The Time stop in the quantum fields fluctuation

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Abstract

We start with discussing the quantum field fluctuation and the propagation, what is the relation between them? Is it possible for the Time to stop? we start with assuming that there is a zero range quantum fluctuated Fields(x_1=x_2 damping fields) which are fixed Fields, so these Fields never propagate, they are fixed in the space-time points, so they are hidden, but their effects exist, it reflects the fact that it is possible for the Time to stop.

we have a problem about their quantum fluctuation, that is because the quantum fluctuation is associated with the propagation phenomena.

So we have a question, Does the Field fluctuate, if it does not propagate? the answer is yes. But if we think that, the quantum fluctuation is global phenomena(relates to global symmetries) and it does not necessity associate with the propagation phenomena.

The field propagation $\Delta(x_2{-}x_1)$ is affected by the symmetry in the space points, continuous transformations between infinity numbers of space points: the point x_1 is source for the point x_2 so the field in x_1 is source for the field in x_2 therefore the field is source for itself, but this is right only for $x_1 \neq x_2$(propagation), we can't imagine a field as a source for itself at a certain moment $x_1=x_2$ because that lets to divergences in the propagation.

For $x_1=x_2$ there must be something different, that difference is: when $x_1=x_2$ there is fields self-connection(the associated energy affects only in $\Delta T=0$ time stop, as in 1.18a), not propagation, because the propagation must occur only for $x_1 \neq x_2$.The self-connection is opposite to the propagation, no Time no Space(no Geometry, just points), it is $r=0$ problem solving($r=0$ problem is related to $x_1=x_2$ problem, it is $x_1=x_2$ undamped infinity oscillations).

Mathematically it is not easy to distinguish $x_1=x_2$ from $x_1 \neq x_2$ in the propagation, so we suggest zero range fluctuation fields(x_1=x_2 damping fields).

The infinity degrees of freedom of the quantum fields are related to the Symmetry of the space points(between infinity numbers of points: continuous transformations). But the *geometrical Aspects* of theses fields are related to the propagation between the *different space points*, they are $x_1 \neq x_2$ not $x_1=x_2$ so for the geometry the $x_1=x_2$ is not exist, therefore we must remove it. We can remove it by a connection in $x_1=x_2$ oppositely to the usual propagation $x_1 \neq x_2$, with that connection we remove $x_1=x_2$ in the propagation, in the mathematically language of the fields that connection must be delta Dirac

function in the space, but to satisfy the Lorentz invariance, it must be delta Dirac function in the space-time.

In other words we can say for a point in the space, it is itself free from the geometry(0-space dimension), so is the field in it.

At certain space-time point x (it is equivalent to say x_1=x_2 in the propagation) the field is free from the conditions(takes all its values without conditions, it is not given by an equation), so it has infinity degrees of freedom. Therefore we can make the changing eq1.3 (for free electromagnetic field A_μ)

$$A^\mu \to A^\mu + \bar{A}^\mu$$

with $\langle 0|\mathrm{T}\bar{A}_\mu(x_1)\bar{A}_\nu(x_2)|0\rangle = i\beta\delta^4(x_1 - x_2)g_{\mu\nu}$

and $\langle 0|\bar{A}_\mu(x)|0\rangle = 0$

the field A_μ is the usual field, it associates only with $x_1 \neq x_2$ propagation, but the damping field $\bar{A}^\mu$ must associate only with in x_1=x_2 propagation, so this field lets to connection in x_1=x_2 (field A_μ self-connection in x_1=x_2 , the associated energy affects only in ΔT=0 time stop(eq 1.18a), so the energy of the field A_μ for the propagation $x_1 \neq x_2$ is unchanged), that is, the x_1=x_2 damping field $\bar{A}^\mu$ cancels the infinity oscillators only in x_1=x_2 unlike the usual propagation role which is damping only in $x_1 \neq x_2$ the result is(eq 1.20):

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - i\varepsilon} - \frac{\beta g_{\mu\nu}}{1+\beta k^2} = \frac{g_{\mu\nu}}{k^2 - i\varepsilon} - \frac{\mathrm{a}^2 g_{\mu\nu}}{1+\mathrm{a}^2 k^2} \;:\; \beta = \mathrm{a}^2$$

or

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 - i\varepsilon}\left(1 - \frac{\mathrm{a}^2 k^2}{1+\mathrm{a}^2 k^2}\right)$$

Notice: Like the Groups, each element has an inverse with respect to identity element, so may be the propagations like that, the Delta Dirac propagation is the identity element.

