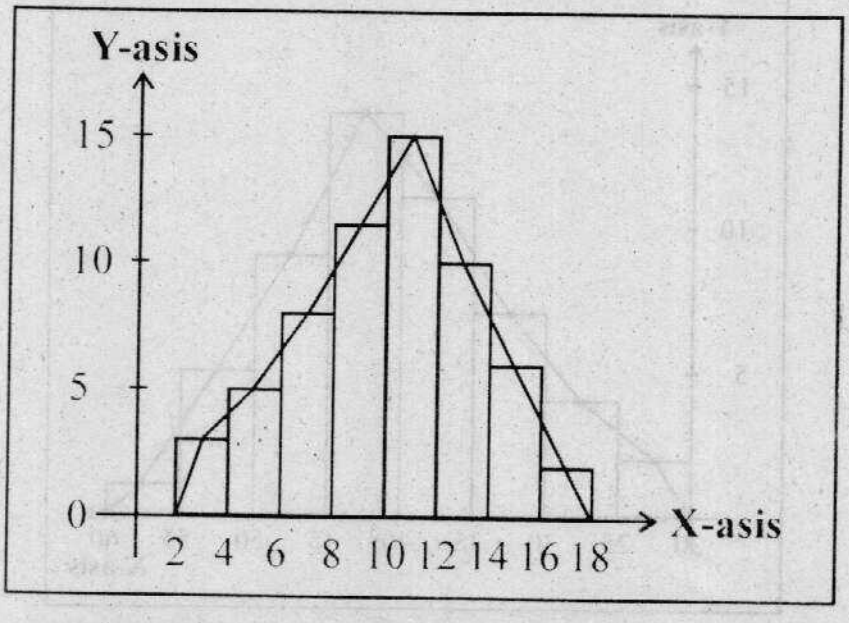
HISTOGRAM



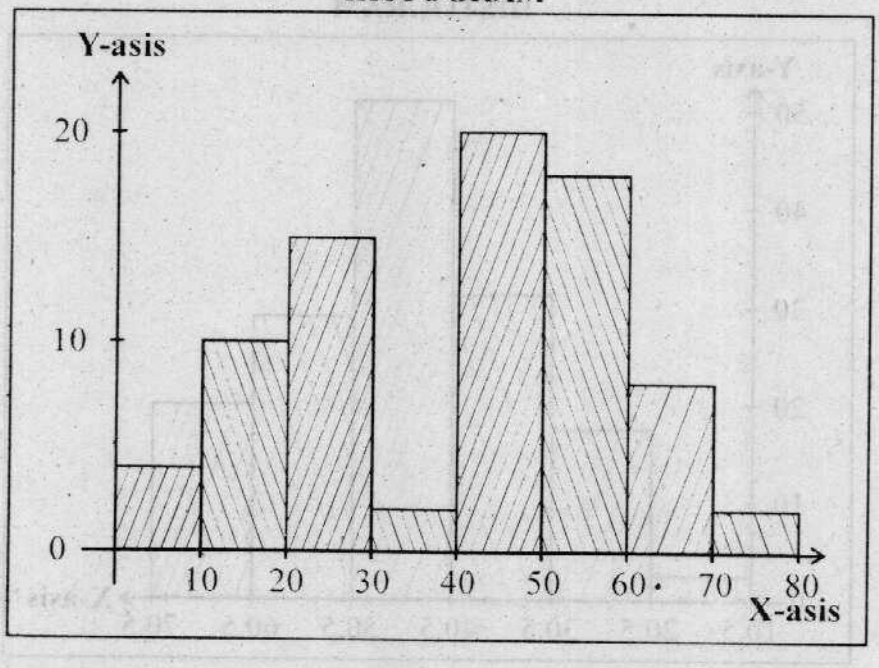
-:2.40:-
HISTOGRAM



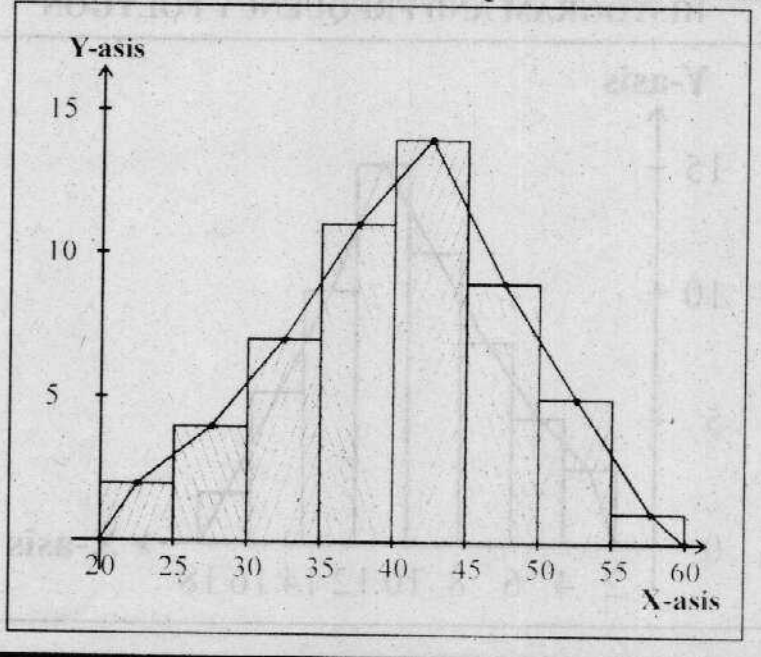
-:2.41:-
HISTOGRAM AND FREQUENCY POLYGON



-:2.42:-
HISTOGRAM



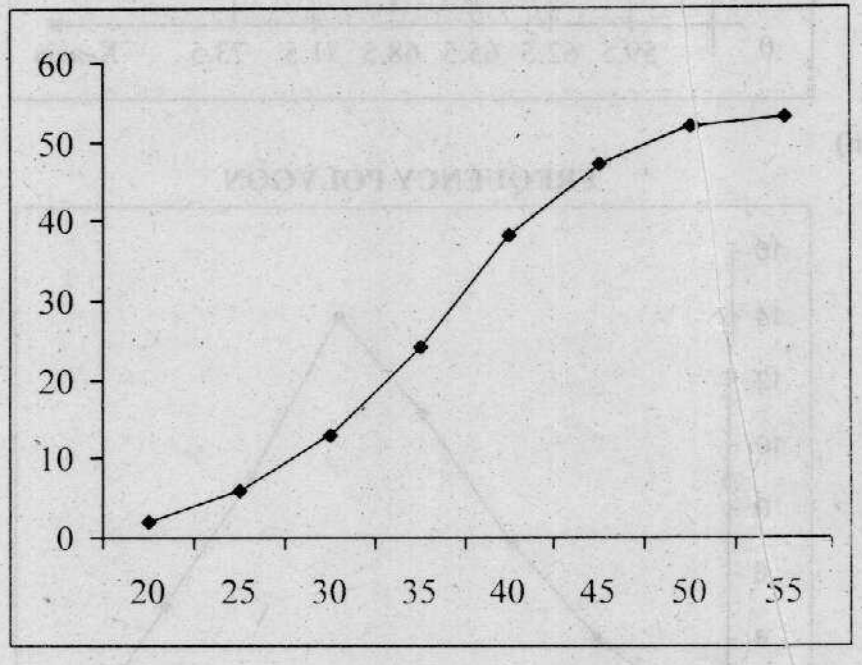
-:2.43:-
(i) & (ii) HISTOGRAM AND FREQUENCY POLYGON



(iii)

Weekly Wages	No of Persons (F)	C.F.
20 – 25	2	2
25 – 30	4	6
30 – 35	7	13
35 – 40	11	24
40 – 45	14	38
45 – 50	9	47
50 – 55	5	52
55 – 60	1	53

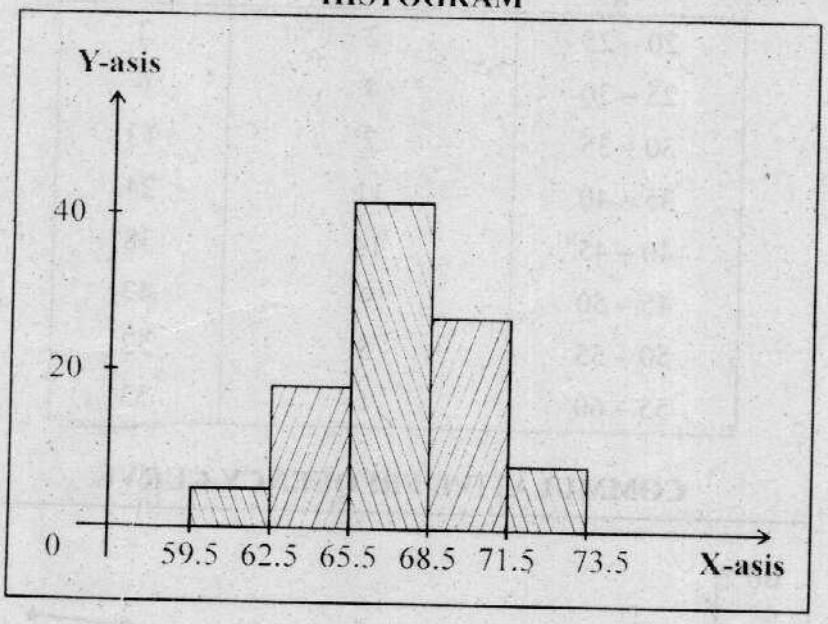
COMMULATIVE FREQUENCY CURVE



-:2.44:-

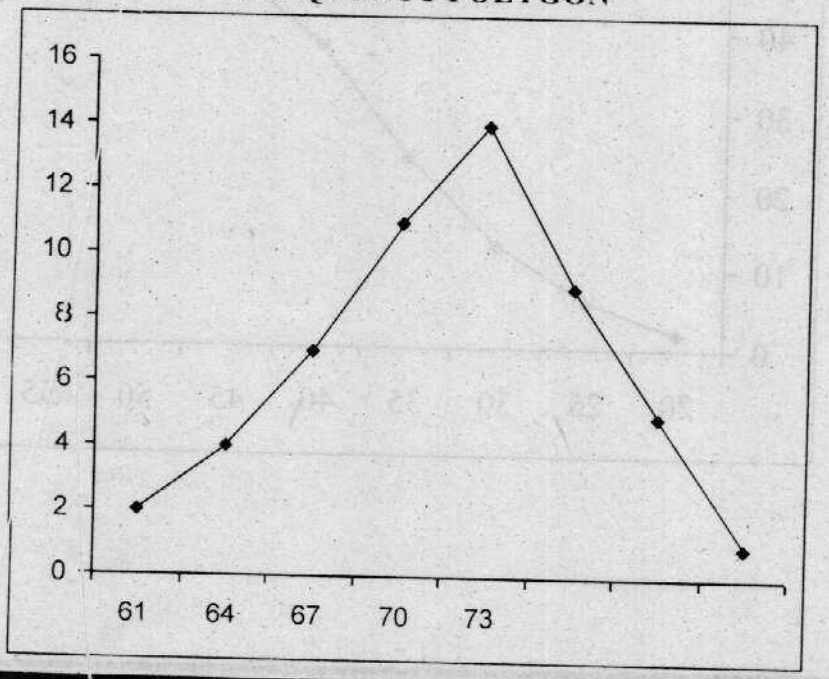
(i)

HISTOGRAM

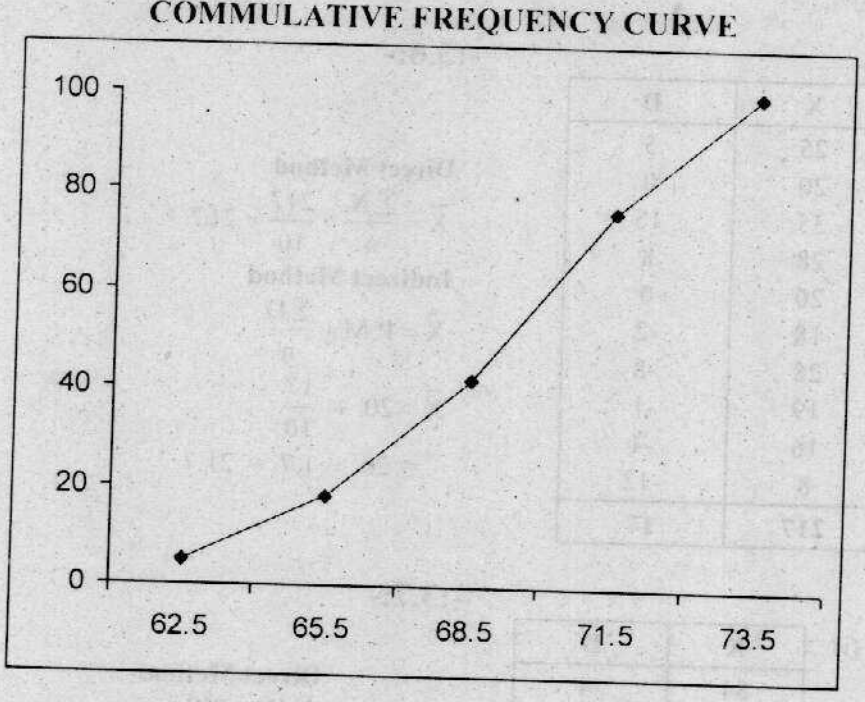


(ii)

FREQUENCY POLYGON



(iii)



EXERCISE NO. 3

-:3.6:-

X	D
25	5
20	0
35	15
28	8
20	0
18	-2
28	-8
19	-1
16	-4
8	-12
217	**17**

Direct Method

$$\bar{X} = \frac{\Sigma X}{n} = \frac{217}{10} = 21.7$$

Indirect Method

$$\bar{X} = P.M + \frac{\Sigma D}{n}$$

$$\bar{X} = 20 + \frac{17}{10}$$

$$= 20 + 1.7 = 21.7$$

-:3.7:-

(i)

X	D
84	-4
92	4
73	-15
67	-21
88	0
74	-14
91	3
16	-72
74	-14
659	**-133**

Direct Method

$$\bar{X} = \frac{\Sigma X}{n} - \frac{659}{9} = 73.22$$

Indirect Method

$$\bar{X} - P.M + \frac{\Sigma D}{n}$$

$$\bar{X} - 88 + \frac{-133}{9}$$

$$= 88 + 14.78 - 73.22$$

(ii)

X
3.41
2.86
-1.84
2.31
-3.80
2.14
0.87
5.95

$$\bar{X} = \frac{\Sigma x}{n}$$

$$\bar{X} = \frac{5.95}{7}$$

$$= 0.85$$

(iii)

X	D
13	3
13	1
16	4
15	3
14	2
15	3
12	0
18	6
16	4
15	3
14	2
18	6
13	1
14	2
15	3
223	**43**

Direct Method

$$\overline{X} = \frac{\sum X}{n} = \frac{223}{15}$$

$$= 14.87$$

Indirect Method

$$\overline{X} = P.M = \frac{\sum D}{n}$$

$$\overline{X} = 12 + \frac{43}{15}$$

$$= 12 + 2.87 = 14.87$$

(iv)

X	D
53	9
46	2
50	6
49	5
52	8
53	9
44	0
55	11
402	**50**

Direct Method

$$\overline{X} = \frac{\sum X}{n} = \frac{402}{8} = 50.25$$

Indirect Method

$$\overline{X} = P.M = \frac{\sum D}{n}$$

$$\overline{X} = 44 + \frac{50}{8} = 44 + 6.25 = 50.25$$

-:3.8:-

X				
216	172	196	180	188
208	203	184	269	198
215	202	194	206	206
189	234	209	178	191
204	195	207	168	206
215	207	217	176	178
165	258	222	168	192
193	234	192	182	174
181	212	192	204	
				8780

$$\bar{X} = \frac{\sum X}{n} = \frac{8760}{44} = 199.5$$

-:3.9:-

Roll No.	History	Statistics	Economics
1	41	46	50
2	35	50	52
3	38	39	41
4	34	50	46
5	30	38	39
Total	**178**	**223**	**228**

$$\bar{X}\text{ (History)} = \frac{\sum X}{n} = \frac{178}{5} = 35.6,$$

$$\bar{X}\text{ (Statistics)} = \frac{\sum X}{n} = \frac{223}{5} = 44.6$$

$$\bar{X}\text{ (Economics)} = \frac{\sum X}{n} = \frac{228}{5} = 45.6$$

Highest knowledge in Economics.

-:3.10:-

(i)

X	D
2000	-350
2100	-250
2200	-150
2300	-50
2400	50
2500	150
2600	250
2700	350
-----	**0**

Here P. M. = 2350

$$\bar{X} = \text{P.M.} + \frac{\sum D}{n}$$

$$= 2350 + \frac{0}{8} = 2350 + 0$$

$$= 2350$$

(ii)

X	D
160	-2
180	18
168	6
172	10
150	-12
155	-7
-----	**13**

Here assumed mean = 162

$$\bar{X} = \text{P.M.} + \frac{\sum D}{n}$$

$$= 162 + \frac{13}{6} = 162 + 2.17$$

$$= 164.17$$

(iii) Here the deviation from X = 300 of 10 values are given as:

D	X = D + P.M.
-140	160
-113	187
-71	229
-55	245
-55	245
0	300
5	305
25	325
75	375
654	954
325	**3325**

By Short Cut Method

$$\bar{X} = P.M. + \frac{\Sigma D}{n}$$

$$= 300 + \frac{325}{10} = 300 + 32.5 = 332.5$$

By Direct Method

$$\bar{X} = \frac{\Sigma X}{n} = \frac{3325}{10} = 332.5$$

(iv) Deviation of 10 values form X = 20 are given

D	X = D + P.M
-5	-5 + 20 = 15
-8	-8 + 20 = 12
-11	-11 + 20 = 9
-5	-5 + 20 = 15
-4	-4 + 20 = 16
-2	-2 + 20 = 18
0	0 + 20 = 20
5	5 + 20 = 25
0	0 + 20 = 20
10	10 + 20 = 30
-----	**180**

By Direct Method

$$\bar{X} = \frac{\Sigma X}{n} = \frac{180}{10} = 18$$

(v) Here P.M. = 10; $\Sigma D = 70$ and $n = 8$

Hence

$$\bar{X} = P.M. + \frac{\Sigma D}{n} = 110 + \frac{70}{8} = 110 + 8.75 = 118.75$$

(vi) Mean of 10 Values

$$\bar{X} = \frac{\Sigma X}{n} = 10 \Rightarrow \frac{\Sigma X}{10} = 10 \Rightarrow \Sigma X = 10 \times 10 = 100$$

Sum of 10 Values = 100

$$\text{Mean of 11 Values} = \frac{\Sigma X}{11} = 10$$

Sum of 11 Values= 10 x 11 = 110

Hence
Eleventh Value is: Sum of 11 Value – Sum of 10 Values
= 110 – 100 = 10

(vii) Average Fine = $\bar{x}$ = Rs. 65
No of Students = n = 8
Sum of the Fine of 8 Students = $\sum x = n\bar{x} = 8 \times 65$ = Rs. 520
Sum of 7 Student's Fine = 30 + 80 + 39 + 100 + 27 + 80 + 64
= Rs. 420
Fine paid by 8th Student = Rs. 520 – Rs. 420 = Rs. 100

(viii) Here $\bar{x}$ = 15, n = 20

$$\frac{\sum X}{n} = 15 \Rightarrow \frac{\sum X}{20} = 15$$

ΣX = Sum of 20 Value = 15 x 20 = 300
Again $\bar{x}$ = 20, n = 21
ΣX = Sum of 21 Value = 20 x 21 = 420
Hence Added Value = 420 – 300 = 120

-:3.11:-

X			
5	17	51	66
7	16	71	63
9	19	81	52
16	28	90	42
13	33	100	44
11	25	99	33
10	24	78	37
8	21	76	40
			1288

$$\bar{X} = \frac{\sum X}{n} = \frac{1288}{32} = 40.25$$

Arranged values are as:
5, 7, 8, 9, 10, 11, 13, 16, 16, 17, 19, 21, 24, 25, 28, 33, 33, 37, 40, 42, 47, 51, 52, 63, 66, 71, 76, 78, 81, 90, 99, 100

Here n = 32 (Even No.)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2}\text{th item} + \text{The value of } \frac{n+2}{2}\text{th item}\right]$$

$$\text{Med} = \frac{1}{2}[\text{The value of } \frac{32}{2} \text{th item} + \text{The value of } \frac{32+2}{2} \text{th item}]$$

$$\text{Med} = \frac{1}{2}[\text{The value of 16th item} + \text{The value of 17th item}]$$

$$\text{Med} = \frac{1}{2}[33 + 33] = 33$$

Q_1 = The value of $\frac{n+1}{4}$ th item = The value of $\frac{32+1}{4}$ th item.

= The value of 8.25th item

= The value of 8th item + 0.25 [9the item - 8th item]

= 16 + 0.25 [16 - 16] = 16 + 0.25(0) = 16 + 0 = 16

Q_3 = The value of $\frac{3(n+1)}{4}$ th item = The value of $\frac{3(32+1)}{4}$ th item.

= The value of 24.75th item.

= The value of 24th item + 0.75 [25th item - 24th item]

= 63 + 0.75 [66 - 63] = 63 + 0.75(3) = 63 + 2.25 = 65.25

D_9 = The value of $\frac{9(n+1)}{10}$ th item = The value of $\frac{9(32+1)}{10}$ th item

=Tthe value of 29.7th item.

= The value 29th item +0.7 [30th item -29th] item]

= 81 + 0.7 [90 -81] = 81 + 0.7(9) = 81 + 6.3 = 87.3

P_{55} = The value of $\frac{55(n+1)}{100}$ th item = The value of $\frac{55(32+1)}{100}$ th item

= The value of 18.15th item

= The value of 18th item + 0.15 [19th item -18th item]

= 37 + 0.15 [40 -37] = 37 + 0.15(3) = 37 + 0.45 = 37.45

Hence Mean = 40.25, Median = 33

$Q_1 = 16$, $Q_3 = 65.25$, $D_9 = 87.3$, $P_{55} = 37.45$

-:3.12:-

Classes	No of Students	X	fx	D	fD
0 – 10	4	5	20	-20	-80
10 – 20	8	15	120	-10	-80
20 – 30	16	25	400	0	0
30 – 40	7	35	245	10	70
40 – 50	5	45	225	20	100
-----	**40**	-----	**1010**	-----	**10**

By Direct Method

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{1010}{40} = 25.25$$

By Short Cut Method

$$\bar{X} = \text{P.M.} + \frac{\sum fD}{\sum f} = 25 + \frac{10}{40} = 25 + 0.25 = 25.25$$

-:3.13:-

Age (years)	No of Cows	x	fx
1 – 4	3	2.5	7.5
5 – 8	7	6.5	45.5
9 – 12	10	10.5	105.0
13 – 16	8	14.5	116.0
17 – 20	2	18.5	37.0
-----	**30**	-----	**311.0**

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{311}{30} = 10.37$$

-:3.14:-

Marks	No of Students	x	fx
3 – 5	8	4	32
5 – 7	12	6	72
7 – 9	20	8	160
9 – 11	9	10	90
11 – 13	1	12	12
-----	**50**	-----	**366**

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{366}{50} = 7.32$$

-:3.15:-

C-1	f	X	fX
1-6	7	3	21
6-10	10	8	80
11-15	16	13	208
16-20	32	18	576
21-25	24	23	552
26-30	18	28	504
31-35	10	33	330
36-40	5	38	190
41-45	1	43	43
-----	**123**	-----	**2504**

D	fD	U	fU
-20	-140	-4	-28
-15	-150	-3	-30
-10	-160	-2	-32
-5	-160	1	-32
0	0	0	0
5	90	1	18
10	100	2	20
15	75	3	15
20	20	4	4
-----	**-325**	-----	**-65**

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{2504}{123} = 20.36$$

$$\bar{X} = P.M + \frac{\sum fD}{\sum f} \text{ (indirect Method)}$$

$$\bar{X} = 23 + \frac{-325}{123} = 23 - 2.64 = 20.36$$

$$\bar{X} = P.M + \frac{\sum fu}{\sum f} \times h \text{ (Step Deviation)}$$

$$\bar{X} = 23 + \frac{-65}{123} \times 5 = 23 + \frac{325}{123} = 20.36$$

-:3.16:-

Scores	F	X
10-24	10	17
25-39	14	32
40-54	23	47
55-69	29	62
70-84	26	77
85-99	16	92
100-114	13	107
115-129	7	122
130-144	1	137
145-159	4	152
-----	**143**	-----

D	U	fU	fX
-60	-4	-40	170
-45	-3	-42	448
-30	-2	-46	1081
-15	-1	-29	1789
0	0	0	2002
15	1	16	1472
30	2	26	1391
45	3	21	854
60	4	4	137
75	5	20	608
-----	-----	**-70**	**9961**

$$\bar{X} = \frac{\sum fx}{\sum f} \text{ (Direct Method)} = \frac{9961}{143} = 69.66. \text{ OR}$$

$$\bar{X} = PM + \frac{\sum fu}{\sum f} \times h \text{ (Step Deviation)} = 77 + \frac{-70}{143} \times 15$$

$$\bar{X} = 77 - \frac{1050}{143} = 77 - 7.34 = 69.66$$

-:3.17:-

Scores	F	X
0-5	4	2.5
5-10	6	7.5
10-15	10	12.5
15-20	16	17.5
20.25	12	22.5
25.30	8	27.5
30.35	4	32.5
-----	**60**	-----

fX	D	U	fX
10	-15	-3	-12
45	-10	-2	-12
125	-5	-1	-10
280	0	0	0
270	5	1	12
220	10	2	16
130	15	3	12
1080	-----	-----	**6**

$$\bar{X} = \frac{\sum fx}{\sum f} \text{ (Direct Method)} = \frac{1080}{143} = 18$$

$$\bar{X} = PM + \frac{\sum fU}{\sum f} \times h \text{ (Step Diviation)}$$

$$\bar{X} = 17.5 + \frac{6}{60} \times 5 = 17.5 + \frac{30}{60} = 17.5 + 0.5 = 18$$

-:3.18:-

C-!	f	X	fX
0-5	5	2.5	12.5
5-15	15	10.0	150.0
15.40	33	27.5	907.5
40.90	65	65.0	4225.0
90-100	76	95.	7220.0
100-130	69	115.0	7935.0
130-150	49	140.0	6861.0
150-200	35	175.0	6125.0
-----	**347**	-----	**33435**

$$A.M = \bar{X} = \frac{\sum fx}{\sum f} = \frac{33435}{347} = 96.354$$

-:3.19:-

(a) **Arranged items are given below.**

21, 35, 46, 49, 50, 54, 57, 58, 59, 63, 64, 67, 68, 70, 71, 72, 73, 76, 77, 80, 84, 84, 89, 93, 98

Number of values = 25 (Odd No.)

$$\text{Med} = \text{The value of } \left(\frac{n+1}{2}\right) \text{th item} = \text{The value of } \left(\frac{25+1}{2}\right) \text{th item}$$

Med = The value of 13th item = 68

(b) (i) $$\bar{X} = \frac{\sum X}{n} = \frac{3+5+2+6+5+9+5+2+8+6}{10} = \frac{51}{10} = 5.1$$

Arrenged values are

2, 2, 3, 5, 5, 5, 6, 6, 8, 9. n = 10 (Even No.)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2} \text{th item} + \text{The value of } \frac{n+2}{2} \text{th item}\right]$$

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{10}{2} \text{th item} + \text{The value of } \frac{10+2}{2} \text{th item}\right]$$

$$\text{Med} = \frac{1}{2} \text{ The value of 5th item + The value of 6th item}$$

$$\text{Med} = \frac{1}{2} \quad 5+5 = 5$$

(ii) $$\bar{X} = \frac{\Sigma X}{n} = \frac{51.6+48.7+50.3+48.9}{4} = \frac{199.5}{4} = 49.875$$

Arranged value are

48.7, 48.9, 50.3, 51.6. n = 4 (Even No.)

$$\text{Med} = \frac{1}{2} \left[\text{The value of } \frac{n}{2} \text{th item + The value of } \frac{n+2}{2} \text{th item}\right]$$

$$\text{Med} = \frac{1}{2} \left[\text{The value of } \frac{4}{2} \text{th item + The value of } \frac{4+2}{2} \text{th item}\right]$$

$$\text{Med} = \frac{1}{2} \text{ [The value of 2th item + The value of 3rd item]}$$

$$\text{Med} = \frac{1}{2} [48.9 + 50.31] = \frac{99.2}{2} = 49.6$$

-:3.20:-

(i) **19, 22, 18, 20, 49, 52, 50, 48, 41**

Arrange the values in ascending and descending order

18, 19, 20, 22, 41, 48, 49, 50, 52

Here n = 9

$$Q_1 = \text{The value of } \left(\frac{n+1}{4}\right) \text{th item}$$

$$= \text{The value of } \left(\frac{9+1}{4}\right) \text{th item}$$

= The value of (2.5)th item

= The value of 2nd item + 0.5(3rd item – 2nd item)

= 19 + 0.5(20 - 19) = 19 + 0.5(1) = 19 + 0.5 = 19.5

$$Q_3 = \text{The value of } 3\left(\frac{n+1}{4}\right) \text{th item}$$

$$= \text{The value of } 3\left(\frac{9+1}{4}\right) \text{th item}$$

= The value of 3 (2.5)th item

= The value of 7.5th item

= The value of 7th item + 0.5(8th item – 7th item)

$= 49 + 0.5(50 - 49) = 49 + 0.5(1) = 49 + 0.5 = 49.5$

D_6 = The value of $6\left(\frac{n+1}{10}\right)$th item

= The value of $6\left(\frac{9+1}{10}\right)$th item

= The value of 6 (1)th item

= The value of 6th item = 48

P_{42} = The value of $42\left(\frac{n+1}{100}\right)$th item

= The value of $42\left(\frac{9+1}{100}\right)$th item

= The value of 42 (0.1)th item

= The value of 4.2th item

= The value of 4th item + 0.2(5th item − 4th item)

