Linear Charge Density $\lambda = \frac{Q}{l}$ | Surface Charge Density $\sigma = \frac{Q}{A}$ $\rho = \frac{Q}{A}$ | Volume Charge Density $\rho = \frac{Q}{V}$

Electrostatics

Electric Charges & Electric Fields (Stationary Charges)

Coulomb's Law

SI: N (Newton)

$$\vec{F}_e = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

Vector Qty
Force: Magnitude & Direction

Electric Field Point Charge

SI: N/C

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Vector Field
Force: Magnitude & Direction

Electric Force

SI: N

$\vec{E} = \frac{\vec{F}_e}{q}$, $\vec{F}_e = (K\frac{Q}{r^2}\hat{r})\, q$, $\vec{F}_e = q\vec{E}$

Vector Qty
Force: Magnitude & Direction

Potential Energy

SI: J (Joule)

$$U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Electric Potential

SI: J/C or V

$$V = k\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential Energy per Unit Charge

Electric Potential

SI: V (Volts) or J/C

$V = \frac{U}{q}$, $U = qV$

Gauss's Law & Electric Flux

SI: N·m^2/C

$$\Phi_E = \oiint \vec{E}\cdot \underbrace{d\vec{A}}_{\text{Surface Area}} = \oiint EdA\cos\theta = \frac{Q_{enc}}{\varepsilon_0}$$

Electric Flux Throuh a Gaussin Surface

Potential Gradiant

SI: N/C

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

Calculating the Field from the Potential

Electric Potential

SI: V (Volts) or J/C

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{l}$$

Calculating the Potential from the Field

Electrical Potential Difference

SI: V (Volts) or J/C $\Delta V = \frac{-W}{q} = \frac{U}{q}$, $W = -q\Delta V$

Capactance

C = Geometric Property

SI: F (Farad), or C/V

$$C = \frac{\varepsilon_0 A}{d}$$

Capacity to Store Charge, for Parallel Plate Capacitor

Capacitance With a Dielectric

SI: F (Farad), or C/V

$$C = \kappa C_0 = \kappa\varepsilon_0\frac{A}{d} = \varepsilon\frac{A}{d}$$

k (Kappa) = Dielectric Constant

Potential Energy of a Capacior

SI: J (Joule)

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Does not Depend on Type of Capacitor

Energy Density of a Capacitor

SI: J/m^3

$u = \frac{U}{Ad}$, $u = \frac{1}{2}\varepsilon_0 E^2$

Potential Energy Per Unit Density

Dipole Moment

SI: C·m

$$\vec{p} = q\underbrace{s}_{\text{Seperation Distance}}, \text{from} - \text{ to } +$$

Direction from − to +, q = |q| ≠ ~~2|q|~~

Torque on a Dipole

SI: N·m

$\underbrace{T = pE\sin\theta}_{\text{General}}$ $\underbrace{\vec{\tau} = \vec{p}\times\vec{E}}_{\text{Vector}}$

Potential Energy of a Dipole

SI: J (Joule)

$\underbrace{U = -pE\cos\theta}_{\text{General}}$ $\underbrace{U = -\vec{p}\cdot\vec{E}}_{\text{Vector}}$

Electrodynamics

Electric Currents (Charges in Motion)

Ohms Law, Voltage Across a Resistor

SI: V (Volts) or J/C

$$\Delta V_R = iR$$

Power

SI: W (Watt)

$$P = iV = i^2 R = \frac{V^2}{R}$$

Rate of Electrical Energy Transfer, Resistive Dissipation

Current Density

SI: A/m^2

$i = JA$, $\vec{J} = n_e e\vec{v}_d$

n = Charge Carrier Density (m^-3), e = Electron Charge, v = Drift Velocity

Resistors in Series

SI: Ω (Ohms)

$$R = R_1 + R_2 + R_3 + \cdots$$

$i_1 = i_2$

Current Across all Resistors is Equal

Resistors in Parallel

SI: Ω (Ohms)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

$V_1 = V_2$

Voltage Across all Resistors is Equal

Resistance

R = Geometric Property, ρ = Intrinsic Property

SI: Ω (Ohms)

$$R = \frac{\rho l}{A}$$

ρ = Resistivity (Ω·m), A = Cross Sectional Area (m^2)

Voltage Across a Capacitor

SI: V (Volts) or J/C

$Q = C\Delta V_c$, $\Delta V_C = \frac{Q}{C}$

Capacitors in Parallel

SI: F (Farad), or C/V

$$C = C_1 + C_2 + C_3 + \cdots$$

$V_1 = V_2$

Voltage Across all Capacitors is Equal

Capacitors in Series

SI: F (Farad), or C/V

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

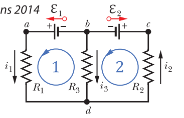
$i_1 = i_2$, $Q_1 = Q_2$

Current & Charge is Equal

Junction Rule (KCL)

$KCL_d:\quad i_1 + i_3 - i_2 = 0$
$KVL_1:\quad \varepsilon - i_1 R_1 + i_3 R_3 = 0$
$KVL_2:\quad -\varepsilon - i_2 R_2 - i_3 R_3 = 0$

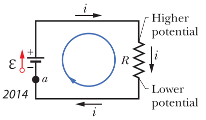
John Wiley & Sons 2014



$$\sum_{\text{junction}} i = 0$$

Loop Rule (KVL)

$KVL: \varepsilon - iR = 0$



John Wiley & Sons 2014

$$\sum_{\text{loop}} (\mathcal{E} + \Delta V_R + \Delta V_C) = 0$$

Capacitor Time Constant

SI: s (Seconds)

$$\tau = RC$$

τ (Tau)

Capacitor Discharging

SI: C (Coulomb)

$Q_0 = C\Delta V$, $Q = Q_0 e^{-t/\tau}$

Capacitor Charging

SI: C (Coulomb) $Q_0 = C\Delta V$, $Q = Q_0\left(1 - e^{-t/\tau}\right)$

t = 0+, Act like Short Circuit

t = ∞, Act like Open Circuit

Magnetostatics

Static Magnetic Fields (B-field from Charges in Motion)

Magnetic Force

RH Rule: + Charge, LH Rule: − Charge

SI: N (Newton)

⊗ IN ⊙ OUT

$\underbrace{F_m = qvB\sin\phi}_{\text{General}}$ $\underbrace{\vec{F}_m = q\vec{v}\times\vec{B}}_{\text{Vector}}$

Magnetic Flux

SI: Wb (Weber) or T·m^2

$\underbrace{\Phi_B = BA\cos\phi}_{\text{General}}$ $\underbrace{\Phi_B = \int \vec{B}\cdot d\vec{A}}_{\text{Vector}}$

Closed Surface Always = 0

Magnetic Force on Current Carrying Wire

SI: N (Newton)

l in Direction of Current

$\underbrace{F = ilB\sin\phi}_{\text{General}}$ $\underbrace{\vec{F} = i\vec{l}\times\vec{B}}_{\text{Vector}}$

Magnetic Dipole Moment (Current Loop)

