1. On the $IS-TR$ model: An economy with fixed prices is described by the following model:

$$Y = DD = C + I + G + TB;$$
$$C = C_0 + C_y \cdot (1-\tau_Y) \cdot Y - C_r \cdot r;$$
$$I = I_0 - I_r \cdot r;$$
$$G = G_0 - G_r \cdot r;$$
$$TB = TB_0 + TB_{ex} \cdot \left(1-\tau_Y^*\right) \cdot Y^* - TB_{im} \cdot (1-\tau_Y) \cdot Y + TB_\varepsilon \cdot \varepsilon;$$
$$\varepsilon = \varepsilon_0 - \varepsilon_r \cdot \left(r - r^*\right);$$
$$r = \overline{r}^{MP} + \phi_y \cdot \left(\frac{Y-\overline{Y}}{\overline{Y}}\right) + \overline{r}^{RP}.$$

Notation and exogeneity assumptions are as in the lectures on the macroeconomy in the short run, except for the parameter $\tau_Y$ denoting the domestic rate of income taxation, the parameter $\tau_Y^*$ denoting the foreign rate of income taxation, and the parameter $G_r$ denoting the interest rate sensitivity of (domestic) government expenditure (that reflects the cost to the government of interest payments on government debt). When numerical calculations are asked for, assume the following parameter values to hold:

$C_0 = 3;\ C_y = 0.6;\ \tau_Y = 0.3;\ C_r = 2;\ I_0 = 1;\ I_r = 15;\ G_0 = 1.5;\ G_r = 6;\ TB_0 = 0.7;$
$TB_{ex} = 0.015;\ Y^* = 312.5;\ \tau_Y^* = 0.2;\ TB_{im} = 0.15;\ TB_\varepsilon = 1;\ \varepsilon_0 = 1;\ \varepsilon_r = 6;\ r^* = 0.0325;$
$\overline{r}^{MP} = 0.02;\ \phi_y = 1;\ \overline{Y} = 15;\ \overline{r}^{RP} = 0.01.$

a) Derive (for general parameter values) the $IS$ – curve and provide a detailed economic rationale for this curve.

b) Derive the short-run values of the interest rate and of the level of output.

a.) quest: IS – curve

$Y = C + I + G + TB$

$Y = C_0 + C_y \cdot (1-\tau_y) \cdot Y - C_r \cdot r + (I_0 - I_r \cdot r) + G_0 - G_r \cdot r$
$+ TB_0 + TB_{ex} \cdot (1-\tau_y^*) \cdot Y^* - TB_{im} \cdot (1-\tau_y) \cdot Y + TB_\varepsilon \cdot \varepsilon$

substitute $\varepsilon$ with $\varepsilon_0 - \varepsilon_r \cdot (r - r^*)$

$Y = C_0 + C_y \cdot (1-\tau_y) \cdot Y - C_r \cdot r + (I_0 - I_r \cdot r) + G_0 - G_r \cdot r$
$+ TB_0 + TB_{ex} \cdot (1-\tau_y^*) \cdot Y^* - TB_{im} \cdot (1-\tau_y) \cdot Y + TB \cdot (\varepsilon_0 - \varepsilon_r \cdot (r - r^*)$

$$\boxed{\begin{aligned} & TB_\varepsilon \cdot (\varepsilon_0 - \varepsilon_r \cdot (r - r^*)) \\ &= TB_\varepsilon \cdot (\varepsilon_0 - \varepsilon_r \cdot r + \varepsilon_r \cdot r^*) \\ &= TB_\varepsilon \cdot \varepsilon_0 - TB_\varepsilon \cdot \varepsilon_r \cdot r + TB_\varepsilon \cdot \varepsilon_r \cdot r^* \\ &= TB_\varepsilon \cdot (\varepsilon_0 + \varepsilon_r \cdot r^*) - TB_\varepsilon \cdot \varepsilon_r \cdot r \end{aligned}}$$

rearrange variables, divide into exogenous and endogenous

$$Y = \overbrace{C_0 + I_0 + G_0 + TB_0 + TB_{ex} \cdot (1 - T_y^*) \cdot Y^* + TB_\varepsilon \cdot (\varepsilon_0 + \varepsilon_r \cdot r^*)}^{=A}$$
$$+ C_y \cdot (1 - T_y) \cdot Y - C_r \cdot r - G_r \cdot r - I_r \cdot r - TB_{IM} \cdot (1 - T_y) \cdot Y$$
$$- TB_\varepsilon \cdot \varepsilon_r \cdot r$$

factorize the right term (I)

$$Y = A + Y \cdot ((C_y \cdot (1 - T_y)) - TB_{IM} \cdot (1 - T_y)) - I_r \cdot r - G_r \cdot r - C_r \cdot r$$
$$- TB_\varepsilon \cdot \varepsilon_r \cdot r$$

factorize the right term (II)

$$Y = A + Y \cdot (1 - T_y) \cdot (C_y - TB_{IM}) - C_r \cdot r - I_r \cdot r - G_r \cdot r - TB_\varepsilon \cdot \varepsilon_r \cdot r$$

