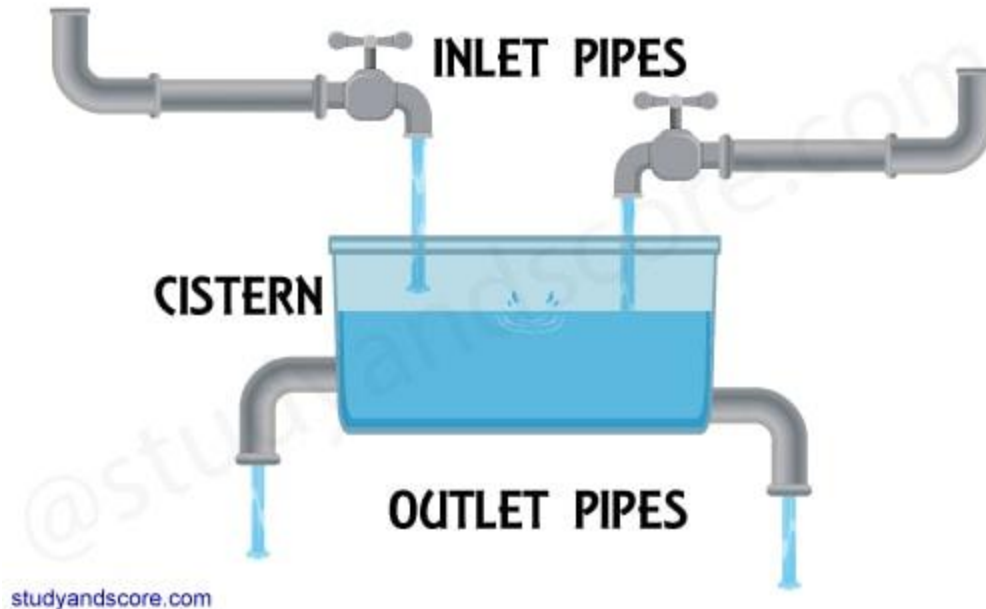


AMAZON APTITUDE TEST

1. Pipes and Cisterns



IMPORTANT FORMULAS

1. **Inlet:**

A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet:

A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

2. If a pipe can fill a tank in x hours, then:

$$\text{part filled in 1 hour} = \frac{1}{x}.$$

3. If a pipe can empty a tank in y hours, then:

$$\text{part emptied in 1 hour} = \frac{1}{y}.$$

4. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, then

$$\text{the net part filled in 1 hour} = \left(\frac{1}{x} - \frac{1}{y} \right).$$

5. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, then

$$\text{the net part emptied in 1 hour} = \left(\frac{1}{y} - \frac{1}{x} \right).$$

Q Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the B is turned off after:

- A. 5 min.
- B. 9 min.
- C. 10 min.
- D. 15 min.

Answer: Option B

Explanation:

Let B be turned off after x minutes. Then,

Part filled by (A + B) in x min. + Part filled by A in $(30 - x)$ min. = 1.

$$\therefore x \left(\frac{2}{75} + \frac{1}{45} \right) + (30 - x) \cdot \frac{2}{75} = 1$$

$$\Rightarrow \frac{11x}{225} + \frac{(60 - 2x)}{75} = 1$$

$$\Rightarrow 11x + 180 - 6x = 225.$$

$$\Rightarrow x = 9.$$

2. Volumes

IMPORTANT FORMULAS

1. CUBOID

Let length = l , breadth = b and height = h units. Then

- Volume = $(l \times b \times h)$ cubic units.
- Surface area = $2(lb + bh + lh)$ sq. units.
- Diagonal = $\sqrt{l^2 + b^2 + h^2}$ units.

2. CUBE

Let each edge of a cube be of length a . Then,

- Volume = a^3 cubic units.
- Surface area = $6a^2$ sq. units.
- Diagonal = $3a$ units.

3. CYLINDER

Let radius of base = r and Height (or length) = h . Then,

- Volume = $(\pi r^2 h)$ cubic units.
- Curved surface area = $(2\pi rh)$ sq. units.
- Total surface area = $2\pi r(h + r)$ sq. units.

4. CONE

Let radius of base = r and Height = h . Then,

- Slant height, $l = \sqrt{h^2 + r^2}$ units.
- Volume = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
- Curved surface area = (πrl) sq. units.
- Total surface area = $(\pi rl + \pi r^2)$ sq. units.

