telescope.

Cassegrain reflecting telescope. Fig. 9.151 shows Cassegrainian type reflecting telescope. It consists of a large concave paraboloidal (primary) mirror having a hole at its centre. There is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is placed on the axis of the telescope near the hole of the primary mirror.

The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to focus at *F*, they are reflected by the small convex mirror and are converged to a point *I* just

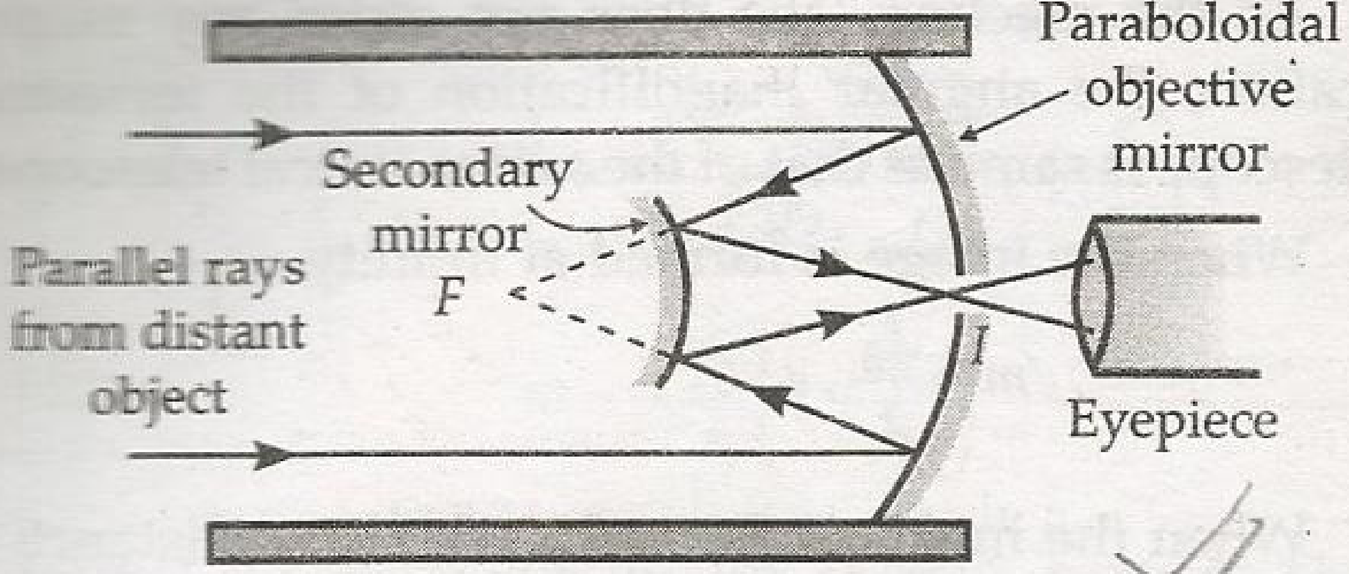


Fig. 9.151 Cassegrain reflecting telescope.

outside the hole. The final image formed at I is viewed through the eyepiece. As the first image at F is inverted with respect to the distant object and the second image I is erect with the respect to the first image F, hence the final image is inverted with respect to the object.

Let f_0 be the focal length of the objective and f_e that of the eyepiece.

For the final image formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

For the final image formed at infinity,

$$m = \frac{f_0}{f_e} = \frac{R/2}{f_e}.$$

Different types of telescope. Broadly, the telescopes can be divided into *two* categories :

1. **Refracting telescopes.** *These make use of lenses to view distant objects.* These are of *two* types :

(a) **Astronomical telescope.** It is used to see heavenly objects like the sun, stars, planets, etc. The final image formed is inverted one which is immaterial in the case of heavenly bodies because of their round shape.

(b) **Terrestrial telescope.** It is used to see distant objects on the surface of the earth. The final image formed is erect one. This is an essential condition of viewing the objects on earth's surface correctly.

2. **Reflecting telescopes.** *These make use of converging mirrors to view the distant objects.* For example, Newtonian and Cassegrain telescopes.

9.46 ASTRONOMICAL TELESCOPE

82. *What is an astronomical telescope ? Give its construction. With the help of ray diagrams, explain its working when it forms final image at the least distance of distinct vision and at infinity. Deduce expression for magnifying power in each case.*

Astronomical telescope. *It is a refracting type telescope used to see heavenly bodies like stars, planets, satellites, etc.*

Construction. It consists of two converging lenses mounted co-axially at the outer ends of two sliding tubes.

1. ***Objective.*** It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant objects, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.

2. ***Eyepiece.*** It is a convex lens of small focal length and small aperture. It faces the eye. The aperture of the eyepiece is taken small so that whole light of the telescope may enter the eye for distinct vision.

Working. (*a*) **When the final image is formed at the least distance of distinct vision.** As shown in Fig. 9.147, the parallel beam of light coming from the distant object falls on the objective at some angle α. The objective focusses the beam in its focal plane and forms a real, inverted and diminished image $A'B'$. This image $A'B'$ acts as an object for the eyepiece. The distance of the eyepiece is so adjusted that the image $A'B'$ lies within its focal length. The eyepiece magnifies this image so that final image $A''B''$ is magnified and inverted with respect to the object. The final image is seen distinctly by the eye at the least distance of distinct vision.

Magnifying power. *The magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the final image formed at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Thus

$$\angle A'OB' = \alpha$$

Also, let $\quad \angle A''EB'' = \beta$

$\therefore$ Magnifying power,

$$m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}.$$

According to the new Cartesian sign convention,

$OB' = +f_0$ = focal length of the objective

$B'E = -u_e$ = distance of $A'B'$ from the eyepiece, acting as an object for it

$$\therefore \qquad m = -\frac{f_0}{u_e}$$

Again, for the eyepiece :

$$u = -u_e \text{ and } v = -D$$

As $\qquad \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

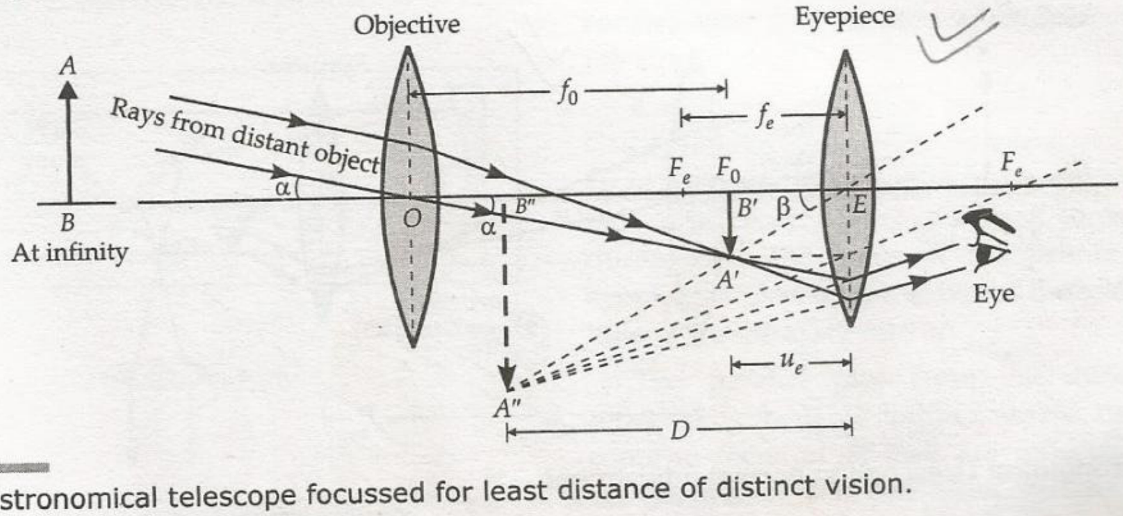


Fig. 9.147 Astronomical telescope focussed for least distance of distinct vision.

$$\therefore \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Hence $$m = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for the magnifying power indicates that the final image formed is *real* and *inverted*.

(*b*) **When the final image is formed at infinity : Normal adjustment.** As shown in Fig. 9.148, when a parallel beam of light is incident on the objective, it forms a real, inverted and diminished image $A'B'$ in its focal plane. The eyepiece is so adjusted that the image $A'B'$ exactly lies at its focus. Therefore, the final image is formed at infinity, and is highly magnified and inverted with respect to the object.

Magnifying power in normal adjustment. *It is defined as the ratio of the angle subtended at the eye by the final image as seen through the telescope to the angle subtended at the eye by the object seen directly, when both the image and the object lie at infinity.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective.

Thus

$$\angle A'OB' = \alpha$$

and let $$\angle A'EB' = \beta$$

∴ Magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

Applying new Cartesian sign convention,

$OB' = + f_0$ = Distance of $A'B'$ from the objective along the incident light

$B'E = - f_e$ = Distance of $A'B'$ from the eyepiece *against* the incident light

$$\therefore \quad m = -\frac{f_0}{f_e}$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for m indicates that the image is *real* and *inverted*.

For Your Knowledge

- In a telescope, the objective has large focal length and large aperture while the eyepiece has small focal length and small aperture.
- A telescope is focussed on the distant object by varying distance between the objective and the eye-piece with the help of rack and pinion arrangement.
- The objective of the telescope should have large aperture because then a much wider beam of light is incident on it and is converged into a small cone which, on entering the eye, produces sufficient illumination on the retina. So even two distant faint stars which cannot be seen by naked eyes, become visible through such a telescope.
- In a telescope, the image is not actually magnified. A telescope simply increases the visual angle. The visual angle β for the image is much larger than the visual angle α for the object. Consequently, the angular magnification β / α is quite large.
- In normal adjustment, the distance between the objective and the eyepiece = $f_0 + f_e$.
  When the final image is formed at the least distance of distinct vision, the magnifying power of the telescope is larger than that in the case of normal adjustment because the factor $\left(1 + \frac{f_e}{D}\right) > 1$.
- An astronomical telescope forms an inverted image. As the celestial objects are oval in shape, so it does not matter whether the final image is inverted or erect.

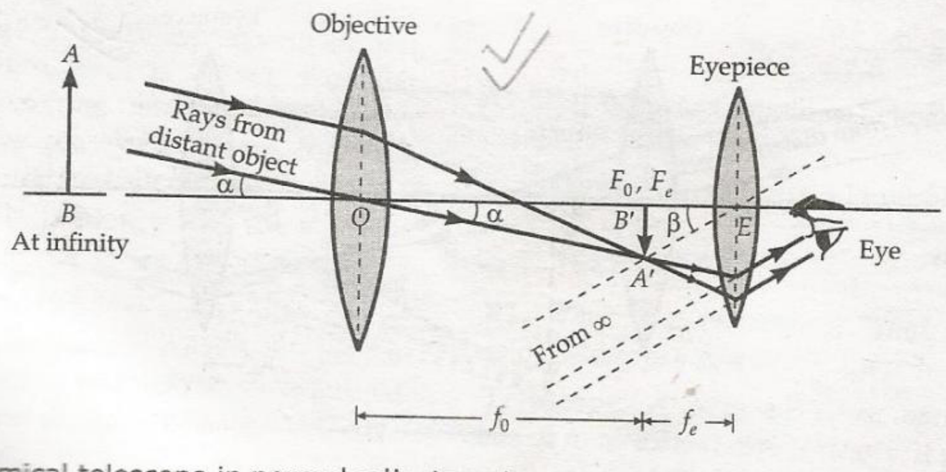


Fig. 9.148 Astronomical telescope in normal adjustment.