First we must give the new term $\frac{\mathrm{a}^2 k^2}{1+\mathrm{a}^2 k^2}$ in the propagation a physical meaning to believe in it. We relate it to field dual behavior: free particle behavior and pairing particle-antiparticles behavior, we assume that the pairing particle-antiparticle behavior is elementary, the single particles behavior makes ability to separate them to be free particles and the pairing particle-antiparticle behavior makes ability to condense them. We see that Nature clearly in the quarks field dual behavior(chapter 9, paper).

Notice: the field dual behavior(modified propagation 1.20 and 2.36) allows us to describe the particle by a wave packet. We cannot describe the particle by a plane wave, because the plane wave propagation velocity is higher than the light speed: $u=\omega/k= \hbar\omega/\hbar k=E/P=mc^2/mv=c^2/v>c$. *But the packet velocity* $u_g= d\omega/dk=dE/dP <c$.

Then we search for situations ka→0 with assuming new conditions, we can impose the mass or the parameters Z_m and Z_2 independent on the energy, or cancelling the self-energies $\sum(\not{p})$and $\Pi^{\mu\nu}(k^2)$ (chapter 5, paper).

The important result we have is the behavior of the length a which must be like the behavior of the coupling constant g in general.

But for free fields there are no conditions on the length a so we can set a→0 but with a≠0(undetermining situation) .

we find the Lagrange parameters Z_i for the electrodynamics interaction, and generate that for the quarks, we can set m_q=0. We find the quarks field dual behavior lets to the scalar particles with mass 1/a which are the pions (chapter 9, paper). .

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1. The electromagnetic Field propagation with $x_1=x_2$ damping(without $x_1=x_2$ fluctuations)

We remove the $x_1=x_2$ in the usual field A_μ propagation by inserting damping $x_1=x_2$ Field $\bar{A}_\mu$ which fluctuates without propagation(Time stop). So they fluctuate with zero range propagation in the space-time points as we imposed that they affect only in $x_1=x_2$.

We start with the free electromagnetic Field Lagrange:

$$L_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} : \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad 1.1$$

With metric tensor $g^{00}=-1$, $g^{ij}=1$
That Lagrange becomes:

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + A_\mu J_g^\mu$$

1.2 where J_g^μ is a $x_1=x_2$ damping current.

we replace the right quantum electromagnetic Field A^μ ($x_1 \neq x_2$ propagation) in 1.1 via:

$$A^\mu \to A^\mu + \bar{A}^\mu \qquad 1.3$$

The term $A_\mu J_g^\mu$ in 1.2 violates the gauge invariance, but we remove it(remove J_g) eq1.17 in Feynman diagrams, then we make these diagrams gauge invariant, for that we suggest the fields dual behavior(chapter 9).

we don't attend to $\bar{A}_\mu$ we use it just for deriving the modified propagator 1.20, we need that propagator for the renormalization, but we see it is more elegant to use 1.20, especially for the quarks static potential.

With some treatments the right Lagrange 1.1 becomes:

$$L = \frac{1}{2}A_\mu[g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]A_\nu + \frac{1}{2}A_\mu[g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\bar{A}_\nu + \frac{1}{2}A_\mu[g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\bar{A}_\nu \qquad 1.4$$

Where we used:

$$\int dx^4 \partial_\rho \left[\bar{A}_\nu(x) A_\mu(x)\right] = 0$$

we remove the term $\frac{1}{2}\bar{A}_\mu[g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\bar{A}_\nu$ which doesn't contain the usual field A^μ .

the zero Range x_1=x_2 damping current J_g^μ satisfies the relation:

$$A_\mu[\, g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\bar{A}_\nu = A_\mu J_g^\mu \qquad 1.5$$

Therefore we have the relation:

$$J_g^\mu = [\, g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\bar{A}_\nu$$

we have the Lagrange:

$$L = \frac{1}{2}A_\mu[g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]A_\nu + A_\mu J_g^\mu \qquad 1.6$$

If we choice the Feynman gauge $\partial^\mu\bar{A}_\mu$=0

Therefore the zero Range current J_g^μ in 1.6 becomes:

$$J_g^\mu = g^{\mu\nu}\partial^2\bar{A}_\nu \qquad 1.7$$

The path integral of the quantum electromagnetic Field A^μ becomes:

$$W\left(J^\mu, J_g^\mu\right) = N\int DAe^{i\int d^4x[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + A_\mu J_g^\mu + A_\mu J^\mu]} \qquad 1.8$$

$$= N\int DAe^{i\int d^4x[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + A_\mu(J_g^\mu + J^\mu)]}$$

Where the integration only over the quantum electromagnetic Field A^μ.

