Option

Gamma

Trading

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Volcube Advanced Options Trading Guides

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About Volcube

Volcube provides a leading options education technology to firms and individuals who want to learn about professional options and volatility trading. The Volcube technology is a web-based option market simulator with embedded, automated teaching tools and a rich learning library.Volcube was founded in 2010.

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About the Volcube Advanced Options Trading Guides series

Clear and concise guides that explore more advanced options trading topics. Check out the other volumes in the series…

volume I : *Option Gamma Trading*

volume II : *Option Volatility Trading : Strategies and Risk*

volume III : *Option Market Making : Part I : An Introduction*

volume IV : *Trading Implied Volatility : An Introduction*

Introduction

This guide explains gamma, gamma trading and gamma hedging in plain English. It introduces gamma as an option Greek and shows how gamma affects a delta-hedged portfolio. It explains why gamma hedging can be necessary to maintain delta-neutrality and how this can have profit and loss implications. Gamma trading is explained as a strategy that looks to balance gamma hedging with option time decay. Throughout, we will emphasise practical techniques and considerations for both gamma trading and gamma hedging.

This ebook is part of the Volcube Advanced Options Trading Guides series and as such expects the reader to have a basic familiarity with options and options trading terminology. A glossary of some terms used in the book is provided. To learn more about the basics of options, visit www.volcube.com/resources.

The book has five main sections.

Section 1 explains option gamma at its most basic.

Section 2 discusses gamma hedging and provides detailed examples to show how gamma and delta interact as the spot price moves. The profit and loss effects are also demonstrated from first principles.

Section 3 discusses gamma trading. Gamma trading is a strategic approach to managing option gamma risk. It can be a distinct profit-seeking strategy in its own right. Some gamma hedging tactics are also discussed.

Section 4 looks at the broader implications of gamma trading and at some of the additional complications that can be encountered by traders. Gamma hedging is also considered in more depth.

Section 5 provides some practical words of wisdom about some of the risks and opportunities that are rarely mentioned in the option theory guides but which can, over time, dramatically impact on a gamma trader's profit and loss for better or worse.

The examples given throughout assume simple option ticks sizes and one to one contract multipliers.

Section 1 : Gamma basics

What is option gamma?

To understand gamma, we must understand delta. All options have a *delta*. The option delta is positive for calls and negative for puts and is usually expressed as a percentage. We can interpret the delta in several ways, one of which is as the probability that an option will expire in-the-money. An at-the-money call option has a delta of 50%, because its chances of expiring in or out-of-the-money are 50:50. The delta of an option depends on several factors and if any of these factors alters, the option delta can change. The most important factor that can affect an option's delta a change in the spot price itself. If the spot price changes, the delta will almost certainly change. This is where gamma comes into play because option gamma tells us by how much option delta changes for a change in the underlying spot price.

Let's look at a very simple example. Suppose the spot is trading at $100. For simplicity, let's ignore forward rate complications and just assume that the $100 strike calls have a delta of 50%; they are at-the-money call options. Now let's suppose the spot rallies in price to $101. How does the delta of the calls change? Well, we know that the chances of the option expiring in-the-money must be higher, given that the spot price is higher. So the delta must be above 50%. But how much above? The option gamma gives us a good way to estimate the change. Suppose that our model tells us the $100 calls have a gamma of 10, with the spot trading at $100. What does this number '10' mean in practice? Typically gamma is 'normalized' so that it can be interpreted as the change in delta for a 1 point move in the underlying. In this case, the spot has rallied by 1 point ($1), so the delta should have changed by about 10. So an estimate of the new call delta with the spot trading at $101 is 60%.

And if the spot had fallen to $99 instead of rallying to $101? In this case, the delta would have *fallen* by 10, to approximately 40%. Gamma is what is known as a 'positive function', meaning that its effect on option delta is positive for increases in the spot price and negative for declines in the spot price, as the example shows. Note also that for put options, the negative sign of the delta remains important. So if our example had pertained to the $100 strike *puts*, their initial delta would be -50%. The increase in spot price to $101 would raise the put delta by about 10, to -40%. Correspondingly, a fall in the spot price to $99 would lower the put delta to -60%, (raising the delta in absolute terms).

Notice that we are careful to say these delta predictions are approximations or estimates. This is because the option gamma itself is a function of the spot price. So although the call option in the example has an initial gamma of 10, this is only precisely true versus the spot price of $100. Versus the new spot price, the gamma will actually be lower (for reasons to be explained later) and so the prediction of the final delta using the initial gamma is only an approximation.

Remember : option gamma is the change in option delta for a change in the spot price. It is often normalized to show the change in option delta for a 1 point move in the spot.

Option gamma is one of the option Greeks. The Greeks are a set of risk measures used by option traders to understand and quantify their option risk. Gamma is a 'higher order' Greek, insofar as it tells us how another Greek (delta) changes rather than telling us how the option value changes with respect to a change in some other factor.

Gamma is a positive function. If we own options, we own gamma. If we are short options, we must have short gamma positions.

Gamma is additive for options struck on the same underlying, regardless of their expiration. This is an important fact for owners of portfolios of options on the same spot product. If we own the $94 puts struck on product $ABCD and we also own the $107 calls struck on the same product, our total portfolio gamma is just the sum of the gammas from the $94 puts and the $107 calls.

Factors that affect gamma

Many factors affect the gamma of an option, although there are three which are most significant.

The strike price relative to the spot price

The closer an option is to being at-the-money, the greater its gamma. At-the-money options have the highest gamma of any option within an expiration. Options whose strikes are a long way from the current spot price will have less or even zero gamma. Why is this the case? Think of gamma as the sensitivity of the option delta to a change in the spot price. The delta of an option very far away from the current spot price is unlikely to change greatly for a small or moderate change in the spot price. Whereas an at-the-money option is on the cusp of being in or out-of-the-money and so it is far more sensitive to such spot price changes. An at-the-money option's delta *must* change (from 50%) when the spot alters. But a 0% delta option (far out-of-the-money) will not noticeably change its character for such a change in the spot. This is simply a way of saying the at-the-money option has higher gamma and the far out-of-the-money option has little or no gamma. The options in between will have progressively less gamma, the further from the spot price their strike lies.

The time to expiration

Other things being equal, options with less time to expiration have greater gamma than longer-dated options. This is because a change in the spot price is more significant for a shorter dated option than a longer dated option and so the effect on the option's delta will be greater. To explain this, (and this often helps when trying to understand option theory) consider two extreme scenarios. An at-the-money call option expires in 30 seconds. Its delta is 50%. Now suppose the spot starts to fall a little. The call delta will start to fall dramatically, perhaps to zero, due to this change in the spot price and the very limited lifespan the option still has. This suggests its gamma at this point is incredibly high (the change in delta for a change in spot price [which is the definition of high gamma] is extremely high). Now consider an at-the-money call option on the same underlying but with 20 years of its life to run. Does the small fall in the spot price effect the option's delta? Barely at all. In the long scheme of things, this change in the spot price is inconsequential for such a long dated option's delta. In between these two extremes (of a 30 second option and a 20 year option) lie the other possibilities, in which gamma will be progressively lower, the longer the option has to live.

The implied volatility of the option

The effect of implied volatility (or changes in implied volatility) on any option Greek are usually comparable to changes in the time to expiration. This is because a fall in implied volatility or a lower level of implied volatility is in some way akin to less time remaining to expiration. If we think of an option with a certain amount of life remaining, then a more volatile underlying product would almost be equivalent to having a longer life, since there is more chance for the option to expire in-the-money in either scenario. With respect to gamma therefore, we can say that the higher the implied volatility, the lower the option gamma, other things being equal. Consider an at-the-money option where implied volatility is very high. Do small changes in the underlying spot price greatly affect the option's delta? (i.e. does the option have high gamma?). The answer is no, because a small change in the spot price is not particularly significant if the product is expected to be highly volatile anyway. For options whose underlying products are expected to exhibit very low levels of volatility (i.e. whose options are valued with low implied volatility), a small move in the underlying is more significant and therefore the option delta will be more greatly affected by the move. In other words, options with lower implied volatility will have higher gamma.