9.44 COMPOUND MICROSCOPE

80. *With the help of a ray diagram, explain the construction and working of a compound microscope. Write an expression for its magnifying power.*

Compound microscope. *A compound microscope is an optical device* used to see magnified images of tiny objects. A good quality compound microscope can produce magnification of the order of 1000.

Construction. It consists of two convex lenses of short focal length, arranged co-axially at the ends of *two* sliding metal tubes.

1. ***Objective.*** It is a convex lens of very short focal length f_0 and small aperture. It is positioned near the object to be magnified.

2. ***Eyepiece or ocular.*** It is a convex lens of comparatively larger focal length f_e and larger aperture than the objective ($f_e > f_0$). It is positioned near the eye for viewing the final image.

The distance between the two lenses can be varied by using rack and pinion arrangement.

Working. (*a*) ***When the final image is formed at the least distance of distinct vision.*** The object AB to be viewed is placed at distance u_0, slightly larger than the focal length f_0 of the objective O. The objective forms a real, inverted and magnified image $A' B'$, of the object AB on the other side of the lens O, as shown in Fig. 9.145. The separation between the objective O and the eyepiece E, is so adjusted that the image $A' B'$ lies within the focal length f_e of the eyepiece. The image $A' B'$ acts as an object for the eyepiece which essentially acts like a simple microscope. The eyepiece E forms a virtual and magnified final image $A'' B''$ of the object AB. Clearly, the final image $A'' B'$ is inverted with respect to the object AB.

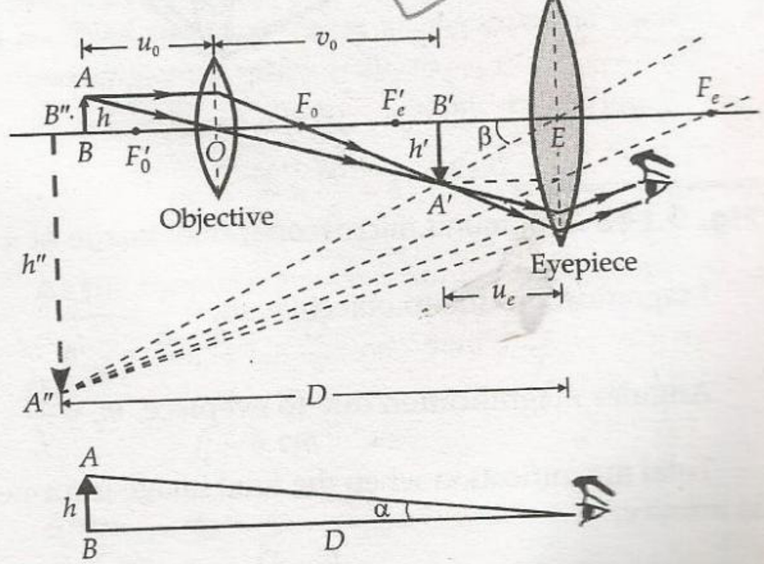


Fig. 9.145 Compound Microscope, final image at D.

Magnifying power. *The magnifying power of a compound microscope is defined as the ratio of the angle* *subtended at the eye by the final virtual image to the angle subtended at the eye by the object, when both are at the least distance of distinct vision from the eye.*

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \frac{h'}{h}\cdot\frac{D}{u_e} = m_0 m_e$$

Here $\quad m_0 = \dfrac{h'}{h} = \dfrac{v_o}{u_0}$.

As the eyepiece acts as a simple microscope, so

$$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e} \quad \therefore \quad m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

As the object AB is placed close to the focus F_0 of the objective, therefore,

$$u_0 \simeq -f_0$$

Also image $A'B'$ is formed close to the eyelens whose focal length is short, therefore $v_0 \simeq L =$ the length of the microscope tube or the distance between the two lenses

$$\therefore \quad m_0 = \frac{v_0}{u_0} = \frac{L}{-f_0}$$

$$\therefore \quad m = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \quad \text{[for final image at } D\text{]}$$

(*b*) **When the final image is formed at infinity.** When the image $A'B'$ lies at the focus F_e' of the eyepiece *i.e.*, $u_e = f_e$, the image $A''B''$ is formed at infinity, as shown in Fig. 9.146.

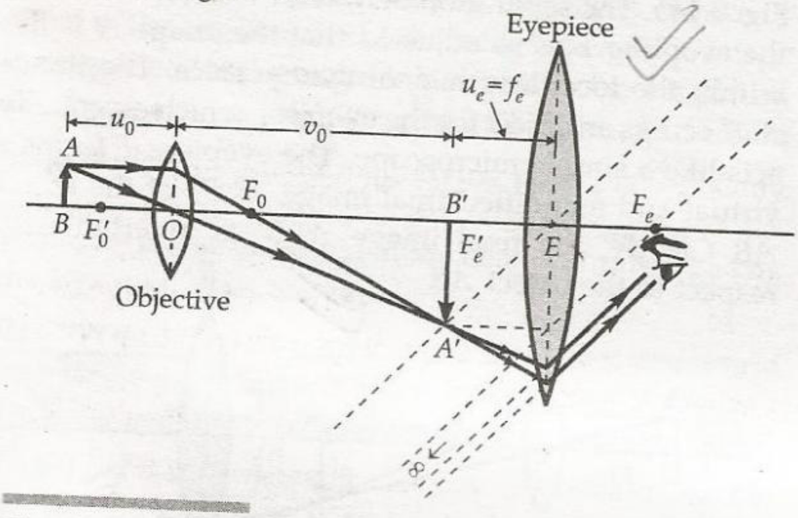


Fig. 9.146 Compound microscope, final image at ∞.

Magnification due to objective, $m_0 = \dfrac{h'}{h} = \dfrac{L}{-f_0}$.

Angular magnification due to eyepiece, $m_e = \dfrac{D}{f_e}$

Total magnification when the final image is formed at infinity,

$$m = m_0 \times m_e = -\frac{L}{f_0} \times \frac{D}{f_e}.$$

Obviously, magnifying power of the compound microscope is large when both f_0 and f_e are small.

9.43 SIMPLE MICROSCOPE

79. *What is a simple microscope ? Give its working principle. Write expressions for its magnifying power when it forms final image at the least distance of distinct vision and at infinity.*

Simple microscope. *A simple microscope or a magnifying glass is just a convex lens of short focal length, held close to the eye.*

Working principle : When the final image is formed at the least distance of distinct vision. When an object *AB* is placed between the focus *F* and optical centre *O* of a convex lens ; a virtual, erect and magnified image *A′ B′* is formed on the same side of the lens as the object. Since a normal eye can see an object clearly at the least distance of distinct vision *D* (=25 cm), the position of the lens is so adjusted that the final image is formed at the distance *D* from the lens, as shown in Fig. 9.143.

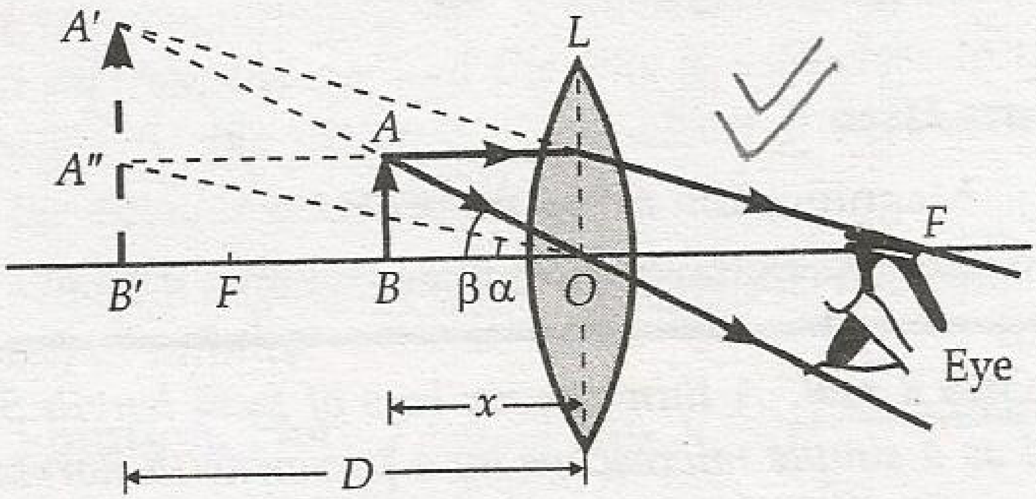


Fig. 9.143 A simple microscope with the eye focussed at the near point.

Magnifying power. *The magnifying power of a simple microscope is defined as the ratio of the angles subtended by* *the image and the object at the eye, when both are at the least distance of distinct vision from the eye.* Thus,

Magnifying power

$$= \frac{\text{Angle subtended by the image at the least distance of distinct vision}}{\text{Angle subtended by the object at the least distance of distinct vision}}$$

As the eye is held close to the lens, the angles subtended at the lens may be taken to be the angles subtended at the eye. The image $A'B'$ is formed at the least distance of distinct vision 'D'. Let $\angle A'OB' = \beta$. Imagine the object AB to be displaced to position $A''B'$ at distance D from the lens. Let $\angle A''OB' = \alpha$. Then magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{AB/OB}{A''B/OB'} = \frac{AB/OB}{AB/OB'} \qquad [\because A''B' = AB]$$

$$= \frac{OB'}{OB} = \frac{-D}{-x}$$

or $$m = \frac{D}{x}$$

Let f be the focal length of the lens. As the image is formed at the least distance of distinct vision from the lens, so

$$v = -D$$

Using *thin lens formula,*

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get, $$\frac{1}{-D} - \frac{1}{-x} = \frac{1}{f}$$

or $$\frac{1}{x} = \frac{1}{D} + \frac{1}{f}$$

or $$\frac{D}{x} = 1 + \frac{D}{f}$$

$\therefore$ $$m = 1 + \frac{D}{f}$$

Thus shorter the focal length of the convex lens, the greater is its magnifying power.

Working principle : When the final image is formed at infinity. When we see an image at the near point, it causes some strain in the eye. Often the object is placed at the focus of the convex lens, so that parallel rays enter the eye, as shown in Fig. 9.144(a). The image is formed at infinity, which is more suitable and comfortable for viewing by the relaxed eye.