$= 22 + 0.2(41 - 22) = 22 + 0.2(19) = 22 + 3.8 = 25.8$

(ii) **48, 28, 92, 22, 69, 37, 69, 92, 88, 82**

Arrange the values in ascending and descending order

22, 28, 37, 48, 69, 69, 82, 88, 92, 92

Here n = 10

Q_1 = The value of $\left(\frac{n+1}{4}\right)$th item

= The value of $\left(\frac{10+1}{4}\right)$th item

= The value of (2.75)th item

= The value of 2nd item + 0.75(3rd item − 2nd item)

$= 28 + 0.75(37 - 28) = 28 + 0.75(9) = 28 + 6.75 = 34.75$

Q_3 = The value of $3\left(\frac{n+1}{4}\right)$th item

= The value of $3\left(\frac{10+1}{4}\right)$th item

= The value of 3 (2.75)th item

= The value of 8.25th item

= The value of 8th item + 0.25(9th item − 8th item)

$= 88 + 0.25(92 - 88) = 88 + 0.25(4) = 88 + 1 = 89$

$$Q_6 = \text{The value of } 6\left(\frac{n+1}{10}\right)\text{th item}$$

$$Q_6 = \text{The value of } 6\left(\frac{10+1}{10}\right)\text{th item}$$

$$= \text{The value of } 6.6\text{th item}$$

$$= \text{The value of } 6\text{th item} + 0.6(7\text{th item} - 6\text{th item})$$

$$= 69 + 0.6(82 - 69) = 69 + 0.6(13) = 69 + 7.8 = 76.8$$

$$P_{42} = \text{The value of } 42\left(\frac{n+1}{100}\right)\text{th item}$$

$$= \text{The value of } 42\left(\frac{10+1}{100}\right)\text{th item}$$

$$= \text{The value of } 4.62\text{th item}$$

$$= \text{The value of } 4\text{th item} + 0.62(5\text{th item} - 4\text{th item})$$

$$= 48 + 0.62(69 - 48) = 48 + 0.62(21) = 48 + 13.02 = 61.02$$

(iii) **60, 45, 43, 52, 49, 50, 38, 30, 44, 55**

Arrange the values in ascending and descending order

30, 38, 43, 44, 45, 49, 50, 52, 55, 60

Here n = 10

$$Q_1 = \text{The value of } \left(\frac{n+1}{4}\right)\text{th item}$$

$$= \text{The value of } \left(\frac{10+1}{4}\right)\text{th item}$$

$$= \text{The value of } \left(\frac{11}{4}\right)\text{th item} = \text{The value of } 2.75\text{th item}$$

$$= \text{The value of } 2\text{nd item} + 0.75(3\text{rd item} - 2\text{nd item})$$

$$= 38 + 0.75(43 - 38) = 38 + 0.75(5) = 38 + 3.75 = 41.75$$

$$Q_3 = \text{The value of } 3\left(\frac{n+1}{4}\right)\text{th item}$$

$$= \text{The value of } 3\left(\frac{10+1}{4}\right)\text{th item}$$

$$= \text{The value of } 8.25\text{th item}$$

$$= \text{The value of } 8\text{th item} + 0.25(9\text{th item} - 8\text{th item})$$

$$= 52 + 0.25(55 - 52) = 52 + 0.25(3) = 52 + 0.75 = 52.75$$

Q_6 = The value of $6\left(\frac{n+1}{10}\right)$th item

Q_6 = The value of $6\left(\frac{10+1}{10}\right)$th item

= The value of 6.6th item

= The value of 6th item + 0.6(7th item − 6th item)

$= 49 + 0.6(50 - 49) = 49 + 0.6(1) = 49 + 0.6 = 49.6$

P_{42} = The value of $42\left(\frac{n+1}{100}\right)$th item

= The value of $42\left(\frac{10+1}{100}\right)$th item

= The value of 4.62th item

= The value of 4th item + 0.62(5th item − 4th item)

$= 44 + 0.62(45 - 44) = 44 + 0.62(1) = 44 + 0.62 = 44.62$

-:3.21:-

Size of Item	F	C-f
4	2	2
5	5	7
6	8	15
7	9	24
8	12	36
9	14	50
10	14	64
11	15	79
12	11	90
13	13	103
14	9	112
15	7	119
16	4	123
17	3	126
-----	**126**	-----

Med = The value of $\left(\frac{n+1}{2}\right)$th item = The value of $\left(\frac{126+1}{2}\right)$th item

Med = The value of 63.5th item = 10

-:3.22:-

No. of Leaves	No. of Branches	C-f
2	10	10
3	70	80
4	110	190
5	300	490
6	320	810
7	120	930
8	3	933
9	20	953
10	8	961
11	7	968
12	5	973
-----	**973**	-----

$$\text{Med} = \text{The value of } \left(\frac{n+1}{2}\right)\text{th item} = \text{The value of } \left(\frac{973+1}{2}\right)\text{th item}$$

Med = The value of 487th item = 5

-:3.23:-

No. of Heads	f	fX	C-f
0	1	0	1
1	9	9	10
2	26	52	36
3	59	177	95
4	72	288	167
5	52	260	219
6	29	174	248
7	7	49	255
8	1	8	256
-----	**256**	**1017**	-----

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{1017}{256} = 3.973 = 4$$

$$\text{Med} = \text{The value of } \left(\frac{n+1}{2}\right)\text{th item} = \text{The value of } \left(\frac{256+1}{2}\right)\text{th item}$$

Med = The value of 128.5th item = 4

$$Q_1 = \text{The value of } \left(\frac{n+1}{4}\right)\text{th item} = \text{The value of } \left(\frac{256+1}{4}\right)\text{th item}$$

Q_1 = The value of 64.25th item

Q_3 = The value of $\frac{3(n+1)}{4}$ th item

Q_1 = The value of $\frac{3(256+1)}{4}$ th item = The value of 192.75th item = 5

Hence

$$\bar{X} = 3.973,\ \text{Med} = 4,\ Q_1 = 3,\ Q_3 = 5$$

-:3.24:-

C – I	F	C – F
35 – 45	8	8
45 – 55	12	20
55 – 65	20	40
65 – 75	11	51
75 – 85	9	60 = n
Total	**60**	-----

Here n = 60; $\frac{n}{2} = \frac{60}{2} = 30$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 30$

L = 55. f = 20, C = 20, h = 10

$$\text{Median} = 55 + \frac{10}{20}(30 - 20) = 55 + \frac{10 \times 10}{20} = 55 + 5 = 60$$

-:3.25:-

Marks	No of Students	C – B	C – F
10 – 19	3	9.5 – 19.5	3
20 – 29	7	19.5 – 29.5	10
30 – 39	20	29.5 – 39.5	30
40 – 49	12	39.5 – 49.5	42
50 – 59	10	49.5 – 59.5	52
60 – 69	8	59.5 – 69.5	60 = n
Total	**60**	-----	

Here n = 60; $\frac{n}{2} = 30$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 30$, L = 29.5, f = 20, C = 10, h = 10

$$\text{Median} = 29.5 + \frac{10}{20}(30 - 10) = 29.5 + \frac{10 \times 20}{20} = 29.5 + 10 = 39.5$$

-:3.26:-

Classes	F	C – B	C – F
10 – 14	10	9.5 – 14.5	10
15 – 19	15	14.5 – 19.5	25
20 – 24	25	19.5 – 24.5	50
25 – 29	18	24.5 – 29.5	68
30 – 34	12	29.5 – 34.5	80
35 – 39	10	34.5 – 39.5	90 = n
-----	**90**	-----	

Here $n = 90;\ \frac{n}{2} = 45$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 45, L = 19.5,\ f = 25,\ C = 25,\ h = 5$

$$\text{Median} = 19.5 + \frac{5}{25}(45 - 25) = 19.5 + \frac{5 \times 20}{25} = 19.5 + 4 = 23.5$$

-:3.27:-

Monthly Wages	f	C-f
50-55	6	6
55-60	10	16
60-65	22	38
65-70	30	68
70-75	16	84
75-80	12	96
80-100	15	111
-----	**111**	-----

Here

$n/2 = 55.5, h = 5, L = 65, f = 30, c = 38$

$$\text{Media} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 65 + \frac{5}{30}(55.5 - 38) = 65 + \frac{17.5}{6}$$

$$= 65 + 2.92 = 67.92$$

-:3.28:-

Groups	F	C-F
0 – 8	3	3
8 – 16	17	20
16 – 24	18	38
24 – 32	22	60
32 – 40	6	66
40 – 48	4	70 = n
-----	70	

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{70}{2} = 35,\ L = 16,\ f = 18,\ C = 20,\ h = 8$$

$$\text{Median} = 16 + \frac{8}{18}(35 - 20) = 16 + \frac{8\times 15}{18} = 16 + \frac{120}{18} = 16 + 6.67 = 22.67$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

$$\frac{n}{4} = 17.5,\ L = 8,\ f = 17,\ C = 3,\ h = 8$$

$$Q_1 = 8 + \frac{8}{17}(17.5 - 3) = 8 + \frac{8\times 14.5}{17} = 8 + \frac{116}{17} = 8 + 6.82 = 14.82$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$$\frac{3n}{4} = 52.5,\ L = 24,\ f = 22,\ C = 38,\ h = 8$$

$$Q_3 = 24 + \frac{8}{22}(52.5 - 38) = 24 + \frac{8\times 14.5}{22} = 24 + \frac{116}{22} = 24 + 5.27 = 29.27$$

$$D_7 = L + \frac{h}{f}\left(\frac{7n}{10} - C\right)$$

$$\frac{7n}{10} = \frac{7\times 70}{10} = 49,\ L = 24,\ f = 22,\ C = 38,\ h = 8$$

$$D_7 = 24 + \frac{8}{22}(49 - 38) = 24 + \frac{8\times 11}{22} = 24 + 4 = 28$$

$$P_{80} = L + \frac{h}{f}\left(\frac{80n}{100} - C\right)$$

$$\frac{80n}{100} = \frac{80\times 70}{100} = 56,\ L = 24,\ f = 22,\ C = 38,\ h = 8$$

$$P_{80} = 24 + \frac{8}{22}(66 - 38) = 24 + \frac{8 \times 18}{22} = 24 + \frac{144}{22} = 24 + 6.55 = 30.55$$

-:3.29:-

X	f	fX	C-B	C-f
2.8	4	11.2	2.7-2.9	4
3.0	15	45.0	2.9-3.1	19
3.2	20	64.0	3.1-3.3	39
3.4	47	159.8	3.3-3.5	86
3.6	63	226.8	3.5-3.7	149
3.8	78	296.4	3.7-3.9	227
4.0	88	352.0	3.9-4.1	315
4.2	60	252.0	4.1-4.3	375
4.4	59	259.6	4.3-4.5	434
4.6	35	161.0	4.5-4.7	469
4.8	10	48.0	4.7-4.9	479
5.0	8	40.0	4.9.5.1	487
5.2	4	20.8	5.1-5.3	491
-----	**491**	**1936.6**	-----	-----

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{1936.6}{491} = 3.94$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Here $h = 0.2$, $n/2 = 245.5$, $L = 3.9$, $f = 88$, $c = 227$

$$\text{Med} = 3.9 + \frac{0.2}{88}(245.5 - 227) = 3.9 + \frac{0.2 \times 18.5}{88}$$

$$= 3.9 + 0.042 = 3.942$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

Here $n/4 = 122.75$, $L = 3.5$, $f = 63$, $C = 86$, $h = 0.2$

$$Q_1 = 3.5 + \frac{0.2}{65}(122.75 = 86) = 3.5 + 0.12 = 3.62$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

Here $3n/4 = 368.25$, $L = 4.1$, $f = 60$, $C = 315$, $h = 0.2$

$$Q_3 = 4.1 + \frac{0.2}{60}(368.25 - 315) = 4.1 + \frac{0.2}{60}(368.25 - 315)$$

$$Q_3 = 4.1 + \frac{0.2 \times 53.25}{60} = 4.1 + 0.17 + 4.27$$

-:3.30:-

Wages in Rs.	f	C-B	C-f
20	8	19.5-20.5	8
21	10	20.5-21.5	18
22	11	21.5-22.5	29
23	16	22.5-23.5	45
24	20	23.5-24.5	65
25	25	24.5-25.5	90
26	15	25.5-26.5	105
27	9	26.5-27.5	114
28	6	27.5-28.5	120
-----	120	-----	-----

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Here $h = 1$, $\frac{n}{2} = 60$, $L = 23.5$, $f = 20$, $C = 45$

$$\text{Med} = 25.5 + \frac{1}{20}(60 - 45) = 23.5 + \frac{15}{20} = 23.5 + 1.75 = 24.25$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

Here $\frac{n}{4} = 30$, $L = 22.5$, $f = 16$, $C = 29$, $h = 1$

$$Q_1 = 22.5 + \frac{1}{16}(30 - 29) = 22.5 + \frac{1}{16} = 22.5 + 0.0625 = 22.56$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

Here $\frac{3n}{4} = 90$, $L = 24.5$, $f = 25$, $C = 95$, $h = 1$

$$Q_3 = 24.5 + \frac{1}{25}(90 - 65) = 24.5 + \frac{25}{25} = 24.5 + 1 = 25.5$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Here $f_m = 25$, $f_1 = 20$, $f_2 = 15$, $L = 24.5$, $h = 1$

$$\text{Mode} = 24.5 + \frac{25 - 20}{(25 - 20) + (25 - 15)} \times 1 = 24.5 + \frac{5}{5 + 10}$$

$$= 24.5 + \frac{5}{15} = 24.5 + 0.33 = 24.83$$

-:3.31:-

Marks	F	C-f
0-10	3	3
10-20	9	12
20-30	15	27
30-40	30	57
40-50	18	75
50-60	5	80
	80	

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Here $h = 10,\ \frac{n}{2} = 40,\ L = 30,\ f = 30,\ C = 27$

$$\text{Med} = 30 + \frac{10}{30}(40 - 27) = 30 + \frac{130}{30} = 30 + 4.33 = 34.33$$

-:3.32:-

Monthly Rent	f	C-f
20-40	6	6
40-60	9	15
60-80	11	26
80-100	14	40
100-120	20	60
120-140	15	75
140-160	10	85
160-180	8	93
180-200	7	100
	100	

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Here $h = 20,\ \frac{n}{2} = 50,\ L = 100,\ f = 20,\ C = 40$

$$\text{Med} = 100 + \frac{20}{20}(50 - 40) = 100 + 10 = 110$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

Here $\frac{n}{4} = 25,\ L = 60,\ f = 11,\ C = 15,\ h = 20$

$$Q_1 = 60 + \frac{20}{11}(25 - 15) = 60 + \frac{20 \times 10}{11} = 60 + 18.18 = 78.18$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$\frac{3n}{4} = 75,\ L = 120,\ f = 15,\ C = 60,\ h = 20$

$$Q_3 = 120 + \frac{20}{15}(75 - 60) = 120 + \frac{20 \times 15}{15} = 120 + 20 = 140$$

$$D_7 = L + \frac{h}{f}\left(\frac{7n}{10} - C\right)$$

$\frac{7n}{10} = 70,\ L = 120,\ f = 15,\ C = 60,\ h = 20$

$$D_7 = 120 + \frac{20}{15}(70 - 60) = 120 + \frac{20 \times 10}{15} = 120 + 13.13 = 133.33$$

-:3.33:-

Batsman A: The mode of the 10 values is 27. The value 27 is the most repeated. Hence mode = 27

Bastman B: The most repeated value is 5. Mode is 5.

Bastman C: The 36 is the repeated value. Mode =36

-:3.34:-

Bowler A: The most repeated value is 4. Hence mode = 4

Bowler B: The value 2 is most frequent value. Mode = 2.

Bowler C: There are two modes, 1 and 3, since each of the two values occurs thrice.

-:3.35:-

(i)

X	1	3	4	0	4	2	3	1	
	2	3	0	1	1	1	5	1	32

$$\bar{X} = \frac{\Sigma X}{n} = \frac{32}{16} = 2$$

Arranged values are

0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 5

n = 16 (Even No.)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2}\text{th item} + \text{The value of } \frac{n+2}{2}\text{th item}\right]$$

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{16}{2}\text{th item} + \text{The value of } \frac{18}{2}\text{th item}\right]$$

$$\text{Med} = \frac{1}{2}\ \text{The value of 8th item} + \text{The value of 9th item}$$

$$\text{Med} = \frac{1}{2}\ 1 + 2 = 1.5 = 1,\ \text{as 1 is the most repeated value.}$$

(ii)

X	8	3	21	10	2	7	0	27	
	19	6	7	4	10	5	7	24	160

$$\bar{X} = \frac{\Sigma X}{n} = \frac{160}{16} = 10$$

Arranged values are

0, 2, 3, 4, 5, 6, 7, 7, 7, 8, 10, 10, 19, 21, 24, 27

n = 16 (Even No.)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2}\text{th item} + \text{The value of } \frac{n+2}{2}\text{th item}\right]$$

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{16}{2}\text{th item} + \text{The value of } \frac{18}{2}\text{th item}\right]$$

$$\text{Med} = \frac{1}{2}\text{ The value of 8th item + The value of 9th item}$$

$$\text{Med} = \frac{1}{2}\ 7+7\ = 7$$

Mode = 7, as 7 is the most repeated value.

-:3.36:-

(a)

Classes	F	x	fx	C-f
10 – 20	10	15	150	10
20 – 30	19	25	475	29
30 – 40	31	35	1085	60
40 – 50	32	45	1440	92
50 – 60	25	55	1375	117
60 – 70	16	65	1040	133 = n
Total	**133**	-----	**5565**	-----

$$\text{Mean} = \text{A.M.} = \frac{\Sigma fx}{\Sigma f} = \frac{5565}{133} = 41.84211$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Here $\frac{n}{2} = \frac{133}{2} = 66.5,\ L = 40,\ f = 32,\ C = 60,\ h = 10$

$$\text{Median} = 40 + \frac{10}{32}(66.5 - 60) = 40 + \frac{10 \times 6.5}{32}$$

$$= 40 + \frac{65}{32} = 40 + 2.03125 = 42.03125$$

Mode = 3 med – 2 mean

= 3(42.03125)–2(41.84211)=126.09375–83.68422=42.40953=42.41

(b) **Arithmetic Mean by Short Cut and Coding Method**

Classes	F	X	D	U	fD	fU
10 – 20	10	15	–20	–2	–200	–20
20 – 30	19	25	–10	–1	–190	–19
30 – 40	31	35	0	0	0	0
40 – 50	32	45	10	1	320	32
50 – 60	25	55	20	2	500	50
60 – 70	16	65	30	3	480	48
Total	**133**	----	----	----	**910**	**91**

(i) By Short Cut Method

$$A.M. = P.M + \frac{\Sigma fD}{\Sigma f}$$

$$= 35 + \frac{910}{133} = 35 + 6.84211 = 41.84211 = 41.84$$

(ii) By Coding Method

$$A.M. = P.M + \frac{\Sigma fu}{\Sigma f} \times h$$

$$= 35 + \frac{91}{133} \times 10 = 35 + \frac{910}{133} = 35 + 6.84211 = 41.84211 = 41.84$$

-:3.37:-

Here Equation $y = 10 - 4x$

x	y = 10 – 4x
–3	10 + 12 = 22
–2	10 + 8 = 18
–1	10 + 4 = 14
0	10 – 0 = 10
1	10 – 4 = 6
2	10 – 8 = 2
3	10 – 12 = –2

$$\bar{Y} = \frac{\Sigma y}{n} = \frac{22+18+14+10+6+2-2}{7} = \frac{70}{7} = 10$$

$$\bar{X} = \frac{\Sigma x}{n} = \frac{-3-2-1+0+1+2+3}{7} = \frac{0}{7} = 0$$

$$\bar{y} = 10 - 4\bar{x} = 10 - 4(0) = 10 - 0 = 10$$

Hence proved that $\bar{y} = 10 - 4\bar{x}$

(b) Median and Mode

For median arrange the values in asending and desending order.

2, 2, 3, 5, 5, 5, 6, 6, 8, 9

Here n = 10 (even number)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2}\text{th item} + \text{The value of } \left(\frac{n+2}{2}\right)\text{th item}\right]$$

$$= \frac{1}{2}\left[\text{The value of } \left(\frac{10}{2}\right)\text{th item} + \text{The value of } \left(\frac{10+2}{2}\right)\text{th item}\right]$$

$$= \frac{1}{2}[\text{The value of 5th item} + \text{The value of 6th item}]$$

$$= \frac{1}{2}[5+5] = \frac{1}{2}(10) = 5$$

Mode is the most repeated number. 5 is the most repeated number.
So Mode = 5

-:3.38:-

(a)

X	D = x – 60
83	23
52	–08
66	06
58	–02
50	–10
47	–13
55	–05
60	00
471	-----

$$\text{A.M.} = \frac{\sum x}{n} = \frac{471}{8}$$

$$\bar{x} = 58.875$$

(b) Compute Median and Mode

Groups	F	C–F
100 – 200	10	10
200 – 300	28	38
300 – 400	35	73
400 – 500	25	98
500 – 600	7	105
-----	**105**	-----

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{105}{2} = 52.5, \quad L = 300, \quad f = 35, \quad C = 38, \quad h = 100$$

$$\text{Med} = 300 + \frac{100}{35}(52.5 - 38) = 300 + \frac{100 \times 14.5}{35}$$

$$= 300 + \frac{1450}{35} = 300 + 41.43 = 341.43$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Here $f_m = 35,\ f_1 = 28,\ f_2 = 25,\ h = 100,\ L = 300$

$$\text{Mode} = 300 + \frac{35-28}{(35-28)+(35-25)} \times 4 = 300 + \frac{7 \times 100}{7+10}$$

$$= 300 + \frac{700}{17} = 300 + 41.18 = 341.18$$

-:3.39:-

(a)

Operators	Income (x)
A	12
B	15
C	18
D	20
E	25
F	30
G	22
H	35
Total	**177**

$$\text{Arithmetic Mean} = \frac{\sum x}{n}$$

$$= \frac{177}{8} = 22.125$$

(b)

Marks	F	C–B	C–F
10 – 19	5	9.5 – 19.5	5
20 – 29	20	19.5 – 29.5	25
30 – 39	35	29.5 – 39.5	60
40 – 49	20	39.5 – 49.5	80
50 – 59	15	49.5 – 59.5	95
60 – 60	9	59.5 – 69.5	104 = n
-----	**104**	-----	

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{104}{2} = 52,\ L = 29.5,\ f = 35,\ C = 25,\ h = 10$$

$$\text{Med} = 29.5 + \frac{10}{35}(52-25) = 29.5 + \frac{10 \times 27}{35}$$

$$= 29.5 + \frac{270}{35} = 29.5 + 7.71 = 37.21$$

-:3.40:-

(a)

Group	f	x	Fx
0 – 5	4	2.5	10
5 – 10	6	7.5	45
10 – 15	8	12.5	100
15 – 20	6	17.5	105
20 – 25	4	22.5	90
25 – 30	2	27.5	55
Total	**30**	-----	**405**

$$\text{A.M.} = \frac{\sum fx}{\sum f} = \frac{405}{30} = 13.5$$

(b)

Groups	f	C–f
30 – 40	3	3
40 – 45	5	8
45 – 55	8	16
55 – 60	4	20
60 – 70	5	25 = n

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{25}{2} = 12.5, \quad L = 45, \quad f = 8, \quad C = 8, \quad h = 10$$

$$\text{Med} = 45 + \frac{10}{8}(12.5 - 8) = 45 + \frac{10 \times 4.5}{8} = 45 + \frac{45}{8}$$

$$= 45 + 5.625 = 50.625$$

-:3.41:-

(a)

Class Limits	f	Class Boundaries
3.0 – 3.9	13	2.95 – 3.95
4.0 – 4.9	27	3.95 – 4.95
5.0 – 5.9	40	4.95 – 5.95
6.0 – 6.9	30	5.95 – 6.95
7.0 – 7.9	16	6.95 – 7.95
8.0 – 8.9	4	7.95 – 8.95

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Here $f_m = 40, \; f_1 = 27, \; f_2 = 30, \; h = 1, \; L = 4.95$

$$\text{Mode} = 4.95 + \frac{40-27}{(40-27)+(40-30)} \times 1 = 4.95 + \frac{13}{13+10}$$

$$= 4.95 + \frac{13}{23} = 4.95 + 0.57 = 5.52$$

(b)

X (Price)	No of Shares (f)	fx
20	120	2400
25	80	2000
-----	200	4400

$$\text{Average Price of Share} = \frac{\Sigma fx}{\Sigma f} = \frac{4400}{200} = \text{Rs.}22$$

-:3.42:-

$$\text{A.M.} = \frac{\Sigma x}{n}$$

$\Sigma x = 40+45+48+50+55+57+55+59+65+70+75+55 = 674$

$n = 12$

Hence $\text{A.M.} = \frac{\Sigma x}{n} = \frac{674}{12} = 56.16$

-:3.43:-

(a)

Marks	f	x	fx
0 – 5	4	2.5	10
5 – 10	6	7.5	45
10 – 15	10	12.5	125
15 – 20	16	17.5	280
20 – 25	12	22.5	270
Total	**48**	-----	**730**

$$\text{A.M.} = \frac{\Sigma fx}{\Sigma f} = \frac{730}{48} = 15.21$$

(b) For median arrange the values in asending and desending order.

10, 12, 13, 13, 15, 17, 18, 19

Here $n = 8$ (even number)

$$\text{Med} = \frac{1}{2}\left[\text{The value of } \frac{n}{2} \text{ th item} + \text{The value of } \left(\frac{n+2}{2}\right) \text{th item}\right]$$

$$= \frac{1}{2}\left[\text{The value of } \left(\frac{8}{2}\right) \text{th item} + \text{The value of } \left(\frac{8+2}{2}\right) \text{th item}\right]$$

$$\text{Med} = \frac{1}{2}[\text{The value of 4th item} + \text{The value of 5th item}]$$

$$= \frac{1}{2}[13+15] = \frac{28}{2} = 14$$

Mode is the most repeated number, 13 is the most repeated number.
So Mode = 13

-:3.44:-

(a) When n = 15, then $\bar{x} = 20$

Sum of 15 values = $\Sigma x = n\bar{x} = 15 \times 20 = 300$

When n = 18, then $\bar{x} = 20$

Sum of 18 values = $\Sigma x = n\bar{x} = 18 \times 20 = 360$

Sum of three newly added values = 360 – 300 = 60

Sum of 3 values = 60

Ratio of three values is 3 : 2 : 1

Sum of ratio = 3 + 2 + 1 = 6

Hence

$$\text{1st Value} = \frac{3}{6} \times 60 = 3 \times 10 = 30$$

$$\text{2nd Value} = \frac{2}{6} \times 60 = 2 \times 10 = 20$$

$$\text{3rd Value} = \frac{1}{6} \times 60 = 1 \times 10 = 10$$

Hence three values are 30, 20, 10

(b)

x	f	Class Boundaries	C–f
3	200	2 – 4	200
5	600	4 – 6	800
7	800	6 – 8	1600
9	1300	8 – 10	2900
11	900	10 – 12	3800
13	700	12 – 14	4500
15	500	14 – 16	5000 = n

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{5000}{2} = 2500, \quad L = 8, \quad f = 1300, \quad C = 1600, \quad h = 2$$

$$\text{Med} = 8 + \frac{2}{1300}(2500 - 1600) = 8 + \frac{2 \times 900}{1300} = 8 + \frac{18}{13}$$

$$= 8 + 1.385 = 9.385$$

-:3.45:-

(a)

Classes	f	C–B	C–f
10 – 14	5	9.5 – 14.5	5
15 – 19	12	14.5 – 19.5	17
20 – 24	30	19.5 – 24.5	47
25 – 29	25	24.5 – 29.5	72
30 – 34	6	29.5 – 34.5	78 = n

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = \frac{78}{2} = 39, \quad L = 19.5, \quad f = 30, \quad C = 17, \quad h = 5$$

$$\text{Med} = 19.5 + \frac{5}{30}(39 - 17) = 19.5 + \frac{22}{6} = 19.5 + 3.675 = 23.17$$

(b) $$\text{A.M.} = \frac{\sum x}{n}$$

$\Sigma x = 1+3+4+0+4+2+3+1+2+3+0+1+1+1+5+1 = 32, \; n = 16$

Hence $$\text{A.M.} = \frac{\sum x}{n} = \frac{32}{16} = 2$$

-:3.46:-

(a) For median, arrange the data in asending and desending order
0.4, 0.5, 1.6, 1.9, 1.9, 1.9. 2.1, 2.5, 2.8, 3.1, 3.1, 3.1 3.2, 3.5, 3.7, 4.8, 4.8, 5.1, 5.2, 6.6, 6.7, 7.7, 8.4
Here n = 23 (odd number)

$$\text{Med} = \text{The value of}\left(\frac{n+1}{2}\right)\text{th item} = \text{The value of}\left(\frac{23+1}{2}\right)\text{th item}$$

$$= \text{The value of}\left(\frac{24}{2}\right)\text{th item} = \text{The value of 12th item} = 3.1$$

Most repeated value is mode. In the data two values that is 1.9 and 3.1 are most repeated. So Mode = 1.9, 3.1

(b)

Income (Rs.)	No of Workers	x	fx
35 – 39	3	37	111
40 – 44	11	42	462
45 – 49	22	47	1034
50 – 54	19	52	988
55 – 59	4	57	228
60 – 64	1	62	62
Total	**60**	-----	**2885**

$$A.M. = \frac{\sum fx}{\sum f} = \frac{2885}{60} = 48.08$$

-:3.47:-

Weights	F
20 – 24	4
24 – 28	16
28 – 32	27
32 – 36	20
36 – 40	3

$$Mode = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Here $f_m = 27,\ f_1 = 16,\ f_2 = 20,\ h = 4,\ L = 28$

$$Mode = 28 + \frac{27 - 16}{(27 - 16) + (27 - 20)} \times 4 = 28 + \frac{11 \times 4}{11 + 7} = 28 + \frac{44}{18}$$

$$= 28 + 2.44 = 30.44$$

-:3.48:-

Groups	F	C – B
50 – 54	5	49.5 – 54.5
55 – 59	18	54.5 – 59.5
60 – 64	22	59.5 – 64.5
65 – 69	15	64.5 – 69.5
70 – 74	10	69.5 – 74.5

Here $f_m = 22,\ f_1 = 18,\ f_2 = 15,\ h = 5,\ L = 59.5$

$$Mode = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 59.5 + \frac{22 - 18}{(22 - 18) + (22 - 15)} \times 5 = 59.5 + \frac{4 \times 5}{4 + 7} = 59.5 + \frac{20}{11}$$

$$= 59.5 + 1.18 = 61.32$$

-:3.49:-

Height	F	C – B
20 – 23	1	19.5 – 23.5
24 – 27	9	23.5 – 27.5
28 – 31	18	27.5 – 31.5
32 – 35	10	31.5 – 35.5
36 – 39	2	35.5 – 39.5

Here $f_m = 18,\ f_1 = 9,\ f_2 = 10,\ h = 4,\ L = 27.5$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 27.5 + \frac{18-9}{(18-9)+(18-10)} \times 4 = 27.5 + \frac{9\times 5}{9+8} = 27.5 + \frac{36}{17}$$

$$= 27.5 + 2.12 = 29.62$$

-:3.50:-

(X) No. Of Persons	(f) No. Of Houses	fX	C-f
1	26	26	26
2	113	226	139
3	120	360	259
4	95	380	354
5	60	300	414
6	42	252	456
7	21	147	477
8	14	113	491
9	5	45	496
10	4	40	500
-----	**500**	**1888**	-----

$$\bar{X} = \frac{\Sigma fX}{\Sigma f} = \frac{1888}{500} = 3.776 = 4$$

Median = 3 as n/2 = 250 lies in 269 value of C-f.
Mode = 3 as 120 is the maximum frequency.