Section 1 Exercises

1.1 Option gamma tells us the change in option delta with respect to the change in …..?

1.2 Option gamma is a first order Greek. True or false?

1.3 Option gamma is usually normalized to show the change in delta for a 1% move in spot, 1 tick move in spot or a 1 point move in the spot?

1.4 What three factors most significantly affect the option gamma?

1.5 Other things being equal, do options with little time to expiration have more or less gamma than longer dated options?

1.6 If I sell short call options and sell short put options on product $XYZ, will I be long, short or flat gamma?

1.7 Is gamma a negative or a positive function of the spot price, with respect its effect on option delta?

1.8 Does the gamma of an option currently at-the-money increase or decrease if the spot price

subsequently changes?

Section 2 : Gamma hedging

What is gamma hedging?

To understand gamma hedging, it is necessary to understand delta hedging. An option's value can be affected by changes in the spot price. The option delta tells us the extent of an option's sensitivity to changes in the spot price; from 0% if the option value is completely unaffected by changes in the spot price, to 100% if every penny change in the spot price leads to a fully penny change in the option value. The delta of an option also shows us therefore the appropriate hedge ratio we should use to hedge the option against changes in the spot price.

A delta hedging example

Let's look at a simple example. Suppose we own 100 at-the-money call options and that each option gives us the right to buy 1 lot of the underlying. Let's also suppose that these call options have a theoretical value of $2.50 and that the underlying product is trading at a price of $75. Now, because these calls are at-the-money options, they must have a 50% delta. This tells us that if the spot falls by say 10 cents, the options will lose about 5 cents in value. If the spot gains 20 cents, the calls will gain about 10 cents. If we wish to eliminate this risk, we can *delta hedge*. This means trading something to remove the risk to our options that is due to changes in the spot price. The underlying product is in fact the simplest instrument to use for delta-hedging purposes. To delta hedge our 100 lots of the 50% delta calls, we need to sell 50 lots of the underlying at the current price of $75. This is because 50% is the hedge ratio and we own 100 lots of the calls, so 50% of 100 implies a delta hedge of 50 lots of the underlying. We need to sell rather than buy the underlying because the calls have a positive delta; by selling the underlying short we create a negative delta position to counter-balance the long deltas from the calls. So let's see whether this position is now delta-neutral (i.e. delta-hedged).

If the spot rallies 10 cents, we know that our calls will increase in value by approximately 5 cents (because they have a 50% delta). So here we make a profit of 5 cents * 100 lots = $5. But we have also sold short 50 lots of the spot at $75, which has now rallied 10 cents. So we make a loss of 10 cents * 50 lots on our delta-hedge, which is a loss of $5. So our delta-hedge has been effective on the upside in that we have not made a profit or a loss in this case.

The exact reverse is true if the spot falls by 10 cents. Our long calls will *lose* about 5 cents in value, but our spot trade will make 10 cents, and since we are long twice as many calls as we are short spot, the two positions again cancel out to leave us p&l neutral.

This is the essence of delta-hedging. By trading so as to reduce our portfolio delta to zero, we no longer have exposure to small moves in the underlying from an overall p&l perspective.

The effect of gamma on a delta-neutral position

Now let's consider the effect option gamma has on a delta-hedged position. Remember that an option’s delta is dependent on the spot price (and gamma quantifies that relationship for us). When we delta-hedged our call options in the preceding example, we used the delta of the options versus a particular spot price (specifically, the delta with the spot trading at $75). But we know that as the spot changes, the delta changes. This means that our original delta hedge is not going to be the ‘correct’ hedge once the spot has moved. To see this, let’s continue with the example above. Initially the spot was trading at $75 and the call had a 50% delta. Suppose that our call options have 10 gamma (and for simplicity let’s also assume that the amount of gamma is constant for changes in the spot price). Let’s assume the spot drops to $74 and assess the situation.

If we knew nothing about gamma, we might expect that the calls would lose 50 cents in value owing to their 50% delta (50% of the $1 that the spot has dropped). This implies a loss on the calls of $50 (50 cents on 100 lots). We might expect that our delta-hedge would make up the difference (the delta-hedge profits by $1 but we have only sold half as many lots of the spot as we are long the calls; 50 lots * $1 = $50). However, this is not the whole picture, owing to the option gamma.

We know that the calls have 10 gamma, so the delta of the calls versus $74 is not 50%. Rather, it has fallen to 40%. This has two implications. Firstly, the value of the calls will not have fallen by as much as we first thought. Rather than losing 50% of $1, they will have lost less than that; approximately 45% of $1 or 45 cents. This is because the calls started with 50% delta, but finish with 40% delta, so we can say that, roughly, whilst the spot was falling $1, the delta of the calls was, on average, 45%. This means that we have in fact made a theoretical profit from the fall in the spot price. Our delta-hedge has made $50 but our call options have only lost $45, hence we are left with a net profit of $5. The second implication is that our current overall portfolio delta is no longer neutral (zero). We are short 50 deltas from our short spot position, but we are only long 40 deltas from our calls. So our net position delta is short 10, meaning we are equivalently short 10 lots of the underlying spot product.

So, intuitively, what is going on here? The answer is that our original delta-hedge was only accurate and appropriate with the spot trading at a price of $75. This was a *static hedge* rather than a *dynamic hedge*. As soon as the spot moved from that price, the delta of the options changed and our original delta-hedge became inaccurate. The further we move from $75, the more inappropriate our hedge becomes.

Notice that we made a theoretical profit when we checked the position with the spot trading at $74. What if the spot had instead rallied to $76? By the same process, we can see that the position makes a profit in this situation too. Because of the gamma of 10, the option delta will increase to 60%, meaning the average delta of the option as the spot rallied was 55%. Hence the calls will increase in value by 55 cents, and given we own 100 lots we profit by $55. Our hedge on the other hand loses $1 on 50 lots. So our net profit is again $5.

The position seems to make money *no matter which was the spot moves*! This can be surprising for two reasons. Firstly, we were supposed to be delta-neutral, so why are we making any profits at all? Again, this is because our original delta-hedge was only statically appropriate (i.e. correct for a particular spot price). Secondly, it would seem like a win-win scenario. This point we will return to in due course but for now simply notice that we owned options in this example and were therefore *long gamma*.

Gamma hedging to restore delta-neutrality

So we now arrive at gamma hedging. We started with a long call position which we delta-hedged. We then looked at how the portfolio's character changed when the spot price fell. Specifically, we noted that we were short 50 deltas from the delta-hedge but that the calls now only had a 40% delta (and since we owned 100, our delta position from the calls was long 40). So we are left with a delta position that is net short 10. In other words, our portfolio is equivalent to being net short 10 lots of the spot product. To restore our delta-neutrality we need to re-hedge the position and this is known as a gamma hedge.

Remember : a gamma hedge is a delta-hedge that has become necessary because the spot price has altered, making the original delta-hedge inaccurate or inappropriate.

In this example, by buying 10 lots of the underlying, we restore delta neutrality. Recall that we initially sold 50 lots, so by repurchasing 10 we will be left short 40. This ties in with the new call delta which, with the spot trading at $74, is 40%.

Gamma profits and losses : exponential, not linear

We saw in the above example that for a $1 fall or rise in the spot price, the profits from gamma hedging were the same ($5). Let's consider an alternative scenario in order to derive an important result regarding gamma profit and losses.