So we fixed $\bar{A}_\mu$ and the zero Range current J_g^μ , J^μ is classical source, we remove it in the final result.

The path integral becomes:

$$W\left(J^\mu, J_g^\mu\right) = Nexp\frac{i}{2}\int d^4xd^4y\left(J^\mu(x) + J_g^\mu(x)\right)\Delta_{\mu\nu}(x-y)\left(J^\nu(y) + J_g^\nu(y)\right) \qquad 1.9$$

Where the free photon propagator is:

$$\Delta_{\mu\nu}(x-y) = \int\frac{d^4k}{(2\pi)^4}\frac{P_{\mu\nu}(k)}{k^2-i\varepsilon}e^{ik(x-y)} \quad : k^2 = \vec{k}^2 - k_0^2 \qquad 1.10$$

And the projection operator:

$$P_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \qquad 1.11$$

We choice the Feynman gauge, the propagator becomes:

$$\Delta_{\mu\nu}(x-y) = \int\frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{k^2-i\varepsilon}e^{ik(x-y)} \qquad 1.12$$

The relation between the vacuum Field expectation and the propagator is:

$$\langle 0|TA^\mu(x)A^\nu(y)|0\rangle = \frac{1}{i}propagator \qquad 1.13$$

We can get that vacuum Field expectation from the path integral as:

$$\langle 0|TA^\mu(x)A^\nu(y)|0\rangle = \frac{1}{i}\frac{\delta}{\delta J_\mu(x)}\frac{1}{i}\frac{\delta}{\delta J_\nu(y)}\, w\left(J^\mu, J_g^\nu\right) : \quad J^\mu = 0 \qquad 1.14$$

Using the relations 1.14, 1.9 and 1.13 in the existence of the Field $\bar{A}_\mu$ the photon propagator becomes

$$\langle 0|TA_\mu(x)A_\nu(y)|0\rangle = \frac{1}{i}\frac{\delta}{\delta J^\mu(x)}\frac{1}{i}\frac{\delta}{\delta J^\nu(y)}\, w\big(J^\mu, J_g^\mu\big)\big|_{J^\mu=0} \quad : \quad J_g^\mu \neq 0 \qquad 1.15$$

$$= \frac{1}{i}\Delta_{\mu\nu}(x-y) + \int d^4x_1 \frac{\Delta_{\mu\rho}(x-x_1)}{i} J_g^\rho(x_1) \int d^4x_2 \frac{\Delta_{\nu\gamma}(x_2-y)}{i} J_g^\gamma(x_2) + \cdots$$

$$= \frac{1}{i}\Delta_{\mu\nu}(x-y) + \int d^4x_1 d^4x_2 \frac{\Delta_{\mu\rho}(x-x_1)}{i}\cdot\frac{\Delta_{\nu\gamma}(x_2-y)}{i} \underbrace{J_g^\rho(x_1) J_g^\gamma(x_2)} + \cdots \qquad 1.16b$$

Where we take the zero Range currents connection.

Then we assume: the zero Range Field($x_1=x_2$ damping) $\bar{A}^\mu$ fluctuates, but with zero range propagation in the space-time, so the propagator of the $x_1=x_2$ damping Fields $\bar{A}^\mu$ must equal the delta Dirac function, that is because the delta Dirac function has zero range in space-time, so:

$$\langle 0|\underbrace{J_g^\rho(x_1) J_g^\gamma(x_2)}|0\rangle = \langle 0|\mathrm{T} g^{\rho\mu}\partial_{x_1}^2 \bar{A}_\mu(x_1)\, g^{\gamma\nu}\partial_{x_2}^2 \bar{A}_\nu(x_2)|0\rangle \; : \; J_g^\mu \neq 0 \qquad 1.17a$$

where

$$\langle 0|\mathrm{T}\bar{A}_\mu(x_1)\bar{A}_\nu(x_2)|0\rangle = i\beta\delta^4(x_1-x_2)g_{\mu\nu} \qquad 1.17b$$

the constant β is to satisfy the unities.

