Laws of reflection on the basis of Huygens' wave theory. As shown in Fig. 10.4, consider a plane wavefront AB incident on the plane reflecting surface XY, both the wavefront and the reflecting surface being perpendicular to the plane of paper.

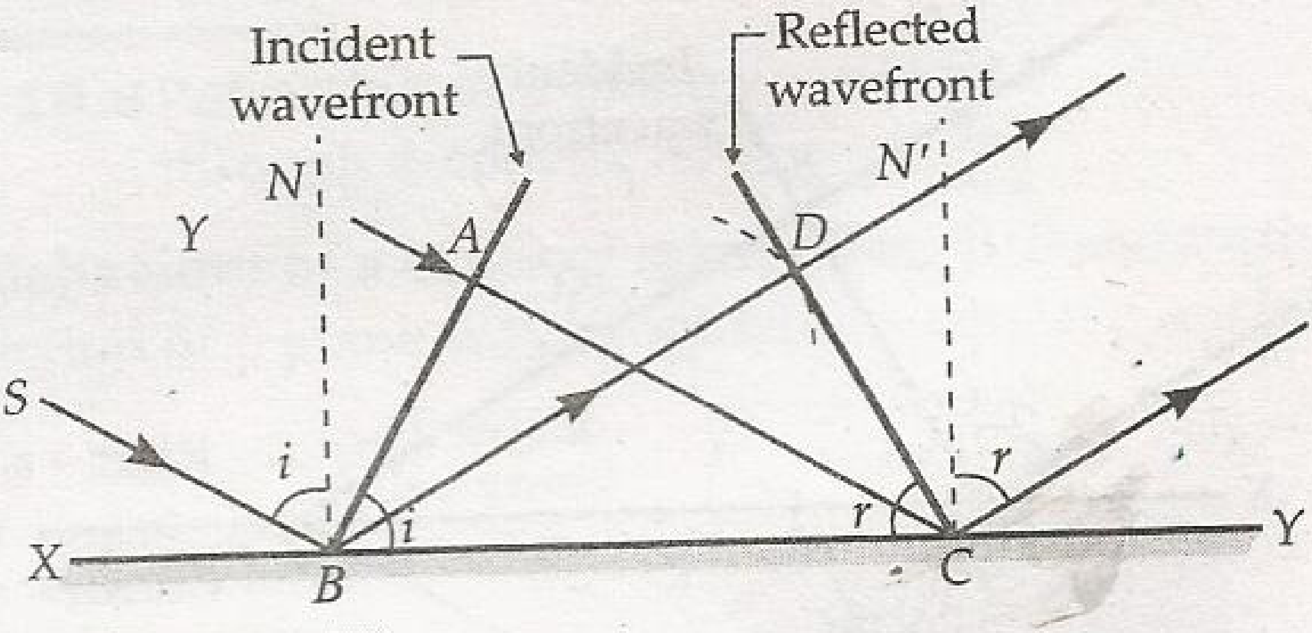

Fig. 10.4 Wavefronts and corresponding rays for reflection from a plane surface.

First the wavefront touches the reflecting surface at B and then at the successive points towards C. In accordance with Huygens' principle, from each point on BC, secondary wavelets start growing with the speed c. During the time the disturbance from A reaches the point C, the secondary wavelets from B must have spread over a hemisphere of radius $BD = AC = ct$, where t is the time taken by the disturbance to travel from A to C. The tangent plane CD drawn from the point C over this hemisphere of radius ct will be the *new reflected wavefront.*

Let angles of incidence and reflection be i and r respectively. In ΔABC and ΔDCB, we have

$$\angle BAC = \angle CDB \quad \text{[Each is } 90°\text{]}$$

$$BC = BC \quad \text{[Common]}$$

$$AC = BD \quad \text{[Each is equal to } ct\text{]}$$

$$\therefore \quad \Delta ABC \cong \Delta DCB$$

Hence $\angle ABC = \angle DCB$

or

$$i = r$$

i.e., the angle of incidence is equal to the angle of reflection. This proves the first law of reflection.

Further, since the incident ray SB, the normal BN and the reflected ray BD are respectively perpendicular to the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD (all of which are perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, *i.e.*, in the same plane. This proves the second law of reflection.

... Huygens' wave theory of light.

Laws of refraction on the basis of Huygens' wave theory. Consider a plane wavefront *AB* incident on a plane surface *XY*, separating two media 1 and 2, as shown in Fig. 10.5. Let v_1 and v_2 be the velocities of light in the two media, with $v_2 < v_1$.