factorize the right term (III)

$$Y = A + Y \cdot (1 - T_y) \cdot (C_y - TB_{IM}) + r\,(- C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)$$

now, solve for Y

$$Y = A + Y \cdot (1 - T_y) \cdot (C_y - TB_{IM}) + r \cdot (- C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)$$
$$\Big| - Y()\ldots$$

$$Y - Y(1 - T_y) \cdot (C_y - TB_{IM}) = A + r \cdot (- C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r) \quad | \text{ factorize}$$

$$Y(1-(1-\tau_y)\cdot(C_y-TB_{IM})) = A + r\cdot(-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r) \quad | :(\;)$$

$$Y = \frac{A + r\cdot(-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)}{(1-(1-\tau_y)\cdot(C_y - TB_{IM}))} \quad | \;T$$

$$Y = \frac{A}{1-(1-\tau_y)\cdot(C_y-TB_{IM})} + \frac{r\cdot(-C_r - I_r - G_r - TB\cdot\varepsilon_r)}{1-(1-\tau_y)\cdot(C_y-TB_{IM})}$$

The IS-curve describes the negative relation between $Y$ and $r$, where the desired aggregate demand for goods and services equals total output produced. An increase (decrease) in the autonomous component, $A$, leads to an increase (decrease) of desired demand and consequently, an increase (decrease) of output $Y$, augmented through the Keynesian multiplier effect. An increase (decrease) in the real interest rate $r$, increases (decreases) the cost of borrowing, therefore consumption, government expenditure and investment decrease (increase). Additionally, it makes domestic goods and services less (more) competitive through the exchange rate channel, and the trade balance decreases (increases). An increase (decrease) in the real interest rate $r$, thereby reduces (increases) desired aggregate demand and consequently output. As for changes in the autonomous component of aggregate demand, the real interest rate effects on output are amplified by the feedback effects of the Keynesian Multiplier.

b) Derive the short-run values of the interest rate and of the level of output.

quest.: $Y$ ; $r$

quick solution

$$r = \bar{r}^{MP} + \phi_y \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right) + \bar{r}^{RP}$$

$\bar{r}^{MP} = 0{,}02$

$\phi_y = 1$

$\bar{r}^{RP} = 0{,}01$

$\bar{Y} = 15$

$$r = 0{,}02 + \left(\frac{Y-15}{15}\right) + 0{,}01$$

solve for $Y$

$$r = 0{,}02 + \left(\frac{Y-15}{15}\right) + 0{,}01 \quad | \; T$$

$$r = 0{,}03 + \left(\frac{Y-15}{15}\right) \quad | -0{,}03$$

$$r - 0{,}03 = \frac{Y-15}{15} \quad | \cdot 15$$

$$15r - 0{,}45 = Y - 15 \quad | +15$$

$$15r + 14{,}55 = Y$$

$$Y = \frac{A}{1-(1-T_y)\cdot(C_y - TB_{IM})} + \frac{r\cdot(-C_r - I_r - G_r - TB\cdot E_r)}{1-(1-T_y)\cdot(C_y - TB_{IM})}$$

substitute Y

$$15r + 14{,}55 = \frac{A + r\cdot(-C_r - I_r - G_r - TB_E\cdot E_r)}{1-(1-T_y)\cdot(C_y - TB_{IM})}$$

$$A = C_0 + I_0 + G_0 + TB_0 + TB_{ex}\cdot(1-T_y^*)\cdot Y^* + TB_E\cdot(E_0 + E_r\cdot r^*)$$

$C_0 = 3$ $\quad$ $C_r = 2$

$I_0 = 1$ $\quad$ $I_r = 15$

$G_0 = 1{,}5$ $\quad$ $G_r = 6$

$TB_0 = 0{,}7$ $\quad$ $C_y = 0{,}6$

$TB_{ex} = 0{,}015$ $\quad$ $T_y = 0{,}3$

$T_y^* = 0{,}2$ $\quad$ $TB_{IM} = 0{,}15$

$Y^* = 312{,}5$ $\quad$ $r^* = 0{,}0325$

$TB_E = 1$

$E_0 = 1$

$E_r = 6$

plug in given parameters

$$A = \underbrace{3+1+1{,}5+0{,}7}_{6{,}2} + \underbrace{0{,}015\cdot(1-0{,}2)\cdot 312{,}5}_{3{,}75} + \underbrace{1\cdot(1+6\cdot 0{,}0325)}_{1{,}195}$$

$$A = 6{,}2 + 3{,}75 + 1{,}195$$

$$A = 11{,}145$$

$$15r + 14{,}55 = \frac{11{,}145 + r\cdot(-2-15-6-1\cdot 6)}{1-(1-0{,}3)\cdot(0{,}6-0{,}15)}$$

$$15r + 14{,}55 = \frac{11{,}145 - 29r}{0{,}685} \quad \Big|\cdot 0{,}685$$

$$10{,}275r + 9{,}96675 = 11{,}145 - 29r \quad \Big| +29r$$

$$39{,}275r + 9{,}96675 = 11{,}145 \quad \Big| -9{,}96675$$

$$39{,}275r = 1{,}17825 \quad \Big| : 39{,}275$$

$$r = 0{,}03$$

use $r=0{,}03$ to calculate $Y$.