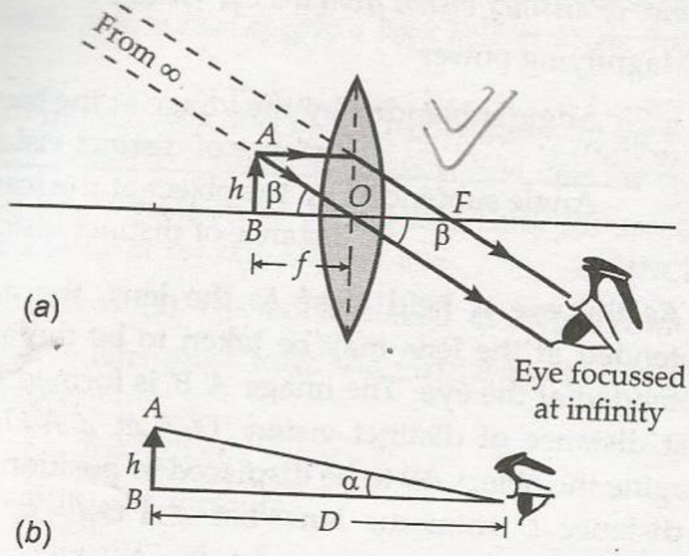


Fig. 9.144 (*a*) With object at *F*, image is at infinity. (*b*) Object at the near point.

Magnifying power. *It is defined as the ratio of the angle formed by the image (when situated at infinity) at the eye to the angle formed by the object at the eye, when situated at the least distance of distinct vision.*

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\alpha, \beta \text{ are small}]$$

From Fig. 9.144(*a*),

$$\tan\beta = \frac{h}{f}$$

From Fig. 9.144(*b*),

$$\tan\alpha = \frac{h}{D}$$

$$\therefore \qquad m = \frac{h/f}{h/D}$$

or

$$m = \frac{D}{f}.$$

This magnification is one less than the magnification when the image is formed at the near point. But viewing is more comfortable when the eye is focussed at infinity.

Uses of simple microscopes :

1. Watch makers and jewellers use a magnifying glass for having a magnified view of the small parts of watches and the fine jewellery work.
2. In magnifying the printed letters in a book, textures of fibres or threads of a cloth, engravings, details of stamp, etc.
3. Magnifying glass is used in science laboratories for reading vernier scales, etc.

9.25 RULES FOR DRAWING IMAGES FORMED BY SPHERICAL LENSES

40. *State the rules used for drawing images formed by spherical lenses.*

Rules for drawing images formed by spherical lenses. The position of the image formed by any spherical lens can be found by considering any two of the following rays of light coming from a point on the object.

(*i*) A ray from the object parallel to the principal axis after refraction passes through the second principal focus F_2 [in a convex lens, as shown in Fig. 9.75(*a*)] or appears to diverge [in a concave lens, as shown in Fig. 9.75(*b*)] from the first principal focus F_1.

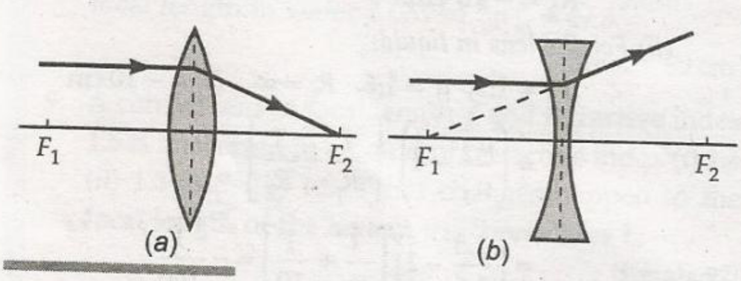


Fig. 9.75 Path of ray incident parallel to the principal axis of (*a*) convex lens (*b*) concave lens.

(*ii*) A ray of light passing through the first principal focus [in a convex lens, as shown in Fig. 9.84(*a*)] or appearing to meet at it [in a concave lens, as shown in Fig. 9.76(*b*)] emerges parallel to the principal axis after refraction.

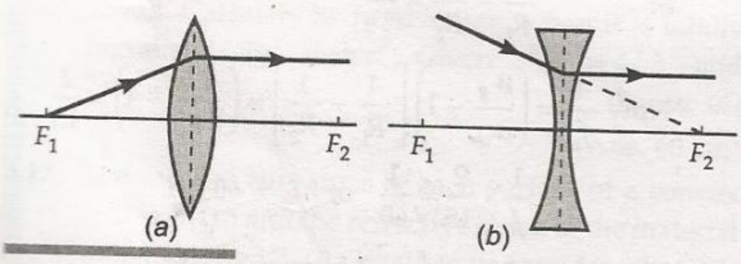


Fig. 9.76 Path of a ray passing through focus of (*a*) convex lens (*b*) concave lens.

(*iii*) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction, as shown in Figs. 9.77(*a*) and (*b*).

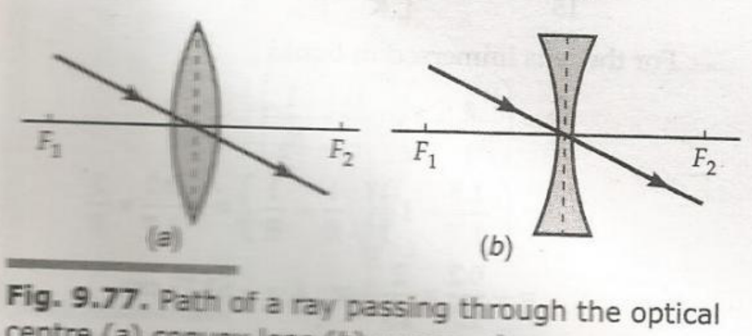
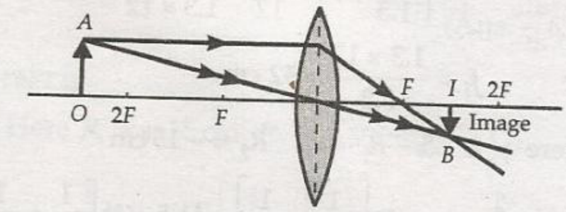


Fig. 9.77. Path of a ray passing through the optical centre (*a*) convex lens (*b*) concave lens.

Formation of images by spherical lenses :

(*a*) **Object beyond 2*F*.** The image is

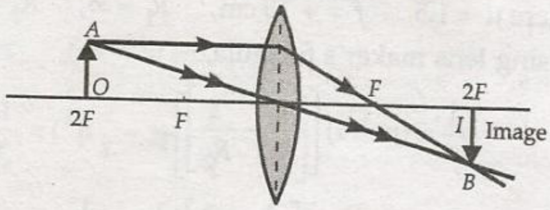
(*i*) between *F* and 2*F* (*ii*) real
(*iii*) inverted (*iv*) smaller



(*a*)

(*b*) **Object at 2*F*.** The image is

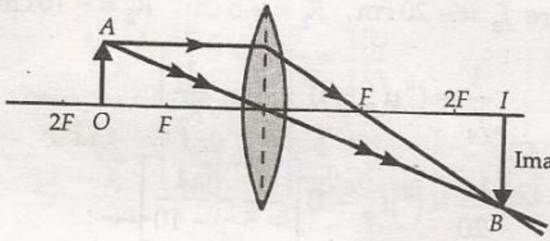
(*i*) at 2*F* (*ii*) real
(*iii*) inverted (*iv*) same size



(*b*)

(*c*) **Object between 2*F* and *F*.** The image is

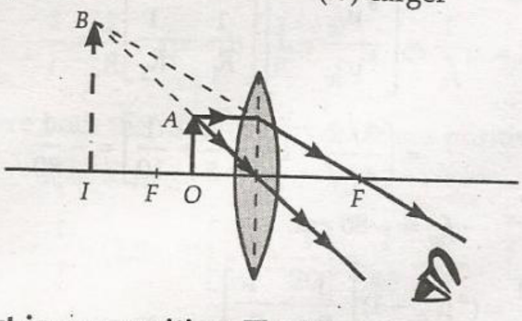
(*i*) beyond 2*F* (*ii*) real
(*iii*) inverted (*iv*) larger



(*c*)

(*d*) **Object between *F* and *O*.** The image is

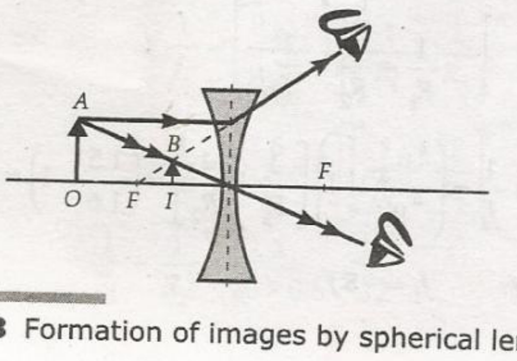
(*i*) behind object (*ii*) virtual
(*iii*) erect (*iv*) larger



(*d*)

(*e*) **Object in any position.** The image is

(*i*) in front of object (*ii*) virtual
(*iii*) erect (*iv*) smaller



(*e*)

Fig. 9.78 Formation of images by spherical lenses.

principal axis.

Derivation of thin lens formula for a convex lens when it forms a real image. As shown in Fig. 9.79, consider an object AB placed perpendicular to the principal axis of a thin convex lens between its F' and C'. A real, inverted and magnified image $A'B'$ is formed beyond C on the other side of the lens.

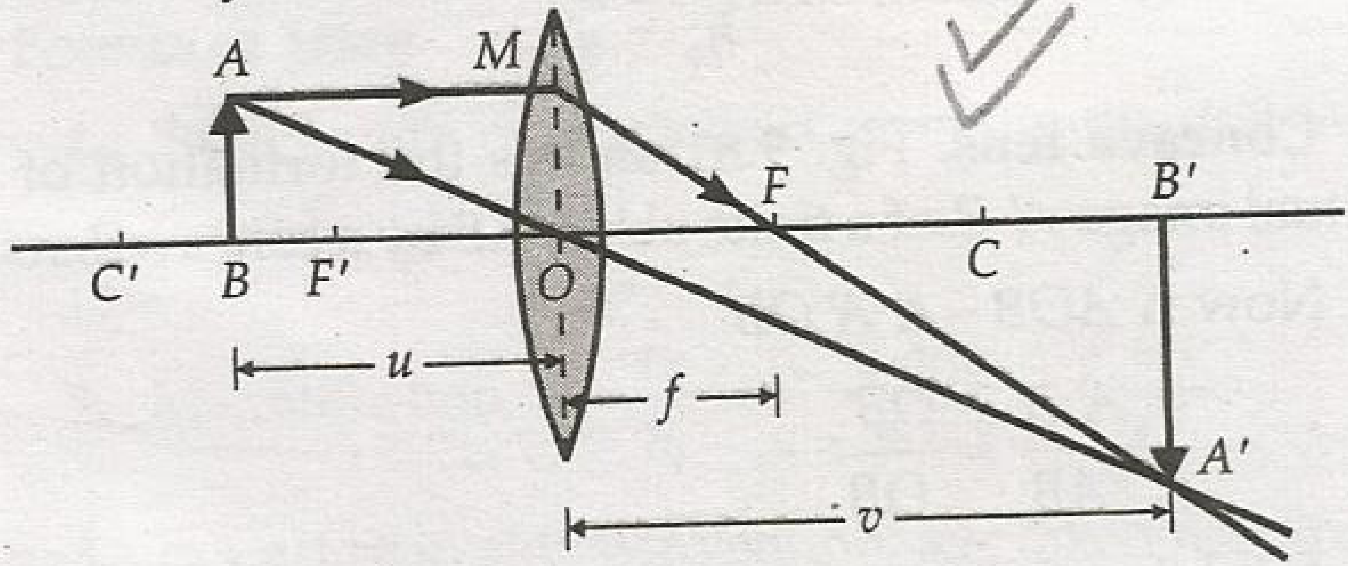


Fig. 9.79 Real image formed by a convex lens.