We let the Field $\bar{A}^\mu$ fluctuate inside 1.16b therefore:

Using 1.17a in 1.16b we get :

$$\langle 0|\mathrm{T}A_\mu(x)A_\nu(y)|0\rangle = \frac{1}{i}\Delta_{\mu\nu}(x-y)$$

$$+ \int d^4x_1 d^4x_2 \frac{\Delta_{\mu\rho}(x-x_1)}{i}\frac{\Delta_{\nu\gamma}(x_2-y)}{i}\langle 0|\mathrm{T} g^{\rho\mu_1}\partial_{x_1}^2 \bar{A}_{\mu_1}(x_1)\, g^{\gamma\vartheta_1}\partial_{x_2}^2 \bar{A}_{\vartheta_1}(x_2)|0\rangle + \cdots$$

Using 1.17b

$$\langle 0|\mathrm{T}A_\mu(x)A_\nu(y)|0\rangle = \frac{1}{i}\Delta_{\mu\nu}(x-y) +$$

$$\int d^4x_1 d^4x_2 \frac{\Delta_{\mu\rho}(x-x_1)}{i}\frac{\Delta_{\nu\gamma}(x_2-y)}{i} g^{\rho\mu_1}\partial_{x_1}^2\, g^{\gamma\vartheta_1}\partial_{x_2}^2 i\beta\delta^4(x_1-x_2)g_{\mu_1\vartheta_1} + \cdots \qquad 1.18a$$

Now the $x_1=x_2$ damping is more clear, the propagation Δ(x-y), Δ(x-x_1) and Δ(x_2-y) are related to the free field propagation(damping only in $x_1 \neq x_2$), the new term(vertex) is $\partial_{x_1}^2\, g^{\gamma\vartheta_1}\partial_{x_2}^2 i\beta\delta^4(x_1-x_2)g_{\mu_1\vartheta_1}$ it affects only in $x_1=x_2$ so in the path integral it cancels the infinity oscillations only in $x_1=x_2$, that is, the associated energy affects only in ΔT=0 (time stop).

Replacing the derivative ∂^2 in the 1.18a with the corresponding $(ik)^2$ and transform it to the momentum space, we find the photon propagator in the momentum space in the existence of the zero Range $x_1=x_2$ damping Field $\bar{A}^\mu$:

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2\text{-}i\varepsilon} + \beta\frac{g_{\mu\rho}}{k^2\text{-}i\varepsilon}(-k^4)g^{\rho\gamma}\frac{g_{\gamma\nu}}{k^2\text{-}i\varepsilon} + \cdots \qquad 1.18b$$

Which is a geometrical series, its sum is[2]:

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2+\beta k^4\text{-}i\varepsilon} = \frac{g_{\mu\nu}}{k^2\text{-}i\varepsilon}\frac{1}{1+\beta k^2} \qquad 1.19$$

That is the photon quantum propagation without $x_1=x_2$ fluctuations. We can write like:

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2-i\varepsilon} - \frac{\beta g_{\mu\nu}}{1+\beta k^2} = \frac{g_{\mu\nu}}{k^2-i\varepsilon} - \frac{\mathrm{a}^2 g_{\mu\nu}}{1+\mathrm{a}^2k^2} \;:\; \beta=\mathrm{a}^2 \; and \;\; k^2 = \vec{k}^2 - k_0^2 \qquad 1.20a$$

Or:

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2-i\varepsilon}\left(1-\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}\right) \qquad 1.20b$$

This $x_1=x_2$ damping propagation makes two phases:

the usual phase ka<<1→k<<1/a : r>a the propagator 1.20 becomes

$$\bar{\Delta}_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2-i\varepsilon}$$

and the high energy phase ka>1→ k>1/a : r<a the term $\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}$ plays an important role.

We try to make Ka→0 (chapter 5) by imposing new conditions, but we must give the length a: $\beta=\mathrm{a}^2=\mathrm{a}_\tau \mathrm{a}_r$ a physical meaning to believe in it. We relate the modifying 1.20 to fields dual behavior: free particle behavior and pairing particle-antiparticles behavior, where we assume that the pairing particle-antiparticles behavior is elementary. As we will see for quarks field dual behavior.

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We have the constant $\beta=\mathrm{a}^2$ in the Lagrange parameters Z_m, Z_2, Z_1 and Z_3, so we can impose the mass or the parameters Z_m, Z_2 independent on the energy.

Or cancelling the self-energies $\sum(\not{p})$ and $\Pi^{\mu\nu}(k^2)$ by cancelling the term $\frac{\mathrm{a}^2k^2}{1+\mathrm{a}^2k^2}$

The important result we must have is the behavior of the lattice side a which must be like the behavior of the coupling constant g in general.

