

Claim: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

Proof:

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \\
&= \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x} && \text{Multiply and divide conjugate} \\
&= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)} && (1 - \cos x)(1 + \cos x) = 1 - \cos^2 x = \sin^2 x \\
&= \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x} \right) && \lim(ab) = \lim(a) \lim(b) \\
&= 1 \cdot \frac{0}{1 + 1} && \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \\
&= \boxed{0}
\end{aligned}$$

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{1 - \sqrt{2} \sin(2x + \pi/4)}{x} \\
&= \lim_{x \rightarrow 0} \frac{1 - \sqrt{2}(\sin 2x \cos(\pi/4) + \cos 2x \sin(\pi/4))}{x} && \sin(A + B) = \sin A \cos B + \cos A \sin B \\
&= \lim_{x \rightarrow 0} \frac{1 - \sin 2x - \cos 2x}{x} && \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \\
&= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x} - \lim_{x \rightarrow 0} \frac{\sin 2x}{x} && \lim(a + b) = \lim(a) + \lim(b) \\
&= 2 \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{2x} - 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} && \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1; \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \\
&= 2(0) - 2(1) \\
&= \boxed{-2}
\end{aligned}$$