$\Delta A'B'O$ and ΔABO are similar,

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{BO} \qquad \text{...(1)}$$

Also $\Delta A'B'F$ and ΔMOF are similar,

$$\therefore \quad \frac{A'B'}{MO} = \frac{FB'}{OF}$$

But $MO = AB,$

$$\therefore \quad \frac{A'B'}{AB} = \frac{FB'}{OF} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{OB'}{BO} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$$

Using new Cartesian sign convention, we get

Object distance, $BO = -u$

Image distance, $OB' = +v$

Focal length, $OF = +f$

$$\therefore \quad \frac{v}{-u} = \frac{v-f}{f}$$

or $$vf = -uv + uf \quad \text{or} \quad uv = uf - vf$$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the lens formula for a convex lens when it forms a real image.

it forms a virtual image.

Derivation of thin lens formula for a convex lens when it forms a virtual image. As shown in Fig. 9.80, when an object AB is placed between the optical centre O and the focus F of a convex lens, the image $A'B'$ formed by the convex lens is virtual, erect and magnified.

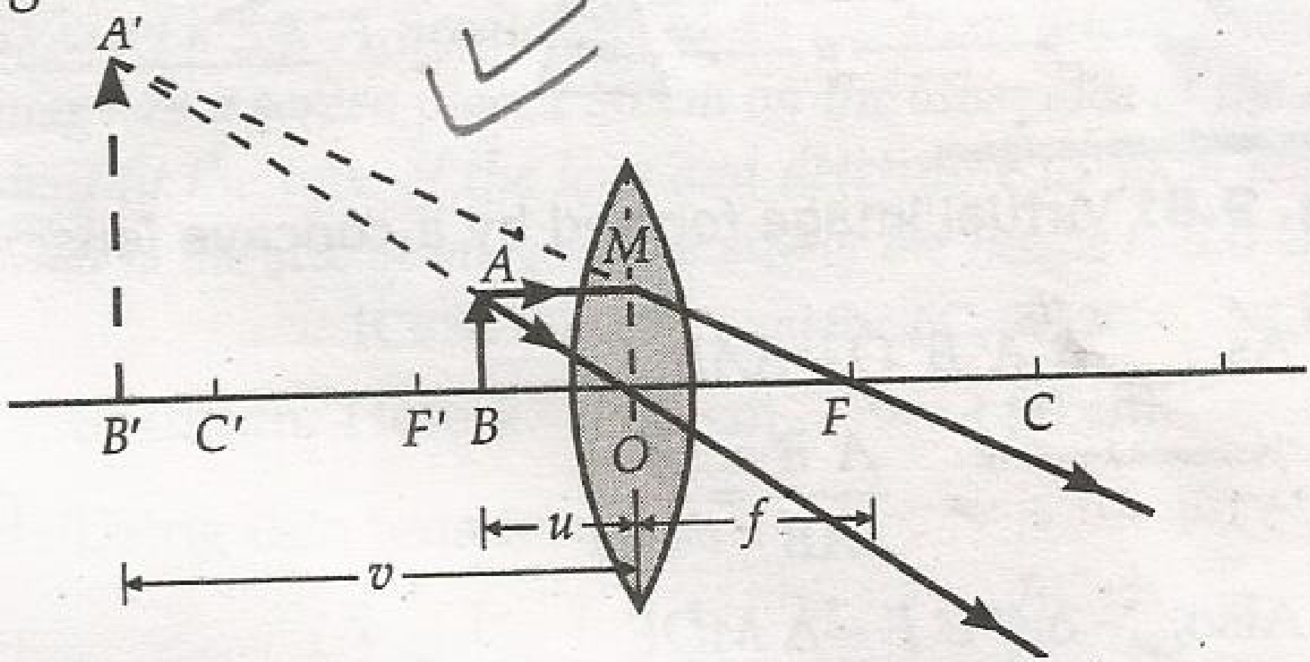


Fig. 9.80 Virtual image formed by a convex lens.

Triangles $A'B'O$ and ABO are similar.

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'O}{BO} \qquad ...(1)$$

Also, triangles $A'B'F$ and MOF are similar.

$$\therefore \quad \frac{A'B'}{MO} = \frac{B'F}{OF}$$

But $MO = AB$, therefore

$$\frac{A'B'}{AB} = \frac{B'F}{OF} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{B'F}{OF} = \frac{B'O + OF}{OF}$$

Using new cartesian sign convention,

$$BO = -u, \quad B'O = -v, \quad OF = +f$$

$$\therefore \qquad \frac{-v}{-u} = \frac{-v+f}{f}$$

or $$-vf = uv - uf$$

or $$uv = uf - vf$$

Dividing both sides by *uvf*, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a convex lens when it forms a virtual image.

Derivation of thin lens formula for a concave lens. As shown in Fig. 9.81, suppose O be the optical centre and F be the principal focus of concave lens of focal length f. AB is an object placed perpendicular to its principal axis. A virtual, erect and diminished image $A'B'$ is formed due to refraction through the lens.

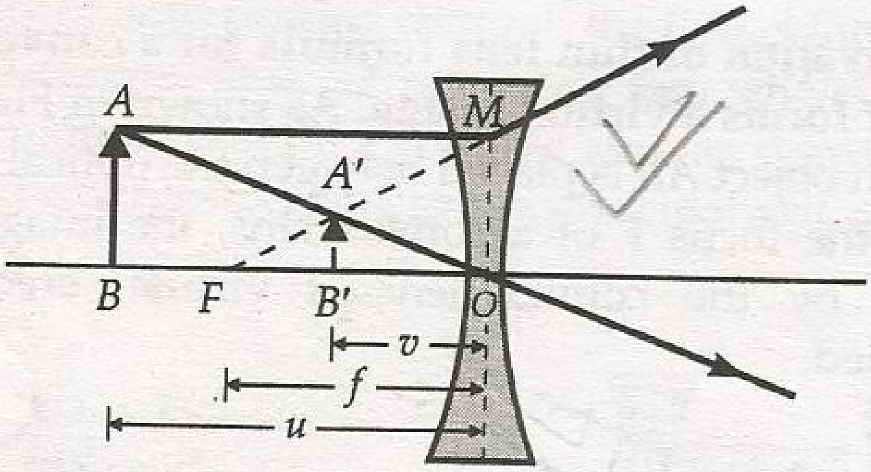


Fig. 9.81 Virtual image formed by a concave lens.

As $\quad \Delta A'B'O \sim \Delta ABO$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'O}{BO} \qquad \text{...(1)}$$

Also, $\quad \Delta A'B'F \sim \Delta MOF$

$$\therefore \quad \frac{A'B'}{MO} = \frac{FB'}{FO}$$

But $MO = AB$, therefore

$$\frac{A'B'}{AB} = \frac{FB'}{FO} \qquad \text{...(2)}$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$$

Using new Cartesian sign convention, we get

$$BO = -u, \quad B'O = -v, \quad FO = -f$$

$$\therefore \qquad \frac{-v}{-u} = \frac{-f+v}{-f}$$

or $$vf = uf - uv \quad \text{or} \quad uv = uf - vf$$

Dividing both sides by *uvf*, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a concave lens.

9.27 ▼ LINEAR MAGNIFICATION

45. *Define linear magnification produced by a lens. Derive expressions for the magnification produced by convex and concave lenses.*

Linear magnification. *The linear magnification produced by a lens is defined as the ratio of the size of the image formed by the lens to the size of the object.* It is denoted by m. Thus

$$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h_2}{h_1}$$

Convex lens. Earlier Fig. 9.79 shows a ray diagram for the formation of image $A'B'$ of a finite object AB by a convex lens.

Now $\quad \Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2$ (*Downward image height*)

$AB = +h_1$ (*Upward object height*)

$OB = -u$ (*Image distance on left*)

$OB' = +v$ (*Image distance on right*)

$$\therefore \quad \frac{-h_2}{+h_1} = \frac{+v}{-u} \quad \text{or} \quad \frac{h_2}{h_1} = \frac{v}{u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Concave lens. Fig. 9.81 shows the formation of a virtual image $A'B'$ of a finite object AB by a concave lens.

Now $\Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}.$$

Applying the new cartesian sign convention, we get

$$A'B' = +h_2, \quad AB = +h_1$$

$$OB' = -v, \quad OB = -u$$

$$\therefore \quad \frac{+h_2}{+h_1} = \frac{-v}{-u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Linear magnification in terms of u and f. The thin lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Multiplying both sides by u, we get

$$\frac{u}{v}-1=\frac{u}{f} \quad \text{or} \quad \frac{u}{v}=1+\frac{u}{f}=\frac{f+u}{f}$$

$$\therefore \qquad m=\frac{v}{u}=\frac{f}{f+u}$$

Linear magnification in terms of v and f. The thin lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Multiplying both sides by v, we get

$$1-\frac{v}{u}=\frac{v}{f} \quad \therefore \quad m=\frac{v}{u}=1-\frac{v}{f}=\frac{f-v}{f}$$

Hence $$m=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$$

Lens maker's formula for a double convex lens. As shown in Fig. 9.70, consider a thin double convex lens of refractive index μ_2 placed in a medium of refractive index μ_1. Here $\mu_1 < \mu_2$. Let B and D be the poles, C_1 and C_2 be the centres of curvature, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and ADC, respectively.

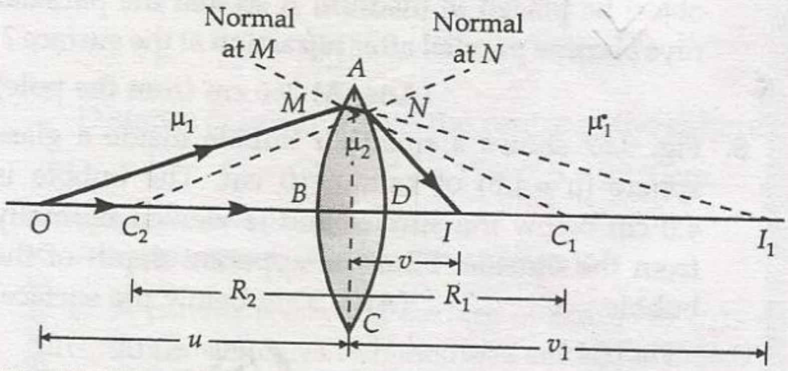


Fig. 9.70 Refraction through a double convex lens.

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. The ray OM is incident on the first surface ABC. It is refracted along MN, bending towards the normal at this surface. If the second surface ADC were absent, the ray MN would have met the principal axis at I_1. So we can treat I_1 as the real image formed by first surface ABC in the medium of refractive index μ_2.

