

Improper integrals

Ex.1. If it is possible, calculate the improper integral.

(1) $\int_0^{\infty} e^{-x} dx$

(9) $\int_0^{\infty} \frac{dx}{5x+2}$

(17) $\int_{-\infty}^0 \frac{\arctan x}{x^2+1} dx$

(2) $\int_0^{\infty} e^{-201x} dx$

(10) $\int_{-\infty}^0 \frac{dx}{4x-3}$

(18) $\int_{\sqrt{3}}^{\infty} \frac{dx}{(x^2+1)\arctan^2 x}$

(3) $\int_0^{\infty} 2xe^{-3x} dx$

(11) $\int_{-\infty}^0 \frac{dx}{2x^2+3}$

(19) $\int_{-\infty}^{-1} \frac{dx}{(x^2+1)\arctan^4 x}$

(4) $\int_0^{\infty} x^2 e^{-2x} dx$

(12) $\int_0^{\infty} \frac{dx}{x^2+8x+18}$

(20) $\int_1^{\infty} \frac{\ln x}{x} dx$

(5) $\int_0^{\infty} x^2 e^{-x^3} dx$

(13) $\int_0^{\infty} \frac{-1-3x}{4x^2-4x} dx$

(21) $\int_e^{\infty} \frac{1}{x \ln x} dx$

(6) $\int_0^{\infty} \frac{e^x}{e^{2x}+1} dx$

(14) $\int_3^{\infty} \frac{dx}{x^3-4x^2+4x}$

(22) $\int_e^{\infty} \frac{dx}{x \ln^3 x}$

(7) $\int_0^{\infty} \frac{e^x}{100e^x+1} dx$

(15) $\int_{-\infty}^0 \frac{4x}{x^4+1} dx$

(23) * $\int_1^{\infty} \frac{10 \ln^2 x}{x(1+\ln^6 x)} dx$

(8) $\int_0^{\infty} \frac{e^{2x}}{(e^x+1)^3} dx$

(16) $\int_0^{\infty} \frac{dx}{\sqrt{x}+9}$

(24) $\int_1^{\infty} \frac{dx}{x^4+x^2}$

Ex.2. If it is possible, calculate the improper integral.

(1) $\int_0^1 \frac{dx}{x}$

(6) $\int_0^{\frac{\pi}{2}} \frac{1}{\cos^2 x} dx$

(11) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(2) $\int_{-3}^0 \frac{dx}{x+3}$

(7) $\int_0^1 \ln 4x dx$

(12) $\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x - \frac{1}{4}} dx$

(3) $\int_0^3 \frac{dx}{3x-2}$

(8) $\int_0^1 x \ln(1-x^2) dx$

(13) $\int_0^3 \frac{x}{\sqrt{3-x}} dx$

(4) $\int_0^1 \frac{dx}{x^3}$

(9) $\int_0^1 \frac{dx}{x \ln^2 x}$

(14) $\int_0^1 \frac{dx}{x^2+2x-3}$

(5) $\int_0^1 \frac{dx}{\sqrt{x}}$

(10) $\int_0^1 \frac{3x}{\sqrt{1-x^2}} dx$

(14) $\int_0^1 \frac{dx}{x^2+2x-3}$

Extra exercises

Ex.1. Find the area of the region bounded by the given curve and its asymptote in $+\infty$.

$$(1) y = e^{-4x} \text{ dla } x \geq 0 \quad (2) y = \frac{e^x}{(e^x+1)^2} \text{ dla } x \geq 0 \quad (3) y = \frac{\ln^2 x}{x^4} \text{ dla } x \geq 1$$

Ex.2. For which values of the parameter $p \in \mathbb{R}$ the below integrals are convergent?

$$(1) \int_0^{\infty} e^{px} dx$$

$$(2) \int_0^{\infty} \frac{px+1}{x^2+1} dx$$