SI: A·m^2

$$\vec{\mu} = i\vec{A}$$

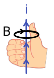
Magnetic Field Point Charge

SI: T (Tesla), or N/A·m

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic Field Small Secion of Wire

SI: T (Tesla), or N/A·m



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{l}\times\hat{r}}{r^2}$$

Biot-Savart Law

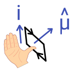
Ampere's Law

$\mu_0\, i = \mu_0\, JA$, $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$

Line Integral, Magnetic Field from Current Enclosed

Torque on a Magnetic Dipole (Current Loop)

SI: N·m



$\underbrace{\tau = il^2 B\sin\phi}_{\text{General}}$ $\underbrace{\vec{\tau} = \vec{\mu}\times\vec{B}}_{\text{Vector}}$

Potential Energy of a Magnetic Dipole (Current Loop)

SI: J (Joule)

$\underbrace{U = -uB\cos\phi}_{\text{General}}$ $\underbrace{U = -\vec{\mu}\cdot\vec{B}}_{\text{Vector}}$

Hall Voltage

SI: V (Volts) or J/C

$$\Delta V_H = \frac{iB}{lne}$$

Cycloton Motion Radius

SI: m (Meters)

$$r = \frac{mv}{qB}$$

m = Mass of Particle, v = Velocity of Particle

Cycloton Motion Period

SI: s (Seconds)

$$T = \frac{2\pi m}{qB}$$

Time to complete 1 orbit

Varying Magnetic Fields

Magnetodynamics

Faraday's Law - Induction Induced (EMF)

$$\varepsilon = \underbrace{-\frac{d}{dt}(BA\cos\phi)}_{\Delta \text{ Flux}} = \underbrace{-\frac{dB}{dt}A\cos\phi}_{\Delta B}\underbrace{-\frac{dA}{dt}B\cos\phi}_{\Delta A}\underbrace{-\frac{d\phi}{dt}BA\sin\phi}_{\Delta \phi} \quad \mathcal{E} = -\frac{d\Phi_B}{dt}$$

SI: V (Volts)

$$\mathcal{E} = \int_a^b (\vec{v}\times\vec{B})\cdot d\vec{l}$$

Self Inductance Second Loop

SI: V (Volts), or Wb/s

$$\mathcal{E}_2 = -N_2\frac{d\Phi_{B2}}{dt}$$

Mutual Inductance Second Loop

SI: V (Volts), or H/A·s

$$\mathcal{E}_2 = -M\frac{di_1}{dt}$$

Mutual Inductance

M = Geometric Property

Proportionality Constant

SI: H (Henry) or T·m^2/A

$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

Self Inductance Self Induced EMF

SI: V (Volts), or HA/s

$$\mathcal{E} = -L\frac{di}{dt}$$

Inductance

L = Geometric Property

SI: H (Henry), or Wb/A

$$L = \frac{N\Phi_B}{i}$$

Capacity to Store Energy (EMF)

Magnetic Energy of a Inductor

SI: J (Joule), or H·A^2

$$U = \frac{1}{2}Li^2$$

Does not Depend on Type of Capacitor

Magnetic Energy Density of a Inductor

SI: J/m^3

$$u = \frac{1}{2\mu_0}B^2$$

Inductor Time Constant

SI: s (Seconds)

$$\tau = \frac{L}{R}$$

τ (Tau)

Inductor Discharging

SI: A (Amp)

$$i_0 = \frac{V}{R}, \quad i = i_0 e^{-t/\tau}$$

Inductor Charging

SI: A (Amp)

$$i_0 = \frac{V}{R}, \quad i = i_0\left(1 - e^{-t/\tau}\right)$$

t = 0+, Act like Open Circuit

t = ∞, Act like Short Circuit

Fundamental Constants

Coulomb / Electrostatic Constant

$$\frac{1}{4\pi\varepsilon_0} = k = 8.99\cdot10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$$

Permittivity Constant

$$\varepsilon_0 = 8.85\cdot10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

Permittivity of Free Space

Vaccum Permeability / Magnetic Constant

$$\mu_0 = 4\pi\cdot10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}$$

Electron Charge

$$q_e = -1.602\cdot10^{-19}\,\mathrm{C}$$

⊖

Electron Mass

$$m_e = 9.11\cdot10^{-31}\,\mathrm{kg}$$

ⓔ

Proton Mass

$$m_p = 1.67\cdot10^{-27}\,\mathrm{kg}$$

ⓟ

Gravity

$$g = 9.81\,\mathrm{N/kg}$$

Gravity

$$G = 6.67\cdot10^{-11}\,\frac{\mathrm{Nm^2}}{\mathrm{kg}}$$

Kinematics and Dynamics

Newton's Second Law

$$\sum\vec{F} = m\vec{a}$$

Kinematic Equation(1)

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$$

Position as a Function of Time

Kinematic Equation(2)

$$v_x = v_{0x} + a_xt$$

Velocity as a Function of Time

Kinematic Equation(3)

$$v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x$$

Velocity as a Function of Displacement

Centripetal Acceleration

$$a_r = \frac{v^2}{r}$$

Conservation of Energy

$$U_i + K_i = U_f + K_f$$

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Mathematical Formulae & Prefixes

SI Prefix

m milli $(m) = 10^{-3}$

k kilo (k) = 10^3

SI Prefix

u micro $(\mu) = 10^{-6}$

M mega (M) = 10^6

SI Prefix

n nano $(n) = 10^{-9}$

G giga (G) = 10^9

SI Prefix

p pico $(p) = 10^{-12}$

T tera (T) = 10^{12}

Circle Circumference

$$C = 2\pi r$$

Surface Area of Circle

$$A_{CIRCLE} = \pi r^2$$

Surface Area of Sphere

$$A_{SPHERE} = 4\pi r^2$$

Volume of Sphere

$$V_{SPHERE} = \frac{4}{3}\pi r^3$$

Surface Area of Cylinder

$$A_{CYL} = 2\pi rL$$

Volume of Cylinder

$$V_{CYL} = \pi r^2L$$

Integral

∫

$$\int\frac{dx}{x} = \ln x$$

Integral

∫

$$\int x^n dx = \frac{1}{n+1}x^{n+1}$$

Quadratic Formula

∪

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Integral

∫

$$\int\frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$$

Integral

∫

$$\int\frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}$$

Integral

∫

$$\int\frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}$$

Integral

∫

$$\int\frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

Integral

∫

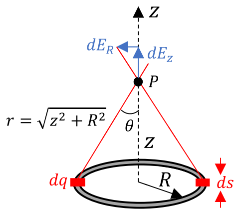
$$\int\frac{xdx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

Binomial Expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \quad if\ x \ll 1, then\ (1+x)^n \cong 1 + nx$$

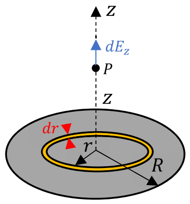
E-Field & F Derivations

Ring of Charge



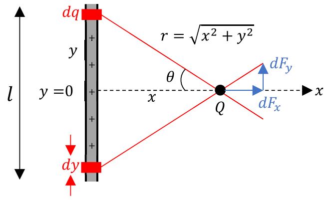
$$E = k\frac{Qz}{(z^2+R^2)^{3/2}}$$

Disk of Charge



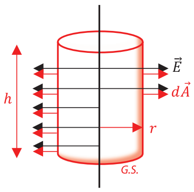
$$E = \frac{\sigma}{2\varepsilon_o}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