Suppose that the spot had fallen $2 instead of $1 and that we had not gamma hedged at any point. The spot is now at $73. Let's continue to assume constant gamma of 10, which implies the call option delta will have fallen to 30%. So we can assume the call delta averaged 40% (half of 50% and 30%) during the fall in the spot price and that calls have lost 80 cents in value (change in option value = change in spot price * option delta = $2 fall in spot * 40%) We now make the same profit and loss assessment as before. The long call option position has lost 80 cents, on 100 lots, or $80. The original delta hedge was a short position of 50 lots. This has profited by $2 per lot or $100. This leaves a net profit of $20. Compare this to the profit when the spot had fallen only $1, namely a win of $5. The profit is four times as great for a spot move only twice as great. Do not skim over this calculations. It is imperative to understand how this works if a firm grasp of gamma is to be achieved.

Remember : gamma profits (and losses) are an exponential function of the change in the spot price.

Double the move in the spot price and the gamma profits/losses *quadruple*. Gamma profit and loss is explosive.

Section 2 Exercises

2.1 John buys 50 puts with (-)30% delta and 5 gamma. He becomes delta-neutral by buying some of the underlying. Is this a gamma hedge?

2.2 How many deltas (i.e. lots of the underlying) did John buy to hedge, assuming each option gives the right to sell one lot of the underlying?

2.3 The spot market underlying John's options rallies by $1. What is the approximate change in the value of his puts?

2.4 What is the profit or loss on John's original hedge following the change in the spot price?

2.5 What is the profit or loss on John's puts following the change in the spot price?

2.6 To gamma hedge, does John need to buy or sell the underlying?

2.7 What is the net profit and loss on John's position if he gamma hedges with the spot $1 higher than the original price?

2.8 Suppose the spot had rallied $2 instead of $1. What would be the profit/loss from the gamma hedge?

2.9 Suppose the spot had rallied $4 instead of $1. What would be the profit/loss from the gamma hedge?

2.10 Suppose John had had the precise opposite of this position (short puts, short spot as a delta hedge). Would he buy or sell the spot as a gamma hedge and would this lock in a profit or a loss?

Section 3 : Gamma trading

Option theta as the flip side of option gamma

In the example in Section 2, we saw how a delta-hedged long call position seemed to make a profit no matter where the spot price moved. Spot up or spot down, when we analysed the change in the delta of the options versus the original delta against which we had delta-hedged, we found the situation had moved in our favour. This, we explained, was due to our owning gamma. By virtue of being long options, we were long gamma and therefore profited from any movement in the spot.

So why not always be long gamma? If it is guaranteed to make a profit whenever the spot price changes and if that profit grows *exponentially* the further the spot moves, surely this is a nice position to assume? And why would anyone ever hold the reverse position, being short options and therefore short gamma and exposed to losses as soon as the spot price changes?

The brief answer to all these questions is option time decay. Other things being equal, options erode in value over time. Consider an out-of-the-money option with a month until expiration. This option may still be valuable even though it is out-of-the-money. This is because it may still have a *chance* of expiring in-the-money. But as time passes, this possibility diminishes and the option's value declines. We measure this decline in option value by the Greek option theta. Theta is typically normalized to show the fall in option value in ticks per day.

So there is a broader picture to consider than just how much profit we can make from owning gamma, since this gamma comes at a cost. To be long gamma, and have the possibility of profiting from changes in spot price, we must be long options. This entails being 'long' theta i.e. paying a time decay theta bill, as the options erode in value.

In the example in Section 2, we owned 50 calls which had 10 gamma and we calculated the potential profits from gamma hedging at spot price change intervals of $1 or $2. Now suppose that these calls have a theta of 20 cents. This means that, other things being equal, the calls will be worth 20 cents *less* tomorrow than they are worth today, implying a loss on the position of $10. This puts the decision about whether to gamma gamma and where to gamma hedge in a different light. Now it is clearly a trade-off between how far we think the spot will move versus how much we expect to lose on the long call position owing to theta.

Also notice that the reverse position now becomes more comprehensible. Why would anyone ever be short delta-hedged options and therefore short gamma? Because they are hoping to *collect* the option time-decay whilst *not* losing money by having to short gamma hedge. The short gamma player is hoping that the spot does not move much at all because any gamma hedge he executes will serve to lock in a loss (just as the long gamma player locks in a profit wherever he gamma hedges). The short gamma player expects the options that he is short to be worth less tomorrow than they are worth today and this is his potential reward for the risk of being short gamma.

Assessing and acting on the trade-off between gamma hedging and theta is the essence of gamma trading.

What is gamma trading?

Remember : gamma trading refers to any strategy that aims to profit from a gamma/theta position by gamma hedging effectively, in relation to the portfolio theta.

To trade gamma, an options position, typically delta-neutral, is established. The position will either be long gamma or short gamma; which, is determined by whether the gamma from the long option positions is greater than or less than the short gamma arising from short option positions.

Let’s take an example. Suppose the spot is trading at $300 and that we are short 100 lots of the $300 strike calls AND puts. In other words, we are short 100 of the $300 strike straddle. The straddle has a theoretical value of $4, gamma of 20 and theta of 25. Because an at-the-money straddle is delta-neutral, this position is statically delta-hedged without our needing to trade the spot. In summary;

Option position : Short 100 of the $300 calls. Short 100 of the $300 puts.

Spot position : Flat.

Spot price: $300

Straddle value: $4

Position gamma: 20

Position theta: 25

Position delta: 0

Gamma trading involves managing this position to try to profit from the theta-gamma trade-off. Presumably we have entered this position either because a) we actively thought it was a good idea or b) we are market makers and were forcibly handed this position. Either way, now we need to gamma trade to make the best of things. We now that the straddle will lose 25 cents in value overnight due to theta. **When we are short gamma, this is our maximum upside profit potential.** The best case scenario is that the spot does not move at all and we ‘collect’ the theta as the straddle erodes over time. However, it is extremely unlikely that the spot will be entirely motionless. So the next best hope is that even if it moves, it returns and settles at $300 *without our having gamma hedged*. As soon as the spot starts to move, we begin to face a dilemma, due to the exponential profits and losses associated with gamma hedging. The closer we hedge to $300, the smaller the losses will be, but the number of times we may hedge could be high. So we could accumulate a large number of small losses by hedging ‘tight’ i.e. hedging close to $300. Alternatively, we could 'let our gamma run' in the market parlance. By not hedging frequently and tightly, we run the risk that the spot market will trend in one direction. If this happens, when we ultimately do hedge, we may find our loss on this hedge to be large. The question is whether this single loss is larger than the sum of lots of little losses. This is the dilemma faced by gamma traders, whether long or short gamma; hedge little and often or hold out for fewer hedges, perhaps further away.

Looking at the example above, suppose we decide in our trading strategy meeting on a hedging rule that dictates we will gamma hedge every time the spot moves 10 cents. Remember in this example these would be *short gamma hedges* (as we are short gamma) so each one locks in a loss. How big a loss? Go back to the numbers. The straddle has left us short 20 gamma, so a 10 cent move changes the delta by 2. Hence we need to sell 2 deltas if the spot *falls* 10 cents and buy 2 deltas if the spot *rallies* 10 cents. Intuitively, the need to sell or buy must be this way round because short gamma hedging is a 'loser'; so we need to sell when the market falls and buy after it has rallied. Selling 2 deltas 10 cents lower makes a loss of 20 cents. But, the actual loss is only around half this amount. This is because we were not long 2 deltas all the way down from $300 to $299.90 nor short 2 from $300 up to $300.10. Remember that we were delta-neutral versus $300. We accrue our long deltas, due to our short gamma, gradually as the spot falls and corresponding pick up short deltas gradually in a spot market rally. So a good approximation of the extent of our loss is that we lose *half* this amount. A way to interpret this is to say that, on average, we carried 1 delta down for the whole drop in the spot. We started delta-neutral and were long 2 deltas by the end. Therefore, on balance, we were roughly long 1 delta over the move. Our loss on the gamma hedge is therefore approximately 10 cents.