The same thing we expect for the quantum propagator of the spinor Fields 2.35:

$$\bar{S}(p)=\frac{-\not{p}+m}{p^2+m^2-i\varepsilon+\beta p^4}_{\beta\to 0} \quad or \quad \frac{-\not{p}+m}{p^2+m^2-i\varepsilon}\frac{1}{1+\beta p^2} \qquad 1.21$$

If we use the equations 1.19 and 1.20 in the calculation of electron self-energy, photon self-energy and in the electron photon vertex we get results without ultraviolet diverges .

Using the propagator 1.20b we find the two electrons scattering amplitude for photons exchanging with $\omega=k_0=0$ (Born approximation to scattering amplitude in non-relativistic quantum mechanics[1]), so we can find the static electric potential by Fourier transform of V(q) to the space x, y, z and putting $\omega=k_0=0$ with

$$\langle p'|iT|p\rangle=-i\tilde{V}(q)(2\pi)\delta(\varepsilon_{p'}-\varepsilon_p) \;\; with\;\; S-matrix\;\; like\;\; S\approx 1+iT$$

By that we have the electric potential

$$U(x)=\int\frac{d^3k}{(2\pi)^3}\tilde{V}(k)e^{ik\cdot x}=\pm\int\frac{d^3k}{(2\pi)^3}\frac{e^2}{k^2}\left(1-\frac{a^2k^2}{1+a^2k^2}\right)e^{ik\cdot x} \;:\; \beta=a^2 \;\; and \;\; k^2=\vec{k}^2-k_0^2$$

That becomes

$$U(r)=\pm\frac{e^2}{4\pi r}\left[1-\exp(-\frac{r}{a})\right] \quad : \; r^2=x^2+y^2+z^2 \qquad 1.22$$

For r<a that potential is written like

$$U(r)\to a_0+a_1r+a_2r \;\; for\;\; r<a \qquad 1.23$$

And so the electric potential $U(r)$ in the space splits to two phases:

$$U(r)\sim\pm\frac{1}{r} \qquad for\;\; \frac{r}{a}>>1 \qquad 1.24$$

$$U(r)=a_0+a_1r+a_2r^2+\ldots.. \quad for\;\; \frac{r}{a}<<1$$

So $U(0)$ =constant.

2. The spinor Field propagation with $x_1=x_2$ damping(without $x_1=x_2$ fluctuations)

The free spinor Field propagator S(x-y) is given by the usual vacuum expectation:

$$\langle 0|T\Psi(x)\bar{\Psi}(y)\;|0\rangle=\frac{1}{i}S_0(x-y)=\frac{1}{i}\int\frac{d^4p}{(2\pi)^4}\bar{S}_0(\not{p})e^{ip(x-y)} \qquad 2.1$$

$$where \qquad \bar{S}_0(\not{p})=\frac{1}{\not{p}+m} \quad and \quad px=\vec{p}\cdot\vec{x}-p_0x_0 \qquad 2.2$$

We use the same ideas, we used for A_μ .

For that purpose we suggest a zero range propagation $x_1=x_2$ damping spinor Field ψ_g which doesn't propagate although it fluctuates, then we remove it in the Feynman diagrams, in a way preserves the usual conserved currents.

its vacuum expectation is:

$$\langle 0|T\psi_g(x)\bar{\psi}_g(y)\,|0\rangle = \beta\not{\partial}_x\delta^4(x-y)\ :\ \beta=a^2 \qquad 2.3$$
$$\langle 0|\psi_g|0\rangle = 0$$

The usual fermions Lagrange is:

$$L_0 = \bar{\psi}(i\not{\partial} - m)\psi \qquad 2.4$$

We split the fermions Field ψ such:

$$\psi \rightarrow \psi + \psi_g \qquad 2.5$$

so the Lagrange 2.4 becomes:

$$L(\psi,\psi_g) = (\bar{\psi}+\bar{\psi}_g)(i\not{\partial} - m)(\psi+\psi_g) \qquad 2.6$$

Therefore we have:

$$L = \bar{\psi}(i\not{\partial} - m)\psi + \bar{\psi}(i\not{\partial} - m)\psi_g + \bar{\psi}_g(i\not{\partial} - m)\psi + \bar{\psi}_g(i\not{\partial} - m)\psi_g \qquad 2.7$$

