3U Preliminary and HSC Mathematics
Sample Resources

Contact Us
admin@prestigetuition.com.au
(02) 8798 8977

1. If $\tan\frac{x}{2} = t$, express each of the following in terms of t:
   a. $sinx + \cos x$
   b. $\sin x \; - \cos x$
   c. $3\sin x + 4\cos x$
   d. $\frac{cotx - \tan x}{\cot x + \tan x}$
   e. $1 - \frac{1}{2}\sin x \tan\frac{x}{2}$
   f. $\frac{\tan x - \tan\frac{x}{2}}{\cot\frac{x}{2} + \tan x}$
   g. $\frac{1 + \sin x + \cos x}{1 + \sin x - \cos x}$
   h. $\frac{\sin x + \sin\frac{x}{2}}{1 + \cos x + \cos\frac{x}{2}}$
   i. $\frac{1 - \cos x}{\sin x}$
2. If $\tan\frac{x}{2} = t$, solve for t the equation $\tan x = c$
3. If $\tan\frac{x}{2} = t$, solve for t the equation $12\,tanx = 5$

Challenge Question:

Solve (5 marks):

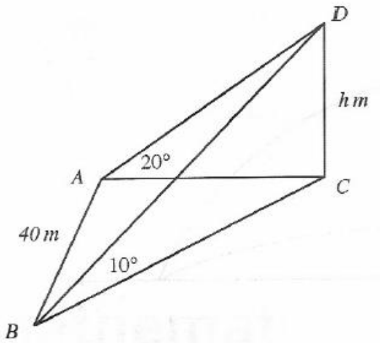
$176\cos x + 64\sin x = 75\cos 2x + 80\sin 2x + 101.$

Show that $\frac{1+\cos x}{\sin x} = \frac{1}{t}$ for $t = \tan\frac{x}{2}$. **2**

1. A ship, sailing in a direction 065°T, observes two lighthouses in a line due north. After travelling 4kn, one of the lighthouses bears 285°T and the other 315°T. Show that the distance between the lighthouses is:
$$4\left(\frac{\sin 70^\circ}{\sin 45^\circ} - \frac{\sin 40^\circ}{\sin 75^\circ}\right)$$
2. A and B are the feet of two vertical towers each 30m high. B lies due north of A. From a point P, due east of A and in the same horizontal plane, the angles of the elevation of the tops of the towers A and B are 45° and 36°52' respectively. Calculate the distance
    a. From P to the foot of each tower
    b. Between the two towers
3. From a point P, an observer finds that the angle of elevation of the top of a vertical tower is α°. After walking *x* metres horizontally towards the foot of the tower, the observer finds that the angle of elevation is β°. If the height of the tower is *h* metres, show that
$$x = h(\cot\alpha^\circ - \cot\beta^\circ)$$
4. A flagpole 5m high stands on top of a vertical tower. From a point on the ground, the angles of elevation of the top and bottom are 68° and 62° respectively. Show that the height of the tower, *h*m, is given by
$$h = \frac{5\tan 62^\circ}{\tan 68^\circ - \tan 62^\circ}$$
5. Two buildings of equal height are 40m apart and at a point on the horizontal line joining their feet the angles of elevation of the tops of the buildings are 47° and 28°. Show that the height of the buildings is given by:
$$h = \frac{40\tan 47^\circ \tan 28^\circ}{\tan 47^\circ + \tan 28^\circ}$$
6. The angle of elevation of the top of a building from a point P due east of it is 40° and from a point Q due south of P, the angle of elevation is 30°. If the distance from P to Q is 20 metres, find the height of the building.
7. From a point A, two points B and C are in line in a direction 049°T. From a point D 100m from A, in a direction 139°T, B is in a direction 352°T and C is in a direction 022°T. Calculate the distance between B & C.
8. From a point A, the angle of elevation of the top of a tower due north of it is 20°. From B, due east of the tower, the angle of elevation is 18°. A and B are 100m apart. Show that the height of the tower is given by:
$$h = \frac{100}{\sqrt{(\tan^2 72^\circ + \tan^2 70^\circ)}}$$

Trial Schools HSC Papers:

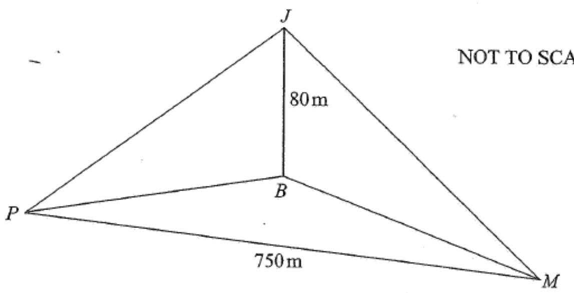
(a)