-:3.51:-

Height	f	X	fX	C-B	C-f
60-62	5	61	305	59.5-62.5	5
63-65	18	64	1152	62.5-65.5	23
66-68	42	67	2814	65.5-68.5	65
69-71	27	70	1890	68.5-71.5	92
72--74	8	73	584	71.5-74.5	100
-----	**100**	-----	**6745**	-----	-----

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{6745}{100} = 67.45$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 65.5 + \frac{3}{42}(50 - 23) = 65.5 + 1.92 = 67.42$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 65.5 + \frac{52 - 18}{(42 - 18) + (42 - 27)} \times 3$$

$$= 65.5 + \frac{72}{39} = 65.5 + 1.85 = 67.35$$

-:3.52:-

Height	f	fX	C-B	C-f
47	2	94	46.5-47.5	2
48	5	240	47.5--48.5	7
49	9	441	48.5-49.5	16
50	10	500	49.5-50.5	36
51	12	612	50.5-51.5	38
52	15	780	51.5-52.5	53
53	20	1060	52.5-53.5	73
54	17	918	53.5-54.5	90
55	6	330	54.5-55.5	96
56	3	168	55.5-56.5	99
57	1	57	56.5-57.5	100
-----	**100**	**5200**	-----	-----

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{5200}{100} = 52$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 51.5 + \frac{1}{15}(50 - 38) = 51.5 + \frac{12}{15} = 51.5 + 0.8 = 52.3$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 52.5 + \frac{20 - 15}{(20 - 15) + (20 - 17)} \times 1 = 52.5 + \frac{5}{8} = 52.5 + 0.625 = 53.125$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right) = 49.5 + \frac{1}{10}(25 - 16) = 49.5 + \frac{9}{10} = 49.5 + 0.9 = 50.4$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right) = 53.5 + \frac{1}{17}(75 - 73) = 53.5 + \frac{2}{17} = 53.5 + 0.11 = 53.61$$

-:3.53:-

Marks	f	X	fX	C-f
10-25	6	17.5	105.0	6
25-40	20	32.5	650.0	26
40-55	44	47.5	2090.0	70
55-70	26	62.5	1625.0	96
70-85	3	77.5	232.0	99
85-100	1	92.5	92.5	100
-----	**100**	-----	**4795**	-----

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{4795}{100} = 47.95$$

$$\text{Med} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 40 + \frac{15}{44}(50 - 26) = 40 + \frac{15 \times 24}{44}$$

$$= 40 + \frac{360}{44} = 40 + 8.18 = 48.18$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 40 + \frac{44 - 20}{(44 - 20) + (44 - 26)} \times 15 = 40 + \frac{24 \times 15}{42}$$

$$= 40 + \frac{360}{42} = 40 + 18.57 = 48.57$$

-:3.54:-

Income (Rs.)	f	x	fx
118 – 126	3	122	366
127 – 135	5	131	655
136 – 144	12	140	1680
145 – 153	6	149	894
154 – 162	4	158	632
Total	**30**	-----	**4227**

$$\text{A.M.} = \frac{\Sigma fx}{\Sigma f} = \frac{4227}{30} = \text{Rs.}140.9$$

-:3.55:-

Groups	f	X	D
0-4	2	2	-16
4-8	5	6	-12
8-12	8	10	-8
12-16	11	14	-4
16-20	12	18	0
20-24	9	22	4
24-28	4	26	8
28-32	1	30	12
-----	**52**	-----	-----

U	fU	C-f	fX
-4	-8	2	4
-3	-15	7	30
-2	-16	15	80
-1	-11	26	154
0	0	38	216
1	9	47	198
2	8	51	104
3	3	52	30
-----	**-30**	-----	**816**

$$\bar{X} = \frac{816}{52} = 15.7$$

$$\bar{X} = P.M + \frac{\sum fu}{\sum f} \times h \text{ (Step Deviation)}$$

$$\bar{X} = 18 + \frac{-30}{52} \times 4 = 18 - \frac{120}{52} = 18 - 2.3 = 15.7$$

$$Med = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = 26,\ L = 12,\ f = 11,\ C = 15,\ h = 4$$

$$Med = 12 + \frac{4}{11}(26 - 15) = 12 + \frac{4 \times 11}{11} = 12 + 4 = 16$$

$$Mode = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$f_m = 12,\ f_1 = 11,\ f_2 = 9,\ L = 16,\ h = 4$$

$$Mode = 16 + \frac{12 - 11}{(12 - 11) + (12 - 9)} \times 4$$

$$= 16 + \frac{4}{4} = 16 + 1 = 17$$

Empirical Relation for Mode

Mode = 3 Med -2 Mean

Mode = 3(16) - 2(15.7) = 48 - 31.4 = 16.6

-:3.56:-

Height	f	fX	C-B	C-f
63	4	252	62.5-63.5	4
64	6	284	63.5-64.5	10
65	10	650	64.5-65.5	20
66	20	1320	65.5-66.5	40
67	30	2010	66.5-67.5	70
68	13	884	67.5-68.5	83
69	12	828	68.5-69.5	95
70	3	210	69.5-70.5	98
71	2	142	70.5-71.5	100
----	**100**	**6680**	----	----

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{6680}{100} = 66.8$$

$$\text{Med} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = 50,\ L = 66.5,\ f = 30,\ C = 48,\ h = 1$$

$$\text{Med} = 66.5 + \frac{1}{30}(50 - 40) = 66.5 + \frac{10}{30} = 66.5 + 0.33 = 66.33$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$f_m = 30,\ f_1 = 20,\ f_2 = 13,\ L = 66.5,\ h = 1$$

$$\text{Mode} = 66.5 + \frac{30 - 20}{(30 - 20) + (30 - 13)} \times 1 = 66.5 + \frac{10}{27}$$

$$= 66.5 + 0.37 = 66.87$$

-:3.57:-

From the data given in !. 3.35 Median = 66.83

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right),$$

$$\frac{n}{2} = 25,\ L = 65.5,\ f = 20,\ C = 20,\ h = 1$$

$$Q_1 = 65.5 + \frac{1}{20}(25 - 20) = 65.5 + \frac{5}{20} = 65.5 + 0.25 = 65.75$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$\frac{3n}{4} = 75,\ L = 67.5,\ f = 13,\ C = 70,\ h = 1$

$$Q_3 = 67.5 + \frac{1}{13}(75 - 70) = 67.5 + \frac{5}{13} = 67.5 + 0.38 = 67.88$$

-:3.58:-

Height	f	C-B	C-f
53-55	15	52.5-55.5	15
56-58	111	55.5-58.5	126
59-61	182	58.5-61.5	308
62-64	105	61.5-64.5	413
65-67	19	64.5-67.5	432
68-70	7	67.5-70.5	439
-----	**439**	-----	-----

Median By formula

$$Med = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 219.5,\ L = 58.5,\ f = 182,\ C = 126,\ h = 3$

$$Med = 58.5 + \frac{3}{182}(219.5 - 126) = 58.5 + 1.54 = 60.04$$

-:3.59:-

	f	C-B	C-f
0-4.9	3	-0.05-4.95	3
5-9.9	4	4.95-9.95	7
10-14.9	9	9.95-14.95	16
15-19.9	11	14.95-19.95	27
20-24.9	15	19.95-24.95	42
25-29.9	13	24.95-29.95	55
30-34.9	7	29.95-34.95	62
35-39.9	5	34.95-39.95	67
40-44.9	2	39.95-44.95	69
45-49.9	1	44.95-49.95	70
-----	**70**	-----	-----

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

$\frac{n}{4} = 17.5,\ L = 14.95,\ f = 11,\ C = 16,\ h = 5$

$$Q_1 = 14.95 + \frac{5}{11}(17.5 - 16) = 14.95 + 0.68 + 0.68 = 15.63$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$\frac{3n}{2} = 52.5,\ L = 24.95,\ f = 13,\ C = 42,\ h = 5$

$$Q_3 = 24.95 + \frac{5}{13}(52.5 - 42) = 24.95 + 4.03 = 28.98$$

$$P_5 = L + \frac{h}{f}\left(\frac{5n}{100} - C\right)$$

$\frac{5n}{100} = 3.5,\ L = 4.95,\ f = 4,\ C = 3,\ h = 5$

$$P_5 = 4.95 + \frac{5}{4}(3.5 - 3) = 4.95 + 0.83 = 5.78$$

-:3.60:-

Class	f	X
0.7312-0.7313	10	0.73125
0.7314-0.7315	15	0.73145
0.7316-0.7317	20	0.73165
0.7318-0.7319	25	0.73185
0.7320-0.7321	30	0.73205
0.7322-0.7323	8	0.73225
0.7324-0.7325	2	0.73245
80.49790	**110**	-----

fX	C-B	C-f
7.31250	0.73115-0.73135	10
10.97175	0.73135-0.73155	25
14.63300	0.73155-0.73175	45
18.29625	0.73175-0.73195	70
21.96150	0.73195-0.73215	100
5.85800	0.7315-0.73235	108
1.46490	0.73235-0.73255	110

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{80.4979}{110} = 0.7318$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right),$$

$$\frac{n}{2} = 55,\ L = 0.73175,\ f = 25,\ C = 45,\ h = 0.0002$$

$$\text{Med} = 0.73175 + \frac{0.0002}{25}(55 - 45) = 0.73175 + 0.00008 = 0.73183$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h,$$

$$f_m = 30,\ f_1 = 25,\ f_2 = 8,\ L = 0.73195,\ h = 0.0002$$

$$\text{Mode} = 0.73195 + \frac{30 - 25}{(30 - 25) + (30 - 8)} \times 0.0002 = 0.73195 + \frac{0.001}{27}$$

$$= 0.73198 + 0.0003 = 0.73198$$

$$D_6 = L + \frac{h}{f}\left(\frac{6n}{10} - C\right)$$

$$\frac{6n}{10} = 66,\ L = 0.73175,\ f = 25,\ C = 45,\ h = 0.0002$$

$$D_6 = 0.73175 + \frac{0.0002}{25}(66 - 45) = 0.73175 + 0.0042 = 0.73595$$

$$P_{74} = L + \frac{h}{f}\left(\frac{74n}{100} - C\right)$$

$$\frac{74n}{100} = 81.4,\ L = 0.73175,\ f = 30,\ C = 70,\ h = 0.0002$$

$$P_{74} = 0.73175 + \frac{0.0002}{30}(81.4 - 70) = 0.73175 + 0.000076 = 0.732026$$

-:3.61:-

Groups	f	X	fX	C-B	C-F
93-97	2	95	190	92.5-97.5	2
98-102	5	100	500	97.5-102.5	7
103-107	12	105	1260	102.5-107.5	19
108-112	17	110	1870	107.5-112.5	36
113-117	14	115	1610	112.5-117.5	50
118-122	6	120	720	117.5-122.5	56
123-127	3	125	375	122.5-127.5	59
128-132	1	130	130	127.5-132.5	60
-----	**60**	-----	**6655**	-----	-----

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{6655}{60} = 110.92$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$\frac{n}{2} = 30,\ L = 107.5,\ f = 17,\ C = 19,\ h = 5$$

$$\text{Med} = 107.5 + \frac{5}{17}(30 - 19) = 107.5 + \frac{55}{17} = 107.5 + 2.23 = 110.73$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 107.5 + \frac{17 - 12}{(17 - 12) + (17 - 14)} \times 5 = 107.5 + \frac{5x5}{5 + 3}$$

$$= 107.5 + 3.125 = 110.625$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

$$\frac{n}{4} = 15,\ L = 102.5,\ f = 12,\ C = 7,\ h = 5$$

$$Q_1 = 105.5 + \frac{5}{12}(15 - 7) = 102.5 + \frac{40}{12} = 102.5 + 3.33 = 105.83$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$$\frac{3n}{4} = 45,\ L = 112.5,\ f = 14,\ C = 36,\ h = 5$$

$$Q_3 = 112.5 + \frac{5}{14}(45 - 36) = 112.5 + \frac{45}{14} = 112.5 + 3.21 = 115.71$$

-:3.62:-

C-1	f	C-f	C-B	x	fx
100-104	10	10	99.5-104.5	102	1020
105-109	15	25	104.5-109.5	107	1605
110-114	20	45	109.5-114.5	112	2240
115-119	25	70	114.5-119.5	117	2925
120-124	30	100	119.5-124.5	122	3660
125-129	20	120	124.5-129.5	127	2540
130-134	16	136	129.5-134.5	132	2112
135-139	14	150	134.5-139.5	137	1918
140-144	10	160	139.5-144.5	142	1420
					19440

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{19440}{160} = 121.5$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 119.5 + \frac{5}{30}(80 - 70)$$

$$= 119.5 + \frac{50}{30} = 119.5 + 1.67 = 121.17$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 119.5 + \frac{30 - 25}{(30 - 25) + (30 - 20)} \times 5$$

$$= 119.5 + \frac{25}{15} = 119.5 + 1.67 = 121.17$$

$$Q_1 = L + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

$\frac{n}{4} = 15$, $L = 102.5$, $f = 12$, $C = 7$, $h = 5$

$$Q_1 = 102.5 + \frac{5}{12}(15 - 7) = 102.5 + \frac{40}{12}$$

$$= 102.5 + 3.33 = 105.93$$

$$Q_3 = L + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$\frac{3n}{4} = 45$, $L = 112.5$, $f = 14$, $C = 36$, $h = 5$

$$Q_3 = 112.5 + \frac{5}{14}(45 - 36) = 112.5 + \frac{45}{14}$$

$$= 112.5 + 3.21 = 115.71$$

-:3.63:-

Marks	f	X	fX	C-B	
10-19	5	14.5	72.5	9.5-19.5	5
20-29	25	24.5	612.5	19.5-29.5	30
30-39	40	34-5	1380.0	29.5-39.5	70
40-49	20	44.5	890-0	39.5-49.5	90
50-59	10	54.5	545-0	49.5-59.5	100
-----	**100**	-----	**3500**	-----	-----

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{3500}{100} = 35$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 29.5 + \frac{10}{40}(50 - 30)$$

$$= 29.5 + \frac{20}{4} = 29.5 + 5 = 34.5$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 29.5 + \frac{40 - 25}{(40 - 25) + (40 - 20)} \times 10$$

$$= 29.5 + \frac{150}{35} = 29.5 + 4.29 = 33.79$$

-:3.64:-

Marks	f	X	fX	C-B	C-f
30-39	6	34.5	207.0	29.5-39.5	6
40-49	10	44.5	445.0	39.5-49.5	16
50-59	11	54.5	599.5	49.5-59.5	27
60-69	12	64.5	774.0	59.5-69.5	39
70-79	22	74.5	1639.0	69.5-79.5	61
80-89	18	84.5	1521.0	79.5-89.5	79
90-99	8	94.5	759.0	89.5-99.5	87
-----	**87**	-----	**5941.5**	-----	-----

$$A.M = \frac{\sum fx}{\sum f} = \frac{5941.5}{87} = 68.29$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 69.5 + \frac{10}{22}(43.5 - 39)$$

$$= 69.5 + \frac{45}{22} = 69.5 + 2.045 = 71.545$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 69.5 + \frac{22 - 12}{(22 - 12) + (22 - 18)} \times 10$$

$$= 69.5 + \frac{100}{14} = 69.5 + 7.143 = 76.643$$

-:3.65:-

Weekly Income	f	X	fX	C-B	C-f
35-39	15	37	555	34.5-39.5	15
40-44	13	42	546	39.5-44.5	28
45-49	17	47	799	44.5-49.5	45
50-54	25	52	1300	49.5-54.5	70
55-59	15	57	855	54.5-59.5	85
60-64	10	62	620	59.5-64.5	95
65-69	5	67	335	64.5-69.5	100
-----	**100**		**5010**	-----	-----

$$A.M = \frac{\sum fx}{\sum f} = \frac{5010}{100} = 50.1$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right) = 49.5 + \frac{5}{25}(50 - 45)$$

$$= 49.5 + \frac{25}{25} = 49.5 + 1 = 50.5$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 49.5 + \frac{25 - 17}{(25 - 17) + (25 - 15)} \times 5$$

$$= 49.5 + \frac{40}{18} = 49.5 + 2.22 = 51.72$$

-:3.66:-

C-I	f	X	fX	C-F
0-5	25	2.5	62.5	25
5-10	45	7.5	337.5	70
10-15	50	12.5	625.5	120
15-20	70	17.5	1225.0	190
20-25	90	22.5	2025.0	280
25-30	99	27.5	2722.5	379
30-35	91	32.5	2957.5	470
35-40	75	37.5	2812.5	545
40-45	51	42.5	2167.5	596
45-50	34	47.5	1615.5	630
-----	**630**	-----	**16550**	-----

$$A.M. = \frac{\sum fx}{\sum f} = \frac{16550}{630} = 26.26$$

$$Median = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 315,\ L = 25,\ f = 99,\ C = 280,\ h = 5$

$$Med = 25 + \frac{5}{99}(315 - 280) = 25 + \frac{175}{99} = 25 + 1.76 = 26.76$$

$$Mode = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h =$$

$f_m = 99,\ f_1 = 90,\ f_2 = 91,\ L = 25,\ h = 5$

$$Mode = 25 + \frac{99 - 90}{(99 - 90) + (99 - 91)} \times 5$$

$$= 25 + \frac{45}{17} = 25 + 3.64 = 27.64$$

$$D_7 = L + \frac{h}{f}\left(\frac{7n}{10} - C\right)$$

$\frac{7n}{10} = 441,\ L = 30,\ f = 91,\ C = 379,\ h = 5$

$$D_7 = 30 + \frac{5}{91}(441 - 379) = 30 + \frac{310}{91} = 30 + 3.4 = 33.4$$

-:3.67:-

W-W	f	X	fX
0-40	6	20	120
40-80	15	60	900
80-120	22	100	2200
120-160	30	140	4200
160-200	45	180	8100
200-240	27	220	5940
240-280	13	260	3380
280-320	6	300	1800
-----	**164**	-----	**26640**

$$A.M = \frac{\sum fx}{\sum f} = \frac{26640}{164} = 162.44$$

$$\text{Median} = L + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$\frac{n}{2} = 82,\ L = 160,\ f = 45,\ C = 73,\ h = 40$

$$\text{Med} = 160 + \frac{40}{45}(82 - 73) = 160 + 8 = 168$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$f_m = 45,\ f_1 = 30,\ f_2 = 27,\ L = 160,\ h = 40$

$$\text{Mode} = 160 + \frac{45 - 30}{(45 - 30) + (45 - 27)} \times 40 = 160 + 18.18 = 178.18$$

EXERCISE NO. 4

-:4.3:-

Year	Price
1930	15
1991	14
1932	19
1935	21
1934	24
1935	23
1936	25
1937	22
1938	26

(a) 1. Nos. (1930 as base)	(b) 1. No. (1935 as base)
$\frac{15}{15} \times 100 = 100$	$\frac{15}{23} \times 100 = 65.22$
$\frac{45}{15} \times 100 = 93.33$	$\frac{14}{23} \times 100 = 60.87$
$\frac{19}{15} \times 100 = 126.67$	$\frac{19}{23} \times 100 = 82.61$
$\frac{21}{15} \times 100 = 140$	$\frac{21}{23} \times 100 = 91.30$
$\frac{24}{15} \times 100 = 160$	$\frac{24}{23} \times 100 = 104.35$
$\frac{23}{15} \times 100 = 153.33$	$\frac{23}{23} \times 100 = 100$
$\frac{25}{15} \times 100 = 166.67$	$\frac{25}{23} \times 100 = 108.7$
$\frac{22}{15} \times 100 = 146.67$	$\frac{22}{23} \times 100 = 95.65$
$\frac{26}{15} \times 100 = 173.33$	$\frac{26}{23} \times 100 = 113.04$

-:4.4:-

(I)

Year	Prices	Index Number (1974 as base year)
1975	20	= 100
1975	18	$\frac{18}{20} \times 100 = 90$
1976	23	$\frac{23}{20} \times 100 = 90$
1977	24	$\frac{24}{20} \times 100 = 120$
1978	25	$\frac{25}{20} \times 100 = 125$
1979	27	$\frac{27}{20} \times 100 = 135$
1980	28	$\frac{28}{20} \times 100 = 140$
1981	30	$\frac{30}{20} \times 100 = 150$
1982	32	$\frac{32}{20} \times 100 = 160$
1983	33	$\frac{33}{20} \times 100 = 165$

Average of first five year $= \frac{20+18+23+24+25}{5} = 22$

Average of all 10 years $= \frac{20+18+23+24+25+27+28+30+32+33}{10} = 26$

(ii)

Index Numbers Taking average of first five years as base	Index Numbers Taking average of first five years as base
$\frac{20}{22} \times 100 = 90.90$	$\frac{27}{22} \times 100 = 122.73$
$\frac{18}{22} \times 100 = 81.82$	$\frac{28}{22} \times 100 = 127.37$
$\frac{23}{22} \times 100 = 104.54$	$\frac{30}{22} \times 100 = 135.36$
$\frac{24}{22} \times 100 = 109.09$	$\frac{32}{22} \times 100 = 145.45$
$\frac{25}{22} \times 100 = 113.64$	$\frac{33}{22} \times 100 = 150.00$

(iii)

Index Numbers Taking average of whole data as base	Index Numbers Taking average of whole data as base
$\frac{20}{26} \times 100 = 76.92$	$\frac{27}{26} \times 100 = 103.85$
$\frac{18}{26} \times 100 = 69.23$	$\frac{28}{26} \times 100 = 107.69$
$\frac{23}{26} \times 100 = 92.46$	$\frac{30}{26} \times 100 = 115.38$
$\frac{24}{26} \times 100 = 96.20$	$\frac{32}{26} \times 100 = 123.07$
$\frac{25}{26} \times 100 = 96.15$	$\frac{33}{26} \times 100 = 126.92$

-:4.5:-

Year	Price	I. No. (1953 as base)
1954	122.5	$\frac{122.5}{122.5} \times 100 = 100$
1955	116	$\frac{116}{122.5} \times 100 = 94.69$
1956	120	$\frac{120}{122.5} \times 100 = 97.96$
1957	137	$\frac{137}{122.5} \times 100 = 111.84$
1958	149	$\frac{149}{122.5} \times 100 = 121.63$
1959	156.8	$\frac{156.8}{122.5} \times 100 = 128.0$
1960	162	$\frac{162}{122.5} \times 100 = 132.24$
1961	149	$\frac{149}{122.5} \times 100 = 121.63$
1962	160.6	$\frac{160.6}{122.5} \times 100 = 131.1$
1963	153.1	$\frac{153.1}{122.5} \times 100 = 124.98$

-:4.6:-

Year	Price
1930	4
1931	5
1932	6
1933	7
1934	8
1935	10
1936	9
1937	10
1938	11

(a) 1. No. (1930 as base)	(b) 1. No. (1935 as base)
$\frac{4}{4} \times 100 = 100$	$\frac{4}{10} \times 100 = 40$
$\frac{5}{4} \times 100 = 125$	$\frac{5}{10} \times 100 = 50$
$\frac{6}{4} \times 100 = 150$	$\frac{6}{10} \times 100 = 60$
$\frac{7}{4} \times 100 = 175$	$\frac{7}{10} \times 100 = 70$
$\frac{8}{4} \times 100 = 200$	$\frac{8}{10} \times 100 = 80$
$\frac{10}{4} \times 100 = 250$	$\frac{10}{10} \times 100 = 100$
$\frac{9}{4} \times 100 = 225$	$\frac{9}{10} \times 100 = 90$
$\frac{10}{4} \times 100 = 250$	$\frac{10}{10} \times 100 = 100$
$\frac{11}{4} \times 100 = 275$	$\frac{11}{10} \times 100 = 110$

-:4.7:-

Year	Price	Index Nos. (1919 as base)
1919	120	$\frac{120}{120} \times 100 = 100$
1920	122	$\frac{122}{120} \times 100 = 101.7$
1921	116	$\frac{116}{120} \times 100 = 96.7$
1922	120	$\frac{120}{120} \times 100 = 100$
1923	120	$\frac{120}{120} \times 100 = 100$
1924	137	$\frac{137}{120} \times 100 = 114.2$
1925	136	$\frac{136}{120} \times 100 = 113.3$
1926	149	$\frac{149}{120} \times 100 = 124.2$
1927	156	$\frac{156}{120} \times 100 = 130$
1928	137	$\frac{137}{120} \times 100 = 114.2$
1929	162	$\frac{162}{120} \times 100 = 135$
1930	149	$\frac{149}{120} \times 100 = 124.2$
1931	160	$\frac{160}{120} \times 100 = 133.33$
1932	160	$\frac{160}{120} \times 100 = 133.33$

-:4.8:-

Year	Price
1977	22.5
1978	25.0
1979	27.5
1980	30.0
1981	35.0
1982	32.5
1983	37.5
1984	47.5
1984	47.5
1985	45.0

I. Nos.(1977 as base)	I. Nos. (average as base)
$\frac{22.5}{22.5} \times 100 = 100$	$\frac{22.5}{33.61} \times 100 = 66.94$
$\frac{25.0}{22.5} \times 100 = 111.11$	$\frac{25.0}{33.61} \times 100 = 74.38$
$\frac{27.5}{22.5} \times 100 = 122.22$	$\frac{27.5}{33.61} \times 100 = 81.82$
$\frac{30.0}{22.5} \times 100 = 133.33$	$\frac{30.0}{33.61} \times 100 = 89.26$
$\frac{35.0}{22.5} \times 100 = 155.56$	$\frac{35.0}{33.61} \times 100 = 104.14$
$\frac{32.5}{22.5} \times 100 = 144.44$	$\frac{32.5}{33.61} \times 100 = 96.69$
$\frac{37.5}{22.5} \times 100 = 166.67$	$\frac{37.5}{33.61} \times 100 = 111.51$
$\frac{47.5}{22.5} \times 100 = 211.11$	$\frac{47.5}{33.61} \times 100 = 141.33$
$\frac{45.5}{22.5} \times 100 = 200.00$	$\frac{45.5}{33.61} \times 100 = 133.89$

Mean of 9 values $= \frac{302.5}{9} = 33.61$

-:4.9:-

Year	Price	Index Nos. (1970 as base)
1965	10.50	$\frac{10.50}{14.00} \times 100 = 75$
1966	11.75	$\frac{11.75}{14.00} \times 100 = 83.93$
1967	12.00	$\frac{12.00}{14.00} \times 100 = 85.71$
1968	12.75	$\frac{12.75}{14.00} \times 100 = 91.07$
1969	13.00	$\frac{13.00}{14.00} \times 100 = 92.86$
1970	14.00	$\frac{14.00}{14.00} \times 100 = 100$
1971	14.00	$\frac{14.00}{14.00} \times 100 = 100$
1972	15.75	$\frac{15.75}{14.00} \times 100 = 112.5$
1973	16.50	$\frac{16.50}{14.00} \times 100 = 112.5$
1974	18.00	$\frac{18.00}{14.00} \times 100 = 128.57$
1975	20.00	$\frac{20.00}{14.00} \times 100 = 142.86$

-:4.10:-

Year	Price
1970	9
1971	6
1972	9
1973	11
1974	10
1975	13
1976	14
1977	16

(a) I. No.	(b) I. No.
$\frac{9}{8}\times100=112.5$	$\frac{9}{11}\times100=81.82$
$\frac{6}{8}\times100=75$	$\frac{6}{11}\times100=54.54$
$\frac{9}{8}\times100=112.5$	$\frac{9}{11}\times100=81.82$
$\frac{11}{8}\times100=137.5$	$\frac{11}{11}\times100=100$
$\frac{10}{8}\times100=125$	$\frac{10}{11}\times100=90.91$
$\frac{13}{8}\times100=162.5$	$\frac{13}{11}\times100=118.19$
$\frac{14}{8}\times100=175$	$\frac{14}{11}\times100=127.27$
$\frac{16}{8}\times100=200$	$\frac{16}{11}\times100=145.45$

Mean of first 3 years $=\frac{9+6+9}{3}=\frac{24}{3}=8$

Mean of all 8 years $=\frac{9+6+9+11+10+13+14+16}{8}=\frac{88}{8}=11$

-:4.11:-

Year	Price	(i) Link Relative	(ii) Chain Relative
1970	30.5	$\frac{30.5}{30.5}\times100=100$	$=100$
1971	32.8	$\frac{32.8}{30.5}\times100=107.54$	$\frac{100\times107.54}{100}=107.51$
1972	40.2	$\frac{40.2}{32.8}\times100=122.56$	$\frac{107.54\times122.56}{100}=131.80$
1973	43.5	$\frac{43.5}{40.2}\times100=108.20$	$\frac{131.80\times108.20}{100}=142.60$
1974	44.5	$\frac{44.5}{43.5}\times100=102.29$	$\frac{142.60\times102.29}{100}=145.87$
1975	48.2	$\frac{48.2}{44.5}\times100=108.31$	$\frac{145.87\times108.31}{100}=157.99$