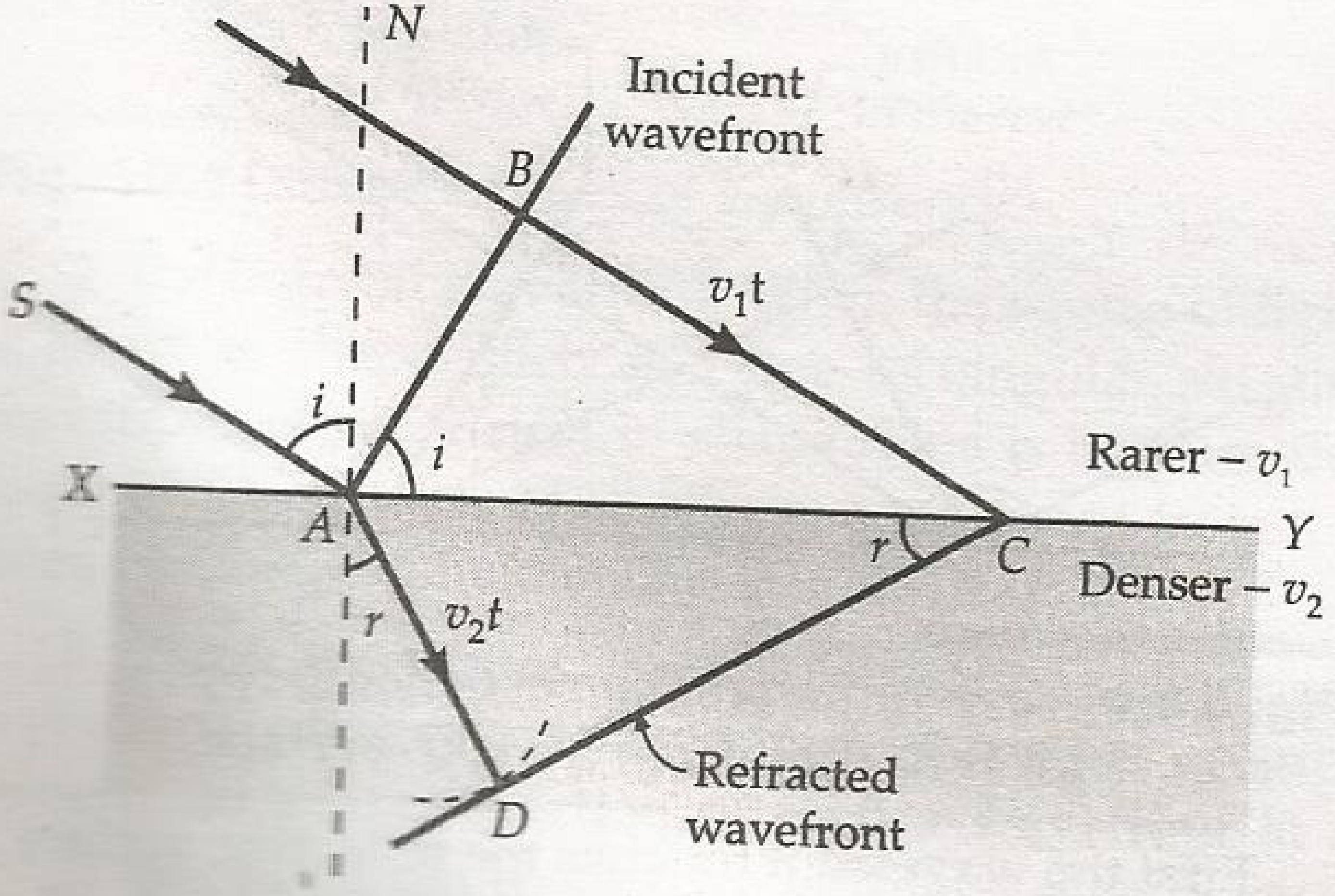

Fig. 10.5 wavefronts and corresponding rays for refraction by a plane surface separating two media.

The wavefront first strikes at point A and then at the successive points towards C. According to Huygens' principle, from each point on AC, the secondary wavelets start growing in the second medium with speed v_2. Let the disturbance take time t to travel from B to C, then $BC = v_1 t$. During the time the disturbance from B reaches the point C, the secondary wavelets from point A must have spread over a hemisphere of *radius* $AD = v_2 t$ *in the second medium.* The tangent plane CD drawn from point C over this hemisphere of radius $v_2 t$ will be the new refracted wavefront.

Let the angles of incidence and refraction be i and r respectively.

From right ΔABC, we have

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔADC, we have

$$\sin \angle DCA = \sin r = \frac{AD}{AC}$$

$$\therefore \qquad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t}$$

or

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2 \qquad \text{(a constant)}$$

This proves ***Snell's law of refraction***. The constant ${}^1\mu_2$ is called the *refractive index* of the second medium with respect to first medium.

Further, since the incident ray SA, the normal AN and the refracted ray AD are respectively perpendicular to the incident wavefront AB, the dividing surface XY and the refracted wavefront CD (all perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, *i.e.*, in the same plane. This proves another law of refraction.

... law of refraction.

of **Refraction at a rarer medium.** Fig. 10.6 shows the refraction of a plane wavefront at a rarer medium *i.e.,*

