Deterministic Generation Of Maximally Discordant Mixed States By Dissipation

Entanglement can be considered as a special quantum correlation, but not the only kind. It is allowed to exist non-classical correlations even for a separable quantum system. Here we propose two dissipative schemes for generating a maximally correlated state of two qubits in the absence of quantum entanglement, which was raised by [F. Galve, G. L. Giorgi, and R. Zambrini, \colorbluePhys. Rev. A 83, 012102 (2011)]. These protocols take full advantages of the interaction between four-level atoms and strongly lossy optical cavities. In the first scenario, we alternatively change the phases of Rabi frequencies of two classical driving fields, while the second proposal introduces a strongly lossy coupled-cavity system. Both schemes can realize all Lindblad terms required by the dissipative dynamics, guaranteeing the maximally quantum dissonant state to be the unique steady state for a certain subspace of system. Moreover, since the target state is a mixed state, the performance of our method is evaluated by the definition of super-fidelity G(1,2)subscript1subscript2G(\rho_1,\rho_2)italic_G (italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT , italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT), and the strictly numerical simulations indicate that fidelity outstripping 99%percent9999\%99 % of the quantum dissonant state is achievable with the current cavity quantum electrodynamics parameters.

As one of the most striking features in quantum theory, quantum entanglement is recognized as the essential resource for quantum information processing Knill and Laflamme (1998). For instance, it is widely used in quantum key distribution Ekert (1991), superdense coding Bennett and Wiesner (1992), quantum teleportation Bennett et al. (1993), and quantum computation Walther et al. (2005). Although the best performance of such tasks requires maximally entangled states (Bell states), the decoherence effects due to the environment make the pure entangled state into a statistical mixture and degrade quantum entanglement in reality. For these entangled mixed states, there are more complicated and lesser understood than pure states. Werner state is a typical mixed state defined by a class of two-body quantum mixtures, which is invariant under the unitary transformation Werner (1989). It has been used in the description of noisy quantum channels, such as nonadditivity claims, and in the study of deterministic purification Lyons et al. .

Quantum discord (QD), a measure of the total quantum correlations, is defined as the difference between the quantum mutual information and the classical correlations at the quantum level Ollivier and Zurek (2001). GAMING NEWS attempts to quantify all quantum correlations including entanglement. The study of QD has a crucial importance for the full development of new quantum technology because it is more robust than entanglement against the effects of decoherence Modi et al. (2012); Yune et al. (2015). Now it has been shown experimentally how to encode quantum information in separable Gaussian states, which introduces an operational protocol to use quantum discord as a resource even in the absence of entanglement Gu et al. (2012). In 2011, Glave et.al found some mixed states have greater values of QD than pure states Galve et al. (2011), and they identified the family

of mixed states which maximize the discord for a given value of the classical correlations. On the basis of this work, López et.al mathematically described a method to produce the maximally correlated states without entanglement López et al. (2017) and gave an example of unitary dynamic process, which places restrictions on the evolution time of system.

It is well known that the quantum dissipation characterized by a Lindblad generator in Markovian quantum master equations is induced by the weak coupling between quantum systems and environment. Traditionally, it has been considered only having detrimental effects on quantum information processing. Nevertheless, recent studies show that the environment can be used as a resource for quantum computation and entanglement generation Plenio et al. (1999); Plenio and Huelga (2002); Vacanti and Beige (2009); Kastoryano et al. (2011); Lin et al. (2013); Carr and Saffman (2013); Shao et al. (2014); Shen et al. (2014); Shao et al. (2017); Shao (2018); Li and Shao (2018, 2019); Su et al. (2014, 2015); Qin et al. (2018); Chen et al. (2018). In particular, Kastoryano et.al Kastoryano et al. (2011) discussed the possibility of preparing highly entangled states via the loss of photon from an optical cavity. In Ref. Carr and Saffman (2013), the authors proposed a dissipative scheme to generate a maximally entanglement between two Rydberg atoms, where the spontaneous emissions of atoms play a positive role during the dynamic evolution. And Emanuele et.al presented and analyzed a new approach for the generation of atomic spin-squeezed states using the interaction between four-level atoms and a single-mode cavity Dalla Torre et al. (2013).

Enlightened by the work of Ref. Dalla Torre et al. (2013), we construct two physical models by taking the environment as a resource to generate the maximally quantum dissonant state. This approach has the following advantages: (i) Compared with the unitary dynamic evolution, the dissipative process is not limited by time. (ii) The initial state is not strictly required by both schemes, and the target state can be successfully prepared as long as the state |-=1/2(|01-|10)ketsuperscript12ket01ket10|\Psi^-\rangle=1/\sqrt2(|01\rangle-|10\rangle)| roman_ start_POSTSUPERSCRIPT - end_POSTSUPERSCRIPT = 1 / square-root start_ARG 2 end_ARG (| 01 - | 10) is not populated initially. (iii) The investigated systems make full use of the cavity decay \kappaitalic_ while suppress the spontaneous emission \gammaitalic_ of atoms. Therefore, the parameters \kappaitalic_ and \gammaitalic_ are permitted to have a wide range of values to improve the experimental feasibility.

The remainder of the paper is organized as follows. In Sec. II, we briefly review the properties of maximally correlation states. In Sec. III, we construct one physical model with a pair of four-level atoms trapped in a strongly loss optical cavity. Under the large decay of cavity and alternatively changing the Rabi frequencies of classical fields, we derive an effective master equation and numerically simulate the effects of relevant parameters on the prepared state. In Sec. IV, we introduce another physical model which requires a coupled-cavity with atoms separately trapped in each cavity. In Sec. V and Sec. VI we discuss the potential experimental feasibility and make a brief summary of the work, respectively.

II Brief Review of the maximally discordant mixed states

The states we are interested in are found within the set of separable states. It has been shown that the most nonclassical two-qubit states, i.e., the family with maximal quantum discord versus classical correlations, were formed by mixed states of rank 2 and 3, which are named maximally discordant mixed states (MDMS). The class of states of rank 3 thus defined by Galve et al. (2011)

=|++|+(1-)[x|0101|+(1-x)|1010|],italic-ketsuperscriptbrasuperscript1italic-delimited-[]ket01quantum-operator-product01110bra10\rho=\epsilon|\Phi^+\rangle\langle\Phi^+|+(1-\epsilon)[x|01\rangle\langle 0% 1|+(1-x)|10\rangle\langle 10|],italic_ = italic_ | roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT | + (1 - italic_) [italic_x | 01 01 | + (1 - italic_x) | 10 10 |] , (1)
where |+=(|00+|11)/2ketsuperscriptket00ket112|\Phi^+\rangle=(|00\rangle+|11\rangle)/\sqrt2| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT = (| 00 + | 11) / square-root start_ARG 2 end_ARG.

The definition of quantum correlation is I=S(A)+S(B)-S(AB)subscriptsubscriptsubscriptI=S(\rho_A)+S(\rho_B)-S(\rho_AB)italic_I = italic_S (italic_ start_POSTSUBSCRIPT italic_A end_POSTSUBSCRIPT) + italic_S (italic_ start_POSTSUBSCRIPT italic_B end_POSTSUBSCRIPT) - italic_S (italic_ start_POSTSUBSCRIPT italic_A italic_B end_POSTSUBSCRIPT) and the classical correlation is C(AB)=S(A)-S(AsubscriptsubscriptsubscriptconditionalC(\rho_AB)=\bf max\S(\rho_A)-S(\rho_B)\italic_C (italic_ start_POSTSUBSCRIPT italic_A italic_B end_POSTSUBSCRIPT) = bold_max italic_B end_POSTSUBSCRIPT) , where S(A|B)subscriptconditionalS(\rho_B)italic_S (italic_ start_POSTSUBSCRIPT italic_A | italic_B end_POSTSUBSCRIPT) is the conditional entropy of AAitalic_A given a measurement on the system BBitalic_B while the optimization is over all possible projective measurement on system BBitalic_B. Refer to Ali-Rau-Alber results of the conditional entropy Ali et al. (2010), it is not difficult to find that for x=1/212x=1/2italic_x = 1 / 2 and =1/3italic-13\epsilon=1/3italic_ = 1 / 3 the quantum mutual information is maximized and the classical correlation is minimized. By changing the basis vector of the second qubit, i.e. |0|1ket0ket1|0\rangle\leftrightarrow|1\rangle| 0 | 1 , we obtain the target state:

=13(|++|+|0000|+|1111|),13ketsuperscriptbrasuperscriptket00bra00ket11bra11\rho=\frac13(|\Psi^+\rangle\langle\Psi^+|+|00\rangle\langle 00|+|11% \rangle\langle 11|),italic_ = divide start_ARG 1 end_ARG start_ARG 3 end_ARG (| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT | + | 00 00 | + | 11 11 |) , (2)
where |+=(|01+|10)/2ketsuperscriptket01ket102|\Psi^+\rangle=(|01\rangle+|10\rangle)/\sqrt2| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT = (| 01 + | 10) / square-root start_ARG 2 end_ARG.