$$Y = \frac{A + r\cdot(-C_r - I_r - G_r - TB_E\cdot E_r)}{1-(1-T_y)\cdot(C_y - TB_{im})}$$

$$Y = \frac{11{,}145 + 0{,}03\cdot(-29)}{0{,}685}$$

$$\underline{\underline{Y = 15}} \qquad r = 0{,}038667736$$

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solution without plugging in parameters:

$$Y = \frac{A}{1-(1-T_y)\cdot(C_y-TB_{im})} + \frac{r\cdot(-C_r-I_r-G_r-TB\cdot E_r)}{1-(1-T_y)\cdot(C_y-TB_{im})}$$

$$r = \bar{r}^{MP} + \phi\cdot\left(\frac{Y-\bar{Y}}{\bar{Y}}\right) + \bar{r}^{RP} \quad \Big| -\bar{r}^{MP} \Big| -\bar{r}^{RP}$$

Solve for Y

$$r - \bar{r}^{MP} - \bar{r}^{RP} = \phi \cdot \left( \frac{Y - \bar{Y}}{\bar{Y}} \right) \quad | : \phi$$

$$\frac{Y - \bar{Y}}{\bar{Y}} = \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\phi} \quad | \cdot \bar{Y}$$

$$Y - \bar{Y} = \frac{\bar{Y} \, (r - \bar{r}^{MP} - \bar{r}^{RP})}{\phi} \quad | + \bar{Y}$$

$$Y = \bar{Y} \cdot \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\phi} + \bar{Y} \quad | \text{ factorize}$$

$$Y = \bar{Y} \cdot \left( \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\phi} + 1 \right)$$

substitute Y:

$$\bar{Y} \cdot \left( \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\phi} + 1 \right) = \frac{A + r \cdot (-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)}{1 - (1 - T_y) \cdot (C_y - TB_{IM})} \quad | \, T$$

$$\left( \frac{\bar{Y}}{\phi} \cdot (r - \bar{r}^{MP} - \bar{r}^{RP}) \right) + \bar{Y} = \frac{A + r \cdot (-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)}{1 - (1 - T_y) \cdot (C_y - TB_{IM})}$$

| partially factorize

$$\frac{\bar{Y}}{\phi} \cdot r + \frac{\bar{Y}}{\phi} \cdot (-\bar{r}^{MP} - \bar{r}^{RP}) + \bar{Y} = \frac{A + r \cdot (-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)}{1 - (1 - T_y) \cdot (C_y - TB_{IM})}$$

$$| - \frac{\bar{Y}}{\phi} \cdot r$$

$$\frac{\bar{Y}}{\phi} \cdot (-\bar{r}^{MP} - \bar{r}^{RP}) + \bar{Y} = \frac{A + r \cdot (-C_r - I_r - G_r - TB_\varepsilon \cdot \varepsilon_r)}{1 - (1 - T_y) \cdot (C_y - TB_{IM})} - \frac{\bar{Y}}{\phi} \cdot r$$

$$| \cdot 1 - (1 - T_y) \cdot (C_y - \ldots)$$

$$\left(\frac{\bar{Y}}{\theta}\cdot\left(-\bar{r}^{MP}-\bar{r}^{RP}\right)+\bar{Y}\right)\cdot\left(1-(1-T_y)\cdot(C_y-TB_{|M})\right)=A+r\cdot\left(-C_r-I_r-G_r-TB_\varepsilon\cdot\varepsilon_r\right)-\frac{\bar{Y}}{\theta}\cdot r\cdot\left(1-(1-T_y)\cdot(C_y-TB)\right)$$

$| -A$, factorize

$$\left(\frac{\bar{Y}}{\theta}\cdot\left(-\bar{r}^{MP}-\bar{r}^{RP}\right)+\bar{Y}\right)\cdot\left(1-(1-T_y)\cdot(C_y-TB_{|M})\right)-A=r\cdot\left(-C_r-I_r-G_r-TB_\varepsilon\cdot\varepsilon_r-\frac{\bar{Y}}{\theta}\cdot\left(1-(1-T_y)\cdot(C_y-TB)\right)\right)$$

$$r=\frac{-A+\left(\frac{\bar{Y}}{\theta}\cdot\left(-\bar{r}^{MP}-\bar{r}^{RP}\right)+\bar{Y}\right)\cdot\left(1-(1-T_y)\cdot(C_y-TB_{|M})\right)}{-C_r-I_r-G_r-TB_\varepsilon\cdot\varepsilon_r-\frac{\bar{Y}}{\theta}\cdot\left(1-(1-T_y)\cdot(C_y-TB_{|M})\right)}$$