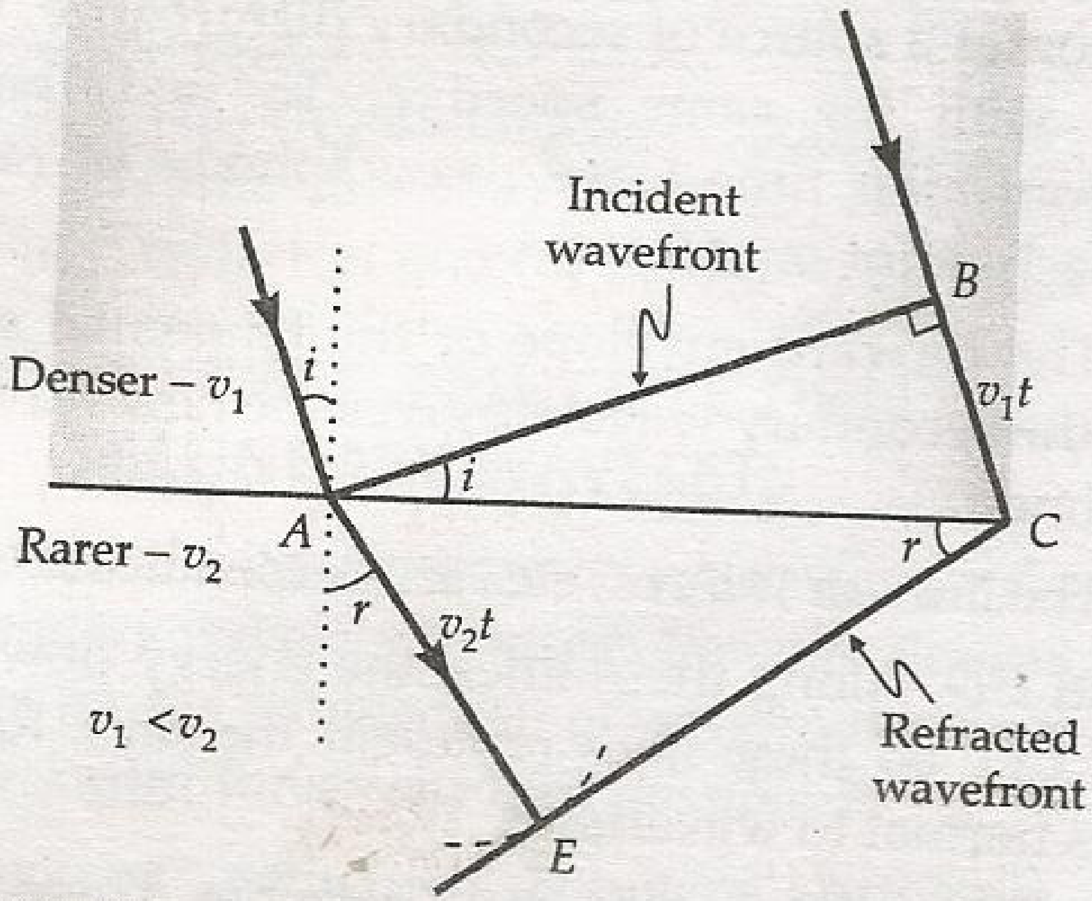

Fig. 10.6 Refraction of a plane wave incident on a rarer medium for which $v_2 > v_1$. The plane wave bends away from the refracting surface.

$v_2 > v_1$. The incident and refracted wavefronts are shown in Fig. 10.6. In this case, the angle of refraction is greater than the angle of incidence. Here also the Snell's law of refraction is valid. That is

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \qquad \text{(a constant)}$$

Behaviour of a prism. Fig. 10.7 shows the refraction of a plane wavefront through a thin prism. Since the speed of light in glass is smaller than that in air, therefore, the lower part *C* of the plane wavefront which travels through the greatest thickness of the glass prism is slowed down the most and the upper part *A*, which travels through the minimum thickness of the glass prism, is slowed down the least. This explains the tilting of a plane wavefront after refraction through a glass prism.

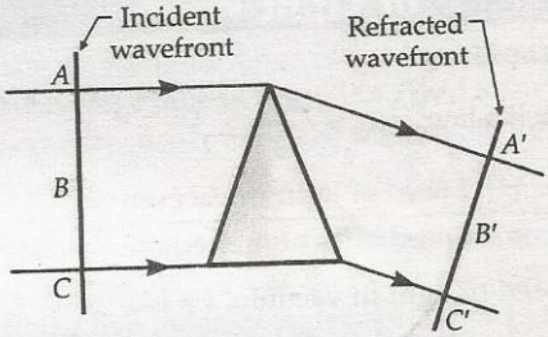

Fig. 10.7 Refraction of a plane wavefront through a prism.

Behaviour of a convex lens. Fig. 10.8 shows the refraction of a plane wavefront through a convex lens. The central part *B* of the plane wavefront travels through the greatest thickness of the lens and is, therefore, slowed down the most. The marginal parts *A* and *C* of the wavefront travel through a minimum thickness of the lens and are, therefore, slowed down the least. So the emerging wavefront is spherical and converges to a focus *F*.

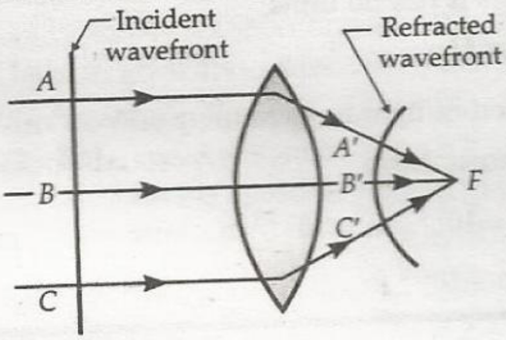

Fig. 10.8 Refraction of a plane wavefront through a convex lens.

Behaviour of a concave mirror. Fig. 10.9 shows the reflection of a plane wavefront from a concave mirror.

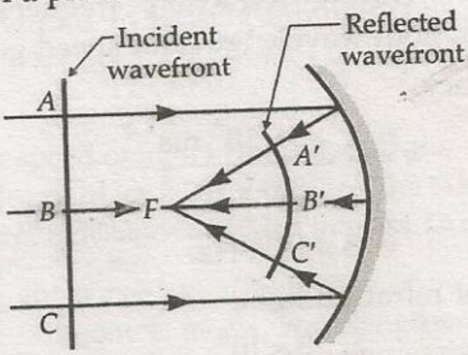

Fig. 10.9 Reflection of a plane wavefront from a concave mirror.

The central part B of the incident wavefront has to travel the greatest distance before getting reflected, compared to the marginal parts A and C. Therefore, the central portion B' of the reflected wavefront is closer to the mirror than the marginal portions A' and C'. Hence the reflected wavefront is spherical and converges to a focus.

10.10 CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

10. *Derive an expression for the intensity at any point on the observation screen in Young's double slit experiment. Hence write the conditions for constructive and destructive interference.*

Expression for intensity at any point in interference pattern. Suppose the displacements of two light waves from two coherent sources S_1 and S_2 at point P on the observation screen at any time t are given by

$$y_1 = a_1 \sin \omega t$$

and $$y_2 = a_2 \sin (\omega t + \phi)$$

where a_1 and a_2 are the amplitudes of the two waves, ϕ is the constant phase difference between the two waves. By the superposition principle, the resultant displacement at point P is

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

or $$y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t$$

Put $\quad a_1 + a_2 \cos\phi = A\cos\theta$ ...(1)

and $\quad a_2 \sin\phi = A\sin\theta$ ...(2)

Then $\quad y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$

or $\quad y = A\sin(\omega t + \theta)$

Thus, the resultant wave is also a harmonic wave of amplitude A and it leads the first harmonic wave by phase angle θ. To determine A, squaring and adding equations (1) and (2), we get