Replacing:

$$\int d^4x\bar{\psi}_g(i\not{\partial} - m)\psi_g \rightarrow \int d^4x\bar{\psi}_g(-i\overleftarrow{\not{\partial}} - m)\psi_g \qquad 2.8$$

we have the Lagrange:

$$L(\bar{\psi},\psi,\bar{\psi}_g,\psi_g) = \bar{\psi}(i\not{\partial} - m)\psi + \bar{\psi}(i\not{\partial} - m)\psi_g + \bar{\psi}_g(-i\overleftarrow{\not{\partial}} - m)\psi \qquad 2.9$$

The Lagrange 2.9 is not gauge invariant: but we remove the term $\bar{\psi}_g(i\not{\partial} - m)\psi_g$ which doesn't contain the usual field ψ and the term $\bar{\psi}\eta_g + \bar{\eta}_g\psi$ would be removed in the Feynman diagrams, so we must make these diagrams gauge invariant(field dual behavior).

we defined the $x_1=x_2$ damping spinor sources:

$$\eta_g = (i\not{\partial} - m)\psi_g \quad and \quad \bar{\eta}_g = \bar{\psi}_g(-i\overleftarrow{\not{\partial}} - m) \qquad 2.10$$

The Lagrange 2.7 for the right spinor field ψ:

$$L(\bar{\psi},\psi,\bar{\eta}_g,\eta_g) = \bar{\psi}(i\not{\partial} - m)\psi + \bar{\psi}\eta_g + \bar{\eta}_g\psi \qquad 2.11$$

The usual path integral for the fermions Field is given by:

$$Z_0(\eta,\ \bar{\eta}) = N\int D\psi\ D\bar{\psi}\ \exp\left[i\int d^4x\,(L_0 + \bar{\eta}\psi + \bar{\psi}\eta)\right] \qquad 2.12$$
$$= N\int D\psi\ D\bar{\psi}\ \exp\left[i\int d^4x(\ \bar{\psi}(i\not{\partial} - m)\psi + \bar{\eta}\psi + \bar{\psi}\eta)\right]$$

$$\textit{the usual Lagrange}\ L_0 = \bar{\psi}(i\not{\partial} - m)\psi \qquad 2.13$$

η is a classical source which must equal to zero at the final results, it is just to help us in the calculation, such:

$$\langle 0|T\psi_\alpha(x)\bar{\psi}_\beta(y)\,|0\rangle = \frac{1}{i}\,\frac{\delta}{\delta\bar{\eta}_\alpha(x)}\,i\frac{\delta}{\delta\eta_\beta(y)}\;\; Z_0(\eta,\,\bar{\eta})\Big|_{\eta=\bar{\eta}=0} \qquad 2.14$$

$$= \frac{1}{i}S_0(x-y)$$

$$(-i\partial\!\!\!/_x + m)S_0(x-y) = \delta^4(x-y) \qquad 2.15$$

Now we replace the Lagrange L_0 2.4 with the Lagrange 2.11 therefore the fermions path integral becomes:

$$Z_0(\eta,\,\bar{\eta},\eta_g,\bar{\eta}_g) = N\int D\psi\, D\bar{\psi}\, \exp\left[i\int d^4x\,(L(\bar{\psi},\psi,\bar{\psi}_g,\psi_g) + \bar{\eta}\psi + \bar{\psi}\eta)\right] \qquad 2.16$$

$$= N\int D\psi D\bar{\psi}\, \exp\left[i\int d^4x(\,\bar{\psi}(i\partial\!\!\!/ - m)\psi + \bar{\psi}\eta_g + \bar{\eta}_g\psi + \bar{\eta}\psi + \bar{\psi}\eta)\right]$$

Therefore the fermions Field vacuum expectation and so the fermions propagator in the existence of the x_1=x_2 damping spinor source η_g is given by:

$$\langle 0|T\psi_\alpha(x)\bar{\psi}_\beta(y)\,|0\rangle_{\eta_g} = \frac{1}{i}\,\frac{\delta}{\delta\bar{\eta}_\alpha(x)}\,i\frac{\delta}{\delta\eta_\beta(y)}\;\; Z_0(\eta,\,\bar{\eta},\eta_g,\bar{\eta}_g)\Big|_{\substack{\eta=\bar{\eta}=0\\ \eta_g,\bar{\eta}_g\neq 0}} \qquad 2.17$$