Using the basis of Bell states |±=1/2(|00±|11),|±=1/2(|01±|10)formulae-

sequenceketsuperscriptplus-or-minus12plus-or-minusket00ket11ketsuperscriptplus-or-minus12plus-or-minusket01ket10|\Phi^\pm\rangle=1/\sqrt2(|00\rangle\pm|11\rangle),|\Psi^\pm\rangle=1/%\sqrt2(|01\rangle\pm|10\rangle)| roman_ start_POSTSUPERSCRIPT ± end_POSTSUPERSCRIPT = 1 / square-root start_ARG 2 end_ARG (| 00 ± | 11) , | roman_ start_POSTSUPERSCRIPT ± end_POSTSUPERSCRIPT = 1 / square-root start_ARG 2 end_ARG (| 01 ± | 10) López et al. (2017), the target state could be rewritten as:

=13(|++|+|--|+|++|).13ketsuperscriptbrasuperscriptketsuperscriptbrasuperscriptketsuperscriptbrasuperscript\rho=\frac13(|\Phi^+\rangle\langle\Phi^+|+|\Phi^-\rangle\langle\Phi^%-|+|\Psi^+\rangle\langle\Psi^+|).italic_ = divide start_ARG 1 end_ARG start_ARG 3 end_ARG (| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT | + | roman_ start_POSTSUPERSCRIPT - end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT - end_POSTSUPERSCRIPT | + | roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT |) . (3)
If we have a system characterized by the following master equation

=x(Sx)+y(z)(Sy(z)),subscriptsubscriptsubscriptsubscriptsubscriptsubscript\dot\rho=\mathscrL_\gamma_x(S_x)\rho+\mathscrL_\gamma_y(z)(S_%y(z))\rho,over start_ARG italic_ end_ARG = script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) italic_ + script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y (italic_z) end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y (italic_z) end_POSTSUBSCRIPT) italic_ , (4)
where Sx=(x1+x2),Sy=(y1+y2)formulae-sequencesubscriptsuperscriptsubscript1superscriptsubscript2subscriptsuperscriptsubscript1superscriptsubscript2S_x=(\sigma_x^1+\sigma_x^2),S_y=(\sigma_y^1+\sigma_y^2)italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT = (italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT) , italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT = (italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT), and
Sz=(z1+z2)subscriptsuperscriptsubscript1superscriptsubscript2S_z=(\sigma_z^1+\sigma_z^2)italic_S start_POSTSUBSCRIPT italic_z end_POSTSUBSCRIPT = (italic_ start_POSTSUBSCRIPT italic_z end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_ start_POSTSUBSCRIPT italic_z end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT)

(x,y,zsubscript\sigma_x,y,zitalic_ start_POSTSUBSCRIPT italic_x , italic_y , italic_z end_POSTSUBSCRIPT are spin operators), the state described by Eq. (3) will be the steady state of this system. However, it is difficult to find a natural system with the above form of the master equation. Thus we consider to design an effective physical model which is equivalent to Eq. (4) under the appropriate approximations, and we will discuss our method detailedly in the next section.

III two four-level atoms in a lossy cavity

The central idea of our work can be understood by considering a pair of atoms interacting with a strongly loss optical cavity characterized in Fig. 1. The atoms are simultaneously driven by the laser fields with complex Rabi frequencies 1(2)ei1(2)subscript12superscriptsubscript12\Omega_1(2)e^i\varphi_1(2)roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT end_POSTSUPERSCRIPT and the quantum field with coupling strength ggitalic_g. The Hamiltonian under the Schrödinger picture can be written as (=1Planck-constant-over-2-pi1\hbar=1roman_ = 1):

Hssubscript\displaystyle H_sitalic_H start_POSTSUBSCRIPT italic_s end_POSTSUBSCRIPT =\displaystyle== H0+Vs,subscript0subscript\displaystyle H_0+V_s,italic_H start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT + italic_V start_POSTSUBSCRIPT italic_s end_POSTSUBSCRIPT , (5)

H0subscript0\displaystyle H_0italic_H start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT =\displaystyle== i=120|0i0|+1|1i1|+e|eie|superscriptsubscript12subscript0subscriptket0subscriptquantum-operator-product0subscript11subscriptquantum-operator-product1subscriptbra\displaystyle\sum_i=1^2\omega_0|0\rangle_i\langle 0|+\omega_1|1%\rangle_i\langle 1|+\omega_e|e\rangle_i\langle e| start_POSTSUBSCRIPT italic_i = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT | 0 start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | + italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT | 1 start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | + italic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT | italic_e start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT italic_e |

+r|rir|+a†a,subscriptsubscriptketbrasuperscript†\displaystyle+\omega_r|r\rangle_i\langle r|+ua^\daggera,+ italic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT | italic_r start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT italic_r | + italic_ italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a ,

Vssubscript\displaystyle V_sitalic_V start_POSTSUBSCRIPT italic_s end_POSTSUBSCRIPT =\displaystyle== i=12g(|ei0|+|ri1|)a+1ei1|ei1|e-

i1tsuperscriptsubscript12subscriptketbra0subscriptketbra1subscript1superscriptsubscript1subscriptketbra1superscriptsubscript1\displaystyle\sum_i=1^2g(|e\rangle_i\langle 0|+|r\rangle_i\langle 1|)a% +\Omega_1e^i\varphi_1|e\rangle_i\langle 1|e^-i\mu_1t start_POSTSUBSCRIPT italic_i = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g (| italic_e start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | + | italic_r start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 |) italic_a + roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | italic_e start_POSTSUPERSCRIPT - italic_i italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT

+2ei2|ri0|e-i2t+H.c.,formulae-sequencesubscript2superscriptsubscript2subscriptketbra0superscriptsubscript2Hc\displaystyle+\Omega_2e^i\varphi_2|r\rangle_i\langle 0|e^-i\mu_2t+% \rm H.c.,+ roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | italic_e start_POSTSUPERSCRIPT - italic_i italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_H . roman_c . , where 0subscript0\omega_0italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT, 1subscript1\omega_1italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT, esubscript\omega_eitalic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT, and rsubscript\omega_ritalic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT are the eigenfrequencies of the lower states |0ket0|0\rangle| 0 , |1ket1|1\rangle| 1 and upper states |eket|e\rangle| italic_e , |rket|r\rangle| italic_r , respectively, while uitalic_ and 1(2)subscript12\mu_1(2)italic_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT are the frequencies of quantum and classical fields. a†superscript†a^\daggeritalic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT and aaitalic_a are the creation and annihilation operators of the optical cavity mode. In addition, the ground states transition is dipole-forbidden. For simplicity, we assume all parameters are real. In the interaction picture, the Hamiltonian of the system reads:

HI=H1+H2,subscriptsubscript1subscript2\displaystyle H_I=H_1+H_2,italic_H start_POSTSUBSCRIPT italic_I end_POSTSUBSCRIPT = italic_H start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT + italic_H start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT , (6)

H1=i=12ga|ei0|ei1t+1ei1|ei1|ei2t+H.c.,formulae-sequencesubscript1superscriptsubscript12subscriptketsubscriptquantum-operator-product0superscriptsubscript1subscript1superscriptsubscript1bra1superscriptsubscript2Hc\displaystyle H_1=\sum_i=1^2ga|e\rangle_i\langle 0|e^i\delta_1t+% \Omega_1e^i\varphi_1|e\rangle_i\langle 1|e^i\delta_2t+\rm H.c.,italic_H start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = start_POSTSUBSCRIPT italic_i = 1

end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g italic_a | italic_e start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_H . roman_c . ,