5. SPHERE

Let the radius of the sphere be r . Then,

- Volume = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
- Surface area = $(4\pi r^2)$ sq. units.

6. HEMISPHERE

Let the radius of a hemisphere be r . Then,

- Volume = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
- Curved surface area = $(2\pi r^2)$ sq. units.
- Total surface area = $(3\pi r^2)$ sq. units.

Note: 1 litre = 1000 cm³.

Q A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of the areas of four walls, the volume of the hall is:

- 720
- 900
- 1200
- 1800

Answer: Option C

Explanation:

$$2(15 + 12) \times h = 2(15 \times 12)$$

$$\Rightarrow h = \frac{180}{27} \text{m} = \frac{20}{3} \text{m}.$$

$$\therefore \text{Volume} = \left(15 \times 12 \times \frac{20}{3} \right) \text{m}^3 = 1200 \text{m}^3.$$

3. Profit and Loss

IMPORTANT FORMULAS

Cost Price:

The price, at which an article is purchased, is called its **cost price**, abbreviated as **C.P.**

Selling Price:

The price, at which an article is sold, is called its **selling prices**, abbreviated as **S.P.**

Profit or Gain:

If S.P. is greater than C.P., the seller is said to have a **profit or gain**.

Loss:

If S.P. is less than C.P., the seller is said to have incurred a **loss**.

IMPORTANT FORMULAE

1. $\text{Gain} = (\text{S.P.}) - (\text{C.P.})$

2. $\text{Loss} = (\text{C.P.}) - (\text{S.P.})$

3. Loss or gain is always reckoned on C.P.

4. Gain Percentage: (Gain %)

$$\text{Gain \%} = \left(\frac{\text{Gain} \times 100}{\text{C.P.}} \right)$$

5. Loss Percentage: (Loss %)

$$\text{Loss \%} = \left(\frac{\text{Loss} \times 100}{\text{C.P.}} \right)$$

6. Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 + \text{Gain \%})}{100} \times \text{C.P.} \right]$$

7. Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 - \text{Loss \%})}{100} \times \text{C.P.} \right]$$

8. Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{(100 + \text{Gain \%})} \times \text{S.P.} \right]$$

9. Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{\quad} \times \text{S.P.} \right]$$

(100 - Loss %)

10. If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.
11. If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.
12. When a person sells two similar items, one at a gain of say $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by:

$$\text{Loss \%} = \left(\frac{\text{Common Loss and Gain \%}}{10} \right)^2 = \left(\frac{x}{10} \right)^2.$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100 \right] \%$$

Q The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is:

- A. 15
- B. 16
- C. 18
- D. 25

Answer: Option B

Explanation:

Let C.P. of each article be Re. 1 C.P. of x articles = Rs. x .

S.P. of x articles = Rs. 20.

Profit = Rs. (20 - x).

$$\therefore \left(\frac{20 - x}{x} \times 100 = 25 \right)$$

$$\Rightarrow 2000 - 100x = 25x$$

$$125x = 2000$$

$$\Rightarrow x = 16.$$

4. Simple & Compound Interest

IMPORTANT FORMULAS

1. Let Principal = P, Rate = R% per annum, Time = n years.

2. When interest is compounded Annually:

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

3. When interest is compounded Half-yearly:

$$\text{Amount} = P \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

4. When interest is compounded Quarterly:

$$\text{Amount} = P \left[1 + \frac{(R/4)}{100} \right]^{4n}$$

5. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

6. When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right).$$

7. Present worth of Rs. x due n years hence is given by:

Present Worth =

$$\frac{x}{\left(1 + \frac{R}{100} \right)^n}$$

Simple Interest

Principal:

The money borrowed or lent out for a certain period is called the **principal** or the **sum**.

Interest:

Extra money paid for using other's money is called **interest**.

Simple Interest (S.I.):

If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called **simple interest**.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then

$$(i). \text{ Simple Interest} = \left(\frac{P \times R \times T}{100} \right)$$

$$(ii). P = \left(\frac{100 \times \text{S.I.}}{R \times T} \right); R = \left(\frac{100 \times \text{S.I.}}{P \times T} \right) \text{ and } T = \left(\frac{100 \times \text{S.I.}}{P \times R} \right).$$

Q There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of Rs. 12,000 after 3 years at the same rate?

