Maxwell's modification of Ampere's law : Displacement current. To modify Ampere's law, Maxwell followed a symmetry consideration. By Faraday's law, a changing magnetic field induces an electric field, hence a changing electric field must induce a magnetic field. As currents are the usual sources of magnetic fields, a changing electric field must be associated with a current. Maxwell called this current *as the displacement current* to distinguish it from the usual conduction current caused by the drift of electrons.

Displacement current *is that current which comes into existence, in addition to the conduction current, whenever the electric field and hence the electric flux changes with time.*

To maintain the dimensional consistency, the displacement current is given the form :

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

where ϕ_E = electric field × area = EA, is the electric flux across the loop.

∴ Total current across the closed loop

$$= I_c + I_d = I_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

Hence the modified form of the Ampere's law is

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left[I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right] \qquad ...(5)$$

Unlike the conduction current, the displacement current exists whenever the electric field and hence the electric flux is changing with time. Thus according to Maxwell, the source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time-varying electric field. Hence the total current I is the sum of the conduction current I_c and displacement current I_d

$$I = I_c + I_d = I_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

Consistency of modified Ampere's law. For loop C_1, there is no electric flux ($\phi_E = 0$). Therefore, from equation (5) we have

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{...(6)}$$

For loop C_2, conduction current $I = 0$ but $I_d \neq 0$, because a time-varying electric field exists in the region between the capacitor plates. Hence

$$\oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} \qquad \text{...(7)}$$

If A be the area of the capacitor plates and q be the charge on the plates at any instant t during the charging process, then the electric field in the gap will be

$$E = \frac{q}{\varepsilon_0 A}$$

or $$EA = \frac{q}{\varepsilon_0}$$

or Flux $$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore \oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \mu_0 \frac{dq}{dt}$$

or $$\oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \left[\because I = \frac{dq}{dt}\right]$$

This agrees with the equation (6), proving the consistency of the Ampere's modified law (5).

Property of continuity. The sum $(I_c + I_d)$ has the important property of continuity along any closed path even when individually I_c and I_d may not be continuous. In Fig. 8.1, for example, a current I_c enters one plate and leaves the other plate of the capacitor. The conduction current

$$I_c = \frac{dq}{dt}$$

is not continuous across the capacitor gap as no charge is transported across this gap. The displacement current I_d is zero outside the capacitor plates and in the gap, it has the value

$$\varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{dq}{dt} \qquad \text{...(8)}$$

which is exactly the value of the conduction current in the lead wires. Thus the displacement current satisfies the basic condition that the current is continuous.

The sum $I_c + \varepsilon_0 \frac{d\phi_E}{dt}$ has the same value along the entire path (both inside and outside the capacitor plates), although individually the two currents are discontinuous. Clearly, outside the capacitor plates, we have only conduction current $I_c = I$, and there is no displacement current ($I_d = 0$). While inside the capacitor plates, there is only displacement current $I_d = I$, and there is no conduction current ($I_c = 0$). But in any general medium, both I_c and I_d are present. However, I_c is larger than I_d in a conducting medium while I_d is larger than I_c in an insulating medium.