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + a_2^2\sin^2\phi$$

or $\quad A^2 = a_1^2 + a_2^2(\cos^2\phi + \sin^2\phi) + 2a_1a_2\cos\phi$

or $\quad A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$ ...(3)

But intensity of a wave $\propto$ (amplitude)^2

We write $I = kA^2$, $I_1 = ka_1^2$ and $I_2 = ka_2^2$

where k is proportionality constant. The equation (3) can be written as

$$kA^2 = ka_1^2 + ka_2^2 + 2\sqrt{k}a_1\sqrt{k}a_2\cos\phi$$

or $\quad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ ...(4)

This equation gives the total intensity at a point where the phase difference is ϕ. Here I_1 and I_2 are the intensities which the two individual sources produce on their own. The total intensity also contains a third term $2\sqrt{I_1 I_2}\cos\phi$. It is called ***interference term.***

Constructive interference. The resultant intensity at the point P will be maximum when

$$\cos\phi = 1 \quad \text{or} \quad \phi = 0, 2\pi, 4\pi, \ldots$$

Since a phase difference of 2π corresponds to a path difference of λ, therefore, if p is the path difference between the two superposing waves, then

$$\frac{2\pi p}{\lambda} = 0,\ 2\pi,\ 4\pi, \ldots$$

or $\quad p = 0,\ \lambda,\ 2\lambda,\ 3\lambda, \ldots = n\lambda$

Hence the resultant intensity at a point is maximum when the phase difference between the two superposing waves is an even multiple of π or path difference is an integral multiple of wavelength λ. This is the condition of constructive interference.

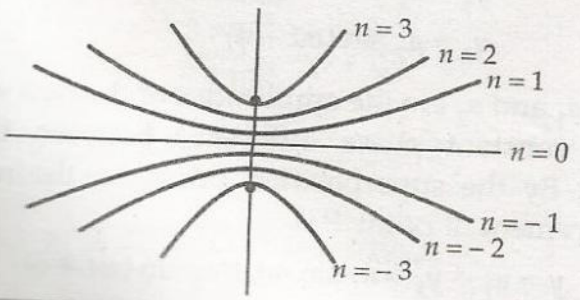

Fig. 10.13 Locus of points for which $S_1P - S_2P$ is equal to $0, \pm\lambda, \pm 2\lambda, \pm 3\lambda$.

Destructive interference. The resultant intensity at the point P will be minimum when

$$\cos\phi = -1 \quad \text{or} \quad \phi = \pi,\ 3\pi,\ 5\pi, \ldots$$

or

$$\frac{2\pi p}{\lambda} = \pi,\ 3\pi,\ 5\pi, \ldots$$

or

$$p = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots = (2n-1)\frac{\lambda}{2}$$

Hence the resultant intensity at a point is minimum when the phase difference between the two superposing waves is an odd multiple of π or the path difference is an odd multiple of λ/2. This is the condition of destructive interference.

10.12 THEORY OF INTERFERENCE FRINGES : FRINGE WIDTH

13. *Deduce an expression for fringe width in Young's double slit experiment. How can the wavelength of monochromatic light be found by this experiment ?*

Expression for fringe width in Young's double slit experiment. As shown in Fig. 10.14, suppose a narrow slit S is illuminated by monochromatic light of wavelength λ. S_1 and S_2 are two narrow slits at equal distance from S. Being derived from the same parent source S, the slits S_1 and S_2 act as two *coherent sources*, separated by a small distance d. Interference fringes are obtained on a screen placed at distance D from the sources S_1 and S_2.

Consider a point P on the screen at distance x from the centre O. The nature of the interference at the point P depends on path difference,

$$p = S_2P - S_1P$$

From right-angled ΔS_2BP and ΔS_1AP,

$$S_2P^2 - S_1P^2 = [S_2B^2 + PB^2] - [S_1A^2 + PA^2]$$

$$= \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

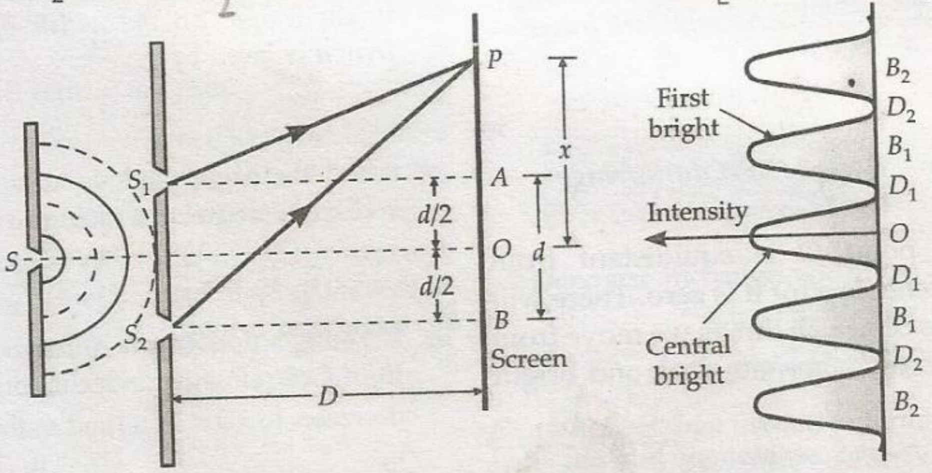

Fig. 10.14 Position of bright and dark fringes in Young's double slit experiment.

or $$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

or $$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

In practice, the point P lies very close to O, therefore $S_1P \simeq S_2P \simeq D$. Hence

$$p = S_2P - S_1P = \frac{2\ xd}{2\ D}$$

or $$p = \frac{xd}{D}$$

Positions of bright fringes. For constructive interference,

$$p = \frac{xd}{D} = n\lambda$$

or $$x = \frac{nD\lambda}{d} \quad \text{where } n = 0, 1, 2, 3, \ldots..$$

Clearly, the positions of various bright fringes are as follows :

For $n = 0$, $x_0 = 0$ *Central bright fringe*

For $n = 1$, $x_1 = \frac{D\lambda}{d}$ *First bright fringe*

For $n = 2$, $x_2 = \frac{2\ D\lambda}{d}$ *Second bright fringe*

....