$(1-(1-T_y)\cdot(C_y-TB_{|M}))$

$=(1-(1-0{,}3)\cdot(0{,}6-0{,}15))$

$=0{,}685$

$A=11{,}145$

$C_r+I_r+G_r+TB_\varepsilon\cdot\varepsilon_r$

$=2+15+6+6$

$=29$

$$r=\frac{-11{,}145+(15\cdot(-0{,}03)+15)\cdot 0{,}685}{-29-15\cdot 0{,}685}$$

$$\underline{\underline{r=0{,}03}}$$

⋮

$$Y = \bar{Y} \cdot \left( \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\theta} + 1 \right)$$

$$Y = 15 \cdot \left( \frac{\cancel{0{,}03 - 0{,}03}}{1} + 1 \right)$$

$$Y = 15 \cdot 1$$

$$\underline{\underline{Y = 15}}$$

c) Calculate the effects on domestic short-run output of an increase in the foreign income tax rate, $\tau_Y^*$, by 0.05. Decompose the overall effect on domestic output into its components (that is, the Keynesian multiplier induced effect and the interest rate induced crowding out/in effects). Make sure to provide, in addition to numerical results, a detailed economic rationale as to how the various effects arise.

1

Quest.: $\Delta Y$

now: $T_Y^*$ increases by 0,05

what implies $\Delta T_Y^* = 0{,}05$?

foreign taxes increases, so autonomous components decrease.

If $A$ decreases, $r$ does so, too.

in general:

$$KM = \frac{\partial Y}{\partial A}$$

$$\Delta Y = KM \cdot \Delta A + \frac{\partial Y}{\partial r} \cdot \Delta r$$

$$\Delta Y = \frac{\partial Y}{\partial A} \cdot \Delta A + \frac{\partial Y}{\partial r} \cdot \Delta r$$

$$\frac{\partial Y}{\partial A} = \frac{1}{1-(1-T_y)\cdot(C_y - TB_{(Y)})} = \frac{1}{0{,}685} = \underline{\underline{1{,}45985}}$$

$$\Delta A = \frac{\partial A}{\partial T_y^*} \cdot \Delta T_y^*$$

$$A = C_0 + I_0 + G_0 + TB_0 + TB_{ex} \cdot (1-T_y^*) \cdot Y^* + TB_\varepsilon \cdot (\varepsilon_0 + \varepsilon_r \cdot r^*)$$

$$\frac{\partial A}{\partial T_y^*} = TB_{ex} \cdot Y^* \cdot (-1)$$

$$\frac{\partial A}{\partial T_y^*} = -TB_{ex} \cdot Y^*$$

$$\Delta A = -0{,}05 \cdot TB_{ex} \cdot Y^*$$

$$\Delta A = -0{,}05 \cdot 0{,}015 \cdot 312{,}5$$

$$\Delta A = \underline{\underline{-0{,}2343}}$$

$$\frac{\partial Y}{\partial r} = -\frac{C_r + I_r + G_r + TB_E \cdot E_r}{1-(1-T_Y)\cdot(C_Y - TB_{(M)})}$$

$$\frac{\partial Y}{\partial r} = -\frac{29}{0{,}685}$$

$$\frac{\partial Y}{\partial r} = \underline{\underline{-42{,}335766}}$$

$$\Delta r = \frac{\partial r}{\partial A} \cdot \Delta A$$

$$r = \frac{-A + \left(\frac{\bar{Y}}{\phi}\cdot(-\bar{r}^{MP} - \bar{r}^{RP}) + \bar{Y}\right)\cdot(1-(1-T_Y)\cdot(C_Y - TB_{(M)})}{-C_r - I_r - G_r - TB_E \cdot E_r - \frac{\bar{Y}}{\phi}\cdot(1-(1-T_Y)\cdot(C_Y - TB_{(M)})}$$

$$r = A \cdot \frac{1}{C_r + I_r + G_r + TB_E \cdot E_r + \frac{\bar{Y}}{\phi}\cdot(1-(1-T_Y)\cdot(C_Y - TB_{(M)})} \underbrace{+ \cdots\cdots}_{\text{irrelevant, if derived} = 0.}$$

$$\frac{\partial r}{\partial A} = \frac{1}{C_r + I_r + G_r + TB_E \cdot E_r + \frac{Y}{\varnothing} \cdot (1-(1-T_y) \cdot (C_y - TB_{1M}))}$$

$$\Delta r = \frac{1}{C_r + I_r + G_r + TB_E \cdot E_r + \frac{Y}{\varnothing} \cdot (1-(1-T_y) \cdot (C_y - TB_{1M}))} \cdot (-0{,}05) \cdot TB_{E_x} \cdot Y^* \cdot (1-T_y^*)$$

$$\Delta r = \frac{1}{29 + \frac{15}{1} \cdot 0{,}685} \cdot (-0{,}2343)$$

$$\Delta r = \underline{\underline{-0{,}00596563}}$$

$$\Delta Y = 1{,}45985 \cdot (-0{,}2343) - 42{,}335766 \cdot (-0{,}00596563)$$

$$\Delta Y = \underline{\underline{-0{,}08947}}$$

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Sub-components :

we divid the Output equation:

$$Y = \frac{1}{1-(1-T_y) \cdot (C_y - TB_{1M})} \cdot A + \frac{-C_r - I_r - G_r - TB_E \cdot E_r}{1-(1-T_y) \cdot (C_y - TB_{1M})} \cdot r$$