-:4.12:-

Year	Price	Link Relative	Chain Indices
1994	187	$\frac{187}{187} \times 100 = 100$	$= 100$
1995	161	$\frac{161}{187} \times 100 = 86.10$	$\frac{100 \times 86.10}{100} = 86.10$
1996	149	$\frac{149}{161} \times 100 = 92.55$	$\frac{86.10 \times 92.55}{100} = 79.69$
1997	142	$\frac{142}{149} \times 100 = 95.30$	$\frac{79.69 \times 95.30}{100} = 75.94$
1998	125	$\frac{125}{142} \times 100 = 88.03$	$\frac{75.94 \times 88.03}{100} = 66.85$
1999	129	$\frac{129}{125} \times 100 = 103.20$	$\frac{66.85 \times 103.20}{100} = 68.99$
2000	133	$\frac{133}{129} \times 100 = 103.10$	$\frac{68.99 \times 103.10}{100} = 71.13$

-:4.13:-

Year	Price	Link Relative	Chain Indices
1990	30	$\frac{30}{30} \times 100 = 100$	$= 100$
1991	32	$\frac{32}{30} \times 100 = 106\ 67$	$\frac{100 \times 106.67}{100} = 106.67$
1992	40	$\frac{40}{32} \times 100 = 125$	$\frac{106.67 \times 125}{100} = 133.34$
1993	43	$\frac{43}{40} \times 100 = 107.5$	$\frac{133.34 \times 107.5}{100} = 143.34$
1994	44	$\frac{44}{43} \times 100 = 102.33$	$\frac{143.34 \times 102.33}{100} = 146.68$
1995	48	$\frac{48}{44} \times 100 = 109.09$	$\frac{146.68 \times 109.09}{100} = 160.01$

-:4.14:-

Year	Price	Index No. (i) $P_0 = 22.50, \left(\frac{P_n}{P_0} \times 100\right)$	Index No. (ii) $P_0 = 30.07, \left(\frac{P_n}{P_0} \times 100\right)$
1977	22.50	$\frac{22.50}{22.50} \times 100 = 100$	$\frac{22.50}{30.07} \times 100 = 74.82$
1978	25.00	$\frac{25.00}{22.50} \times 100 = 111.11$	$\frac{25.00}{30.07} \times 100 = 83.13$
1979	27.50	$\frac{27.50}{22.50} \times 100 = 122.22$	$\frac{27.50}{30.07} \times 100 = 91.45$
1980	30.00	$\frac{30.00}{22.50} \times 100 = 1333.33$	$\frac{30.00}{30.07} \times 100 = 99.76$
1981	35.50	$\frac{35.50}{22.50} \times 100 = 157.78$	$\frac{35.50}{30.07} \times 100 = 118.05$
1982	32.50	$\frac{32.50}{22.50} \times 100 = 144.44$	$\frac{32.50}{30.07} \times 100 = 108.08$
1983	37.50	$\frac{37.50}{22.50} \times 100 = 166.67$	$\frac{37.50}{30.07} \times 100 = 124.70$

Average of Prices:

$$\frac{22.50 + 25.00 + 27.50 + 30.00 + 35.50 + 32.50 + 37.50}{7} = \frac{210.5}{7} = 30.07$$

-:4.15:-

Year	Price	Index No. (i) $\left(\frac{P_n}{P_0} \times 100\right)$	Index No. (ii) $\left(\frac{P_n}{P_0} \times 100\right)$
1990	65	$\frac{65}{65} \times 100 = 100$	$\frac{65}{70.67} \times 100 = 91.98$
1991	72	$\frac{72}{65} \times 100 = 110.77$	$\frac{72}{70.67} \times 100 = 101.88$
1992	75	$\frac{75}{65} \times 100 = 115.38$	$\frac{75}{70.67} \times 100 = 106.13$
1993	80	$\frac{80}{65} \times 100 = 123.08$	$\frac{80}{70.67} \times 100 = 113.20$
1994	85	$\frac{85}{65} \times 100 = 130.77$	$\frac{85}{70.67} \times 100 = 120.28$
1995	82	$\frac{82}{65} \times 100 = 126.15$	$\frac{82}{70.67} \times 100 = 116.03$

Average of Price of First 3 years:

$$\frac{65+72+75}{3}=\frac{212}{3}=70.67$$

-:4.16:-

Year	Price	(i) Price Relatives $\left(\frac{P_n}{P_0}\times 100\right)$	(ii) Link Relatives $\left(\frac{P_n}{P_{n-1}}\times 100\right)$
2001	1000	$\frac{1000}{1000}\times 100=100$	$\frac{1000}{1000}\times 100=100$
2002	1020	$\frac{1020}{1000}\times 100=102$	$\frac{1020}{1000}\times 100=102$
2003	1050	$\frac{1050}{1000}\times 100=105$	$\frac{1050}{1020}\times 100=102.94$
2004	1060	$\frac{1060}{1000}\times 100=106$	$\frac{1060}{1050}\times 100=100.95$
2005	1100	$\frac{1100}{1000}\times 100=110$	$\frac{1100}{1060}\times 100=103.77$

-:4.17:-

Year	Price	Index Nos. (1962 as base)
1960	50	$\frac{50}{55}\times 100=90.91$
1961	52	$\frac{52}{55}\times 100=94.55$
1962	55	$\frac{55}{55}\times 100=100$
1963	57	$\frac{57}{55}\times 100=103.64$
1964	62	$\frac{62}{55}\times 100=112.73$
1965	72	$\frac{72}{55}\times 100=130.91$
1966	73	$\frac{73}{55}\times 100=132.73$

Year	Price	Index Nos. (1962 as base)
1967	75	$\frac{75}{55} \times 100 = 136.36$
1968	71	$\frac{71}{55} \times 100 = 129.09$
1969	70	$\frac{70}{55} \times 100 = 127.27$

-:4.18:-

Year	Price
1964	20
1965	18
1966	23
1967	24
1968	25
1969	27
1970	28
1971	30
1972	32
1973	33

(i) I. No. (1964 as base)	(ii) Average of 5 years as base
$\frac{20}{20} \times 100 = 100$	$\frac{20}{22} \times 100 = 90.9$
$\frac{18}{20} \times 100 = 90$	$\frac{18}{22} \times 100 = 81.8$
$\frac{23}{20} \times 100 = 115$	$\frac{23}{22} \times 100 = 104.5$
$\frac{24}{20} \times 100 = 120$	$\frac{24}{22} \times 100 = 109.1$
$\frac{25}{20} \times 100 = 125$	$\frac{25}{22} \times 100 = 113.6$
$\frac{27}{20} \times 100 = 135$	$\frac{27}{22} \times 100 = 122.7$

(i) I. No. (1964 as base)	(ii) Average of 5 years as base
$\frac{28}{20}\times100=140$	$\frac{28}{22}\times100=127.3$
$\frac{30}{20}\times100=150$	$\frac{30}{22}\times100=136.4$
$\frac{32}{20}\times100=160$	$\frac{32}{22}\times100=145.5$
$\frac{33}{20}\times100=165$	$\frac{33}{22}\times100=150$

(i) Mean of first 5 years $= \frac{20+18+23+24+25}{5} = \frac{110}{5} = 22$

(ii) Mean of all values $= \frac{260}{10} = 26$

(iii) Average of all values as base	(iv) Link Relative	Chain Index Nos.
$\frac{20}{26}\times100=76.9$	$\frac{20}{20}\times100=100$	$=100$
$\frac{18}{26}\times100=69.2$	$\frac{18}{20}\times100=90$	$\frac{100\times90}{100}=90$
$\frac{23}{26}\times100=88.5$	$\frac{28}{18}\times100=127.7$	$\frac{90\times127.7}{100}=114.93$
$\frac{24}{26}\times100=92.3$	$\frac{24}{23}\times100=104.3$	$\frac{114.93\times104.3}{100}=119.87$
$\frac{25}{26}\times100=96.2$	$\frac{25}{24}\times100=104.17$	$\frac{119.87\times104.17}{100}=124.87$
$\frac{27}{26}\times100=103.8$	$\frac{27}{25}\times100=108$	$\frac{124.87\times108}{100}=134.49$
$\frac{28}{26}\times100=115.4$	$\frac{28}{27}\times100=103.7$	$\frac{134.49\times103.7}{100}=139.47$
$\frac{30}{26}\times100=115.4$	$\frac{30}{28}\times100=107.14$	$\frac{139.47\times107.14}{100}=149.43$

(iii) Average of all values as base	(iv) Link Relative	Chain Index Nos.
$\frac{32}{26}\times100=123.1$	$\frac{32}{30}\times100=106.69$	$\frac{149.43\times106.69}{100}=159.94$
$\frac{33}{26}\times100=126.9$	$\frac{33}{32}\times100=103.13$	$\frac{159.94\times103.13}{100}=164.95$

-:4.19:-

(a)

Year	Price	I. Nos. (1953 as base)
1953	14.95	$\frac{14.95}{14.95}\times100=100$
1954	14.95	$\frac{14.95}{14.95}\times100=100$
1955	15.10	$\frac{15.10}{14.95}\times100=101$
1956	15.65	$\frac{15.65}{14.95}\times100=104.7$
1957	16.28	$\frac{16.28}{14.95}\times100=108.9$
1958	16.53	$\frac{16.53}{14.95}\times100=110.6$

(b)

Year	Price	Link Relative	Chain Index Nos.
1953	14.95	= 100	= 100
1954	14.95	$\frac{14.95}{14.95}\times100=100$	$\frac{100\times100}{100}=100$
1955	15.10	$\frac{15.10}{14.95}\times100=101$	$\frac{100\times101}{100}=101$
1956	15.65	$\frac{15.65}{15.10}\times100=103.64$	$\frac{101\times103.64}{100}=104.68$
1957	16.28	$\frac{16.28}{15.65}\times100=104$	$\frac{104.68\times104}{100}=108.87$
1958	16.53	$\frac{16.53}{16.28}\times100=101.54$	$\frac{108.87\times101.54}{100}=110.55$

-:4.20:-

Year	Price	Link Relative	Chain Index Nos.
1960	2.4	$\frac{2.4}{2.4} \times 100 = 100$	$= 100$
1961	2.5	$\frac{2.5}{2.4} \times 100 = 104.2$	$\frac{100 \times 104.2}{100} = 104.20$
1962	2.8	$\frac{2.8}{2.5} \times 100 = 112$	$\frac{104.2 \times 112}{100} = 116.70$
1963	2.9	$\frac{2.9}{2.5} \times 100 = 103.6$	$\frac{116.70 \times 103.6}{100} = 120.90$
1964	3.1	$\frac{3.1}{2.9} \times 100 = 106.9$	$\frac{120.90 \times 106.9}{100} = 129.24$
1965	3.3	$\frac{3.3}{3.1} \times 100 = 106.5$	$\frac{129.24 \times 106.5}{100} = 137.76$
1966	3.5	$\frac{3.5}{3.3} \times 100 = 106.1$	$\frac{137.76 \times 106.1}{100} = 146.16$
1967	4.2	$\frac{4.2}{3.5} \times 100 = 120$	$\frac{146.16 \times 120}{100} = 175.39$
1968	5.1	$\frac{5.1}{4.2} \times 100 = 121.4$	$\frac{175.39 \times 121.4}{100} = 212.92$
1969	6.4	$\frac{6.4}{5.1} \times 100 = 125.5$	$\frac{212.92 \times 125.5}{100} = 267.21$
1970	5.7	$\frac{5.7}{6.4} \times 100 = 89.1$	$\frac{267.21 \times 89.1}{100} = 238.04$

-:4.21:-

Year	Price	Link Relative	Chain Index Nos. (1975 as base)
1975	58	$\frac{58}{58} \times 100 = 100$	$= 100$
1976	62	$\frac{62}{58} \times 100 = 106.9$	$\frac{100 \times 106.9}{100} = 106.9$
1977	64	$\frac{64}{62} \times 100 = 103.23$	$\frac{106.9 \times 103.23}{100} = 110.35$
1978	66.5	$\frac{66.5}{64} \times 100 = 103.91$	$\frac{110.35 \times 103.91}{100} = 114.67$

Year	Price	Link Relative	Chain Index Nos. (1975 as base)
1979	67	$\frac{67}{66.5} \times 100 = 100.75$	$\frac{114.67 \times 100.75}{100} = 115.53$
1980	69	$\frac{69}{67} \times 100 = 102.99$	$\frac{115.53 \times 102.99}{100} = 118.98$
1981	67	$\frac{67}{69} \times 100 = 97.10$	$\frac{118.98 \times 97.10}{100} = 115.53$
1982	70	$\frac{70}{67} \times 100 = 104.48$	$\frac{115.53 \times 104.48}{100} = 120.71$

-:4.22:-

Year	Price	Link Relative	Chain Induces
1970	9	$\frac{9}{9} \times 100 = 100$	$= 100$
1971	11	$\frac{11}{9} \times 100 = 122.22$	$\frac{100 \times 122.22}{100} = 122.22$
1972	10	$\frac{10}{11} \times 100 = 90.91$	$\frac{122.22 \times 90.91}{100} = 111.11$
1973	13	$\frac{13}{10} \times 100 = 130$	$\frac{111.11 \times 130}{100} = 144.44$
1974	14.5	$\frac{14.5}{13} \times 100 = 111.54$	$\frac{144.44 \times 111.54}{100} = 161.11$
1975	16	$\frac{16}{14.5} \times 100 = 110.34$	$\frac{161.11 \times 110.34}{100} = 177.77$
1976	18	$\frac{18}{16} \times 100 = 112.50$	$\frac{177.77 \times 112.50}{100} = 199.99$

-:4.23:-

Year	Price	Link Relative	Index Nos. (1919 as base)
1960	40	$\frac{40}{40} \times 100 = 100$	$= 100$
1961	45	$\frac{45}{40} \times 100 = 112.5$	$\frac{100 \times 112.5}{100} = 112.5$
1962	48	$\frac{48}{45} \times 100 = 106.5$	$\frac{112.5 \times 106.5}{100} = 119.81$

Year	Price	Link Relative	Index Nos. (1919 as base)
1963	50	$\frac{50}{48} \times 100 = 104.2$	$\frac{119.81 \times 104.2}{100} = 124.84$
1964	52	$\frac{52}{50} \times 100 = 104.0$	$\frac{124.84 \times 104}{100} = 129.83$
1965	54	$\frac{54}{52} \times 100 = 103.8$	$\frac{129.83 \times 103.8}{100} = 134.76$
1966	56	$\frac{56}{54} \times 100 = 103.7$	$\frac{134.76 \times 103.7}{100} = 139.75$
1967	60	$\frac{60}{56} \times 100 = 107.1$	$\frac{139.75 \times 107.1}{100} = 149.67$

-:4.24:-

Year	Price Relatives			
	Wheat	Rice	Cotton	Ghee
1960	100	100	100	100
1961	105	106.67	110	112.5
1962	110	110	116.67	125
1963	120	116.67	120	140.63
1964	125	120	125	150

Total	Index Nos.
400	100
434.17	108.5
461.67	115.4
497.3	124.3
520	130.0

-:4.25:-

The necessary calculations are shown below:

Year	Price Relatives			
	A	B	C	D
1964	100	100	100	100
1965	116.67	116.67	107.14	120
1966	133.33	125	116.07	128.51
1967	150	133.33	121.43	135.71

Index Nos.	
(i)	(ii)
100	100
115.1	116.7
125.8	126.8
135.1	134.5

-:4.26:-

Year	Price Relatives		
	Gold	Wheat	Cotton
1912	100	100	100
1913	121.74	83.82	69.23
1914	132.02	28.32	85.89
1915	140.32	32.95	71.79
1916	139.53	98.84	92.31
1917	142.29	67.1	130.77

Index Nos.	
Total	Mean
300	100
274.79	91.6
241.23	82.1
245.06	81.7
370.68	110.2
340.16	113.4

-:4.27:-

Year	Price Relatives			
	Wheat	Rice	Cotton	Sugar
1980	100	100	100	100
1981	108.33	115.38	114.28	107.69
1982	120	123.08	128.57	115.38
1983	133.33	153.85	139.05	132.69
1984	141.67	184.61	147.62	138.46

Index Nos.		
(i) Mean	(ii) Median	(iii) G.M.
100	100	100
111.42	111.85	111.37
121.75	121.54	121.66
139.73	136.19	139.48
153.09	144.65	152.05

-:4.28:-

Year	Price Relative				Index nos.
	A	B	C	D	G.M.
1985	100	100	100	100	100
1986	128	117.142	122.5	124.44	125.299
1987	204	164.285	150	142.22	163.5
1988	280	211.428	187.5	160	205.26

-:4.29:-

Year	Price Relatives				Index Nos.
	Blades	Cigarettes	Soap	Salt	Mean
1985	100	100	100	100	100
1986	101.67	110.33	117.52	108.18	110
1987	108	119.17	124	114.54	117
1988	108.33	117.52	123.5	113.64	116
1869	115	133.33	130.5	122.73	124

-:4.30:-

(a)

Year	Price Relatives			Index Nos.
	Wheat	Rice	Maize	Med.
1970	100	100	100	100
1971	93.3	131.4	107.4	107.4
1972	133.3	139.3	142.6	139.3
1973	131.7	135.7	162.9	135.7
1974	133.3	144.3	157.4	144.3

(b)

Year	Link Relatives		
	Wheat	Rice	Maize
1970	100	100	100
1971	93.3	131.4	107.4
1972	121.45	105.98	132.76
1973	116.17	97.44	114.86
1974	101.26	106.32	96.59

Mean	Chain Index Nos.
100	= 100
110.6	$\frac{100 \times 110.6}{100} = 110.6$
120.1	$\frac{110.6 \times 120.1}{100} = 132.8$
109.49	$\frac{132.8 \times 109.49}{100} = 145.4$
101.39	$\frac{101.39 \times 145.49}{100} = 147.0$

-:4.31:-

Year	Link Relatives			
	A	B	C	D
1910	100	100	100	100
1911	76.54	70.13	116.36	94.55
1912	167.74	161.11	86.72	192.31
1913	89.42	86.20	138.73	96.00
1914	64.52	57.33	107.14	91.66

G.M.	Chain Index Nos.
100	= 100
87.66	$\frac{100 \times 87.66}{100} = 87.66$
145.7	$\frac{87.66 \times 145.7}{100} = 127.72$
100.66	$\frac{127.72 \times 100.06}{100} = 128.56$
77.63	$\frac{128.56 \times 77.63}{100} = 99.8$

-:4.32:-

Year	Link Relatives			
	Wheat	Sugar	Ghee	Cotton
1941	100	100	100	100
1942	125	125	125	120
1943	90	140	96	133.331
1944	138.89	114.29	25	125

(i)

A.M.	Chain Index Nos.
100	= 100
123.75	$\frac{100 \times 123.75}{100} = 124$
114.33	$\frac{124 \times 114.33}{100} = 114.67$
125.86	$\frac{114.67 \times 125.86}{100} = 125$

(ii)

Median	Chain Index Nos.
100	= 100
125	$\frac{100 \times 125}{100} = 125$
114.33	$\frac{125 \times 114.67}{100} = 112.67$
125	$\frac{112.67 \times 125}{100} = 125.5$

(iii)

G.M.	Chain Index Nos.
100	= 100
123.7	$\frac{100 \times 123.7}{100} = 123.7$
112.67	$\frac{123.7 \times 112.67}{100} = 139.4$
125.5	$\frac{139.4 \times 125.5}{100} = 174.9$

-:4.33:-

Year	Link Relatives			
	Sugar	Milk	Coffee	Tea
1955	100	100	100	100
1956	101.23	70.13	107.56	154.55
1957	126.83	161.11	86.72	117.65
1958	89.42	86.21	138.74	96.00
1959	64.52	57.33	107.14	91.67
1960	100.00	102.33	96.36	101.14
1961	103.33	106.82	87.74	94.38

Mean	Chain Indelces
100	$= 100$
108.37	$\frac{100 \times 108.37}{100} = 108.37$
123.08	$\frac{108.37 \times 123.08}{100} = 133.38$
102.59	$\frac{133.38 \times 102.59}{100} = 136.84$
80.17	$\frac{136.84 \times 80.17}{100} = 109.70$
99.96	$\frac{109.70 \times 99.96}{100} = 109.66$
98.07	$\frac{109.66 \times 98.07}{100} = 107.54$

-:4.34:-

Year	Link Relatives		
	A	B	C
1944	100	100	100
1945	121.74	94.12	87.16
1946	108.44	87.50	102.33
1947	106.29	110.71	108.33
1948	99.44	103.23	100.00
1949	101.98	121.88	105.59
1950	100.00	76.92	92.72
1951	105.00	113.33	123.57

Mean	Chain Indices
100	$= 100$
101.1	$\frac{100 \times 101.1}{100} = 101.1$
99.42	$\frac{101.1 \times 99.42}{100} = 100.42$
108.44	$\frac{108.90 \times 100.89}{100} = 109.87$
100.89	$\frac{108.90 \times 100.89}{100} = 109.87$

Mean	Chain Indices
109.82	$\frac{109.87 \times 109.82}{100} = 120.66$
89.88	$\frac{120.66 \times 89.88}{100} = 108.45$
123.57	$\frac{108.45 \times 123.57}{100} = 123.60$

-:4.35:-

Commodity	1960		1961	
	p_0	q_0	p_1	q_1
A	8	50	20	60
B	2	15	6	10
C	1	20	2	25
D	2	10	5	8
E	1	40	3	30

p_1q_0	p_0q_0	p_1q_1	p_0q_1
1000	400	1200	480
90	30	60	20
40	20	50	25
50	20	40	16
120	40	90	30
1300	**510**	**1440**	**571**

(i) **Laspeyre's Index No.**

$$P_{0n} = \frac{\sum p_n q_0}{\sum p_0 q_0} \times 100$$

Laspeyre's Index No. for 1961.

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{1300}{510} \times 100 = 254.9$$

(ii) **Paasche's Index No.**

$$P_{0n} = \frac{\sum p_n q_n}{\sum p_0 q_1} \times 100$$

Paasche's Index No. for 1961.

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{1440}{571} = 100 = 252.19$$

(iii) **Fisher's Ideal Index No.**

$$P_{0n} = \sqrt{\frac{\sum P_n q_0}{\sum P_0 q_0} \times \frac{\sum P_n q_n}{\sum P_0 q_n}} \times 100$$

Fisher's Ideal Index No. for1961.

$$P_{01} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100 = P_{01} = \sqrt{\frac{1300}{510} \times \frac{1440}{571}} \times 100$$

$$P_{01} = \sqrt{2.5490 \times 2.5219} \times 100 = 253.54$$

(iv) **Marshall Edgeworth Index No.**

$$P_{0n} = \frac{\sum P_nq_0 + \sum P_nq_n}{\sum P_0q_0 + \sum P_0q_n} \times 100$$

Marshall Edgeworth ndex No for 1961.

$$P_{01} = \frac{\sum P_1q_0 + \sum P_1q_1}{\sum P_0q_0 + \sum P_0q_1} \times = 100 = \frac{1300 + 1440}{510 + 571} \times 100$$

$$= \frac{274000}{1081} = 253.47$$

-:4.36:-

Commodity	1985		1995		P_1q_1	P_0q_1
	Price p_0	Quantity q_0	Price p_1	Quantity q_1		
Wheat	6	45	10	40	400	240
Rice	7	50	12	35	420	245
Cotton	8	60	16	30	480	240
					1300	**725**

Paache's Index No.

$$P_{0n} = \frac{\sum P_nq_n}{\sum P_0q_n} \times 100$$

Paache's Index No. For 1995

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{1300}{725} \times 100 = 179.31$$

-:4.37:-

Commodity	2004		2005	
	p_0	q_0	P_1	q_1
A	2	50	10	40
B	3	10	8	50
C	4	60	4	80
Total	-----	-----	-----	-----

p_1q_0	p_0q_0	p_1q_1	p_0q_1
500	100	400	80
80	30	400	150
240	240	320	320
820	**370**	**1120**	**550**

Fisher's Ideal Index No.

$$P_{0n} = \sqrt{\frac{\sum P_n q_n}{\sum P_0 q_0} \times \frac{\sum P_n q_n}{\sum P_0 q_n}} \times 100$$

Fisher's Ideal Index No. For 2005

$$P_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$$

$$= \sqrt{\frac{820}{370} \times \frac{1120}{550}} \times 100 = \sqrt{2.21 \times 2.03} \times 100 = \sqrt{4.48} \times 100$$

$$= 2.1166 \times 100 = 211.66$$

-:4.38:-

Commodity	1973		1975	
	p_0	q_0	P_1	q_1
Wheat	40	180	42	95
Rice	10	75	11	70
Bajra	12	110	14	125
Total	-----	-----	-----	-----

p_1q_0	p_0q_0	p_1q_1	p_0q_1
7560	7200	3990	3800
825	750	770	700
1540	1320	1750	1500
9925	**9270**	**6510**	**6000**

(i) **Laspeyre's Index No.**

$$P_{0n} = \frac{\sum P_n q_0}{\sum P_0 q_0} \times 100$$

Laspeyre's Index No. for 1973.

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{9925}{9270} \times 100 = 107.07$$

(ii) **Paasche's Index No.**

$$P_{0n} = \frac{\sum P_n q_n}{\sum P_0 q_n} \times 100$$

Paasche's Index No. for 1973.

$$P_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 = \frac{6510}{6000} \times 100 = 108.05$$

(iii) Fisher's Ideal Index No.

$$P_{0n} = \sqrt{\frac{\sum P_n q_0}{\sum P_0 q_0} \times \frac{\sum P_n q_n}{\sum P_0 q_n}} = 100$$

Fisher's Ideal Index No. for 1973.

$$P_{0n} = \sqrt{\frac{9925}{9270} \times \frac{6510}{6000}} = 100 = P_{0n} = \sqrt{1.070 \times 1.085} \times 100 = 107.8$$

(iv) Marshall Edgeworth Index No.

$$P_{0n} = \frac{\sum P_n q_0 + \sum P_n q_n}{\sum P_0 q_0 + \sum P_0 q_n} \times 100$$

Marshall Edgeworth Index No. for1973.

$$P_{01} = \frac{\sum P_1 q_0 + \sum P_1 q_1}{\sum P_0 q_0 + \sum P_0 q_1} \times 100 = P_{01} = \frac{9925 + 6410}{9270 + 6000} \times 100$$

$$= \frac{16435}{15270} \times 100 = 107.6$$

-:4.39:-

Commodity	1973		1963	
	p_0	q_0	p_1	q_1
A	2	50	10	40
B	3	10	8	5
C	4	5	4	5
Total	-----	-----	-----	-----

P_1q_0	p_0q_1	p_1q_1	p_0q_1
500	100	400	80
80	30	40	15
20	20	20	20
600	**150**	**460**	**115**

(i) Laspeyre's Index No. for 1963.

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{600}{150} \times 100 = 400$$

(ii) Paasche's Index No. for1963.

$$P_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 = \frac{460}{115} \times 100 = 400$$

(iiii) Fisher's Ideal Index No. for1963.

$$P_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100 = P_{01} = \sqrt{\frac{600}{150} \times \frac{460}{115}} \times 100$$

$$P_{01} = \sqrt{4 \times 4} \times 100 = 400$$

-:4.40:-

Commodity	1975		1980	
	p_0	q_0	p_1	q_1
Wheat	55	100	63	130
Jawar	25	55	29	45
Gur	27	130	32	85
Total	----	----	----	----

p_1q_0	p_0q_0	p_1q_1	p_0q_1
6300	5500	8190	7150
1595	1375	1305	1125
4160	3510	2720	2295
12055	**10385**	**12215**	**10570**

(i) Laspeyre's Index No. for 1980.