Finite Line of Charge



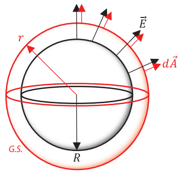
$$F_{net} = k\frac{Q\lambda l}{x\sqrt{x^2+\left(\frac{l}{2}\right)^2}}$$

Infinite Line of Charge



$$E = \frac{\lambda}{2\pi r\varepsilon_o} = k\frac{2\lambda}{r}$$

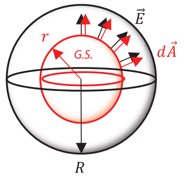
Hollow Shell (Outside)



$r > R$
(Outside Shell)

$$E = k\frac{Q}{r^2}$$

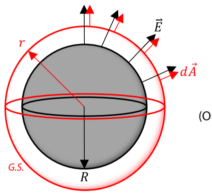
Hollow Shell (Inside)



$r < R$
(Inside Shell)

$$E = 0$$

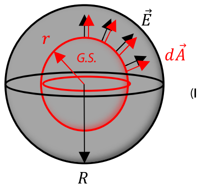
Solid Sphere (Outside)



$r > R$
(Outside Sphere)

$$E = k\frac{Q}{r^2}$$

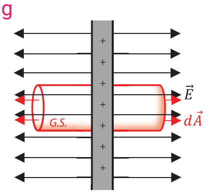
Solid Sphere (Inside)



$r < R$
(Inside Sphere)

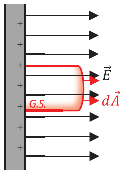
$$E = k\frac{Qr}{R^3}$$

Non-Conducting Infinite Sheet



$$E = \frac{\sigma}{2\varepsilon_0}$$

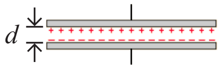
Conducting Infinite Sheet



$$E = \frac{\sigma}{\varepsilon_0}$$

Capacitors

Parallel Plate Capacitor

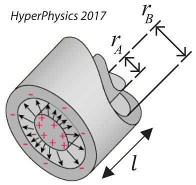


HyperPhysics 2017

$$C = \frac{\varepsilon_0 A}{d}$$

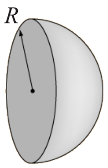
Cylindrical Capacitor

HyperPhysics 2017



$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{r_B}{r_A}\right)}$$

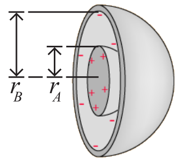
Isolated Spherical Capacitor



HyperPhysics 2017

$$C = 4\pi\varepsilon_0 R$$

Spherical Capacitor

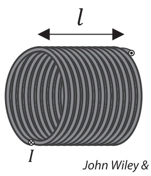


HyperPhysics 2017

$$C = \frac{4\pi\varepsilon_0 r_b r_a}{r_b - r_a} = \frac{4\pi\varepsilon_0}{\frac{1}{r_a}-\frac{1}{r_b}}$$

Inductors

Ideal Solenoid



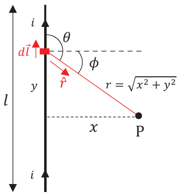
John Wiley & Sons 2014

$$L = \mu_0 n^2 lA$$

n = loop density, n = N/l

B-Field Derivations

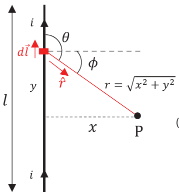
Finite Line of Current



(Finite Line)

$$\vec{B} = \frac{\mu_o i l}{4\pi x\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}$$

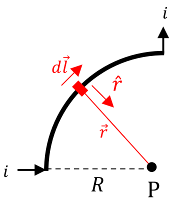
Infinite Line of Current



$L \gg x$
(Infinite Line)

$$\vec{B} = \frac{\mu_o i}{2\pi x}$$

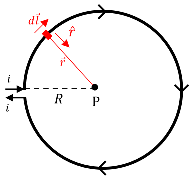
Arc of Current



$$\vec{B} = \frac{\mu_o i}{4\pi R}\Delta\theta \ \ ,\Delta\theta\epsilon[0,2\pi]$$

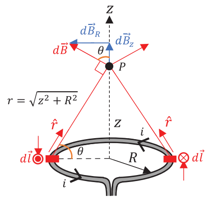
θ in Radians

Single Coil of Current



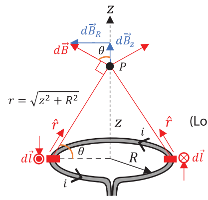
$$\vec{B} = \frac{\mu_o i}{2R}$$

Single Coil of Current at Distance Z



$$\vec{B} = \frac{\mu_o i R^2 N}{2(z^2 + R^2)^{3/2}}$$

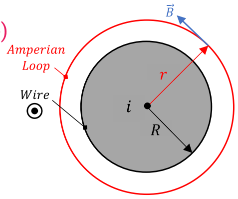
Magnetic Dipole



$z \gg R$
(Looks Like a Magnetic Dipole)

$$\vec{B} = \frac{\mu_o \vec{\mu}}{2\pi z^3} \ ,\vec{\mu} = Ni\vec{A}$$

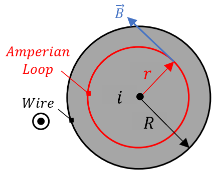
Wire (Outside)



$r > R$
(Outside Wire)

$$B = \frac{\mu_0 i}{2\pi r}$$

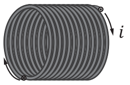
Wire (Inside)



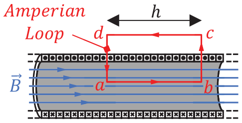
$r < R$
(Inside Wire)

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

Ideal Solenoid



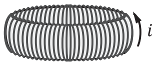
John Wiley & Sons 2014



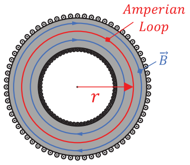
$$B = \mu_0 i n \ \ ,n = \frac{N}{l}$$

n = loop density, n = N/*l*

Ideal Toroid



John Wiley & Sons 2014

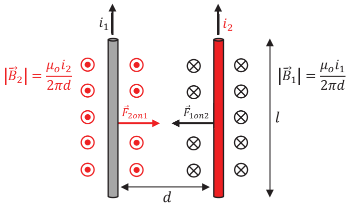


$$B = \frac{\mu_0 i N}{2\pi r}$$

N = Total Loops

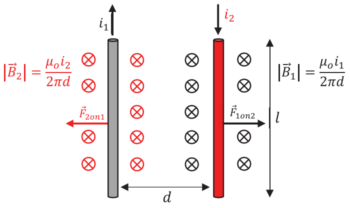
Current Carrying Wires

Parallel Wires



Current in same Direction

Parallel Wires

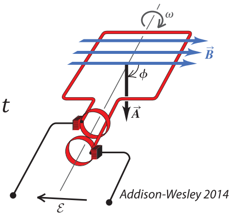


Current in Opposite Directions

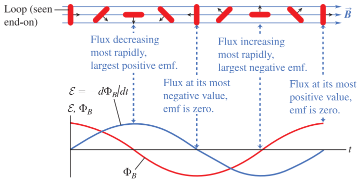
$$|\vec{F}| = \frac{\mu_o l i_1 i_2}{2\pi d} \ \ ,|\vec{F}_{2on1}| = |\vec{F}_{1on2}|$$