For refraction at surface ABC, we can write the relation between the object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad ...(1)$$

But actually the ray MN suffers another refraction at surface ADC, bending away from the normal at point N. The emergent ray meets the principal axis at point I which is the final image of O formed by the lens. For refraction at second surface, I_1 acts as a virtual object placed in the medium of refractive index μ_2 and I is the real image formed in the medium of refractive index μ_1. Therefore, the relation between the object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or

$$\frac{1}{v} - \frac{1}{u} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(3)$$

If the object is placed at infinity ($u = \infty$), the image will be formed at the focus, *i.e.*, $v = f$. Therefore,

$$\frac{1}{f} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(4)$$

This is ***lens maker's formula.***

When the lens is placed in *air*, $\mu_1 = 1$, and $\mu_2 = \mu$. The lens maker's formula takes the form :

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

From equations (3) and (4), we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This is the *thin lens formula* which gives relationship between u, v and f of a lens.

concave lens.

Lens maker's formula for a double concave lens. As shown in Fig. 9.71, consider a thin double concave lens of refractive index μ_2 placed in a medium of refractive μ_1. Here $\mu_1 < \mu_2$. Let B and E be the poles, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and DEF, respectively.

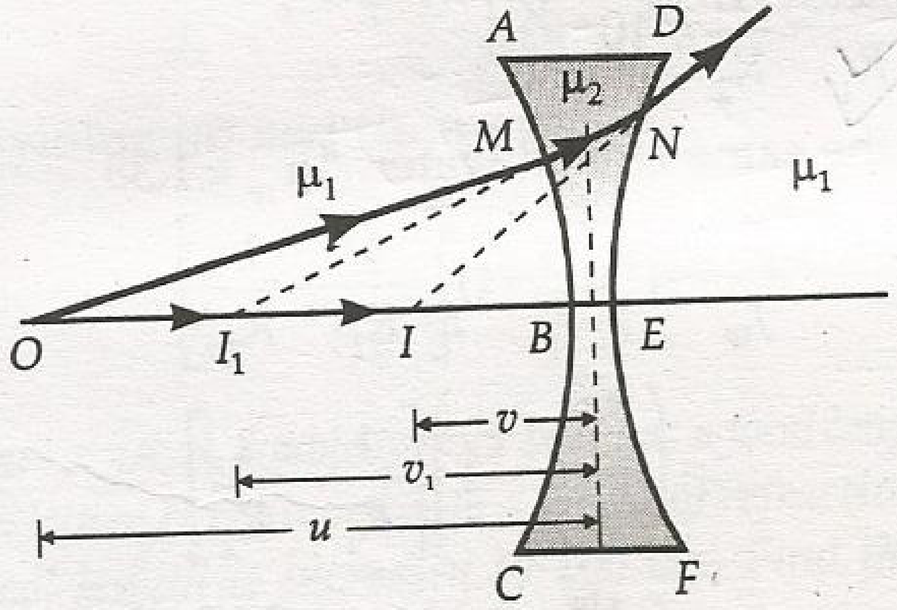


Fig. 9.71 Refraction through a double concave lens.

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. First the spherical surface ABC forms its virtual image I_1. As refraction occurs from rarer to denser medium, so we can write the relation between object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad ...(1)$$

But the lens material is not continuous. The ray MN suffers another refraction at N and emerges along IN. So I is the final virtual image of the point object O. The image I_1 acts as an object for refraction at surface DEF from denser to rarer medium. So the relation between object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or

$$\frac{1}{v} - \frac{1}{u} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

If an object is placed at infinity, then the image is formed at the focus *i.e.*, $v = f$, so

$$\frac{1}{f} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

This is *lens **maker's formula***.

When the lens is placed in air, $\mu_1 = 1$ and $\mu_2 = \mu$. The lens maker's formula takes the form :

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Refraction at a convex spherical surface :

(i) ***The object lies in rarer medium and the image formed is real.*** In Fig. 9.55, *APB* is a convex refracting surface which separates a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2. Let *P* be the pole, *C* be the centre of curvature and $R = PC$ be the radius of curvature of this surface. Suppose a point object *O* is placed on the principal axis in the rarer medium. Starting from the point object *O*, a ray *ON* is incident at an angle *i*. After refraction, it bends towards the normal *CN* at an angle of refraction *r*. Another ray *OP* is incident normally on the convex surface and passes undeviated. The two refracted rays meet at point *I*. So *I* is the real image of point object *O*.

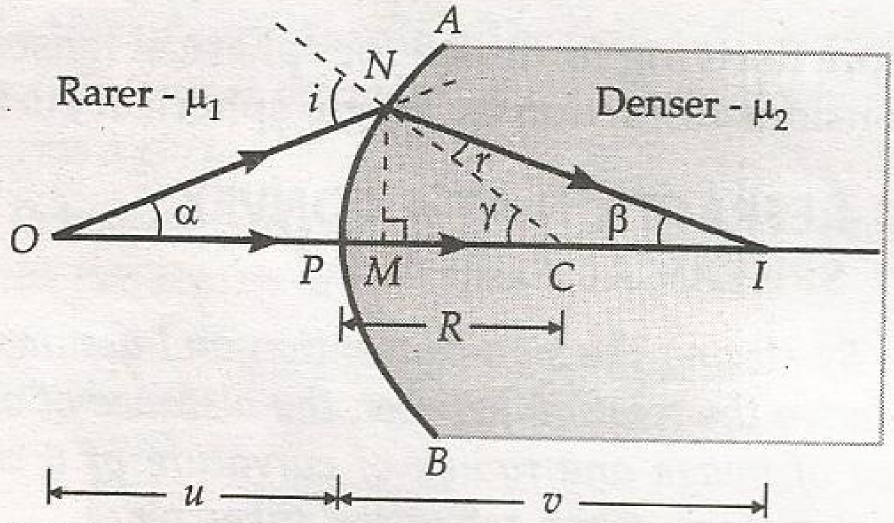


Fig. 9.55 Refraction from rarer to denser medium, when the image is real.

Draw *NM* perpendicular to the principal axis. Let α, β and γ be the angles, as shown in Fig. 9.55.

In ΔNOC, *i* is an exterior angle, therefore,

$$i = \alpha + \gamma$$

Similarly, from $\Delta\, NIC$, we have

$$\gamma = r + \beta$$

or $$r = \gamma - \beta$$

Suppose all the rays are paraxial. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because\ P \text{ is close to } M]$$

$$\beta \simeq \tan\beta = \frac{NM}{MI} \simeq \frac{NM}{PI}$$

and $$\gamma \simeq \tan\gamma = \frac{NM}{MC} \simeq \frac{NM}{PC}$$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and r are small, therefore

$$\frac{i}{r} = \frac{\mu_2}{\mu_1}$$

or $$\mu_1 i = \mu_2 r$$

or $$\mu_1 [\alpha + \gamma] = \mu_2 [\gamma - \beta]$$

or $$\mu_1 \left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_2 \left[\frac{NM}{PC} - \frac{NM}{PI}\right]$$

or $$\mu_1 \left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_2 \left[\frac{1}{PC} - \frac{1}{PI}\right]$$

or $$\frac{\mu_1}{OP} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP = -u$
Image distance, $PI = +v$
Radius of curvature, $PC = +R$

$$\therefore \quad \frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Note If first medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$, we have

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}.$$

(ii) The object lies in the rarer medium and the image formed is virtual. When the object O in the

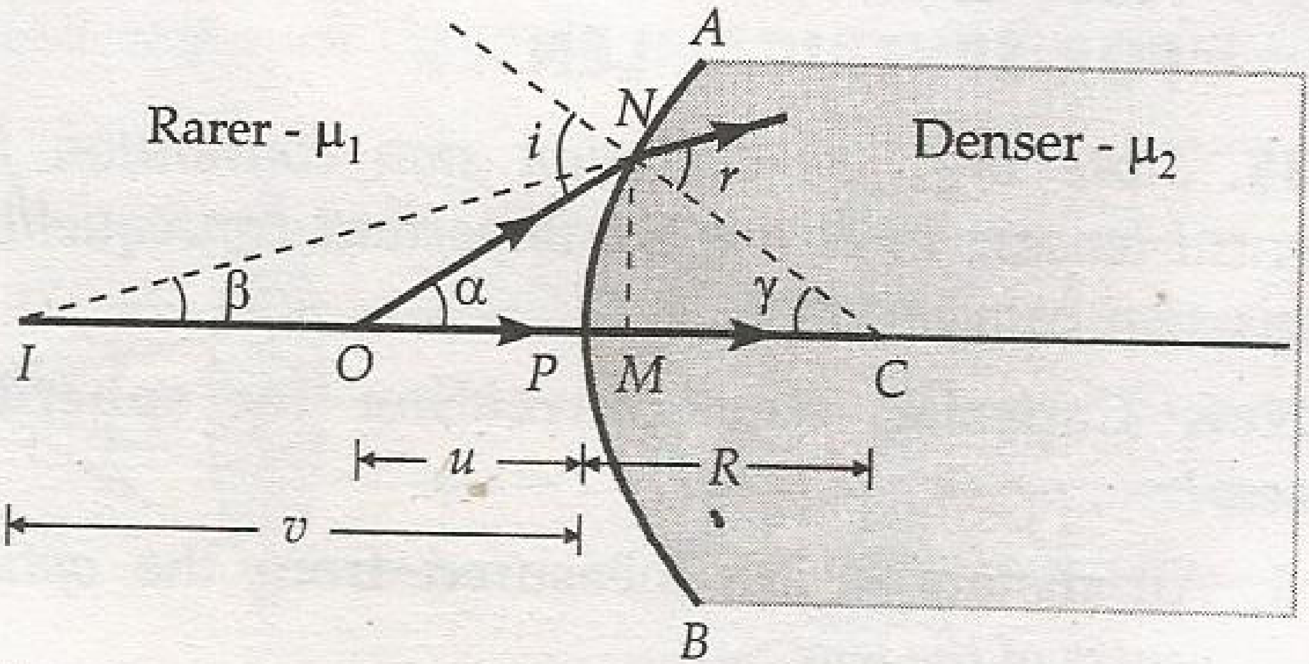


Fig. 9.56 Refraction from rarer to denser medium, when the image is virtual.

rarer medium lies close to the pole P of the convex refracting surface, the two refracted rays appear to diverge from a point I on the principal axis, as shown in Fig. 9.56. So I is the virtual image of the point object O.

From ΔNOC, $\quad i = \alpha + \gamma$

From ΔNCI, $\quad r = \beta + \gamma$.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small

$$\therefore \quad \alpha \simeq \tan \alpha = \frac{NM}{OM} = \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan \beta = \frac{NM}{IM} = \frac{NM}{IP}$$

and $$\gamma \simeq \tan \gamma = \frac{NM}{MC} = \frac{NM}{PC}$$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and r are small, so

$$\frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \mu_1 i = \mu_2 r$$

or $$\mu_1 (\alpha + \gamma) = \mu_2 (\beta + \gamma)$$

or $$\mu_1 \left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_2 \left[\frac{NM}{IP} + \frac{NM}{PC}\right]$$

or $$\mu_1 \left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_2 \left[\frac{1}{IP} + \frac{1}{PC}\right]$$

or $$\frac{\mu_1}{OP} - \frac{\mu_2}{IP} = \frac{\mu_2 - \mu_1}{PC}$$

Using new Cartesian sign convention, we find that

Object distance, $\quad OP = -u$

Image distance, $\quad IP = -v$

Radius of curvature, $\quad PC = +R$

$$\therefore \quad \frac{\mu_1}{-u} - \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(*iii*) ***The object lies in the denser medium and the image formed is real.*** Fig. 9.57 shows a convex refracting surface which is convex towards the rarer medium. The point object *O* lies in the denser medium. The two refracted rays meet at point *I*. So *I* is the real image of the point object *O*.

