Integrals – Applications

Ex.1. Find the area of teh region bounded by the graph of a function f and X-axis.

(1) $f(x) = \frac{1}{\sin x}$, for $x \in \langle \frac{\pi}{3}, \frac{\pi}{2} \rangle$

(2) $f(x) = x \arctan x$, for $x \in \langle 0, 1 \rangle$

(3) $f(x) = x^3 e^x$, $x \in \langle 0, 3 \rangle$

(4) $f(x) = \frac{36 \ln x}{x^7}$, $x \in \langle 1, 2 \rangle$

(5) $f(x) = \frac{\ln x}{\sqrt{x}}$, $x \in \langle 1, 4 \rangle$

(6) $f(x) = \frac{\sqrt{x}}{\sqrt{x}+1}$, $x \in \langle 1, 4 \rangle$

(7) $f(x) = 2x\sqrt{x+3}$

(8) $f(x) = \frac{(x-3)\sqrt{x-1}}{x}$

(9) $f(x) = x^2 \sin^2 x$, $x \in \langle 0, \pi \rangle$

(10) $f(x) = x^4 + 2x^3 + x^2 + 2x$

(11) $f(x) = x^3(x^4 - 2)^3$

(12) $f(x) = \frac{3}{x^2+4}$, $x \in \langle 0, 2 \rangle$

(13) $f(x) = \frac{1}{x^2(x+2)}$, $x \in \langle -\frac{3}{2}, -1 \rangle$

Ex.2. Find the area of the region bounded by the graphs of f and g.

(1) $f(x) = x^2$, $g(x) = x$

(2) $f(x) = x^2$, $g(x) = 3 - x^2$

(3) $f(x) = x^2 - 3x$, $g(x) = 3 - x$

(4) $f(x) = x^4 - x^2$, $g(x) = x^2 - 1$

(5) $f(x) = x^2$, $g(x) = \frac{4}{x^2}$ for $x \geq 1$

(6) $f(x) = x^3$, $g(x) = \frac{16}{x}$ for $x \geq 1$

(7) $f(x) = |x| - 2$, $g(x) = 2 - \frac{1}{2}x^2$

(8) $f(x) = e^{2x}$, $g(x) = e^{-x}$ for $x \in \langle 0, 1 \rangle$

(9) $f(x) = e^x$, $g(x) = \frac{-3}{e^x - 4}$

(10) $f(x) = \sqrt{x+1} + 1$, $g(x) = x$

(11) $f(x) = \ln^2 x$, $g(x) = 3 \ln x - 2$

(12) $f(x) = \frac{4}{x^2+1}$, $g(x) = \frac{x^2}{5}$

(13) $f(x) = -x^2 + 2x$, $g(x) = x^3 - 6x^2 + 12x - 8$ and the line $x = 0$

(14) the line $y = \frac{-5x-7}{2}$, $f(x) = \frac{1}{2}(x+1)^2 + 1$ and the tangent to f in $x_0 = 1$

Ex.3. Find the length of the curve.

(1) $y = x\sqrt{x}$, $x \in \langle 0, 4 \rangle$

(2) $y = \sqrt{x+1}$, $x \in \langle 0, 2 \rangle$

(3) $y = \ln(\sin x)$, $x \in \langle \frac{\pi}{4}, \frac{3\pi}{4} \rangle$

(4) $y = \ln(1 - x^2)$, $x \in \langle 0, \frac{1}{2} \rangle$

(5) $y = \ln \frac{e^x+1}{e^x-1}$, $x \in \langle 1, 2 \rangle$

(6) $y = \arcsin x + \sqrt{1 - x^2}$, $x \in \langle 0, \frac{1}{2} \rangle$

Ex.4. Find the area of the region bounded by the parametric curve.

(1) $\begin{cases} x(t) = t^2 \\ y(t) = 1 - t^5 \end{cases}$, $t \in \langle 0, 1 \rangle$

(2) $\begin{cases} x(t) = t^4 - 1 \\ y(t) = t^4 - t^2 \end{cases}$ and the line $y = x$

(3) $\begin{cases} x(t) = t + \frac{1}{t} \\ y(t) = t + \frac{3}{t} \end{cases}$ and the line $y = 4$

(4) $\begin{cases} x(t) = e^{3t} - 1 \\ y(t) = e^{2t} + 1 \end{cases}$ and the lines $x = 0$ and $y = e + 1$

(5) $\begin{cases} x(t) = \cos^3 t \\ y(t) = \sin^3 t \end{cases}$ for $0 \le t \le 2\pi$

(6) $\begin{cases} x(t) = \ln(t^2 + 1) \\ y(t) = t^2 \end{cases}$ and the line $y = 1$

Ex.5. Find the lenght of the parametric curve.

(1) $\begin{cases} x(t) = 1 - t^3 \\ y(t) = 2 - t^3 \end{cases}$, $t \in \langle 0, 1 \rangle$

(2) $\begin{cases} x(t) = e^t \cos t \\ y(t) = e^t \sin t \end{cases}$, $t \in \langle 0, 2 \rangle$

(3) $\begin{cases} x(t) = t + \sin t \\ y(t) = 1 - \cos t \end{cases}$, $t \in \langle 0, \pi \rangle$

(4) $\begin{cases} x(t) = \sin^3 t \\ y(t) = \cos^3 t \end{cases}$, $t \in \langle \frac{\pi}{2}, \frac{3\pi}{4} \rangle$

(5) $\begin{cases} x(t) = \sin^3 t \\ y(t) = \cos^3 t \end{cases}$, $t \in \langle 0, \frac{\pi}{4} \rangle$

(6) $\begin{cases} x(t) = \arctan \sqrt{t} \\ y(t) = \ln(t + 1) \end{cases}$, $t \in \langle 1, 2 \rangle$

(7) $\begin{cases} x(t) = e^t(\cos t + \sin t) \\ y(t) = e^t(\cos t - \sin t) \end{cases}$, $t \in \langle 0, 1 \rangle$

Ex.6. For which values of the parameter $p \in \mathbb{R}_+$ the area of the region bounded by the graphs $y = px^2$ and $y = x\sqrt{x}$ equals $\frac{1}{15}$?