Simple Alternator

$$\phi = \omega t$$



Addison-Wesley 2014



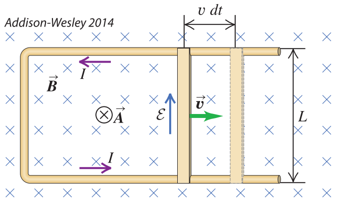
Addison-Wesley 2014

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = BA\cos(\omega t)$$

$$\varepsilon = -\frac{d\ \Phi_B}{dt} = \omega BA\sin(\omega t)$$

Slide Wire Generator

Addison-Wesley 2014



$$\varepsilon = -\frac{d\ \Phi_B}{dt} = -Blv$$

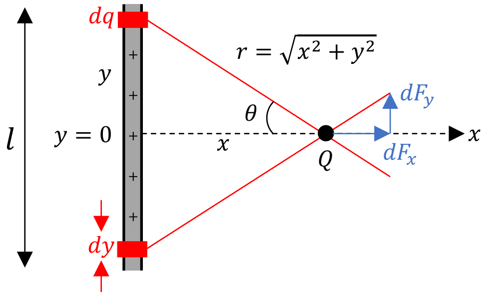
$$P_{dissipated} = i^2 R = \frac{B^2 l^2 v^2}{R}$$

$$\vec{F} = i\vec{l}\times\vec{B} = \frac{B^2 l^2 v}{R}$$

$$P_{applied} = Fv = \frac{B^2 l^2 v^2}{R}$$

Section 1. Electric Force

Finite Line of Charge



$$\lambda = \frac{q}{l}$$

$$dF_x = dF \cos\theta$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\boldsymbol{x \gg l}$$

(Looks Like a Point Charge)

$$F_{net} = \frac{kQ\lambda l}{x\sqrt{x^2 + \cancelto{0}{\left(\frac{L}{2}\right)^2}}}$$

$$F_{net} = \frac{kQ\lambda l}{x\sqrt{x^2}}$$

$$\boxed{F_{net} = k\frac{Qq}{x^2}}, q = \lambda l$$

$$\boldsymbol{x \ll l}$$

(Looks Like an Infinite Line)

$$F_{net} = \frac{kQ\lambda l}{x\sqrt{\cancelto{0}{x^2} + \left(\frac{l}{2}\right)^2}}$$

$$F_{net} = \frac{kQ\lambda l}{x\sqrt{\left(\frac{l}{2}\right)^2}}$$

$$F_{net} = \frac{kQ\lambda l}{x\frac{l}{2}}$$

(Infinite Line) $\boxed{F_{net} = k\frac{2\lambda Q}{x}}$

$$F = k\frac{Q_1 Q_2}{r^2}$$

$$q = \lambda l$$

$$dq = \lambda dy$$

$$dF = k\frac{Q\,dq}{r^2}$$

$$dF = k\frac{Q\,dq}{x^2 + y^2}$$

$$dF_{net} = 2dF_x$$

$$dF_x = 2k\frac{Q\,dq}{x^2 + y^2}\cos\theta$$

$$dF_x = 2k\frac{Q\lambda\,dy}{x^2 + y^2}\frac{x}{\sqrt{x^2 + y^2}}$$

$$dF_x = \frac{2kQ\lambda x\,dy}{(x^2 + y^2)^{3/2}}$$

$$F_{net} = \int_0^{l/2} \frac{2kQ\lambda x\,dy}{(x^2 + y^2)^{3/2}}$$

$$F_{net} = 2kQ\lambda x \int_0^{l/2} \frac{1}{(x^2 + y^2)^{3/2}}\,dy$$

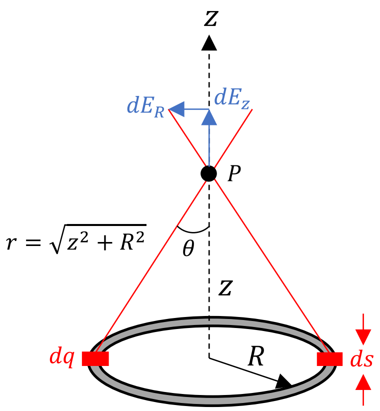
$$F_{net} = 2kQ\lambda x \frac{y}{x^2\sqrt{x^2 + y^2}}$$

$$F_{net} = \frac{2kQ\lambda xy}{x^2\sqrt{x^2 + y^2}}$$

$$F_{net} = \frac{2kQ\lambda\left(\frac{l}{2}\right)}{x\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}$$

(Finite Line) $\boxed{F_{net} = k\frac{Q\lambda l}{x\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}}$

Ring of Charge



$$\lambda = \frac{Q}{l}$$

$$l = 2\pi R$$

$$dE_Z = dE \cos\theta$$

$$\cos\theta = \frac{adj}{hyp} = \frac{z}{r} = \frac{z}{\sqrt{z^2 + R^2}}$$

$z \gg R$
(Looks Like a Point Charge)

$$E_{net} = k\frac{QZ}{(Z^2 + \cancelto{0}{R^2})^{\frac{3}{2}}}$$

$$E_{net} = k\frac{Q\cancel{Z}}{Z^{\cancel{3}2}}$$

$$\boxed{E = k\frac{Q}{Z^2}}$$

$z \ll R$
(Close to the center)

$$E_{net} = k\frac{Qz}{(\cancelto{0}{z^2} + R^2)^{\frac{3}{2}}}$$

$$E_{net} = k\frac{Qz}{(R^2)^{\frac{3}{2}}}$$

$$\boxed{E = k\frac{Qz}{R^3}}$$

$$E = k\frac{Q}{r^2}$$

$$q = \lambda l$$

$$dq = \lambda ds$$

$$dE = k\frac{dq}{r^2}$$

$$dE = k\frac{dq}{z^2 + R^2}$$

$$dE_Z = 2dE_Z$$

$$dE_Z = 2k\frac{dq}{z^2 + R^2}\cos\theta$$

$$dE_Z = 2k\frac{\lambda\, ds}{z^2 + R^2}\frac{Z}{\sqrt{z^2 + R^2}}$$

$$dE_Z = \frac{2k\lambda z\, ds}{(z^2 + R^2)^{3/2}}$$

$$E = \int_0^{\pi R}\frac{2k\lambda z\, ds}{(z^2 + R^2)^{3/2}}$$

$$E = \frac{2k\lambda z}{(z^2 + R^2)^{3/2}}\int_0^{\pi R} ds$$

$$E = \frac{2k\lambda z}{(z^2 + R^2)^{3/2}}\pi R$$

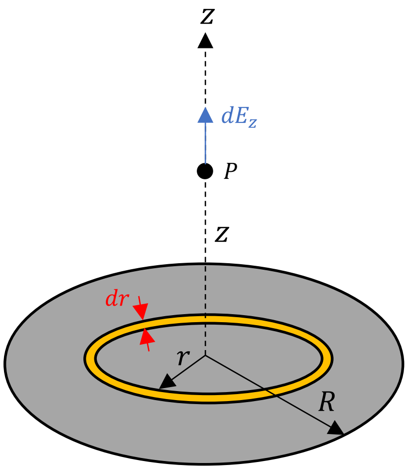
$$\lambda = \frac{Q}{l}$$

$$\lambda = \frac{Q}{2\pi R}$$

$$E = \frac{\cancel{2}kz\frac{Q}{\cancel{2\pi R}}}{(z^2 + R^2)^{3/2}}\cancel{\pi R}$$