Changes in $\Delta Y$ occure due to :

1. Keynesian multiplier effect :

$$\frac{1}{1-(1-T_y) \cdot (C_y - TB_{1M})} \cdot \Delta A$$

$$= 1{,}45985 \cdot (-0{,}2343)$$

$$= \underline{\underline{-0{,}34204}}$$

2. crowding-in of consumption

$$\frac{-C_r}{1-(1-T_Y)\cdot(C_Y-TB_{(Y)})}\cdot\Delta r$$

$$=\frac{-2}{0{,}685}\cdot(-0{,}00596563)$$

$$=\underline{\underline{0{,}0174179}}$$

3. crowding-in of investment

$$\frac{-I_r}{1-(1-T_Y)\cdot(C_Y-TB_{(Y)})}\cdot\Delta r$$

$$=\frac{-15}{0{,}685}\cdot(-0{,}00596563)$$

$$=\underline{\underline{0{,}130634}}$$

4. crowding-in of government expenditure

$$\frac{-G_r}{1-(1-T_Y)\cdot(C_Y-TB_{(Y)})}\cdot\Delta r$$

$$=\frac{-6}{0{,}685}\cdot(-0{,}00596563)$$

$$=\underline{\underline{0{,}0522537}}$$

5. crowding-in of trade balance

$$\frac{-TB_E\cdot E_r}{1-(1-T_Y)\cdot(C_Y-TB_{(Y)})}\cdot\Delta r$$

$$= \frac{-1.6}{0,685} \cdot (-0,00536563)$$

$$= \underline{\underline{0,0522537}}$$

The increase in foreign tax rate, $\tau_y^*$, causes a decrease of desired aggregate demand. The Keynesian Multiplier will cause a further decrease of output, $Y$. The central bank will observe the lower level of output and decrease the monetary policy rate according to the Taylor rule. Since the risk premium set by commercial banks is exogenous, a decrease in the monetary policy rate must be followed by one-to-one decrease in the real interest rate. As the real interest rate decreases, households will increase consumption expenditure (crowding-in from the consumption channel), firms will increase investment in physical capital (crowding-in from the investment channel), and the government will increase expenditure (crowding-in from the government channel). Moreover, the interest parity implication predicts a depreciation of the domestic currency, implying an increase in the trade balance (crowding-in from the exchange rate channel).

d) Suppose the domestic government responds by decreasing the domestic income tax rate, $\tau_Y$, by 0.05 (with the foreign income tax rate, $\tau_Y^*$, remaining at 0.25). Calculate the overall effect this fiscal policy change has on domestic short-run output (relative to the level of output obtained in c)). Decompose this overall effect into its components (that is, the Keynesian multiplier induced effect and the interest rate induced crowding out/in effects). Make sure to provide, in addition to numerical results, a detailed economic rationale as to how the various effects arise.

now, $T_y$ decreases by 0,05

$\hookrightarrow$ new $T_y = 0,25$

$T_y^* = 0,25$ because it was set in c)

quest.: $\Delta Y$

firstly, calculate new r, then new Y.

Then compare $Y_{new}$ with $Y_{old}$ to obtain $\Delta Y$.

$$r = \frac{-A + \left( \frac{\bar{Y}}{\theta} \cdot (-\bar{r}^{MP} - \bar{r}^{RP}) + \bar{Y} \right) \cdot (1-(1-T_y) \cdot (C_y - TB_{(M)}))}{-C_r - I_r - G_r - TB_E \cdot E_r - \frac{\bar{Y}}{\theta} \cdot (1-(1-T_y) \cdot (C_y - TB_{(M)}))}$$

$1-(1-T_y) \cdot (C_y - TB_{(M)})$

$= 1-(1-0{,}25) \cdot (0{,}6 - 0{,}15)$

$= 0{,}6625$

new A:

$A = C_0 + I_0 + G_0 + TB_0 + TB_{ex} \cdot (1-T_y^*) \cdot Y^* + TB_E \cdot (E_0 + E_r \cdot r^*)$

$A = 3 + 1 + 1{,}5 + 0{,}7 + 0{,}015 \cdot (0{,}75) \cdot 312{,}5 + 1 + 6 \cdot 0{,}0325$

$A = 10{,}910625$

$$r_{new} = \frac{-10{,}910625 + (15 \cdot (-0{,}03) + 15) \cdot 0{,}6625}{-29 - 15 \cdot 0{,}6625}$$

$r_{new} = 0{,}0327$

$$Y = \bar{Y} \cdot \left( \frac{r - \bar{r}^{MP} - \bar{r}^{RP}}{\theta} + 1 \right)$$