This may sound pretty small, seeing as we are collecting $25 overnight from the position. It suggests we could hedge every 10 cents up to 250 times in one day before we are making a net loss. The elephant in the room of this trading strategy meeting is the expected volatility of the underlying. Whether or not that hedging strategy makes sense, entirely depends on the volatility of the spot market. If the spot market moves 10 cents every minute, we could be hedging 60 times per hour and after 6 or 8 or 10 hour trading day and our $25 theta collection will soon be overwhelmed by our gamma losses.

A gamma trading plan converts the expected volatility in the underlying into a likely profit and loss due to gamma hedging and compares this to the expected profit and loss due to the theta associated with owning, or being short, options. A gamma *hedging* strategy looks to hedge in a way deemed most likely to bring about profits. This may mean hedging small but often or it may mean hedging less frequently but more likely further away.

What is the difference between gamma trading and gamma hedging?

The difference between gamma trading and gamma hedging should now be clear, although sometimes the two terms are used interchangeably. Gamma *hedging*, also sometimes known as gamma scalping, refers to the actual delta re-hedge of a portfolio. Gamma *trading* sees gamma hedging in the context of the portfolio's theta and the volatility of the underlying. Gamma trading is more a high-level view; the strategy as a whole that tries to profit from gamma being too cheap or too rich. If gamma is too cheap, it means that there is profit to be made from owning gamma and by gamma hedging. It means that the option theta bill (i.e. the cost of owning options) is too low relative to the quantity of gamma that can be owned and gamma hedged. If gamma is rich, it means that *shorting* options could be a good strategy. In this case, the time decay that can be collected from being short options may exceed the losses that might be incurred from short gamma hedges.

Gamma hedge p&l rule of thumb

For the gamma component of the position, the trader will often use a simple rule of thumb when making calculations. The p&l from a gamma hedge is approximately equal to a half times the gamma times x^2, where x is the change in the spot price in ticks.

Gamma hedge P&L = ½ * Γ * x^2

where Γ is position gamma and x is change in spot price

The 'half gamma x squared' formula is a very handy estimator for gamma hedge profit and loss. It has a simple enough intuitive explanation. Suppose we have 50 gamma and the spot moves $2 and then we gamma hedge. This is no different to the examples worked through above. If the spot moves $2, we will pick up 100 deltas in this scenario. Remember that 50 gamma by definition means we pick up 50 deltas for every 1 point move in the spot. So the new portfolio delta is the gamma multiplied by the move in the spot i.e. $\Gamma * x$. The profit or loss from these deltas is *half* the number of the deltas (½ * Γ * x) multiplied by the change in the spot price (x), giving us the formula above. Remember that our delta changed from zero to 100, but we only picked up these deltas gradually. So the p&l is, roughly, ½ * 100 * $2 = $100.

Notice also the exponential component to the formula (x^2). This ties in with our earlier discovery that the profit and loss from gamma is not a linear function of the change in the spot price. Rather it is an exponential function and this is precisely why gamma p&l can be explosively large for large spot price moves.

One particular use of this formula (or rather a simple re-arrangement thereon) is to calculate the breakeven gamma hedge levels. Suppose in the above example we are paying $75 per day in theta to hold the 50 gamma. Of interest would be the amount the spot price has to move in order for us to profit sufficiently to 'pay off' our theta debt with a single gamma hedge. We can rearrange the formula to be in terms of x, the change in spot.

$$x = \sqrt{\frac{2 * \text{Gamma P\&L}}{\square}}$$

and now set the Gamma P&L that we are interested in to be equal to the theta bill. So in the example we find the price change, x is :

$$x = \sqrt{\frac{2 * \$75}{50}} = \sqrt{3} \approx \$1.73$$

So in this case, we would need to gamma hedge $1.73 above or below the current spot price in order to make a profit of $75 and break-even versus our theta bill.

Section 3 Exercises

3.1 An option portfolio is long gamma. Do gamma hedges for this position ever lose money?

3.2 An option portfolio is collecting theta. Is the trader long or short options? Is the trader long or short gamma?

3.3 A trader's portfolio is losing $75 per day in theta. The trader is long 50 gamma. If the spot moves $1, what is the profit from a gamma hedge at this level?

3.4 How many such gamma hedges does the trader in 3.3 need to cover his daily theta bill?

3.5 Suppose the trader decides to hedge more 'tightly' (i.e. when the spot has moved less far). If the trader gamma hedges every 50 cents, how many hedges will be required to cover the theta bill of $75?

3.6 Suppose the trader is late for work and when he arrives the spot has already moved $3, without his having gamma hedged at all. What is the profit from the gamma hedge and the net profit from the gamma trading (i.e. when the daily theta bill is deducted)?

Section 4 : Advanced gamma trading for option portfolios

Having explained gamma, gamma hedging and gamma trading, we now consider some more advanced ideas about the practicalities of gamma trading an options portfolio.

The difference between short gamma and long gamma

An important distinction needs to be drawn between short and long gamma positions. Unlike say long and short positions in a particular spot product whose payoff profiles are to all intents and purposes symmetrical, long and short gamma players face fundamentally different prospects.

Long gamma payoff profile

The long gamma player faces a certain risk, but an uncertain reward. His position will decline in value over time due to theta. This is known in advance and with certainty. This is also the limit of his downside loss (excluding vega effects which are a separate matter discussed below). Remember this well; *the long gamma player is exposed to limited losses*. His possible rewards however are unknown. They depend entirely on the volatility that the underlying product exhibits and the effectiveness of his gamma hedging strategy. There is the possibility of making no profit at all from gamma hedging, if no gamma hedging opportunities present themselves or alternatively if they are not taken when they do. So the loss from theta may not be mitigated in the slightest. At the other extreme, there is the possibility for explosively good profitability. If the spot moves dramatically and/or suddenly, then due to the exponential returns to gamma hedging for spot moves, profits can be very large.

Short gamma payoff profile

The short gamma player faces a known, maximum level of reward from his position and his potential losses are uncertain. The most the short gamma player can win from his gamma trade is the theta his position collects overnight. This may be offset (and indeed out-stripped) by losses due to his *negative* gamma hedges. These losses could be explosively large if the spot moves dramatically and/or suddenly and the trader elects not to, or has no time to, gamma hedge.

Written thus, it may seem that a long gamma position makes more sense than a short gamma position. But such a generalisation would be entirely misguided. Whilst short gamma losses can be large or very large, the daily payoff from collecting theta can mount up over time. Likewise whilst long gamma explosive super-profits may sound appealing, the daily ‘bleed’ of a theta bill can eat up such profits over time. In short, whether to be long or short gamma (or indeed flat gamma) is decision the trader must make for his own good reasons. Nevertheless, understanding this fundamental distinction between the two positions is critical.

Gamma hedging strategies

A gamma hedging strategy is a plan for how to gamma hedge in order to maximise the profits of gamma trading. For long gamma players, it looks to maximise the profit from the gamma hedges. For short gamma players, its intention is to minimise the losses. This area has been studied fairly extensively but no consensus appears to have been reached as to whether an optimal strategy exists. So here are some of the ideas currently employed by gamma traders to make the best of things.

Gamma hedging timeframes

There is a seemingly natural tendency for most traders to consider gamma trading as a daily activity. Whilst they may take on a gamma position with a view to say holding it for a couple of weeks, when it comes to the actual gamma hedging several factors draw traders to thinking day by day. Two obvious reasons for this are that theta is normalized and measured as a daily decay number. So one half of the gamma trading risk/reward equation is already denominated in dollars *per* 24 hour period. The other reason is that the trader's own working experience is divided into days. Each trading day has its own start and end, open and close and clear daily demarcation. Much therefore points towards a quotidien starting point for gamma hedging. Of course days are broken into hours and some traders have pursued hourly (or even more frequent) gamma hedging policies. But the results achieved have been unclear and the work involved is undoubtedly higher. So the majority of gamma traders still think in terms of their daily theta and the moves the spot will make during each day's trading session(s).