$$\boxed{E = k\frac{Qz}{(z^2+R^2)^{3/2}}}$$

Disk of Charge



$$\sigma = \frac{Q}{A}$$

$$SA = \pi R^2$$

$$E_{Ring} = k\frac{Qz}{(Z^2 + R^2)^{3/2}}$$

$$\sigma = \frac{Q}{A}$$

$$Q = \sigma A$$

$$Q = \sigma \pi R^2$$

$$dQ = \sigma \pi 2r\, dr$$

$$E_{Ring} = k\frac{Qz}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$dE = \frac{k\, dQ\, z}{(z^2 + R^2)^{3/2}}$$

$$dE = \frac{2k\sigma\pi rz\, dr}{(z^2 + R^2)^{3/2}}$$

$$E = \int_0^R \frac{2k\sigma\pi rz\, dr}{(z^2 + R^2)^{\frac{3}{2}}}$$

$$E = 2k\sigma\pi z \int_0^R \frac{r\, dr}{(z^2 + R^2)^{3/2}}$$

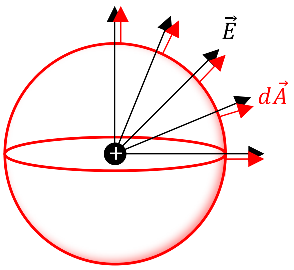
$$E = 2k\sigma\pi z \left(-\frac{1}{\sqrt{z^2 + R^2}}\right)\Bigg|_0^R$$

$$E = 2k\sigma\pi z \left(\frac{1}{z} - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)}$$

Section 3. Gauss's Law Electric Field

Point Charge



$SA_{Sphere\ G.S.} = 4\pi r^2$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

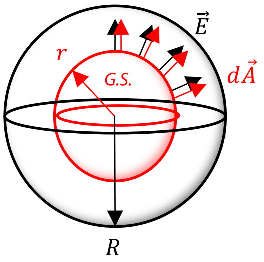
$$E = \frac{q_{enc}}{A\varepsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

$$\boxed{E = k\frac{Q}{r^2}}$$

Hollow Shell of Charge – Inside Shell

$\boldsymbol{r < R}$
(Inside Shell)



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

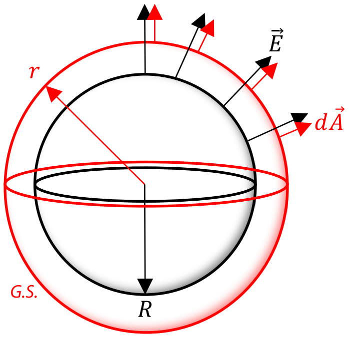
$$q_{enc} = 0$$

$\boldsymbol{r < R}$
(Inside Shell)

$$\boxed{E = 0}$$

Hollow Shell of Charge – Outside Shell

$\boldsymbol{r > R}$
(Outside Shell)



$SA_{Sphere\ G.S.} = 4\pi r^2$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{q_{enc}}{A\varepsilon_0}$$

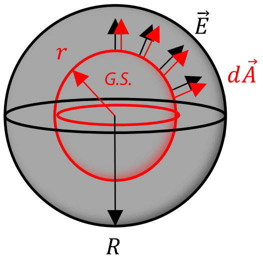
$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

$\boldsymbol{r > R}$
(Outside Shell)

$$\boxed{E = k\frac{Q}{r^2}}$$

Solid Sphere – Inside Sphere

$\boldsymbol{r < R}$
(Inside Sphere)



$$SA_{Sphere\ G.S.} = 4\pi r^2$$

$$V_{Sphere} = \frac{4}{3}\pi R^3$$

$$\rho = \frac{Q}{V}$$

$$\rho = \frac{Q}{V}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$q_{enc} = \rho V_{G.S.}$$

$$q_{enc} = \left(\frac{Q}{\cancel{\frac{4}{3}}\cancel{\pi}R^3}\right)\cancel{\frac{4}{3}}\cancel{\pi}r^3$$

$$q_{enc} = \frac{Qr^3}{R^3}$$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{q_{enc}}{A\varepsilon_0}$$

$$E = \frac{\left(\frac{Qr^3}{R^3}\right)}{4\pi r^2\varepsilon_0}$$

$$E = \frac{Qr^{\cancel{3}}}{4\pi \cancel{r^2}\varepsilon_0 R^3}$$

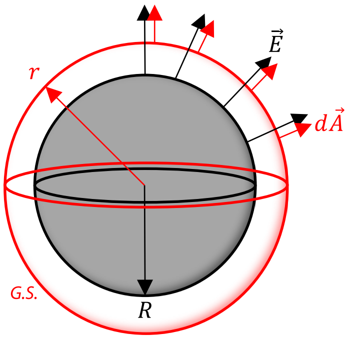
$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$\boldsymbol{r < R}$
(Inside Sphere)

$$\boxed{E = k\frac{Qr}{R^3}}$$

Solid Sphere – Outside Sphere

$\boldsymbol{r > R}$
(Outside Sphere)



$$SA_{Sphere\ G.S.} = 4\pi r^2$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{q_{enc}}{A\varepsilon_0}$$

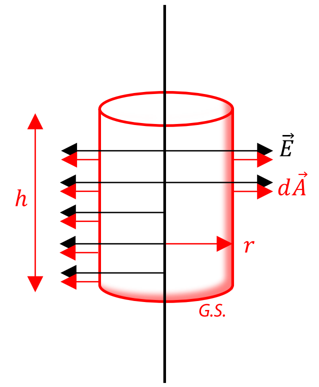
$$E = \frac{Q}{4\pi r^2\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$\boldsymbol{r > R}$
(Outside Sphere)

$$\boxed{E = k\frac{Q}{r^2}}$$

Infinite Line of Charge



$SA_{\text{Sphere Side G.S.}} = 2\pi rh$

$$\lambda = \frac{Q}{h}$$

$$Q = \lambda h$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

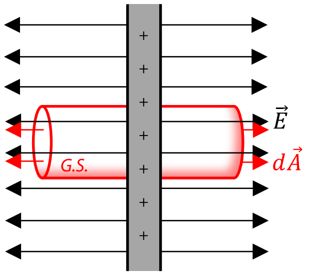
$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{\lambda h}{2\pi rh\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_o}}$$

$$or \; \boxed{E = k\frac{2\lambda}{r}}$$

Non-Conducting Infinite Sheet



Surface Charge Density $\sigma = \frac{Q}{A}$

$$Q = \sigma A$$

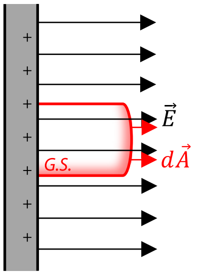
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$2E = \frac{\sigma A}{A\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Conducting Infinite Sheet