- A. Rs. 2160
- B. Rs. 3120
- C. Rs. 3972
- D. Rs. 6240
- E. None of these

Answer: Option C

Explanation:

Let P = Rs. 100. Then, S.I. Rs. 60 and T = 6 years.

$$\therefore R = \left(\frac{100 \times 60}{100 \times 6} \right) = 10\% \text{ p.a.}$$

Now, P = Rs. 12000. T = 3 years and R = 10% p.a.

$$\begin{aligned} \therefore \text{C.I.} &= \text{Rs.} \left[12000 \times \left\{ \left(1 + \frac{10}{100} \right)^3 - 1 \right\} \right] \\ &= \text{Rs.} \left(12000 \times \frac{331}{1000} \right) \\ &= 3972. \end{aligned}$$

5. Indices and Surds

IMPORTANT FORMULAS

1. Laws of Indices:

i. $a^m \times a^n = a^{m+n}$

ii.

$$\frac{a^m}{a^n} = a^{m-n}$$

iii. $(a^m)^n = a^{mn}$

iv. $(ab)^n = a^n b^n$

v.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

vi. $a^0 = 1$

2. Surds:

Let a be rational number and n be a positive integer such that $a^{(1/n)} = a$
Then, a is called a surd of order n .

3. Laws of Surds:

i. $a = a^{(1/n)}$

ii. $ab = a \times b$

iii.

$$\sqrt[n]{\frac{a}{b}} = \frac{a}{b}$$

iv. $(a)^n = a$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

vi. $(a)^m = a^m$

Q If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is:

A. $\frac{1}{2}$

B. 1

C. 2

D. $\frac{7}{2}$

Answer: Option C

Explanation:

Given $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$

$$\Rightarrow \left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-(x-3)} = \left(\frac{a}{b}\right)^{(3-x)}$$

$$\begin{aligned} & \quad \quad \quad b \quad \quad \quad b \quad \quad \quad b \\ \Rightarrow x - 1 &= 3 - x \\ \Rightarrow 2x &= 4 \\ \Rightarrow x &= 2. \end{aligned}$$

6. Ratio and Proportions

IMPORTANT FORMULAS

1. Ratio:

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$. In the ratio $a : b$, we call a as the first term or **antecedent** and b , the second term or **consequent**.

Eg. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. $4 : 5 = 8 : 10 = 12 : 15$. Also, $4 : 6 = 2 : 3$.

2. Proportion:

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. Fourth Proportional:

If $a : b = c : d$, then d is called the fourth proportional to a, b, c .

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b .

Mean Proportional:

Mean proportional between a and b is ab .

4. Comparison of Ratios:

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

5. Compounded Ratio:

The compounded ratio of the ratios: $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

7. Duplicate Ratios:

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(a : b)$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

8. Variations:

We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y , if $xy = k$ for some constant k and

we write, $x \propto \frac{1}{y}$.

$$\frac{1}{y}$$

Q Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

- A. 2 : 5
- B. 3 : 5
- C. 4 : 5
- D. 6 : 7

Answer: Option C

Explanation:

Let the third number be x .

$$\text{Then, first number} = 120\% \text{ of } x = \frac{120x}{100} = \frac{6x}{5}$$

$$\text{Second number} = 150\% \text{ of } x = \frac{150x}{100} = \frac{3x}{2}$$

$$\therefore \text{Ratio of first two numbers} = \left(\frac{6x}{5} : \frac{3x}{2} \right) = 12x : 15x = 4 : 5.$$

7. Time and Distance

IMPORTANT FORMULAS

1. **Speed, Time and Distance:**

$$\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}} \right), \text{ Time} = \left(\frac{\text{Distance}}{\text{Speed}} \right), \text{ Distance} = (\text{Speed} \times \text{Time}).$$

2. **km/hr to m/sec conversion:**

$$x \text{ km/hr} = \left(x \times \frac{5}{18} \right) \text{ m/sec.}$$

3. **m/sec to km/hr conversion:**

$$x \text{ m/sec} = \left(x \times \frac{18}{5} \right) \text{ km/hr.}$$

4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

5. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right)$ km/hr.