Some traders like to analyse the break-even points (as discussed in Section 3) and use this as the basis for gamma hedging levels. For example a long gamma player may think that if he can achieve a gamma hedge at the break-even point early in the session, any subsequent gamma hedges will be pure profit. A short gamma player may remember the rule that a gamma hedge half as far away as another, only generates ¼ of the profit/loss. So the short gamma player may decide to short gamma hedge half way towards either break-even point and the same distance thereafter in any direction. He knows that he can stomach four such negative gamma hedges before he will incur losses greater the theta he intends to collect on his short options overnight.

Gamma hedging and standard deviation

Another often used reference point is the implied standard deviation. Slightly theoretical and more than a little suspect, but nevertheless commonly used, some traders will multiply the spot price by the implied volatility (usually of the at-the-money options) and divide the result by 15.8 (the square root of 250 which is very roughly the number of trading days in a year). This, allegedly, gives some measure of the standard deviation of the spot price. In other words a statistical gauge of the magnitude of spot moves that one might expect to see just over half of the days. This is all assuming particular probability distributions for the spot market returns, which in reality they are not thought to adhere to closely. Nevertheless, this measure of standard deviation does have some merit, not least that it is easy to compute and does generally bear some relation to how much products tend move day to day. With respect to gamma hedging, traders will tend to use the standard deviation as a benchmark as to how large moves in the spot price can be considered.

Here's an example. The spot product closed yesterday at $100. The at-the-money implied volatility is 25%. A trader is long gamma and, unforgivably, arrives at his desk 10 minutes after the market has opened. The spot is already trading at $98. Is this $2 fall a large move and should the trader gamma hedge?

As to whether this is ‘a large move’ in the spot, of course this question is incomplete. Large relative to what? Considering the $2 move relative to the theoretical standard deviation does give us some grounds for assessment. The daily standard deviation implied by at-the-money options is 0.25 * 100 / 15.8 = $1.58. This is the sort of daily change in price we might expect to see a little over half the time if the actual volatility matches the implied. So from this quick calculation we might say that the $2 price change is reasonably good on a day-to-day measure from a long gamma hedging perspective. Gamma hedging at this point would in all likelihood more than pay for the daily theta decay of the portfolio. However, being only approximately 1.25 larger than the daily standard deviation, the excess profits are unlikely to be extreme. It’s a nice start to the day, but not made the trader’s week or month.

As to whether the trader should gamma hedge. This is broadly a subjective matter. Presumably some kind of back-testing might be performed on the underlying product and/or related instruments with tick data to try to estimate the likelihood of the spot continuing its move south following a fall on the open. This would be of interest because ‘letting the deltas run’ i.e. not hedging the gamma and letting the overall portfolio delta become shorter and shorter *is* a way to generate very large profits. Of course, this runs the risk of the spot rallying back to $100 without the position having been gamma hedged at all. All the potential gains from hedging versus $98 will have been lost in this scenario. In essence, we are into the realms of forecasting the spot and if the trader has an edge in this department, he might want to consider dispensing with fiddly options trading and just trade the spot!

Partial gamma hedging

To gamma hedge or not has a binary ring to it and it would be wrong to think these two actions (or rather one action and one inaction) is the end to the matter. A third way involves gamma hedging *partially*. If a portfolio has 20 gamma and the spot moves 1 point, we will have 20 deltas with which to play. But there is no hard and fast rule stating we must trade all or none of the deltas. We could trade a fraction such as half the deltas. This might be attractive in various scenarios. Perhaps we are long gamma and have seen a large fall in the spot on the open. We may feel strongly that the spot is likely to move further in the same direction. If we choose not to gamma hedge and are proven correct, our profits may be very large. If we fail to gamma hedge and are mistaken, we will give back the theoretical profit made thus far. *Remember that no gamma profits or losses are locked in until we gamma hedge.* Suppose instead of an all or nothing approach, we hedge half our gamma deltas. If the spot continues to fall, we are still carrying half our short deltas plus we are accruing new short deltas from our gamma. If on the other hand the spot retraces to its original starting price, we will have made some money from the move owing to the deltas we purchased. Such is the nature of a partial gamma hedge. Of course, analogous reasoning could be applied in a short gamma situation.

Gamma trading and expiration

Gamma trading at option expiration is not for the faint of heart. Indeed, many traders who actively pursue gamma trading strategies will look to avoid options whose lives are ending. For market makers, it is rare that they can be so choosy as they are often left with a rump position towards expiration that still requires careful risk management, and gamma trading is an important part of this process. There are two main reasons why expiration time brings special considerations in play. The first is that the essential gamma character of options close to expiration changes markedly into something far more binary, which tends to increase risk. The second is that pin risk may become a factor. We consider both in turn.

In the moments before an at-the-money option expires, its gamma approaches infinity. This is because its delta at expiry must be either 100 or 0 and therefore the change in delta (as it flips between the two numbers) for changes in the spot price (i.e. the gamma), is massive. Gamma *trading* tends to be a play of realised volatility against implied volatilities. It tends to be a strategy that looks to capture, via gamma hedging, a discrepancy between the actual volatility of the spot and the price of options as measured by the implied volatility. All of this plays out over a period of time. The 'problem' with gamma trading at expiration is that time is in short supply. Individual trading sessions of a day or even just an afternoon, can have an enormous significance on the outcome of the strategy as a whole, since they can occur when the gamma profile of the portfolio is at its most elevated. This is no different to the idea that common option pricing models tend to be less meaningful at expiration time. They too are predicated on long-run behaviour in the spot price generating mechanism. This is not to say that gamma trading at expiration is simply a fools errand. It is just to highlight that the risk can be magnified and that it is not perhaps the ideal playground for any gamma trading strategy reliant on long run average price volatility.

Pin risk is one of those phenomena whose existence is sometimes doubted by academics but rarely by practitioners. A market pins when the spot seems drawn inescapably to a particular option strike in the hours or days before expiration. Once at the strike it seems to be pinned there. It is often suggested that the strike to which the spot gravitates is one with considerable option open interest and especially one which market makers are *long*. The idea is that if market makers are, collectively, long the strike then they will be, *de facto*, long gamma. They will attempt to hedge their gamma by selling deltas above the strike and buying them below the strike. The effect of their collective gamma hedging may be to add selling pressure to the spot when it trades above the strike and buying pressure when it is below. Both serve to ensure the spot struggles to move any great distance from the strike. This seems to ignore the possible pressures of other market forces (non-option traders busy trading the spot for their own purposes). The argument however is that option traders can have so great an effect as to make pinning a reality. The point here is not really to debate the reality of pin risk. Rather it is to make the point that if pin risk is more than a figment of unfortunate option traders' imaginations, it provides an added distortion to the market that is not obviously conducive to a regular gamma trading strategy. That said, a believer in pin risk may argue that, on the contrary, this distortion is ideally suited to a *short* gamma trading strategy which has the added possibility of success if pin risk a) exists and b) occurs.

Gamma trading and portfolios of options

Gamma trading a simple position such as a delta-hedged call or put, or perhaps a straddle or strangle, can be reasonably straightforward, in so far as the gamma of the position, whilst unlikely to be a constant, at least always remains either positive or negative. A long straddle position can only lead to gamma ranging from zero (for straddles whose strike is far from the current spot price) to a positive number for straddles closer to the action. This is true for any long only options position. Correspondingly, short only options positions must have gamma ranging from zero to a *negative* number.