Surface Charge Density $\sigma = \frac{Q}{A}$

Charge lies on the surface

$$Q = \sigma A$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

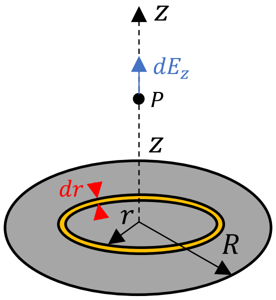
$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{\sigma A}{A\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

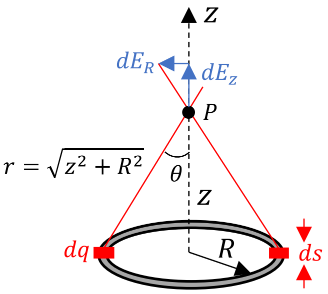
Section 4. Summary

Disk of Charge



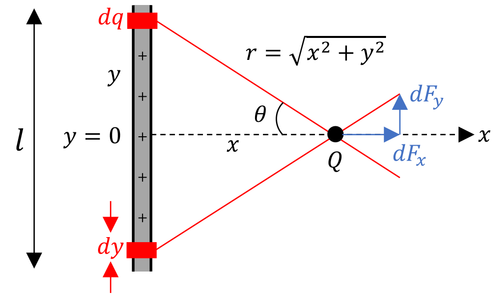
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)}$$

Ring of Charge



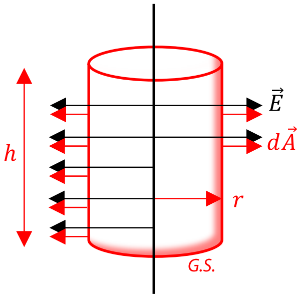
$$\boxed{E = k\frac{Qz}{(z^2 + R^2)^{3/2}}}$$

Finite Line of Charge



$$\boxed{F_{net} = k\frac{Q\lambda l}{x\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}}$$

Infinite Line of Charge

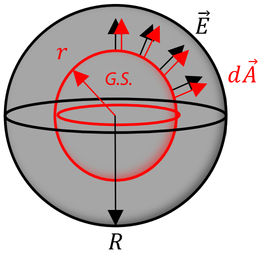


$$\boxed{E = \frac{\lambda}{2\pi r\varepsilon_0}}$$

$$or\ \boxed{E = k\frac{2\lambda}{r}}$$

Solid Sphere – Inside Sphere

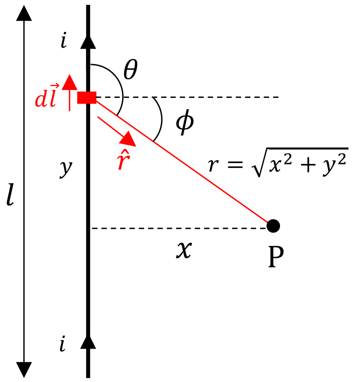
$\boldsymbol{r < R}$
(Inside Sphere)



$$\boxed{E = k\frac{Qr}{R^3}}$$

Section 5. Magnetic Field

Line of Current



$$\theta = 90 + \phi$$

$$\sin\theta = (90 + \phi)$$

$$\sin(90 + \phi) = \cancelto{1}{\sin 90}\cos\phi + \cancelto{0}{\cos 90}\sin\phi$$

$$\sin\theta = \cos\phi$$

$$\cos\phi = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{x}{r}$$

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,\Delta\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$$\left|d\vec{l}\times\hat{r}\right| = dl\sin\theta$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{r^2}dl\sin\theta$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{r^2}\frac{x}{r}dl$$

$$d\vec{B} = \frac{\mu_o ix}{4\pi r^3}dl \quad , r = \sqrt{x^2 + y^2}\,, dl = dy$$

$$d\vec{B} = \frac{\mu_o ix}{4\pi(x^2 + y^2)^{3/2}}dy$$

$$\vec{B} = \frac{\mu_o ix}{4\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{dy}{(x^2 + y^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_o ix}{4\pi}\left[\frac{y}{x^2}\frac{1}{(x^2 + y^2)^{3/2}}\right]\Bigg|_{-\frac{l}{2}}^{\frac{l}{2}}$$

$$\vec{B} = \frac{\mu_o ix}{4\pi}\left[\frac{\left(\frac{l}{2}\right)}{x^2}\frac{1}{\left(x^2 + \left(\frac{l}{2}\right)^2\right)^{3/2}} - \frac{\left(-\frac{l}{2}\right)}{x^2}\frac{1}{\left(x^2 + \left(-\frac{l}{2}\right)^2\right)^{3/2}}\right]$$

$$\vec{B} = \frac{\mu_o ix}{4\pi}\left[\frac{l}{x^2}\frac{1}{\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}\right]$$

(Finite Line) $\boxed{\vec{B} = \frac{\mu_o il}{4\pi x\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}}$

$L \gg x$
(Infinite Line)

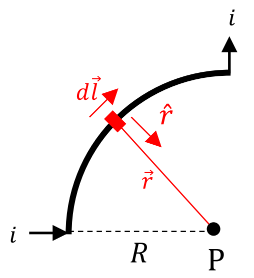
$$\vec{B} = \frac{\mu_o il}{4\pi x\sqrt{\cancelto{0}{x^2} + \left(\frac{l}{2}\right)^2}}$$

$$\vec{B} = \frac{\mu_o il}{4\pi x\sqrt{\left(\frac{l}{2}\right)^2}}$$

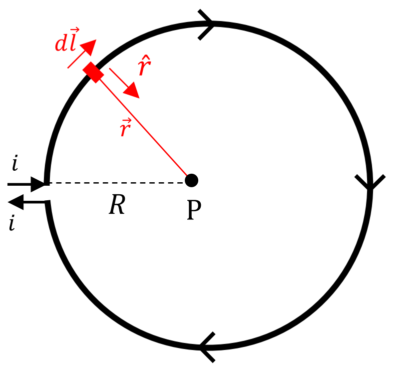
$$\vec{B} = \frac{\mu_o il}{4\pi x\left(\frac{l}{2}\right)}$$

(Infinite Line) $\boxed{\vec{B} = \frac{\mu_o i}{2\pi x}}$

Arc of Current



Arc has a radius R
Arc has a current i
B-field at point P at the centre



if $\Delta\theta = 2\pi$ (Entire Loop)

$$\vec{B} = \frac{\mu_o i}{{}_2\cancel{4\pi}R}\cancel{2\pi}$$

(Equation of an Entire Loop) $\boxed{\vec{B} = \frac{\mu_o i}{2R}}$

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,\Delta\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$\vec{B}$ points into the page $\otimes$
(true for all elements along the arc)