For $n = n$, $x_n = \frac{nD\lambda}{d}$ **nth bright fringe**

Positions of dark fringes. For destructive interference,

$$p = \frac{xd}{D} = (2n-1)\frac{\lambda}{2}$$

or $$x = (2n-1)\frac{D\lambda}{2d} \quad \text{where } n = 1, 2, 3, \ldots...$$

Clearly, the positions of various dark fringes are as follows :

For $n = 1$, $x'_1 = \frac{1}{2}\frac{D\lambda}{d}$ *First dark fringe*

For $n = 2$, $x'_2 = \frac{3}{2}\frac{D\lambda}{d}$ *Second dark fringe*

....

For $n = n$, $x'_n = (2n-1)\frac{D\lambda}{2d}$ ***n*th dark fringe**

Since the central point O is equidistant from S_1 and S_2, the path difference p for it is zero. There will be a bright fringe at the centre O. But as we move from O upwards or downwards, alternate dark and bright fringes are formed.

Fringe width. *It is the separation between two successive bright or dark fringes,* Width of a dark fringe = Separation between two consecutive bright fringes

$$= x_n - x_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)\,D\lambda}{d} = \frac{D\lambda}{d}$$

Width of a bright fringe

= Separation between two consecutive dark fringes

$$= x'_n - x'_{n-1}$$

$$= (2n-1)\frac{D\lambda}{2d} - [2\,(n-1)-1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

Clearly, both the bright and dark fringes are of equal width.

Hence the expression for the fringe width in Young's double slit experiment can be written as

$$\beta = \frac{D\lambda}{d}$$

As β is independent of n (the order of fringe), therefore, all the fringes are of equal width. In the case of light, λ is extremely small, D should be much larger than d, so that the fringe width β may be appreciable and hence observable.

Measurement of wavelength. Young's double slit experiment can be used to determine the wavelength of a monochromatic light. The interference pattern is obtained in the focal plane of a micrometer eyepiece and with its help fringe width β is measured. By measuring the distance d between the two coherent sources and their distance D from the eyepiece, the value of wavelength λ can be calculated as

$$\lambda = \frac{\beta d}{D}$$

10.16 COMPARISON OF INTENSITIES AT MAXIMA AND MINIMA

17. *Derive an expression for the ratio of intensities at maxima and minima in an interference pattern.*

Comparison of intensities at maxima and minima. Let a_1 and a_2 be the amplitudes and I_1 and I_2 be the intensities of light waves from two different sources.

As Intensity $\propto$ Amplitude^2 $\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$.

Amplitude at a maximum in interference pattern

$$= a_1 + a_2$$

Amplitude at a minimum in interference pattern

$$= a_1 - a_2$$

Therefore, the ratio of intensities at maxima and minima is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(\frac{a_1}{a_2} + 1\right)^2}{\left(\frac{a_1}{a_2} - 1\right)^2} \quad \text{or} \quad \frac{I_{max}}{I_{min}} = \left[\frac{r+1}{r-1}\right]^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = *amplitude ratio* of the two waves.

10.22 DIFFRACTION AT A SINGLE SLIT

24. *Explain the phenomenon of diffraction of light at a single slit to show the formation of diffraction fringes. Show graphically the variation of intensity with angle in this diffraction pattern. Why secondary maxima are less intense than the central maximum ?*

Diffraction at a single slit. As shown in Fig. 10.21, a source *S* of monochromatic light is placed at the focus of a convex lens L_1. A parallel beam of light and hence a plane wavefront *WW* gets incident on a narrow rectangular slit *AB* of width *d*.

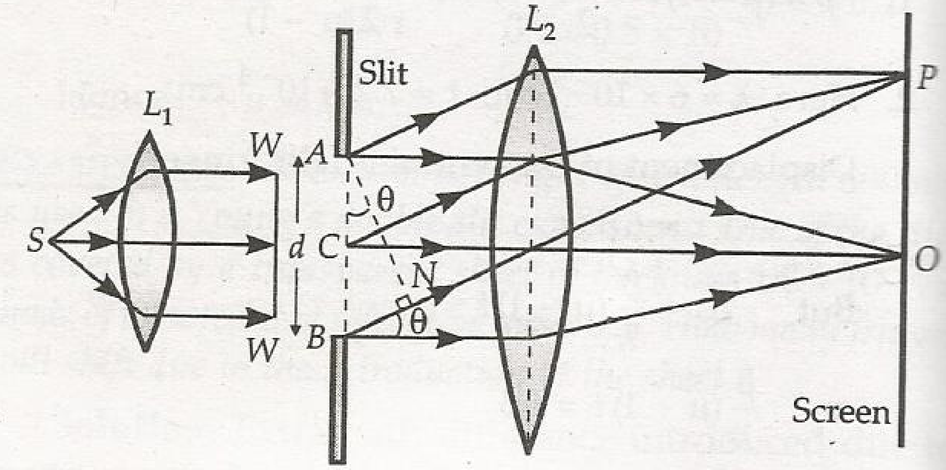

Fig. 10.21 Diffraction at a single slit.

The incident wavefront disturbs all parts of the slit *AB* simultaneously. According to Huygens' theory, all parts of the slit *AB* will become source of secondary wavelets, which all start in the same phase. These wavelets spread out in all directions, thus causing diffraction of light after it emerges through slit *AB*. Suppose the diffraction pattern is focussed by a convex lens L_2 on a screen placed in its focal plane.

Central maximum. All the secondary wavelets going straight across the slit *AB* are focussed at the central point *O* of the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point *O* in the same phase, they add constructively to produce a *central bright fringe*. For detailed explanation of diffraction fringes, see *For Your Knowledge* box on page 10.32.