H2=i=12ga|ri1|ei1t+2ei2|ri0|ei2t+H.c.,formulae-sequencesubscript2superscriptsubscript12subscriptketsubscriptquantum-operator-product1superscriptsubscriptsuperscript1subscript2superscriptsubscript2bra0superscriptsubscriptsuperscript2Hc\displaystyle H_2=\sum_i=1^2ga|r\rangle_i\langle 1|e^i\delta^\prime%_1t+\Omega_2e^i\varphi_2|r\rangle_i\langle 0|e^i\delta^\prime_2% t+\rm H.c.,italic_H start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = start_POSTSUBSCRIPT italic_i = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g italic_a | italic_r start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_H . roman_c . , where 1(2)=e-0(1)-(1)subscript12subscriptsubscript01subscript1\delta_1(2)=\omega_e-\omega_0(1)-u(\mu_1)italic_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT 0 (1) end_POSTSUBSCRIPT - italic_ (italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT) and 1(2)=r-1(0)-(2)subscriptsuperscript12subscriptsubscript10subscript2\delta^\prime_1(2)=\omega_r-\omega_1(0)-u(\mu_2)italic_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT 1 (0) end_POSTSUBSCRIPT - italic_ (italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT). We further suppose 1=2=1subscript1subscript2subscript1\delta_1=\delta_2=\Delta_1italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT and 1=2=2subscriptsuperscript1subscriptsuperscript2subscript2\delta^\prime_1=\delta^\prime_2=\Delta_2italic_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT. Now we consider the process of

constructing the collective decay operator Sy=y1+y2subscriptsuperscriptsubscript1superscriptsubscript2S_y=\sigma_y^1+\sigma_y^2italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT. Taking 1ei1=-i1subscript1superscriptsubscript1subscript1\Omega_1e^i\varphi_1=-i\Omega_1roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT = - italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT, and 2ei2=i2subscript2superscriptsubscript2subscript2\Omega_2e^i\varphi_2=i\Omega_2roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT end_POSTSUPERSCRIPT = italic_i roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT, and in the regime of large detuning |1(2)|g,1(2)much-greater-thansubscript12subscript12|\Delta_1(2)|\gg\g,\Omega_1(2)\| roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT | italic_g , roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT , we may safely eliminate the upper states |eket|e\rangle| italic_e and |rket|r\rangle| italic_r , then the above Hamiltonian reduces to

Heffsubscripteff\displaystyle H_\textmdeffitalic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT =\displaystyle== Heff1+Heff2,superscriptsubscripteff1superscriptsubscripteff2\displaystyle H_\textmdeff^1+H_\textmdeff^2,italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT , (7) where

Heff1superscriptsubscripteff1\displaystyle H_\textmdeff^1italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT =\displaystyle== G1J-a†+H.c.+i=12geff1a†a|0i0|+eff1|1i1|,formulae-sequencesubscript1subscriptsuperscript†Hcsuperscriptsubscript12superscriptsubscripteff1superscript†subscriptket0subscriptquantum-operator-product0superscriptsubscripteff11bra1\displaystyle G_1J_-a^\dagger+\rm H.c.+\sum_i=1^2g_\textmdeff% ^1a^\daggera|0\rangle_i\langle 0|+\Omega_\textmdeff^1|1\rangle_% i\langle 1|,italic_G start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT + roman_H . roman_c . + start_POSTSUBSCRIPT italic_i = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a | 0

start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | + roman_ start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT | 1 start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | ,

Heff2superscriptsubscripteff2\displaystyle H_\textmdeff^2italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT =\displaystyle== G2J+a†+H.c.+i=12geff2a†a|1i1|+eff2|0i0|,formulae-sequencesubscript2subscriptsuperscript†Hcsuperscriptsubscript12superscriptsubscripteff2superscript†subscriptket1subscriptquantum-operator-product1superscriptsubscripteff20bra0\displaystyle G_2J_+a^\dagger+\rm H.c.+\sum_i=1^2g_\textmdeff% ^2a^\daggera|1\rangle_i\langle 1|+\Omega_\textmdeff^2|0\rangle_% i\langle 0|,italic_G start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT + roman_H . roman_c . + start_POSTSUBSCRIPT italic_i = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a | 1 start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 1 | + roman_ start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT | 0 start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT 0 | ,

where G1(2)=g1(2)/1(2)subscript12subscript12subscript12G_1(2)=g\Omega_1(2)/\Delta_1(2)italic_G start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT = italic_g roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT, geff1(2)=-g2/1(2)superscriptsubscripteff12superscript2subscript12g_\textmdeff^1(2)=-g^2/\Delta_1(2)italic_g start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 (2) end_POSTSUPERSCRIPT = - italic_g start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT / roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT, and eff1(2)=1(2)2/1(2)superscriptsubscripteff12superscriptsubscript122subscript12\Omega_\textmdeff^1(2)=\Omega_1(2)^2/\Delta_1(2)roman_ start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 (2) end_POSTSUPERSCRIPT = roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT / roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT. J+=-i(|110|+|120|)subscriptsubscriptket11bra0subscriptket12bra0J_+=-i(|1\rangle_1\langle 0|+|1\rangle_2\langle 0|)italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT = - italic_i (| 1 start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT 0 | + | 1 start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 0 |) and J-=i(|011|+|021|)subscriptsubscriptket01bra1subscriptket02bra1J_-=i(|0\rangle_1\langle 1|+|0\rangle_2\langle 1|)italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT = italic_i (

| 0 start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT 1 | + | 0 start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 1 |) are the collective ascending and descending operators. If we further assume G1=G2=Gsubscript1subscript2G_1=G_2=Gitalic_G start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_G start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = italic_G, i.e. 1/1=2/2subscript1subscript1subscript2subscript2\Omega_1/\Delta_1=\Omega_2/\Delta_2roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT, and omit the Stark shifts of the ground states induced by the quantum field and the classical fields, the above Hamiltonian could be simplified as

Heff=G(J-+J+)a†+H.c..formulae-sequencesubscripteffsubscriptsubscriptsuperscript†HcH_\textmdeff=G(J_-+J_+)a^\dagger+\rm H.c..italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT = italic_G (italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT + italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT) italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT + roman_H . roman_c . . (8)
Since the effective system only includes the ground states, the spontaneous emissions of atoms are greatly suppressed, and the master equation could be written as

\displaystyle\dot\rhoover start_ARG italic_ end_ARG =\displaystyle== -i[G(J-+J+)a†+G(J-+J+)a,]subscriptsubscriptsuperscript†subscriptsubscript\displaystyle-i[G(J_-+J_+)a^\dagger+G(J_-+J_+)a,\rho]- italic_i [italic_G (italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT + italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT) italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT + italic_G (italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT + italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT) italic_a , italic_] (9)

+2(2aa†-a†a-a†a).22superscript†superscript†superscript†\displaystyle+\frac\kappa2(2a\rhoa^\dagger-a^\daggera\rho-% \rhoa^\daggera).+ divide start_ARG italic_ end_ARG start_ARG 2 end_ARG (2 italic_a italic_ italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT - italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a italic_ - italic_ italic_a start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a) .
In the limitation of large decay rate Gmuch-greater-than\kappa\ggGitalic_ italic_G, the cavity mode can also be neglected, and we obtain the master equation characterizing the system of atoms as:

=(D[J-]+D[J+])=y(Sy),delimited-[]subscriptdelimited-[]subscriptsubscriptsubscriptsubscript\dot\rho=\Gamma(D[J_-]\rho+D[J_+]\rho)=\mathscrL_\gamma_y(S_y)\rho,over start_ARG italic_ end_ARG = roman_ (italic_D [italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT] italic_ + italic_D [italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT] italic_) = script_L

start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT) italic_ , (10)
where =4G2/4superscript2\Gamma=4G^2/\kapparoman_ = 4 italic_G start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT / italic_ is the collective decay rate of the atoms, and the superoperator DDitalic_D is defined as D[O]=(2OO†-O†O-O†O)/2delimited-[]2superscript†superscript†superscript†2D[O]=(2O\rhoO^\dagger-O^\daggerO\rho-\rhoO^\daggerO)/2italic_D [italic_O] = (2 italic_O italic_ italic_O start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT - italic_O start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_O italic_ - italic_ italic_O start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_O) / 2.