A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due West of C and B is a point on the ground 40 metres due South of A. From A and B the angles of elevation of the top D of the flagpole are 20° and 10° respectively. Find the height of the flagpole correct to the nearest metre. **4**

(d) Janus, *J*, is on top of an 80 metre cliff, watching the Sydney to Hobart yacht race. **3**

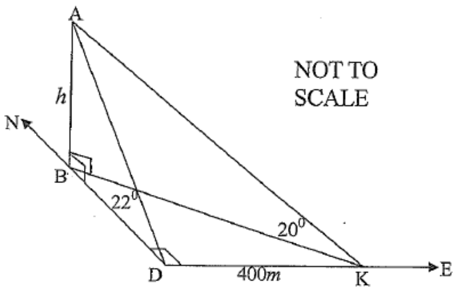
From the base of the cliff, *B*, directly below Janus, *Poseidon, P,* is on a bearing of 202° and *Majorca, M,* is on a bearing of 140°. *Majorca* is 750 m from *Poseidon* on a bearing of 110°.



Copy or trace this diagram into your writing booklet.

Find the angle of depression of *Poseidon, P,* from Janus, *J*.

(d) Donna is standing at D and observes the angle of elevation of the tip of a flagpole A, on top of a building to be 22^0. Her friend Kate, who is standing at K, 400 *metres* due east of Donna, finds the angle of elevation of the tip of the flagpole to be 20^0. The building is due north of Donna and B is the base of the building. The points B, D and K are all on level ground.

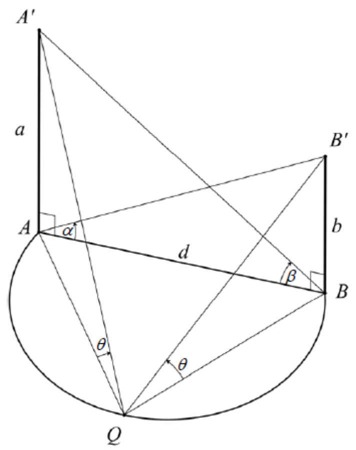


(i) Show that the height (h) of the flagpole above the ground is given by: **3**

$$h = \frac{400}{\sqrt{\cot^2 20^0 - \cot^2 22^0}}$$

(ii) Find the value of h, correct to 3 significant figures. **1**

(b) AQB is a semi-circle in the horizontal plane with diameter AB of length d metres. There are two vertical posts AA' and BB' of heights a and b respectively. From Q, the angle of elevation to the tops of both posts A' and B' is θ. From A the angle of elevation to B' is α and from B the angle of elevation to A' is β.



(i) Prove that $d^2 = \dfrac{a^2 + b^2}{\tan^2 \theta}$. **3**

(ii) Show that $\tan^2 \alpha + \tan^2 \beta = \tan^2 \theta$. **1**

Year 11 Maths Lesson Plan – Trigonometry 3U

Property of Prestige Tuition

- THREE DIMENSIONAL TRIGONOMETRY
  - Trigonometry essentially deals with triangles, which are two-dimensional objects. Hence when trigonometry is applied to a three-dimensional problem, the diagram must be broken up into a collection of triangles in space, and trigonometry used for each in turn.
  - Method to Approach Questions: When Triangles can be solved
    1. Draw a careful sketch of the situation.
    2. Note carefully all the triangles in the figure.
    3. Mark all right angles in these triangles.
    4. Always state which triangle you are working with.
  - **Worked Example** (From Cambridge): The rectangular prism ABCDEFGH sketched below has sides of length AB = 5cm, BC = 4cm and AE = 3 cm.
    (a) Find the lengths of the three diagonals AC, AF and FC.
    (b) Find the angle CAF between the diagonals AC and AF.
    (c) Find the length of the space diagonal AG.
    (d) Find the angle between the space diagonal AG and the edge AB.
  - **Question for the Class** → pick two from Cambridge Exercise 2G, or Fitzpatrick Textbook
  - Method to Approach Questions: when triangles cannot be solved (need for pronumeral)
    1. Draw a careful diagram of the situation, marking all right angles.
    2. A plan diagram, looking down, is usually a great help.
    3. Identify every triangle in the diagram, to see whether it can be solved.
    4. If one triangle can be solved, then work from it around the diagram until the problem is solved.
    5. If no triangle can be solved, assign a pronumeral to what is to be found, then work around the diagram until an equation in that pronumeral can be formed and solved.
  - **Worked Example**: The elevation from a hill at a place P due east of it is 48 degrees and at a place Q due south of P the elevation is 30 degrees. If the distance from P to Q is 500m, find the height of the hill.
  - **Question for Class**: pick one or two from Cambridge Exercise 2H or Fitzpatrick 3u.