$Y_{new} = 15 \cdot (0{,}0326485 - 0{,}03 + 1)$

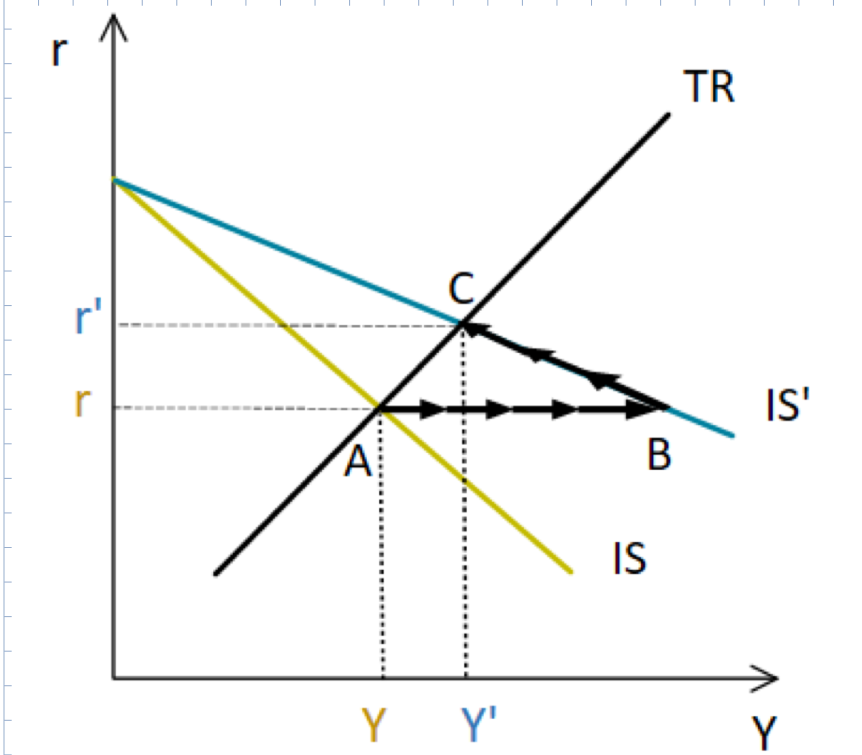
$Y_{new} = 15 \cdot 1{,}00264848$

$Y_{new} = \underline{\underline{15{,}0397}}$

$\Delta Y = 15{,}0397 - 15$

$\underline{\Delta Y = 0{,}0397}$

The decrease in the domestic tax rate $\tau_y$, causes a change in the Keynesian Multiplier and a flattening of the IS curve. Lower taxes imply higher disposable income, and more consumption of goods and services. Consequently, desired aggregate demand and output increases but the central bank rises the monetary policy rate and output crowds out through the consumption, investment, government expenditure and exchange rate channels.



2. On the Extended $TR$ – curve: Consider a central bank that uses the following extended monetary policy rule to set its monetary policy rate:

$$r^{MP} = \max\left\{0, \bar{r}^{MP} + \phi_y \cdot \left(\frac{Y-\bar{Y}}{\bar{Y}}\right) + \phi_{TB} \cdot \left(TB - TB^T\right)\right\},$$

where

$$TB = TB_{ex} \cdot \left(Y^* - T^*\right) - TB_{im} \cdot (Y - T) - TB_\varepsilon \cdot \varepsilon,$$

$TB^T$ denotes the central bank's target level for the trade balance, and

$$\varepsilon = \varepsilon_0 .$$

We also have

$$r = r^{MP} + r^{RP},$$

and

$$r^{RP} = \bar{r}^{RP} - r_y \cdot \left(Y - Y^{ZLB}\right) \cdot I\left(Y \le Y^{ZLB}\right).$$

a) Derive the highest level of output at which the central bank, following its monetary policy rule, would set $r^{MP} = 0$ ("$Y^{ZLB}$").

Quest.: $Y^{ZLB}$

$r^{MP} = 0$

plug in $TB_{ex} \cdot (Y^* - T^*) - TB_{IM} \cdot (Y - T) - TB_\varepsilon \cdot \varepsilon$ for $TB$.

$$r^{MP} = \bar{r}^{MP} + \phi_y \cdot \left(\frac{Y^{ZLB} - \bar{Y}}{\bar{Y}}\right) + \phi_{TB} \cdot \left(\left(TB_{ex} \cdot (Y^* - T^*) - TB_{IM} \cdot (Y^{ZLB} - T) - TB_\varepsilon \cdot \varepsilon\right) - TB^T\right) = 0$$

solve for $Y^{ZLB}$

$$\phi_y \cdot \left(\frac{Y^{ZLB} - \bar{Y}}{\bar{Y}}\right) = \phi_y \cdot \left(\frac{Y^{ZLB}}{\bar{Y}} - \frac{\bar{Y}}{\bar{Y}}\right) = \frac{\phi_y \cdot Y^{ZLB}}{\bar{Y}} - \phi_y$$