Q A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is:

- A. 100 kmph
- B. 110 kmph
- C. 120 kmph
- D. 130 kmph

Answer: Option C

Explanation:

Let speed of the car be x kmph.

Then, speed of the train = $\frac{150}{100}x = \left(\frac{3}{2}x\right)$ kmph.

$$\therefore \frac{75}{x} - \frac{75}{(3/2)x} = \frac{125}{10 \times 60}$$

$$\Rightarrow \frac{75}{x} - \frac{50}{x} = \frac{5}{24}$$

$$\Rightarrow x = \left(\frac{25 \times 24}{5}\right) = 120 \text{ kmph.}$$

8. Problems on Trains

IMPORTANT FORMULAS

1. **km/hr to m/s conversion:**

$$a \text{ km/hr} = \left(a \times \frac{5}{18}\right) \text{ m/s.}$$

2. **m/s to km/hr conversion:**

$$a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{ km/hr.}$$

3. **Formulas for finding Speed, Time and Distance**

4. Time taken by a train of length l metres to pass a pole or standing man or a signal post is equal to the time taken by the train to cover l metres.

5. Time taken by a train of length l metres to pass a stationery object of length b metres is the time taken by the train to cover $(l + b)$ metres.

6. Suppose two trains or two objects bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relative speed is = $(u - v)$ m/s.

7. Suppose two trains or two objects bodies are moving in opposite directions at u m/s and v m/s, then their relative speed is = $(u + v)$ m/s.

8. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then:

The time taken by the trains to cross each other = $(a + b)$ sec.

- $(u + v)$
9. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then:

The time taken by the faster train to cross the slower train = $\frac{(a + b)}{(u - v)}$ sec.

10. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then:
(A's speed) : (B's speed) = $(b : a)$

Q A train 125 m long passes a man, running at 5 km/hr in the same direction in which the train is going, in 10 seconds. The speed of the train is:

- A. 45 km/hr
- B. 50 km/hr
- C. 54 km/hr
- D. 55 km/hr

Answer: Option B

Explanation:

Speed of the train relative to man = $\left(\frac{125}{10}\right)$ m/sec

$$= \left(\frac{25}{2}\right) \text{ m/sec.}$$

$$= \left(\frac{25}{2} \times \frac{18}{5}\right) \text{ km/hr}$$

$$= 45 \text{ km/hr.}$$

Let the speed of the train be x km/hr. Then, relative speed = $(x - 5)$ km/hr.

$$\therefore x - 5 = 45 \Rightarrow x = 50 \text{ km/hr.}$$

9. Areas

IMPORTANT FORMULAS

1. 1. Area of a rectangle = (Length x Breadth).

$$\therefore \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}}\right) \text{ and Breadth} = \left(\frac{\text{Area}}{\text{Length}}\right).$$

2. 2. Perimeter of a rectangle = $2(\text{Length} + \text{Breadth})$.

3. Area of a square = $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

4. Area of 4 walls of a room = $2(\text{Length} + \text{Breadth}) \times \text{Height}$.

5. 1. Area of a triangle = $\frac{1}{2}$ x Base x Height.
 2. Area of a triangle = $s(s-a)(s-b)(s-c)$
 where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a + b + c)$.
 3. Area of an equilateral triangle = $\frac{3}{4}$ x (side)².
 4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.
 5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.
 6. Radius of incircle of a triangle of area Δ and semi-perimeter $r = \frac{\Delta}{s}$.
6. 1. Area of parallelogram = (Base x Height).
 2. Area of a rhombus = $\frac{1}{2}$ x (Product of diagonals).
 3. Area of a trapezium = $\frac{1}{2}$ x (sum of parallel sides) x distance between them.
7. 1. Area of a circle = πR^2 , where R is the radius.
 2. Circumference of a circle = $2\pi R$.
 3. Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle.
 4. Area of a sector = $\frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2\theta}{360}$.
8. 1. Circumference of a semi-circle = πR .
 2. Area of semi-circle = $\frac{\pi R^2}{2}$.

10. Averages

IMPORTANT FORMULAS

1. **Average:**

$$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observations}} \right)$$

2. **Average Speed:**

Suppose a man covers a certain distance at x kmph and an equal distance at y kmph.

Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right)$ kmph.

Q The average weight of 8 person's increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person?