More complicated are positions where the gamma switches from positive to negative (or *vice versa*), depending on the current spot price. Consider for example a simple long put spread position. A long put spread is an option strategy whereby we own a put of one strike and are short a put of a lower strike, in equal quantity. Suppose the spot is trading at $100 and we are long the $95/$90 put spread. This means we are long the $95 puts and short an equal number of the $90 strike. What is the gamma profile of this position as the spot moves? We know that at-the-money options have the highest gamma of any option in a particular month. We also know that option gamma declines, the further the option's strike is from the current spot. So with the spot trading at $100, we know that owning the $95/$90 put spread must render us long gamma, since the $95 puts (our long strike) must have more gamma than the $90 puts (our short). If now the spot rallies, the gamma associated with both the $95 and $90 puts will decline (because they are becoming further away from the at-the-money strike), but we know that the position will never have negative gamma in this situation. This is because the gamma from the $95 puts will always exceed that of the $90 puts, *in a spot market rally*. So we can gamma hedge as and when on the upside and not be concerned about our gamma switching signs.

However, to the downside things are more complicated. If the spot falls to $95, the $95 puts become at-the-money options and their gamma will definitely be higher than the $90 puts. Should the spot slip further south, the situation starts to change. At some spot price (likely enough around $92.5) our long and short option positions will have *equal* gamma and our portfolio gamma will be neutral. As the spot moves even further down, there will come a point when our gamma is distinctly negative, as the short gamma from the $90 puts will exceed the longs from the $95 puts. This will certainly be the case at a spot price of $90, where the $90 puts become at-the-money and therefore have higher gamma than any other strike on the board.

Remember this is as simple a long/short position as we could generate, with just two option strikes. The principles nevertheless hold no matter how complex the position becomes. The gamma of a mixed portfolio may switch back and forth between positive and negative over large spot moves. So what are the profit and loss implications and how do traders manage this complexity? To track the profit and loss, we follow the portfolio delta. Delta can be understood as the portfolio's equivalent position in the underlying. So to understand the profit and loss that gamma creates, always check the delta. Let's work through the above example in close detail.

In the case of a rally from $100 upwards, the long $95/$90 put spread position will simply make profits from gamma hedges. As the spot rallies, our long gamma makes us long delta, which is obviously good news in a climbing market. The overall p&l of course would need to take account of the theta decay on our long put spread.

Should the spot move south from $100, our delta changes are less straightforward. To begin with, we pick up *short* deltas. Our positive gamma from the put spread makes us short deltas as the spot market falls. This is profitable and we become shorter and shorter deltas as the spot falls until our long gamma 'runs out'. This will happen at a spot price of around $92.50. Here, we are as short delta as we will ever be due to gamma. If the spot continues to fall, things change. We become short gamma because the gamma from our short $90 puts exceeds the gamma from our long $95 puts. A short gamma position in a falling market becomes *longer* deltas. So our negative portfolio delta starts to evaporate as we pick up long deltas by being short gamma in a declining spot market. Eventually, it is possible for all our short deltas to be gobbled up and we could still be short gamma, meaning that if the spot continues to fall we could become, overall, *long* delta.

Let's track the profit from all these delta changes. For now, we ignore the effect of theta. To the upside, the gamma simply makes profits. To the downside, the gamma initially makes money because we pick up short deltas in a falling market. Eventually this profit reaches a maximum (at the point where we become delta neutral again, because here we have carried down as many short deltas as we possibly can). Then we start incurring losses from being short gamma. Ultimately the losses from the short gamma could eliminate all the gains and lead to outright losses (if the spot falls far enough).

How do traders deal with situations such as these, from a gamma hedging perspective? Well, one thing to note is that if a trader has a complex long-short option position, he probably did not acquire the position with gamma trading as his primary motivation. More likely he is gamma trading this complex book because either he is a market maker who has been handed the position or he has initiated the position for reasons other than gamma trading, but now finds he has gamma/theta issues to manage. As to how to deal with such portfolios, there is no straightforward solution. In the example above, the upside scenario is less of a minefield than the downside. In a small down move, the trader (who, remember, is paying theta at this point) will see gamma profits available but also be aware that if he gamma hedges (by buying deltas) he may be storing up trouble. If the spot falls further he needs to realise that he will, eventually, naturally become long deltas anyway (due to his newly uncovered short gamma). So by buying deltas here, he risks having to sell them (and possibly additional deltas) lower down. Buying high and selling low is nobody's Plan A. The other horn of the trader's dilemma is that if he fails to gamma hedge after a relatively small down move, he may see the spot bounce. He will have locked in no gamma profits to offset his theta bill on the long put spread.

In practice, it comes down to a subjective judgement that balances several factors. Eventually the trader has to make a call about the likelihood of the spot moving to different prices, relative to his other positional considerations. For example, the trader may reason that if the spot falls further, implied volatility is likely to rise and this will benefit the put spread giving him an opportunity to sell the position out. So he may then argue it is worth gamma hedging (buying deltas) after a modest fall in the spot price, in case of a retracement, supposing that the risk of a large fall to the downside (causing him to eventually make losses from short gamma) is outweighed by the his chances of exiting the position at a profit before this becomes an issue. There are many other possible plans of attack, but this should offer a flavour. When gamma switches between long and short scenarios, the portfolio delta can do likewise over large stretches of spot prices. This can be tricky to manage and experience is probably the only real guide.

Gamma trading and vega concerns

Gamma trading may appear to be a straight fight between the p&l from gamma hedging and the daily theta payment or collection. Sometimes, this is indeed the case. However there is another potential complication involving a third Greek; namely option vega. Let's consider a simple portfolio that is short delta-hedged options and therefore short gamma and short (i.e. collecting) theta. This position will also be short vega. The owner of this portfolio is hoping the spot will move little, that he will not have to gamma hedge (negatively) and he will collect his theta decay. Suppose however things do not go the trader's way. The spot market decides to collapse and he gets savaged on negative gamma hedges, continually having to sell deltas at lower and lower prices, with the losses exceeding the amount the trader hoped to make from collecting theta. The trader was short gamma and it has not worked out on this occasion.

Alas, this sorry tale may not be the end of the trader's woes. By virtue of holding a net short option position, the trader will, as mentioned, also be short vega. In other words, he has exposure to the current level of implied volatility. In the case of a dramatic re-location of the spot price, the market may well decide to re-evaluate implied volatility. There are several possible causes and effects at work here. It could be that several traders have been 'caught short' gamma and try to buy back options to cover their risk. This will bid up the price of options which in turn will be reflected in higher implied volatility levels. For the short gamma/vega players, this represents another loss to their portfolio. Being short vega, they lose money when implied volatility rises. Another possible path is for the market to decide that perhaps implied volatility is too cheap relative to the actual realized volatility on display. Again, this could encourage the buying of options and an increase in implied volatility. So the short player may find he is on the receiving end of a second hiding. Following his losses on gamma hedging, he may see the implied volatility move against him too.

Of course, there are other possible combinations. The implied volatility may *soften* (i.e. fall) after a large move in the spot price. This may sound counter-intuitive but certainly happens in some instances. For example, consider stocks over earnings announcements. They can move sharply when their earnings are released but often the implied volatility will then collapse; the reason being that the news that was creating high uncertainty (=high implied volatility) is now old news and a potential source of spot movement has been eliminated. In this case the short gamma player may find his short vega position is a blessing.

The point being made is that often the situation is more complex than a simple gamma hedging versus theta bill analysis allows for. Because even this type of analysis involves realized volatility (on the gamma hedging side) and implied volatility (in terms of the price of options) on the theta side. Which in turn brings to the fore issues around the position's vega.