→ plugging that in

$$0 = \bar{r}^{MP} + \frac{Y^{ZLB} \cdot \phi}{\bar{Y}} - \phi_y + \phi_{TB} \cdot \left(\left(TB_{ex} \cdot (Y^* - T^*) - TB_{IM}(Y^{ZLB} - T) - TB_\varepsilon \cdot \varepsilon\right)\right.$$

$$-TB^{T})$$

| partially factorize

$$0=\bar{r}^{MP}+Y^{ZLB}\cdot\frac{\phi}{y}-\phi_y-\phi_{TB}\cdot TB_{IM}\cdot(Y^{ZLB}-T)+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)$$

$$\Big|-Y^{ZLB}\cdot\frac{\phi}{y}\quad\Big|+\phi_{TB}\cdot TB_{IM}\cdot(Y^{ZLB}-T)$$

$$-Y^{ZLB}\cdot\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}\cdot(Y^{ZLB}-T)=\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)$$

| rear.

$$-Y^{ZLB}\cdot\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}\cdot Y^{ZLB}-\phi_{TB}\cdot TB_{IM}\cdot T=\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)$$

| factorize

$$Y^{ZLB}\left(-\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}\right)-\phi_{TB}\cdot TB_{IM}\cdot T=\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)$$

$$\Big|+\phi_{TB}\cdot TB_{IM}\cdot T$$

$$Y^{ZLB}\cdot\left(-\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}\right)=\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)+\phi_{TB}\cdot TB_{IM}\cdot T$$

$$\Big|:\left(-\frac{\phi}{y}\dots\right)$$

$$Y^{ZLB}=\frac{\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon)+\phi_{TB}\cdot TB_{IM}\cdot T}{-\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}}$$

| factorize

$$Y^{ZLB}=\frac{\bar{r}^{MP}-\phi_y+\phi_{TB}\cdot(TB_{ex}\cdot(Y^*-T^*)-TB_{\varepsilon}\cdot\varepsilon+TB_{IM}\cdot T)}{-\frac{\phi}{y}+\phi_{TB}\cdot TB_{IM}}$$

| factorize $(-)$
$\varepsilon = \varepsilon_0$

$$Y^{ZLB} = \frac{\cancel{-}\left(\phi_y - \bar{r}^{MP} - \phi_{TB} \cdot \left(TB_{ex} \cdot (Y^* - T^*) + TB_{IM} \cdot T - TB_{\varepsilon} \cdot \varepsilon_0\right)\right)}{\cancel{-}\left(\frac{\phi}{y} - \phi_{TB} \cdot TB_{IM}\right)}$$

$$Y^{ZLB} = \frac{\phi_y - \bar{r}^{MP} - \phi_{TB} \cdot \left(TB_{ex} \cdot (Y^* - T^*) + TB_{IM} \cdot T - TB_{\varepsilon} \cdot \varepsilon_0\right)}{\frac{\phi}{y} - \phi_{TB} \cdot TB_{IM}}$$

As long as the central bank cares sufficiently more about deviations of output from long-run output compared to deviations in the trade balance from its target value, then we can reasonably assume that both numerator and denominator of the previous expression are positive; therefore, $Y^{ZLB}$ is positive.

b) Graph the extended monetary policy rule and the extended $TR$ – curve.

$$r = \max\left\{0, \bar{r}^{MP} + \phi_y \cdot \left(\frac{Y - \bar{Y}}{\bar{Y}}\right) + \phi_{TB} \cdot (TB - TB^T)\right\} + \bar{r}^{RP} - r_y \cdot (Y - Y^{ZLB}) \cdot I\left(Y < Y^{ZLB}\right)$$

$$r = \begin{cases} \bar{r}^{RP} - r_y \cdot (Y - Y^{ZLB}) & if \quad Y < Y^{ZLB} \\ \bar{r}^{RP} & if \quad Y = Y^{ZLB} \\ \bar{r}^{MP} + \phi_y \cdot \left(\frac{Y - \bar{Y}}{\bar{Y}}\right) + \phi_{TB} \cdot (TB - TB^T) + \bar{r}^{RP} & if \quad Y > Y^{ZLB} \end{cases}$$

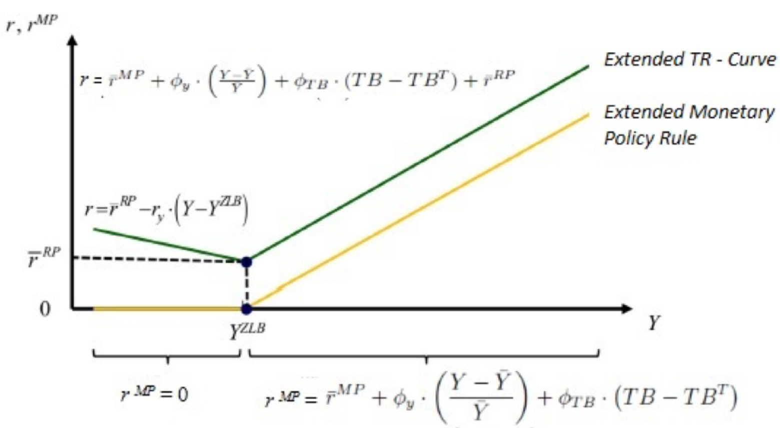



Figure 2: Extended TR-Curve and Extended Monetary Policy Rule

c) How does a decrease of $Y^*$ affect $Y^{ZLB}$ ? Provide an economic rationale for your analytical derivation. Also, graph the new extended monetary policy rule and the new extended $TR$ – curve following the decrease of $Y^*$.