$d\vec{l}$ and $\hat{r}$ are $\perp$ **so**: $d\vec{l}\times\hat{r} = dl$

$$\vec{A}\times\vec{B} = AB\sin\theta$$

$$\hat{r} = Unit\ Vector\ of\ Length\ 1$$

$$d\vec{l}\times\hat{r} = dl\,r\sin\theta$$

$$d\vec{l}\times\hat{r} = dl\,(1)\sin(90)$$

$$d\vec{l}\times\hat{r} = dl$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,dl}{R^2}$$

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{R^2}\int dl$$

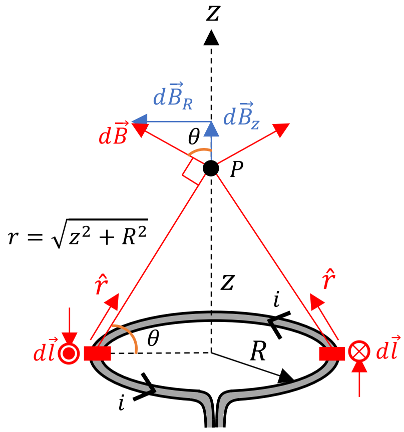
Arc Lengh Equation where $\Delta\theta\epsilon[0,2\pi]$

$$\int dl = R\Delta\theta$$

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{R^2}R\Delta\theta$$

(General Equation) $\boxed{\vec{B} = \frac{\mu_o i}{4\pi R}\Delta\theta}$ $, \Delta\theta\epsilon[0,2\pi]$

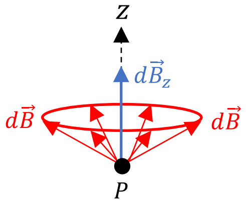
Ring of Current



$$l = 2\pi R$$

$$d\vec{B}_Z = d\vec{B}\cos\theta$$

$$\cos\theta = \frac{adj}{hyp} = \frac{R}{r} = \frac{R}{\sqrt{z^2 + R^2}}$$



$z \gg R$
(Looks Like a Magnetic Dipole)

$$\vec{B} = \frac{\mu_o iR^2}{2(z^2 + R^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_o iR^2}{2(z^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_o iR^2}{2z^3}$$

$$A = \pi R^2, N = Loops$$

$$\vec{B} = \frac{\mu_o NiA}{2z^3}$$

$$\vec{B} \parallel \vec{\mu}, \mu = NiA$$

(Magnetic Field from a dipole) $\boxed{\vec{B} = \frac{\mu_o\vec{\mu}}{2\pi z^3}}, \vec{\mu} = Ni\vec{A}$

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,\Delta\vec{s}\times\hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$d\vec{l}$ and $\hat{r}$ are $\perp$ **so**: $d\vec{l}\times\hat{r} = dl$

$$\vec{A}\times\vec{B} = AB\sin\theta$$
$$\hat{r} = Unit\ Vector\ of\ Length\ 1$$
$$d\vec{l}\times\hat{r} = dl\,r\sin\theta$$
$$d\vec{l}\times\hat{r} = dl\,(1)\sin(90)$$
$$d\vec{l}\times\hat{r} = dl$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{r^2}dl$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i}{(z^2+R^2)}dl$$

Resulting Magnetic Field is going to be along the Z axis because other components cancel (Need to calculate Z component from $d\vec{B}$)

$$d\vec{B}_Z = \frac{\mu_o}{4\pi}\frac{i}{(z^2 + R^2)}dl\cos\theta$$

$$d\vec{B}_Z = \frac{\mu_o}{4\pi}\frac{i}{(z^2 + R^2)}\frac{R}{\sqrt{z^2 + R^2}}dl$$

$$d\vec{B}_Z = \frac{\mu_o iR}{4\pi(z^2 + R^2)^{3/2}}dl$$

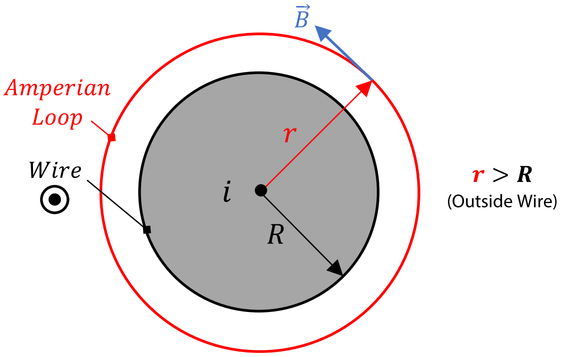
$$\vec{B} = \frac{\mu_o iR}{4\pi(z^2 + R^2)^{3/2}}\int dl$$

$$\vec{B} = \frac{\mu_o iR}{4\pi(z^2 + R^2)^{3/2}}l$$

$$\vec{B} = \frac{\mu_o iR}{{}_2\cancel{4}\cancel{\pi}(z^2 + R^2)^{3/2}}\cancel{2}\cancel{\pi}R$$

$$\boxed{\vec{B} = \frac{\mu_o iR^2 N}{2(z^2 + R^2)^{3/2}}}, N = Loops$$

Solid Current Carrying Wire – Outside Wire



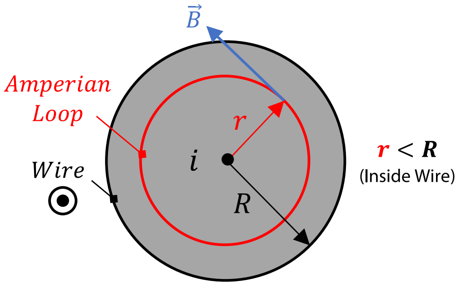
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc}$$

$$BL = \mu_0 i_{enc}$$

$$B2\pi r = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

Solid Current Carrying Wire – Inside Wire



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc}$$

$$BL = \mu_0 i_{enc}\,, \qquad i = JA$$

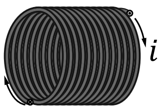
$$BL = \mu_0 JA\,, \qquad J = \frac{i}{A}$$

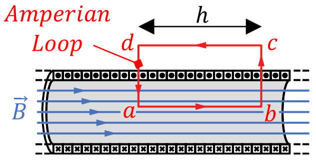
$$BL = \mu_0 \frac{i}{A} A$$

$$B2\pi r = \mu_0 \frac{i}{\pi R^2} \pi r^2$$

$$\boxed{B = \frac{\mu_0 i r}{2\pi R^2}}$$

Ideal Solenoid





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$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

$$\int_a^b \vec{B}\cdot d\vec{l} + \underset{\vec{B}\perp d\vec{l}}{\cancelto{0}{\int_b^c \vec{B}\cdot d\vec{l}}} + \underset{\vec{B}=0}{\cancelto{0}{\int_c^d \vec{B}\cdot d\vec{l}}} + \underset{\vec{B}\perp d\vec{l}}{\cancelto{0}{\int_d^a \vec{B}\cdot d\vec{l}}} = \mu_0 i_{enc}$$