We calculate the path integral 2.16 for ψ:

$$Z_0(\eta,\,\bar{\eta},\eta_g,\bar{\eta}_g) = N\exp\left(i\int d^4x\, d^4y\,[\bar{\eta}(x)+\bar{\eta}_g(x)]S_0(x-y)[\eta(y)+\eta_g(y)]\right) \qquad 2.18$$

Where the usual $S_0(x\text{–}y)$ is given by:

$$(-i\partial\!\!\!/_x + m)S_0(x - y) = \delta^4(x - y) \qquad 2.19$$

The fermions Field vacuum expectation using the 2.18 is:

$$\langle 0|T\psi(x)\bar{\psi}(y)\,|0\rangle_{\eta_g} = \frac{1}{i}\,\frac{\delta}{\delta\bar{\eta}_\alpha(x)}\,i\frac{\delta}{\delta\eta_\beta(y)}\;\; Z_0(\eta,\,\bar{\eta},\eta_g,\bar{\eta}_g)\Big|_{\substack{\eta=\bar{\eta}=0\\ \eta_g,\bar{\eta}_g\neq 0}} \qquad 2.20$$

$$= \frac{1}{i}S_0(x-y) + \int d^4x_1\frac{1}{i}S_0(x-x_1)\eta_g(x_1)\int d^4x_2\bar{\eta}_g(x_2)\frac{1}{i}S_0(x_2-y) + \ldots\ldots$$

We make a vacuum connection between the two x_1=x_2 damping sources $\eta_g\,(x_1)$, $\eta_g(x_2)$ so we have:

$$\langle 0|T\psi(x)\bar{\psi}(y)\,|0\rangle_{\eta_g} = \frac{1}{i}\,\frac{\delta}{\delta\bar{\eta}_\alpha(x)}\,i\frac{\delta}{\delta\eta_\beta(y)}\;\; Z_0(\eta,\,\bar{\eta},\eta_g,\bar{\eta}_g)\Big|_{\substack{\eta=\bar{\eta}=0\\ \eta_g,\bar{\eta}_g\neq 0}} \qquad 2.21$$

$$= \frac{1}{i}S_0(x-y) + \int d^4x_1 d^4x_2\,\frac{1}{i}S_0(x-x_1)\underbrace{\eta_g(x_1)\bar{\eta}_g}(x_2)\frac{1}{i}S_0(x_2-y) + \ldots\ldots$$

Using the relations

$$\eta_g = (i\partial\!\!\!/ - m)\psi_g \quad and \quad \bar{\eta}_g = \bar{\psi}_g(-i\overleftarrow{\partial\!\!\!/} - m) \qquad 2.22$$

We have the connection:

$$\underbrace{\eta_g(x_1)\bar{\eta}_g(x_2)} = (i\partial\!\!\!/_{x_1} - m)\underbrace{\psi_g(x_1)\;\bar{\psi}_g(x_2)}(-i\overleftarrow{\partial\!\!\!/}_{x_2} - m) \qquad 2.23$$

using the connection between ψ_g (x_1) and $\psi_g(x_2)$ which has zero range propagation:

$$\underbrace{\psi_g(x_1)\ \bar{\psi}_g}(x_2) = \langle 0|T\psi_g(x_1)\bar{\psi}_g(x_2)\ |0\rangle = \beta\not{\partial}_{x_1}\delta^4(x_1 - x_2) \qquad 2.24$$

Using 2.24 in 2.23, for the massless fermions Field, the connection between *$\eta_g(x_1)$* and *$\eta_g(x_2)$* is:

$$\underbrace{\eta_g(x_1)\bar{\eta}_g}(x_2) = (i\not{\partial}_{x_1})\beta\not{\partial}_{x_1}\delta^4(x_1 - x_2)(-i\overleftarrow{\not{\partial}}_{x_2})$$

$$= (i\not{\partial}_{x_1})\beta\not{\partial}_{x_1}\int\frac{d^4k}{(2\pi)^4}e^{ik(x_1-x_2)}(-i\overleftarrow{\not{\partial}}_{x_2}) \qquad 2.25$$

Now we use that result in the 2.21 we get:

$$\langle 0|T\psi(x)\bar{\psi}(y)\ |0\rangle_{\eta_g} = \frac{1}{i}S_0(x-y) \qquad 2.26$$

$$+\int d^4x_1 d^4x_2\frac{1}{i}S_0(x-x_1)\ (i\not{\partial}_{x_1})\beta\not{\partial}_{x_1}\int\frac{d^4k}{(2\pi)^4}e^{ik(x_1-x_2)}(-i\overleftarrow{\not{\partial}}_{x_2})\frac{1}{i}S_0(x_2-y)\ +\ \ldots\ldots$$