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{12055}{10385} \times 100 = 116.08$$

(ii) Paaschr's Index No. for 1980

$$P_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 = \frac{12215}{10570} \times 100 = 115.56$$

(iii) Fisher's Ideal Index No. for 1980

$$P_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100 = P_{01} = \sqrt{\frac{12055}{10385} \times \frac{12215}{10570}} \times 100$$

$$P_{01} = \sqrt{1.161 \times 1.556} \times 100 = 115.82$$

-:4.41:-

Commodity	1980		1981	
	p_0	q_0	p_1	q_1
A	10	25	13	21
B	9	27	12	22
C	4	10	3	14

p_1q_0	p_0q_0	p_1q_1	p_0q_1
225	250	243	270
252	273	264	286
30	40	42	56
507	563	549	612

(i) Laspeyre's Index No. for 1981

$$P_{01} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100 = \frac{507}{563} \times 100 = 90.05$$

(ii) Paasche's Indes No. for 1981

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{549}{612} \times 100 = 89.71$$

(iii) Fisher's Ideal Index No for 1981

$$P_{0n} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100 = P_{01} = \sqrt{\frac{507}{563} \times \frac{549}{612}} \times 100$$

$$P_{01} = \sqrt{0.898794} \times 100 = 89.88$$

-:4.42:-

Commodity	1960		1964	
	p_0	q_0	p_1	q_1
Milk	3.95	9675	4.25	10436
Cheese	34.80	78	38.90	83
Butter	61.56	118	59.70	116

p_1q_0	p_0q_0	p_1q_1	p_0q_1
41118.75	38216.25	44353.0	41222.2
3034.20	2714.40	3228.7	2888.4
7044.60	7264.08	6925.2	7140.96
51197.55	**48194.73**	**54506.9**	**51251.56**

(i) Laspeyre's Index No. for 1964

$$P_{01} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100 = \frac{51197.55}{48194.73} \times 100 = 106.23$$

(ii) Paasche's Index No. for 1964

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{54506.9}{51251.56} \times 100 = 106.35$$

-:4.43:-

Commodity	1970		1971		1972	
	p_0	q_0	p_1	q_1	q_2	q_3
Wheat	4	50	4	50	5	40
Gram	3	100	8	50	5	80
Tobacco	2	500	10	400	8	200

p_1q_0	p_0q_0	p_1q_1	p_0q_1
200	250	200	200
800	500	300	400
5000	4000	1000	4000
6000	**4750**	**1500**	**4600**

p_2q_2	p_0q_1	p_2q_1
200	200	160
400	150	240
1600	800	400
2200	**1150**	**800**

(i) Laspeyre's Index No. for 1971

$$P_{01} = \frac{\sum P_1q_0}{\sum P_0q_0} \times 100 = \frac{6000}{1500} \times 100 = 400$$

Laspeyre's Index No. for 1972

$$P_{02} = \frac{\sum P_2q_0}{\sum P_0q_0} \times 100 = \frac{4750}{1500} \times 100 = 316.67$$

(ii) Paasche's Index No. for 1971

$$P_{01} = \frac{\sum P_1q_1}{\sum P_0q_1} \times 100 = \frac{4600}{1500} \times 100 = 400$$

Paasche's Index No for 1972

$$P_{02} = \frac{\sum P_2q_2}{\sum P_0q_2} \times 100 = \frac{2200}{800} \times 100 = 275$$

(iii) Fisher's Ideal Index No. for 1971

$$P_{01} = \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100 = P_{01} = \sqrt{\frac{6000}{1500} \times \frac{4600}{1150}} \times 100$$

$$P_{01} = \sqrt{4 \times 4} \times 100 = 400$$

Fisher's Ideal Index No. for 1972.

$$P_{02} = \sqrt{\frac{\sum P_2q_0}{\sum P_0q_0} \times \frac{\sum P_2q_2}{\sum P_0q_2}} \times 100 = P_{01} = \sqrt{\frac{4750}{1500} \times \frac{2200}{800}} \times 100$$

$$= 295.00$$

-:4.44:-

	1986		1987		1988	
	p_0	q_0	p_1	q_1	p_2	q_2
A	5	100	6	120	10	56
B	7	120	10	80	21	20
C	10	80	12	80	6	60
D	4	50	5	60	12	24
E	8	70	8	80	12	36

p_0q_1	p_1q_0	p_1q_1	p_0q_1	p_0q_2	p_2q_2	p_2q_0
500	600	720	600	560	280	1000
840	1200	800	500	240	840	240
800	960	960	800	360	600	480
200	250	300	240	288	96	600
560	560	640	640	432	288	840
2900	**3570**	**3420**	**2840**	**1880**	**2104**	**3160**

1. Laspeyre's Index or Base Yer Weighted Index

$$P_{0n} = \frac{\sum p_n q_0}{\sum p_0 q_0} \times 100$$

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{3570}{2900} \times 100 = 123.10$$

$$P_{02} = \frac{\sum p_2 q_0}{\sum p_0 q_0} \times 100 = \frac{3160}{2900} \times 100 = 108.96$$

2. Paasche's Index or Current Year Weighted Index

$$P_{0n} = \frac{\sum p_n q_n}{\sum p_0 q_n} \times 100$$

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{3420}{2840} \times 100 = 120.42$$

$$P_{02} = \frac{\sum p_2 q_2}{\sum p_0 q_2} \times 100 = \frac{1680}{1404} \times 100 = 119.66$$

3. Fisher's Ideal Index

$$P_{01} = \sqrt{\frac{\sum p_n q_0}{\sum p_0 q_0} \times \frac{\sum p_n q_n}{\sum p_0 q_n}} \times 100$$

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100 = \sqrt{\frac{3580}{2800} \times \frac{3420}{2840}} \times 100 = 121.93$$

$$P_{02} = \sqrt{\frac{\sum p_2 q_0}{\sum p_0 q_0} \times \frac{\sum p_2 q_2}{\sum p_0 q_2}} \times 100 = \sqrt{\frac{3160}{2900} \times \frac{1680}{1404}} \times 100 = 114.18$$

4. Marshall Edgeworth Index

$$P_{0n} = \frac{\sum p_n(q_0+q_n)}{\sum p_0(q_0+q_n)} \times 100 = \frac{\sum p_n q_0 + \sum p_n q_n}{\sum p_0 q_0 + \sum p_0 q_n} \times 100$$

$$P_{01} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100 = \frac{3580+3420}{2900+2840} \times 100 = \frac{7000}{5740} \times 100 = 121.95$$

$$P_{02} = \frac{\sum p_2 q_0 + \sum p_2 q_2}{\sum p_0 q_0 + \sum p_0 q_2} \times 100 = \frac{3160+1680}{2900+1404} \times 100 = \frac{4840}{53404} \times 100 = 112.45$$

-:4.45:-

Commodity	146		1950		1951	
	p_0	q_0	p_1	q_1	p_2	q_2
A	64	270	75	276	80	290
B	40	124	45	118	41	144
C	18	130	21	121	20	137
D	58	185	68	167	56	355

p_1q_0	p_2q_0	p_0q_0	p_1q_1
20250	21600	17280	20700
5580	5084	4960	5310
2780	2600	2340	2541
12580	10360	10730	11356
41140	**39644**	**35310**	**39907**

p_2q_2	p_0q_1	p_0q_2
23200	17664	18560
5904	4720	5760
2740	2178	2466
19880	9686	20590
51724	**34248**	**47376**

(i) Laspeyre's Index No. for 1950

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100 = \frac{41140}{35310} \times 100 = 116.5$$

Laspeyre's Index No. for 1951

$$P_{02} = \frac{\sum P_2 q_0}{\sum P_0 q_0} \times 100 = \frac{39644}{35310} \times 100 = 112.3$$

(ii) Paasche's Index No. for 1950

$$P_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100 = \frac{39907}{34248} \times 100 = 166.5$$

Paasche's Index No. for 1951

$$P_{02} = \frac{\sum P_2 q_2}{\sum P_0 q_2} \times 100 = \frac{51724}{47376} \times 100 = 109.2$$

(iii) **Fisher's Ideal Index No. for 1950**

$$P_{0n} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100 = P_{01} = \sqrt{\frac{41140}{35310} \times \frac{39907}{34248}} \times 100 = 116.5$$

Fisher's Ideal Index No. for 1951

$$P_{02} = \sqrt{\frac{\sum P_2 q_0}{\sum P_0 q_0} \times \frac{\sum P_2 q_2}{\sum P_0 q_2}} \times 100 = P_{01} = \sqrt{\frac{39644}{35310} \times \frac{51724}{47376}} \times 100 = 110.7$$

-:4.46:-

Commodity	Quantity Consumed in 1939	Unit of Price
	q_0	
Rice	6 mds	pr md
Wheat	6 mds	"
Gram	1 md	"
Sugar	1 md	"
Arhar	6 mds	"
Ghee	4 seers	per seer

Price in 1939	Price in 1940
p_0	p_1
5.75	6.00
5.00	8.00
6.00	9.00
20.00	15.00
8.00	10..00
2.00	1.50

Aggregate Expenditure in base year	Aggregate Expenditure in Current year
$p_0q_0 = w$	p_1q_0
34.50	36.00
30.00	48.00
6.00	9.00
20.00	15.00
48.00	60.00
8.00	6.00
146.50	**174.00**

Price Relative $\frac{p_1}{p_0} \times 100 = I$	W x I
104.3	3598.36
160.0	4800.00
150.0	900.00
75.0	500.00
125.0	6000.00
75.0	600.00
-----	17398.35

(i) **Aggregate Expenditure Method**

$$P_{0n} = \frac{\sum p_n q_0}{\sum p_0 q_0} \times 100$$

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{174}{146.50} \times 100 = 118.8$$

(ii) **Family Budget Method**

$$= \frac{\sum W \times I}{\sum W} = \frac{17398.36}{146.50} = 118.8$$

-:4.47:-

Commodity	Quantity Consumed in 1939 q_0	Unit of Price
Wheat	0.50 md	per md
Rice	0.20 "	"
Sugar	0.05 "	"
Ghee	0.025 "	"
Milk	25 seer	Per seer
Vegetables	16 seer	"
Mutton	5 seer	"
Fuel	4 md	per md

Price in 1956 p_0	Price in 1962 p_1
12	15
40	45
40	50
110	120
0.50	0.75
0.25	0.50
3.00	4.00
2.50	3.00

Aggregate Expenditure in base year	Aggregate Expenditure in Current year
$p_0q_0 = W$	p_1q_0
6.00	7.50
8.00	9.00
2.00	2.50
2.75	3.00
12.50	18.75
4.00	8.00
15.00	20.00
10.00	12.00
59	**80.75**

Price Relative $\frac{p_1}{p_0} \times 100 = I$	W x I
1250.00	750.00
1125.00	900.00
125.00	250.00
109.09	230.00
150.00	1875.00
200.00	800.00
133.33	1999.95
120.00	800.495

(i) Aggregate Expenditure Method

$$P_{0n} = \frac{\sum p_n q_0}{\sum p_0 q_0} \times 100$$

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{80.75}{59} \times 100 = 136$$

(ii) Family Budget Method

$$P_{01} = \frac{\sum W \times I}{\sum W} = \frac{8004.95}{59} = 136$$

EXERCISE NO. 5

-:5.1:-

(i) $11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 39916800$

(ii) $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$

(iii) $\frac{12!}{8!0!} = \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 0!} = 12 \times 11 \times 10 \times 9 = 11880$

(iv) $\frac{12!8!}{9!6!} = \frac{12 \times 11 \times 10 \times 9 \times 8! \times 6!}{9! \times 6!} = 12 \times 11 \times 10 \times 8 \times 7 = 73920$

(v) $\frac{10!8!}{9!5!} = \frac{(10 \times 9!) \times (8 \times 7 \times 6 \times 5!)}{9! \times 5!} = 10 \times 8 \times 7 \times 6 = 3360$

(vi) $\frac{13!4!}{5!2!} = \frac{13! \times 4 \times 3 \times 2 \times 1!}{15 \times 14 \times 13! \times 2 \times 1!} = \frac{4 \times 3}{15 \times 14} = \frac{2}{35}$

(vii) $\frac{10!3!}{13!} = \frac{10! \times 3 \times 2 \times 1}{13 \times 12 \times 11 \times 10!} = \frac{3 \times 2 \times 1}{13 \times 12 \times 11} = \frac{6}{1716} = \frac{1}{286}$

(viii) $\frac{13!}{10!3!} = \frac{13 \times 12 \times 11 \times 10!}{10! \times 3 \times 2 \times 1} = 286$

-:5.2:-

(i) ${}^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = 10 \times 9 \times 8 = 720$

(ii) ${}^{9}P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$

(iii) ${}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!}$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$

(iv) ${}^{8}P_7 = \frac{8!}{(8-7)!} = \frac{8!}{1!} = 8 \times 7 \times 6 \times 5 \times 4 \times 2 \times 1! = 40320$

(v) ${}^{25}P_5 = \frac{25!}{(25-5)!} = \frac{25!}{20!} = \frac{25 \times 24 \times 23 \times 22 \times 21 \times 20!}{20!}$

$= 25 \times 24 \times 23 \times 22 \times 21 = 6375600$

-:5.3:-

The number of digits are 6. The numbers of 3 digits can be formed in the following ways:

$$^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3} = 6 \times 5 \times 4 = 120$$

-:5.4:-

The number of travelers are three and number of hotels are four. They can take their quarters each in the following ways:

$$^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4 \times 3 \times 2 \times 1 = 24$$

-:5.5:-

The number of students = 4, The number of doors = 6, 4 students can enter in 6 doors in following different ways.

$$^nP_r = {}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 6 \times 5 \times 4 \times 3 = 360$$

-:5.6:-

Here n = 8, r = 4

$$^nP_r = {}^8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$$

-:5.7:-

The word Statistics contains 10 letters in all, of these the 3S's are alike of one kind, the 3T's are alike of one kind, 2I's are alike. There is one C and One A. Thus we have

Total No. N = 10

No. of S's = 3, No. of T's = 3, No. of C's = 1, No. of A = 1, No of I = 2

Hence required number of arrangements are given below:

$$P = \frac{10!}{3!3!2!1!1!} = 75600$$

-:5.8:-

The word FAISALABAD contains 10 letters, we have

Total No. n = 10

No. of A's = 4, No. of B's = 1, No. of D's = 1, No. of S's = 1, No. of F's = 1, No. of L's = 1, No. of I's = 1

Hence required number of arrangements are given below:

$$P = \frac{10!}{3!1!1!1!1!1!} = 151200$$

-:5.9:-

Here n = 6 Flags. Red Flags = 3, Green Flags = 2, Yellow Flags = 1
Required number of arrangements are given

$$P = \frac{6!}{3!2!1!} = 60$$

-:5.10:-

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

(i) ${}^{8}C_{5}$ Here n = 8, r = 5

$${}^{n}C_{r} = {}^{8}C_{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6 \times 5!}{5 \times 3 \times 2 \times 1} = 8 \times 7 = 56$$

(ii) ${}^{7}C_{3}$ Here n = 7, r = 3

$${}^{n}C_{r} = {}^{7}C_{3} = \frac{7}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

(iii) ${}^{24}C_{4}$ Here n = 24, r = 4

$${}^{n}C_{r} = {}^{24}C_{4} = \frac{24!}{4!(24-4)!} = \frac{24!}{4!20!} = \frac{24 \times 23 \times 22 \times 21 \times 20!}{4 \times 3 \times 2 \times 1 \times 20!}$$

$$= 23 \times 22 \times 21 = 10626$$

(iv) ${}^{19}C_{4}$ Here n = 19, r = 4

$${}^{n}C_{r} = {}^{19}C_{4} = \frac{19!}{4!(19-4)!} = \frac{19!}{4!5!} = \frac{19 \times 18 \times 17 \times 16 \times 15!}{4 \times 3 \times 2 \times 1 \times 15!}$$

$$= 19 \times 6 \times 7 = 3876$$

(v) ${}^{52}C_{50}$ Here n = 52, r = 50

$${}^{n}C_{r} = {}^{52}C_{50} = \frac{52!}{50!(52-50)!} = \frac{52!}{50!2!} = \frac{52 \times 51 \times 50!}{50! \times 2 \times 1}$$

$$= 1326$$

(vi) ${}^{2n}C_{r+1}$ Here 2n = r, r = r + 1

$${}^{n}C_{r} = {}^{2n}C_{r+1} = \frac{2n!}{(r+1)!(2n-r-1)!}$$

-:5.11:-

Here n = 8, r = 6

$${}^{n}C_{r} = {}^{8}C_{6} = \frac{8!}{6!(8-6)!} = \frac{8}{6!2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = 28$$

-:5.12:-

Here n = 12, r = 5, 5 books including one specified book can be selected in the following ways:

$$^{11}C_4 x\, ^1C_1 = \frac{11!}{4!(11-4)!} = \frac{11!}{4!1!} = \frac{11 \times 10 \times 9 \times 8 \times 7!}{4 \times 3 \times 2 \times 1 \times 7!} = 330 \text{ ways}$$

-:5.13:-

2 balls out of 4 white and 6 black balls can drawn in the following ways:
Here n = 4 + 6 = 10, r = 4

$$^nC_r = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 5 \times 9 = 45 \text{ ways}$$

-:5.14:-

The committee of 3 men and 2 women from 6 men and 4 women can be formed in the following ways:

$$^6C_3 = {}^4C_2 = \frac{6!}{3!(6-3)!} x \frac{4!}{2!(4-2)!} = \frac{6!}{3!3!} x \frac{4!}{2!2!}$$

$$= \frac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \times 3!} x \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = (5 \times 4)(2 \times 3)$$

$$= 20 \times 6 = 120 \text{ ways}$$

-:5.15:-

There are 9 balls of which 5 are yellow and 4 black. A group of 3 yellow and 3 black balls from 9 balls can be drawn in the following ways:

$$^5C_3 x\, ^4C_2 = \frac{5!}{3!(5-3)!} x \frac{4!}{2!(4-2)!} = \frac{5!}{3!2!} x \frac{4!}{2!2!}$$

$$= \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} x \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 10 \times 6 = 60 \text{ ways}$$

-:5.16:-

Expand $(x + y)^7$

$$(x+y)^7 = x^7 + {}^7C_1 x^6 y + {}^7C_2 x^5 y^2 + {}^7C_3 x^4 y^3 + {}^7C_4 x^3 y^4 + {}^7C_5 x^2 y^5 + {}^7C_6 x y^6 + y^7$$

$$= x^7 + 7x^6 y + 21x^5 y^2 + 35x^4 y^3 + 35x^3 y^4 + 21x^2 y^5 + 7xy^6 + y^7$$

-:5.17:-

Expand $(x - 2y)^4$

$$(x-2y)^4 = x^4 + {}^4C_1x^3(-2y) + {}^4C_2x^2(-2y)^2 + {}^4C_3x(-2y)^3 + (-2y)^4$$
$$= x^4 + 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$$

-:5.18:-

Expand $(x - 3y)^5$

$$(x-3y)^5 = x^5 + {}^5C_1x^4(-3y) + {}^5C_2x^3(-3y)^2 + {}^5C_3x(-3y)^3 + {}^5C_4x(-3y)^4 + (-3y)^5$$
$$= x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$$

-:5.19:-

Expand $(2x - y)^5$

$$(2x-y)^5 = (2x)^5 + {}^5C_1(2x)^4(-y) + {}^5C_2(2x)^3(-y)^2 + {}^5C_3(2x)^2(-y)^3$$
$$+ {}^5C_4(2x)(-y)^4 + (-y)^5$$
$$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(-y)^2 + 10(4x^2)(-y)^3 + 5(2x)(-y) + (-y)^5$$
$$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

-:5.20:-

Expand $(1 - 3a^2)^6$

$$(1-3a^2)^6 = 1^6 + {}^6C_1 1^5(-3a^2) + {}^6C_2 1^4(-3a^2)^2 + {}^6C_3 1^3(-3a^2)^3 + {}^6C_4 1^2$$
$$(-3a^2)^4 + {}^6C_5 1(-3a^2)^5 + (-3a^2)^6$$
$$= 1 + 6(1)(-3a^2) + 15(1)(-3a^2)^2 + 20(1)(-3a^2)^3 + 15(1)(-3a^2)^4$$
$$+ 6(1)(-3a^2)^5 + (-3a^2)^6$$
$$= 1 - 18a^2 + 135a^4 - 540a^6 + 1215a^8 - 145a^{10} + 729a^{12}$$

-:5.21:-

Expand $\left(a - \frac{1}{n}\right)^6$

$$(a - \frac{1}{a})^6 = a^6 + {}^6C_1a^5(-\frac{1}{a}) + {}^6C_2a^4(-\frac{1}{a})^2 + {}^6C_3a^3(-\frac{1}{a})^3 + {}^6C_4a^2(-\frac{1}{a})^4$$
$$+ {}^6C_5a(-\frac{1}{a})^5 + (-\frac{1}{a})^6$$
$$= a^6 + 6a^5(-\frac{1}{a}) + 15a^4(-\frac{1}{a})^2 + 20a^3(-\frac{1}{a}) + 15a^2(-\frac{1}{a})^4$$
$$+ 6a(-\frac{1}{a})^5 + (-\frac{1}{a})^6$$
$$= a^4 - 6a^4 + 15a^2 - 20 + \frac{15}{a^2} - \frac{6}{a^4} + \frac{1}{a^4}$$

-:5.22-:

Find the 4th term of $(x-5)^{13}$

Here n = 13, r =3, a = x, h = -5

$T_{r+1} = {}^{n}C_r\, a^{n-r} b^r$

$T_4 = T_{3+1} = {}^{13}C_3 x^{13-3}(-5)^3 = 286x^{10}(-125) = -35750x^{10}$

-:5.23:-

Find the 5th term of $(a + 2x^3)^{17}$

Here n = 17, r =4, a = a, b =$2x^3$

$T_{r+1} = {}^{n}C_r a^{n-r} b^r$

$T_5 = T_{4+1} = {}^{17}C_4 x^{17-4}(2x^3)^4 = 2380a^{13}(16x^{12}) = 38080a^{13}x^{12}$

-:5.24:-

Find the middle term of $\left(\frac{a}{x}+\frac{x}{a}\right)^{10}$

Here n = 10

The total number of terms will be 11 and the middle term is 6th. Therefore,

$$T_6 = T_{5-1} = {}^{10}C_5\left(\frac{a}{x}\right)^{10-5}\left(\frac{x}{a}\right)^5 = 252\left(\frac{a}{x}\right)^5\left(\frac{x}{a}\right)^5 = 252 = \frac{a^5}{x^5}+\frac{x^5}{a^5} = 252$$

-:5.25:-

Find the 7th term in the expansion of $(x-2)^{15}$

Here n = 15, r = 6, a = x, b = -2

$T_{r+1} = {}^{n}C_r\, a^{n-r} b^r$

$T_7 = T_{6+1} = {}^{15}C_6\, x^{15-6}(-2)^6 = 5005x^9(64) = 5005(64)x^9 = 320320x^9$

-:5.26:-

$$\sqrt{1.07} = \sqrt{1+0.07} = (1+0.07)^{\frac{1}{2}}$$

$$= (1)^{\frac{1}{2}} + (\tfrac{1}{2})(1)^{-\frac{1}{2}}(0.07) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(1)^{-\frac{1}{2}}(0.07)^2$$

$$+ \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(1)^{-\frac{5}{2}}(0.07)^3$$

$$= 1 + 0.035 - 0.0006125 + 0.000214 + \ldots\ldots = 1.0356339$$

-:5.27:-

Find 23 correct to 3 places of decimals

$$\sqrt{23} = (25-2)^{\frac{1}{2}} = 5(1-\frac{2}{25})^{\frac{1}{2}} = 5(1-0.08)^{\frac{1}{2}}$$

$$5(1-0.08)^{\frac{1}{2}} = 5[1+\frac{1}{2}(-0.08)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-0.08)^2+$$

$$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-0.08)^3+....]$$

$$= 5[1-0.047864] = 5[0.959136] = 4.79568 = 4.796$$

-:5.28:-

Find 0.97 to 6 places of decimals

$$\sqrt{0.97} = \sqrt{1-0.03} = (1-0.03)^{\frac{1}{2}}$$

$$(1-0.03)^{\frac{1}{2}} = 1+\frac{1}{2}(-0.03)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(-0.03)^2$$

$$+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-0.03)^3+.....$$

$$= 1+\frac{1}{2}(-0.03)+\frac{\frac{1}{2}(-\frac{1}{2})}{2!}(-0.03)^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(-0.03)^3+.....$$

$$= 1-0.015-0.0001125-0.0000016+.....= 1-0.151141$$

$$= 0.9848859 = 0.984886$$

-:5.29:-

Find $(1.2)^7$ correct to four decimals places.

$$(1.2)^7 = (1+0.2)^7$$

$$(1+0.2)^7 = (1)^7+{}^7c_1(1)^6(0.2)+{}^7c_2(1)^5(0.2)^2+{}^7c_3(1)^4(0.2)^3+{}^7c_4(1)^5(0.2)^4$$
$$+{}^7c_5(1)^2(0.2)^5+{}^7c_6(1)(0.2)^6+(0.2)^7$$
$$= 1+7(1)(0.2)+21(1)(0.2)^2+35(1)(0.2)^3+35(1)(0.2)^4+21(1)(0.2)^5$$
$$+7(1)(0.5)^6+(0.2)^7$$
$$= 1+7(0.2)+21(0.04)+35(0.008)+35(0.0016)+21(0.00032)$$
$$+7(0.00064)+0.0000128$$
$$= 1+1.4+0.84+0.28+0.056+0.00672+0.000448+0.0000128$$
$$= 3.5831808 = 3.5832$$

-:5.45:-

(i) Sample Space = S = {1, 2, 3, 4, 5, 6}, n(S) = 6
Let A represent the event, an odd number appears in a single throw. n(A) = 3

$$P(\text{Having Odd No.}) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) n(S) = 6

Let A represent the event of an ace appears. n(A)=1

$$P(\text{Having an ace}) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(iii) n(S) = 6

Let A represent the event, a five appears. n(A)=1

$$P(\text{Having a 5}) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(iv) The two dice can be thrown in 6x6 = 36 ways. The number of sample points in sample space are n(S) = 36

Let A represent the event that sum of faces is 8.

A = (6,2), (5,3), (4,4), (3,5), (2,6); n(A) = 5

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(v) There are two possible outcomes, head and tail. n(S)=2

Let A represent the event that head occur. n(A)=1

$$P(\text{Head}) = P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(vi) The sample space in three tosses of a fair coin is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

n(S) = 8

Let A represent the event that at least one head appears.

n(A)=7

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

(vii) Once card from 52 cards can be drawn 52 ways. n(S)=52

Let A represent the event of drawing a king, ace, jack of clubs or queen of diamonds. n(A)=10

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{52} = \frac{5}{26}$$

(viii) n(S)=6

Let A represent the event, an even number appears. n(A)=3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

-:5.46:-

(i) Let A represent the event that the card drawn is an ace. The number of ace cards is 4. n(A)=4

Hence

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let B represent the event that the card drawn is heart. The number of outcomes favourable to the event B is 13, n(B)=13
Hence

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C represent the event that card drawn is a red. The number of outcomes favourable to event C is 26. n(C)=26

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(iv) Let D represent the event that the card drawn is a pictured card. The number of outcomes favourable to the event D is 16. n(D)=16

$$P(D) = \frac{n(D)}{n(S)} = \frac{16}{52} = \frac{3}{13}$$

(v) Let E represent the event that the card drawn is a king of heart. The favourable case to the event E is one, n(E)=1

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

(vi) Let F represent the event that card drawn is a black card. The number of outcomes favourable to the event F is 26, n(F)=26

$$P(F) = \frac{n(F)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(vii) Let H represent the event that the card drawn is a card of diamond. The number of outcomes favourable to the event H is 13, n(H)=13

$$P(H) = \frac{n(H)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(viii) Let G represent the event that card drawn is a face card. The number of outcomes favourable to event G is 16, n(G)=16

$$P(G) = \frac{n(G)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

-:5.47:-

(a) When 2 dice are thrown, the number of various total number of dots are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

(b) The two dice can be thrown 6x6=36 ways. There are 36 sample points in sample space.
Let A represent the event that the sum of faces is 10 with 2 dice.
A = (6,4), (5,5), (4,6); n(A)=3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$$

(c) The two dice can be thrown 6x6=36 ways, n(S)=36
Let B represent the event that the sum of faces is 11 with 2 dice
B = (6,5), (5,6) n(B) = 2

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$$

(d) There are 36 sample points in sample space i.e. n(S) = 36
Let A represent the event that a total of more than 7 occurs. Then the event A has 15 following outcomes.
A = (2,6), (3,6), (4,6), (5,6), (6,6), (3,5), (4,5), (5,5), (6,5), (4,4), (5,4), (6,4), (5,3), (6,3), (6,2)
n(A) = 15

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{36}$$

Let B represent the event that total of less than 7 occurs. Then event B has 15 following outcomes.
B = (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)
n(B) = 15

$$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36}$$

Hence P(A) = P(B)

(e) Two dice can be thrown in 6x6 = 36 n(S) = 36
Let A represent the event that is sum shown is 8, and B represent the event that two show the same number then

A = (6,2), (5,3), (4,4), (3,5), (2,6)
n(A) = 5 and
B = (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)
n(B) = 6

Hence

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

-:5.48:-

(a) There are 35 students of which 15 are boys and 20 are girls. One student is chosen. The number of sample points in the sample space are $n(S) = {}^{35}C_1 = 35$
Let A represent the event of choosing a girls, $n(A) = {}^{20}C_1 = 20$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{35} = \frac{4}{7}$$

(b) The number of sample points in the sample space are

$$n(S) = {}^{52}C_1 = 52$$

Let A represent the event of drawing a spade card.

$$n(A) = {}^{13}C_1 = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(c) There are 9 chips of which 5 are black and 4 are pink. One chip is drawn. The number of sample points in the sample space are

$$n(S)\ {}^{9}C_1 = 9$$

(i) Let A represent the event of drawing black chip.

$$n(A) = {}^{5}C_1 = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

(ii) Let B represent the event of drawing pink chip

$$n(B) = {}^{4}C_1 = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{5}$$

-:5.49:-

The three dice can be thrown in 6x6x6 = 216 ways.

(a) Possible outcomes to get 7 with 3 dice

Outcomes	No of Outcomes
(1,1,5)	3
(1,2,4)	6
(1,3,3)	3
(2,2,3)	3
	15

Let A represent the event of getting 7 with 3 dice, $n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{216} = 0.069$$

(b) Let B represent event of 9 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	8	5
2	7	6
3	6	4
4	5	3
5	4	2
6	3	1
		25

$n(B) = 25, \quad n(S) = 216$

$$P(B) = \frac{n(B)}{n(S)} = \frac{25}{216}$$

(c) . Let C represent the event of getting 15 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
3	12	1
4	11	2
5	10	3
6	9	4
		10

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{216}$$

(d) Let D represent the event of getting 13 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	12	1
2	11	2
3	10	3
4	9	4
5	8	5
6	7	6
		21

$n(D) = 21$

$$P(D) = \frac{n(D)}{n(S)} = \frac{21}{216}$$

(e) We know from (b) that

$$P(A) = \frac{25}{216}$$

Let B represent 11 with 3 dice

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	10	3
2	9	4
3	8	5
4	7	6
5	6	5
6	5	4
		27

$$n(B) = 27$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{27}{216}$$

By using addition law of probability

$$P(9 \text{ or } 11) = P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{25}{216} + \frac{27}{216} = \frac{13}{54}$$

-:5.50:-

The two dice can be thrown 6x6 = 36 ways, $n(S) = 36$

(i) Let A represent the event that the product of the number on dice is between 8 and 16.

A = (4,2), (3,3), (2,4), (5,2), (4,3), (3,4), (2,5), (6,2), (5,3), (4,4), (3,5), (2,6)

$$n(A) = 12$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = \frac{1}{13}$$

(ii) Let B represent the event that the product of the numbers on the dice is divisible by 4.