- A. 76 kg
- B. 76.5 kg
- C. 85 kg
- D. Data inadequate
- E. None of these

Answer: Option C

Explanation:

Total weight increased = (8×2.5) kg = 20 kg.

Weight of new person = $(65 + 20)$ kg = 85 kg.

11. Time and Work

IMPORTANT FORMULAS

1. **Work from Days:**

If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

2. **Days from Work:**

If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

3. **Ratio:**

If A is thrice as good a workman as B, then:

Ratio of work done by A and B = 3 : 1.

Ratio of times taken by A and B to finish a work = 1 : 3.

Q A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day?

- A. 12 days
- B. 15 days
- C. 16 days
- D. 18 days

Answer: Option B

Explanation:

A's 2 day's work = $\left(\frac{1}{20} \times 2\right) = \frac{1}{10}$.

$$(A + B + C)\text{'s 1 day's work} = \left(\frac{1}{20} + \frac{1}{30} + \frac{1}{60}\right) = \frac{6}{60} = \frac{1}{10}$$

$$\text{Work done in 3 days} = \left(\frac{1}{10} + \frac{1}{10}\right) = \frac{1}{5}$$

Now, $\frac{1}{5}$ work is done in 3 days.

\therefore Whole work will be done in $(3 \times 5) = 15$ days.

12. Boats and Streams

IMPORTANT FORMULAS

1. Downstream/Upstream:

In water, the direction along the stream is called **downstream**. And, the direction against the stream is called **upstream**.

2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then:

Speed downstream = $(u + v)$ km/hr.

Speed upstream = $(u - v)$ km/hr.

3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then:

Speed in still water = $\frac{1}{2}(a + b)$ km/hr.

$$\text{Rate of stream} = \frac{1}{2}(a - b) \text{ km/hr.}$$

A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

A. 2 : 1

B. 3 : 2

C. 8 : 3

D. Cannot be determined

E. None of these

Answer: Option C

Explanation:

Let the man's rate upstream be x kmph and that downstream be y kmph.

Then, distance covered upstream in 8 hrs 48 min = Distance covered downstream in 4 hrs.

$$\Rightarrow \left(x \times 8\frac{4}{5}\right) = (y \times 4)$$

$$\Rightarrow \frac{44}{5}x = 4y$$

$$\Rightarrow y = \frac{11}{5}x$$

$$\begin{aligned} \therefore \text{Required ratio} &= \left(\frac{y+x}{2}\right) : \left(\frac{y-x}{2}\right) \\ &= \left(\frac{16x}{2} \times \frac{1}{2}\right) : \left(\frac{6x}{2} \times \frac{1}{2}\right) \end{aligned}$$

$$= \frac{8^5}{5^2} \div \frac{3^5}{5^2}$$

$$= 8 : 3.$$

13. Problems on LCM and HCF

IMPORTANT FORMULAS

1. Factors and Multiples:

If number a divided another number b exactly, we say that a is a **factor** of b .
In this case, b is called a **multiple** of a .

2. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers:

- I. **Factorization Method:** Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
- II. **Division Method:** Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.

Similarly, the H.C.F. of more than three numbers may be obtained.

3. Least Common Multiple (L.C.M.):

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

There are two methods of finding the L.C.M. of a given set of numbers:

- I. **Factorization Method:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.
- II. **Division Method (short-cut):** Arrange the given numbers in a row in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

4. Product of two numbers = Product of their H.C.F. and L.C.M.

5. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.

6. H.C.F. and L.C.M. of Fractions:

$$1. \text{ H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$2. \text{ L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

8. H.C.F. and L.C.M. of Decimal Fractions:

In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

9. Comparison of Fractions:

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Q The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is:

- A. 276
- B. 299
- C. 322
- D. 345

Answer: Option C

Explanation:

Clearly, the numbers are (23×13) and (23×14) .

∴ Larger number = $(23 \times 14) = 322$.