Otherwise, if we attempt to construct the collective decay operator Sx=x1+x2subscriptsuperscriptsubscript1superscriptsubscript2S_x=\sigma_x^1+\sigma_x^2italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 1 end_POSTSUPERSCRIPT + italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT, we may simply take 1=2=0subscript1subscript20\varphi_1=\varphi_2=0italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = 0, then after a series of similar derivations, the effective master equation reads

=(D[J-]+D[J+])=x(Sx),delimited-[]superscriptsubscriptdelimited-[]superscriptsubscriptsubscriptsubscriptsubscript\dot\rho=\Gamma(D[J_-^\prime]\rho+D[J_+^\prime]\rho)=% \mathscrL_\gamma_x(S_x)\rho,over start_ARG italic_ end_ARG = roman_ (italic_D [italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT start_POSTSUPERSCRIPT start_FLOATSUPERSCRIPT end_FLOATSUPERSCRIPT end_POSTSUPERSCRIPT] italic_ + italic_D [italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT start_POSTSUPERSCRIPT start_FLOATSUPERSCRIPT end_FLOATSUPERSCRIPT end_POSTSUPERSCRIPT] italic_) = script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) italic_ , (11)
where J+=|110|+|120|superscriptsubscriptsubscriptket11bra0subscriptket12bra0J_+^\prime=|1\rangle_1\langle 0|+|1\rangle_2\langle 0|italic_J start_POSTSUBSCRIPT + end_POSTSUBSCRIPT start_POSTSUPERSCRIPT start_FLOATSUPERSCRIPT end_FLOATSUPERSCRIPT end_POSTSUPERSCRIPT = | 1 start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT 0 | + | 1 start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 0 | and J-=|011|+|021|superscriptsubscriptsubscriptket01bra1subscriptket02bra1J_-^\prime=|0\rangle_1\langle 1|+|0\rangle_2\langle 1|italic_J start_POSTSUBSCRIPT - end_POSTSUBSCRIPT start_POSTSUPERSCRIPT start_FLOATSUPERSCRIPT end_FLOATSUPERSCRIPT end_POSTSUPERSCRIPT = | 0 start_POSTSUBSCRIPT 1

end_POSTSUBSCRIPT 1 | + | 0 start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 1 |.

Up to present, we have shown how to generate the collective operators SxsubscriptS_xitalic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT and SysubscriptS_yitalic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT respectively. But the stability of Eq. (3) requires there should be x(Sx)subscriptsubscriptsubscript\mathscrL_\gamma_x(S_x)script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) and y(Sy)subscriptsubscriptsubscript\mathscrL_\gamma_y(S_y)script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT) in the master equation at the same time. Fortunately, drawing lessons from the spin echoes effect, our model is able to simulate the effective master equation of Eq. (4) apart from a coefficient 1/2121/21 / 2, as long as the phases of the classical fields 1subscript1\varphi_1italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT and 2subscript2\varphi_2italic_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT are interchanged fast enough. The result is obtained by using the Trotter product formula (see Corollary 5.85.85.85.8 in Chap. III of Ref. Engel and Nagel (2000))

limN[ex(Sx)T2/Ney(Sy)T2/N]N=e12[x(Sx)+y(Sy)]T,subscriptsuperscriptdelimited-[]superscriptsubscriptsubscriptsubscript2superscriptsubscriptsubscriptsubscript2superscript12delimited-[]subscriptsubscriptsubscriptsubscriptsubscript\lim_N\rightarrow\infty[e^\mathscrL_\gamma_x(S_x)\fracT2/Ne^%
\mathscrL_\gamma_y(S_y)\fracT2/N]^N=e^\frac12[\mathscrL_%
\gamma_x(S_x)+\mathscrL_\gamma_y(S_y)]T,roman_lim start_POSTSUBSCRIPT italic_N end_POSTSUBSCRIPT [italic_e start_POSTSUPERSCRIPT script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) divide start_ARG italic_T end_ARG start_ARG 2 end_ARG / italic_N end_POSTSUPERSCRIPT italic_e start_POSTSUPERSCRIPT script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT) divide start_ARG italic_T end_ARG start_ARG 2 end_ARG / italic_N end_POSTSUPERSCRIPT] start_POSTSUPERSCRIPT italic_N end_POSTSUPERSCRIPT = italic_e start_POSTSUPERSCRIPT divide start_ARG 1 end_ARG start_ARG 2 end_ARG [script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) + script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT)] italic_T end_POSTSUPERSCRIPT , (12)
where TTitalic_T is the total evolution time. Then the effective master equation is

=12[x(Sx)+y(Sy)].12delimited-[]subscriptsubscriptsubscriptsubscriptsubscriptsubscript\dot\rho=\frac12[\mathscrL_\gamma_x(S_x)+\mathscrL_\gamma_y% (S_y)]\rho.over start_ARG italic_ end_ARG = divide start_ARG 1 end_ARG start_ARG 2 end_ARG [script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) + script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT)] italic_ . (13)

Fig. 2 shows the population of |+ketsuperscript|\Psi^+\rangle| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT under different evolution processes with initial state |00|0cket00subscriptket0|00\rangle|0\rangle_c| 00 | 0 start_POSTSUBSCRIPT italic_c end_POSTSUBSCRIPT. The evolution of the effective master equation (13) is shown with empty circles and the other lines are the switching evolution results of the master equation with Hamiltonian (6) together considering the cavity decay \kappaitalic_. The total evolution time is gt=80008000gt=8000italic_g italic_t = 8000. Different lines correspond to the results with different switching number NNitalic_N. With the increasing of NNitalic_N, line gets nearly to the empty circles. But since the cavity decay is a resource for states generation, the switching number NNitalic_N has an upper limit keeping the interval time much larger than 1/11/\kappa1 / italic_, which guarantees the role of \kappaitalic_ in each process. And the longer operation time 1/11/\Gamma1 / roman_ determines the minimum value of NNitalic_N ensuring the interval time far less than 1/11/\Gamma1 / roman_. Thus we choose N=200,gt=8000formulae-sequence2008000N=200,gt=8000italic_N = 200 , italic_g italic_t = 8000 in the following simulations.

In quantum information theory, distinguishing two quantum states is a fundamental task. One of the main tools used in distinguishability theory is quantum fidelity Nilsen and Chuang (2000); Chen et al. (2011), which is widely used and has been found applications in solving some problems like quantifying entanglement Vedral et al. (1997); Vedral and Plenio (1998), quantum error correction Kosut et al. (2008), quantum chaos Giorda and Zanardi (2010) and so on. In order to measure the distance between quantum states including mixed states, we adopt the definition of super-fidelity Miszczak et al. (2009)

G(,)\displaystyle G(\rho,\sigma)italic_G (italic_ , italic_) =\displaystyle== Tr[(t)]+[1-Tr(t)2](1-Tr2),Trdelimited-[]delimited-[]1Trsuperscript21Trsuperscript2\displaystyle\textmdTr[\rho(t)\sigma]+\sqrt[1-\textmdTr\rho(t)^2](1-% \textmdTr\sigma^2),Tr [italic_ (italic_t) italic_] + square-root start_ARG [1 - Tr italic_ (italic_t) start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT] (1 - Tr italic_ start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT) end_ARG , (14) with \sigmaitalic_ being the density operator of the target state as =1/3(|++|+|0000|+|1111|)13ketsuperscriptbrasuperscriptket00bra00ket11bra11\sigma=1/3(|\Psi^+\rangle\langle\Psi^+|+|00\rangle\langle 00|+|11\rangle% \langle 11|)italic_ = 1 / 3 (|

roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT | + | 00 00 | + | 11 11 |). We initialize the system into state |00|0cket00subscriptket0|00\rangle|0\rangle_c| 00 | 0 start_POSTSUBSCRIPT italic_c end_POSTSUBSCRIPT and plot the fidelity of the target-state under the switching evolution of the master equations with full Hamiltonian (6). Figure 3(a), 3(b), and 3(c) respectively discuss the effects of parameters ,\kappa,\gammaitalic_ , italic_, and \Omegaroman_ on the preparation of the target state. Fig. 3(a) shows the fidelity as a function of the cavity decay \kappaitalic_ with parameters g=1,1,2=100gformulae-sequence1subscript12100g=1,\Delta_1,2=100gitalic_g = 1 , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 100 italic_g, and 1,2=0.5gsubscript120.5\Omega_1,2=0.5groman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.5 italic_g. The increase of \kappaitalic_ will prolong the convergence time. It can be explained by Eq. (10). To obtain the target state, the coupling strength 4G2/4superscript24G^2/\kappa4 italic_G start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT / italic_ will increase as \kappaitalic_ decreasing, which results in a short convergence time. But if \kappaitalic_ is too small, it will destroy the condition Gmuch-greater-than\kappa\gg Gitalic_ italic_G and fail to generate the target state.