The propagation is defined as:

$$\langle 0|T\psi(x)\bar{\psi}(y)\ |0\rangle_{\eta_g} = \frac{1}{i}S(x-y) \qquad 2.27$$

using 2.26 we have:

$$\langle 0|T\psi(x)\bar{\psi}(y)\ |0\rangle_{\eta_g} = \frac{1}{i}S(x-y) \qquad 2.28$$

$$+\int d^4x_1 d^4x_2\frac{1}{i}S_0(x-x_1)\ (-\beta)\int\frac{d^4k}{(2\pi)^4}e^{ik(x_1-x_2)}(i\not{k})(k^2)\frac{1}{i}S_0(x_2-y)\ +\ \ldots\ldots$$

Where the free propagator:

$$S_0(x-y) = \int\frac{d^4k}{(2\pi)^4}\bar{S}_0(\not{p})e^{ip(x-y)} \qquad 2.29$$

$$and\ \ \bar{S}_0(\not{p}) = \frac{1}{\not{p}}$$

The result is :

$$S(x-y) = \int\frac{d^4k}{(2\pi)^4}\bar{S}(\not{p})e^{ip(x-y)} \qquad 2.30$$

$$with\ \ \bar{S}(\not{p}) = \frac{1}{\not{p} - \Sigma(\not{p})} \qquad 2.31$$

And

$$\frac{1}{i}\bar{S}(\not{p}) = \frac{1}{i}\bar{S}_0(\not{p}) + \frac{1}{i}\bar{S}_0(\not{p})i\Sigma(\not{p})\frac{1}{i}\bar{S}_0(\not{p})\ +\ \ldots\ldots \qquad 2.32$$

Transformation 2.28 to four momentum space we have:

$$\frac{1}{i}\bar{S}(\not{p}) = \frac{1}{i}\bar{S}_0(\not{p}) + \frac{1}{i}\bar{S}_0(\not{p})i(-\beta)(p^2)\not{p}\frac{1}{i}\bar{S}_0(\not{p})\ +\ \ldots\ldots \qquad 2.33$$

The self-energy for these processes is

$$i\Sigma(\not p) = -i\beta(p^2)\not p$$
$$\Sigma(\not p) = -\beta(p^2)\not p \qquad 2.34$$

The $x_1=x_2$ damping fermions propagator 2.33 becomes:

$$\bar{S}(\not p) = \frac{1}{\not p - \Sigma(\not p)} = \frac{1}{\not p + \beta p^2 \not p} = \frac{-\not p}{p^2}\frac{1}{1+a^2p^2} \;:\; \beta = a^2 \qquad 2.35$$

Which can be written like:

$$\bar{S}(\not p) = \frac{-\not p}{p^2 - i\varepsilon} - \frac{a^2(-\not p)}{1+a^2p^2} \quad \rightarrow \quad \bar{S}(\not p) = \frac{-\not p}{p^2 - i\varepsilon} - \frac{-\not p}{p^2 + \frac{1}{a^2}} \qquad 2.36a$$

Or:

$$\bar{S}(\not p) = \frac{-\not p}{p^2 - i\varepsilon}\left(1 - \frac{a^2p^2}{1+a^2p^2}\right) \qquad 2.36b$$

For free fields there are no conditions on the length a , so no problem to set $a\rightarrow 0$ but with $a\neq 0$

In general this $x_1=x_2$ damping propagation makes two phases:

the usual phase $pa \ll 1 \rightarrow p \ll 1/a$: $r \gg a$ where the propagator 2.36 becomes $\bar{S}(\not p) = \frac{-\not p}{p^2 - i\varepsilon}$

And the phase $pa>1 \rightarrow p>1/a$: $r<a$ the term $\frac{a^2p^2}{1+a^2p^2}$ plays an important role.

We see that the 1/a equals a scalar particle mass, for the massive particles, which appear in the Feynman diagrams as pairing particle-antiparticle, as we see for the quarks. So we relate the modifying in 2.36 and 1.20 to fields dual behavior: separated free particles behavior and joined(pairing particle-antiparticle with mass 1/a) behavior, where we assume the condensed particles(pairing) behavior is elementary. As we will see for quarks field dual behavior(chapter 9).

See: http://iiste.org/Journals/index.php/APTA/article/view/26837