14. Permutations and Combinations

IMPORTANT FORMULAS

1. Factorial Notation:

Let n be a positive integer. Then, factorial n , denoted $n!$ is defined as:

$$n! = n(n - 1)(n - 2) \dots \text{3.2.1.}$$

Examples:

- i. We define $0! = 1$.
- ii. $4! = (4 \times 3 \times 2 \times 1) = 24$.
- iii. $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

2. Permutations:

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

- i. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb) .

ii. All permutations made with the letters a, b, c taking all at a time are:
 ($abc, acb, bac, bca, cab, cba$)

3. **Number of Permutations:**

Number of all permutations of n things, taken r at a time, is given by:

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

Examples:

i. ${}^6 P_2 = (6 \times 5) = 30$.

ii. ${}^7 P_3 = (7 \times 6 \times 5) = 210$.

iii. **Cor. number of all permutations of n things, taken all at a time = $n!$.**

4. **An Important Result:**

If there are n subjects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r^{th} kind, such that $(p_1 + p_2 + \dots + p_r) = n$.

Then, number of permutations of these n objects is = $\frac{n!}{(p_1!)(p_2!) \dots (p_r!)}$

5. **Combinations:**

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a **combination**.

Examples:

1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note: AB and BA represent the same selection.

2. All the combinations formed by a, b, c taking ab, bc, ca .

3. The only combination that can be formed of three letters a, b, c taken all at a time is abc .

4. Various groups of 2 out of four persons A, B, C, D are:

AB, AC, AD, BC, BD, CD.

5. Note that ab ba are two different permutations but they represent the same combination.

6. **Number of Combinations:**

The number of all combinations of n things, taken r at a time is:

$${}^n C_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2) \dots \text{to } r \text{ factors}}{r!}$$

Note:

${}^n C_n = 1$ and ${}^n C_0 = 1$.

i. ${}^n C_r = {}^n C_{(n-r)}$

Examples:

i. ${}^{11} C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$.

ii. ${}^{16} C_{13} = {}^{16} C_{(16-13)} = {}^{16} C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$.

Q From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

A. 564

- B. 645
- C. 735
- D. 756
- E. None of these

Answer: Option D

Explanation:

We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only).

$$\begin{aligned}
 \therefore \text{Required number of ways} &= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5) \\
 &= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{6 \times 5}{2 \times 1} \right) + ({}^7C_3 \times {}^6C_1) + ({}^7C_2) \\
 &= 525 + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6 \right) + \left(\frac{7 \times 6}{2 \times 1} \right) \\
 &= (525 + 210 + 21) \\
 &= 756.
 \end{aligned}$$

15. Probability

IMPORTANT FORMULAS

1. Experiment:

An operation which can produce some well-defined outcomes is called an experiment.

2. Random Experiment:

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

- i. Rolling an unbiased dice.
 - ii. Tossing a fair coin.
 - iii. Drawing a card from a pack of well-shuffled cards.
 - iv. Picking up a ball of certain colour from a bag containing balls of different colours.
- Details:**
- v. When we throw a coin, then either a Head (H) or a Tail (T) appears.
 - vi. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

- vii. A pack of cards has 52 cards.
It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.
Cards of spades and clubs are **black cards**.
Cards of hearts and diamonds are **red cards**.

There are 4 honours of each unit.

There are **Kings, Queens and Jacks**. These are all called **face cards**.

3. **Sample Space:**

When we perform an experiment, then the set S of all possible outcomes is called the **sample space**.

Examples:

1. In tossing a coin, $S = \{H, T\}$
2. If two coins are tossed, the $S = \{HH, HT, TH, TT\}$.
3. In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.
4. **Event:**
Any subset of a sample space is called an **event**.
5. **Probability of Occurrence of an Event:**

Let S be the sample and let E be an event.

Then, $E \subseteq S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

6. **Results on Probability:**

- . $P(S) = 1$
- i. $0 \leq P(E) \leq 1$
- ii. $P(\Phi) = 0$
- iii. For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- iv. If A denotes (not-A), then $P(A) = 1 - P(\bar{A})$.

Q In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

A. $\frac{1}{3}$

B. $\frac{3}{4}$

C. $\frac{7}{19}$

D. $\frac{8}{19}$

21

E. $\frac{9}{21}$

Answer: Option A

Explanation:

Total number of balls = $(8 + 7 + 6) = 21$.

Let E = event that the ball drawn is neither red nor green

= event that the ball drawn is blue.

$$\therefore n(E) = 7.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{21} = \frac{1}{3}.$$

16. Mixture and Alligation

IMPORTANT FORMULAS

1. Alligation:

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.

2. Mean Price:

The cost of a unit quantity of the mixture is called the mean price.

