

Examen final de MATH 3

Corrigé Type

$P_1 \longrightarrow$ pas forcément 02 pt

$P_2 \longrightarrow$ Intégrale de Riemann $\alpha = \frac{5}{4} > 1, I = [0, 1]$

Donc elle DV.

EX02

1) \longrightarrow système de coordonnées Sphériques

$$2) \longrightarrow J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$= \begin{vmatrix} \sin \phi \cos \theta & r \sin \phi \sin \theta & r \cos \phi \\ -r \sin \phi \sin \theta & r \sin \phi \cos \theta & 0 \\ r \cos \phi \cos \theta & r \cos \phi \sin \theta & -r \sin \phi \end{vmatrix} \begin{array}{l} \text{suivant} \\ \text{la 3}^{\text{em}} \\ \text{colonne.} \end{array}$$

$$J = \cos \phi \left[-r^2 \sin \phi \sin \theta \cos \phi \sin \theta - r^2 \sin \phi \cos \theta \cos \phi \cos \theta \right] + 0 - r \sin \phi \left[\sin \phi \cos \theta r \sin \phi \cos \theta + \sin \phi \sin \theta r \sin \phi \sin \theta \right]$$

$$= -\cos \phi r^2 \left[\sin \phi \cos \phi \sin^2 \theta + \sin \phi \cos \phi \cos^2 \theta \right]$$

$$- r^2 \sin \phi \left[\sin^2 \phi \cos^2 \theta + \sin^2 \phi \sin^2 \theta \right]$$

$$= -\cos \phi r^2 \left[\sin \phi \cos \phi \right] - r^2 \sin \phi \left[\sin^2 \phi \right]$$

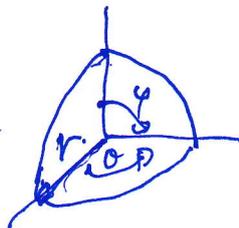
(1)

$$J = -r^2 \sin \phi \cdot \cos^2 \phi - r^2 \sin \phi \sin^2 \phi$$

$$= -r^2 \sin \phi (\cos^2 \phi + \sin^2 \phi) = -r^2 \sin \phi$$

$$J = -r^2 \sin \phi \rightarrow |J| = r^2 \sin \phi$$

3) $V \rightarrow 0 < r < 2, 0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{2}$



Donc $\iiint_V \frac{dx dy dz}{x^2 + y^2 + z^2} = \iiint_V \frac{r^2 \sin \phi dr d\theta d\phi}{r^2}$

$$= \int_0^2 dr \int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\pi}{2}} \sin \phi d\phi$$

$$= 2 \cdot \frac{\pi}{2} \cdot \left[-\cos \phi \Big|_0^{\frac{\pi}{2}} \right] = 2 \cdot \frac{\pi}{2} = \pi$$

Exo 3

1. $\lim_{n \rightarrow +\infty} u_n = 0$ condition nécessaire non suffisante on peut rien dire.

2. $\sum_0^{+\infty} \frac{1}{n^{\frac{8}{7}}}$, série de Riemann $\alpha = \frac{8}{7} > 1$ CV

3. $\sum_1^{+\infty} \left[\sin\left(\frac{1}{n}\right) \right]^2 \rightarrow$ pour n assez grand $\frac{1}{n} \rightarrow 0$ donc $\sin \frac{1}{n} \approx \frac{1}{n} \rightarrow \left[\sin \frac{1}{n} \right]^2 \approx \frac{1}{n^2}$ et $\sum \frac{1}{n^2}$

Série de Riemann $\alpha = 2$ CV Donc 3 CV.

$$4. \sum_0^{+\infty} (-1)^n \frac{n!}{n(n+1)(n+2) \dots 2n}, \text{ on a } |u_n| = \frac{n!}{n(n+1) \dots 2n}$$

D'Après D'Alembert $\frac{u_{n+1}}{u_n} = \frac{(n+1)!}{(n+1)(n+2) \dots 2n \cdot (2n+1)(2n+2)} \cdot \frac{n(n+1) \dots 2n}{n!}$

$$= \frac{(n+1)n!}{(n+1)(n+2) \dots 2n \cdot (2n+1)(2n+2)} \cdot \frac{n(n+1) \dots 2n}{n!}$$

$$= \frac{(n+1) \cdot n}{(2n+1)(2n+2)} = \frac{n^2 + n}{4n^2 + 4n + 2n + 2}$$

Donc $\lim_{n \rightarrow +\infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow +\infty} \frac{n^2}{4n^2} = \frac{1}{4} < 1$

Donc $\sum |u_n|$ cv Donc $\sum u_n$ est

Absolument cv

$$5. \sum_0^{+\infty} (-1)^n \frac{(2n)^n}{(n+1)^{n^2}}, \text{ on a } |u_n| = \frac{(2n)^n}{(n+1)^{n^2}}$$

D'Après Cauchy $|u_n|^{\frac{1}{n}} = \frac{2n}{(n+1)^n}$

$$\lim_{n \rightarrow +\infty} |u_n|^{\frac{1}{n}} = \lim_{n \rightarrow +\infty} \frac{2n}{(n+1)^n} = 0 < 1 \text{ Donc}$$

est Absolument cv

(3)