Calculation of path difference. Suppose the secondary wavelets diffracted at an angle θ are focussed at point *P*. The secondary wavelets start from different parts of the slit in same phase but they reach the point P in different phases. Draw perpendicular AN from A on to the ray from B. Then the path difference between the wavelets from A and B will be

$$p = BP - AP = BN = AB \sin\theta = d \sin\theta.$$

Positions of minima. Let the point P be so located on the screen that the path difference, $p = \lambda$ and angle $\theta = \theta_1$. Then from the above equation, we get

$$d \sin\theta_1 = \lambda$$

We can divide the slit AB into two halves AC and CB. Then the path difference between the wavelets from A and C will be $\frac{\lambda}{2}$. Similarly, corresponding to every point in the upper half AC, there is a point in the lower half CB for which the path difference is $\frac{\lambda}{2}$. Hence the wavelets from the two halves reach the point P always in opposite phases. They interfere destructively so as to produce a minimum.

Thus the condition for ***first dark fringe*** is

$$d \sin\theta_1 = \lambda$$

Similarly, the condition for ***second dark fringe*** will be

$$d \sin\theta_2 = 2\lambda$$

Hence the condition for ***nth dark fringe*** can be written as

$$d \sin\theta_n = n\lambda, \quad n = 1, 2, 3, \ldots$$

The directions of various minima are given by

$$\theta_n \approx \sin\theta_n = n\frac{\lambda}{d}$$

[As $\lambda << d$, so $\sin\theta_n \approx \theta_n$]

Positions of secondary maxima. Suppose the point P is so located that $p = \frac{3\lambda}{2}$

When $\theta = \theta_1'$, then $\quad d \sin\theta_1' = \frac{3}{2}\lambda$

We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\frac{\lambda}{2}$. The wavelets from these points will interfere destructively. However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum.

Thus the condition for the ***first secondary maximum*** is

$$d \sin\theta_1' = \frac{3}{2}\lambda$$

Similarly, the condition for the ***second secondary maximum*** is

$$d \sin\theta_2' = \frac{5}{2}\lambda$$

Hence the condition for *nth secondary* maximum can be written as

$$d \sin \theta'_n = (2n+1)\frac{\lambda}{2}, n = 1, 2, 3, ...$$

The directions of secondary maxima are given by

$$\theta'_n \simeq \sin \theta'_n = (2n+1)\frac{\lambda}{2d}$$

The intensity of secondary maxima decreases as n increases.

Intensity distribution curve. If we plot a graph between the intensities of maxima and minima against the diffraction angle θ, we get a graph of the type shown in Fig. 10.22. It has a broad central maximum in the direction ($\theta = 0°$) of incident light. On either side of the central maximum, it has secondary maxima of decreasing intensity at positions,

$$\theta = \pm(2n+1)\frac{\lambda}{2d}$$

and minima at positions,

$$\theta = \pm n\frac{\lambda}{d} \qquad (n \neq 0).$$

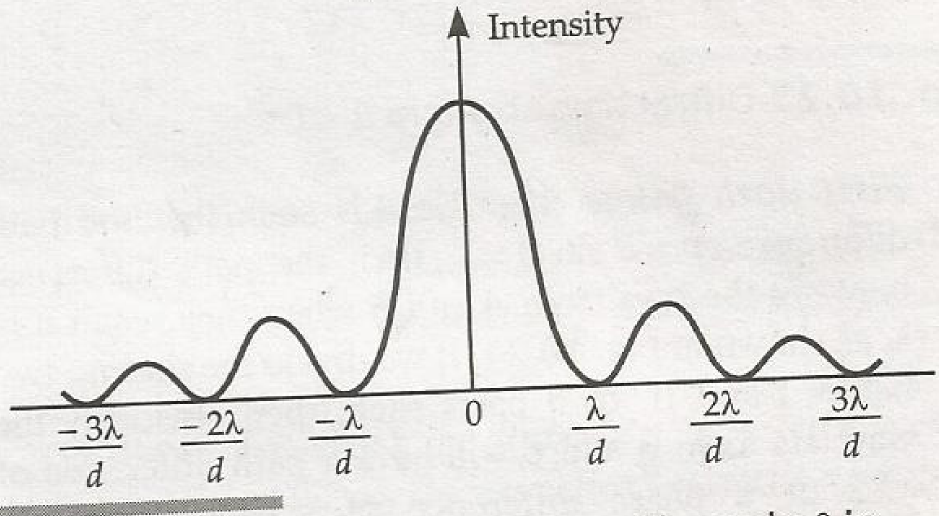

Fig. 10.22 Variation of intensity with angle θ in single slit diffraction.

The intensities of secondary maxima relating to the intensity of central maximum are in ratio,

$$1 : \frac{1}{21} : \frac{1}{61} : \frac{1}{121} .....$$

Thus the intensity of the first secondary maximum is just 4% of that of the central maximum.

25. *Deduce expressions for (i) angular width of central maximum (ii) linear width of central maximum and (iii) linear width of a secondary maximum.*

Angular width of central maximum. *The angular width of the central maximum is the angular separation between the directions of the first minima on the two sides of the central maximum,* as shown in Fig. 10.26.