- RADIAN MEASURE
  - The radian is the standard unit of angular measure. It is equivalent to the angle subtended at the centre of a circle by an arc equal in length to the radius.
  - $1\,radian = \frac{180°}{\pi} \quad therefore \quad \pi = 180°$
  - Exercise 1: converting radians into degrees
    a) Pi/4 = 45 degrees
    b) 5pi/3 = 300 degrees
    c) 2 radians = 114 degrees 35 minutes
    d) 4.1 radians = 235 degrees
  - Exercise 2: converting degrees to radians
    a) 25 degrees = 5pi/36

    - b) 270 degrees = 3pi/2
  - Exercise 3: Using the calculator to evaluate radians
    - a) Sin1.5 = 0.9974949866
    - b) Cos 3pi/5 = -0.3090169944
    - c) Tan theta = 0.673 → theta = 0.5923744119

- <u>ADDITION/SUBTRACTION THEOREM + DOUBLE ANGLE THEOREM</u>
  - Compound Angle Formulae
  - THE COMPOUND-ANGLE FORMULAE:

    A. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

    B. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

    C. $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

    D. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

    E. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

    F. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
  - Double-Angle Formulae

    THE DOUBLE-ANGLE FORMULAE:
    $$\sin 2\theta = 2\sin\theta\cos\theta$$
    $$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
    $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
  - Cos 2 theta Formulae
  - THE $\cos 2\theta$ FORMULAE:
    $$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
  - **Exercise 1**: Cambridge Exercise 14D (Yr11)
  - **Exercise 2**: <u>Cambridge</u> Exercise 2A (Yr 12)

- <u>The t-FORMULAE</u>
  - The t-formulae express sinθ, cosθ and tanθ as algebraic functions of the single trigonometric function tan ½ θ.
  - THE t-FORMULAE: Let t = tan ½ θ. Then: sinθ = 2t/(1+t^2) ; cosθ = (1−t^2)/(1+t^2) ; tanθ = 2t/(1−t^2)
  - Proof:

    PROOF:

    Let $t = \tan\frac{1}{2}\theta$. We seek to express $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of t.

    First, $\tan\theta = \dfrac{2\tan\frac{1}{2}\theta}{1-\tan^2\frac{1}{2}\theta}$, by the double-angle formula,
    $$= \frac{2t}{1-t^2}. \qquad (1)$$

    Secondly, $\cos\theta = \cos^2\frac{1}{2}\theta - \sin^2\frac{1}{2}\theta$, by the double-angle formula,

    $= \dfrac{\cos^2\frac{1}{2}\theta - \sin^2\frac{1}{2}\theta}{\cos^2\frac{1}{2}\theta + \sin^2\frac{1}{2}\theta}$, by the Pythagorean identity,

    $= \dfrac{1-\tan^2\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$, dividing through by $\cos^2\frac{1}{2}\theta$,
  -

$$= \frac{1-t^2}{1+t^2}\,. \qquad (2)$$

Thirdly, $\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$, by the double-angle formula,

$$= \frac{2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{\cos^2\frac{1}{2}\theta+\sin^2\frac{1}{2}\theta}, \text{ by the Pythagorean identity,}$$

$$= \frac{2\tan\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}, \text{ dividing through by } \cos^2\frac{1}{2}\theta,$$

$$= \frac{2t}{1+t^2}\,. \qquad (3)$$

- Exercise 1:
  1. Solve:
     a) $8cos\theta - sin\theta = 4 \;\; 0° < \theta < 360° \;\; Ans = 53° \; or \; 293°$
  2. Prove each of the following using the t-formulae
     a) cosθ(tanθ−tan ½ θ) = tan ½ θ
     b) $\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{1}{2}\theta$
     c) $\frac{\cos\theta+\sin\theta-1}{\cos\theta-\sin\theta+1} = \tan\frac{1}{2}\theta$
     d) $\frac{\tan 2\alpha+\cot\alpha}{\tan 2\alpha-\tan\alpha} = \cot^2\alpha$
     e) $\tan(\frac{1}{2}x+\frac{\pi}{4})+\tan(\frac{1}{2}x-\frac{\pi}{4}) = 2\tan x$

- AUXILIARY ANGLE METHOD x
  - The form f(x) = asinx + bcosx can be written as a single function, thereby making it easier to solve. The process is done by expanding the standard form and equating coefficients of sinx and cosx.
  - AUXILIARY-ANGLE METHOD:
    - Any function of the form $f(x) = a\sin x + b\cos x$, where a and b are constants (not both zero), can be written in any one of the four forms:

      $y = R\sin(x-\alpha) \qquad y = R\cos(x-\alpha)$

      $y = R\sin(x+\alpha) \qquad y = R\cos(x+\alpha)$

      where $R > 0$ and $0° \le \alpha < 360°$. The constant $R = \sqrt{a^2+b^2}$ is the same for all forms, but the *auxiliary angle* α depends on which form is chosen.
    - To begin the process, expand the standard form and equate coefficients of $\sin x$ and $\cos x$. Be careful to identify the quadrant in which α lies.
  - SOLVING EQUATIONS OF THE FORM $a\sin x + b\cos x = c$:
    - THE AUXILIARY-ANGLE METHOD: Get the LHS into one of the forms

      $R\sin(x+\alpha), \quad R\sin(x-\alpha), \quad R\cos(x+\alpha) \quad \text{or} \quad R\cos(x-\alpha),$

      then solve the resulting equation.
  - Note: expand the four formulas out
  - Example:
    1. Rewrite $\sqrt{3}\,sinx - cosx$ in the form Rsin(x – a) such that R>0 and a is acute
       = 2sin(x – pi/6)
    2. Solve $\sqrt{3}\,sinx - cosx = -\sqrt{2}$ for 0< x < 2Π
       x= 17pi/12 , 23pi/12
  - Exercise 1: Solve
    1. 2cosx + sinx = 1
    2. √3sin2x – cos2x =2
    3. sin4x+cos4x =1
    4. √3sinx−cosx = pi

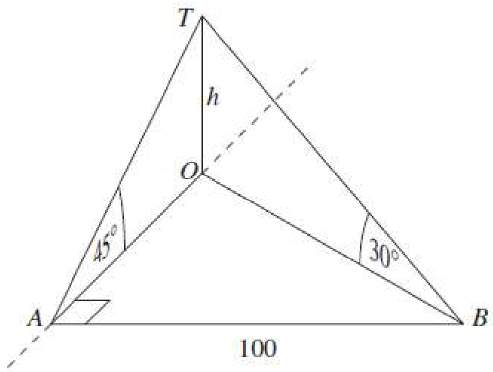
HSC Extension 1 Mathematics Trigonometry 3U

Past Papers
2000 – 2015

2000:

(c) Solve the equation $\cos 2\theta = \sin\theta, \quad 0 \le \theta \le 2\pi.$ **4**

(c) **5**



A surveyor stands at a point A, which is due south of a tower OT of height h m. The angle of elevation of the top of the tower from A is 45°. The surveyor then walks 100 m due east to point B, from where she measures the angle of elevation of the top of the tower to be 30°.

(i) Express the length of OB in terms of h.

(ii) Show that $h = 50\sqrt{2}$.

(iii) Calculate the bearing of B from the base of the tower.

2001:

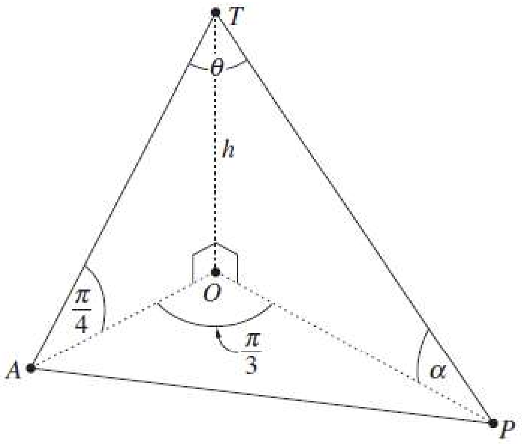
(c) (i) Starting from the identity $\sin(\theta + 2\theta) = \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$, and using the double angle formulae, prove the identity **2**

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(ii) Hence solve the equation **3**

$$\sin 3\theta = 2\sin\theta \quad \text{for} \quad 0 \le \theta \le 2\pi.$$

(b)



Consider the diagram, which shows a vertical tower OT of height h metres, a fixed point A, and a variable point P that is constrained to move so that angle AOP is $\frac{\pi}{3}$ radians. The angle of elevation of T from A is $\frac{\pi}{4}$ radians.

Let the angle of elevation of T from P be α radians and let angle ATP be θ radians.

(i) By considering triangle AOP, show that **1**

$$AP^2 = h^2 + h^2\cot^2\alpha - h^2\cot\alpha .$$

(ii) By finding a second expression for AP^2, deduce that **3**

$$\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha .$$

(iii) Sketch a graph of θ for $0 < \alpha < \frac{\pi}{2}$, identifying and classifying any turning points. Discuss the behaviour of θ as $\alpha \to 0$ and as $\alpha \to \frac{\pi}{2}$. **4**

2002:

(b) Find the general solution to $2\cos x = \sqrt{3}$. **2**

Express your answer in terms of π.

2003:

(e) (i) Express $\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where α is in radians. **2**

(ii) Hence, or otherwise, sketch the graph of $y = \cos x - \sin x$ for $0 \le x \le 2\pi$. **2**