B = (4,1), (1,4), (4,2), (2,4), (2,6), (6,2), (4,4), (2,6), (4,5), (4,3), (2,2), (3,4), (5,4), (6,4), (4,6), (6,6)

$$n(B) = 15$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36}$$

-:5.51:-

The number of sample points in the sample space are

$$n(S) = {}^{52}C_2 = 1326$$

Let A represent the event that one card is king and the other card is queen.

$$n(A) = {}^{4}C_1 \times {}^{4}C_1 = 4 \times 4 = 16 \text{ ways}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{16}{1326} = \frac{8}{663}$$

-:5.52:-

There are 9 balls of which 4 are white and 5 are black. Three are drawn. The number of sample points in the sample space are

$$n(S) = {}^{9}C_3 = 84 \text{ ways}$$

Let A represent the event of drawing 3 black balls.

$$n(A) = {}^{5}C_3 = 10 \text{ ways}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{84}$$

-:5.53:-

There are 13 balls of which 5 are white and 8 are black. Four balls are drawn at random. The number of sample points in the sample space are.

$$n(S) = {}^{13}C_4 = 715$$

Let A represent the event of drawing 4 black balls.

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{715} = \frac{1}{143}$$

-:5.54:-

There are 8 cards of which one joker and seven other cards. Five cards are chosen at random. The number of sample points in sample space are

$$n(S) = {}^{8}C_5 = 56$$

A can choose one joker and 4 other cards.

$$n(S) = {}^{1}C_1 \times {}^{7}C_4 = 35$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{35}{56} = \frac{5}{8}$$

-:5.55:-

There are 9 persons of which 3 men, 2 women and 4 children. Four persons are chosen at random. The number of sample points in sample space are

$${}^{9}C_4 = 126$$

Let A represent the event to choose 4 persons of which exactly 2 are children. Four persons can be chosen by the following ways.

(A_1)	2 men	0 women	2 children
(A_2)	0 men	2 women	2 children
(A_3)	1 man	1 woman	2 children

$n(A_1) = {}^{3}C_2 \times {}^{2}C_0 \times {}^{4}C_2 = 3 \times 1 \times 6 = 18$

$n(A_2) = {}^{3}C_0 \times {}^{2}C_2 \times {}^{4}C_2 = 1 \times 1 \times 6 = 6$

$n(A_3) = {}^{3}C_1 \times {}^{2}C_1 \times {}^{4}C_2 = 3 \times 2 \times 4 = 36$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{18}{126}$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{6}{126}$$

$$P(A_3) = \frac{n(A_3)}{n(S)} = \frac{36}{126}$$

Hence

$$P(A) = P(A_1) + P(A_2) + P(A_3)$$

$$= \frac{18}{126} + \frac{6}{126} + \frac{36}{126} = \frac{60}{126} = \frac{10}{21}$$

-:5.56:-

There are 15 balls of which 5 white, 5 red and 4 black. Three balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{15}C_3 = 455$$

(i) Let A represent the event of drawing all the three white balls.

$$n(A) = {}^{5}C_3 \times {}^{6}C_0 \times {}^{4}C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{455} = \frac{2}{91}$$

(ii) let A represent the white balls and B represent the red balls. Then

$$n(A \text{ or } B) = {}^{5}C_3 \times {}^{6}C_0 \times {}^{4}C_0 + {}^{5}C_0 \times {}^{6}C_3 \times {}^{4}C_0$$

$$= 10 + 20 = 30$$

$$P(A \text{ or } B) = P(\text{White or black})$$

$$P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{30}{455} = \frac{6}{91}$$

(iii) Let C represent the event of drawing one red and 2 black ball.

$$n(C) = {}^{6}C_1 \times {}^{4}C_2 = 6 \times 6 = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{455}$$

(iv) Let D represent the event of drawing one ball of each colour.

$$n(D) = {}^{5}C_1 \times {}^{6}C_1 \times {}^{4}C_1 = 5 \times 6 \times 4 = 120$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{120}{455} = \frac{24}{91}$$

-:5.57:-

There are 15 balls of which 4 white, 5 red and 6 black balls. Three balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{15}C_3 = 455$$

(i) Let A represent the event of drawing no black ball.

$$n(A) = {}^{9}C_3 = 84$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{84}{455} = \frac{12}{65}$$

(ii) Let B represent the event of drawing exactly 2 black and one other ball.

$$n(B) = {}^{6}C_2 \times {}^{9}C_1 = 135$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{135}{455} = \frac{27}{91}$$

(iii) Let C represent the event of drawing all the same colour

$$n(C) = {}^{4}C_1 + {}^{5}C_3 + {}^{6}C_3$$

$$= 4+10+20 = 34 \text{ ways.}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{34}{455}$$

-:5.58:-

There are 50 lamps in which 3 are defective and 47 are non-defective. Five lamps are chosen at random. The number of sample points in sample space are

$$n(S) = {}^{50}C_5 = 2118760$$

(a) Let A represent the event that non of lamp will be defective.

$$n(A) = {}^{47}C_5 = 1533939$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1533939}{2118760} = 0.734$$

(b) Let B be the event of choosing exactly 2 defective and three other non defective lamps.

$$n(B) = {}^{3}C_2 \times {}^{47}C_3$$
$$= 3 \times 16215 = 48645$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{48645}{2118760} = 0.023$$

-:5.59:-

There are 15 balls in which 6 white 5 red and 4 black. Four balls are drawn at random. The number of sample points in sample space are

$$n(A) = {}^{15}C_4 = 1365$$

Let A represent the event of getting at least 2 red balls.

$$n(A) = {}^{5}C_2 \times {}^{10}C_2 + {}^{5}C_3 \times {}^{10}C_1 + {}^{5}C_4 \times {}^{10}C_0$$
$$= 450 + 100 + 5 = 555$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{555}{1365} = \frac{57}{91}$$

-:5.60:-

There are 12 balls of which 5 white and 7 black. 3 balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{12}C_3 = 220$$

(a) Let A represent the event of drawing all white balls.

$$n(A) = {}^{5}C_3 \times {}^{7}C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{220} = \frac{1}{22}$$

(b) Let B represent the event of drawing 2 white and one black ball

$$n(B) = {}^{5}C_2 \times {}^{7}C_1 = 70$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{70}{220} = \frac{7}{22}$$

(c) Let C represent the event of drawing all 3 of the same colour

$$n(C) = {}^5C_3 \times {}^7C_0 + {}^5C_0 \times {}^7C_3$$
$$= 10 + 35 = 45$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{45}{220} = \frac{9}{44}$$

-:5.61:-

There are 14 balls of which 4 are red, 5 black and 5 white. Six balls are drawn at random. The number of sample points in sample are

$$n(S) = {}^{14}C_6 = 3003$$

(i) Let A represent the event of drawing 3 red and 3 other balls

$$n(A) = {}^4C_3 \times {}^{10}C_3 = 4 \times 120 = 480$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{480}{3003}$$

(ii) Let B represent the event of drawing at least 2 white balls

$$n(B) = {}^5C_2 \times {}^9C_2 + {}^5C_3 \times {}^9C_3 + {}^5C_4 \times {}^9C_2 \times {}^5C_5 \times {}^9C_1$$
$$= 1260 + 840 + 180 + 9 = 2289$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{2289}{3003}$$

-:5.62:-

There are 14 balls of which 3 white 2 red, 5 green and 4 yellow. Four balls are selected at random. The number of sample points in sample space are

$$n(S) = {}^{14}C_4 = 1001$$

Let A represent the event of drawing 4 balls of all colour. As there are 4 colours, one ball of each colour is drawn.

$$n(A) = {}^3C_1 \times {}^2C_1 \times {}^5C_1 \times {}^4C_1$$
$$= 3 \times 2 \times 5 \times 4 = 120$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{120}{1001}$$

-:5.63:-

There are two bags, one bag contains 6 balls of which 4 white and 2 black and the other bag contains 8 balls of which 3 white and 5 black. One ball is drawn from each bag.

(i) Let A represent the event drawing both white ball
Probability of drawing one white ball from first, one white ball from second bag is

$$P(A) = \frac{{}^4C_1 \times {}^2C_0}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_0}{{}^8C_1} = \frac{4}{6} \times \frac{3}{8} = \frac{12}{48} = \frac{1}{4}$$

(ii) Let B represent the event of drawing both black ball.
The probability of drawing one black ball from first, one black ball from second bag is

$$P(B) = \frac{{}^4C_0 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_1}{{}^8C_1} = \frac{2}{6} \times \frac{5}{8} = \frac{10}{48} = \frac{5}{24}$$

(iii) Let C represent the event of drawing one white, one black ball.
One white and one black ball can be drawn by the following ways.

	Bag I		Bag II	
	White 4	Black 2	White 2	Black 5
Case I	White ball		Black ball	
Case II	Black ball		White ball	

Probability of drawing one white from first, one black from the second bag is

$$P_1 = \frac{{}^4C_1 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_1}{{}^8C_1} = \frac{4}{6} \times \frac{5}{8} = \frac{5}{12}$$

Probability of drawing one black from first and one black from second bag is

$$P_2 = \frac{{}^4C_0 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_0}{{}^8C_1} = \frac{2}{6} \times \frac{3}{8} = \frac{1}{8}$$

Hence the required probability is

$$P(C) = P_1 + P_2 = \frac{5}{12} + \frac{1}{8} = \frac{13}{24}$$

-:5.64:-

There are three bags A, B, C. Bag A contains 12 balls of which 5 are white, 7 are black. Bag B contains 10 balls of which 4 are white, 6 black. Bag C contains 6 balls of which 2 are white and 4 black. One ball is drawn from a bag selected at random white ball is selected.

There are three bags and each these has an equal chance of being selected. The probability of each bag is equal to 1/3.

The following three possible cases in the selection of bags are

i. First bag may be selected.
ii. Second bag may be selected.
iii. Third bag may be selected.

The probability of a white ball from first bag

$$= \frac{{}^5C_1 \times {}^7C_0}{{}^{12}C_1} = \frac{5}{12}$$

The probability of a white ball with the probability of selection of first bag.

$$P_1 = \frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

The probability of a white ball from second bag

$$= \frac{{}^4C_1 \times {}^6C_0}{{}^{10}C_1} = \frac{4}{10}$$

The probability of a white ball with the probability of selection of second bag

$$P_2 = \frac{1}{3} \times \frac{4}{10} = \frac{4}{30}$$

Similarly for the probability of a white ball from third bag with the probability of the selection of third bag is

$$= \frac{1}{3} \times \frac{2}{6} = \frac{2}{18}$$

$$P_1 = \frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

Hence the required probability is

$$P = P_1 + P_2 + P_3 = \frac{5}{36} + \frac{4}{30} + \frac{2}{18} = \frac{69}{180}$$

-:5.65:-

There are 13 balls of which 5 white and 8 black. Two drawings each of 3 balls are made.

Let A represent the event that first drawing contains 3 white balls. B represent the event that second drawing contains 3 black balls. The number of sample points in sample space are

$$n(S) = {}^{13}C_3 = 286$$

$$n(A) = {}^5C_3 \times {}^8C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{286}$$

$$n(B) = {}^5C_0 \times {}^8C_0 = 56$$

To find the probability of B; when the white balls drawn in the first drawing are not being replaced i.e. P(B/A), the sample space S is reduced.

$$n(S) = {}^{10}C_3 = 120$$

$$P(B/A) = \frac{n(B)}{n(S)} = \frac{56}{120}$$

We have to find

$$P(A \cap B) = P(A).P(B/A) = \frac{10}{286} \times \frac{56}{120} = \frac{7}{249}$$

-:5.66:-

(i) With Replacement:

$$n(S) = {}^{52}C_1 = 52$$

Let A_1 represent the event of drawing first ace card

$$n(A_1) = 4$$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{4}{52}$$

Let A_2 represent the event of drawing second ace

$$n(A_2) = 4$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{4}{52}$$

Hence

$$P(A_1 \cap A_2) = P(A_1).P(A_2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

(ii) Without replacement

$$n(S) = 51;\ n(A_2) = 3$$

$$P(A_2 / A_1) = \frac{n(A_2)}{n(S)} = \frac{3}{51}$$

Hence

$$P(A_1 \cap A_2) = P(A_1).P(A_2 / A_1) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{121}$$

-:5.67:-

Let A represent the event of football players and B represent the event of basketball players. It is given that

$$P(A) = \frac{60}{100};\quad P(B) = \frac{50}{100};\quad P(A \cap B) = \frac{30}{100}$$

(i)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{50}{100} - \frac{30}{100} = \frac{80}{100} = 0.8$$

(ii)

$$P(\overline{A}) = 1 - \frac{60}{100} = \frac{40}{100},\quad P(\overline{B}) = 1 - \frac{50}{100} = \frac{50}{100}$$

$$P(\overline{A} \cap \overline{B}) = P(\overline{A}).P(\overline{B}) = \frac{40}{100} \times \frac{50}{100} = 0.2$$

-:5.68:-

There are 5 balls in a bag of which 2 red and 3 white. The number of sampling points in sample space are

$$n(S) = {}^5C_1 = 5$$

Let A_1 represent the first drawing of a red ball

$$n(A_1) = {}^2C_1 = 2$$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{2}{5}$$

let A_2 represent the second drawing of red ball. First ball is being replaced. Hence

$$n(S) = 5; \quad n(A_2) = {}^2C_1 = 2$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{2}{5}$$

$$P(A_1 \cap A_2) = P(A_1).P(A_2) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

-:5.69:-

When the first red ball is not replaced, then the sample space reduced.

$$n(S) = 4$$

The remaining red ball is one, hence

$$P(A_2 / A_1) = \frac{1}{4}$$

$$P(A_1 \cap A_2) = (PA_1).P(A_2 / A_1) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$$

-:5.70:-

Let A represent the event that sum of dots is odd and B represent the event that sum of dots is 7. Then we have to find P(B/A). The number of sample points in sample space are.

$$n(S) = 36; \quad n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

There are 6 sample points in AB

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Hence

$$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

-:5.71:-

Let A_1 represent the event that the employee has accounting back ground and A_2 be the event that employee is an executive.

$$P(A_1) = 0.20; \quad P(A_1/A_2) = 0.05$$

Hence required probability is

$$P(A_2 / A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.05}{0.20} = 0.25$$

-:5.72:-

The sample space S for this experiment consists of the following 36 equally likely outcomes, that is

$$n(S) = 36$$

Let A represent the sum is 7, B represent sum is odd, C represent sum is greater than 6 and D represent the two dice had the same outcome. Then

$$n(A) = 6, \quad P(A) = \frac{6}{36} = \frac{1}{6}$$

(i) B represent the event that sum is odd

$$n(B) = 18$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

$$n(A \cap B) = 6$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

(ii) Let C represent the event that sum is greater than 6

$C = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (2,6), (3,5), (4,4), (5,3), (6,2), (3,6), (4,5), (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6)\}$

$$n(C) = 21$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{21}{36}$$

$$n(A \cap C) = 6$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A / C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{6}}{\frac{21}{36}} = \frac{2}{7}$$

(iii) Let D represent the event having same outcomes.

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$$n(D) = 6$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$A \cap D = \phi$$

$$P(A \cap D) = 0$$

$$P(A / D) = \frac{P(A \cap D)}{P(B)} = \frac{0}{\frac{1}{6}} = 0$$

1 INTRODUCTION

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The science of statistics can be divided in branches	2	✓ 3	5	4
		(Faisalabad Board, 2008)			
2)	The lifetime of a T.V table is	Discrete data	✓ Continuous data	Both types	None
		(Faisalabad Board 2008			
3)	The grouped data is	✓ Secondary data	Primary data	Individually data	None
		(Faisalabad Board 2008			
4)	Classification has important basis	✓ 2	4	5	7
		(Faisalabad Board, 2008			
5)	Frequency is denoted by	C	q	p	✓ f
		(Faisalabad Board, 2008; Lahore Board 2008)			
6)	Now a days, the word statistics can be expressed in how many ways	2	✓ 3	4	5
		(Lahore Board, 2008)			
7)	The data obtained from college record is	Primary	✓ Secondary	Raw	Qualitative
		(Lahore Board 2008			

S#	Questions	Answers			
8)	A measure computed from sample data is called	Parameter	✓ Statistic	Statistics	Data
		(Lahore Board, 2008; Faisalabad Board, 2009)			
9)	Statistics are always	✓ Aggregate of facts and figures	True	Continuous	New
		(Lahore Board, 2008, 2009; Faisalabad Board, 2010)			
10)	The grouped data is always called	Raw data	Primary data	Sample data	✓ Secondary data
11)	Population census is conducted through	Sample survey	✓ Complete enumeration	Registration	Investigation
		(Lahore Board, 2008)			
12)	The word "Statistics" is belived to have been derived from Latin word	Statistic	✓ Status	Statistique	Statista
		(Gujranwala Board 2009; Faisalabad Board 2010)			
13)	One way classification is done considering	✓ One variable	Two variable	Three variable	More than three variable
		(Gujranwala Board, 2009)			
14)	Height of a student is an example of	Discrete variable	✓ Continuous variable	Constant	Qualitative data
		(Gujranwala Board 2009; Lahore Board 2009)			
15)	Questionnaire method is used in collection of	✓ Primary data	Grouped data	Secondary data	None of these
		(Gujranwala Board, 2009)			

S#	Questions	Answers			
16)	Statistics must be	✓ Comparable	Not comparable	Discrete in nature	Qualitative in nature
					(Faisalabad Board, 2009)
17)	Any data in your book for you is	Primary	Raw	✓ Secondary	False
					(Faisalabad Board, 2009)
18)	The number of weeks in a months is an example of	✓ Discrete variable	Continuous variable	Attribute	Constant
					(Lahore Board, 2009)
19)	Collection of data from house to house is by	Mailed questionnaire	Through enumerators	Registration	✓ Both mailed questionnaire and through enumerators
					(Lahore Board, 2009)
20)	Non measureable quantities are	Continuous variable	Discrete variable	Constant	✓ Attribute
					(Lahore Board, 2009)
21)	Census reports published are	Primary data	✓ Secondary data	Raw data	Simple data
					(Faisalabad Board, 2010)
22)	A characteristic that changes from one individual to another is called	✓ Variable	Constant	Statistic	None of these
					(Lahore Board, 2010)
23)	Raw material collected by first hand is	✓ Primary data	Secondary data	Continuous data	None of these
					(Lahore Board, 2010)

S#	Questions	Answers			
24)	Lower class boundary of 25 – 35 will be	20	✓ 25	30	35
					(Lahore Board, 2010)
25)	Identify the attribute	Height	Blood pressure	✓ Eye colour	Marks
					(Gujranwala Board, 2010; Lahore Board, 2010)
26)	The group data is also called	Raw data	Primary data	Qualitative data	✓ Secondary data
					(Lahore Board, 2010)
27)	Statistical laws are valid in the	Short run	Medium run	✓ Long run	Average run
					(Gujranwala Board, 2010)
28)	Parameters are related to	Sample	✓ Population	Mean	Median
					(Gujranwala Board, 2010)
29)	There is no difference between the secondary data and	✓ Grouped data	Primary data	Ungrouped data	Raw data
					(Gujranwala Board, 2010)
30)	In plural sense, statistics means	Methods	✓ Numerical data	Sample value	Population values
					(Rawalpindi Board, 2010)
31)	The number of accidents on a road on 01–01–2010 is	Continuous variable	Constant	Qualitative variable	✓ Discrete variable
					(Rawalpindi Board, 2010)
32)	The data collected from first time is called	✓ Primary data	Secondary data	Discrete data	None of these
					(Sargodha Board, 2010)

S#	Questions	Answers			
33)	The word statistics is derived from	✓ Latin word	Greek word	Italian word	None of these
					(Sargodha Board, 2010)

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The word "statistics" is believed to have been derived from the Latin word:	Statistik	✓ Status	Statistique	Statista
2)	Statistics is a word:	✓ Latin	Punjabi	German	Urdu
3)	Statistics means:	✓ Aggregates of facts	Computer science	Research science	Distribution of wealth
4)	Statistical laws are true on the:	✓ Average	Numerically	Individually	None of these
5)	Statistics is defined as the numerical data in:	Singular sense	✓ Plural sense	Technical sense	None of these
6)	Statistics plays an important role in:	Mathematics	Computer	Accounting	✓ Research
7)	The word statistics is now a days used in:	Two ways	✓ Three ways	Four ways	Five ways
8)	Statistics does study:	✓ Quantitative facts	Qualitative facts	Both quantitative & qualitative	None of these
9)	Statistical laws are true for:	✓ Long term	Short term	Medium term	None of these

S#	Questions	Answers			
10)	Science of systematic collection, presentation, analysis and interpretation of numerical data is called:	Commerce	✓ Statistics	Economics	Mathematics
11)	A measurable quantity which vary from one value to another is:	Constant	✓ Variable	Non of them	Both constant & variable
12)	A variable which can assume any value between two given values is a:	✓ Continuous variable	Discrete variable	Constant	None of these
13)	Data classified by attribute is called:	✓ Qualitative data	Quantitative data	Both qualitative & quantities	None of these
14)	A variable which can not assume any value between two given values is a:	Continuous variable	✓ Discrete variable	Constant	None of these
15)	Non – measurable quantities are:	Continuous variables	Discrete variable	Constant	✓ Attributes
16)	Number of weeks in a month is an example of:	Discrete variable	Continuous variable	Attribute	✓ Constant
17)	A characteristic that does not change is called:	Variable	✓ Constant	Attribute	Both variable & constant
18)	Which of the following is an example of a discrete variable:	Number of Rs. in a pocket	Height of a student in the class	✓ Number of children in a house	Daily income of a shop

S#	Questions	Answers			
19)	Which of the following variable is an example of a continuous variable:	✓ Height of a student in the class	Number of books on the table	Number of students in a class	Number of heads in tossing two coins
20)	The height of a student is an example of:	Discrete data	Qualitative data	✓ Continuous data	None of these
21)	The number of road accidents on Lahore, Faisalabad road is the example of:	Continuous variable	✓ Discrete variable	Qualitative	Non of these
22)	Which one is the continuous variable:	✓ Rainfall on different days in Multan	Number of patients coming in the hospital in a week	Number of flights landing on Lahore airport in a day	None of these
23)	Identify the attribute:	Height of a student	Blood pressure	✓ The eye colour of students	The number of TV sold in a day
24)	Which of the following is an example of discrete variable:	Business expenses	The yearly income of shop keeper	The life time of electric bulbs	✓ The number of flowers on a tree
25)	Number of days in a week is an example of:	✓ Constant	Qualitative variable	Discrete variable	Continuous variable
26)	Collection of data by Patwari is the example of:	✓ Primary data	Secondary data	Both Primary & Secondary	None of these
27)	Collection of data from house to house is by:	Mailed questionnaire	Through enumerators	Registration	✓ Both mailed questionnaire & through enumeration

S#	Questions	Answers			
28)	Publication of central statistical office, ministries of Agriculture, Finance, Communications and Railway etc are examples of:	Primary data	✓ Secondary data	Both primary & secondary	None of these
29)	The data which is not arranged is called:	Grouped data	✓ Raw data	Individual data	None of these
30)	Questionnaire method is used in the collection of:	✓ Primary data	Secondary data	Grouped data	Discrete data
31)	A numerical quantity calculated from sample is called:	Parameter	✓ Statistic	Statistics	None of these
32)	The process by which things are arranged in groups or classes is called:	✓ Classification	Tabulation	Graph	None of these
33)	One way classification is done considering:	✓ One variable	Two variables	Three variables	None of these
34)	Data grouped according to magnitude is called:	Sampling distribution	✓ Frequency distribution	Both sampling & frequency distribution	None of these
35)	Arrangement of data according to some common characteristics is called:	✓ Classification	Tabulation	Both classification & tabulation	None of these
36)	The class mark is defined as:	Class interval	Class boundary	✓ Mid point	Class limits

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q.1: Define Statistics.

(Lahore Board 2010)
(Rawalpindi Board 2010)

Ans: Statistics is defined as the science of collection, presentation, analysis and interpretation of numerical data.

Q.2: What are the meaning of the word statistics in plural sense.

(Sargodha Board, 2010)

Ans: In plural sense, the word statistics refer to the aggregates of numerical facts collected for some purpose in any field of study. For instance, statistics of births and deaths, statistics of agriculture, price statistics, etc.

Q.3: Give the meaning of the word statistics in singular sense.

(Sargodha Board, 2010)

Ans: In Singular sense, the word statistics refer to the science used in the collection, presentation, analysis and interpretation of numerical data.

Q. 4: What are the meaning of the word statistics as plural of the word "Statistic".

Ans: The word statistics is used as plural of the word statistic. By statistic, we mean a value calculated from the sample values.

Q. 5: Define population.

(Lahore Board, 2008)
(Rawalpindi & Gujranwala Boards, 2010)

Ans: Population is defined as the totality of the observations with which we are concerned.

Q. 6: What is sample?

(Lahore Board, 2008)
(Rawalpindi & Gujranwala Boards, 2010)

Ans: A sample is the representative part of the population.

Q. 7: What is parameter?

(Faisalabad Board, 2008)
(Lahore Board, 2010)

Ans: A numerical quantity calculated from population is called parameter.

Q. 8: What is statistic?

(Faisalabad Board, 2008)
(Lahore Board, 2010)

Ans: A numerical quantity calculated from sample is called statistic.

Q. 9: Name the two branches of statistics.

(Sargodha Board, 2010)

Ans: The two branches of statistics are:

(i) Descriptive statistics (ii) Inferential statistics.

Q. 10: Define descriptive statistics.

(Lahore Board, 2008)

Ans: The branch of statistics that deals with collection, presentation and analysis of data is called descriptive statistics.

Q. 11: Define inferential statistics.

(Lahore Board, 2008)

Ans: The branch of statistics that deals with the methods that use sample results to help make decisions about a population.

Q. 12: What do you mean by "Statistical Method".

(Lahore Board, 2009)

Ans: Statistical method means the following steps:

i) Collection of data
ii) Organization and presentation of data
iii) Analysis of data
iv) Interpretation of data
v) Preparation of report

Q. 13: What is presentation?

(Lahore Board, 2010)

Ans: Presentation is the manner or style in which some thing is expresses.

Q. 14: Give three characteristics of statistics.

(Faisalabad Board, 2008 & 2009)

Ans: Statistics are:

(i) Aggregates of facts
(ii) Numerically expressed
(iii) Collected in a systematic manner for a predetermined purpose.

Q. 15: Describe two limitations of statistics.

(Rawalpindi Board, 2010)

Ans: (i) Statistics deals with aggregate of facts.
(ii) Statistical laws are true on the average.

Q. 16: Describe three important uses of statistics in various fields.

(Lahore Board, 2010)

Ans: (i) Statistics are an aid to supervision.
(ii) Planning without statistics can not be imagined.
(iii) Statistics are the eyes of administration.

Q. 17: Define three functions of statistics

(Lahore Board, 2010)

Ans: (i) Presentation of facts in definite form.
(ii) Statistics facilitate comparison.
(iii) Statistics tests the law of other sciences.

Q. 18: What is a constant?

(Lahore Board, 2008 & 2009)
(Faisalabad Board, 2010)

Ans: A quantity that does not change is called constant. For example: Number of days in a week, $\pi = 3.1516$, $e = 2.71828$ etc.

Q. 19: Define a variable. **(Lahore Board, 2008)** **(Gujranwala Board, 2010)**

Ans: A variable is a symbol, such as x, y, z which can assume any of a prescribed set of values.

Q. 20: Give some examples of variable. **(Lahore Board, 2009)**

Ans: The weight of an individual, the number of students in a class, weekly wages, etc. are the examples of a variable.

Q. 21: Define quantitative variable. **(Lahore Board, 2010)**

Ans: A variable that can be measured numerically is called quantitative variable.

Q. 22: Give some examples of a quantitative variable.

Ans: Heights and weights of students, number of children per house, blood pressure, temperature etc. are the examples of quantitative variable.

Q. 23: Define qualitative variable with example. **(Faisalabad Board, 2008)**

Ans: A characteristics which can not be measured numerically is called qualitative variable. For example, religion, beauty, eye colour, educational level, sex etc.

Q. 24: Define discrete variable, giving examples. **(Lahore Board, 2008)** **(Faisalabad Board, 2010)**

Ans: A discrete variable is a variable which can not theoretically assume any value between two given numbers. For example, number of accidents in a month, number of children in a family, number of student in a class.

Q. 25: Define continuous variable, giving examples. **(Lahore Board, 2008)** **(Faisalabad Board 2009)** **(Gujranwala Board 2010)**

Ans: A variable which can assume any possible value with in a range is called a continuous variable. For example, the weight of a boy, the monthly income, daily temperature, age, blood pressure etc.

Q. 26: What is data? **(Faisalabad & Lahore Board, 2009; Rawalpindi Board, 2010)**

Ans: The number recorded as a result of counting or measuring are called data.

Q. 27: What is ungrouped data? **(Lahore Board, 2010)**

Ans: The data which is not presented in the form of a frequency distribution is called ungrouped data.

Q. 28: What is meant by grouped data?

(Lahore Board, 2010)

Ans: The data which is presented in the form of a frequency distribution is called grouped data.

Q. 29: Name the methods used for collection of data.

(Sargodha Board, 2010)

Ans: The data collected may be: (i) Primary and (ii) Secondary

Method for collection of primary data

(a) Observation (b) Telephonic interviews (c) Personal interviews (d) Mailed questionnaires

Method for collection of secondary data

(a) Official sources (b) Semi-official sources (c) Private sources (d) Technical and trade journals (e) Publications of research organisations

Q. 30: What is questionnaire?

(Sargodha Board, 2010)

Ans: Questionnaire is method in which required information is obtained by sending questionnaires to the selected individuals by mail who fill in the questionnaire and return it to the investigators

Q. 31: Define primary data.

(Lahore Board, 2008)
(Lahore Board, 2009)
(Gujranwala Board 2010)

Ans: Primary data is the most original data and has not undergone any statistical treatment. It is the data collected for the first time and forms the raw material for inquiry.

Q. 32: Define secondary data.

(Lahore Board, 2008)
(Faisalabad Board 2009 & 2010)

Ans: Secondary data are those which have gone through the statistical treatment at least once. The data have been collected, tabulated or presented are called secondary data.

Q. 33: Describe the methods used for collection of primary data.

(Lahore & Faisalabad Boards, 2008)

Ans: Methods for collection of primary data are:

(i) Direct personal observation.
(ii) Indirect oral investigation.
(iii) Investigation through questionnaire.
(iv) Investigation through enumerators.
(v) Estimates through local correspondents.
(vi) Registration.