Quest.: $\Delta Y^{ZLB}$

$Y^*$ decreases

$$\Delta Y^{ZLB} = \frac{\partial Y^{ZLB}}{\partial Y^*}$$

$$Y^{ZLB} = \frac{\phi_y - \bar{r}^{MP} - \phi_{TB}\cdot(TB_{ex}\cdot(Y^* - T^*) + TB_{IM}\cdot T - TB_{\varepsilon}\cdot\varepsilon_0)}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}}$$

$$Y^{ZLB} = \frac{-\phi_{TB}\cdot(TB_{ex}\cdot(Y^* - T^*) + TB_{IM}\cdot T - TB_{\varepsilon}\cdot\varepsilon_0)}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}} + \frac{\phi_y - \bar{r}^{MP}}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}}$$

$$Y^{ZLB} = \frac{-\phi_{TB}\cdot TB_{ex}\cdot(Y^* - T^*) - \phi_{TB}\cdot(TB_{IM}\cdot T - TB_{\varepsilon}\cdot\varepsilon_0)}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}} + \frac{\phi_y - \bar{r}^{MP}}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}}$$

$$Y^{ZLB} = \frac{-\phi_{TB}\cdot TB_{ex}\cdot(Y^* - T^*)}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}} - \frac{\phi_{TB}\cdot(TB_{IM} - TB_{\varepsilon}\cdot\varepsilon_0)}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}} + \frac{\phi_y - \bar{r}^{MP}}{\frac{\phi}{y} - \phi_{TB}\cdot TB_{IM}}$$

$$Y^{ZLB} = \frac{-\phi_{TB}\cdot TB_{ex}\cdot Y^{*} + \phi_{TB}\cdot TB_{ex}\cdot T^{*}}{\frac{\phi}{\bar{y}} - \phi_{TB}\cdot TB_{IM}} - \frac{\phi_{TB}\cdot(TB_{IM} - TB_{E}\cdot \varepsilon_0)}{} + \frac{\phi \bar{y}\, \bar{r}^{MP}}{}$$

$$Y^{ZLB} = \frac{-\phi_{TB}\cdot TB_{ex}\cdot Y^{*}}{\frac{\phi}{\bar{y}} - \phi_{TB}\cdot TB_{IM}} + \ldots \text{ (irrelevant, equals 0, if it's derived)}$$

$$\frac{\partial Y^{ZLB}}{\partial Y^{*}} = \frac{-\phi_{TB}\cdot TB_{ex}}{\frac{\phi}{\bar{y}} - \phi_{TB}\cdot TB_{IM}} < 0$$

Consider for example the case when the actual trade balance is above the target trade balance. Then a decrease in $Y^*$ means that the difference between the actual trade balance, $TB$, and the target level of the trade balance, $TB^T$, decreases; hence, the Taylor rule suggests that the central bank sets a lower monetary policy rate for every level of domestic output as long as $Y > Y^{ZLB}$. Therefore, considering falling levels of the output the central bank hits the zero lower bound for a higher level of output.

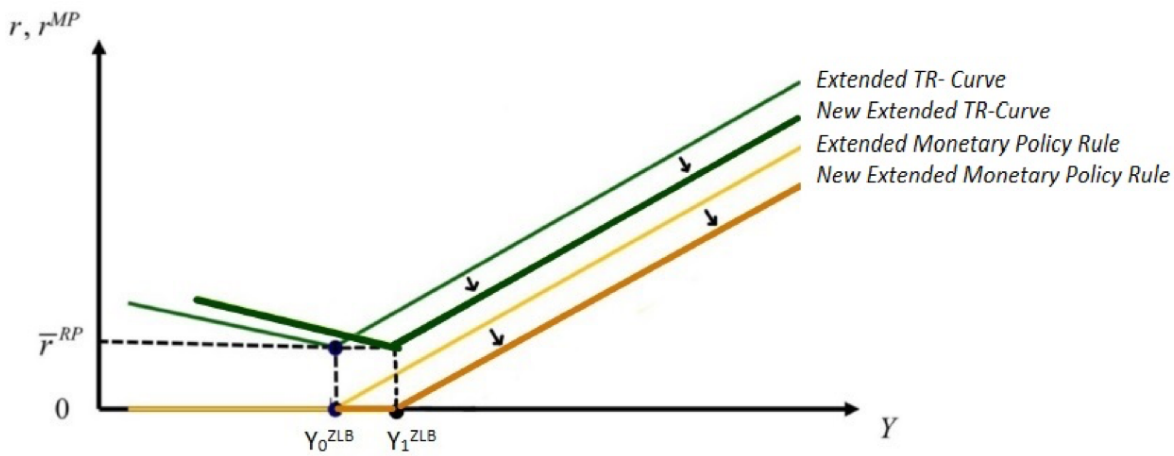



Figure 3: Change in Extended TR-Curve and Extended Monetary Policy Rule