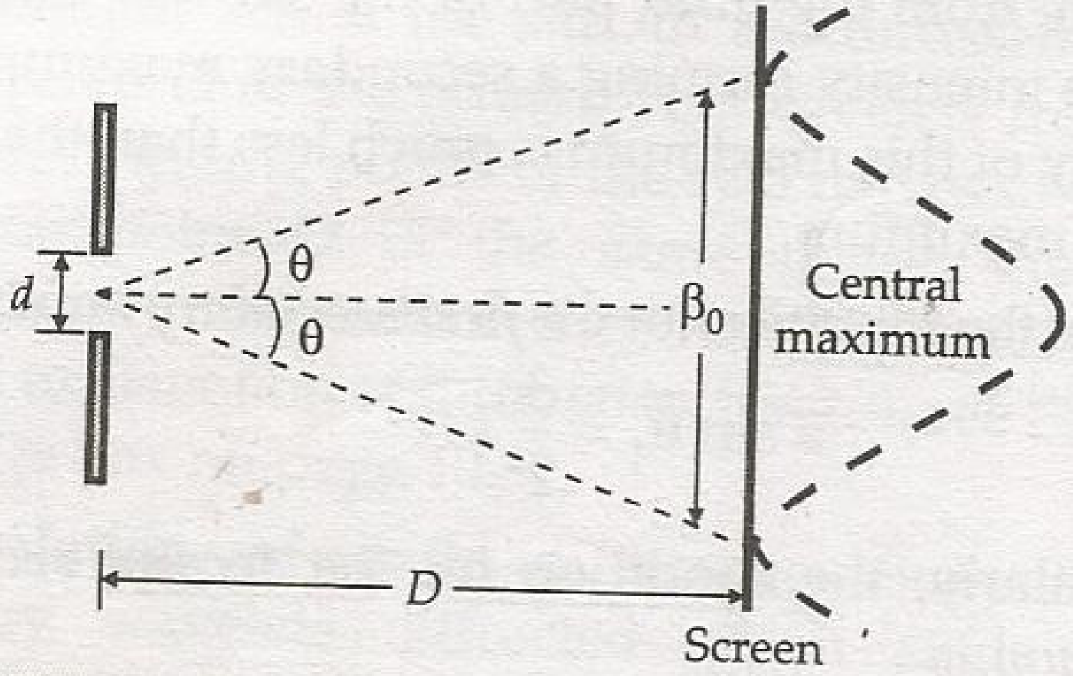

Fig. 10.26 Angular and linear widths of central maximum.

The directions of first minima on either side of central maximum are given by

$$\theta = \frac{\lambda}{d}$$

This angle is called ***half angular width of central maximum.***

$\therefore$ Angular width of central maximum $= 2\theta = \dfrac{2\lambda}{d}$

Linear width of central maximum. If D is the distance of the screen from the single slit, then the linear width of central maximum will be

$$\beta_0 = D \times 2\theta = \frac{2D\lambda}{d}$$

$$\left[2\theta\,(\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{\beta_0}{D}\right]$$

Linear width of a secondary maximum. *The angular width of nth secondary maximum is the angular separation between the directions of nth and (n+1)th minima.*

Direction of nth minimum, $\theta_n = n\,\dfrac{\lambda}{d}$

Direction of $(n+1)$ th minimum,

$$\theta_{n+1} = (n+1)\frac{\lambda}{d}$$

$\therefore$ Angular width of nth secondary maximum

$$= \theta_{n+1} - \theta_n = (n+1)\frac{\lambda}{d} - n\frac{\lambda}{d} = \frac{\lambda}{d}$$

Hence the linear width of nth secondary maximum

$$= \text{Angular width} \times D$$

or

$$\beta = \frac{D\lambda}{d}$$

Clearly, $\beta_0 = 2\beta$

Thus the central maximum of a diffraction pattern is twice as wide as any secondary maximum.

Clearly, width of a secondary maximum

$$\propto \frac{1}{\text{slit width}}$$

As the slit width is increased, the secondary maxima get narrower. If the slit is sufficiently wide, the secondary maxima disappear and only the central maximum is obtained which is the sharp image of the slit. Thus a distinct diffraction pattern is possible only if the slit is very narrow.

distance and size of Fresnel's zone.

Ray optics as a limiting case of wave optics : Fresnel's distance and Fresnel's zone. A parallel beam of light of wavelength λ on passing through an aperture of size d gets diffracted into a beam of angular width,

$$\theta = \frac{\lambda}{d}$$

If a screen is placed at distance D, this beam spreads over a linear width,

$$x = \frac{D\lambda}{d}$$

If the diffraction spread x is small, the concept of ray optics will be valid.

If we have an aperture of size $d = 10$ mm and use light of wavelength $\lambda = 6 \times 10^{-7}$ m, then the beam after travelling a distance of 3 m will get diffracted through a width

$$x = \frac{D\lambda}{d} = \frac{3 \times 6 \times 10^{-7}}{10 \times 10^{-3}}$$

$$= 18 \times 10^{-5} \text{ m} = 0.18 \text{ mm}$$

This diffraction spreading is not quite large. Thus ray optics is valid in many common situations. It is useful here to define what is called Fresnel's distance, D_F.

The distance at which the diffraction spread of a beam is equal to the size of the aperture is called ***Fresnel's distance.***

i.e., when $x = d$, $D = D_F$

$$\therefore \quad d = \frac{D_F \lambda}{d} \quad \text{or} \quad D_F = \frac{d^2}{\lambda}$$

If $D < D_F$, then there will not be too much broadening by diffraction *i.e.,* the light will travel along straight lines and the concepts of ray optics will be valid.

$$\text{As } D < D_F \quad \text{or} \quad D < \frac{d^2}{\lambda} \quad \text{or} \quad d > \sqrt{\lambda D}$$

For a given value of D, the quantity $\sqrt{\lambda D}$ is called the ***size of Fresnel zone*** and is denoted by d_F.

i.e.
$$d_F = \sqrt{\lambda D}$$

Hence the concepts of ray optics can be conveniently used without introducing any appreciable error if the size of the aperture is greater than the size of the Fresnel zone,

i.e.,
$$d > d_F.$$

immersion objective.

Resolving power of a microscope. *The resolving power of a microscope is defined as reciprocal of the smallest distance between two point objects at which they can be just resolved when seen through the microscope.*

The smallest distance between two point objects at which they can be just resolved by the microscope, or the ***limit of resolution,*** is given by

$$d = \frac{\lambda}{2\mu \sin\theta}$$

$$\therefore \text{ Resolving power of a microscope} = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

Here,

λ = the wavelength of light used,

θ = half the angle of cone of light from each point object or the angle subtended by each point object on the radius of the objective[Fig. 10.30].