Q. 34: Describe the methods for collection of secondary data.

(Lahore Board, 2008)

Ans: Methods for collection of secondary data are:

(i) Official sources
(ii) Semi-official sources

(iii) Private sources
(iv) Technical and trade journals
(v) Publications of research organizations

Q. 35: What are the different methods of presentation?

Ans: Different methods of presentation of data are:

(i) Classification (ii) Tabulation
(iii) Diagrams (iv) Graphs

Q. 36: Define the term classification. **(Lahore Board, 2008)**
(Rawalpindi Board 2010)

Ans: Classification is a process by which things are arranged in groups or classes according to some resemblance in the units of each group or class.

Q. 37: Write the two characteristics of a good classification.

Ans: The two characteristics of a good classification are:

(i) It should be unambiguous.
(ii) It should be flexible.

Q. 38: What are the main basis of the classification?

Ans: Classification of data depends upon the following basis:

(i) Spatial (ii) Temporal
(iii) Qualitative (iv) quantitative

Q. 39: Define one way classification.

Ans: When classification is done considering only one variable, it is called one way classification.

Q. 40: Define two way classification.

Ans: When classification is done according to two variables, it is called two way classification.

Q. 41: What is meant by tabulation?

(Faisalabad Board, 2008)

Ans: The systematic arrangement of data in form of rows and columns for the purpose of comparison.

Q. 42: What are the types tabulation?

Ans: There are two types of statistical tables.

(i) General purpose table
(ii) Specified purpose table

Q. 43: What is frequency distribution?

(Lahore Board, 2009)
(Rawalpindi & Sargodha Boards, 2010)

Ans: When data are grouped according to magnitude, the resulting series is called a frequency distribution.

Q. 44 What is relative frequency distribution?

(Faisalabad Board, 2010)

Ans: The relative frequency of a class is the frequency of the class divided by the total frequency of all classes and expressed as percentage.

Q. 45: What are class limits?

(Rawalpindi Board, 2010)

Ans: The class limits are defined as the values of the variables, which explain the classes.

Q. 46: What are the class boundaries?

(Faisalabad Board, 2008)

Ans: The class boundaries are real values, which break up one class from another class.

Q. 47: Define the term class and class frequency.

(Gujranwala Board, 2010)

Ans: A class is a set of objects which are sharing a given characteristic. A class frequency is the number of observations or objects falling into a class.

Q. 48: What do you mean by class mark or mid point?

(Faisalabad & Lahore Boards, 2010)

Ans: The average value of the lower and upper class limits or class boundaries is called class mark or mid point.

Q. 49: What is class interval or class width?

(Lahore Board, 2008)

Ans: The difference between the upper and lower class boundaries, or the difference between two successive class marks is called class interval or class width.

2 DIAGRAMMATIC AND GRAPHIC REPRESENTATION OF DATA

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	What is total angle of pie-chart	90^0	180^0	✓ 360^0	60^0
					(Lahore Board, 2008)
2)	A statistical table has at least\	One part	Two parts	Three parts	✓ Four parts
					(Lahore Board, 2008) (Gujranwala Board, 2010)
3)	A graph of cumulative frequency distribution is called	Histogram	Historigram	Frequency polygon	✓ Ogive
					(Gujranwala Board, 2009) (Lahore Board, 2010)
4)	The data, when plot on graph paper is called	Frequency distribution	Graphic	Ogive	✓ Diagrammatic representation
					(Lahore Board, 2009)
5)	The graph of time series is called	Pie-chart	Ogive	Histogram	✓ Historigram
					(Lahore Board, 2009) (Gujranwala Board 2010)
6)	Graph of frequency distribution is called	Histogram	Historigram	Pie diagram	✓ Frequency curve
					(Faisalabad Board, 2010)
7)	Graph of frequency distribution is	Histogram	Bell shaped	j-shaped	✓ Curve
					(Lahore & Gujranwala Boards, 2010)

S#	Questions	Answers			
8)	An ogive is a	Frequency polygon	✓ Cumulative frequency polygon	Frequency curve	Histogram
					(Lahore Board, 2010)
9)	Total angles of pie chart are	60^0	260^0	✓ 360^0	460^0
					(Rawalpindi Board, 2010)
10)	The data which is plotted on the graph paper is called	Classification	Tabulation	Frequency distribution representation	✓ Graphic representation
					(Sargodha Board, 2010)
11)	Main classes of graphs are (types of graph)	One	✓ Two	Three	Four
					(Sargodha Board, 2010)
12)	In graphic representation data is used:	Individual items	Discrete series	✓ Grouped data	None of these
13)	The data when plot on graph paper it is called:	Frequency distribution	Graphic	Ogive	✓ Diagrammatic representation
14)	Histogram, histogrigram, frequency polygon, ogive are types of:	Diagrammatic representation	✓ Graphic representation	Two dimensional diagram	None of these
15)	In two dimensional diagrams we use:	✓ Length and breadth	Length	Breadth	Volume
16)	In one dimensional diagrams we draw:	✓ Component bar diagrams	Rectangles	Cubes	Pie Diagram
17)	The main classes of graphs are:	Four	Three	✓ Two	Five

S#	Questions	Answers			
18)	Bar chart has:	Two types	✓ Four types	Six types	Ten types
19)	When we construct histogram, which is to be taken along X-axis:	Class intervals	Mid points	✓ Class boundaries	Class limits
20)	The graph of class boundaries and frequency is:	Bar chart	Ogive	✓ Histogram	None of these
21)	When we connect the mid – points of rectangles in a histogram with lines, we get:	Ogive	Histogram	✓ Frequency polygon	Frequency curve
22)	Which one of the following charts is suitable when two or more different variables are presented with total is:	Simple bar chart	✓ Multiple bar chart	Pie chart	None of these
23)	A set of adjacent rectangles is called:	Ogive	✓ Histogram	Multiple bar diagram	Historigram
24)	A circle in which sectors represent various quantities is called:	✓ Pie chart	Simple bar chart	Multiple bar chart	All above
25)	To show the component parts by sectors, the angle for each sector is obtained by:	$\frac{\text{Component part}}{\text{Total}} \times 90^\circ$	$\frac{\text{Component part}}{\text{Total}} \times 180^\circ$	$\frac{\text{Component part}}{\text{Total}} \times 270^\circ$	✓ $\frac{\text{Component part}}{\text{Total}} \times 360^\circ$
26)	The graph of a time series is called:	Histogram	Ogive	✓ Histogrigram	Pie chart

S#	Questions	Answers			
27)	A graph formed by joining the mid-points of the tops of successive bars in a histogram by straight lines is:	✓ Frequency polygon	Histogrigram	Ogive	None of these
28)	A graph of a cumulative frequency distribution is called:	Histogram	Historigram	Frequency polygon	✓ Ogive
29)	The data related to import and export of Pakistan should be presented by:	Simple bar diagram	✓ Multiple bar diagram	An ogive	Histogram
30)	The graph of a symmetrical distribution is:	U shaped	J shaped	✓ Bell shapped	None of these
31)	Presentation of statistical data by geometrical curves is called:	✓ Graphs	Diagrams	Both graph & diagram	None of these
32)	Translation of statistical data into geometrical figures is called:	Graphs	✓ Diagrams	Both graph & diagram	None of these

QUESTION NO. 2

Write the short answer to the following questions in the space provided

Q. 1: What is diagrammatic representation?

Ans: Presentation of statistical data by geometrical figures is called diagrammatic representation.

Q. 2: What are the types of diagrams?

Ans: The following are the different types of diagrammatic representation.

(i) One dimensional diagrams (ii) Two dimensional diagrams
(iii) Three dimensional diagrams (iv) Pie diagrams
(v) Pictograms (vi) Cartograms

Q. 3: What is a chart?

(Faisalabad Board, 2008)

Ans: A chart is a device used for presenting a statistical data in a simple, clear and effective manner.

Q. 4: Write any four types of diagrams.

(Faisalabad Board, 2009)
(Sargodha Board, 2010)

Ans: Following are the different types of diagrams
(i) One dimensional diagrams (ii) Two dimensional diagrams
(iii) Three dimensional diagrams (iv) Pie diagrams
(v) Pictogram (vi) Cartograms

Q. 5: Define a simple bar chart.

Ans: A simple bar chart consists of equally spaced vertical or horizontal bars of the same width, having heights proportional to the corresponding numerical values.

Q. 6: Define a multiple bar chart.

Ans: Multiple bar chart is used to represent two or more related sets of data having some common characteristics in a variable value.

Q. 7: Define subdivided or component bar chart.

Ans: The charts used to present the data which are to be shown in the parts are called subdivided or component bar charts.

Q. 8: Define two dimensional diagrams or charts.

Ans: In two dimensional diagrams two dimensions i.e. length and breadth is taken into account. These are called area diagrams.

Q. 9: What are the types of two dimensional diagram.

Ans: Following are the two dimensional diagrams: (i) Squares (ii) Rectangles (iii) Circles.

Q. 10: Define three dimensional diagrams or charts.

Ans: These diagrams are based on three dimensions i.e. length, breadth and thickness. These are called volume diagram.

Q. 11: What are the types of three dimensional diagrams?

Ans: Following are the three dimensional diagrams (i) Cubes (ii) Blocks (iii) Cylinder.

Q. 12: What is a pie diagram or chart?

(Lahore Board, 2008)
(Faisalabad Board, 2008 & 2010)

Ans: Pie chart consists of a circle divided into sectors whose area are proportional to the various parts into which the total quantity is divided.

Q. 13: What is the formula used to obtain the sector wise degree of angles?

Ans: Sector wise degree of angles can be calculated through the help of following formula.

$$\text{Angle of Sector} = \frac{360 \times \text{Figure of the Sector}}{\text{Aggregate i.e. Total}}$$

Q. 14: Define the pictograms.

Ans: When relative values of items are represented by symbols or pictures, it is called pictograms.

Q. 15: Define cartograms.

Ans: The used of maps in the representation of data is called cartograms.

Q. 16: What are the advantages of diagrams?

(Lahore Board, 2010)

Ans: Following are some advantage of diagrams.

(i) Attractive impression (ii) Effective impression
(iii) Easy to understand (iv) Helpful to economists
(v) Helpful in business and administration
(vi) Helpful to all

Q. 17: Define the graph.

Ans: Presentation of statistical data into geometrical curves is called graphic representation of data.

Q. 18: What are main classes of graphs? Or name the types of graphs?

(Lahore Board, 2008)
(Gujranwala Board, 2010)

Ans: There are two main classes of graphs:

(i) Graphs of time series or historigram
(ii) Graphs of frequency distributions

Q. 19: What is the historigram?

(Faisalabad Board, 2010)

Ans: Historigram is a curve which shows the changes in the value of one or more variables from one period of time to the next.

Q. 20: What are the important graphs of frequency distributions?

(Lahore Board, 2008, 2009 & 2010
(Gujranwala Board, 2010)
(Sargodha Board, 2010)

Ans: Following are the important graphs of frequency distributions:

(i) Histogram
(ii) Frequency polygon
(iii) Frequency curve
(iv) Cumulative frequency polygon or ogive

Q. 21: Define Histogram.

(Lahore Board, 2008, 2009 & 2010)
(Gujranwala & Sargodha Board, 2010)

Ans: A histogram consists of a set of adjacent rectangles having bases along X-axis with centres at mid points and areas proportional to class frequencies.

Q. 22: What is the difference between histogram and historigram.

Ans: Histogram is the graph of frequency distribution where as historigram is the graph of a time series.

Q. 23: Define frequency polygon.

Ans: Frequency polygon is a graph that represent the data by using lines that connect points plotted for the frequencies at the mid points of the classes.

Q. 24: What is meant by frequency curve?

Ans: If the frequency polygon is smoothed, the resulting curve is called a frequency curve.

Q. 25: Define cumulative frequency polygon or ogive.

(Faisalabad Board, 2009)

Ans: An ogive is a graph drawn for the cumulative frequency distribution by joining with straight lines the dots marked above the upper boundaries of classes at heights equal to the cumulative frequencies of respective classes.

Q. 26: What are different methods of representation? Write two names.

(Lahore Board, 2010)

Ans: Following are the two methods of representation.

(i) Graphical
(ii) Tabular

Q. 27: For which distribution the graph is bell shaped?

Ans: For symmetrical distribution the graph is bell shaped.

Q. 28: What do you meant by bivariate frequency distribution?

Ans: The constructed frequency distribution, considering two variables at a time is called bivariate frequency distribution.

3 MEASURES OF LOCATION OR CENTRAL TENDENCY

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The sum of values divided by their numbers is called	Mode	✓ Mean	Median	None of these
					(Faisalabad & Lahore Boards, 2008)
2)	If $Z = X - Y$ then is $\bar{Z}$	$\bar{X} + \bar{Y}$	✓ $\bar{X} - \bar{Y}$	0	$\bar{X}\bar{Y}$
					(Faisalabad Board, 2008)
3)	Sum of deviation from mean is	✓ Zero	Positive	Negative	None of these
					(Lahore Board, 2008)
4)	The most frequent value in the data is called	✓ Mode	Median	G.M	H.M
					(Lahore Board, 2008; Faisalabad Board, 2010)
5)	In symmetrical distribution mean median and mode are always	Unequal	✓ Equal	Different	Negative
					(Lahore Board, 2008)
6)	The mid point of a class ranging from '40' to '60' is	30	40	✓ 50	60
					(Gujranwala Board, 2009)
7)	Mode of the word 'Professor' is	R	S	O	✓ R, S and O
					(Faisalabad Board, 2009)
8)	The mean, median, mode of constant 'a' are	0	$a/2$	✓ a	a^2
					(Faisalabad Board, 2009)

S#	Questions	Answers			
9)	The most frequent (common) value in the data is	Mean	Median	✓ Mode	Both mean and median
					(Lahore Board Group I&II, 2009)
10)	The notation for mean is	X	✓ $\bar{X}$	X	$\overline{X_w}$
					(Lahore Board, 2009)
11)	For classes, 50 – 54, 55 – 59, 60 – 64, ... The class interval is	4	✓ 5	10	None of these
					(Lahore Board, 2009)
12)	The size of class is also called	✓ Class interval	Class frequency	Class mark	Class boundary
					(Lahore Board, 2009)
13)	We must arrange the data, before calculating	Mean	✓ Median	Mode	None of these
					(Lahore Board, 2009)
14)	The median of 3, 4, 5, 6, 9, 10, 12, is	5	✓ 6	9	5.5
					(Faisalabad Board, 2010)
15)	For a certain distribution, if $\Sigma(x-5)-0$ the value of mean is	0	✓ 5	–5	None of these
					(Lahore Board, 2010)
16)	Mode of the series 2, 3, 3, 3, 4, 4, 5, 0 is	2	✓ 3	4	6
					(Lahore Board, 2010)
17)	To find the avcrage of blue colour pieces of cloth, we use	Median	✓ Mode	Mean	All
					(Lahore Board, 2010)

S#	Questions	Answers			
18)	The mean of 10 numbers is 9.2, then the sum of numbers is	72	82	✓ 92	102
		(Lahore Board, 2010)			
19)	Averages are also called measures of	Variation	✓ Location	Skewness	Median
		(Gujranwala Board, 2010)			
20)	The sum of squared deviations is least from	Median	✓ Mean	Mode	None
		(Gujranwala Board, 2010)			
21)	The number of observations in particular class is known as	✓ Frequency	Class interval	Mid point	Middle value
		(Rawalpindi Board, 2010)			
22)	The mode in word 'STATISTICS' is /are	I	T	S	✓ T and S
		(Rawalpindi Board, 2010)			
23)	A data having single mode is	✓ Uni-model	Bi-model	Tri-model	Multi-model
		(Rawalpindi Board, 2010)			
24)	A value obtained by dividing sum of all the values by their number is called	✓ A.M.	Median	Mode	G.M.
		(Sargodha Board, 2010)			
25)	To find out mid-point of class the formula is	✓ $\frac{\text{Upper Limit} + \text{Lower Limit}}{2}$	$\frac{\text{Upper Limit} - \text{Lower Limit}}{2}$	Upper Limit + Lower Limit	Upper Limit − Lower Limit
		(Sargodha Board, 2010)			
26)	Quartiles divide the values in equal parts	2	3	✓ 4	5
		(Sargodha Board, 2010)			

S#	Questions	Answers			
27)	Mid point of class 10 – 20 is	10	20	30	✓ 15

(Sargodha Board, 2010)

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The central value which represents a set of data is called measure of:	✓ Central tendency	Skewness	Dispersion	Symmetry
2)	The value obtained by dividing the sum of all the values by their numbers is called:	Median	Mode	✓ Arithmetic mean	Skewness
3)	The sum of deviations taken from mean is:	✓ Always equal to zero	Some times equal to zero	Never equal to zero	Less than zero
4)	The sum of deviations of observations is zero if deviations are taken from:	Mode	Median	✓ Arithmetic Mean	Provisional Mean
5)	For a data if $\Sigma(x-20) = 14$, $\Sigma(x-25) = 0$, $\Sigma(x-40) = 29$, then the value of A.M. is:	20	✓ 25	40	29
6)	Sum of deviations from arithmetic mean is always:	Less than zero	More than zero	✓ Equal to zero	None of these
7)	The smallest and the largest values of any given class of a frequency distribution are called:	✓ Class limits	Class marks	Class interval	Class frequency

S#	Questions	Answers			
8)	The difference between the upper and lower class boundaries of a class are known as:	Class limits	✓ Class interval	Class marks	Class frequency
9)	The number of observations or objects falling into a particular class is called	✓ Class frequency	Relative frequency	Cumulative frequency	None of these
10)	The average value of the lower and upper limits is called:	✓ Class mark	Class width	Class frequency	Class boundaries
11)	Class mark or class mid point is equal to:	Upper limit − lower limit	Upper limit + lower limit	✓ $\frac{\text{Upper Limit} + \text{Lower Limit}}{2}$	Upper Limit − $\frac{\text{Lower Limit}}{2}$
12)	The mid point of a class ranging from 40 to 60 is:	30	40	✓ 50	60
13)	Given 8 quantities. x_1 to x_8, the correct notation for adding quantities 3 to 6 is:	$\sum_{i=1}^{4} xi$	$\sum_{i=2}^{5} xi$	✓ $\sum_{i=3}^{6} xi$	$\sum_{i=4}^{8} xi$
14)	Given $x_1 = 10$, $x_2 = 20$, $x_3 = 25$, $x_4 = 11$ $\sum_{i=1}^{3} xi$ equal:	45	✓ 55	65	75
15)	We calculate arithmetic mean by:	Direct method	Short cut method	Step deviation	✓ All the above methods
16)	The most common value in a set of data is called:	A.M.	Median	Mode	None of these
17)	The algebric sum of the observations of a set of data from their mean is always equal to:	Three	Two	One	✓ Zero

S#	Questions	Answers			
18)	The sum of squares of the observations from their mean is always:	✓ Minimum	Maximum	Central	Zero
19)	If the mean of 15 observations is 20, then the sum of these numbers is:	100	200	✓ 300	400
20)	The average which is used to find the middle value of data is called;	Arithmetic mean	✓ Median	Mode	None of these
21)	We must arrange the data before calculating:	Mean	✓ Median	Mode	All of these
22)	The combined mean is calculated by the formula:	$\frac{\bar{x}_1 + \bar{x}_2}{n_1 + n_2}$	$\frac{n_1x_1 + n_2x_2}{n_1 + n_2}$	✓ $\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$	$\frac{n_1 + n_2}{\bar{x}_1 + \bar{x}_2}$
23)	Which of the following is not based upon all the values	A.M.	✓ Mode	Both A.M. & mode	None of these
24)	In a symmetrical distribution $Q_1 = 20$, Med = 30, then the value of Q_3 will be:	80	60	✓ 40	10
25)	Upper quartile = Q_3 = _____	✓ P_{75}	D_4	Median	D_7
26)	The formula of median used for discrete grouped frequency distribution is:	$L + \frac{h}{f}(\frac{n}{2} - c)$	$(\frac{n+1}{4})^{th}$ value	$(\frac{n+1}{2})^{th}$ value	✓ None of these
27)	The most frequencent value of data is called:	Mean	Median	✓ Mode	None of these
28)	The mode of the letters in the word 'MATHEMATICS' is:	M	A	T	✓ All M, A, T

S#	Questions	Answers			
29)	The mode of the letters in the word 'STATISTICS' is:	S	T	✓ Both S & T	A
30)	Median is a value which divides an arranged distribution into:	Four parts	Three parts	✓ Two parts	None of these
31)	Cumulative frequency is called:	✓ Increasing	Decreasing	Both increasing & Decreasing	Constant
32)	Mode will lie in a class which is said to be:	✓ Modal class	Middle class	Class boundaries	Class frequency
33)	If the data are arranged, median is central value:	✓ Always	Never	Some time	None of these
34)	Corresponds to maximum frequency:	Median	AM	✓ Mode	Skewness
35)	The relation between mean, median, mode i.e. Mode = 3 Median – 2 mean is called:	✓ Empirical	Symmetrical	Asymmetrical	All of these
36)	For positively skewed distribution Mean ….. Median ….. Mode:	=	<	✓ >	≠
37)	For negatively skewed distribution Mean ….. Median ….. Mode	=	✓ <	>	≠
38)	For a symmetrical distribution Mean ….. Median ….. Mode:	✓ =	<	>	≠

S#	Questions	Answers			
39)	In a symmetrical distribution, the relation between mean, median and mode is:	✓ Mean = Median = Mode	Mean < Median < Mode	Mean > Median > Mode	Mean > Median < Mode
40)	Mean, Median and Mode are equal in	Skewed distribution	✓ Symmetrical distribution	Both skewed & symmetrical distribution	None of these
41)	For positively skewed distribution:	Mean = Median = Mode	✓ Mean > Median > Mode	Mean < Median < Mode	Mean < Med > Mode
42)	For negatively skewed distribution:	Mean = Median = Mode	Mean > Median > Mode	✓ Mean < Median < Mode	Mean < Median > Mode
43)	The arithmetic mean, median, mode are called:	✓ First order averages	Second order averages	Third order averages	Fourth order average

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: What is a statistical average?

(Lahore Board, 2009)
(Rawalpindi Board, 2010)

Ans: An average is a single value that is typical or representative for a group of numbers.

Q. 2: What are other names of average?

Ans: As an average indicates the location or general position of the distribution or an average is some where in the middle of the group, so it is also called measures of central tendency or measures of location or measures of position.

Q. 3: Why are the averages called measures of central tendency?

Ans: As the averages tend to lie in the center of the distribution they are called measures of central tendency.

Q. 4: What is "Measures of central tendency"?

(Lahore Board, 2010)

Ans: Since the averages which mostly fall in the centre of data and used to represent the whole data. So these averages are called measures of central tendency.

Q. 5: What are the objects of measures of central tendency or an average.

Ans: Objects of measures of central tendency are as follows:

(i) To find out a value that can be represent the whole data.
(ii) To help the comparative study.
(iii) To provide a condensed picture of a large group.

Q. 6: What are the important types of averages?

Ans: The most commonly used averages are:

(i) Arithmetic mean (ii) Geometric mean
(iii) Harmonic mean (iv) Median
(v) Mode

Q. 7: What are characteristics of a good average?

(Lahore Board, 2009 & 2010)
(Sargodha Board, 2010)

Ans: It should be:

(i) Easy to calculate (ii) Easy to understand
(iii) Rigidly defined (iv) Based on the values of data
(v) Least affected by extreme values

Q. 8: What type of averages are mathematical in character?

Ans: The arithmetic mean, geometric mean and harmonic mean are mathematical in character.

Q. 9: What type of average are averages of location?

Ans: The median and mode are averages of location.

Q. 10: What does Σ denote for?

Ans: Σ is used to denote the sum of all the values of a series.

Q. 11: Define arithmetic mean.

(Lahore Board, 2009)
(Gujranwala & Sargodha Boards, 2010)

Ans: The arithmetic mean is defined as a value obtained by dividing sum of all the values by their numbers.

Q. 12: Give any three algebric properties of A.M.

(Lahore Board, 2008; Faisalabad Board, 2009; Rawalpindi Board, 2010)

Ans:

(i) $\Sigma(X - \bar{X}) = 0$

(ii) $\Sigma(X - \bar{X})^2$ is minimum

(iii) If $y_i = ax_i + b$ then $\bar{Y} = a\bar{X} + b$

Q. 13: The sum of observations from their mean -----?

Ans: The sum of observations from their mean is zero.

Q. 14: What will be the mean of 36, 36, 36, ..., 36?

Ans: As we know that arithmetic mean of a constant is constant itself, so mean is 36.

Q. 15: Discuss the used of weighted mean is statistics.

Ans: When all the observations are not of equal importance, then we used weighted arithmetic mean.

Q. 16: Where arithmetic mean is preferred to measure center of the data.

Ans: Arithmetic mean is preferred when:

(i) The distribution is approximately symmetrical.
(ii) A measure is needed that reflects the total.
(iii) The measure of central tendency having the greatest stability is required.

Q. 17: Write three merits (advantages) of arithmetic mean.

(Faisalabad Board, 2009)

Ans:

(i) Simple to understand and easy to calculate.
(ii) Rigidly defined.
(iii) Capable of further mathematical treatments.

Q. 18: Define median.

(Lahore Board, 2008, 2009 & 2010)
(Faisalabad & Sargodha Boards, 2010)

Ans: The median is defined as the middle most value of the arranged data.

Q. 19: Give the merits of the median.

(Lahore Board, 2008 & 2009)
(Sargodha Board, 2010)

Ans: Median is:

(i) Simple to understand and easy to calculate
(ii) Not affected by extreme observations
(iii) Possible to locate graphically

Q. 20: What are the demerits of the median?

(Lahore Board, 2008)
(Faisalabad Board, 2009)
(Gujranwala Board, 2010)

Ans: Median is:

(i) Not rigidly defined (ii) Not based on all values
(iii) Not capable of further mathematical treatment

Q. 21: Is sum of absolute deviations of the observations from mean is minimum.

Ans: No, the sum of absolute deviations taken from median is minimum.

Q. 22: What do you mean by quartilies?

Ans: Quartilies are the values, which divide the arranged data into four equal parts.

Q. 23: Define deciles and percentilies?

Ans: Deciles and percentilies are the values, which divide the arranged data into ten and hundred equal parts.

Q. 24: Define mode with example.

(Lahore Board, 2008 & 2010)
(Faisalabad Board 2008, 2009 & 2010)
(Rawalpindi Board, 2010)

Ans: The most repeated or most common or most frequent value in the set of data is called mode. For example: 20, 10, 18,, 10, 18, 7, 10, 30, 15
Here most repeated value is 10, so mode = 10

Q. 25: Which measure of location can measure more than one value for a data.

Ans: The only measure of location that can assume more than one value for a data is mode.

Q. 26: What is the mode of the following data?
10, 20, 30, 40, 50, 60, 70, 80, 90

Ans: No mode, as there is no repeated value.

Q. 27: Define bi-modal distribution.

(Lahore Board, 2009)

Ans: A distribution having two modes is defined bi-modal distribution.

Q. 28: Discuss the relationship between mean, median and mode.

Ans: For a skewed distribution:

$$\text{Mode} = 3\,\text{Med} - 2\,\text{Mean}$$

Which is called empirical relation.
For a symmetrical distribution, the mean, median and mode are equal i.e.

$$\text{Mean} = \text{Median} = \text{Mode}$$

Q. 29: Give the two merits of the mode.

Ans: (i) Mode is simple to understand and easy to calculate
(ii) It is not affected by extreme values.

Q. 30: Give three merits of the mode.

Ans: (i) Mode is not rigidly defined.
(ii) It is not based on all the values.
(iii) It is not capable of further mathematical treatment.

Q. 31: What is mode of this data?
10, 20, 30, 10, 40, 50, 10, 60

(Faisalabad Board, 2009)

Ans: Here most common or most repeated number is 10, so mode = 10

Q. 32: Write down the formula to compute mode for grouped data.

(Faisalabad Board, 2010)

Ans: The formula to compute mode for group data is

$$\text{Mode} = L + \frac{f_m - f_m}{(f_m - f_1) + (f_m - f_2)} \times h$$

Where f_m = Maximum frequency
f_1 = Preceding value of frequency to f_m
f_2 = Next value of frequency to f_m
L = Lower limit of mode – group
h = Interval size

Q. 33: What is meant by uni-modal distribution?

(Faisalabad & Sargodha Boards, 2010)

Ans: A distribution, having single value of mode is called uni-modal distribution.
For example. 55, 58, 57, 58, 60, 50, 53, 58
is a uni-model distribution have mode = 58 only.

Q. 34: What do you understand by "Change of Origin"?

(Lahore Board, 2008)

Ans: The addition or subtraction of any constant value from each value of a variable is called change of origin.

Q. 35: What is modal class?

(Lahore Board, 2010)

Ans: The class in which maximum frequency lies, is called modal class.

Q. 36: State the formula of arithmetic mean (grouped data / ungrouped data). **(Lahore Board, 2010)**

Ans: **Ungrouped Data**

i) $A.M = \frac{\sum X}{n}$ Direct Method

ii) $A.M = P.M + \frac{\sum D}{n}$ Indirect or Short cut Method

Grouped Data

i) $A.M = \frac{\sum fX}{\sum f}$ Direct Method

ii) $A.M = P.M + \frac{\sum fD}{\sum f}$ Indirect or Short cut Method

iii) $A.M = P.M + \frac{\sum fu}{\sum f} \times h$ Coding Method

Q. 37: In a symmetrical distribution arithmetic mean is 10. Find the value of median and mode.

(Gujranwala Board, 2010)

Ans: In a symmetrical distribution mean, med and mode are equal.
So, if mean = 10, then med = 10 and mode = 10

Q. 38: Find median 5, 10, 12, 13, 15, 7, 9

(Lahore Board, 2010)

Ans: For median, arrange the values in ascending and descending order,

5, 7, 9, 10, 12, 13, 15

Here n = 7 (odd number), so

$$\text{Med} = \text{The value of } \left(\frac{n+1}{2}\right)\text{th item} \Rightarrow = \text{The value of } \left(\frac{7+1}{2}\right)\text{th item}$$

$$= \text{The value of } \left(\frac{8}{2}\right)\text{th item} = \text{The value of 4th item} = 10$$

Hence med = 10

Q. 39: Find out arithmetic mean given $\Sigma X = 308$, $n = 7$

(Lahore Board, 2008)

Ans: Here $\Sigma X = 308$, $n = 7$

$$A.M = \frac{\Sigma X}{n} = \frac{308}{7} = 44$$

Q. 40: For a certain frequency, the value of mean is 15, median is 20. What will be the value of mode?

(Lahore Board, 2008)

Ans: Here Mean = 15, Median = 20

We know that

$$\text{Mode} = 3\text{med} - 2\text{mean} \Rightarrow = 3(20) - 2(15) = 60 - 30 = 30$$

Q. 41: What position exit when mean = med = mode?

(Faisalabad Board, 2008)

Ans: A distribution is said to be symmetrical if mean, mode are equal.