From ΔNOC, $\gamma = i + \alpha$ or $i = \gamma - \alpha$

From ΔNIC, $r = \beta + \gamma$

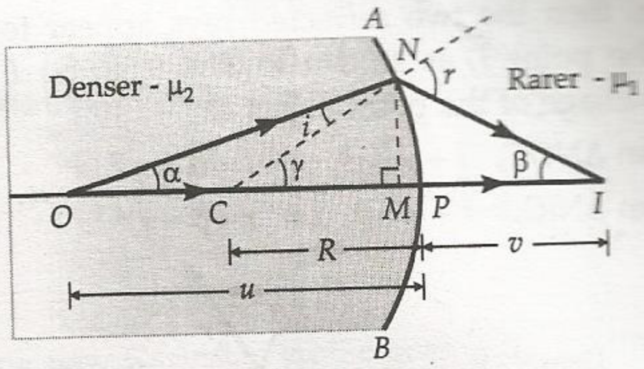


Fig. 9.57 Refraction from denser to rarer medium when the image is real.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NP}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{MI} = \frac{NM}{PI}$$

and $$\gamma \simeq \tan\gamma = \frac{NM}{CM} = \frac{NM}{CP}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2 i = \mu_1 r$$

or $$\mu_2(\gamma - \alpha) = \mu_1(\beta + \gamma)$$

or $$\mu_2\left[\frac{NM}{CP} - \frac{NM}{OP}\right] = \mu_1\left[\frac{NM}{PI} + \frac{NM}{CP}\right]$$

or $$\mu_2\left[\frac{1}{CP} - \frac{1}{OP}\right] = \mu_1\left[\frac{1}{PI} + \frac{1}{CP}\right]$$

or $$-\frac{\mu_1}{PI} - \frac{\mu_2}{OP} = \frac{\mu_1 - \mu_2}{CP}$$

Using the new Cartesian sign convention, we have

Object distance,	$OP = -u$
Image distance,	$PI = +v$
Radius of curvature,	$CP = -R$

$$\therefore \quad -\frac{\mu_1}{v} - \frac{\mu_2}{-u} = \frac{\mu_1 - \mu_2}{-R}$$

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

(*iv*) ***The object lies in the denser medium and the image formed is virtual***. If the point object O placed on the principal axis lies close to the pole of the refracting surface, then the two refracted rays appear to come from the point I, as shown in Fig. 9.58. So I is the virtual image of the point object O.

From ΔNOC, $i + \gamma = \alpha$ or $i = \alpha - \gamma$

From ΔNIC, $r + \gamma = \beta$ or $r = \beta - \gamma$

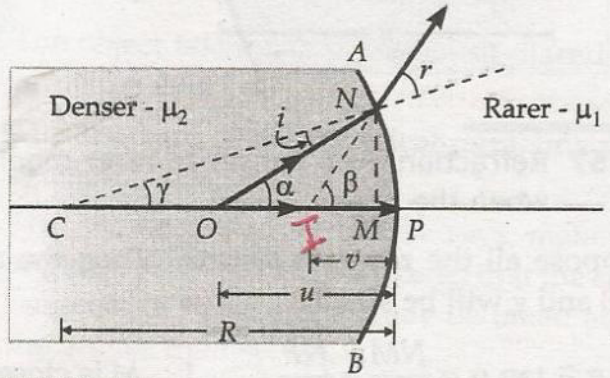


Fig. 9.58 Refraction from denser to rarer medium when the image is virtual.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{IM} = \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{CM} = \frac{NM}{CP}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2 i = \mu_1 r$$

or $$\mu_2(\alpha - \gamma) = \mu_1(\beta - \gamma)$$

or $$\mu_2\left[\frac{NM}{OP} - \frac{NM}{CP}\right] = \mu_1\left[\frac{NM}{IP} - \frac{NM}{CP}\right]$$

or $$\mu_2\left[\frac{1}{OP} - \frac{1}{CP}\right] = \mu_1\left[\frac{1}{IP} - \frac{1}{CP}\right]$$

or $$-\frac{\mu_1}{IP} + \frac{\mu_2}{OP} = -\frac{\mu_1 - \mu_2}{CP}$$

Using the new Cartesian sign convention, we have

Object distance,	$OP = -u$
Image distance,	$IP = -v$
Radius of curvature,	$CP = -R$

$$\therefore \quad -\frac{\mu_1}{-v} + \frac{\mu_2}{-u} = -\frac{\mu_1 - \mu_2}{-R}$$

or $$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at a concave spherical surface.

(*i*) ***The object lies in the rarer medium.*** In Fig. 9.59, *APB* is a concave refracting surface separating two media of refractive indices μ_1 and μ_2.

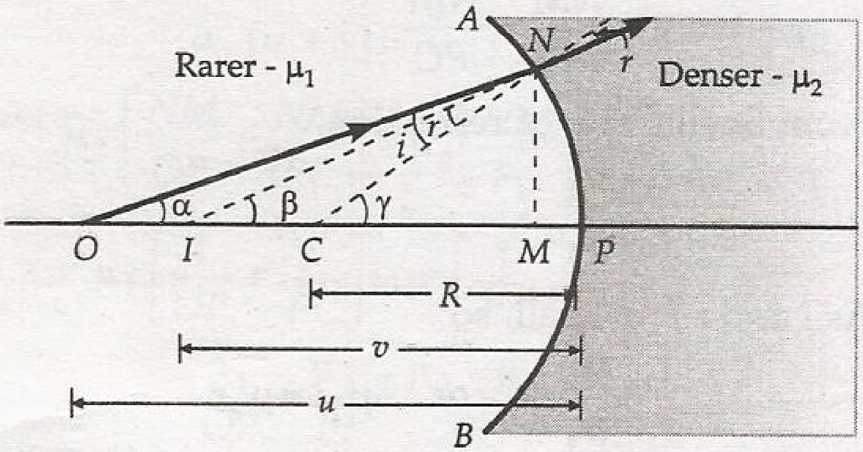


Fig. 9.59 Refraction at a concave surface when the object lies in the rarer medium.

Let

P = Pole of the concave surface APB

C = Centre of curvature of the concave surface

O = Point object placed on the principal axis

I = Virtual image of point object O

In Δ *NOC*, γ is an exterior angle, therefore

$$\gamma = \alpha + i \quad \text{or} \quad i = \gamma - \alpha$$

Similarly, from Δ *NIC*, we have

$$\gamma = \beta + r \quad \text{or} \quad r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan \alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan \beta = \frac{NM}{IM} \simeq \frac{NM}{IP}$$

$$\gamma \simeq \tan \gamma = \frac{NM}{CM} \simeq \frac{NM}{CP}$$

From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

As i and i are small angles, therefore

$$\frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \mu_1 i = \mu_2 r$$

or $$\mu_1[\gamma - \alpha] = \mu_2[\gamma - \beta]$$

or $$\mu_1\left[\frac{NM}{CP} - \frac{NM}{OP}\right] = \mu_2\left[\frac{NM}{CP} - \frac{NM}{IP}\right]$$

or $$\mu_1\left[\frac{1}{CP} - \frac{1}{OP}\right] = \mu_2\left[\frac{1}{CP} - \frac{1}{IP}\right]$$

or $$-\frac{\mu_1}{OP} + \frac{\mu_2}{IP} = \frac{\mu_2 - \mu_1}{CP}$$

Using new Cartesian sign convention, we find

Object distance,	$OP = -u$
Image distance,	$IP = -v$
Radius of curvature,	$CP = -R$

$\therefore$ $$\frac{-\mu_1}{-u} + \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{-R}$$

or $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(*ii*) ***The object lies in the denser medium.*** As shown in Fig. 9.64, when the point object O is placed in the denser medium, the refracted rays appear to diverge from a point I in the denser medium. So I is the virtual image of the point object O.

From ΔNOC, $\quad i = \alpha + \gamma$

From ΔNIC, $\quad r = \beta + \gamma$

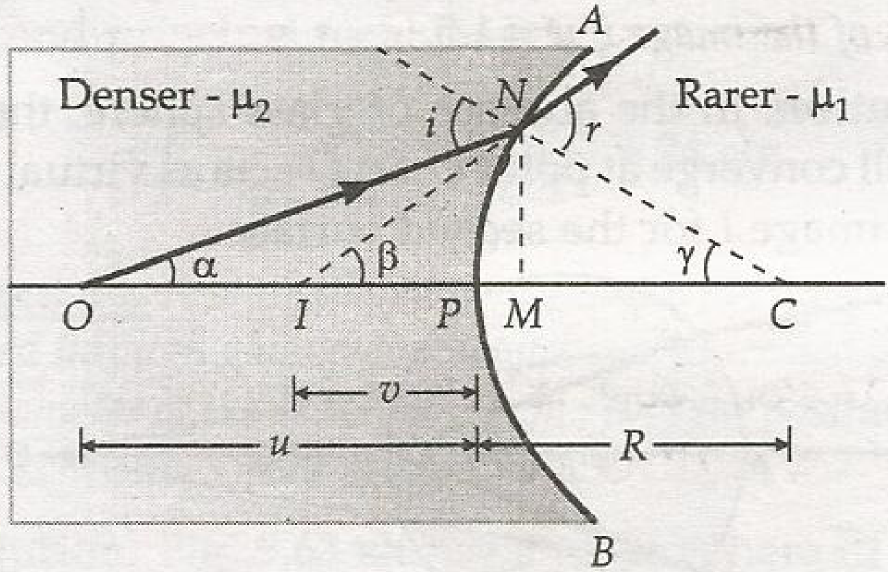


Fig. 9.60 Refraction at a concave surface when the object lies in the denser medium.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan \alpha = \frac{NM}{OM} = \frac{NM}{OP} \qquad [\because \quad M \text{ is close to } P]$$

$$\beta \simeq \tan \beta = \frac{NM}{IM} = \frac{NM}{IP}$$

$$\gamma \simeq \tan \gamma = \frac{NM}{MC} = \frac{NM}{PC}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

As i and r are small angles, so

$$\frac{i}{r} = \frac{\mu_1}{\mu_2} \quad \text{or} \quad \mu_2 i = \mu_1 r$$

or $$\mu_2 [\alpha + \gamma] = \mu_1[\beta + \gamma]$$

or $$\mu_2 \left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_1 \left[\frac{NM}{IP} + \frac{NM}{PC}\right]$$

or $$\mu_2 \left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_1 \left[\frac{1}{IP} + \frac{1}{PC}\right]$$

or $$-\frac{\mu_1}{IP} + \frac{\mu_2}{OP} = \frac{\mu_1 - \mu_2}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP = -u$