The question of course is how to manage these risks. Traders active in an options market will often come to understand how implied volatility tends to react to certain market moves and conditions. They may also have a grasp of the inventory held by various participants (market makers or end clients for example) and factor this in when making estimates. In practice, traders will often try to incorporate information of this type, however informally, into their gamma trading decisions. For instance, in the above example of the short gamma player who sees the market falling fast, he may decide to negatively gamma hedge little and often on the downside. This could be a tactic because he expects implied volatility to increase in the event of a large down move whereas he expects it to fall in a rally. There is a sense here in which he has an asymmetric vega payoff profile (negative if the spot falls, positive if it rallies), and this could simply be viewed as a synthetic delta exposure. By keeping his hedging 'tight' on the downside, he knows he will lock in lots of small gamma hedging losses. But he may estimate the alternative is worse; if the spot market falls precipitously he will experience big gamma hedging losses *and* potentially large vega losses. If instead the spot had rallied, he may have employed different thinking. Reasoning that a rally would be good for the position on the vega front (since implied volatility may fall), he may decide to be more liberal with his negative gamma hedging and allow the deltas to run a little more on the upside. Clearly this depends entirely on his estimates of how the options market will respond to different types of spot price action. Hopefully the point is made that often the gamma/theta trade-off assessment ought to take account of vega/implied volatility issues too.

Gamma-theta ratios

As we saw earlier, an option's gamma is negatively related to its implied volatility. The higher the implied volatility the lower the gamma. Option theta however is a positive function of implied volatility. This is because higher implied volatility means an option has more extrinsic value; given that all extrinsic value erodes away before expiration, a higher option value has further to fall (in the same amount of time) and this is reflected in higher theta. Now, since options on a product with the same expiration date can have different implied volatilities (for example in the case of a volatility smile being present or a steep put skew), this means that there can be differences between such options in the amount of gamma *relative to their theta* that they provide. Put simply, from a gamma perspective some options on the board represent better value than others.

Let's take an example. Consider two options; the $100 strike put (which is at-the-money) and the $83 strike put. The $100 puts have an implied volatility of 25%. The $83 strike put however has an implied vol of 36.4%, since there is a steep put skew in the market. The $100 puts have 5 gamma and an overnight theta decay of 4.3. The $83 puts have just 1 gamma and a theta decay of 1.5 ticks per night.

What is clear here is that the $100 puts provide much better value from a long gamma perspective. Remember that both puts obviously relate to the same underlying product; to gamma hedge either put, we would use the same spot product. If we buy 100 of the $100 puts, we will have 500 gamma and an overnight theta bill of $430. In order to buy 500 gamma via the $83 puts, we need to buy 500 lots but this comes with a theta bill of $750. So with the $83 puts we are either paying more to achieve the same gamma or we are receiving less gamma for a certain theta spend. Neither is particular good news.

In fact it would be possible to generate an unpalatable position in this instance which is short gamma and *paying* theta. Consider being short 100 of the $100 puts and long 400 of the $83 puts. Make the calculations and you will find the position is short 100 gamma and paying $170. Obviously a trader needs another very good reason to hold this position for any period of time since it is all but guaranteed to make losses day to day. Such reasons may exist (such as the trader receives very good edge to enter the position in the first place), but if they do not, the trader perhaps needs to re-think things. It goes without saying that the inverse of this position collects money for being long gamma.

A simple metric to compute and keep track of, particularly for very complex positions running to perhaps thousands of contracts long and short at various parts of the curve, is the gamma-theta ratio. Monitored over time, this can be a useful guide to what may be called the gamma trading efficiency of a portfolio. If the ratio is particularly low relative to previous results and the trader is long gamma, he may want to investigate why he is receiving meagre amounts of gamma for the theta he is paying. There is only really one culprit which is that the longs are coming from expensive options (i.e. high implied volatility options) and the shorts from cheaper options. Consciously looking to adjust this ratio for the better can make good sense if it can be done economically.

Section 4 Exercises

4.1 Is there any limit to a short gamma player's profits, day-to-day and excluding the effect of vega?

4.2 Is there any reason why gamma hedging should not be viewed from a weekly rather than daily or intra-day perspective?

4.3 Before the market opens, a trader calculates his breakeven level, whereby a full gamma hedge at the new spot price will cover all his overnight theta bill. The market then opens and the trader only partially hedges his gamma and yet still covers all his overnight theta bill. What must have happened to the spot market?

4.4 A trader checks the open inventory of the front month options, two weeks prior to their expiration. He then initiates a delta-hedged short position in a strike which has unusually large inventory. Why is he doing this?

4.5 A trader is long 100 at-the-money straddles, short 100 puts with a (-)10% delta and short 100 calls calls with a 10% delta. The position is delta neutral. How might you expect the portfolio's gamma and delta to change as the spot price varies?

4.6 A trader is short gamma, short vega and collecting theta. The market moves sharply and implied volatility subsequently falls. Is this likely to be good, bad or mixed news for the trader?

4.7 A trader is short gamma/collecting theta. He is short 300 gamma and collecting $500 per night. How does this compare to a position where he is short 400 gamma and collecting $700 in theta?

Section 5 : Gamma trading : miscellaneous useful ideas

Futures spreads

Take care with options struck on futures contracts with the same underlying but different expirations. For example March, April and May options on a government bond future may relate to the March future, whereas June options may relate to the June future. Usually the March and June features will be highly correlated and traders will delta or gamma hedge using the front month futures contract since it is the most liquid. However, if the gamma is generated by options based on a different futures contract from that used for gamma hedging, this can create synthetic futures spreads positions. Suppose we are long gamma from June options and we hedge this gamma with March futures. As the futures rally, we need to sell deltas. We may be left delta neutral after gamma hedging, but we are long June deltas and short March deltas. In other words this is a synthetic futures spread position. At some point before expiration, these spreads will need to be traded out of. The spreads can also move for or against the trader independently of other price action.

Making sure exceptional profit opportunities are not missed

When long gamma across a large range of spot prices, it is often sensible to work an order in the spot market a long way away. Suppose spot is trading $100 and we are just long the $100 straddle and expect the spot to move $1 or so a day. Suppose that due to some extreme event the spot drops instantaneously to $90 and then rebounds in the space of a few seconds. Such a relatively huge move could have been an opportunity for massive gamma profits, but by not working orders in the spot market in advance, there can be a high chance that the opportunity is entirely missed, if we are too slow to enter an order to gamma hedge.

A note of caution. Never forget that orders are being worked a long way away. Especially if the gamma position changes (because the long gamma has been sold for example).

The importance of gamma hedging when the gamma has been sold

Another easy mistake to make. Suppose we are long gamma and the spot market has fallen. We become short delta. Now suppose implied volatility/options become better bid and we sell out all or some of our options, and our gamma goes with it. It is important to recognise that we are now simply short deltas *which will not vanish if the spot retraces*, unlike when we were long gamma. Often in this situation, traders will elect to gamma hedge in part or in full to avoid the risk of loss should the spot rally.

On choosing levels for gamma hedges

When long or short gamma, traders will often decide in advance the spot prices at which they intend to hedge. For long players, they may decide to work orders in the spot order book, in order to be higher up the queue. Short players may set up automatic stop-loss orders. There are no right and wrong answers as to how to select the particular levels, but there are a couple of ideas worth mentioning.

It can be wise to avoid large, all or nothing gamma hedges at a single price point

Suppose we are long gamma and decide to hedge 50 ticks above or below the current spot price. Now suppose the spot oscillates all day long between plus and minus 49 ticks and finishes the day unchanged. This illustrates the risk of putting all the gamma hedge eggs into the basket of a single spot price. One way to mitigate this risk, and frankly to avoid the irritation of missing a large gamma hedge by a whisker, is to break the hedge up into multiple smaller orders. Rather than work an order to sell 100 lots of the spot at say $100.50, we could work 25 at $100.48, 50 lots at $100.50 and 25 lots at $100.52. Or any other combination of prices and quantities that we like. If all these orders are filled, we still achieve, on average, our original gamma hedge of 100 lots at $100.50. Breaking up the gamma hedge has the disadvantage of pushing some of the gamma hedge further away than we would like (in this example, out to $100.52), but this is price of eliminating the all-or-nothing scenario.