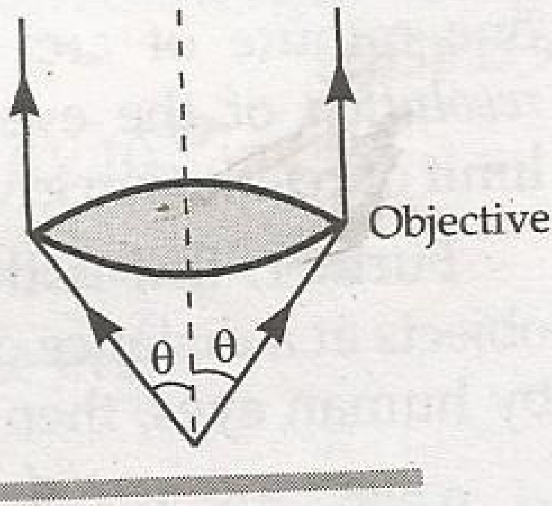

Fig. 10.30

μ = the refractive index of the medium between the point object and the objective of the microscope.

The factor $\mu \sin\theta$ is called the ***numerical aperture*** (for eye, $\mu \sin\theta = 0.004$).

The smaller the limit of resolution *'d'*, the greater will be the resolving power. The resolving power of a microscope increases when an oil of high refractive index (μ) is used between the object and the objective (called the ***oil immersion objective***) of the microscope.

Resolving power of a telescope. *The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images can be just resolved by it.*

The *smallest linear angular separation between two distant objects whose images can be just resolved by the telescope,* or the ***limit of resolution,*** is given by

$$d\theta = \frac{1.22\,\lambda}{D}$$

$$\therefore \text{Resolving power of a telescope} = \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

Here

λ = the wavelength of light,

D = the diameter of the telescope objective, and

$d\theta$ = the angle subtended by the two distant objects at the objective.

Thus larger the aperture of the objective and smaller the wavelength of light used, the greater will be the resolving power of the telescope.

Doppler effect. In class XI, you have already learnt Doppler effect for sound waves. When a source of sound travels towards an observer, the apparent frequency is higher than the frequency actually emitted by the source. When the source moves away, the apparent frequency is lower than the actual frequency. Doppler effect is a basic property of all waves and so occurs in case of light also.

Whenever there is a relative motion between source of light and observer, the frequency of light received by the observer is different from the frequency actually emitted by the source. This phenomenon of the apparent change in the frequency of light is called ***Doppler effect*** *for light.*

Expression for the apparent frequency of light. Suppose a source of light emits waves of frequency ν and wavelength λ. If c is the speed of light, then

$$\lambda = \frac{c}{\nu}$$

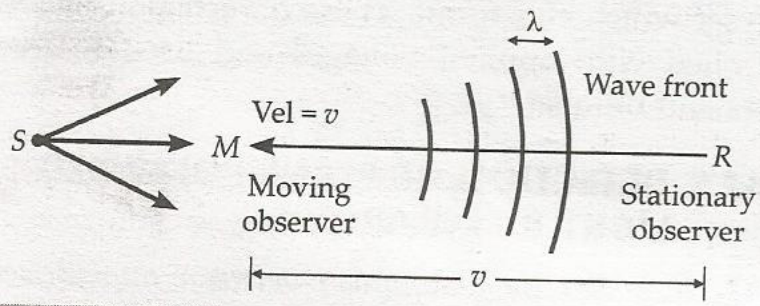

Fig. 10.50

Suppose an observer moves towards the source with velocity v. In one second, the source and observer come closer by a distance v.

$\therefore$ Apparent frequency

= No. of light waves emitted per second by the source + No. of light waves contained in distance v

or $$\nu' = \nu + \frac{v}{\lambda} = \nu + \frac{v}{c/\nu} = \nu + \nu \cdot \frac{v}{c}$$

or $$\nu' = \nu\left(1 + \frac{v}{c}\right) \qquad ...(1)$$

Clearly, $\nu' > \nu$ *i.e.*., the apparent frequency increases when source and observer approach each other.

When source and observer move away from each other, the apparent frequency can be obtained by replacing v by $-v$ in the above equation. Then

$$\nu' = \nu\left(1 - \frac{v}{c}\right) \qquad ...(2)$$

Clearly, $\nu' < \nu$ *i.e.*, the apparent frequency decreases when source and observer move away from each other.

Blue shift and red shift. Equations (1) and (2) can be combined together as

$$\nu' = \nu\left(1 \pm \frac{v}{c}\right)$$

or
$$\nu' - \nu = \pm \frac{v}{c}.\nu$$

The frequency change $\Delta\nu = \nu' - \nu$ is called *Doppler shift*. Putting $\nu = \frac{c}{\lambda}$ and $\nu' = \frac{c}{\lambda'}$, we get

$$\frac{c}{\lambda'} - \frac{c}{\lambda} = \pm \frac{v}{c}.\frac{c}{\lambda}$$

or
$$\frac{\lambda - \lambda'}{\lambda'} = \pm \frac{v}{c}$$

But
$$\frac{\lambda - \lambda'}{\lambda'} \simeq \frac{\lambda - \lambda'}{\lambda}$$

$\therefore$
$$\frac{\lambda - \lambda'}{\lambda} = \pm \frac{v}{c}$$

or
$$\lambda - \lambda' = \pm \frac{v}{c}\lambda$$

(*i*) When source and observer approach each other, positive sign is taken. Then $\lambda - \lambda'$ is positive or $\lambda' < \lambda$, *i.e.*, the wavelengths in the middle part of the visible spectrum shift towards the blue region. This is called **blue shift.**

(*ii*) When source and observer move away from each other, negative sign is taken. Then $\lambda - \lambda'$ is negative or $\lambda' > \lambda$, *i.e.*, the wavelengths in the middle part of the visible spectrum shift towards the red region. This is called **red shift.**