Q. 42: Find A.M given that $X = 10 + 5u$, $\Sigma fu = 46$, and $n = 125$

(Lahore Board, 2008)

Ans: Here $X = 10 + 5u$, $\Sigma fu = 46$, $n = \Sigma f = 125$

We know that

$$\bar{X} = P.M + \frac{\Sigma fu}{\Sigma f} \times h \Rightarrow u = \frac{X - P.M.}{h}$$

We have $X = 10 + 5u$

$5u = X - 10$

$$u = \frac{x - 10}{5} = \frac{X - P.M.}{h}$$

Hence P.M. = 10, h = 5

Now

$$\bar{X} = 10 + \frac{46}{125} \times 5 = 10 \frac{46}{h25} = 10 + 1.84 = 11.84$$

4 INDEX NUMBERS

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	In chain base method the base year is	✓ Variable	Fixed	Both variable & fixed	None of these
					(Faisalabad Board, 2008)
2)	If $\Sigma P_1q_0 = 403$, $\Sigma P_0q_0 = 283$, then IN is	152.40	130.40	122.4	✓ 142.4
					(Faisalabad Board, 2008)
3)	Index number are called the barometers of	Statistics	✓ Economics	Mathematics	None of these
					(Lahore Board, 2008; Faisalabad Board, 2009)
4)	Index number for base period is	100	✓ Always 100	Never 100	None of these
					(Lahore Board, 2008; Gujranwala Board, 2009)
5)	Price relative is percentage ratio of current year price and	✓ Base year price	Base year quantity	Preceding year price	None of these
					(Lahore Base 2008)
6)	For averaging the relatives, we never use	A.M.	G.M.	Median	✓ Mode
					(Lahore Board, 2008)
7)	Link relative are used in	Fixed base method	✓ Chain base method	Both fixed and chain	None of these
					(Gujranwala Board, 2009)

S#	Questions	Answers			
8)	Index number for base period is	✓ 100	One	Fix	None of these
					(Faisalabad & Lahore Boards, 2009)
9)	In chain base method, the base period is	Fixed	Constant	✓ Not fixed	Zero
					(Lahore Board, 2009)
10)	The most suitable average, in connection with index number	A.M.	Median	Mode	✓ G.M.
					(Lahore Board, 2009)
11)	Laspeyre's index number is also called	✓ Base year weight	Current year weight	Ideal index number	None of these
					(Lahore Board, 2009; Faisalabad Board, 2010)
12)	Index number for base period is	✓ 100	200	150	50
					(Faisalabad Board, 2010)
13)	Current year weighted index number is also called	✓ Paache's	Marshall's	Fisher's	Laspyre's
					(Lahore Board, 2010)
14)	In chain base method the base year is	Fixed year	Next year	✓ Previous year	Same year
					(Lahore Board, 2010)
15)	In chain base method the base period is	Fixed	✓ Changed	Constant	None of these
					(Lahore Board, 2010)

S#	Questions	Answers			
16)	$\frac{P_n}{P_0} \times 100$ is equal to	Link relative	Chain indices	✓ Price relative	None of these
					(Lahore Board, 2010)
17)	Which is considered to be the most suitable average for computing index number?	Median	Mean	Mode	✓ None
					(Gujranwala Board, 2010)
18)	Fisher's index number is	$\sqrt{\frac{P}{L}}$	$\sqrt{\frac{L}{P}}$	✓ $\sqrt{L \times P}$	None of these
					(Gujranwala Board, 2010)
19)	Index numbers are called	Statistical barometer	✓ Economic barometer	Mathematical barometer	Physical barometer
					(Rawalpindi Board, 2010)
20)	In fixed base method the base period should be	For away	Abnormal	✓ Normal	Unreliable
					(Rawalpindi Board, 2010)
21)	Price index number calculate changes is	Price	Quantity	✓ Price and quantity	None of these
					(Sargodha Board, 2010)

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	Index number are divided into two types:	Price and quantity	✓ Simple and composite	Weighted and un-weighted	None of these

S#	Questions	Answers			
		index numbers	index numbers	index number	
2)	An index number is called a simple index number when it deal with:	✓ Single variable	Two variables	More than two variables	None of these
3)	If all the values of equal importance, the index numbers are called:	✓ Un-weighted	Weighted	Composite	None of these
4)	An index number, which indicates a relative change in a single variable with respect to a base, is called:	Weighted index number	Composite index number	✓ Simple index number	None of these
5)	The most suitable average in connection with index numbers is:	Arithmetic mean	Median	✓ Geometric mean	H.M.
6)	Price relative is the percentage ratio of current year price and:	✓ Base year price	Preceding year price	Both base & current	None of these
7)	Link relative is the percentage ratio of current year price and:	Base year price	✓ Preceding year price	Both base & preceding year price	None of these
8)	An index number constructed to measure the relative changes in price of a commodity or a group of commodities are:	✓ Price index numbers	Quantity index numbers	Consumer's index numbers	All of these
9)	Index number constructed to measure the relative change in quantity or volume are:	Price index numbers	✓ Quantity index numbers	Consumer's index numbers	Laspeyre Index number

S#	Questions	Answers			
10)	In aggregative index numbers we use:	Only price	Only quantity	✓ Both price and quantity	None of these
11)	When all the values are not of equal importance, we assign certain values and index numbers are called:	✓ Weighted index numbers	Un-weighted index numbers	Simple index number	Composite index number
12)	Index number for base period is always taken:	✓ 100	200	300	400
13)	The prices of wheat are to be compared by:	Quantity index numbers	Aggregative index numbers	✓ Price index numbers	None of these
14)	Import and export of Pakistan is an example of:	✓ Composite index number	Whole sale price index number	Simple index number	None of these
15)	In chain base method, the base period is:	Fixed	✓ Not fixed	Constant	Zero
16)	Other name of consumer's price index number is:	✓ Cost of living index number	Whole sale price index number	Composite index number	Simple index number
17)	Index numbers are called:	Statistical barometer	Mathematical barometer	✓ Economic barometer	✓ All of these
18)	A normal year should be free form:	War	Floods	Strikes	All of the above
19)	The formula used in fixed base method is:	✓ $\frac{P_n}{P_0} \times 100$	$\frac{P_{n-1}}{P_n} \times 100$	$\frac{P_n}{P_{n-1}} \times 100$	$\frac{P_n - 1}{P_0}$

S#	Questions	Answers			
20)	The formula used in chain base method is:	$\frac{P_n}{P_0} \times 100$	✓ $\frac{P_{n-1}}{P_n} \times 100$	$\frac{P_n}{P_{n-1}} \times 100$	$\frac{P_n - 1}{P_0}$
21)	If a price of current year is divided by the price of a fixed particular year, the result will be:	Link relative	Chain relative	✓ Simple relative	All of the above
22)	Price relatives computed by chain base method is called:	Price relatives	Chain indices	✓ Link relative	Non of them
23)	Consumer price index numbers are obtained by:	✓ Laspeyre's formula	Paache's formula	Fisher's formula	Marshal formula
24)	Consumer price index numbers are obtained by:	Laspeyre's formula	Family budget method formula	✓ Both Laspeyre & Family budget method	None of these
25)	Aggregative expenditure method and family budget method always give:	✓ Same results	Different results	Approximate results	Zero results
26)	Laspeyre's index number is also called:	✓ Base year weighted index number	Current year weighted index number	Ideal index numbers	All of the above
27)	Paache's index number is also called:	Base year weighted index number	✓ Current year weighted index number	Ideal index number	Non of the above
28)	Laspeyre's and Paache's index number are:	Simple index numbers	✓ Weighted index	Composite index number	All of these

S#	Questions	Answers			
			numbers		
29)	Current year quantities are used as weighted in:	Laspeyre's index number	✓ Paache's index number	Composite index number	None of these
30)	Which one is the ideal index number	Laspeyre's index number	Paache's index number	Marshall Edgeworth index number	✓ Fisher's index number
31)	Fisher's ideal index number is also obtained by:	Calculating A.M. of Laspeyre's and Paache's index number	✓ Calculating G.M. of Laspeyre's and Paache's index number	Median of Laspeyre's and Paache's index number	None of these

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: Define an index numbers. **(Lahore Board, 2008 & 2010) (Rawalpindi Board, 2010; Sargodha Board, 2010)**

Ans: An index number is a series of numbers which measures relative change in a variable or a group of variables with respect to time or space.

Q. 2: Define price index number. **(Lahore Board, 2008 & 2010)**

Ans: An index number, which measure the relative change in the price of a commodity or an average relative change in the price of group of commodities with respect to a base is called price index number.

Q. 3: What is the purpose of index number? **(Gujranwala Board, 2010)**

Ans: Index number is a statistical measure designed to show changes in a variable or a group of related variables for the purpose of comparison.

Q. 4: Define simple index number.

Ans: An index number is called simple index number if it measure a relative change in a single variable with respect to a base.

Q. 5: Give two examples of simple index number.

Ans: Index number for wages of employees, index number of sugar prices.

Q. 6: Define composite price index number.

(Lahore Board, 2008)
(Faisalabad Board, 2010)

Ans: An index number, which indicates an average relative change in the prices of a group of related commodities with respect to a base.

Q. 7: What are the types of index number as regard to base?

Ans: These are two types of index number.

(i) Fixed base index
(ii) Chain base index

Q. 8: Define fixed base method.

(Lahore Board, 2009)
(Gujranwala & Sargodha Boards, 2010)

Ans: In fixed base method, the base period remains fixed from time to time. The formula for fixed base method is as

$$P_{0n} = \frac{P_n}{P_0} \times 100$$

Where P_n = Current year price &
P_0 = Base year price

Q. 9: Define chain base method.

(Lahore Board, 2008)
(Rawalpindi Board, 2010)

Ans: In chain base method base period does not remain fixed but price (quantity) of each year is compared with preceding year.
The formula for chain base method is as:

$$P_{0n} = \frac{P_n}{P_{n-1}} \times 100$$

Where P_n = Current year price &
P_{n-1} = Preceding year price to current year price

Q. 10: What is price relative? Also give its formula.

(Lahore Board, 2010)

Ans: Price relative is the percentage ratio of the price in current year and the price in a base year.

$$P_{0n} = \frac{P_n}{P_0} \times 100$$

Q. 11: Define link relative.

(Faisalabad & Lahore Boards, 2008 & 2010)

Ans: Link relative is the percentage ratio of the price in current year and the price in the preceding year.

Q. 12: What are the various methods of averaging that can be used in constructing index numbers?

Ans: Following methods are used:

(i) Arithmetic mean (ii) Median
(iii) Geometric mean

Q. 13: What average is considered to be the most suitable in connection with index number?

Ans: Geometric mean is the most suitable average for measuring the average ratio of change in prices, but arithmetic mean is mostly used.

Q. 14: What is weighted aggregative price index number?

(Faisalabad Board, 2009)
(Lahore Board, 2008 & 2010)
(Rawalpindi Board, 2010)

Ans: An index number that measures the change in the prices (or quantities) of a group of commodities when the relative importance of commodities has been taken into account is called weighted aggregative index number.

Q. 15: What are the important weighted aggregative price index number?

(Faisalabad Board, 2008)

Ans:
(i) Laspeyre's index number
(ii) Paache's index number
(iii) Fisher's ideal index number
(iv) Marshall Edgeworth index number

Q. 16: What is the base year weighted index number?

(Faisalabad Board, 2010)

Ans: Laspeyre's index number is called base year weighted index number.

$$P_{0n} = \frac{\sum P_n q_0}{\sum P_n q_0} \times 100$$

Q. 17: What is the current year weighted index number?

(Sargodha Board, 2010)

Ans: Paache's index number is called current year weighted index number.

$$P_{0n} = \frac{\sum P_n q_n}{\sum P_0 q_n} \times 100$$

Q. 18: Write the name of '3' types of index number OR describe the main types of index number.

(Lahore Board, 2008 & 2010)

Ans: Following are the three types of index number.
(i) Prince Index Number (ii) Quantity Index Number
(iii) Value Index Number

Q. 19: What is quantity index number?

(Lahore Board, 2008)

Ans: An index number which measures the change in the quantity of goods produced, or goods exported or imported or goods consumed is called quantity index number.

Q. 20: Define a Laspeyre's index number.

(Lahore Board, 2009)

Ans: An index number in which base year prices / quantities are used as weight is called Laspeyre's index number.

$$P_{0n} = \frac{\sum P_n q_0}{\sum P_0 q_0} \times 100 \quad \text{OR} \quad Q_{0n} = \frac{\sum P_n q_0}{\sum P_0 q_0} \times 100$$

Q. 21: Give Marshall index number formula **(Lahore Board, 2010)**

Ans: The formula for Marshall index number is given below:

$$P_{0n} = \frac{\sum P_n q_0 + \sum P_n q_n}{\sum P_0 q_0 + \sum P_0 q_n} \times 100$$

Q. 22: Why Fisher's index number is called an ideal index number?

Ans: Fisher's index number is called ideal because it confirms to certain tests of consistent that is time reversal test and factor reversal test.

Q. 23: Define Fisher's ideal index number.

(Faisalabad Board, 2009)

Ans: It is the geometric mean of the Laspeyre's and Paache's index number.

$$P_{0n} = \sqrt{L \times P} = \sqrt{\frac{\sum P_n q_0}{\sum P_0 q_0} \times \frac{\sum P_n q_n}{\sum P_0 q_n}} \times 100$$

Q. 24: For what does C.P.I. stand for?

Ans: C.P.I. stand for consumer price index number.

Q. 25: Define consumer's price index number. **(Lahore Board, 2009)**

Ans: Index number which measures the changes in prices of a specified basket of goods and services consumed in the given period relative to the base period.

Q. 26: What do you mean by 'basket" of goods?

Ans: The basket of goods and services contains:

(i) Food (ii) Clothing
(iii) Education (iv) House rent
(v) Misc. etc.

Q. 27: What are the other names of consumer's price index number?

Ans: Consumer's price index number is also called cost of living index number, retail price index number.

Q. 28: How we calculate consumer's index number?

Ans: We calculate the consumer's index umber by the following two methods.

(a) Aggregate expenditure method

$$P_{0n} = \frac{\sum P_n q_0}{\sum P_0 q_0} \times 100$$

(b) Family budget method

$$P_{0n} = \frac{\sum(W \times I)}{\sum W}$$

Q. 29: Give three uses of index number.

(Faisalabad Board, 2008)

Ans: (i) Index number are used as economic barometers.
(ii) Index numbers are used in forecasting
(iii) Several economic and business policies are guided by index number.

Q. 30: Write two limitations of index number.

(Faisalabad Board, 2009)
(Lahore Board, 2010)

Ans: (i) All index numbers are not used for all purposes.
(ii) If a wrong base year is selected the results will be misleading.

Q. 31: What is the base period?

(Lahore & Rawalpindi Boards, 2010)

Ans: The period with which prices of other periods to be compared is called the base period.

Q. 32: Laspeyre's index number = 120, Fisher index = 115. Find Paache's index number.

(Gujranwala Board, 2010)

Ans:

$$\text{Fisher's Index Number} = \sqrt{\text{Laspeyre} \times \text{Paache}}$$

$$115 = \sqrt{120 \times \text{Paache's Index Number}}$$

Taking square of both sides

$$120 \times \text{Paache's} = (115)^2 = 13225$$

$$\text{Paache's} = \frac{13225}{120} = 110.21$$

Q. 33: If Laspeyre's price index number is 119.89 and Paache's index number is 119.65. Find Fisher's ideal index.

(Faisalabad Board, 2010)

Ans: Laspeyre Index Number = 119.89
Paache's Index Number = 119.65
We know that

$$\text{Fisher's Index Number} = \sqrt{\text{Laspeyre} \times \text{Paache}}$$

$$= \sqrt{119.89 \times 119.65} = \sqrt{14344.8385} = 119.77$$

5 PROBABILITY

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	An experiment of three coins are tossed has sample points	4	✓ 8	9	None of these
					(Faisalabad Board, 2008)
2)	The probability of an event always lies between	−1 and +1	−1 and 0	✓ 0 and +1	0 and ∞
					(Lahore Board, 2008 & 2010)
3)	The probability is the measure of	✓ Un-certainty	Certainty	Chances	None of these
					(Lahore Board, 2008)
4)	When two coins are tossed, the possible outcomes are	6	✓ 4	12	3
					(Gujranwala Board, 2009) (Faisalabad Board, 2009 & 2010) (Lahore Board, 2009)
5)	The probability of sure even is	0	100	✓ 1	½
					(Lahore Board, 2009)
6)	If P(A) = 0.70, then $P(\overline{A})$ = is	0.70	✓ 0.30	0	1
					(Faisalabad Board, 2010)
7)	For two mutually exclusive events A and B: P(A) = 0.70; P(B) = 0.3 then P(AUB) is	0	0.40	0.30	✓ 1
					(Lahore Board, 2010)

S#	Questions	Answers			
8)	Four books can be arranged in a shelf is	8 ways	2 ways	16 ways	✓ 24 ways
					(Lahore Board, 2010)
9)	The probability of a black card from pack of 52 cards is	$\frac{1}{52}$	✓ $\frac{1}{2}$	$\frac{13}{52}$	$\frac{3}{4}$
					(Lahore Board, 2010)
10)	The value of probability can not be greater than	Zero	✓ Unity	Sample	Two
					(Gujranwala Board, 2010)
11)	When two coins are tossed simultaneously, the probability of one head is	✓ $\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
					(Rawalpindi Board, 2010)
12)	If n coins are tossed, the possible outcomes are	n	✓ 2n	2^n	n^2
					(Rawalpindi Board, 2010)
13)	The complete sample space is called	Empty	Impossible event	Equally likely event	✓ Sure event
					(Sargodha Board, 2010)

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The idea of probability was first applied in gambling games in:	Pakistan	America	✓ France	India
2)	Today the application of the theory of probability is the most important in making decisions in:	Business	Insurance	Management	✓ All above

S#	Questions	Answers			
3)	n-factorial is denoted by:	✓ n!	m!	r!	P!
4)	$(n+2)! =$ _____	n(n-1)!	(n-1)(n-2)!	✓ (n+2)(n+1)n!	(n x 2)!
5)	Three objects can be arranged in:	3 ways	4 ways	5 ways	✓ 6 ways
6)	The total number of possible combinations of a set of a different objects, taken r at a time is denoted by:	${}^nP_r$	✓ ${}^nC_r$	${}^nD_r$	None of these
7)	${}^nC_r =$ _____	$\frac{n!}{r!}$	$\frac{n!}{(n-r)!}$	✓ $\frac{n!}{r!(n-r)!}$	$\frac{r!}{n!}$
8)	A set having zero as its element is called:	Empty set	Infinite set	Singleton set	Null set
9)	Two books are to be selected at random without replacement out of four books. The number of possible selections are:	✓ 6	2	4	3
10)	Three books of different colours are to be arranged in book-shelf. The possible arrangements are:	3	1	✓ 6	2
11)	A set consisting of all the elements under consideration is called:	Empty set	Disjoint set	✓ Universal set	Sub set
12)	A set consisting of all the elements under consideration is called:	U	λ	✓ φ	μ

S#	Questions	Answers			
13)	Sets having elements in common are called:	✓ Overlapping sets	Disjoint sets	Infinite sets	None of these
14)	The set A is a subset of the set B if every element of A is an element of B. We use the symbol as:	$A \supseteq B$	✓ $A \subseteq B$	$A = B$	$A + B$
15)	The union of two sets in denoted by:	✓ $(A \cup B)$	$(A \cap B)$	$A = B$	$A + B$
16)	The intersection of two sets is denoted by:	$A \cup B$	✓ $A \cap B$	$A \subset B$	$A - B$
17)	An act or the process of obtaining an observation is called:	✓ Experiment	Sample space	Event	Set
18)	If an experiment provides different results under essentially identical conditions, it is called:	Sample space	Simple event	✓ Random experiment	Sub set
19)	A set of all possible outcomes of the random experiment is called:	✓ Sample space	Impossible event	Union of sets	Sub set
20)	Any subset of a sample space is called:	Finite set	Nul set	✓ Event	All of these
21)	An event that contains more than one sample point is called:	Simple event	✓ Compound event	Impossible event	None of these
22)	When a die is rolled, all possible out comes are:	2	4	✓ 6	8
23)	When 2 dice are rolled, all possible out comes are:	4	10	22	✓ 36

S#	Questions	Answers			
24)	When n dice are rolled, the possible sample points in the sample space are:	6^n	n^6	✓ 6^n	None of these
25)	When two coins are tossed, the possible out comes are:	6	✓ 4	36	12
26)	If three coins are tossed, the possible outcomes are:	✓ 8	3	1	10
27)	If n coins are tossed, the possible outcomes are:	n	2n	✓ 2^n	n^2
28)	If two events A and B can not occur together, then they are:	✓ Mutually exclusive	Not mutually exclusive	Independent	All of these
29)	If two events A and B have same chance to occur, then they are:	Mutually exclusive	✓ Equally likely	Exhaustive	None of these
30)	If A and B are mutually exclusive events, P(A)=0.25, P(B) 0.50, then P(AUB) =	✓ 0.75	0.25	0.50	1
31)	The probability of an event always lies between:	–1 and +1	–1 and 0	✓ 0 and 1	None of these
32)	P(AUB) = P(A)+P(B) if A and B are:	✓ Mutually exclusive events	Independent events	Non mutually exclusive events	None of these
33)	The probability of appearing a tail when a fair coin is tossed is:	0	✓ $\frac{1}{2}$	$\frac{1}{4}$	1

S#	Questions	Answers			
34)	The probability of drawing a queen from well-shuffled pack of 52 playing cards is:	✓ $\frac{4}{52}$	$\frac{2}{52}$	$\frac{13}{52}$	$\frac{26}{52}$
35)	If two cards are drawn at random from a standard deck of cards, the number of possible sample points is:	52	52!	✓ 1326	2000
36)	The probability of drawing a jack from a deck of cards is:	✓ $\frac{1}{13}$	$\frac{1}{4}$	$\frac{1}{52}$	$\frac{5}{52}$
37)	The probability of selecting a jack of club from a deck of cards is:	$\frac{13}{52}$	$\frac{4}{52}$	✓ $\frac{1}{52}$	$\frac{2}{52}$
38)	The probability of getting exactly one head when two coins are tossed is:	$\frac{1}{4}$	✓ $\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
39)	Five cards are selected at random from a pack of 52 cards. The possible outcomes are:	52	✓ $^{52}C_5$	52^5	5^{52}
40)	A fair coin is tossed 200 times, the expected number of heads is:	✓ 100	7	50	20
41)	When two dice are rolled, the maximum total on the two faces of the dice will be:	6	36	✓ 12	10
42)	If A and B are independent events then:	$P(A \cup B) = P(A) + P(B)$	$P(A \cap B) = P(A) + P(B)$	✓ $P(A \cap B) = P(A).(P(B)$	None of these

S#	Questions	Answers			
43)	If $P(A/B) = P(A)$ and $P(B/A) = P(B)$, then the events A and B are said to be:	✓ Independent	Dependent	Mutually exclusive	Simple
44)	If $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$, then events A and B are said to be:	Equally likely	Mutually exclusive	✓ Independent	Exhaustive
45)	Probability of an event can never be:	One	✓ Negative	Positive	Zero
46)	The conditional probability P(A/B) is:	$\frac{P(A \cap B)}{(B)}$	$\frac{P(A \cap B)}{(A)}$	✓ $\frac{P(A \cap B)}{P(B)}$	$\frac{P(A \cap B)}{P(A)}$

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: What is difference between permutation and combination?

Ans: A permutation of a number of objects is an arrangement of these objects in some order. A selection of distinct objects without some order is called a combination. symbolically

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$ is a formula for permutation and

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$ is a formula for combination

Q. 2: What do you mean by n!?

Ans: The product of first "n" natural numbers is called n factorial denoted by $n! = n(n-1)(n-2)\ldots\ldots 1$

Q. 3: Define the term experiment.

Ans: An act or the process of obtaining an observation is called experiment.

Q. 4: Define the term trail.

Ans: Performing of an experiment is called a trail.

Q. 5: Define the term random experiment.

(Lahore Board, 2009 & 2010
(Rawalpindi Boar, 2010)
(Sargodha Board, 2010)

Ans: An experiment is said to be random, if it provides different results, if it is repeated a large number of times under identical conditions.

Q. 6: What is a sample space?

Ans: A set of all possible outcomes of a random experiment is called a sample space. It is denoted by S.

Q. 7: Give two examples of a random experiment.

Ans: The tossing of a fair coin and throwing of a balanced die are examples of random experiment.

Q. 8: What is an event?

(Gujranwala Board, 2010)

Ans: Any subset of the sample space is called an event. It is denoted by A, B, C, D etc.

Q. 9: Make a sample space, when a cubical die is rolled.

(Gujranwala Board, 2010)
(Sargodha Board, 2010)

Ans: $S = \{1, 2, 3, 4, 5, 6\}$

Q. 10: Define sample points.

Ans: The elements of the sample space are called sample points.

Q. 11: What are the properties of a random experiment?

Ans: A random experiment has three properties.

(i) The experiment can be repeated
(ii) The experiment always has at least two outcomes
(iii) The outcome of each trail is unpredictable

Q. 12: Define simple event.

(Lahore Board, 2010)
(Rawalpindi Board, 2010)

Ans: A simple event is one which contains only one sample point of the sample space.

Q. 13: Define compound event.

Ans: Compound event contains two or more sample points of the sample space.

Q. 14: What is impossible event?

Ans: An event consisting of the empty set is called impossible event.

Q. 15: Define sure event or certain event.

Ans: An event that consists of the sample space itself is called sure or certain event.

Q. 16: Define mutually exclusive events with example.

Ans: Two events A and B are said to be mutually exclusive if they can not occur together. Thus occurrence of head or tail in a single throw of a coin are mutually exclusive events.

Q. 17: Define not mutually exclusive events with example.

Ans: If two events can occur together, they are not mutually exclusive. If we draw a card from an ordinary deck of 52 cards, it can be both a queen and a card of spade.

Q. 18: Define equally likely events with example.

Ans: Two or more events are said to be equally likely events if they have same chance to occur. If we toss a coin, the head and tail have the same chance to be occur.

Q. 19: Define independent events with example.

(Lahore Board, 2008 & 2009)
(Sargodha Board, 2010)

Ans: Two events A and B are said to be independents if the events in no way affect each other. For example, suppose event A is the drawing of king from a pack of 52 cards and event B is head on tossing a fair coin. The events are independent because drawing of a card does not affect the head when tossing a coin.

Q. 20: Define dependent events.

Ans: Two events are said to be dependent when the happening of any one event is affected by the happening or non happening of the other.

Q. 21: Define the exhaustive events.

(Lahore Board, 2009)

Ans: Two or more events are said to be collectively exhaustive if their union generates the whole sample space.

Q. 22: Define favourable cases.

Ans: The cases which entail the occurrence of an event are said to be favourable cases to the event.

Q. 23: Are independent events also mutually exclusive?

Ans: No, mutually exclusive events are always independent.

Q. 24: What are two approaches to define probability?

Ans: Two approaches to define probability are:

(a) Subjective approach
(b) Objective approach

Q. 25: Give the classical priori definition of probability.

(Gujranwala Board, 2010)

Ans: If there are "n" equally likely, mutually exclusive and exhaustive sample points in a sample space and "m" out of them are favourable to an event A, then

$$P(A) = \frac{m}{n}$$

Q. 26: Give the relative frequency or posteriori definition of probability.

Ans: If a random experiment is repeated a large number of times say n under uniform conditions and the event A is observed m times, then probability of event A, is

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

Q. 27: Give the axiomatic (mathematical approach) of probability.

Ans: The probability that an event A will occur is the number P(A). The number P(A) satisfy the following axioms.

(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Q. 28: When does probability become negative?

Ans: Probability can never be negative.

Q. 29: State the addition law of probability for two mutually exclusive events.

Ans: If A and B are two mutually exclusive events then the probability that anyone of them occur is given by sum of their respective probabilities, that is $P(A \cup B) = P(A) + P(B)$

Q. 30: State the addition law of probability for two not mutually exclusive events.

Ans: If A and B are not mutually exclusive events then the probability that at least one of them occur is given by the sum of their respective probabilities minus the probability of their joint occurrence that is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Q. 31: State the multiplication law of probability for two independent events.

Ans: If A and B are two independent events, then the probability that both occur is given as:

$$P(A \cap B) = P(A).P(B)$$

Q. 32: State multiplication law of probability for two dependent events.

Ans: If A and B are two dependents events, then

$$P(A \cap B) = P(A).P(B/A) \text{ provided } P(A) \neq 0 \text{ OR}$$

$$P(A \cap B) = P(B).P(A/B) \text{ provided } P(B) \neq 0$$

Q. 33: What is conditional probability?

Ans: If A and B are two events, and $P(B) \neq 0$, then conditional probability for event A given that the event B has occurred, denoted by P(A/B), is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q. 34: Define set.

(Lahore Board, 2010)

Ans: Well defined collection of distinct objects is called set. It is denoted by capital letters A, B, C, ...

Q. 35: What is empty set?

(Lahore Board, 2010)

Ans: A set consisting no element is called a null or empty set. It is denoted by ϕ or $\{\}$