In Fig. 3(b), we take into account the spontaneous emissions of the atoms and plot the evolution of the target state with different \gammaitalic_. Even if \gammaitalic_ is extremely large (g)similar-to(\gamma\sim g)(italic_ italic_g), the fidelity is still above 0.990.990.990.99, which demonstrates that our scheme have favorable resistance to spontaneous emission. The inset picture of Fig. 3(b) is the enlarge view of the part indicated by the arrow. It shows that as =00.1g00.1\gamma=0\thicksim 0.1gitalic_ = 0 0.1 italic_g, the spontaneous emission is a positive factor which successfully enhances the fidelity by dissipating some high-level items to the ground states. But when \gammaitalic_ increases to ggitalic_g, the excessive spontaneous emission becomes negative to our model. In addition, the population keeps oscillating at the final time with small amplitude, and stays around a definite value.

Moreover, the convergence time is related to the intensity of the classical field 1(2)subscript12\Omega_1(2)roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT. Fig. 3(c) displays the evolution curves under different \Omegaroman_ with =0.1g0.1\gamma=0.1gitalic_ = 0.1 italic_g and discusses the optimal parameter range of Rabi frequency. The figure shows the optimal range of \Omegaroman_ is about 0.3g0.7g0.30.70.3g\backsim 0.7g0.3 italic_g 0.7 italic_g, which could ensure the fidelity over 0.99. Fig. 3(d) additionally considers the request to the initial state of the system. We can obtain the target state with arbitrary initial state except for the singlet state |-ketsuperscript|\Psi^-\rangle| roman_ start_POSTSUPERSCRIPT - end_POSTSUPERSCRIPT .

To expound the properties peculiar to the target state, we plot the concurrence Wootters (2001), classical correlation and quantum discord of the state with the full master equation in Fig. 4. It is worth mentioning that we directly utilize the results given in Ref. Altintas et al.

(2012) to measure quantum discord (QD), and the calculation of S(A|B)subscriptconditionalS(\rho_A)italic_S (italic_ start_POSTSUBSCRIPT italic_A | italic_B end_POSTSUBSCRIPT) is based on the positive-operator-valued measurements (POVM) locally performed on the subsystem B. The QD and the classical correlation (CC) are given as: QD()=Q1,Q2subscript1subscript2QD(\rho)=\bf min\Q_1,Q_2\italic_Q italic_D (italic_) = bold_min italic_Q start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT , italic_Q start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT , CC()=CC1,CC2subscript1subscript2CC(\rho)=\bf max\CC_1,CC_2\italic_C italic_C (italic_) = bold_max italic_C italic_C start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT , italic_C italic_C start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT , where CCj=h[11+22]-Djsubscriptdelimited-[]subscript11subscript22subscriptCC_j=h[\rho_11+\rho_22]-D_jitalic_C italic_C start_POSTSUBSCRIPT italic_j end_POSTSUBSCRIPT = italic_h [italic_ start_POSTSUBSCRIPT 11 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 22 end_POSTSUBSCRIPT] - italic_D start_POSTSUBSCRIPT italic_j end_POSTSUBSCRIPT and Qj=h[11+33]+k=14klog2k+Djsubscriptdelimited-[]subscript11subscript33superscriptsubscript14subscriptsubscriptlog2subscriptsubscriptQ_j=h[\rho_11+\rho_33]+\sum_k=1^4\lambda_k\rm log_2\lambda_k% +D_jitalic_Q start_POSTSUBSCRIPT italic_j end_POSTSUBSCRIPT = italic_h [italic_ start_POSTSUBSCRIPT 11 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 33 end_POSTSUBSCRIPT] + start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 4 end_POSTSUPERSCRIPT italic_ start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT roman_log start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT + italic_D start_POSTSUBSCRIPT italic_j end_POSTSUBSCRIPT, with ksubscript\lambda_kitalic_ start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT being the eigenvalues of \rhoitalic_ and h[x]delimited-[]h[x]italic_h [italic_x] is the binary entropy defined as h[x]=-xlog2x-(1-x)log2(1-x)delimited-[]subscriptlog21subscriptlog21h[x]=-x\rm log_2x-(1-x)\rm log_2(1-x)italic_h [italic_x] = - italic_x roman_log start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_x - (1 - italic_x) roman_log start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT (1 - italic_x). D1=h[]subscript1delimited-[]D_1=h[\tau]italic_D start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_h [italic_], where =(1+[1-2(33+44)]2+4(|14|+|23|)2)/21superscriptdelimited-[]12subscript33subscript4424superscriptsubscript14subscript2322\tau=(1+\sqrt+)/2italic_ = (1 + square-root start_ARG [1 - 2 (italic_ start_POSTSUBSCRIPT 33 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 44 end_POSTSUBSCRIPT)] start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT + 4 (| italic_ start_POSTSUBSCRIPT 14 end_POSTSUBSCRIPT | + | italic_ start_POSTSUBSCRIPT 23 end_POSTSUBSCRIPT |) start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT end_ARG) / 2 and D2=-k=14kklog2kk-h[11+33]subscript2superscriptsubscript14subscriptsubscriptlog2subscriptdelimited-[]subscript11subscript33D_2=-\sum_k=1^4\rho_kk\rm log_2\rho_kk-h[\rho_11+\rho_33]italic_D start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = - start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 4 end_POSTSUPERSCRIPT italic_ start_POSTSUBSCRIPT italic_k italic_k

end_POSTSUBSCRIPT roman_log start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_k italic_k end_POSTSUBSCRIPT - italic_h [italic_ start_POSTSUBSCRIPT 11 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 33 end_POSTSUBSCRIPT]. Based on Fig. 4, the final state has the maximally quantum discord 1/3131/31 / 3 without entanglement and the classical relation reaches the minimum. The steady state is a maximally quantum dissonant state.

Fig. 5 discusses the effect of the switching number NNitalic_N, where the increasing of NNitalic_N smooths the evolution process. It also illustrates that a high fidelity over 0.980.980.980.98 can be obtained with a wide range of values for NNitalic_N. Even if N=44N=4italic_N = 4 the fidelity can still get over 0.990.990.990.99. Thus, in actual operations, we can properly reduce the value of NNitalic_N to simplify the experiment.

IV two atoms in a lossy coupled-cavity system

The lossy coupled-cavity system Irish et al. (2008); Cho et al. (2008); Liew and Savona (2013); Hartmann et al. (2008); Serafini et al. (2006) is shown in Fig. 6. It consists of two coupled cavities which respectively trapped a four-level atom with ground states |0,|1ket0ket1|0\rangle,|1\rangle| 0 , | 1 and excited states |e,|rketket|e\rangle,|r\rangle| italic_e , | italic_r . The transition between |0ket0|0\rangle| 0 (|1ket1|1\rangle| 1) and |eket|e\rangle| italic_e (|rket|r\rangle| italic_r) is coupled resonantly to the quantum field with coupling constant ggitalic_g, and other non-resonant transitions with detuning ±plus-or-minus\pm\Delta± roman_ are driven by classical field with Rabi frequencies 1(2)subscript12\Omega_1(2)roman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT and 1(2)subscriptsuperscript12\Omega^\prime_1(2)roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT. Thus, the Hamiltonian under the Schrödinger picture can be written as

H\displaystyle Hitalic_H =\displaystyle== H0+Vs,subscript0subscript\displaystyle H_0+V_s,italic_H start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT + italic_V start_POSTSUBSCRIPT italic_s end_POSTSUBSCRIPT , (15)

H0subscript0\displaystyle H_0italic_H start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT =\displaystyle== k=120|0k0|+1|1k1|+e|eke|superscriptsubscript12subscript0subscriptket0subscriptquantum-operator-product0subscript11subscriptquantum-operator-product1subscriptbra\displaystyle\sum_k=1^2\omega_0|0\rangle_k\langle 0|+\omega_1|1\rangle_k\langle 1|+\omega_e|e\rangle_k\langle e| start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT | 0 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT | 1 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 | + italic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT italic_e |