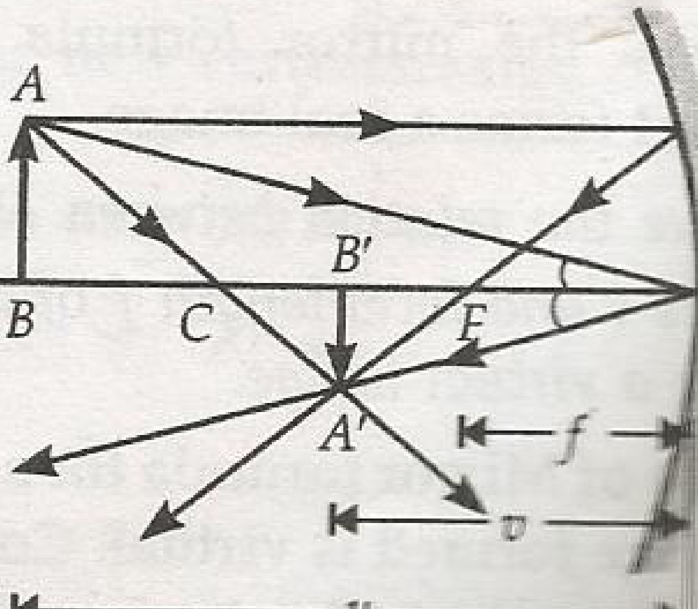
Image distance, $IP = -v$

Radius of curvature, $PC = +R$

$\therefore$ $$\frac{-\mu_1}{-v} + \frac{\mu_2}{-u} = \frac{\mu_1 - \mu_2}{R}$$

or $$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

Derivation of mirror formula for a concave mirror when it forms a real image. Consider an object AB placed on the principal axis beyond the centre of curvature C of a concave mirror of small aperture, as



shown in Fig. 9.13. A ray *AM* from the object travels parallel to the principal axis and after reflection from the mirror it passes through focus *F*. Another ray *AP* is incident on the pole *P* of the mirror and is reflected along *PA'* in accordance with the laws of reflection so that $\angle APB = \angle B'PA'$. The two reflected rays meet at point *A'*. Thus *A'* is the real image of *A*. The image of any point on *AB* will lie on a corresponding point of *A'B'*. Hence *A'B'* is the real image of *AB* formed by reflection from the concave mirror.

Using cartesian sign convention, we find

Object distance,	$BP = -u$
Image distance,	$B'P = -v$
Focal length,	$FP = -f$
Radius of curvature,	$CP = -R = -2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{CB'}{BC} = \frac{CP - B'P}{BP - CP} = \frac{-R + v}{-u + R} \qquad ...(1)$$

As $\angle A'PB' = \angle APB$, therefore,

$\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{B'P}{BP} = \frac{-v}{-u} = \frac{v}{u} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{-R + v}{-u + R} = \frac{v}{u}$$

or

$$-uR + uv = -uv + vR$$

or

$$vR + uR = 2uv$$

Dividing both sides by *uvR*, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But

$$R = 2f$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a concave mirror, when it forms a real image.

Derivation of Mirror formula for a concave mirror when the image formed is virtual. Consider an object *AB* placed on the principal axis of a concave mirror (of small aperture) between its pole *P* and focus *F*. As shown in Fig. 9.14, a virtual and erect image $A'B'$ is formed behind the mirror, after reflection from the concave mirror.

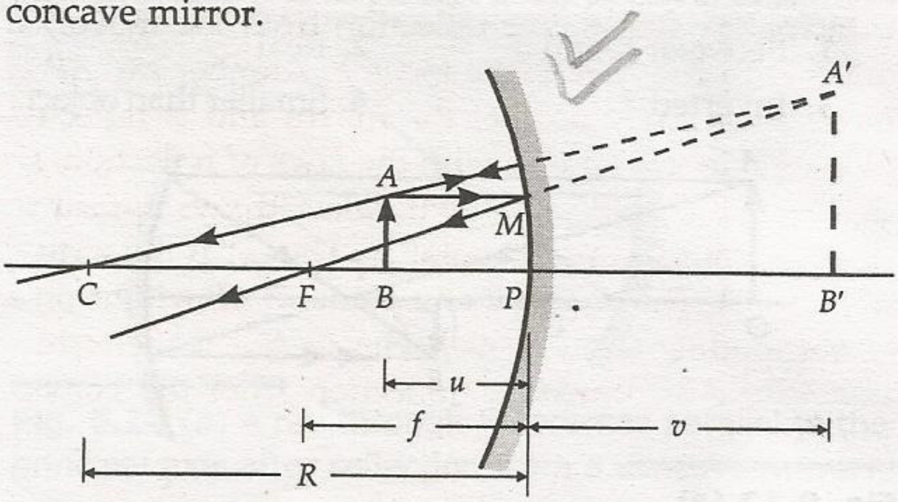


Fig. 9.14 Image formed by a concave mirror when the object lies between F and P.

Using the cartesian sign convention, we find that

Object distance,	$BP = -u$
Image distance,	$PB' = v$
Focal length,	$FP = -f$
Radius of curvature,	$CP = -R = -2f$

Now $\Delta ABC \sim \Delta A'B'C$, therefore

$$\frac{AB}{A'B'} = \frac{CB}{CB'} = \frac{CP - BP}{CP + PB'} = \frac{-2f + u}{-2f + v} \quad ...(1)$$

Also $\Delta MPF \sim \Delta A'B'F$, therefore,

$$\frac{MP}{A'B'} = \frac{FP}{FB'} = \frac{FP}{FP + PB'}$$

or
$$\frac{AB}{A'B'} = \frac{-f}{-f + v} \quad ...(2)$$

From equations (1) and (2), we get

$$\frac{-2f + u}{-2f + v} = \frac{-f}{-f + v}$$

or $2f^2 - fu - 2fv + uv = 2f^2 - fv$

or
$$-fv - fu + uv = 0$$

or
$$uv = fv + fu$$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This proves the mirror formula for a concave mirror when it forms a virtual image.

10. *Establish the relationship between object distance, image distance and radius of curvature for a convex mirror.*

Derivation of mirror formula for a convex mirror. Consider an object *AB* placed on the principal axis of a convex mirror of small aperture, as shown in Fig. 9.15. A ray AM from the object travels parallel to the principal axis and after reflection from the mirror, it appears to come from the focus F. Another ray AP is incident on the pole P of the mirror and is reflected along PQ in accordance with the laws of reflection, so that $\angle APB = \angle BPQ$. The two reflected rays appear to diverge from a common point A'. Thus A' is the virtual image of A. The image of any point on AB will lie on a corresponding point of $A'B'$. Hence $A'B'$ is the virtual image of AB formed by reflection from the convex mirror.

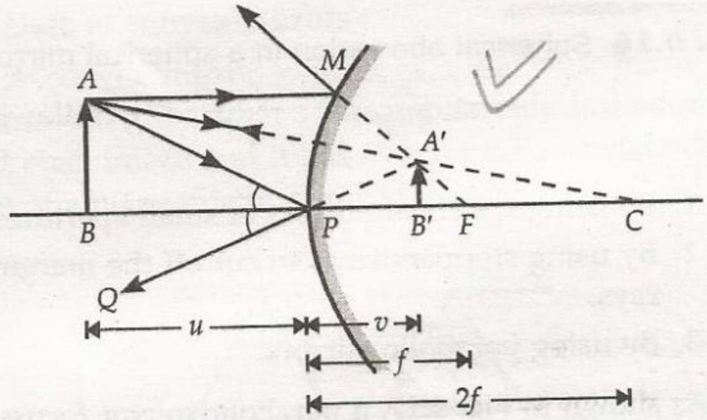


Fig. 9.15 To derive mirror formula for a convex mirror.

Using cartesian sign convention, we find

Object distance,	$BP = -u$
Image distance,	$PB' = +v$
Focal length,	$FP = +f$
Radius of curvature,	$PC = +R = +2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'C}{BC} = \frac{PC - PB'}{BP + PC} = \frac{R - v}{-u + R} \qquad ...(1)$$

As $\angle A'PB' = \angle BPQ = \angle APB$,

Therefore, $\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{PB'}{BP} = \frac{v}{-u} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

or $$-uR + uv = -uv + vR$$

or $$vR + uR = 2uv$$

Dividing both sides by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But $$R = 2f$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a convex mirror.

11. *Define magnification. Write the expressions for magnification for (i) a concave mirror and (ii) a convex mirror. Express m in terms of u, v and f.*

Linear magnification. *The ratio of the height of the image to that of the object is called* ***linear*** *or* ***transverse magnification*** *or just* ***magnification*** and is denoted by *m*.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1}$$

Concave mirror. Fig. 9.13 shows the ray diagram for the formation of image $A' B'$ of a finite object AB by a concave mirror.

Now, $\Delta APB \sim \Delta A' PB'$

$$\therefore \quad \frac{A' B'}{AB} = \frac{B' P}{BP}$$

Applying the new cartesian sign convention, we get

$$A' B' = -h_2 \qquad \text{(Downward image height)}$$

$$AB = +h_1 \qquad \text{(Upward object height)}$$

$$B' P = -v \qquad \text{(Image distance on left)}$$

$$BP = -u \qquad \text{(Object distance on left)}$$

$$\therefore \quad \frac{-h_2}{h_1} = \frac{-v}{-u}$$

Magnification,

$$m = \frac{h_2}{h_1} = -\frac{v}{u}.$$

Convex mirror. Fig. 9.15 shows the formation of image $A' B'$ of a finite object AB by a convex mirror.

Now, $\Delta A' B' P \sim \Delta ABP$

$$\therefore \quad \frac{A' B'}{AB} = \frac{PB'}{BP}$$

Applying the new cartesian sign convention, we get

$$A' B' = +h_2, \quad AB = +h_1$$

$$PB' = +v, \quad BP = -u$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{v}{-u}$$

Magnification, $m = \dfrac{h_2}{h_1} = -\dfrac{v}{u}$

Linear Magnification in terms of u and f. The mirror formula is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying both sides by u, we get

$$1+\frac{u}{v}=\frac{u}{f}$$

or $$-\frac{u}{v}=1-\frac{u}{f}=\frac{f-u}{f}$$

$\therefore$ $$m=-\frac{v}{u}=\frac{f}{f-u}$$

Linear magnification in terms of v and f. As

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying both sides by v, we get

$$\frac{v}{u}+1=\frac{v}{f}$$

or $$-\frac{v}{u}=1-\frac{v}{f}=\frac{f-v}{f}$$

$\therefore$ $$m=-\frac{v}{u}=\frac{f-v}{f}$$

9.12 REFRACTION THROUGH A RECTANGULAR GLASS SLAB AND LATERAL SHIFT

21. *Discuss the refraction through a glass-slab and show that emergent ray is parallel to the incident ray but laterally displaced.*

Refraction through a rectangular glass slab. Consider a rectangular glass slab *PQRS*, as shown in Fig. 9.26. A ray *AB* is incident on the face *PQ* at an angle of incidence i_1. On entering the glass slab, it bends towards normal and travels along *BC* at an angle of refraction r_1. The refracted ray *BC* is incident on face *SR* at an angle of incidence i_2. The emergent ray *CD* bends away from the normal at an angle of refraction r_2.