It may be wise to be aware of certain technical levels in the spot market

Technical trading and the belief in support and resistance levels remains a matter of contention. For gamma traders who have some degree of faith in technical trading, even if it is only belief in the capacity of such prophecies to become self-fulfilling, an awareness of technical levels that ‘the market’ in general seems to be calling, can influence the levels chosen for gamma hedging purposes. For example, suppose we are short gamma and have decided that we shall short gamma hedge 50 ticks lower or higher. However, suppose that market commentators are collectively calling a large support level in the underlying 52 ticks below the current spot. Some gamma traders may decide that rather than slavishly hedge 50 ticks lower, they will let the position run slightly further and test the support level. After all, the idea of a support level is that it is a price in the spot where extra support may be found and a rebound in the price may be a stronger possibility. Hence the short gamma player may hope this will prove to be true and delay his hedge until, say, 55 ticks lower or wherever he feels that the support level has definitively been breached.

Again, this is a matter of faith and more art than science. But there is no questioning how frustrating it is to negatively gamma hedge just before a support or resistance level of which one was ignorant and then see the spot retrace sharply. Likewise for the long gamma player, to work a long gamma hedge just *outside* the range of support and resistance levels which are not then breached and to see the spot retrace without having hedged, can be infuriating.

Appendix : Solutions to exercises

Section 1

1.1 Spot price.

1.2 False. It is a second order Greek telling us the change in a first order Greek with respect to changes in the spot price.

1.3 A 1 point move in the spot.

1.4 The spot price relative to strike price, the time to expiration and the implied volatility of the option.

1.5 More gamma.

1.6 Short gamma.

1.7 A positive function.

1.8 It must decrease, since at-the-money options have the highest gamma of any option in a given expiration. If the spot price moves, the option is no longer at-the-money and will therefore have lower gamma than it had before.

Section 2

2.1 No. John is delta-hedging.

2.2 15 lots bought. 30% * 50 lots of the puts.

2.3 The puts will be worth less because the spot has rallied. After the rally, the puts have a delta of (-)25%, given a gamma of 5. So the average delta over the move in the spot will be (-)27.5%. Hence the puts are worth approximately 27.5 cents less.

2.4 A profit of $15. 15 lots carried up $1.

2.5 A loss of $13.75. (50 times $0.275).

2.6 John needs to sell. He is long 15 deltas from the original hedge but the 50 options only have (-)25% delta, implying shorts of 12.5 delta. Selling 2.5 deltas (if half lots could be traded!) would be the gamma hedge.

2.7 $1.25. i.e. Answer to 2.4 minus answer to 2.5.

2.8 $5. i.e. Four times as large a profit.

2.9 $20. ie Sixteen times as large a profit.

2.10 Everything is reversed. John's gamma hedge would now be a 'short' gamma hedge that locks in a loss rather than a profit. He would need to buy deltas in a rally as a gamma hedge. Needing to buy in a rallying market is the reason he makes losses in this scenario.

Section 3

3.1 No.

3.2 In all likelihood, short options and gamma. See Section 4 for rare exceptions.

3.3 Approximately $25.

3.4. Three.

3.5 Hedging half as far away, means only ¼ of the profit i.e $6.25. So four times as many gamma hedges are required, namely twelve to make $75.

3.6 $225 from the gamma hedge, meaning a net profit from gamma trading of $150.

Section 4

4.1 Yes. The overnight theta bill.

4.2 Not really. Some traders may indeed hedge on longer term time frames, although their risk manager will need to be understanding regarding their net delta position which at times may become excessively large due to (un-hedged) gamma.

4.3 The spot market has opened with a move larger than the breakeven level. The partial hedge is still sufficient to cover the theta bill because it is more profitable, being executed after a greater price move.

4.4 He may believe that the strike has a good chance of pinning come expiration and he wishes to be short gamma and short the strike. Or maybe he knows something else that no-one else knows.

4.5 It rather depends on the exact gamma of the options, but we can still guess at the likely gamma and delta profiles. The portfolio (which is technically an iron butterfly) will certainly be long gamma with the spot trading near to the strike of the straddles; since the at-the-money options have higher gamma than any other options on the board, the straddle gamma must outweigh that of the 10% put and call. We know the trader is delta neutral versus the current spot price. In fact we can surmise that the trader has not used the underlying to delta hedge because the portfolio of options is itself delta-neutral at the current spot price. The straddle is delta neutral and the put and call positions form a delta-flat short strangle. If the spot falls, the trader will mostly remain long gamma. Thus, he will become shorter deltas as the spot falls. Eventually he will become flatter gamma and ultimately short gamma as the spot approaches, and passes through, the strike of the short put. As this put becomes at-the-money, its gamma may exceed that of the straddle (because the straddle will be relatively far away). The short gamma will start to eradicate the short deltas. If the spot falls far enough the position will become delta neutral again. To see this, imagine the spot falling to zero. The put from the straddle and the put from the strangle both have deltas of (-)100%, which cancel out as the trader is long one and short the other. The calls have both have zero delta. There was no position in the underlying (we surmised) so the portfolio deltas will tend to zero in an extreme move to the downside.

To the upside, the portfolio is initially long gamma and becoming longer deltas. Eventually the gamma turns negative and the deltas will start to disappear leaving the trader delta-neutral.

4.6 It sounds mixed. The sharp move in the spot is not something the short gamma player wishes to see. But falling implied volatility is good news for the short vega player.

4.7 The alternative (short 400, collecting $700) is marginally more efficient but also somewhat larger. Short gamma players prefer a lower gamma/theta ratio to one that is higher. In other words, to be collecting a lot in theta without being short too much gamma. Long gamma players prefer the opposite; lots of gamma at a low theta cost.

Glossary

At-the-money : Something equal to the current spot price. An at-the-money option has a strike equal to the current spot price.

Delta : The change in an option's value for a change in the spot price. Alternatively interpreted as the equivalent position in the underlying. "I am long 100 deltas" means "My position in options, the underlying and anything else that is relevant, combined, is equivalent to my being long 100 lots of the underlying".

Delta hedge : a trade in an instrument with a delta, such as the spot product or an option, that aims to reduce the delta exposure of the portfolio to zero.

Delta neutral : a portfolio that has no exposure to changes in the spot price

Gamma : The change in option's delta for a change in the spot price.

Gamma hedge : a delta-hedge that has become necessary because the spot price has altered, making the original delta-hedge inaccurate or inappropriate.

Market maker: A market participant is prepared to buy **and** sell financial instruments at all times.

Payoff profile : The returns from a portfolio shown for different spot prices.

Pin risk : The risk that normal spot market price action can be distorted by gamma hedging activity originating from the related options market, whereby the spot gravitates to a strike with large option inventory in the hours or days before an option expiration.

Put spread : An option strategy involving a put of one strike and a second put of a lower strike. A long put spread involves purchasing the higher strike put and selling the lower strike put. A short put spread involves the reverse.

Straddle : An option strategy involving a put and a call of the same strike traded in equal quantity and direction.

Strangle : An option strategy involving an out-of-the-money put and an out-of-the-money call traded in equal quantity and direction. For example, "Long 100 of the 95/105 strangle" means long 100 lots of the 95 strike puts and long 100 lots of the 105 strike calls.

Vega : The change in option value for a change in implied volatility. Per option, it is often normalized to show the change in ticks for a 1% change in implied volatility. For portfolios, it is often normalized to show the dollar change in the portfolio for a 1% change in implied volatility.