+r|rkr|+cak†ak,subscriptsubscriptketbrasubscriptsuperscriptsubscript†subscript\displaystyle+\omega_r|r\rangle_k\langle r|+\omega_ca_k^\daggera_k,+ italic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT italic_r | + italic_ start_POSTSUBSCRIPT italic_c end_POSTSUBSCRIPT italic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT ,

Vssubscript\displaystyle V_sitalic_V start_POSTSUBSCRIPT italic_s end_POSTSUBSCRIPT =\displaystyle== k=12g(|ek0|+|rk1|)ak+1|ek1|e-iLtsuperscriptsubscript12subscriptketbra0subscriptketbra1subscriptsuperscriptsubscript1subscriptketbra1superscriptsuperscriptsubscript\displaystyle\sum_k=1^2g(|e\rangle_k\langle 0|+|r\rangle_k\langle 1|)a% _k+\Omega_1^\prime|e\rangle_k\langle 1|e^-i\omega_L^\primet start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g (| italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |) italic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT + roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 | italic_e start_POSTSUPERSCRIPT - italic_i italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT italic_t end_POSTSUPERSCRIPT

+2|rk0|e-iLt-i1(|e11|-|e21|)e-iLtsuperscriptsubscript2subscriptketbra0superscriptsuperscriptsubscriptsubscript1subscriptket1bra1subscriptket2bra1superscriptsubscript\displaystyle+\Omega_2^\prime|r\rangle_k\langle 0|e^-i\omega_L^% \primet-i\Omega_1(|e\rangle_1\langle 1|-|e\rangle_2\langle 1|)e^-i% \omega_Lt+ roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | italic_e start_POSTSUPERSCRIPT - italic_i italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT italic_t end_POSTSUPERSCRIPT - italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT (| italic_e start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT 1 | - | italic_e start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 1 |) italic_e start_POSTSUPERSCRIPT - italic_i italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT

+i2(|r10|-|r20|)e-iLt+H.c.formulae-sequencesubscript2subscriptket1bra0subscriptket2bra0superscriptsubscriptHc\displaystyle+i\Omega_2(|r\rangle_1\langle 0|-|r\rangle_2\langle 0|)e^-% i\omega_Lt+\mathrmH.c.+ italic_i roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT (| italic_r start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT 0 | - | italic_r start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 0 |) italic_e start_POSTSUPERSCRIPT - italic_i italic_

start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT + roman_H . roman_c .

+A(a1†a2+a2†a1),superscriptsubscript1†subscript2superscriptsubscript2†subscript1\displaystyle+A(a_1^\daggera_2+a_2^\daggera_1),+ italic_A (italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT + italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT) ,
where isubscript\omega_iitalic_ start_POSTSUBSCRIPT italic_i end_POSTSUBSCRIPT (i=0,1,e,r)01(i=0,1,e,r)(italic_i = 0 , 1 , italic_e , italic_r) are the eigenfrequencies of ground and excited states for each atom, csubscript\omega_citalic_ start_POSTSUBSCRIPT italic_c end_POSTSUBSCRIPT is the frequency of quantum field, ak†superscriptsubscript†a_k^\daggeritalic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT and ak(k=1,2)subscript12a_k~(k=1,2)italic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT (italic_k = 1 , 2) are creation and annihilation operators of cavity mode kkitalic_k, Lsubscript\omega_Litalic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT and Lsuperscriptsubscript\omega_L^\primeitalic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT are frequencies of classical fields. Then we switch the Hamiltonian from Schrödinger picture to the interaction picture and obtain

HIsubscript\displaystyle H_Iitalic_H start_POSTSUBSCRIPT italic_I end_POSTSUBSCRIPT =\displaystyle== k=12g(|ek0|+|rk1|)ak+i1ei1t(-1)k|ek1|superscriptsubscript12subscriptketbra0subscriptketbra1subscriptsubscript1superscriptsubscript1superscript1subscriptketbra1\displaystyle\sum_k=1^2g(|e\rangle_k\langle 0|+|r\rangle_k\langle 1|)a% _k+i\Omega_1e^i\Delta_1t(-1)^k|e\rangle_k\langle 1| start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT italic_g (| italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |) italic_a start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT + italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT (- 1) start_POSTSUPERSCRIPT italic_k end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 | (16)

+i2ei2t(-1)k-1|rk0|+1e-i1t|ek1|subscript2superscriptsubscript2superscript11subscriptketsubscriptquantum-operator-product0superscriptsubscript1superscriptsubscript1bra1\displaystyle+i\Omega_2e^i\Delta_2t(-1)^k-1|r\rangle_k\langle 0|+% \Omega_1^\primee^-i\Delta_1t|e\rangle_k\langle 1|+ italic_i roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i roman_ start_POSTSUBSCRIPT 2

end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT (- 1) start_POSTSUPERSCRIPT italic_k - 1 end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT italic_e start_POSTSUPERSCRIPT - italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |

+2e-i2t|rk0|+H.c.+A(a1†a2+a2†a1),formulae-sequencesuperscriptsubscript2superscriptsubscript2subscriptketbra0Hcsuperscriptsubscript1†subscript2superscriptsubscript2†subscript1\displaystyle+\Omega_2^\primee^-i\Delta_2t|r\rangle_k\langle 0|+% \mathrmH.c.+A(a_1^\daggera_2+a_2^\daggera_1),+ roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT italic_e start_POSTSUPERSCRIPT - italic_i roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_t end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + roman_H . roman_c . + italic_A (italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT + italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT) ,
where 1=e-1-L=1+L-esubscript1subscriptsubscript1subscriptsubscript1superscriptsubscriptsubscript\Delta_1=\omega_e-\omega_1-\omega_L=\omega_1+\omega_L^\prime-% \omega_eroman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT - italic_ start_POSTSUBSCRIPT italic_e end_POSTSUBSCRIPT, 2=r-0-L=0+L-rsubscript2subscriptsubscript0subscriptsubscript0superscriptsubscriptsubscript\Delta_2=\omega_r-\omega_0-\omega_L=\omega_0+\omega_L^\prime-% \omega_rroman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT - italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT = italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT italic_L end_POSTSUBSCRIPT start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT - italic_ start_POSTSUBSCRIPT italic_r end_POSTSUBSCRIPT, and we suppose 1=2=subscript1subscript2\Delta_1=\Delta_2=\Deltaroman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT = roman_. Now we introduce a pair of delocalized bosonic modes in order to remove the localized modes as follows Serafini et al. (2006),

m1subscript1absent\displaystyle m_1\equivitalic_m start_POSTSUBSCRIPT 1

end_POSTSUBSCRIPT 12(a1-a2),m212subscript1subscript2subscript2absent\displaystyle\frac1\sqrt2(a_1-a_2),\ \ m_2\equivdivide start_ARG 1 end_ARG start_ARG square-root start_ARG 2 end_ARG end_ARG (italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT - italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT) , italic_m start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT 12(a1+a2).12subscript1subscript2\displaystyle\frac1\sqrt2(a_1+a_2).divide start_ARG 1 end_ARG start_ARG square-root start_ARG 2 end_ARG end_ARG (italic_a start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT + italic_a start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT) . (17)
Then we have

HIsubscript\displaystyle H_Iitalic_H start_POSTSUBSCRIPT italic_I end_POSTSUBSCRIPT =\displaystyle== k=12g2m1eiAt(-1)k-1(|ek0|+|rk1|)superscriptsubscript122subscript1superscriptsuperscript11subscriptketbra0subscriptketbra1\displaystyle\sum_k=1^2\fracg\sqrt2m_1e^iAt(-1)^k-1(|e\rangle_% k\langle 0|+|r\rangle_k\langle 1|) start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT divide start_ARG italic_g end_ARG start_ARG square-root start_ARG 2 end_ARG end_ARG italic_m start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i italic_A italic_t end_POSTSUPERSCRIPT (- 1) start_POSTSUPERSCRIPT italic_k - 1 end_POSTSUPERSCRIPT (| italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |) (18)