3. Rule of Alligation:

If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} \right) = \left(\frac{\text{C.P. of dearer} - \text{Mean Price}}{\text{Mean price} - \text{C.P. of cheaper}} \right)$$

We present as under:

C.P. of a unit quantity
of cheaper C.P. of a unit quantity
of dearer

$$\begin{array}{ccc} (c) & \text{Mean Price} & (d) \\ (d - m) & (m) & (m - c) \end{array}$$

$$\therefore (\text{Cheaper quantity}) : (\text{Dearer quantity}) = (d - m) : (m - c).$$

4. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid = $\left[x \left(1 - \frac{y}{x} \right)^n \right]$ units.

Q A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?

- A. 4 litres, 8 litres
- B. 6 litres, 6 litres
- C. 5 litres, 7 litres
- D. 7 litres, 5 litres

Answer: Option B

Let the cost of 1 litre milk be Re. 1

Milk in 1 litre mix. in 1st can = $\frac{3}{4}$ litre, C.P. of 1 litre mix. in 1st can Re. $\frac{3}{4}$

Milk in 1 litre mix. in 2nd can = $\frac{1}{2}$ litre, C.P. of 1 litre mix. in 2nd can Re. $\frac{1}{2}$

Milk in 1 litre of final mix. = $\frac{5}{8}$ litre, Mean price = Re. $\frac{5}{8}$

By the rule of alligation, we have:

C.P. of 1 litre mixture in 1st can C.P. of 1 litre mixture in 2nd can

$\frac{3}{4}$	Mean Price	$\frac{1}{2}$
	$\frac{5}{8}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

∴ Ratio of two mixtures = $\frac{1}{8} : \frac{1}{8} = 1 : 1$.

So, quantity of mixture taken from each can = $\left(\frac{1}{2} \times 12\right) = 6$ litres.

17. Percentages

IMPORTANT FORMULAS

1. Concept of Percentage:

By a certain **percent**, we mean that many hundredths.

Thus, x percent means x hundredths, written as x%.

To express x% as a fraction: We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$.

To express \underline{a} as a percent: We have, $\underline{a} = (\underline{a} \times 100)\%$

$$\text{Thus, } \frac{1}{4} = \left(\frac{1}{4} \times 100 \right) \% = 25\%.$$

2. Percentage Increase/Decrease:

If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100 \right] \%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100 \right] \%$$

3. Results on Population:

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

$$1. \text{ Population after } n \text{ years} = P \left(1 + \frac{R}{100} \right)^n$$

$$2. \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{R}{100} \right)^n}$$

4. Results on Depreciation:

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

$$1. \text{ Value of the machine after } n \text{ years} = P \left(1 - \frac{R}{100} \right)^n$$

$$2. \text{ Value of the machine } n \text{ years ago} = \frac{P}{\left(1 - \frac{R}{100} \right)^n}$$

$$3. \text{ If A is R\% more than B, then B is less than A by } \left[\frac{R}{(100 + R)} \times 100 \right] \%$$

$$4. \text{ If A is R\% less than B, then B is more than A by } \left[\frac{R}{(100 - R)} \times 100 \right] \%$$

Q A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had:

- A. 588 apples
- B. 600 apples
- C. 672 apples
- D. 700 apples

Answer: Option D

Explanation:

Suppose originally he had x apples.
Then, $(100 - 40)\%$ of $x = 420$.

$$\Rightarrow \frac{60}{100} \times x = 420$$

$$\Rightarrow x = \left(\frac{420 \times 100}{60} \right) = 700.$$

18. Equations

IMPORTANT FORMULAS

1. Some Basic Formulae:

- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Q The sum of first 45 natural numbers is:

- 1035
- 1280
- 2070
- 2140

Answer: Option A

Explanation:

Let $S_n = (1 + 2 + 3 + \dots + 45)$. This is an A.P. in which $a = 1$, $d = 1$, $n = 45$.

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{45}{2} \times [2 \times 1 + (45 - 1) \times 1] = \left(\frac{45}{2} \times 46 \right) = (45 \times 23)$$

$$= 45 \times (20 + 3)$$

$$= 45 \times 20 + 45 \times 3$$

$$= 900 + 135$$

$$= 1035.$$

Shortcut Method:

$$S_n = \frac{n(n + 1)}{2} = \frac{45(45 + 1)}{2} = 1035.$$