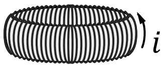
$$\int_a^b \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

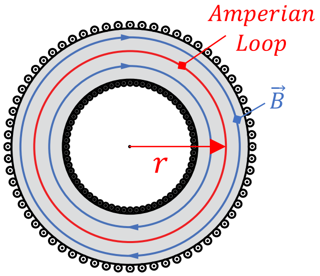
$$Bh = \mu_0 i_{enc}, \qquad i_{enc} = i(nh)$$

$$B\cancel{h} = \mu_0 in\cancel{h}$$

$$\boxed{B = \mu_0 in}\,, n = \frac{N}{l}$$

Ideal Toroid





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$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enc}$$

$$BL = \mu_0 i_{enc}$$

$$B2\pi r = \mu_0 iN$$

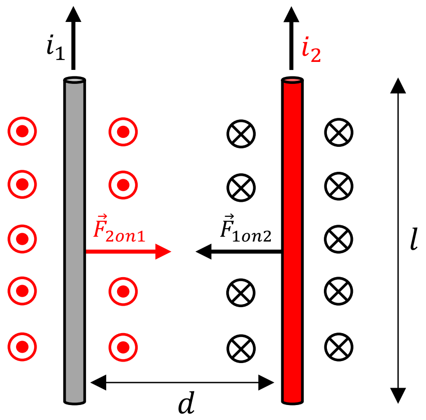
$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r}$$

$$\boxed{B = \frac{\mu_0 iN}{2\pi r}}$$

Force Exerted Parallel Wires

Current in Same Direction

$$|\vec{B}_2| = \frac{\mu_o i_2}{2\pi d} \qquad |\vec{B}_1| = \frac{\mu_o i_1}{2\pi d}$$



$$|\vec{B}_1| = \frac{\mu_o i_1}{2\pi d}$$

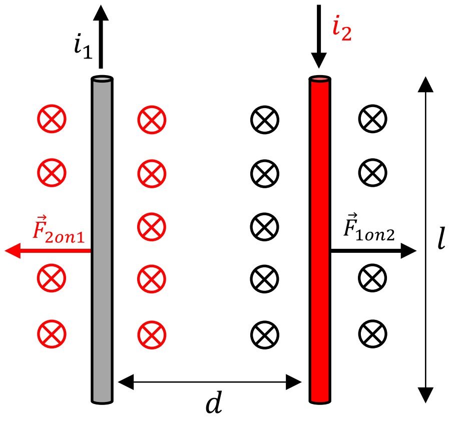
$$|\vec{B}_2| = \frac{\mu_o i_2}{2\pi d}$$

$$\vec{F}_{2on1} = i_1 \vec{l} \times \vec{B}_2$$

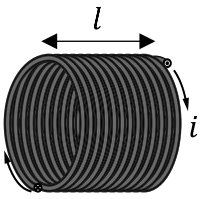
$$|\vec{F}_{2on1}| = i_1 l \frac{\mu_o I_2}{2\pi d}$$

$$\boxed{|\vec{F}| = \frac{\mu_o l i_1 i_2}{2\pi d}}\ , |\vec{F}_{2on1}| = |\vec{F}_{1on2}|$$

Current in Opposite Directions



Inductance of a Solenoid



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$$B = \mu_0 i n$$

$$L = \frac{N\Phi_B}{i}$$

$$L = \frac{(nl)\mathrm{BA}}{i}$$

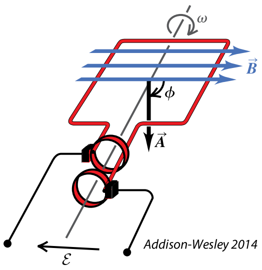
$$L = \frac{(nl)(\mu_0 \cancel{i} n)\mathrm{A}}{\cancel{I}}$$

$$\boxed{L = \mu_0 n^2 l\mathrm{A}}$$

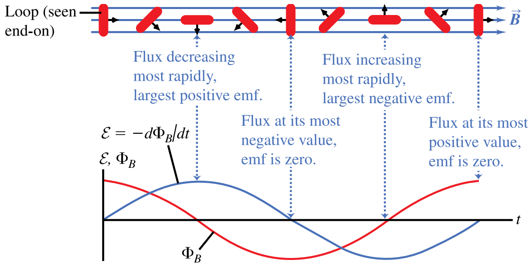
$$n = \frac{N}{l} \qquad \Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$N = nl \qquad \Phi_B = BA$$

Simple Alternator



Addison-Wesley 2014



Addison-Wesley 2014

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\Phi_B = BA\cos\phi$$

$$\phi = \omega t$$

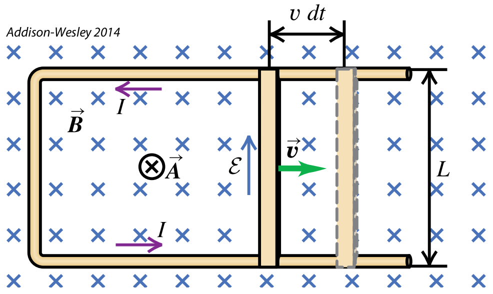
$$\boxed{\Phi_B = BA\cos(\omega t)}$$

$$\varepsilon = -\frac{d\,\Phi_B}{dt}$$

$$\varepsilon = -\frac{d}{dt}\,(BA\cos(\omega t))$$

$$\boxed{\varepsilon = \omega BA\sin(\omega t)}$$

Slidewire Generator



$$\varepsilon = -\frac{d\,\Phi_B}{dt}$$

$$\varepsilon = -\frac{d}{dt}(BA\cos\phi)$$

$$\varepsilon = -\cancelto{0}{\frac{dB}{dt}(A\cos\phi)} - \frac{dA}{dt}(B\cos\phi) - \cancelto{0}{\frac{d\phi}{dt}(BA\sin\phi)}$$

$$\varepsilon = -\frac{dA}{dt}(B\cos\phi)$$

$$dA = lv\,dt$$

$$\varepsilon = -\frac{lv\,dt}{dt}(B\cos\phi)$$

$$\boxed{\varepsilon = -Blv}$$

$$|\varepsilon| = iR$$

$$i = \frac{|\varepsilon|}{R}$$

$$P_{dissipated} = i^2R$$

$$P_{dissipated} = \left(\frac{Blv}{R}\right)^2 R$$

$$P_{dissipated} = \left(\frac{B^2l^2v^2}{R^{\cancel{2}}}\right)\cancel{R}$$

$$\boxed{P_{dissipated} = i^2R = \frac{B^2l^2v^2}{R}}$$

$$\vec{F} = i\vec{l} \times \vec{B}$$

$$F = ilB\sin\phi$$

$$F = ilB$$

$$F = \left(\frac{Blv}{R}\right)lB$$

$$\boxed{F = \frac{B^2l^2v}{R}}$$

$$P_{applied} = Fv$$

$$P_{applied} = \left(\frac{B^2l^2v}{R}\right)v$$

$$\boxed{P_{applied} = Fv = \frac{B^2l^2v^2}{R}}$$