+g2m2e-iAt(|ek0|+|rk1|)+1e-it|ek1|2subscript2superscriptsubscriptketbra0subscriptketbra1subscriptsuperscript1superscriptsubscriptketbra1\displaystyle+\fracg\sqrt2m_2e^-iAt(|e\rangle_k\langle 0|+|r% \rangle_k\langle 1|)+\Omega^\prime_1e^-i\Delta t|e\rangle_k\langle 1|+ divide start_ARG italic_g end_ARG start_ARG square-root start_ARG 2 end_ARG end_ARG italic_m start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT - italic_i italic_A italic_t end_POSTSUPERSCRIPT (| italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |) + roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT - italic_i roman_ italic_t end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |

+2e-it|rk0|+i1eit(-1)k|ek1|subscriptsuperscript2superscriptsubscriptketsubscriptquantum-operator-product0subscript1superscriptsuperscript1bra1\displaystyle+\Omega^\prime_2e^-i\Delta t|r\rangle_k\langle 0|+i\Omega% _1e^i\Delta t(-1)^k|e\rangle_k\langle 1|+ roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT - italic_i roman_ italic_t end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_k

end_POSTSUBSCRIPT 0 | + italic_i roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i roman_ italic_t end_POSTSUPERSCRIPT (- 1) start_POSTSUPERSCRIPT italic_k end_POSTSUPERSCRIPT | italic_e start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 |

+i2eit(-1)k-1|rk0|+H.c..formulae-sequencesubscript2superscriptsuperscript11subscriptketbra0Hc\displaystyle+i\Omega_2e^i\Delta t(-1)^k-1|r\rangle_k\langle 0|+% \mathrmH.c..+ italic_i roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_e start_POSTSUPERSCRIPT italic_i roman_ italic_t end_POSTSUPERSCRIPT (- 1) start_POSTSUPERSCRIPT italic_k - 1 end_POSTSUPERSCRIPT | italic_r start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 | + roman_H . roman_c . .
We set A=A=\Deltaitalic_A = roman_ to guarantee the two-photon process resonance, and choose 1(2)=1(2)=subscript12subscriptsuperscript12\Omega_1(2)=\Omega^\prime_1(2)=\Omegaroman_ start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT = roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 (2) end_POSTSUBSCRIPT = roman_. Under the large detuning condition, i.e. ||,gmuch-greater-than|\Delta|\gg\left\\Omega,g\right\| roman_ | roman_ , italic_g , and neglecting the Stark-shift terms, the effective Hamiltonian reads

Heffsubscripteff\displaystyle H_\textmdeffitalic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT =\displaystyle== k=12g2m1(-i|0k1|+i|1k0|)superscriptsubscript122subscript1subscriptket0subscriptquantum-operator-product11bra0\displaystyle\sum_k=1^2\fracg\Omega\sqrt2\Deltam_1(-i|0\rangle_k% \langle 1|+i|1\rangle_k\langle 0|) start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT divide start_ARG italic_g roman_ end_ARG start_ARG square-root start_ARG 2 end_ARG roman_ end_ARG italic_m start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT (- italic_i | 0 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 | + italic_i | 1 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 |) (19)

+g2m2(|0k1|+|1k0|)+H.c..formulae-sequence2subscript2subscriptket0bra1subscriptket1bra0Hc\displaystyle+\fracg\Omega\sqrt2\Deltam_2(|0\rangle_k\langle 1|+|1% \rangle_k\langle 0|)+\mathrmH.c..+ divide start_ARG italic_g roman_ end_ARG start_ARG square-root start_ARG 2 end_ARG roman_ end_ARG italic_m start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT (| 0 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 1 | + | 1 start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT 0 |) + roman_H . roman_c . .
Based on the definition of collective spin operators Sx=(1x+2x),Sy=(1y+2y)formulae-sequencesubscriptsubscript1subscript2subscriptsubscript1subscript2S_x=(\sigma_1x+\sigma_2x),S_y=(\sigma_1y+\sigma_2y)italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT = (italic_ start_POSTSUBSCRIPT 1 italic_x

end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 2 italic_x end_POSTSUBSCRIPT) , italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT = (italic_ start_POSTSUBSCRIPT 1 italic_y end_POSTSUBSCRIPT + italic_ start_POSTSUBSCRIPT 2 italic_y end_POSTSUBSCRIPT), the effective Hamiltonian can be rewritten as

Heff=Gm1Sy+Gm2Sx+H.c.,formulae-sequencesubscripteffsubscript1subscriptsubscript2subscriptHcH_\textmdeff=Gm_1S_y+Gm_2S_x+\mathrmH.c.,italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT = italic_G italic_m start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT + italic_G italic_m start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT + roman_H . roman_c . , (20)
where G=g/22G=g\Omega/\sqrt2\Deltaitalic_G = italic_g roman_ / square-root start_ARG 2 end_ARG roman_. It can be seen that the current system only involves couplings between ground states and delocalized cavity modes.

Therefore, the dissipative dynamics of system can be considered as governed by the following master equation

=i[,Heff]+k=122(2mkmk†-mk†mk-mk†mk).subscripteffsuperscriptsubscript1222subscriptsuperscriptsubscript†superscriptsubscript†subscriptsuperscriptsubscript†subscript\dot\rho=i[\rho,H_\textmdeff]+\sum_k=1^2\frac\kappa2(2m_k\rho m% _k^\dagger-m_k^\daggerm_k\rho-\rho m_k^\daggerm_k).over start_ARG italic_ end_ARG = italic_i [italic_ , italic_H start_POSTSUBSCRIPT eff end_POSTSUBSCRIPT] + start_POSTSUBSCRIPT italic_k = 1 end_POSTSUBSCRIPT start_POSTSUPERSCRIPT 2 end_POSTSUPERSCRIPT divide start_ARG italic_ end_ARG start_ARG 2 end_ARG (2 italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT italic_ italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT - italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT italic_ - italic_ italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT start_POSTSUPERSCRIPT † end_POSTSUPERSCRIPT italic_m start_POSTSUBSCRIPT italic_k end_POSTSUBSCRIPT) . (21)
In the limit |G|much-greater-than\kappa\gg|G|italic_ | italic_G |, we can adiabatically eliminating the delocalized cavity modes, and obtain the effective master equation,

=x(Sx)+y(Sy).subscriptsubscriptsubscriptsubscriptsubscriptsubscript\dot\rho=\mathscrL_\gamma_x(S_x)\rho+\mathscrL_\gamma_y(S_y)\rho.over start_ARG italic_ end_ARG = script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) italic_ + script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT) italic_ . (22)

Compared with the previous model, the coupled-cavity system provides the mean to realize x(Sx)subscriptsubscriptsubscript\mathscrL_\gamma_x(S_x)script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_x end_POSTSUBSCRIPT) and y(Sy)subscriptsubscriptsubscript\mathscrL_\gamma_y(S_y)script_L start_POSTSUBSCRIPT italic_ start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT end_POSTSUBSCRIPT (italic_S start_POSTSUBSCRIPT italic_y end_POSTSUBSCRIPT) simultaneously. Thus the target state =1/3(|++|+|0000|+|1111|)13ketsuperscriptbrasuperscriptket00bra00ket11bra11\rho=1/3(|\Psi^+\rangle\langle\Psi^+|+|00\rangle\langle 00|+|11\rangle% \langle 11|)italic_ = 1 / 3 (| roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT roman_ start_POSTSUPERSCRIPT + end_POSTSUPERSCRIPT | + | 00 00 | + | 11 11 |) can be generated using the driven-dissipative dynamics.

To verify the effectiveness of our scheme in generating MDMS, we respectively plot the populations and the fidelity of the target state with the initial state |00|00cket00subscriptket00|00\rangle|00\rangle_c| 00 | 00 start_POSTSUBSCRIPT italic_c end_POSTSUBSCRIPT under the full and effective Hamiltonian in Fig. 7. We can find that these two lines perfectly coincide with each other and the state prepared by our scheme can maintain high fidelity after reaching steady state as 0.9986 in the case of steady state after about 13000/g1300013000/g13000 / italic_g. The selections of numerical simulation parameters are =0.1g0.1\kappa=0.1gitalic_ = 0.1 italic_g, =0.2g0.2\Omega=0.2groman_ = 0.2 italic_g, and =100g100\Delta=100groman_ = 100 italic_g.