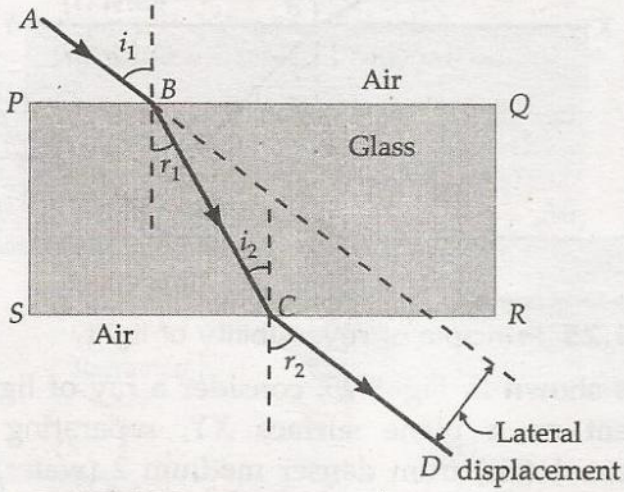


Fig. 9.26

Using Snell's law for refraction at face *PQ*,

$$\frac{\sin i_1}{\sin r_1} = {}^a\mu_g \qquad ...(1)$$

For refraction at face *SR*,

$$\frac{\sin i_2}{\sin r_2} = {}^g\mu_a = \frac{1}{{}^a\mu_g} \qquad ...(2)$$

Multiplying (1) and (2), we get

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

As $PQ \parallel SR$, therefore, $i_2 = r_1$; hence

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = 1$$

or $$\sin i_1 = \sin r_2 \quad \text{or} \quad i_1 = r_2$$

Thus the emergent ray *CD* is parallel to the incident ray *AB*, but it has been laterally displaced with respect to the incident ray. *This shift in the path of light on emerging from a refracting medium with parallel faces is called lateral displacement.*

Hence ***lateral shift*** *is the perpendicular distance between the incident and emergent rays, when light is incident obliquely on a refracting slab with parallel faces.*

22. *A ray of light is incident at angle i on a rectangular slab of thickness t and refractive index μ. Obtain an expression for the lateral displacement of the emergent ray. Can lateral displacement exceed t ?*

Expression for lateral displacement. Fig. 9.27 shows the path of the ray undergoing refraction through the slab *PQRS*. Let *t* be the thickness of the slab and *x*, the lateral displacement of the emergent ray. Then from right Δ *BEC*, we have

$$\frac{x}{BC} = \sin(i-r) \quad \text{or} \quad x = BC \sin(i-r)$$

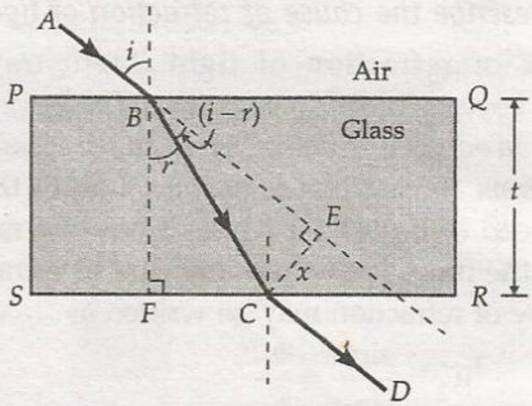


Fig. 9.27 Calculation of lateral displacement.

From right Δ *BFC*, we have

$$\frac{BF}{BC} = \cos r \quad \text{or} \quad BC = \frac{BF}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad x = \frac{t}{\cos r} \sin(i-r) \qquad \text{...(1)}$$

$$= \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$$

$$= t\left[\sin i - \frac{\cos i \sin r}{\cos r}\right]$$

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \sin r = \frac{\sin i}{\mu}$$

and $$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2 i}{\mu^2}}$$

Hence $$x = t\left[\sin i - \frac{\cos i . \sin i}{\mu\left(1-\frac{\sin^2 i}{\mu^2}\right)^{1/2}}\right]$$

or $$x = t \sin i\left[1-\frac{\cos i}{(\mu^2-\sin^2 i)^{1/2}}\right] \qquad \text{...(2)}$$

Clearly, *x* tends to a maximum value when $i \to 90°$, so that $\sin i \to 1$ and $\cos i \to 0$. Thus

$$x_{max} = t \sin 90° = t$$

i.e., the displacement of the emergent ray cannot exceed the thickness of the glass slab.

combination of lenses is called an equivalent lens.

Equivalent focal length and power of two thin lenses in contact. As shown in Fig. 9.100, let L_1 and L_2 be two thin lenses of focal length f_1 and f_2 respectively, placed coaxially in contact with one another. Let O be a point object on the principal axis of the lens system.

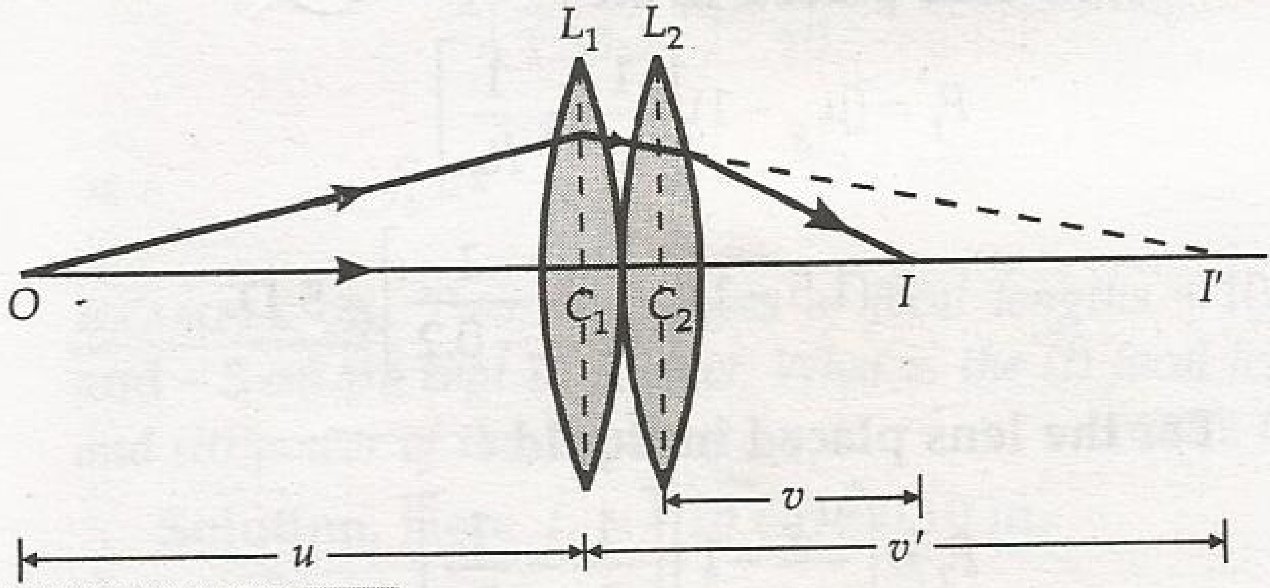


Fig. 9.100 Two thin lenses in contact.

Let $OC_1 = u$. In the absence of second lens L_2, the first lens L_1 will form a real image I' of O at distance $C_1I' = v'$. Using thin lens formula,

$$\frac{1}{f_1} = \frac{1}{v'} - \frac{1}{u} \qquad ...(1)$$

The image I' acts as a virtual object ($u = v'$) for the second lens L_2 which finally forms its real image I at distance v. Thus

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \qquad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad ...(3)$$

For the combination of thin lenses in contact, if f is the equivalent focal length, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{...(4)}$$

From equations (3) and (4), we find that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\therefore$ Equivalent power,

$$P = P_1 + P_2$$

For n thin lenses in contact, we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots \frac{1}{f_n}$$

$\therefore$ Equivalent power,

$$P = P_1 + P_2 + P_3 + \ldots\ldots + P_n$$

Thin lenses separated by a small distance. As shown in Fig. 9.101, consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2, respectively. The two lenses are placed coaxially, distance 'd' apart.

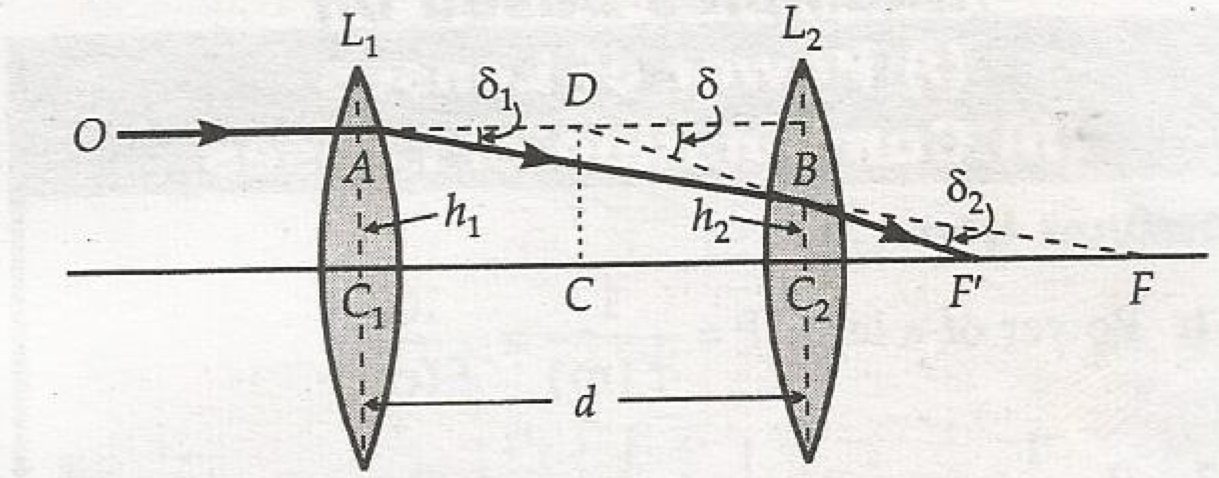


Fig. 9.101 Two thin lenses separated by a small distance

Suppose a ray *OA* traversing parallel to the principal axis is incident on lens L_1. It is refracted along *AF*, *F* being the second principal focus of L_1. The deviation produced by L_1 is

$$\delta_1 \simeq \tan \delta_1 = \frac{h_1}{f_1}.$$

The emergent ray is further refracted by second lens L_2 along BF'. Since the incident ray *OA* is parallel to the principal axis, F' should be second principal focus of the combination. The deviation produced by the second lens L_2 is

$$\delta_2 \simeq \tan \delta_2 = \frac{h_2}{f_2}.$$

The final emergent ray BF', when produced backwards, meets the incident ray at point *D*. Obviously, δ is the final deviation produced. A single thin lens placed at *C* will produce the same deviation as by the combination of two lenses. Thus distance CF' is the second focal length of the combination. If f is the focal length of the combination, then

$$\delta = \frac{h_1}{f}$$

It is obvious from Fig. 9.101, that

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

As $\Delta AC_1F \sim \Delta BC_2F$, therefore, we have

$$\frac{AC_1}{C_1F} = \frac{BC_2}{C_2F} \quad \text{or} \quad \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or

$$h_2 = \frac{f_1 - d}{f_1} \cdot h_1$$

Hence

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{f_1 - d}{f_1 f_2} \cdot h_1$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of powers of the lenses,

$$P = P_1 + P_2 - d \cdot P_1 \cdot P_2$$