Then we make the same discussions as Fig. 3 in the coupled-cavity system. The results are shown in Fig. 8, which show the similar behaviors of ,0,subscript0\kappa,\rho_0,\gammaitalic_ , italic_ start_POSTSUBSCRIPT 0 end_POSTSUBSCRIPT , italic_ and \Omegaroman_. Compared with the first scenario, the fidelity is higher and the final population is stable after a longer evolution time.

Now, we discuss about the basic elements that maybe candidate for the intended experiment. The possible realizations of these physical models could be set up in Rb87superscriptRb87^87\textmdRbstart_FLOATSUPERSCRIPT 87 end_FLOATSUPERSCRIPT Rb using the clock states |F=1,mF=0ketformulae-sequence1subscript0|F=1,m_F=0\rangle| italic_F = 1 , italic_m start_POSTSUBSCRIPT italic_F end_POSTSUBSCRIPT = 0 and |F=2,mF=0ketformulae-sequence2subscript0|F=2,m_F=0\rangle| italic_F = 2 , italic_m start_POSTSUBSCRIPT italic_F end_POSTSUBSCRIPT = 0 in the 5S1/25subscript125S_1/25 italic_S start_POSTSUBSCRIPT 1 / 2 end_POSTSUBSCRIPT ground-state manifold as two-lower levels |0ket0|0\rangle| 0 and |1ket1|1\rangle| 1 . While in addition, the states to the |F=1,mF=+1ketformulae-sequence1subscript1|F=1,m_F=+1\rangle| italic_F = 1 , italic_m start_POSTSUBSCRIPT italic_F end_POSTSUBSCRIPT = + 1 , and |F=2,mF=+1ketformulae-sequence2subscript1|F=2,m_F=+1\rangle| italic_F = 2 , italic_m start_POSTSUBSCRIPT italic_F end_POSTSUBSCRIPT = + 1 of the

5P1/25subscript125P_1/25 italic_P start_POSTSUBSCRIPT 1 / 2 end_POSTSUBSCRIPT manifold as two-higher levels |eket|e\rangle| italic_e and |rket|r\rangle| italic_r Dalla Torre et al. (2013). Moreover, in the first scheme, to construct the model we need the condition 1/1=2/2subscript1subscript1subscript2subscript2\Omega_1/\Delta_1=\Omega_2/\Delta_2roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT, which can be realized by adjusting the value of 1,2subscript12\Delta_1,2roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT and 1,2subscript12\Omega_1,2roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT. Identically, the second scheme needs similar conditions 1/1=2/2subscript1subscript1subscript2subscript2\Omega_1/\Delta_1=\Omega_2/\Delta_2roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT and 1/1=2/2subscriptsuperscript1subscript1subscriptsuperscript2subscript2\Omega^\prime_1/\Delta_1=\Omega^\prime_2/\Delta_2roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 1 end_POSTSUBSCRIPT = roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT / roman_ start_POSTSUBSCRIPT 2 end_POSTSUBSCRIPT to construct the model.

According to past works Brennecke et al. (2007); Guerlin et al. (2010); Zhang et al. (2013); Grankin et al. (2014), the transition between the atomic ground level 5S1/25subscript125S_1/25 italic_S start_POSTSUBSCRIPT 1 / 2 end_POSTSUBSCRIPT and the optical level 5P1/25subscript125P_1/25 italic_P start_POSTSUBSCRIPT 1 / 2 end_POSTSUBSCRIPT of Rb87superscriptRb87^87\textmdRbstart_FLOATSUPERSCRIPT 87 end_FLOATSUPERSCRIPT Rb atom is coupled to the quantized cavity mode with strength g=2×14.4MHz214.4MHzg=2\pi\times 14.4~\textmdMHzitalic_g = 2 italic_ × 14.4 MHz. The decay rates from higher levels to lower ones and the cavity mode are =2×3MHz23MHz\gamma=2\pi\times 3~\textmdMHzitalic_ = 2 italic_ × 3 MHz and =2×0.66MHz20.66MHz\kappa=2\pi\times 0.66~\textmdMHzitalic_ = 2 italic_ × 0.66 MHz, respectively. The Rabi frequencies 1,2subscript12\Omega_1,2roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT can be tuned continuously and for the first scheme we adopt parameters 1,2=0.3g,1,2=76g,N=200formulae-sequencesubscript120.3formulae-sequencesubscript1276200\Omega_1,2=0.3g,\Delta_1,2=76g,N=200roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.3 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 76 italic_g , italic_N = 200, the fidelity of the target state is 99.41%percent99.4199.41\%99.41 %. For the second one, we set 1,2=1,2=0.1g,1,2=50gformulae-sequencesubscript12subscriptsuperscript120.1subscript1250\Omega_1,2=\Omega^\prime_1,2=0.1g,\Delta_1,2=50groman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.1 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2

end_POSTSUBSCRIPT = 50 italic_g and the fidelity is 99.56%percent99.5699.56\%99.56 %.

In addition, Ref. Spillane et al. (2005) reported the projected limits for a Fabry---Perot cavity, which the coupling coefficient g=2×7702770g=2\pi\times 770italic_g = 2 italic_ × 770 MHz. Based on the corresponding critical photon number and critical atom number, we obtain (,)=2×(21.7,2.6)MHz221.72.6MHz(\kappa,\gamma)=2\pi\times(21.7,2.6)~\textmdMHz(italic_ , italic_) = 2 italic_ × (21.7 , 2.6) MHz. The fidelity FFitalic_F reaches 99.10%percent99.1099.10\%99.10 % for the first scheme with the other relevant parameters are selected as 1,2=0.2g,1,2=72g,N=200formulae-sequencesubscript120.2formulae-sequencesubscript1272200\Omega_1,2=0.2g,\Delta_1,2=72g,N=200roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.2 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 72 italic_g , italic_N = 200. And for the second one, the fidelity is 99.67%percent99.6799.67\%99.67 %, while other parameters are 1,2=1,2=0.12g,1,2=50gformulae-sequencesubscript12subscriptsuperscript120.12subscript1250\Omega_1,2=\Omega^\prime_1,2=0.12g,\Delta_1,2=50groman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.12 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 50 italic_g. Moreover, in a microscopic optical resonator Dayan et al. (2008), the parameters of an atom interacting with an evanescent field are (g,,)=2×(70,5,1)MHz27051MHz(g,\kappa,\gamma)=2\pi\times(70,5,1)~\textmdMHz(italic_g , italic_ , italic_) = 2 italic_ × (70 , 5 , 1) MHz, which correspond to the fidelity F=99.18%percent99.18F=99.18\%italic_F = 99.18 % with parameters 1,2=0.3g,1,2=43g,N=200formulae-sequencesubscript120.3formulae-sequencesubscript1243200\Omega_1,2=0.3g,\Delta_1,2=43g,N=200roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.3 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 43 italic_g , italic_N = 200 in the first scheme and F=99.34%percent99.34F=99.34\%italic_F = 99.34 % with parameters 1,2=1,2=0.1g,1,2=50gformulae-sequencesubscript12subscriptsuperscript120.1subscript1250\Omega_1,2=\Omega^\prime_1,2=0.1g,\Delta_1,2=50groman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = roman_ start_POSTSUPERSCRIPT end_POSTSUPERSCRIPT start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 0.1 italic_g , roman_ start_POSTSUBSCRIPT 1 , 2 end_POSTSUBSCRIPT = 50 italic_g in the second scheme.

VI Summary

In summary, our work provides two schemes to dissipatively produce the maximal discordant mixed state where the environment becomes a resource for states generation and breaks the time limit of the unitary dynamics. In the first scheme, by alternatively changing the phase of Rabi frequencies, the target state turns into the unique steady state of the whole process while the second one leaves out the alternating evolutionary process by introducing a lossy coupled-cavity system. We make a comparison between two schemes. Both of them have advantages and disadvantages. For the first one, it takes shorter time to achieve the target

state with the fidelity oscillated around a certain value. For the second one, although it takes longer to achieve the target state, the fidelity is stability and higher. Meanwhile, both systems have favorable resistance to the spontaneous emission of atoms, and the target state can be obtained with arbitrary initial state except for the singlet state |-ketsuperscript|\Psi^-\rangle| roman_ start_POSTSUPERSCRIPT - end_POSTSUPERSCRIPT . We also discuss the relevant parameters under current experimental data and obtain high fidelities over 0.990.990.990.99. We hope the work may be useful for the experimental realization on quantum correlation in the near future.